Further sigma notation

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Summary

Sigma notation is used to express many additions at once. Understanding what this notation is, how it works, and how to manipulate them is a valuable skill to learn for use in almost any area of mathematics.

## Properties

In this section you will learn about a few properties of sigma notation which means you’ll have a toolkit to rearrange sums!

The first property you’ll learn about sigma notation is *distribuitivity*. This property allows you to take constants from inside the sigma notation to outside the summation.

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| Distribuitivity |
| Let be a sequence of numbers (where and are integers with ) and be any constant. Then |

You can see this is true by writing the entire sum out, like this:

|  |  |
| --- | --- |
|  | **Example 7**  What is the value of ?  Using distributivity, . From Example 2, you know that . Therefore, . |

## Double sums

Sometimes, you’ll want to multiply two sums together. This can be written succinctly using something called *double sums*.

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|  | **Double sums**  Let and be two sequences of numbers (where , and are integers with and ). Then the **double sum** is defined as |

|  |
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| Tip |
| You might find it easier to remember the above by thinking of as |

You will now see how this relates to multiplying two sums together. Suppose that and are like above, and consider the product . Writing it all out and performing the multiplication, you get

You can write this as a result:

|  |
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| Double sums and products of two sums |
| Let and be two sequences of numbers (where , and are integers with and ). Then |

|  |  |
| --- | --- |
|  | **Example 10**  Write as a double sum and as a product of two sums.  First, notice you can write out the above expression in the form  From the definition above you may now rewrite the expression to the double sum  using the distrubitivity property this can be written as  This can then be written using the product of two sums rule above to  It is evident that the two sums are the same with different index variables this means that they can be combined to form  has been used to differentiate the new sum from the ones involving and before but as always the choice of index variable is relatively unimportant |

## Quick check problems

1. What is the value of .

Answer: The value of the above is: \_\_.

1. Given Identify the index of the sum.

Answer: The index is \_

1. You are given several statements below based on the properties of sums. Identify whether they are true or false.
2. The sum can be expressed as Answer: TRUE / FALSE.
3. The sum can be expressed as Answer: TRUE / FALSE.
4. Answer: TRUE / FALSE.
5. Answer: TRUE / FALSE.
6. Answer: TRUE / FALSE.
7. Answer: TRUE / FALSE.
8. You are given several statements below based on the properties of sums. Identify whether they are true or false.
9. can be expressed as Answer: TRUE / FALSE.
10. can be expressed as $ Answer: TRUE / FALSE.
11. The sum can be expressed as $ Answer: TRUE / FALSE.

## Further reading