

Fast and optimal change-point algorithms

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Why are change-point algorithms so interesting?

In statistics, we have data, and we want good parameter estimates:

- ▶ Linear model coefficients
- ▶ Neural networks weights
- ▶ Change-points

	Linear	Neural Net	Change-point
Problem:	Convex	Non-convex	Non-convex
Algorithm:	Gradient	Gradient	Dynamic programming
Optimality:	Global	Local	Global

- ▶ Change-point detection is a non-convex problem
- ▶ with efficient dynamic programming algorithms
- ▶ that return the globally optimal model parameters!

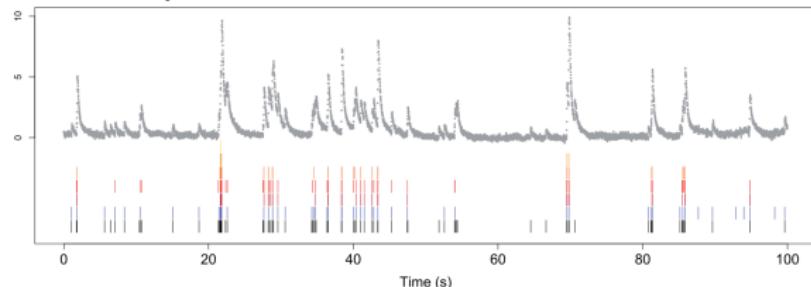
This talk: recent computational advances.

1. Drawbacks of classic algorithms
2. Linear time optimal algorithms with pruning
3. Computing constrained models using penalized solver

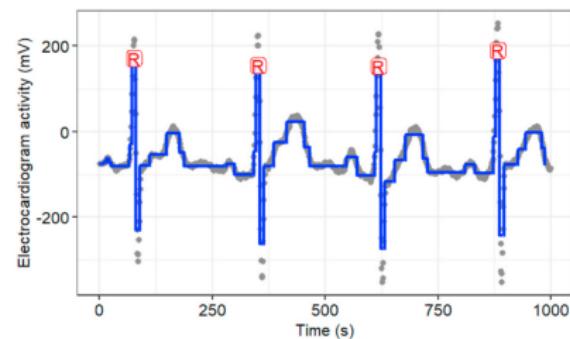
Summary and Discussion

Changepoint detection algorithms for data over time

Neuron spikes, Jewell *et al.*, Biostatistics 2019.

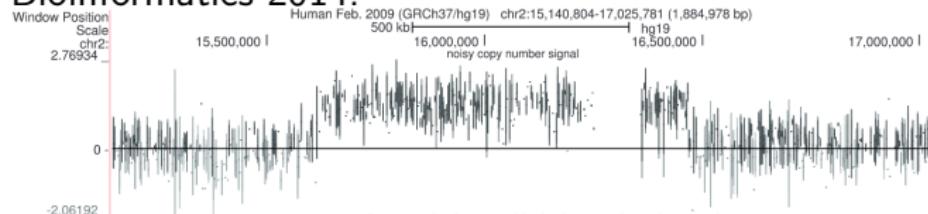


Electrocardiograms (heart monitoring), Fotoohinasab *et al.*, Asilomar conference 2020.

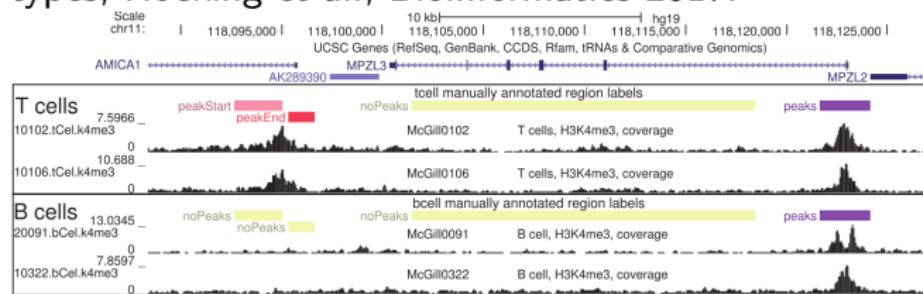


Changepoint detection algorithms for data over space

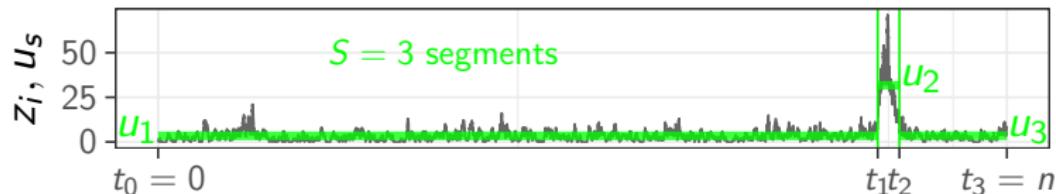
DNA copy number data for cancer diagnosis, Hocking *et al.*, Bioinformatics 2014.



ChIP-seq data for annotating active/inactive regions in different cell types, Hocking *et al.*, Bioinformatics 2017.



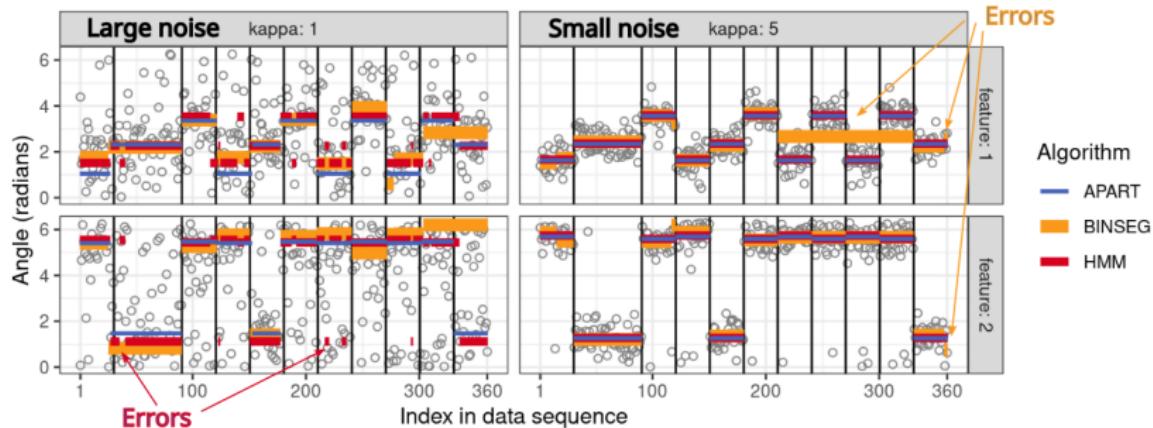
Optimal changepoint detection problem



$$\min_{\substack{u \in \mathbb{R}^S \\ 0 = t_0 < t_1 < \dots < t_{S-1} < t_S = n}} \sum_{s=1}^S \sum_{i=t_{s-1}+1}^{t_s} \ell(u_s, z_i)$$

- ▶ Algorithm inputs n data z_1, \dots, z_n and # of segments S .
- ▶ Loss function ℓ measures fit of means u_s to data z_i (Gaussian, Poisson, exponential, ...).
- ▶ Goal is to compute best $S - 1$ changepoints $t_1 < \dots < t_{S-1}$ and S segment parameters u_1, \dots, u_S .
- ▶ Non-convex optimization problem, naively $O(n^S)$ time.
- ▶ Fast approximate heuristic algorithms?

Fast heuristics can yield inaccurate change-points



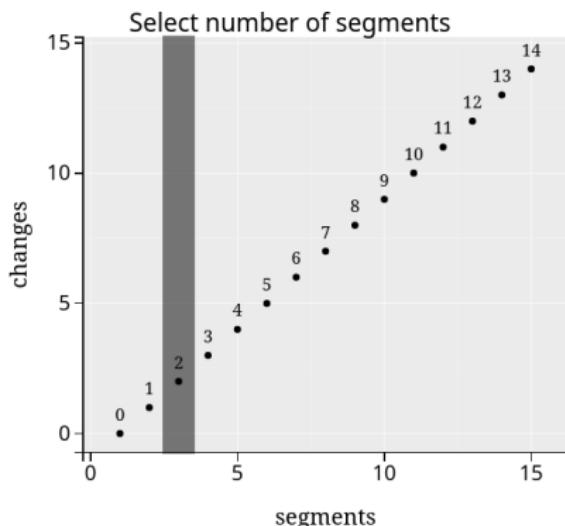
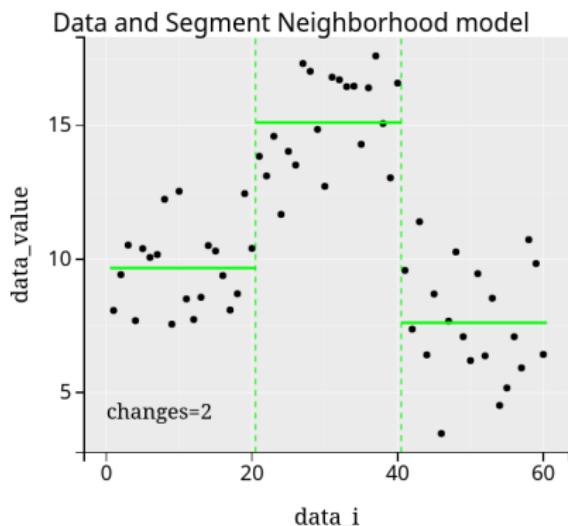
- ▶ Simulation: angular data, 2 dimensions/features.
- ▶ Heuristics are approximate optimization algorithms.
- ▶ Fast: linear $O(n)$ time for n data.
- ▶ Not guaranteed to compute change-points with best loss.
- ▶ BINSEG = Binary segmentation (Scott and Knott, 1974).
- ▶ HMM = Hidden Markov Model (Rabiner, 1989).

Constrained problem, classic dynamic programming solution

Constrained optimal change-point problem:

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} \quad \text{subject to}$$

$$\underbrace{\sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}} \leq S.$$



Constrained problem, classic dynamic programming solution

Constrained optimal change-point problem:

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} \quad \text{subject to} \quad \underbrace{\sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}} \leq S.$$

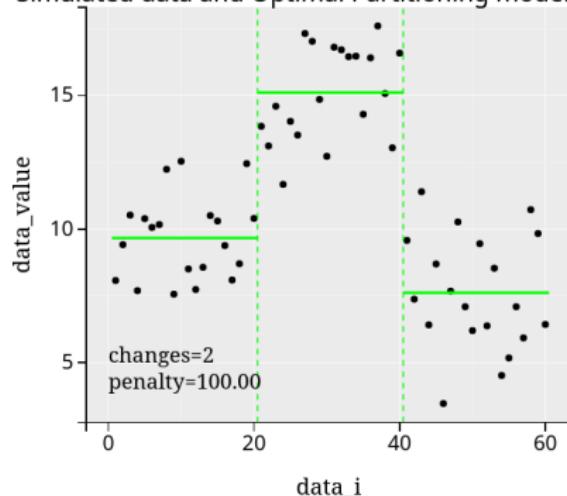
- ▶ Auger and Lawrence, Algorithms for the optimal identification of **segment neighborhoods**, Bull Math Biol (1989).
- ▶ Optimal recursive updates (dynamic programming algorithm).
- ▶ Let $C_{n,S}$ be best cost up to n data and S segments.
- ▶ Start by computing $C_{1,1}$ to $C_{1,n}$ (cum sum).
- ▶ Then compute $C_{2,2}$ to $C_{2,n}$, $C_{3,3}, \dots, C_{3,n}$, etc.
- ▶ Output all optimal models from 1 to S segments.
- ▶ Time complexity $O(Sn^2)$ — faster than naïve $O(n^S)$.
- ▶ Still too slow for large data n and model sizes S .

