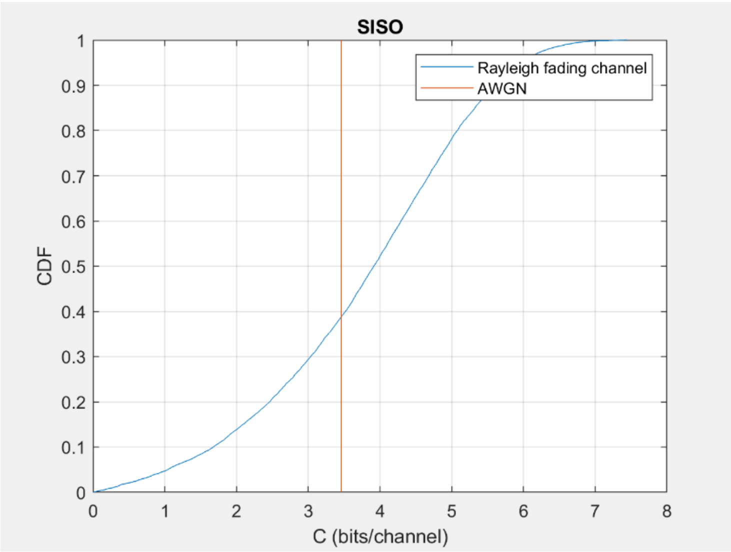
Information Theory: Lab experiment

Shannon capacity study for multi-antenna channels

You can find our Matlab code in annex, at the end of the report. Plots are obtained from Matlab and remarkable values are read with Matlab graph tools.

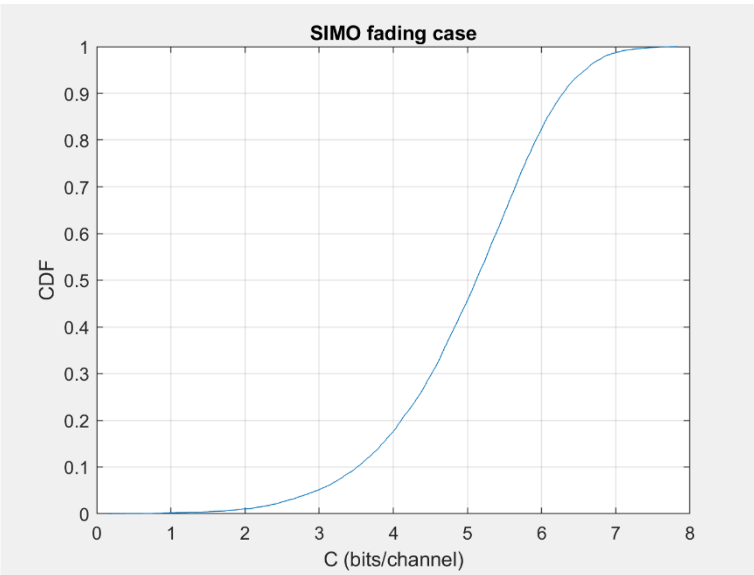
Task 1



For AWGN: $C_{0.5} = 3.5$; $C_{0.1} = 0$.

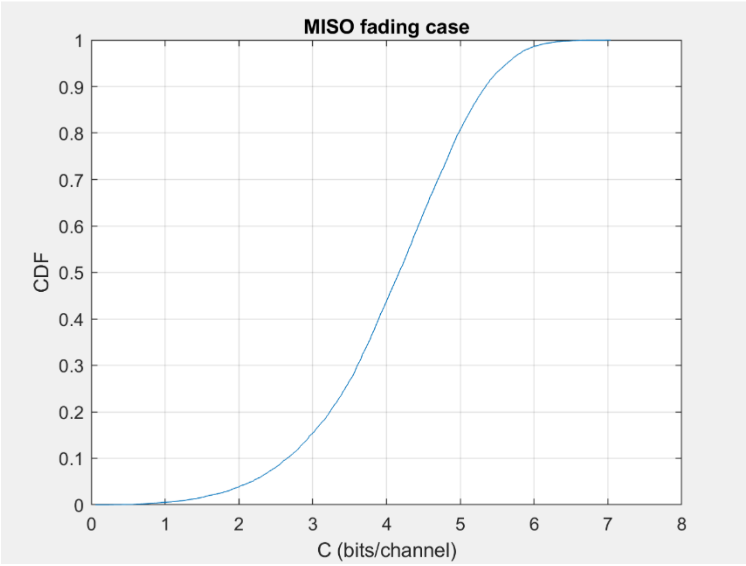
For Rayleigh fading channel: $C_{0.5} = 3.9$; $C_{0.1} = 1.7$.

