Auto-encoders

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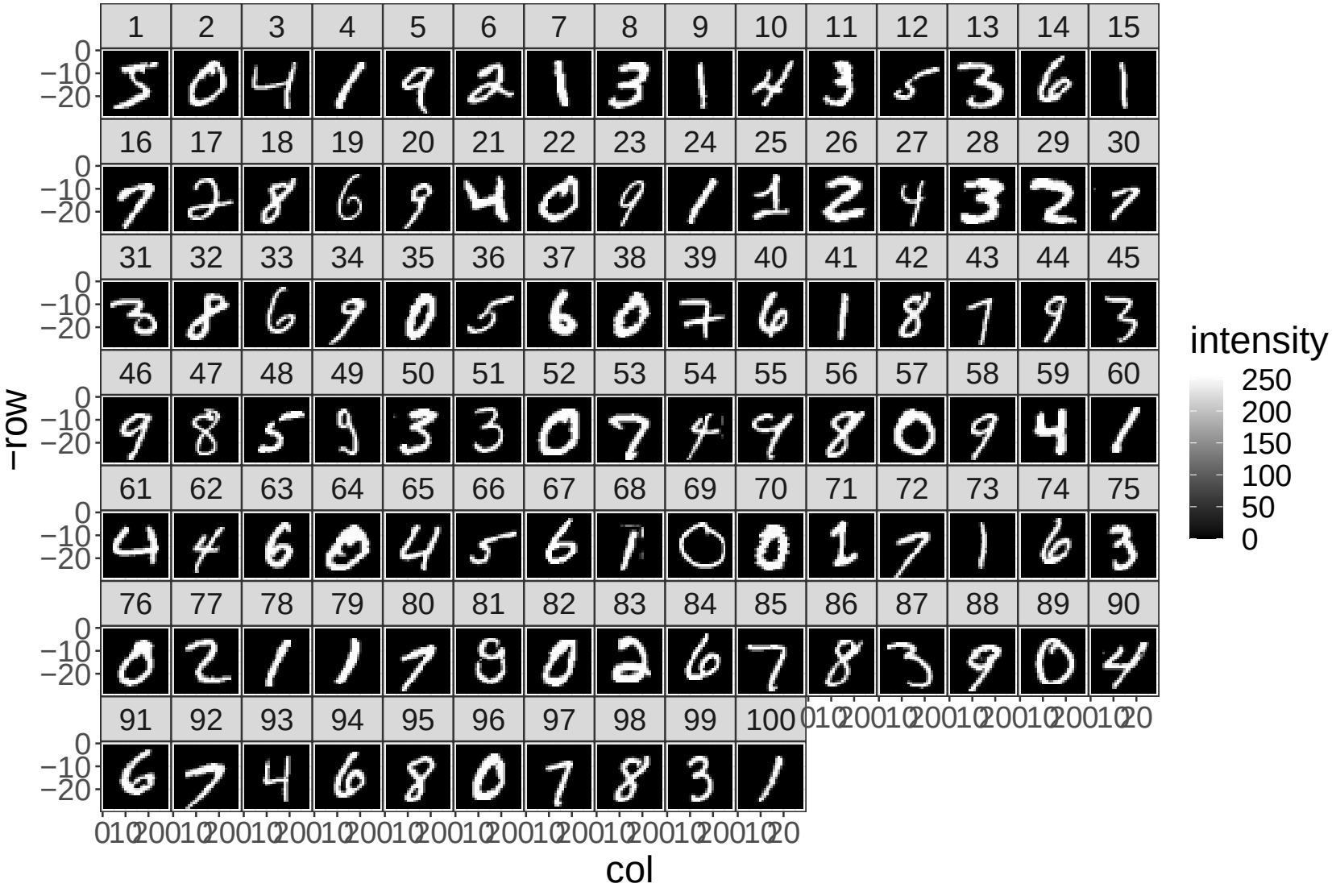
Motivation: MNIST digits data



Set of digits is represented as a matrix

- Each digit image in MNIST data set is a matrix of 28×28 pixel intensity values, $x_i \in \{0, \ldots, 255\}^{784}$.
- Each of the images is a row in the data matrix.
- Each of the columns is a pixel.
- All images on last slide represented by a data matrix with $n = 100$ rows/images and $p = 784$ columns/pixels.