Penalized problem, classic dynamic programming solution

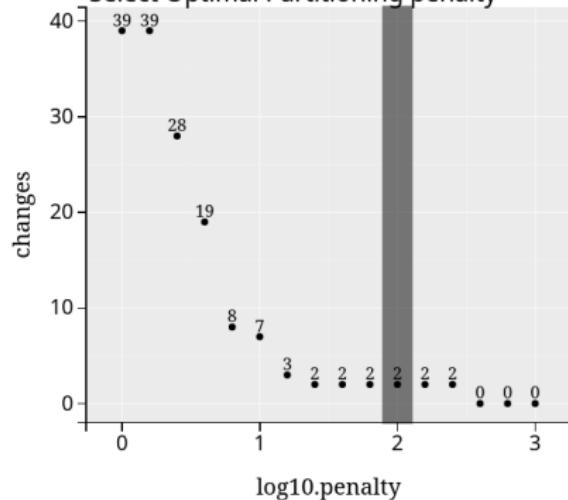
Penalized optimal change-point problem:

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \underbrace{\lambda \sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}}.$$

Simulated data and Optimal Partitioning model



Select Optimal Partitioning penalty



Penalized problem, classic dynamic programming solution

Penalized optimal change-point problem:

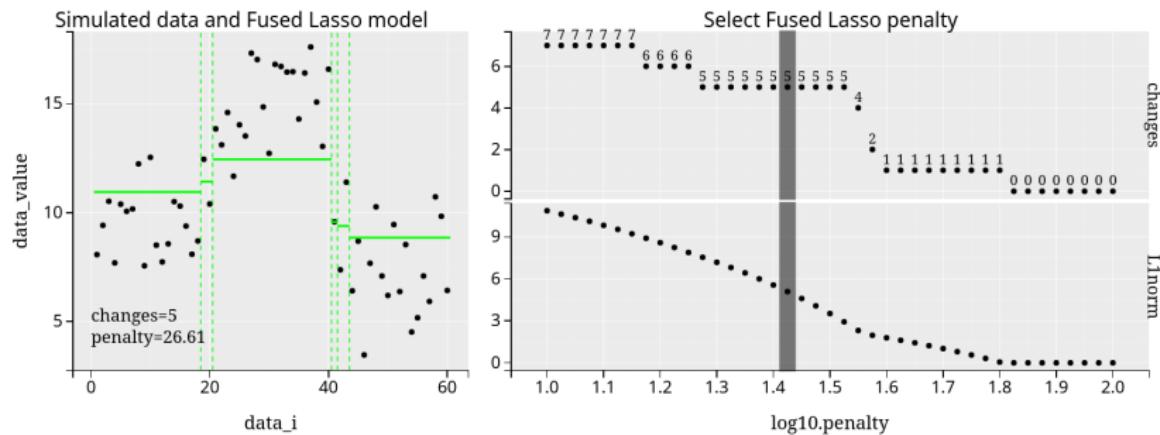
$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \underbrace{\lambda \sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}}.$$

- ▶ Hyper-parameter is non-negative penalty $\lambda \geq 0$.
- ▶ Jackson, et al., An algorithm for **optimal partitioning** of data on an interval, IEEE Sig Proc Lett (2005).
- ▶ Optimal recursive updates (dynamic programming algorithm).
- ▶ Let C_n be best cost up to n data.
- ▶ Recursively compute C_1, C_2, \dots, C_n .
- ▶ Time complexity $O(n^2)$ — faster than $O(Sn^2)$.
- ▶ User can not directly specify number of segments S .
- ▶ Output one optimal change-point model (not all in $1, \dots, S$).
- ▶ Quadratic time is still too slow for large data n .

Penalized problem, Fused Lasso heuristic

Penalized relaxed change-point problem (fused lasso):

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \lambda \underbrace{\sum_{i=2}^n |\mu_{i-1} - \mu_i|}_{\text{Absolute differences (regularization)}}$$



<https://tdhock.github.io/FL-demo/>
library(flsa) in R.

Penalized problem, Fused Lasso heuristic

Penalized relaxed change-point problem (fused lasso):

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \lambda \underbrace{\sum_{i=2}^n |\mu_{i-1} - \mu_i|}_{\text{Absolute differences (regularization)}}$$

- ▶ Hyper-parameter is non-negative penalty $\lambda \geq 0$.
- ▶ Indicator function $I[\cdot] \in \{0, 1\}$ is non-convex.
- ▶ Substitute absolute difference penalty: $\sum |\cdot|$.
- ▶ Fused Lasso for change-points (Tibshirani *et al.*, JRSSB 2005).
- ▶ Gradient-based linear time algorithm.
- ▶ Lasso selects too many variables (false positive change-points).

Summary of classic algorithms

Some fast heuristics:

- ▶ Binary segmentation (Scott and Knott, 1974).
- ▶ Hidden Markov Models (Rabiner, 1989).
- ▶ Fused Lasso (Tibshirani, 2005).

Quadratic time optimal algorithms:

- ▶ Segment neighborhood (Auger and Lawrence, 1989).
- ▶ Optimal partitioning (Jackson, *et al.*, 2005).

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Summary and Discussion

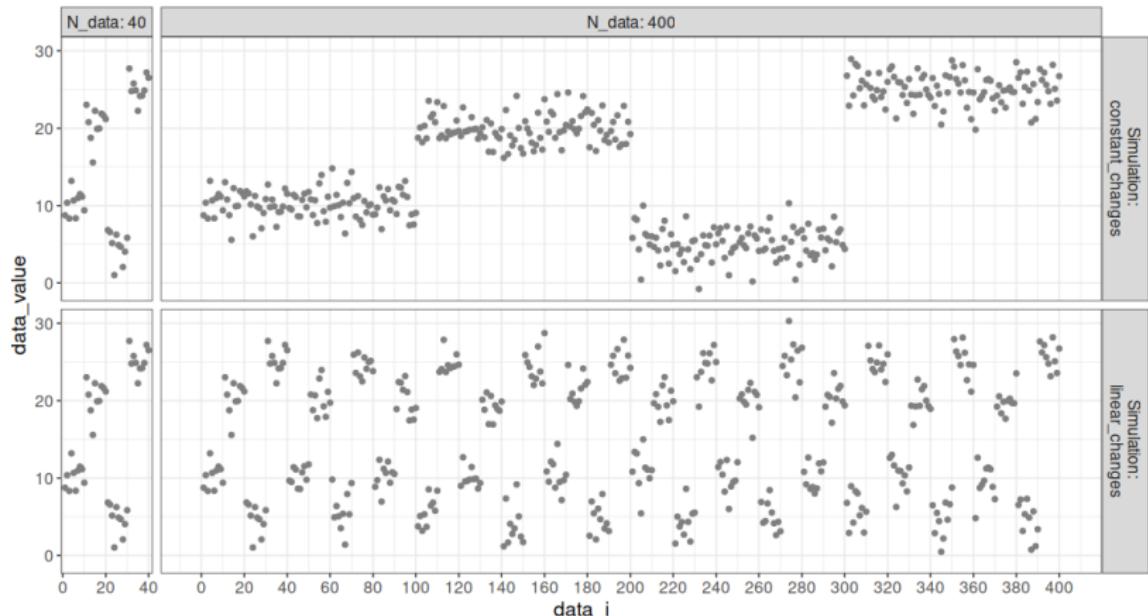
Penalized problem, PELT algorithm

Penalized optimal change-point problem:

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- ▶ Recursively compute C_1, C_2, \dots, C_n .
- ▶ Classic DP considers all previous change-points.
- ▶ At time n , candidates **all of** $\{1, \dots, n-1\}$.
- ▶ Pruned Exact Linear Time algo: Killick, et al., JASA (2012).
- ▶ At time n , candidates **subset of** $\{1, \dots, n-1\}$.
- ▶ Easy to implement, with only 1 additional line of code!
- ▶ Same output: one optimal change-point model.
- ▶ Time complexity: best $O(n)$, worst $O(n^2)$.
- ▶ library(changepoint) in R.

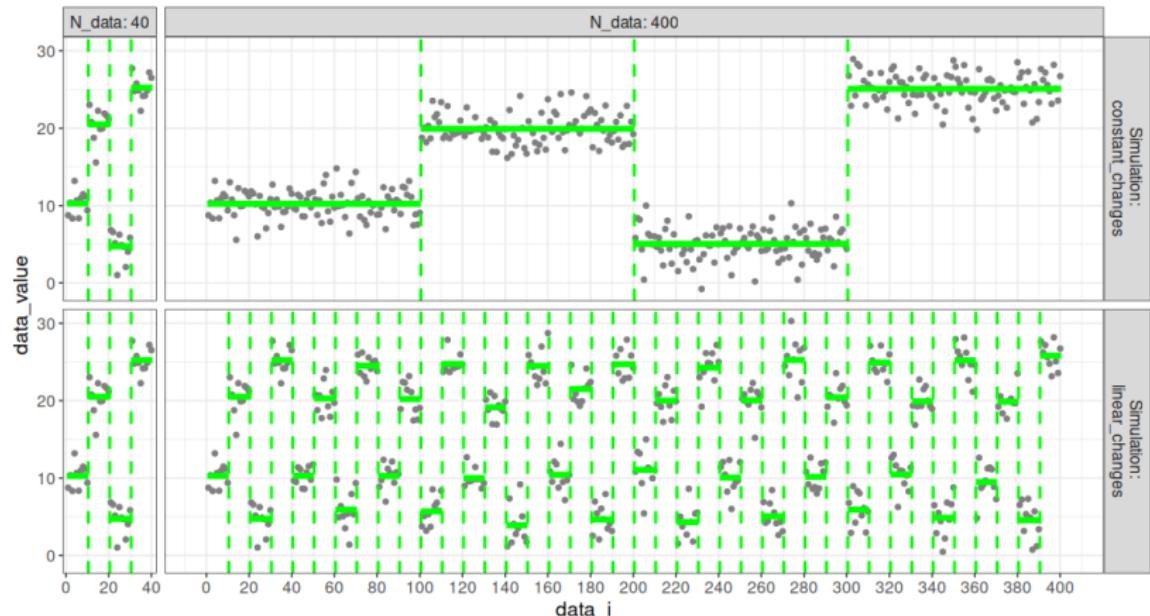
Simulated data with constant and linear changes



- ▶ Constant changes: always 4 segments (for any data size n).
- ▶ Linear: change every 10 data.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