Task 2



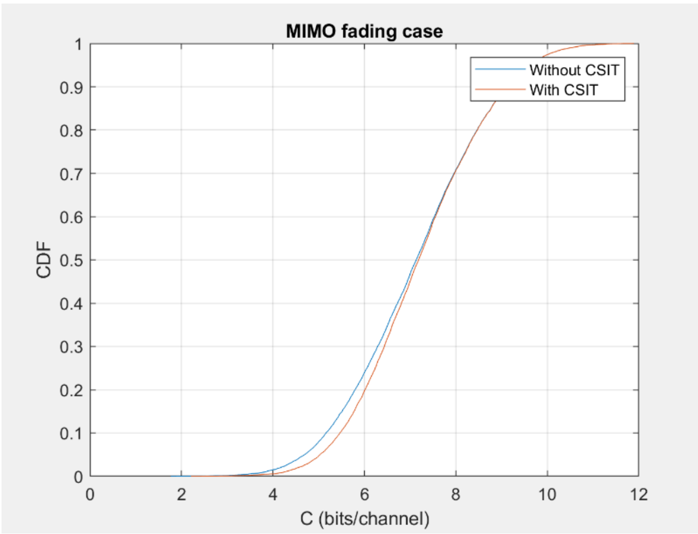
Measures: $C_{0.5} = 5.1$; $C_{0.1} = 3.5$.

Task 3



Measures: $C_{0.5} = 4.2$; $C_{0.1} = 2.7$.

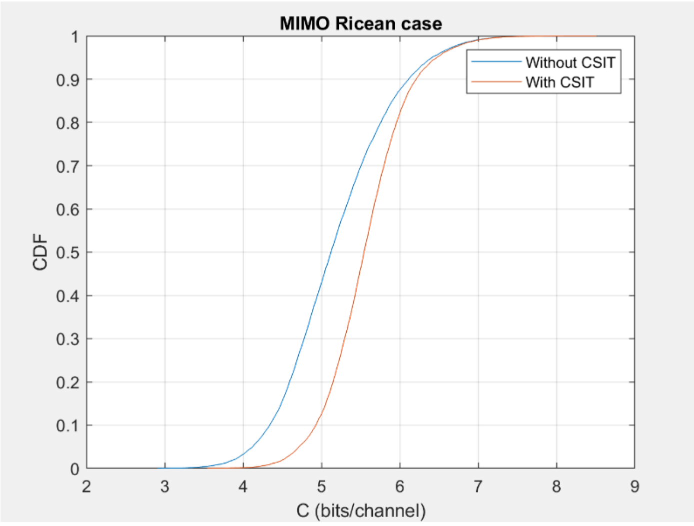
Task 4



Without CSIT: $C_{0.5} = 7.12$; $C_{0.1} = 5.17$.

With CSIT: $C_{0.5} = 7.18$; $C_{0.1} = 5.47$.

Task 5



Without CSIT: $C_{0.5} = 5.12$; $C_{0.1} = 4.34$.
With CSIT: $C_{0.5} = 5.54$; $C_{0.1} = 4.93$.

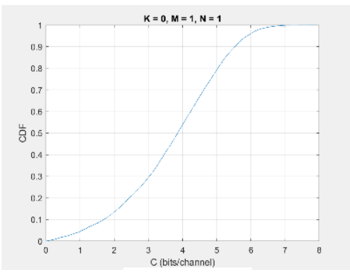
Task 6: Comparisons and interpretations

- **System's average throughput**: it can be interpreted from the curves by taking the value of $C_{0.5}$.

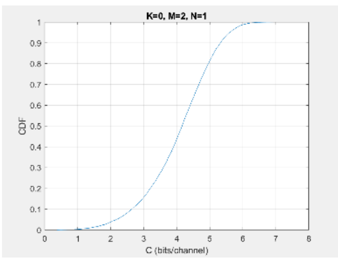
- **System's reliability**: it can be interpreted from the curves by taking the value of $C_{0.1}$.

- **Relation between the average throughput and the number of antennas**

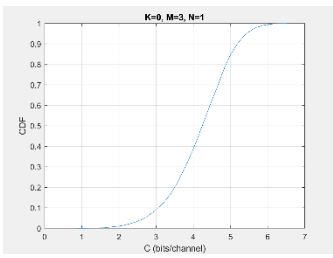
  From the previous tasks, we can deduce that the average throughputs increases with the number of antennas. Indeed, let's compare task 1 and task 2. In task 1, for $K = 0$, and $N + M = 2$, we have $C_{0.5} = 3.9$. In task 2, for $K = 0$ and $N + M = 3$, we have $C_{0.5} = 5.9$ and in task 3, for $K = 0$ and $N + M = 3$, we have $C_{0.5} = 5.1$. So average throughput is clearly increasing with the number of antennas. Now let's show it by different simulations for $K = 0$, $N = 1$, and by increasing M (so $N + M$ = number of antennas will also increase) and with no CSIT.