Background/motivation: dimensionality reduction

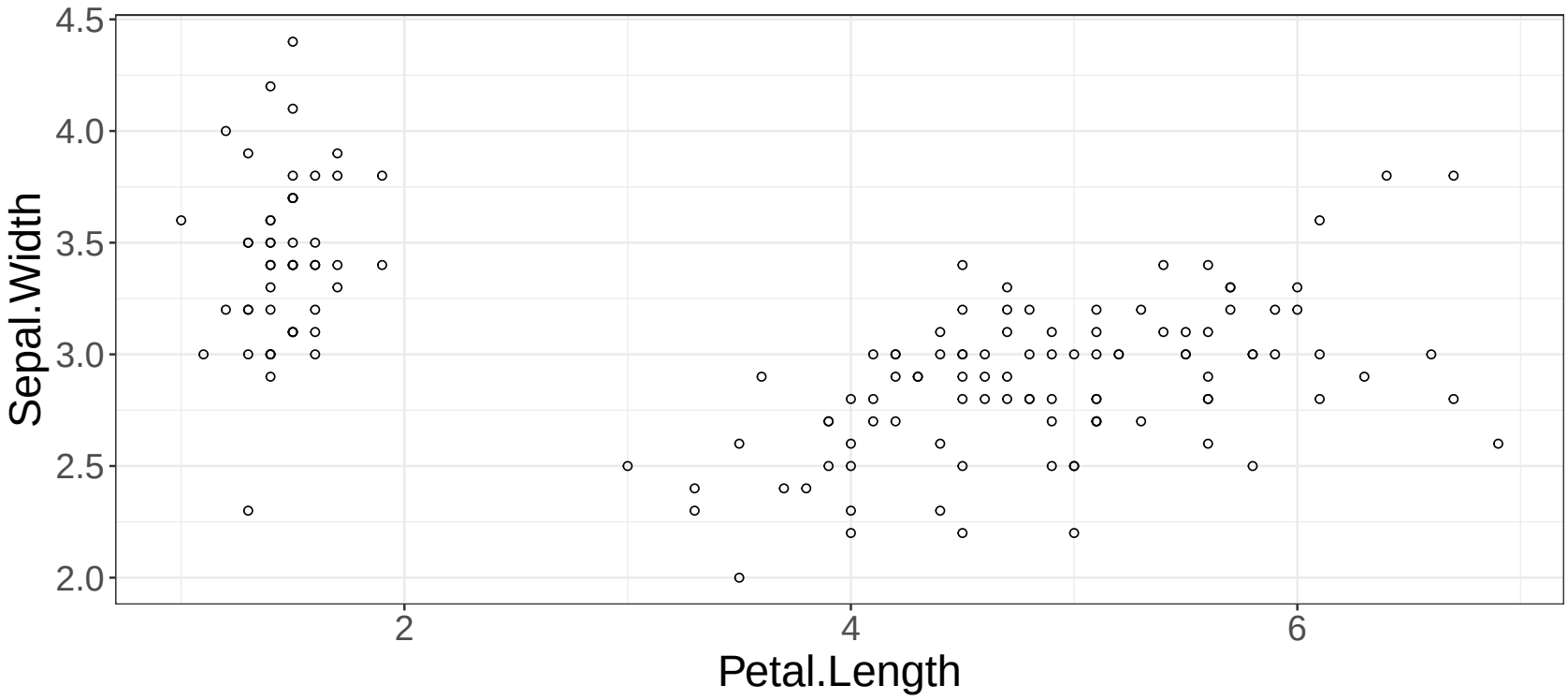
- High dimensional data are difficult to visualize.
- For example each observation/example in the MNIST data is of dimension 28 x 28 = 784 pixels.
- We would like to map each observation into a lower-dimensional space for visualization / understanding patterns in the data.

Example: 2d iris data

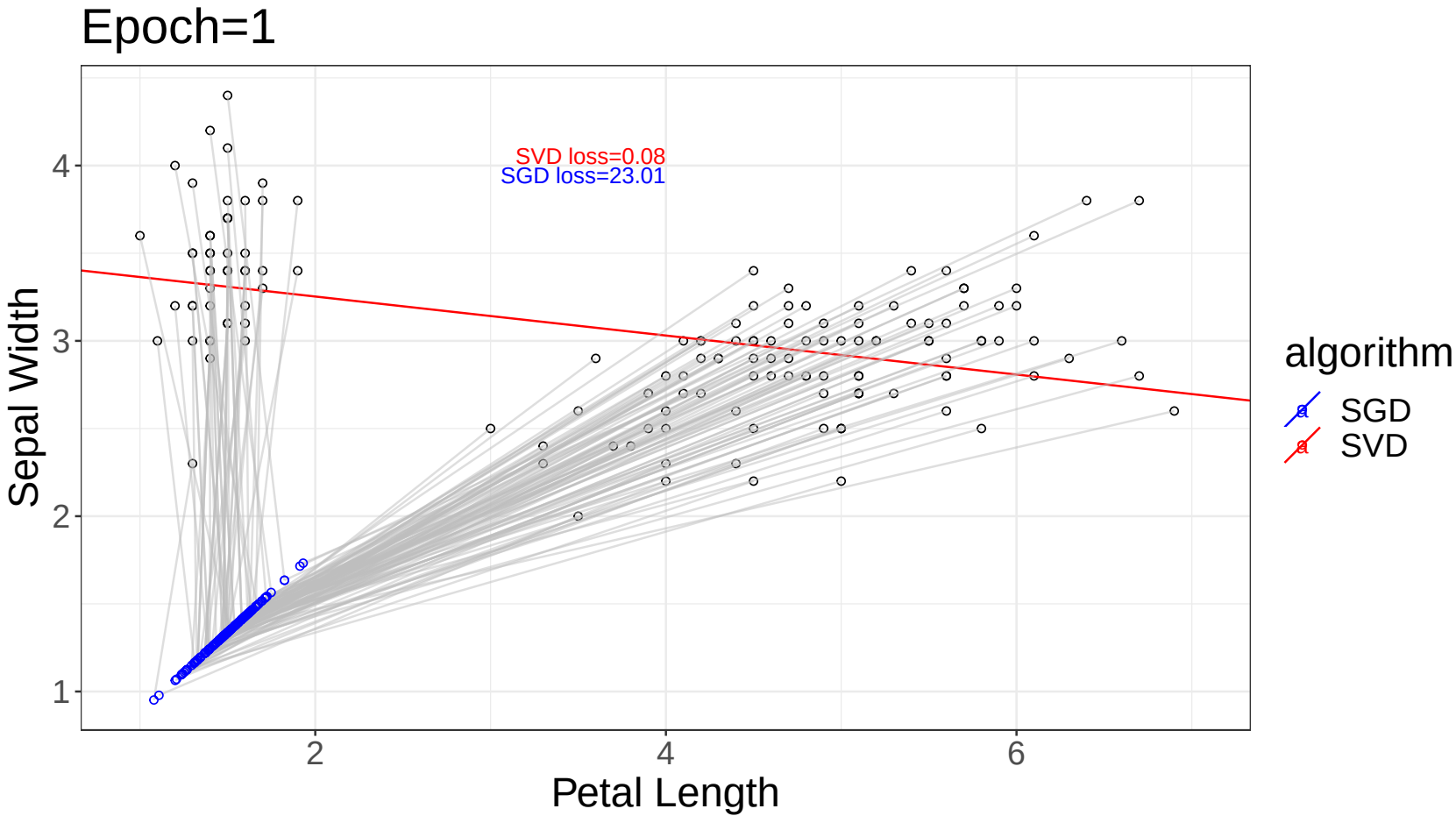
- Simpler example: iris.
- One row for each flower (only 6 of 150 shown below).
- One column for each measurement/dimension.

```
##   Sepal.Width Petal.Length
## 1         3.5          1.4
## 2         3.0          1.4
## 3         3.2          1.3
## 4         3.1          1.5
## 5         3.6          1.4
## 6         3.9          1.7
```