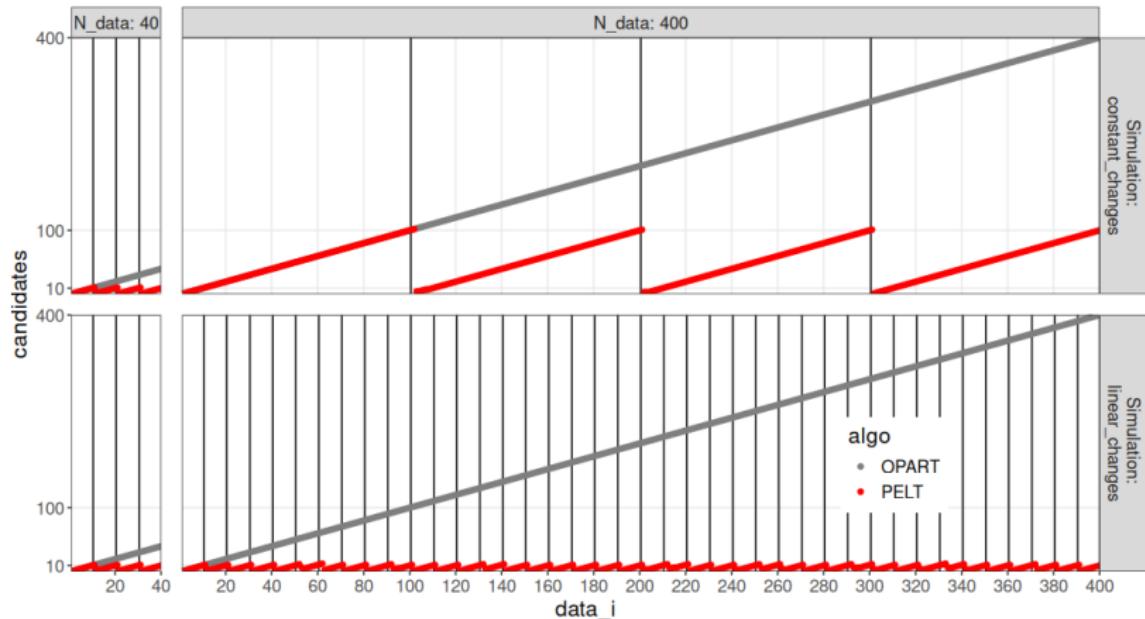
Simulated data with constant and linear changes



- ▶ Constant changes: always 4 segments (for any data size n).
- ▶ Linear: change every 10 data.

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PELT algorithm demonstration



- ▶ After each change in data, PELT prunes prior candidates.
- ▶ More changes—more pruning—faster.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

Penalized problem, PELT algorithm

# changes in data	Algorithm	# candidates per data	Overall time
Constant $O(1)$	OPART	$O(n)$	$O(n^2)$
	PELT	$O(n)$	$O(n^2)$
Linear $O(n)$	OPART	$O(n)$	$O(n^2)$
	PELT	$O(1)$	$O(n)$

- ▶ PELT fast/linear for data with frequent changes.
- ▶ But slow/quadratic for long runs of data without changes.
- ▶ Can we go faster when there are no changes?

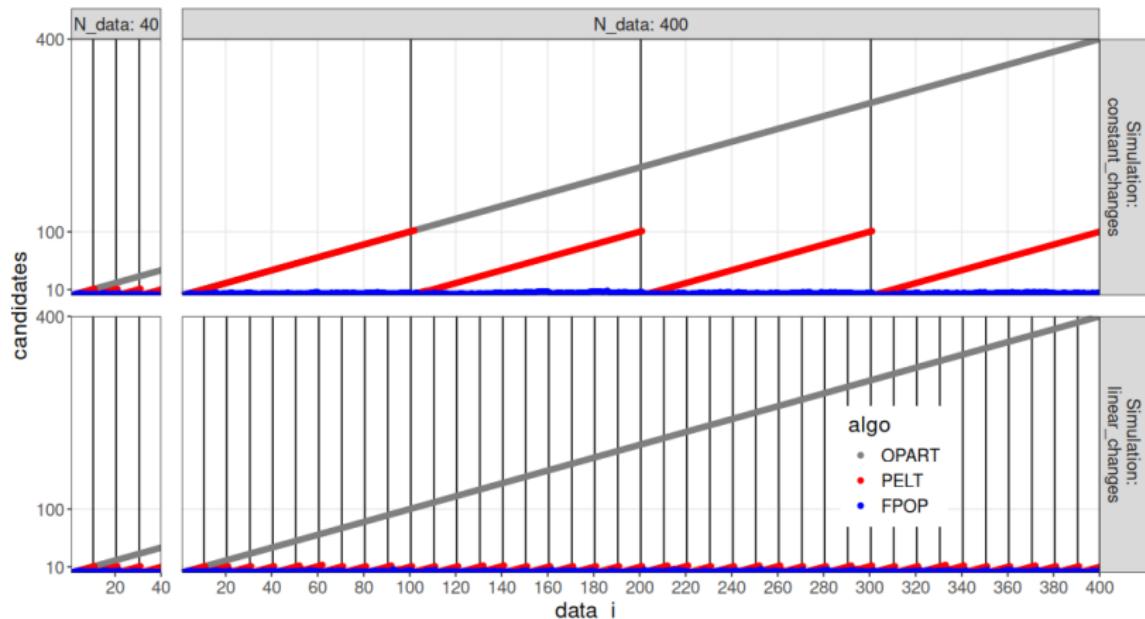
Yes, faster with FPOP algorithm!

Penalized optimal change-point problem:

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \underbrace{\lambda \sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}}.$$

- ▶ FPOP algo: Maidstone, *et al.*, Stat. and Comp. (2017).
- ▶ Let $C_n(m)$ be the best cost with mean m at n data.
- ▶ Recursively compute functions $C_1(m), C_2(m), \dots, C_n(m)$.
- ▶ **Functional pruning** considers smaller subset of candidates.
- ▶ Same output: one optimal change-point model.
- ▶ Same time as PELT in theory: best $O(n)$, worst $O(n^2)$.
- ▶ But much faster in practice! (never quadratic)
- ▶ library(fpopw) in R.

FPOP algorithm demonstration



- ▶ FPOP always prunes, whether or not there are changes in data.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

Penalized problem, FPOP algorithm

# changes in data	Algorithm	# candidates per data	Overall time
Constant $O(1)$	OPART	$O(n)$	$O(n^2)$
	PELT	$O(n)$	$O(n^2)$
	FPOP	$O(\log n)$	$O(n \log n)$
Linear $O(n)$	OPART	$O(n)$	$O(n^2)$
	PELT	$O(1)$	$O(n)$
	FPOP	$O(1)$	$O(n)$

- ▶ FPOP always fast in 1d data, no matter how many changes.
- ▶ Worst case $O(n^2)$ only happens in pathological data.

Penalized problem, FPOP extensions

Functional pruning can handle:

- ▶ Robust loss functions: Fearnhead and Rigaill, JASA (2019).
- ▶ Auto-regressive models: Romano *et al.*, JASA (2021).
- ▶ Storage on disk: Hocking *et al.*, JSS (2022).
- ▶ Inequality/graph constraints: Runge *et al.*, JSS 2023.
- ▶ Multi-variate time series: Pishchagina *et al.*, Computo (2024).
- ▶ Multi-scale penalties: Lehrmann and Rigaill, JCGS (2025).

But:

- ▶ Difficult to implement (100s of lines of C++ code).
- ▶ Especially difficult for high dimensional data/models.

Is it possible to go faster?

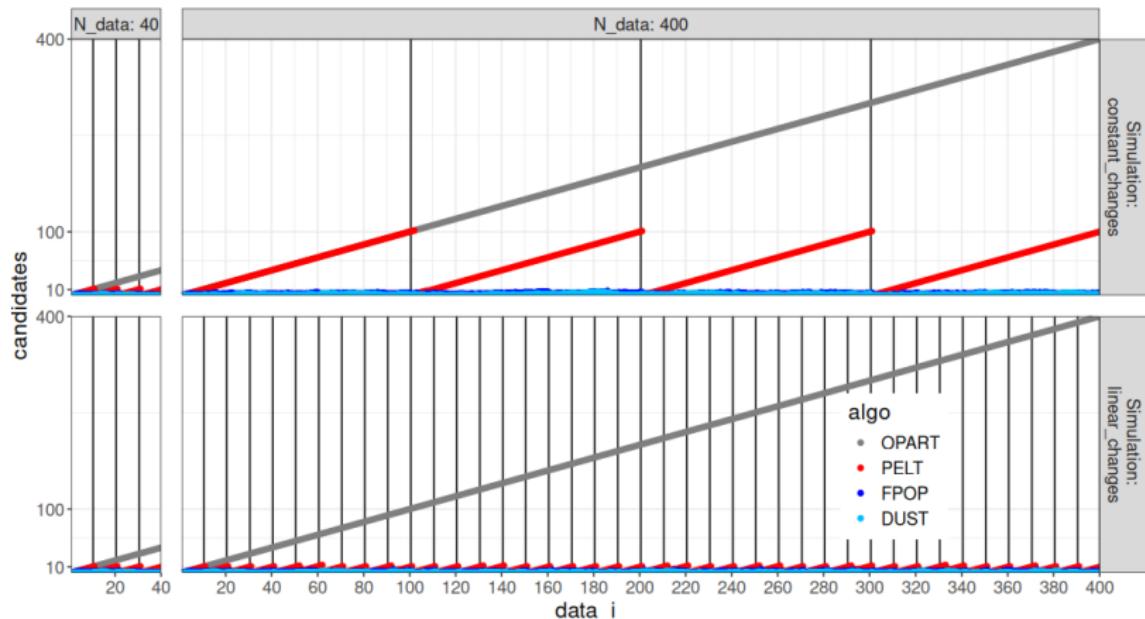
Yes, faster with DUST algorithm!

Penalized optimal change-point problem:

$$\min_{\mu \in \mathbb{R}^n} \underbrace{\sum_{i=1}^n \ell(\mu_i, z_i)}_{\text{Loss (data-fitting)}} + \underbrace{\lambda \sum_{i=2}^n I[\mu_{i-1} \neq \mu_i]}_{\text{Number of change-points (regularization)}}.$$

- ▶ Truong and Runge, An Efficient Algorithm for Exact Segmentation of Large Compositional and Categorical Time Series, Stat (2024).
- ▶ DUST algo: DUality Simple Test.
- ▶ Combines ideas from PELT and FPOP algos.
- ▶ Solve a Langrange dual problem to prune change-points.
- ▶ Easier to code than FPOP.
- ▶ Same time as FPOP in theory: best $O(n)$, worst $O(n^2)$.
- ▶ But much faster in practice! (constant factors)
- ▶ library(dust) in R.