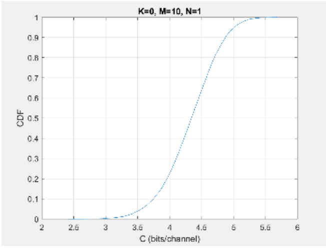
$C_{0.5} = 3.8$



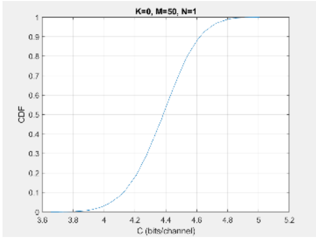
$C_{0.5} = 4.2$



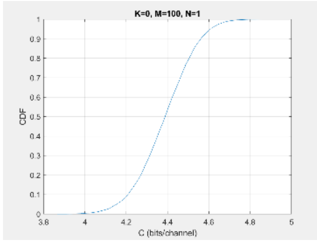
$C_{0.5} = 4.22$



$C_{0.5} = 4.35$



$C_{0.5} = 4.38$



$C_{0.5} = 4.39$

Finally, we can observe that the average throughput is increasing with the number of antennas.

- <u>**Is it good or bad to have a LOS component?**</u>

At the first approach, according to the previous results, it is detrimental to have a LOS component. Indeed, if we compare the two extreme cases K=0 and K=+∞ for SISO, the throughput is higher without LOS component. The same result can be observed comparing tasks 4 and 5.

For the reliability, it is also the case. Indeed, if we compare results of tasks 4 and 5, we can observe that reliability is at a lower level when $K = 10 > 0$.

Therefore, we can observe that LOS decreases the average throughput and the reliability. **Globally, it is not beneficial**.

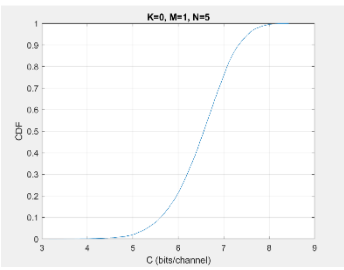
However, having a LOS component presents two major advantages:

1. The channel H **is no more only random**
2. There is **less energy dissipation and information is better directed**

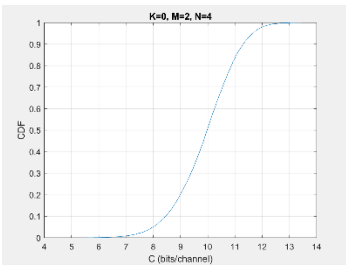
- <u>**In which case do we have the biggest gain from using a feedback channel? Why?**</u>

We use a feedback channel in tasks 4 and 5. Clearly, according to the plots, the biggest gain is for the MIMO fading case. It confirms what we said: having a LOS ($K = 10 > 0$) is not beneficial for the gain and for the average throughput.

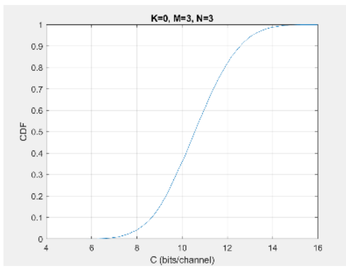
- <u>**Maximize average throughput when there is no CSIT with $M + N = 6$**</u>



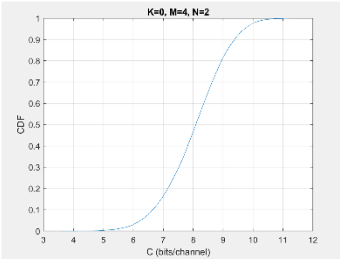
$C_{0.5} = 6.6$



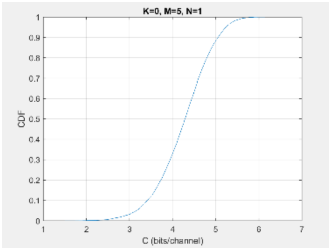
$C_{0.5} = 10$



$C_{0.5} = 10.6$



$C_{0.5} = 8.1$



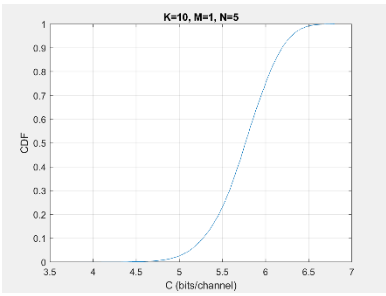
$C_{0.5} = 4.3$

From these plots, we can deduce that the optimal breakdown of M and N when there is no CSIT is: M= 3 and N= 3 with K = 0 and we find $C_{0.5}$ = 10.6.