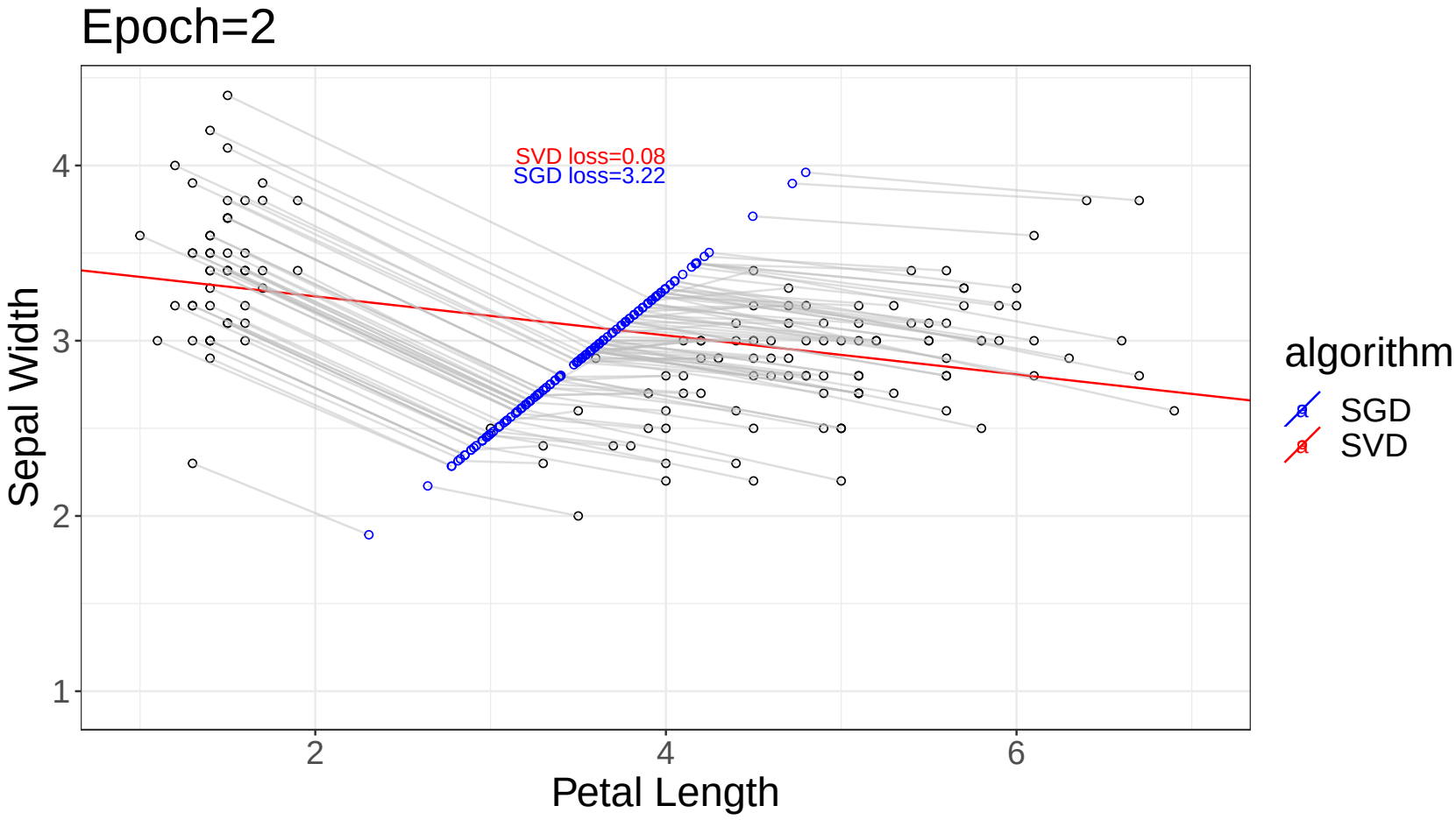
Example: 2d iris data



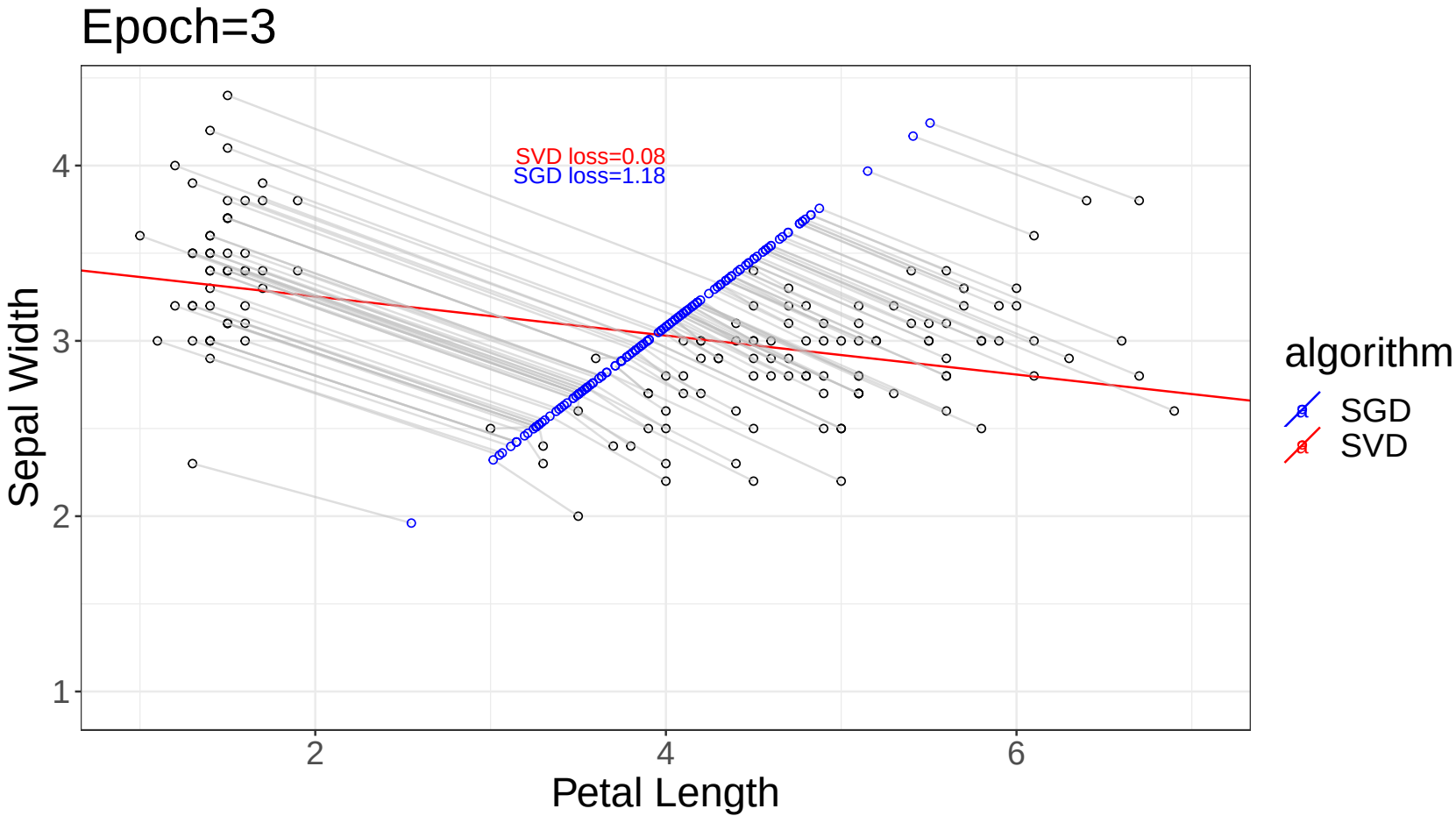
Project 2d data onto 1d subspace (line)