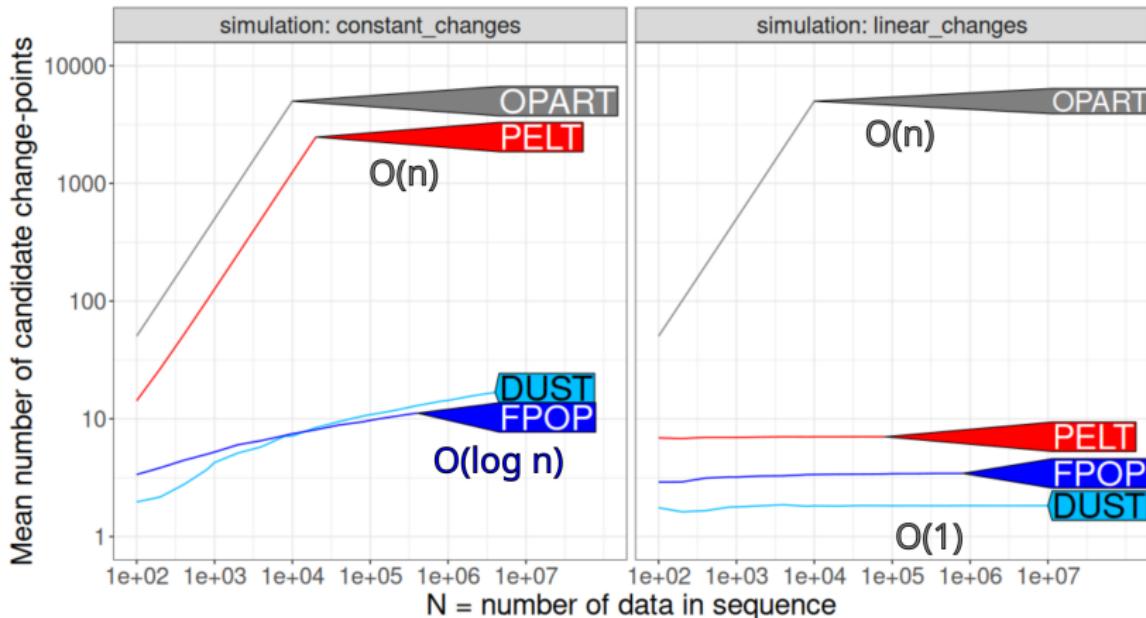
DUST algorithm demonstration



- ▶ DUST always prunes, whether or not there are changes in data.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

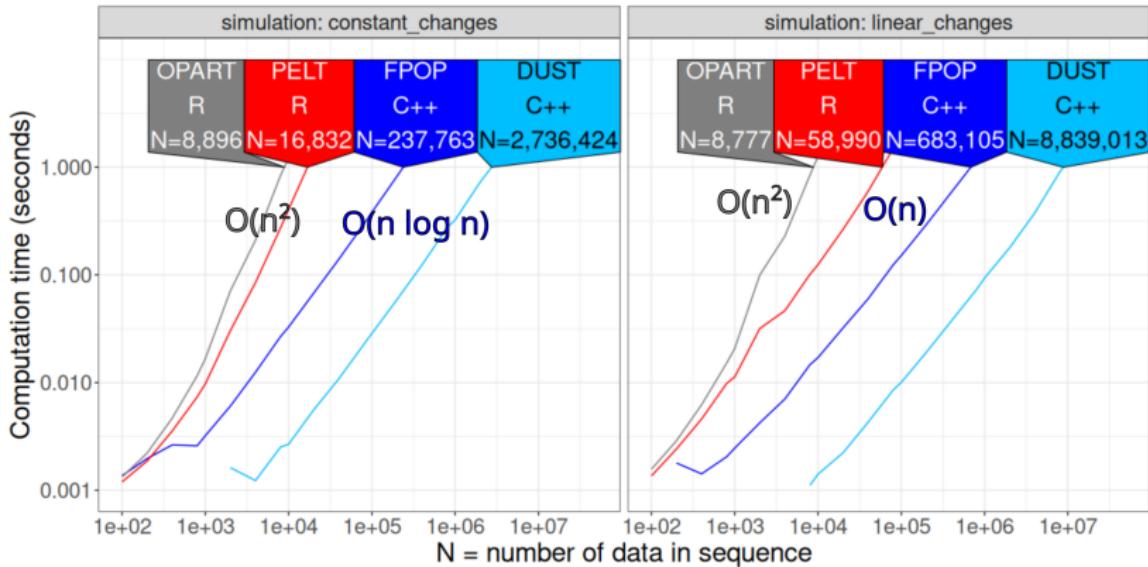
Number of candidates considered



- Slope on log-log plot indicates asymptotic complexity class.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

Overall computation time



- ▶ N= data size computable in time limit of 1 second.
- ▶ FPOP 10x faster than PELT.
- ▶ DUST 10x faster than FPOP.
- ▶ library(atime) in R for asymptotic analysis.

<https://tdhock.github.io/blog/2025/PELT-vs-fpopw/>

Penalized problem, DUST algorithm

Penalized optimal change-point problem:

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- ▶ DUST always fast in 1d data, no matter how many changes.
- ▶ Is it possible to go faster and stay optimal?

Penalized problem, DUST algorithm

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- ▶ DUST always fast in 1d data, no matter how many changes.
- ▶ Is it possible to go faster and stay optimal?
- ▶ Conjecture: only by constant factors.

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Summary and Discussion

Computing one constrained model using penalized solver

How to compute the best model with P^* change-points?

- ▶ Example: want $P^* = 100$ change-points.

Pruned dynamic programming algo: Rigaill, JSFDS (2015).

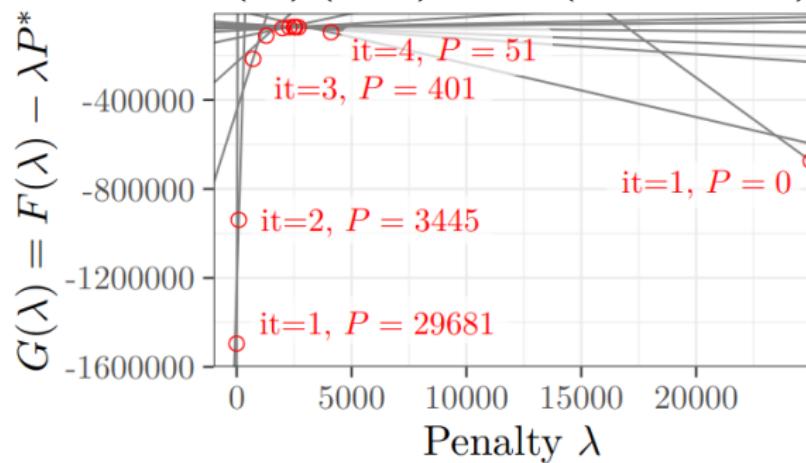
- ▶ library(fpopw) in R.
- ▶ Run **constrained** solver to get all model sizes from 0 to 100.
- ▶ $O(P^* n \log n)$ time—fast in n .
- ▶ Slow in P^* : we do not want model sizes 0 to 99.

Can we go faster?

- ▶ FPOP/DUST are $O(n \log n)$.
- ▶ But need to input penalty $\lambda \geq 0$
- ▶ and λ that yields P^* is unknown.

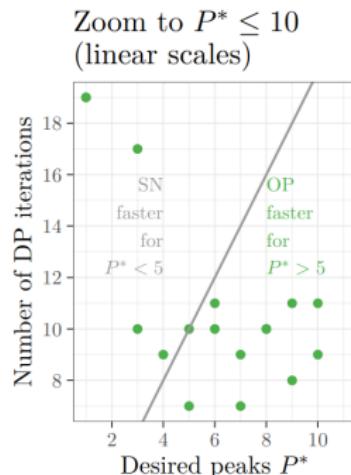
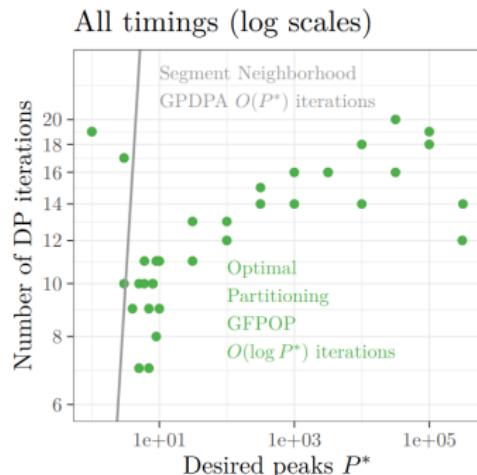
Computing one constrained model using penalized solver

Sequential search algo: Hocking *et al.*, Journal of Statistical Software 101(10) (2022), library(PeakSegDisk) in R.



- ▶ Run **penalized** solver for an special sequence of λ values.
- ▶ Maximizes $G(\lambda) = \min$ of finite number of linear functions.
- ▶ $O(\log P^* n \log n)$ time—fast in n and P^* ! (empirically)

Penalized (OP) is faster than constrained (SN)



- ▶ Figure: genomic data, $N \approx 10^6$,
- ▶ Sequential search repeated runs penalized (OP) solver.
- ▶ Example: for $N = 10^7$, desired change-points $P^* = 3000$.
- ▶ Constrained (SN) solver: $\approx 100\text{TB}$ storage, 10 weeks.
- ▶ Penalized (OP) solver: 100GB storage, 10 hours.

Hocking *et al.*, Journal of Statistical Software 101(10) (2022).

Fast computation of large model sizes and ranges

How to compute all models with penalties $\lambda \in [\underline{\lambda}, \bar{\lambda}]$?

- ▶ Example: want all models with $\lambda \in [0.1, 10.5]$.
- ▶ CROPS: Change-points for a Range Of PenaltieS.
- ▶ Haynes, et al. Journal of Computational and Graphical Statistics, 26(1), 134-143 (2017).
- ▶ `library(changepoint)` in R.

How to compute all models with number of change-points

$P \in [\underline{P}, \bar{P}]$?

- ▶ Example: want all models from 50 to 60 change-points.
- ▶ CROCS: Change-points for a Range Of ComplexitieS.
- ▶ Liehrmann et al., BMC Bioinformatics 22(323) (2021).
- ▶ `library(CROCS)` in R.

Both linear time in the number of models!

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Summary and Discussion

Summary and Discussion

- ▶ Optimal changepoint detection in n data is a non-convex problem, naïvely a $O(n^S)$ computation for S segments.
- ▶ Pruning algorithms compute a globally optimal change-point model much faster, $O(n \log n)$ or even $O(n)$.

Future work:

- ▶ Even faster versions of DUST.
- ▶ Fast pruning algorithms for piecewise loss functions (L1 etc).
- ▶ Proving worst case time for sequential search?

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Merci !!

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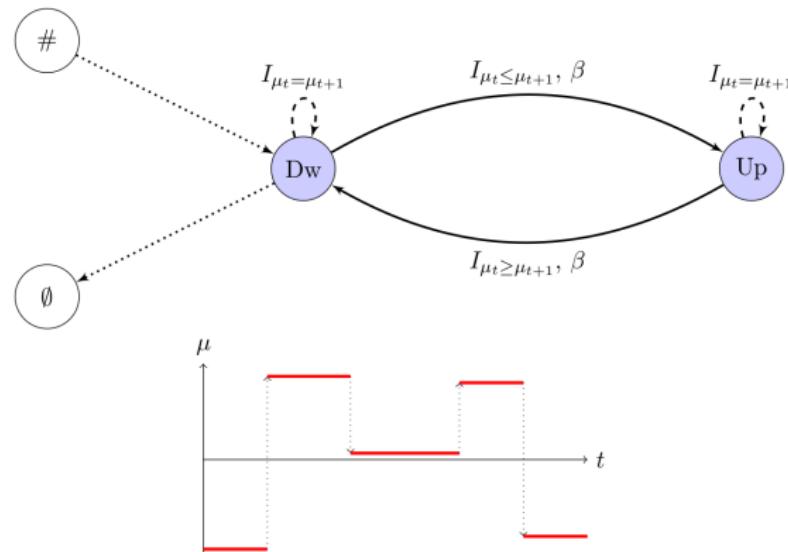
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- ▶ **Hocking TD**, Srivastava A. Labeled Optimal Partitioning. Computational Statistics 38 (2023).
- ▶ Fotoohinasab A, **Hocking TD**, Afghah F. A Greedy Graph Search Algorithm Based on Changepoint Analysis for Automatic QRS-Complex Detection. Computers in Biology and Medicine 130 (2021).

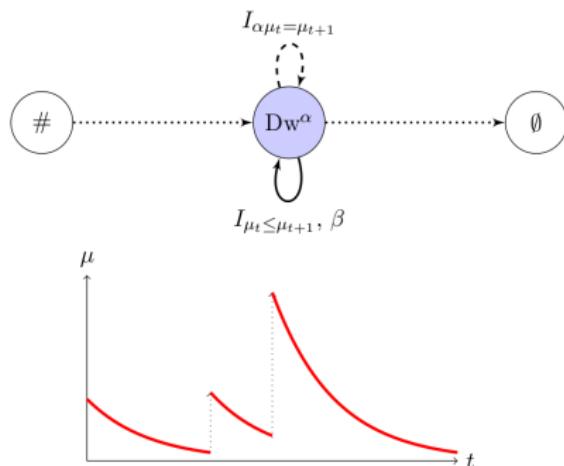
Optimization constraints defined using a graph

Runge *et al.*, Journal of Statistical Software 2023.

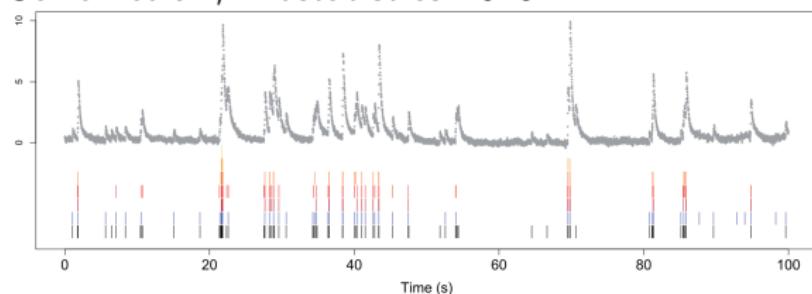


- ▶ Purple Dw/Up nodes represent hidden states.
- ▶ #/ \emptyset nodes constrain start/end state.
- ▶ Edges represent possible state transitions.
- ▶ `library(gfpop)` in R computes optimal change-points for user-defined constraint graphs.

All up changes, exponential decaying segments

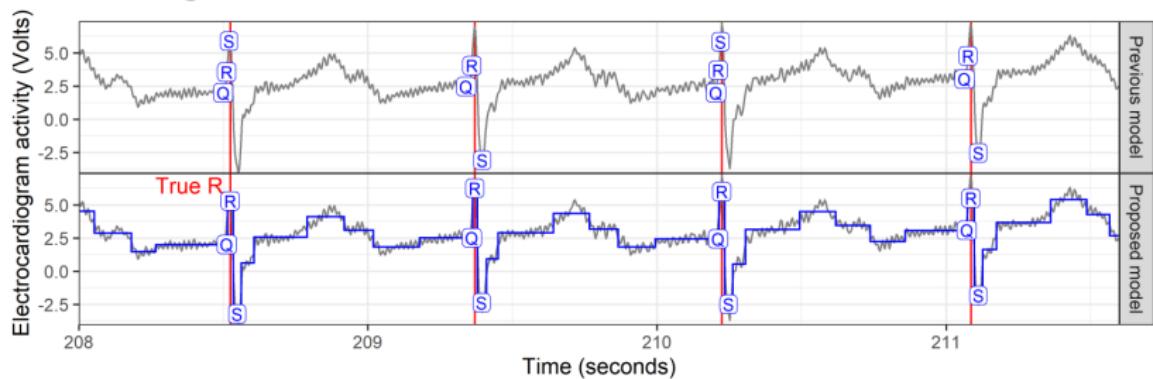
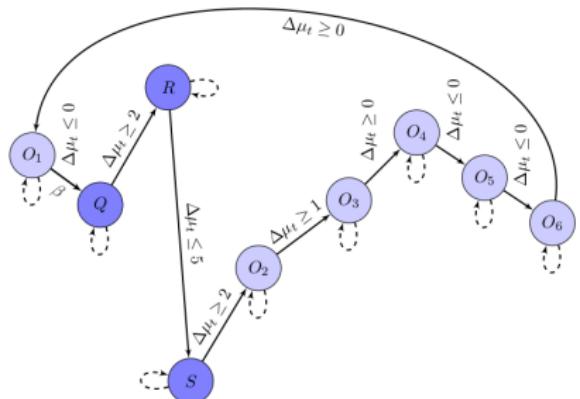


Jewell et al., Biostatistics 2019.



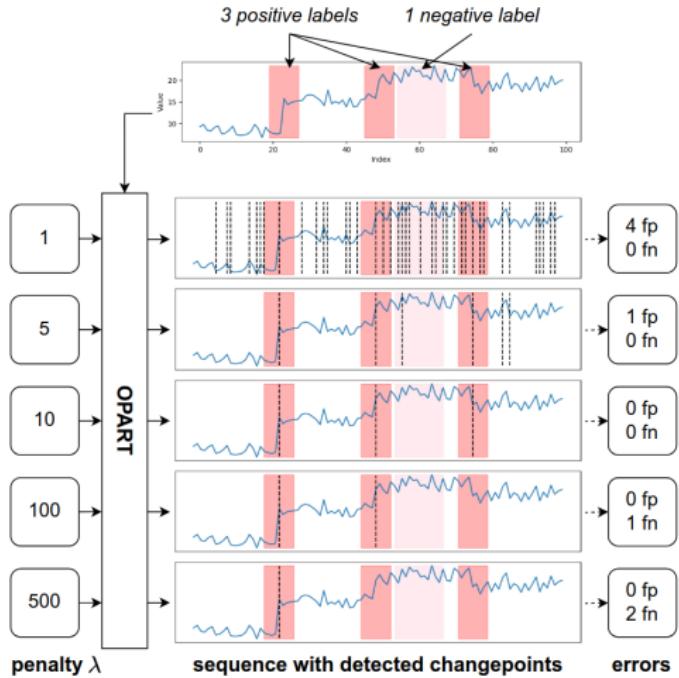
library(FastLZeroSpikeInference) in R.

Complex graph for electrocardiogram data



Fotoohinasab et al., Asilomar conference 2020.

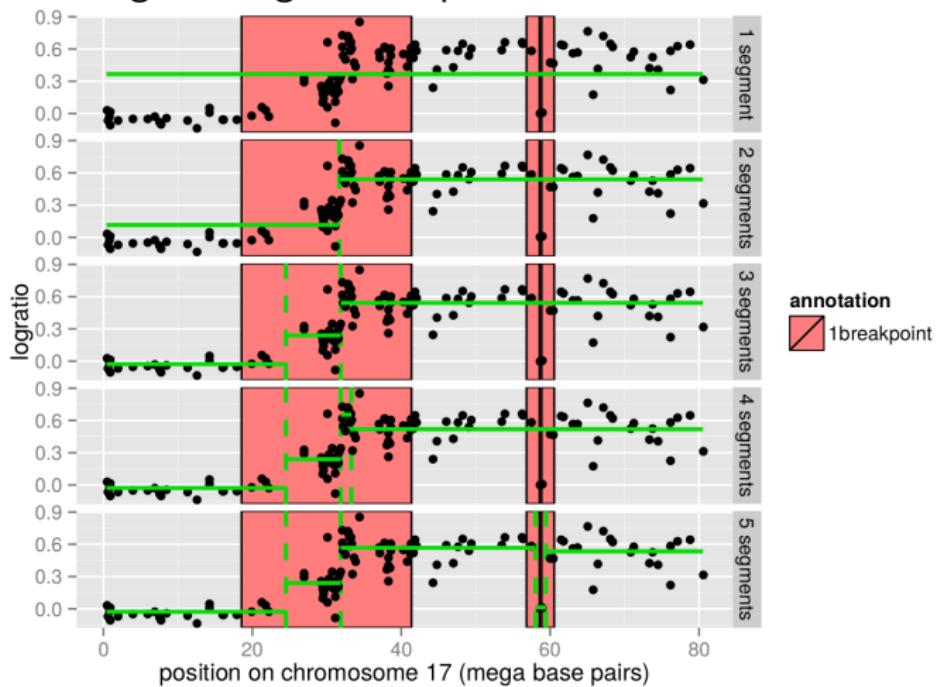
Fitting change-points to labels from an expert



- ▶ Figure: Nguyen and Hocking, arXiv:2505.07413.
- ▶ fp=false positive (too many changes).
- ▶ fn=false negative (too few changes).
- ▶ Try different λ values until we find a model with no errors.

What if no models agree with expert labels?

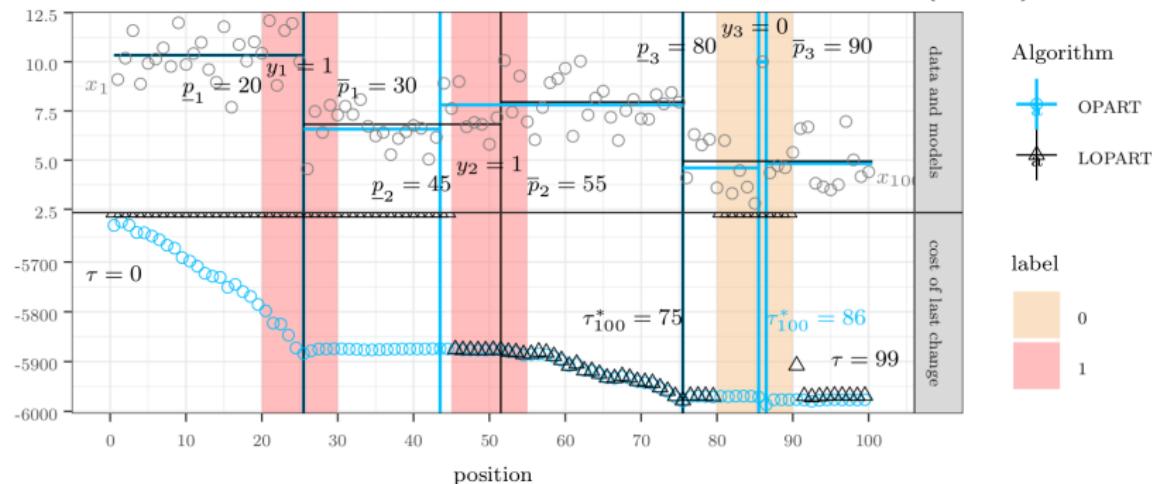
Hocking and Rigaill, Pre-print hal-00759129.



- ▶ Expert wants: one changepoint in each label (red rectangle).
- ▶ No model is consistent with all three labels.