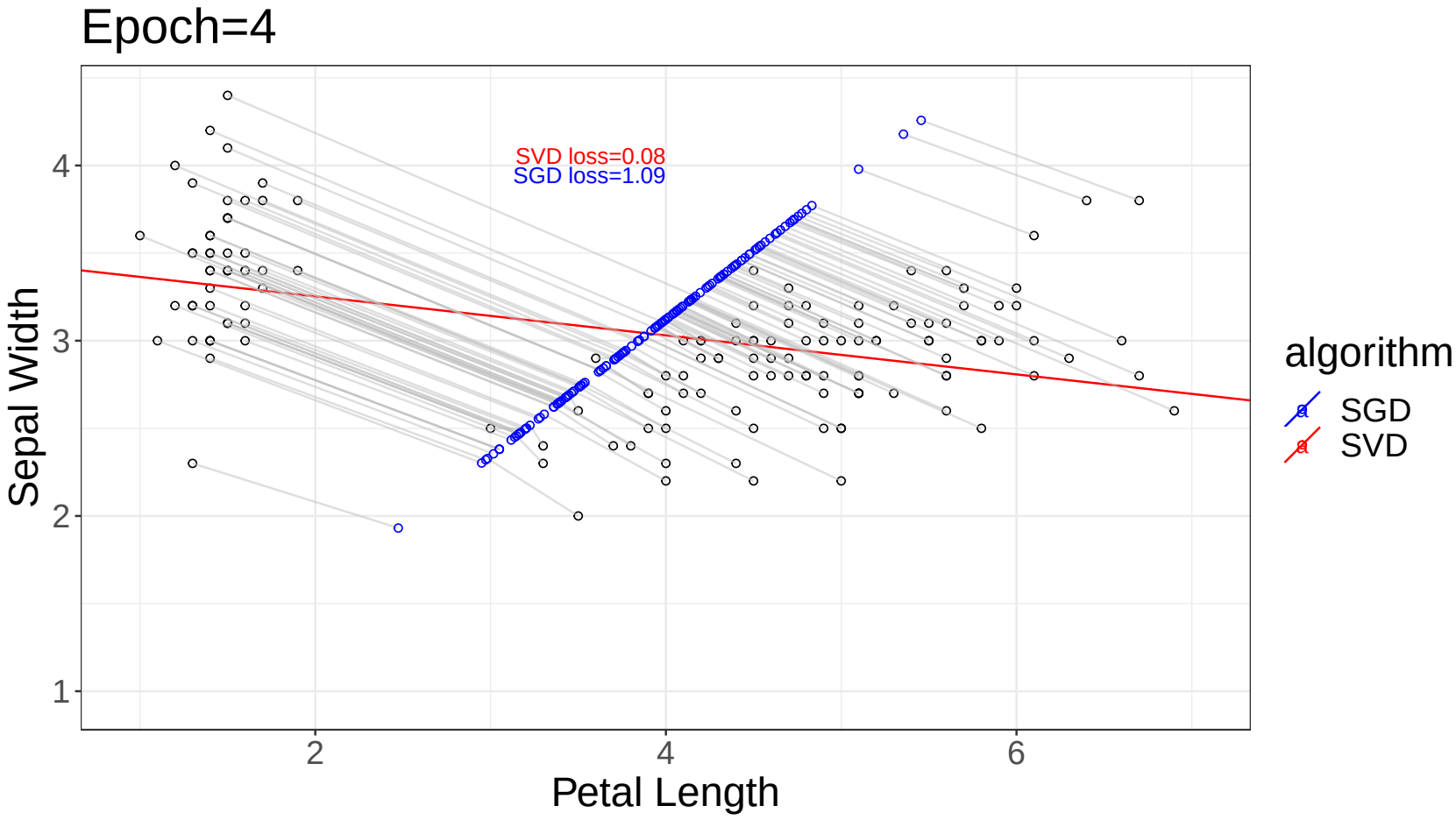
Visualization of predicted values



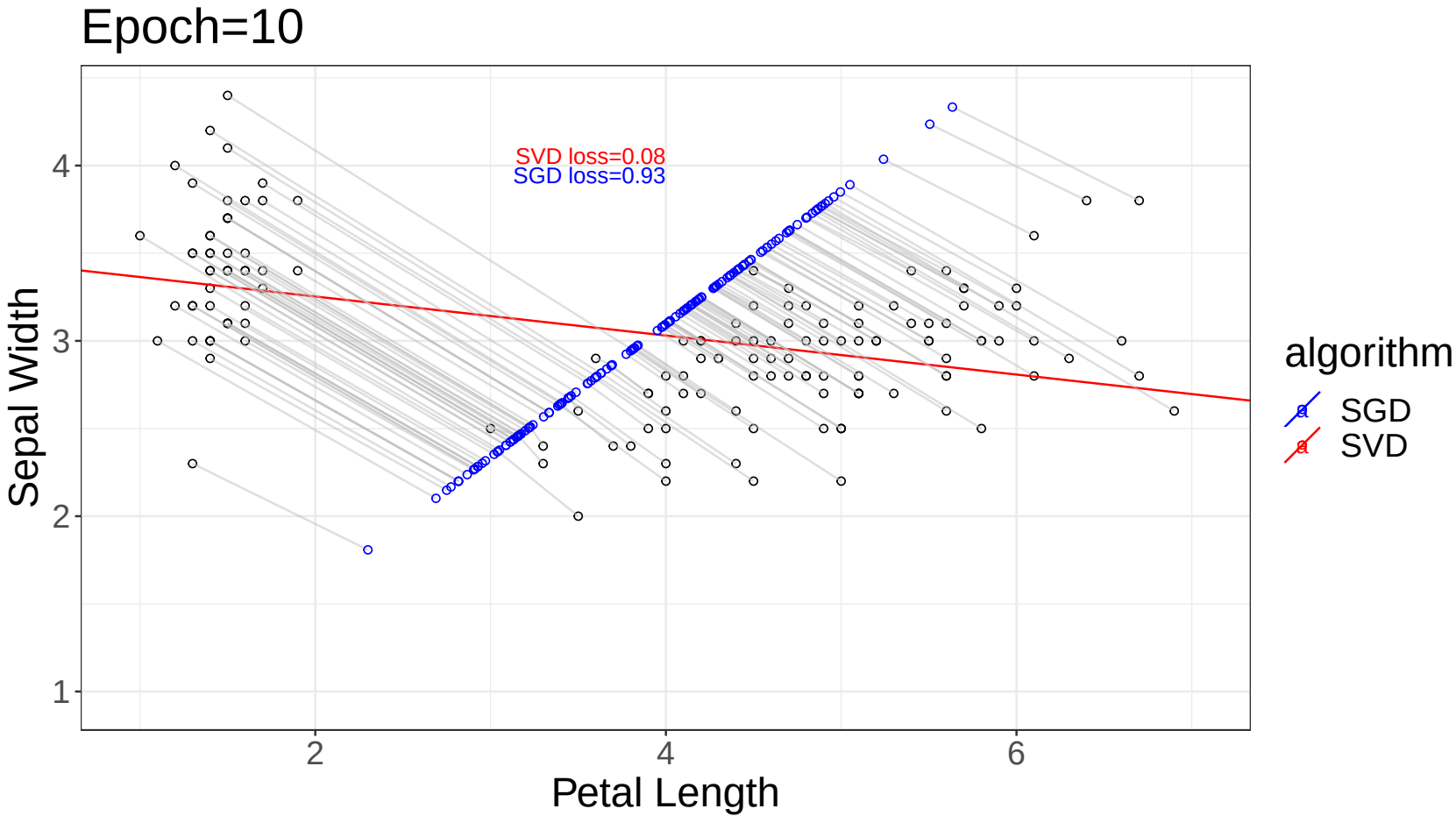
Visualization of predicted values



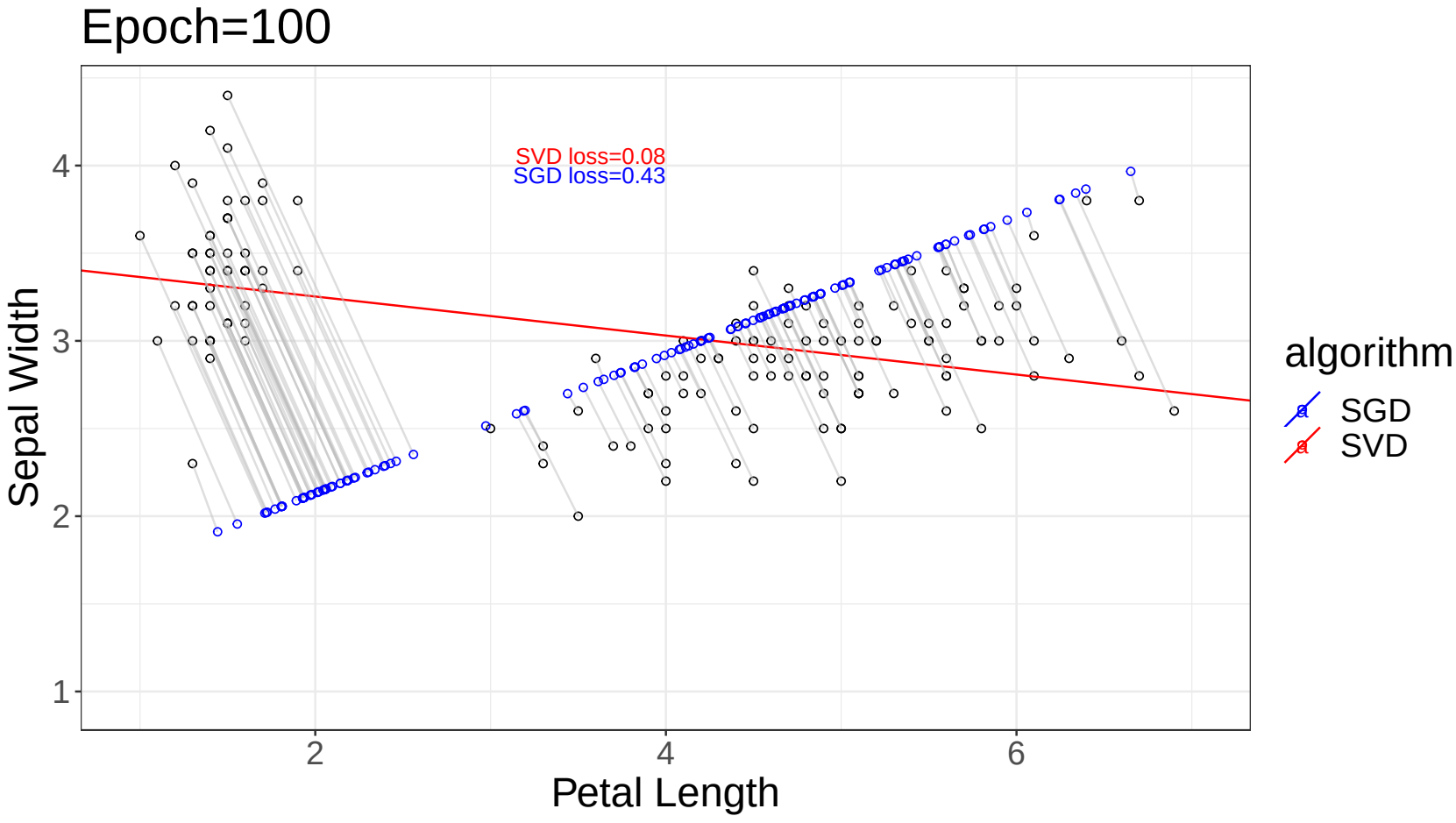
Visualization of predicted values



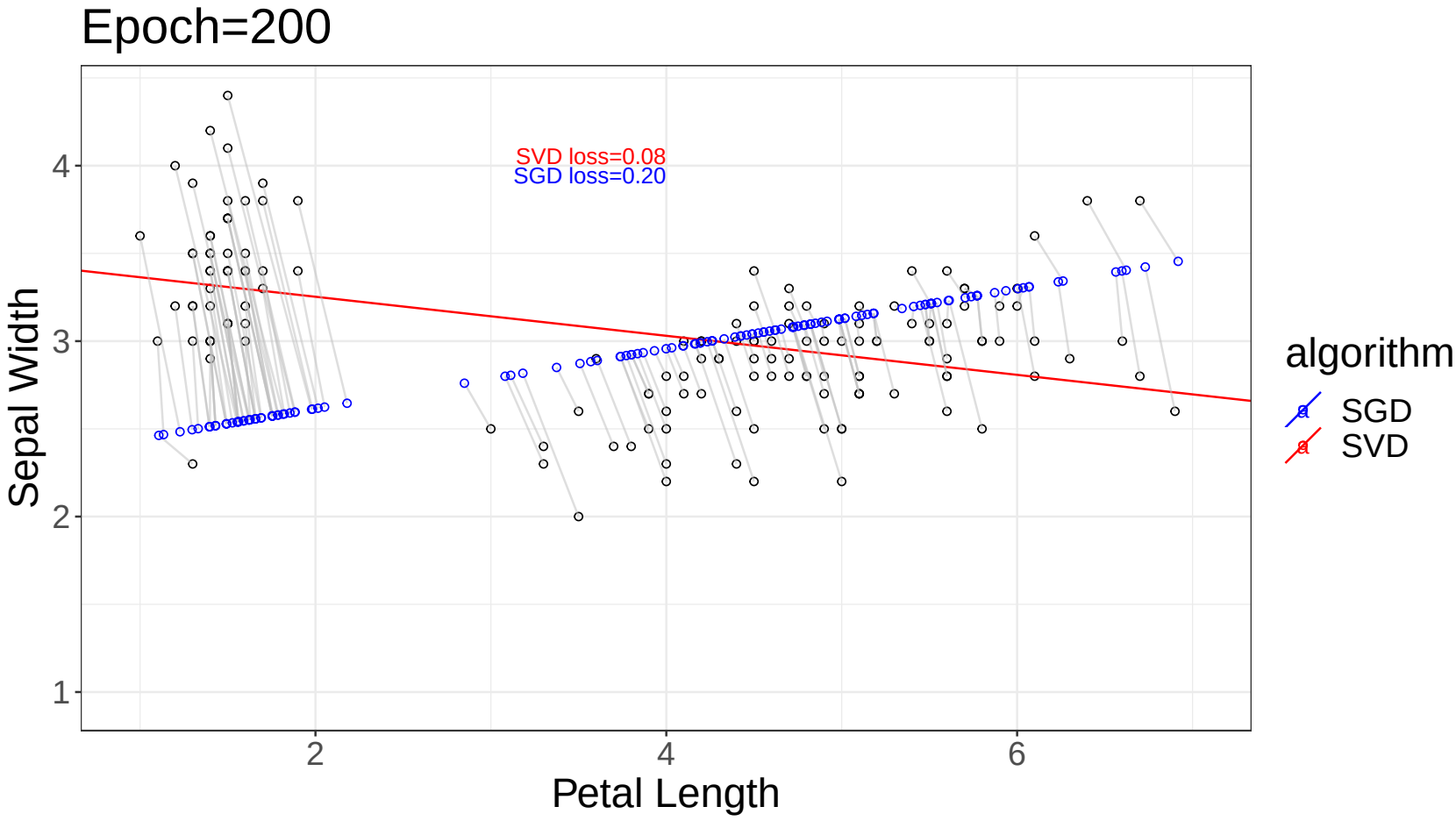
Visualization of predicted values



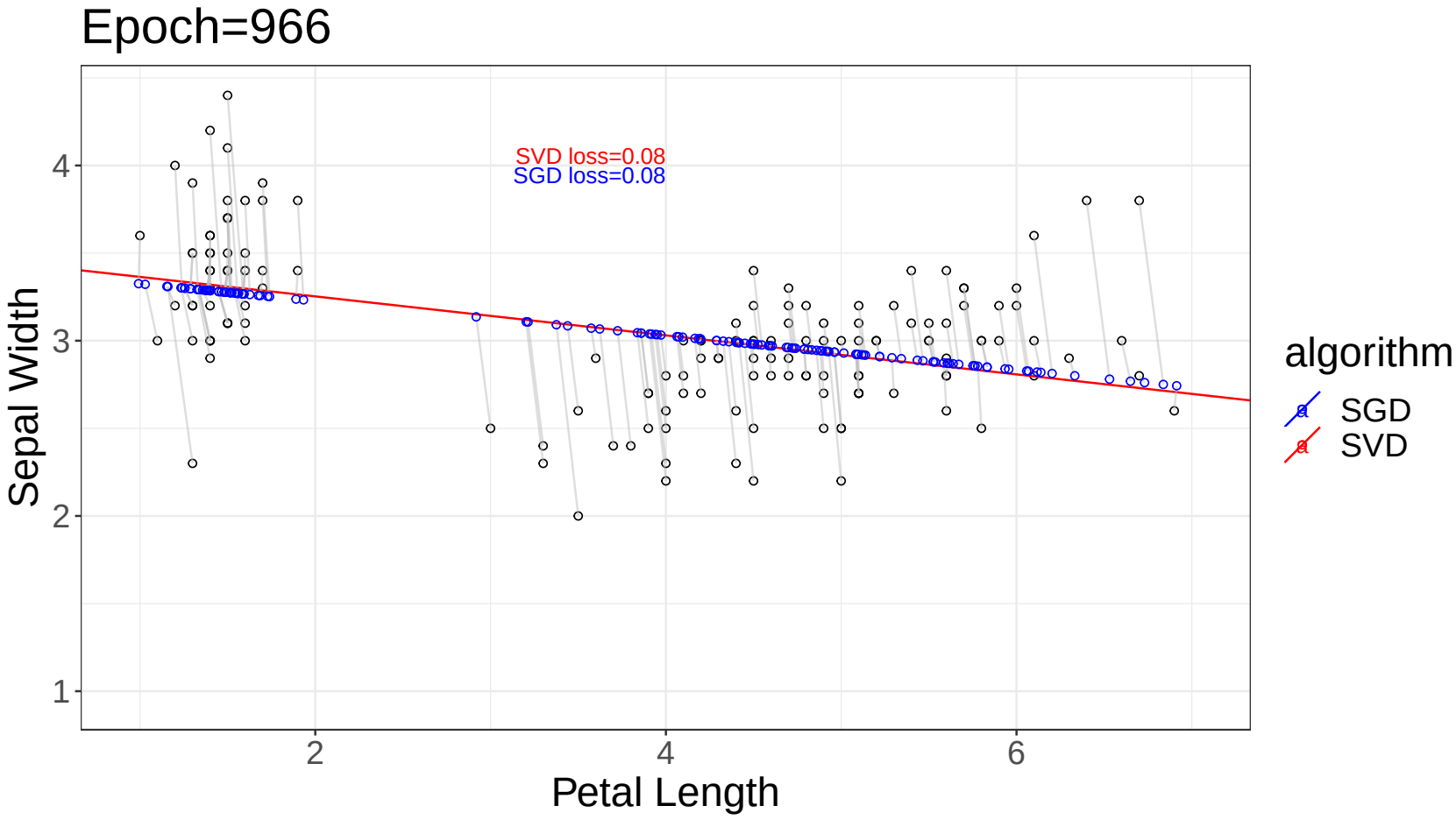