Using expert labels as optimization constraints

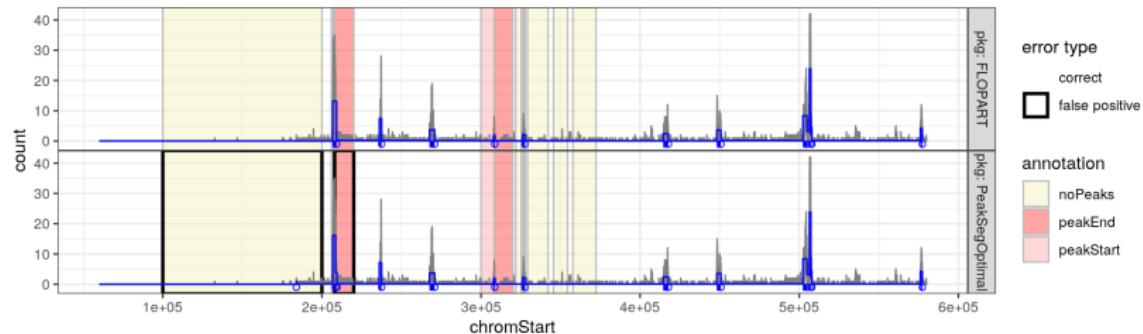
Hocking and Srivastava, Computational Statistics 38 (2023).



- ▶ Previous OPART model (blue) ignores labels (two errors).
- ▶ Main idea: add optimization constraints to ensure that there is the right number of changepoints predicted in each label.
- ▶ Proposed LOPART model (black) consistent with labels.
- ▶ library(LOPART) in R.

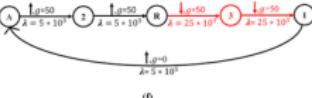
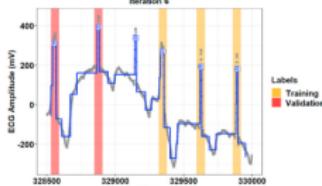
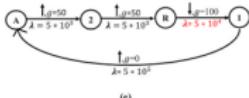
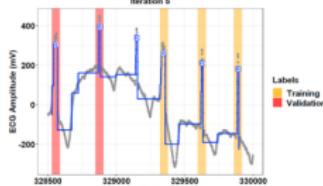
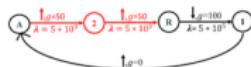
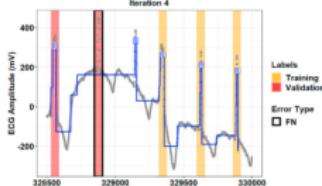
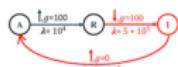
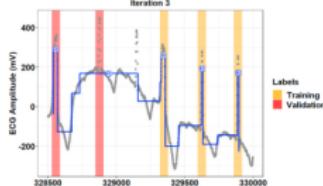
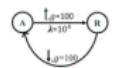
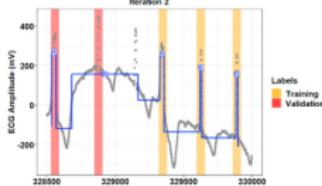
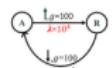
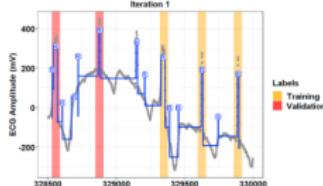
Label constraints and directional constraints

Kaufman *et al.*, Journal of Computational and Graphical Statistics
33(4) (2024).



- ▶ Previous PeakSegOptimal algorithm (bottom) ignores labels (two errors).
- ▶ Proposed FLOPART model (top) consistent with labels, and interpretable in terms of up changes to peaks and down changes to background noise.
- ▶ library(FLOPART) in R.

Learning a complex graph using labeled regions



- ▶ Fotoohinasab *et al.*, 2021.
- ▶ Simple initial graph is iteratively edited (red) to agree with expert labeled regions (orange rectangles).
- ▶ Easier for expert to provide labels than graph.

How to predict the number of changes?

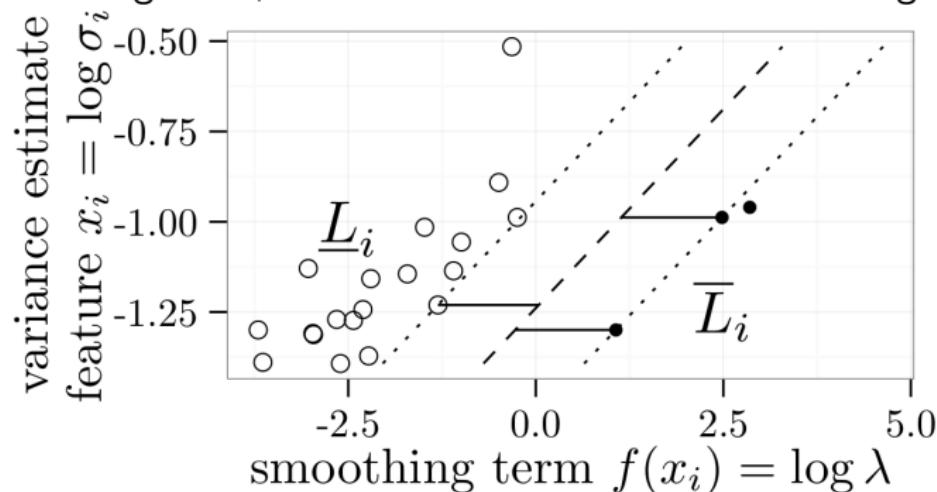
TODO figure.

In practice the penalty λ is unknown — what value should we use?

- ▶ $z = z_1, \dots, z_n$ is the data sequence to segment.
- ▶ Let $x = \phi(z)$ be a feature vector (fixed, not learned).
- ▶ Learn $f(x) = \log \lambda$, predicted penalty.
- ▶ Goal: minimize error with respect to labeled regions in test set.

Learning to predict number of changes similar to SVM

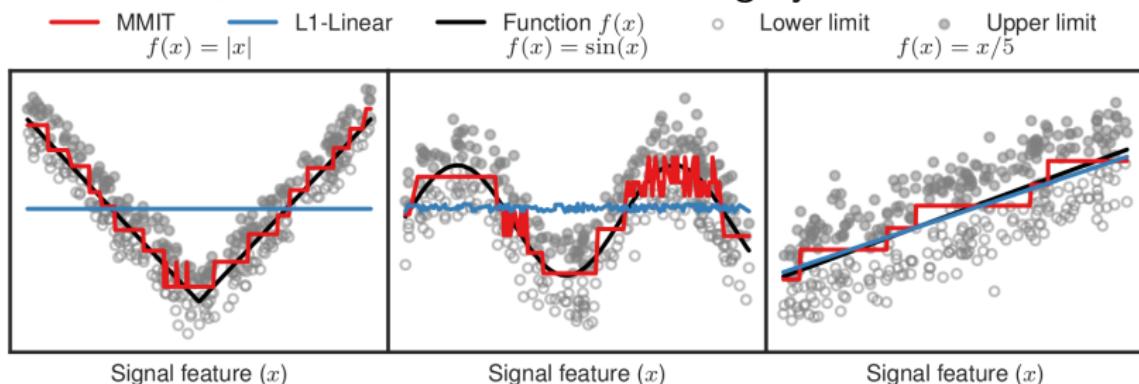
Hocking et al., Int'l Conference on Machine Learning 2013.



- ▶ Train on several data sequences with labels (dots).
- ▶ Want to compute function between white and black dots.
- ▶ SVM margin is multi-dimensional (diagonal).
- ▶ Here margin to maximize is one-dimensional (horizontal).
- ▶ L1 regularized linear model, library(penaltyLearning) in R.

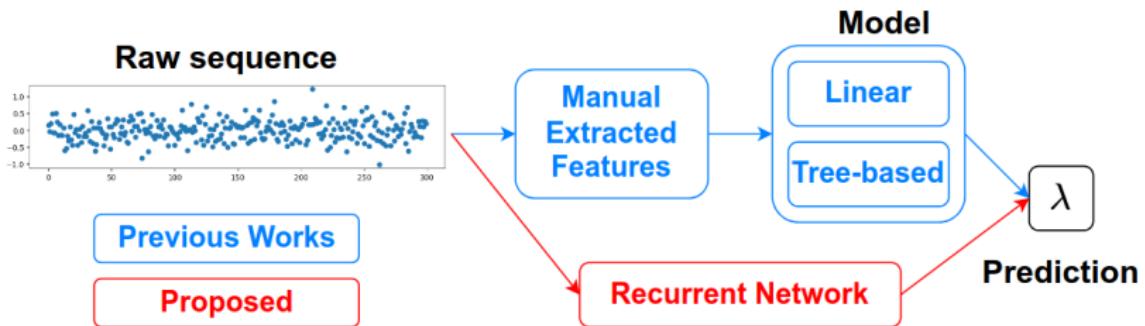
Decision tree learns non-linear function of inputs

Drouin et al., Neural Information Processing Systems 2017.



- ▶ Generalization of classical CART regression tree learning algorithm.
- ▶ Can learn non-linear functions of inputs.
- ▶ More recently we implemented a similar idea in xgboost, Barnwal et al., Journal of Computational and Graphical Statistics 31(4) (2022).
- ▶ `library(mmit)` and `library(xgboost)` in R.

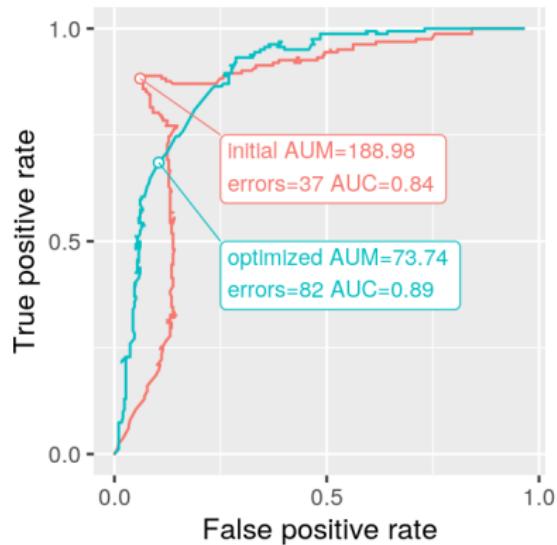
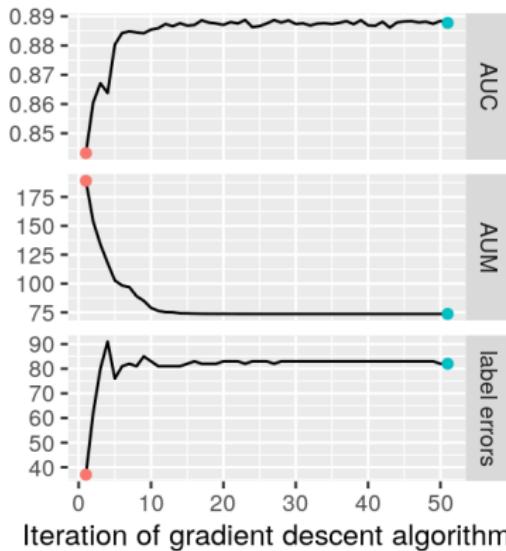
Penalty prediction using recurrent neural network



- ▶ Nguyen and Hocking, arXiv:2505.07413.