Visualization of predicted values



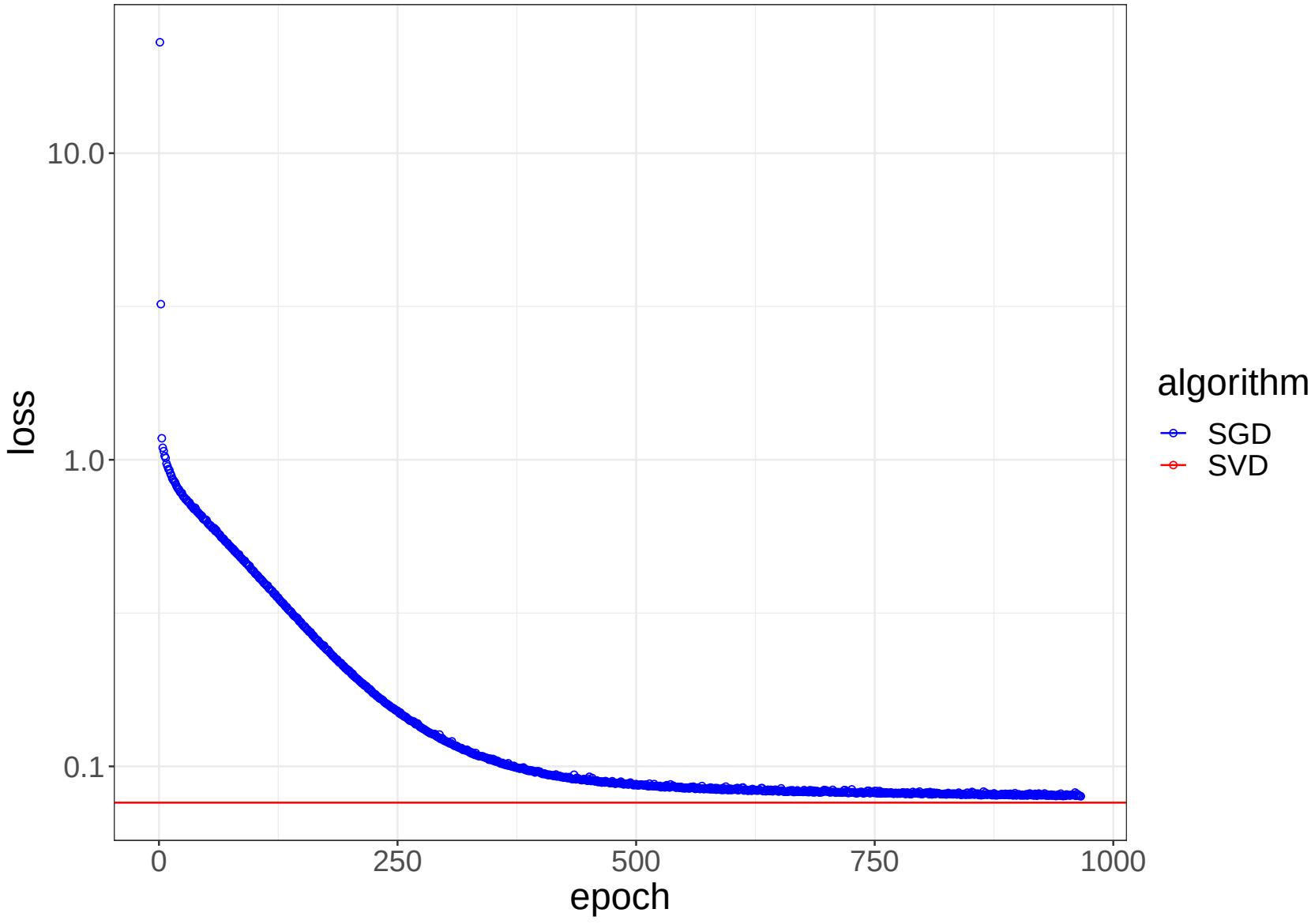
Visualization of predicted values



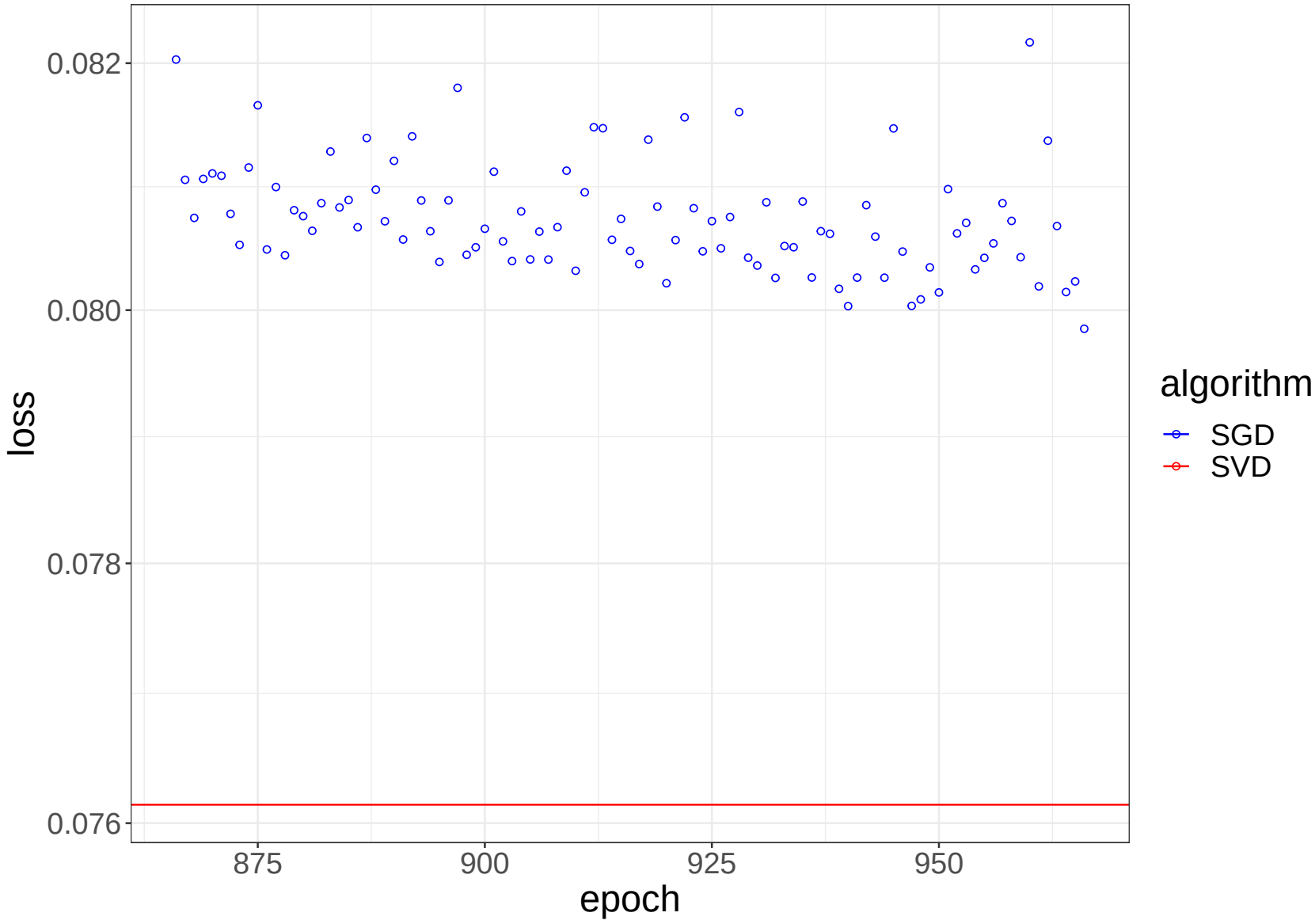
Visualization of predicted values



Loss decreases with number of epochs



Zoom to last 100 epochs



Visualization of predicted values

Possible exam questions

- When is the max number of principal components equal to the number of rows of the data matrix?
- When is the max number of principal components equal to the number of columns of the data matrix?