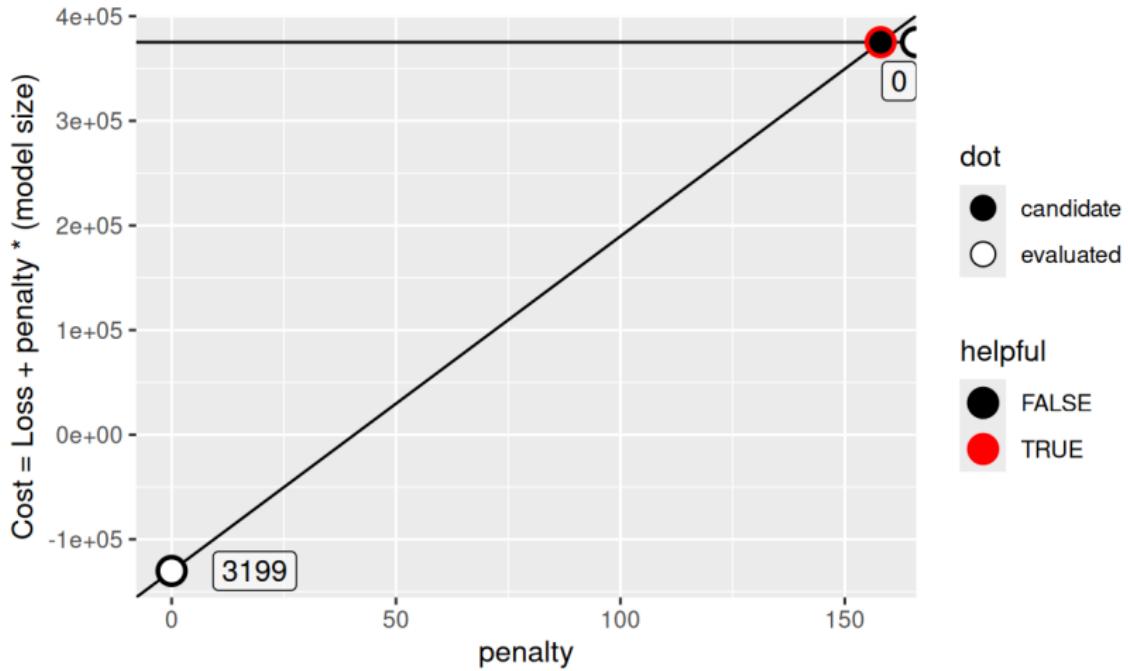
AUM gradient descent algorithm optimizes AUC

Hillman and Hocking, *Journal of Machine Learning Research* 2023.

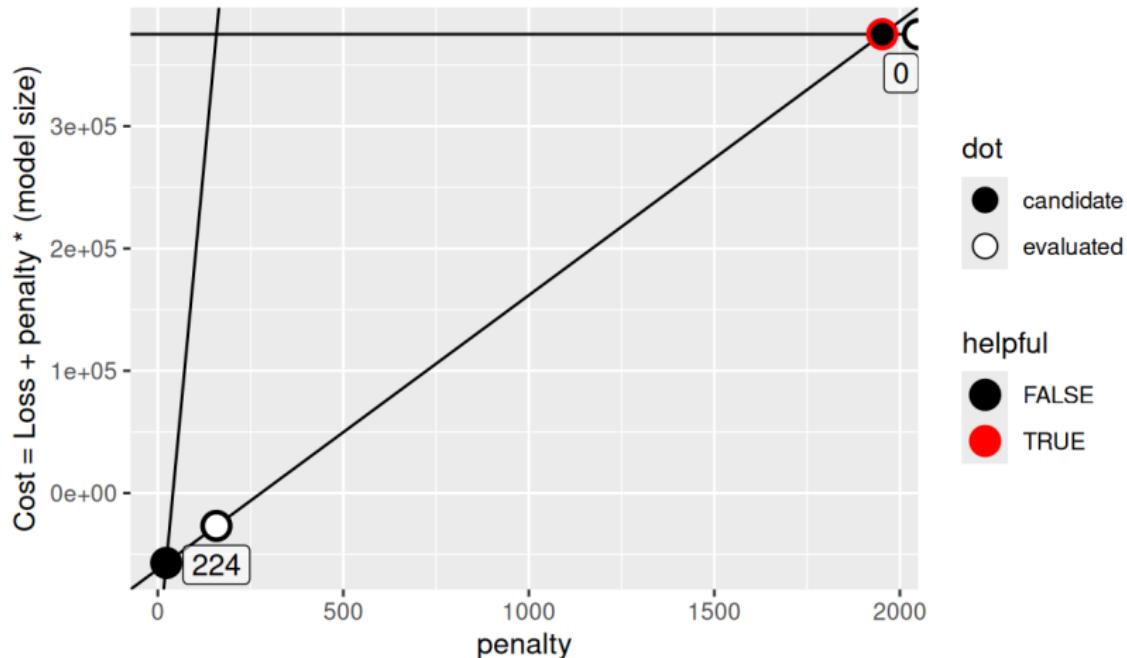


- ▶ Initial predictions: minimum label errors.
- ▶ Optimized ROC curves are more regular/monotonic.
- ▶ Trade-off between AUC and label error optimization.
- ▶ library(aum) in R.

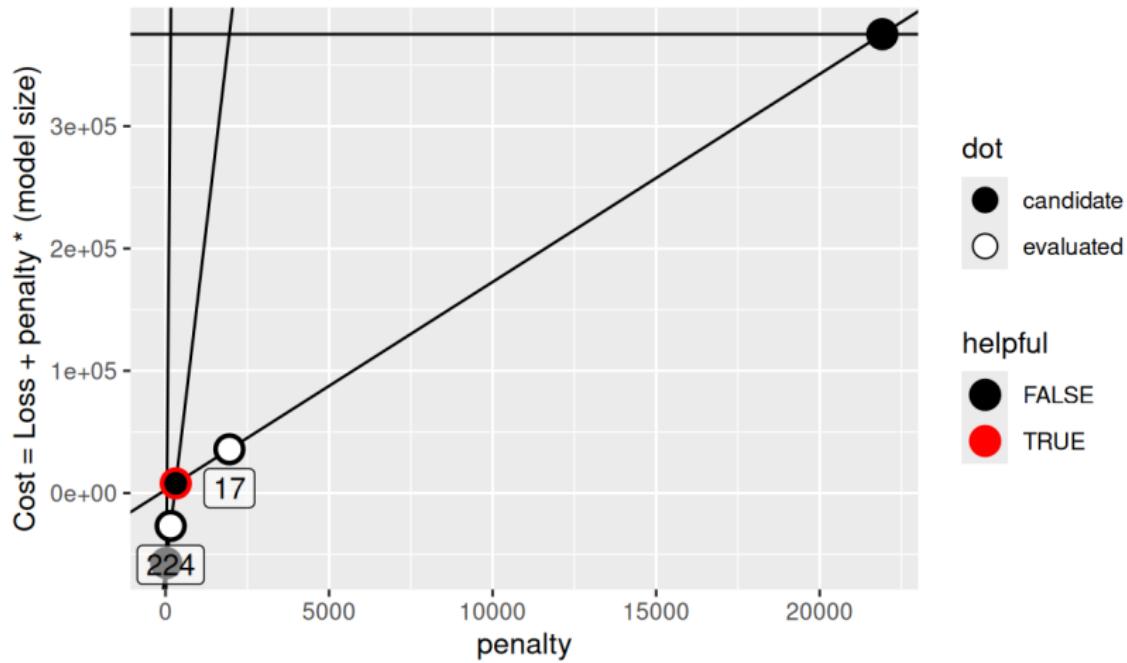
Goal size=100, iteration=2, model sizes computed so far:
3199, 0



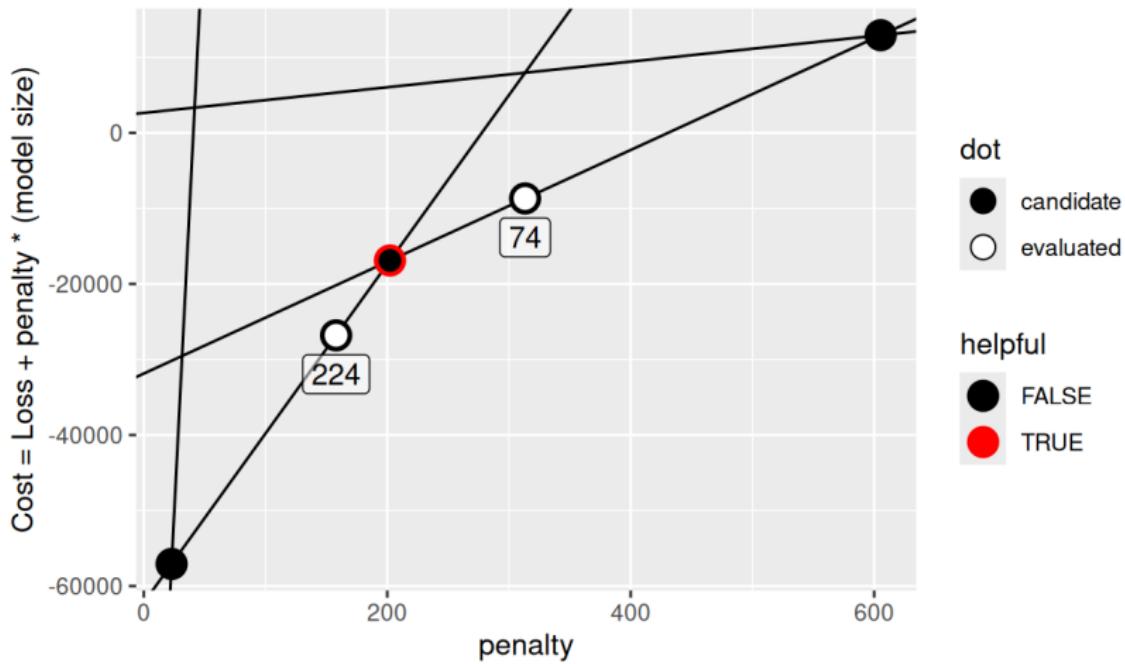
Goal size=100, iteration=3, model sizes computed so far:
3199, 0, 224



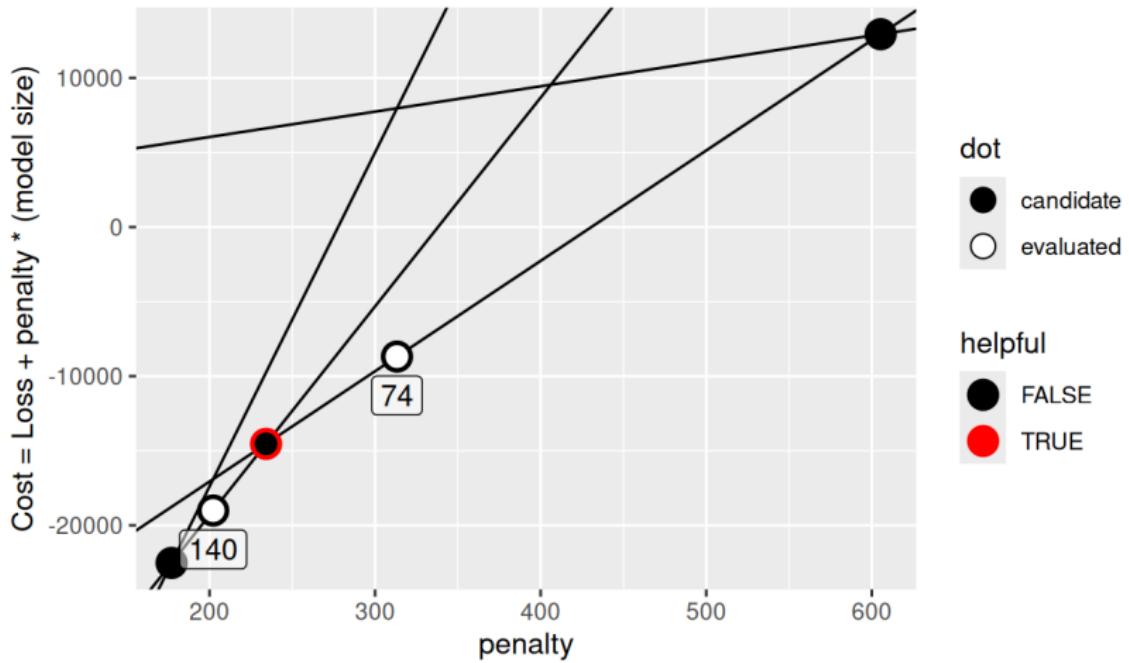
Goal size=100, iteration=4, model sizes computed so far:
3199, 0, 224, 17



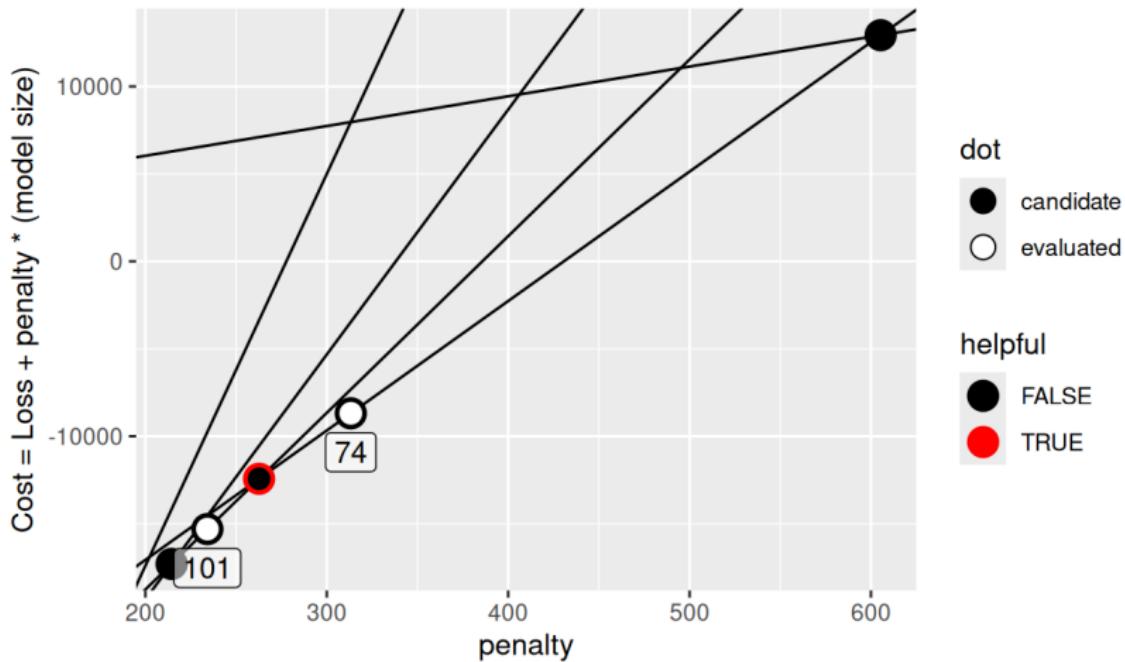
Goal size=100, iteration=5, model sizes computed so far:
3199, 0, 224, 17, 74



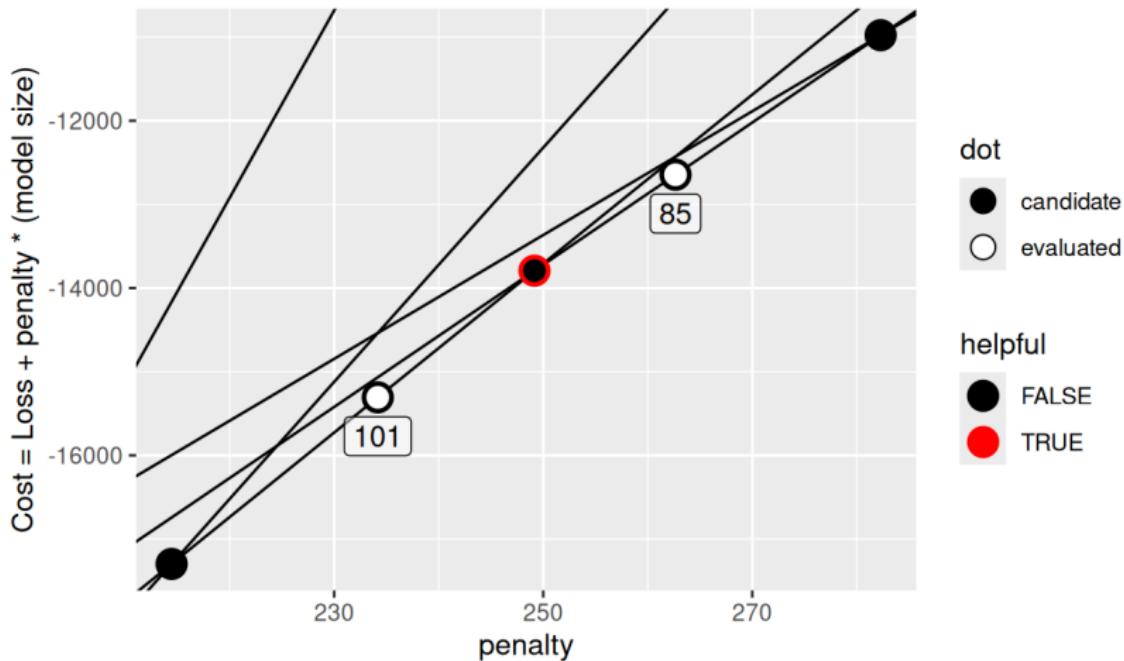
Goal size=100, iteration=6, model sizes computed so far:
3199, 0, 224, 17, 74, 140



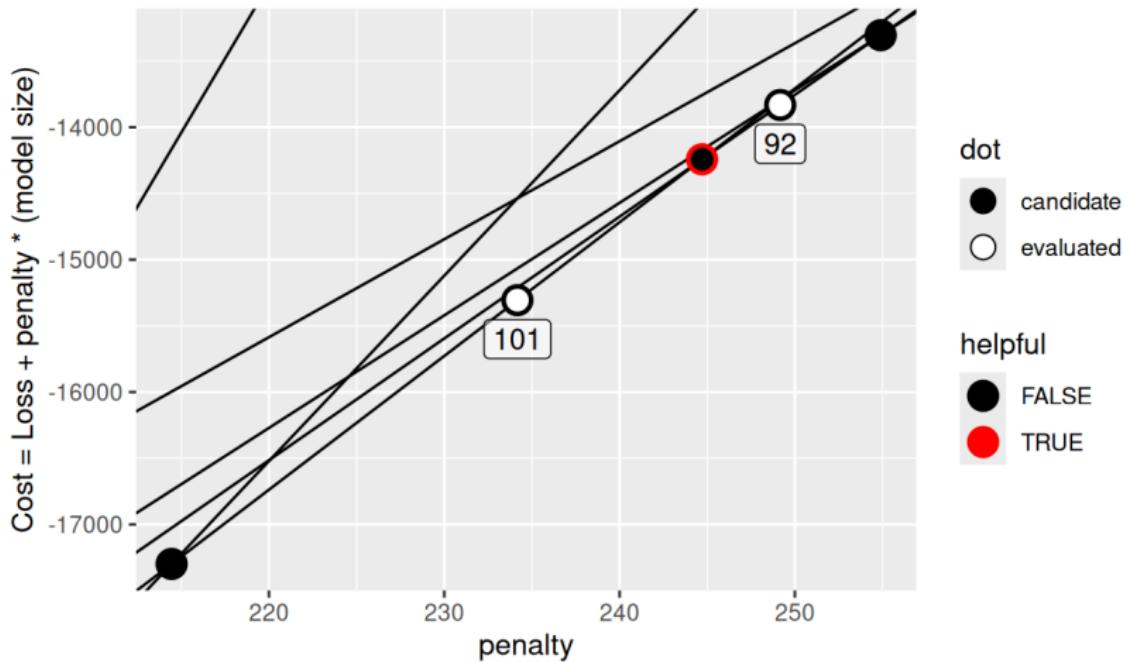
Goal size=100, iteration=7, model sizes computed so far:
3199, 0, 224, 17, 74, 140, 101



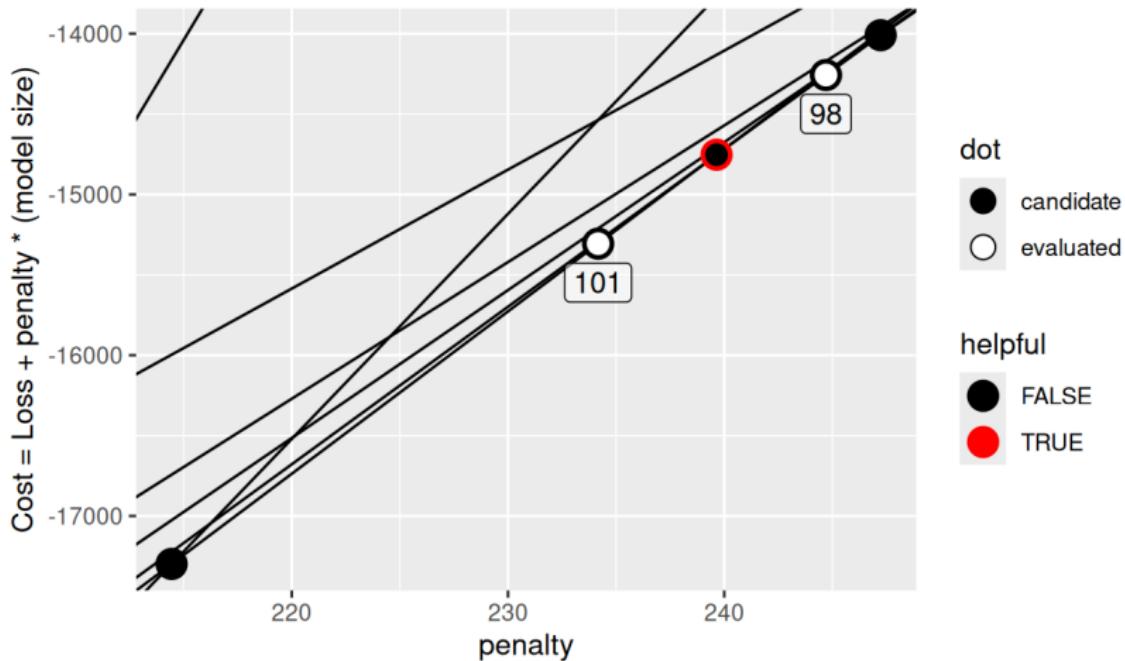
Goal size=100, iteration=8, model sizes computed so far:
3199, 0, 224, 17, 74, 140, 101, 85



Goal size=100, iteration=9, model sizes computed so far:
3199, 0, 224, 17, 74, 140, 101, 85, 92



Goal size=100, iteration=10, model sizes computed so far:
3199, 0, 224, 17, 74, 140, 101, 85, 92, 98



Goal size=100, iteration=11, model sizes computed so far:
3199, 0, 224, 17, 74, 140, 101, 85, 92, 98, 100