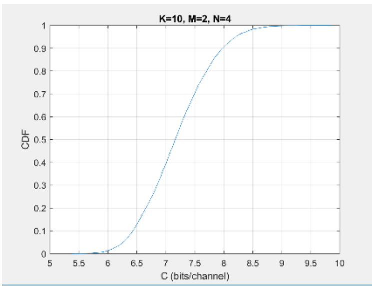
Remark: we can take $K \neq 0$, in order to study a normal "case". We find exactly the same breakdown, M=N=3.

- **Maximize average throughput when there is CSIT with M + N = 6**

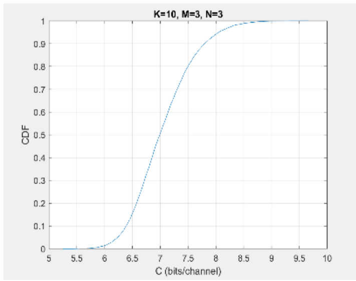
Here we will take $K = 10$. Again, we compute the three possible cases ($M > N$ are two useless cases with CSIT). Plots are displayed below.



$C_{0.5} = 5.8$



$C_{0.5} = 7.2$



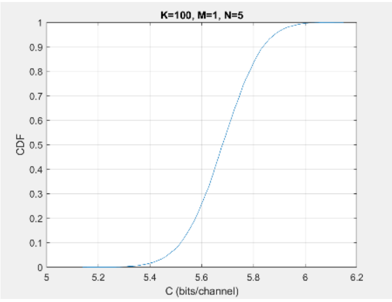
$C_{0.5} = 7$

When there is CSIT, it is useless to compute $M > N$ cases, because A is the null matrix, and the Shannon capacity will be equal to zero.

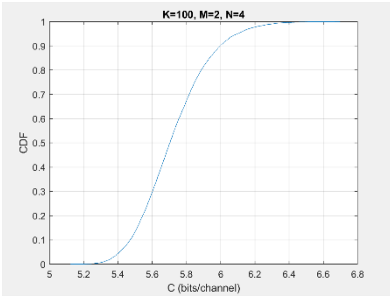
According to the plots, with CSIT, the optimal breakdowns of M and N, which maximize average throughput, are: M=2, N=4 and M=N=3. More precisely, the optimal breakdown is M=2 and N=4.

- **<u>Maximize throughput at a high reliability level when there is strong LOS and no CSIT</u>**

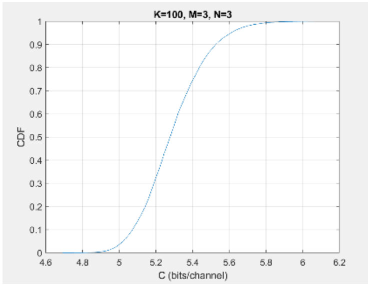
For the strong LOS, we will take K = 100. Now we compute all the five possible cases, when there is no CSIT.



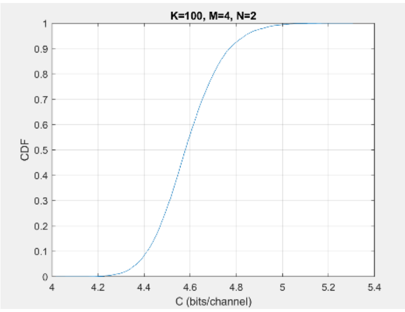
$C_{0.1} = 5.52$



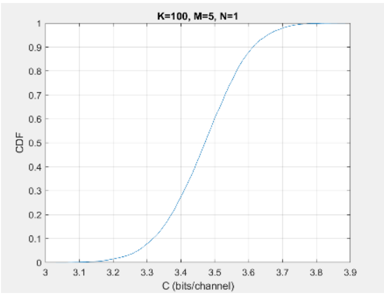
$C_{0.1} = 5.47$



$C_{0.1} = 5.01$



$C_{0.1} = 4.41$



$C_{0.1} = 3.32$

According to the plots, when there is no CSIT and a strong LOS, the optimal breakdown for M and N is: M = 1, N=5.

Annex

This following code has been published with MATLAB® R2019b.

```
% NO CSIT

% VARIABLES
M = 1;
N = 1;
K = 10;
H_los = ones(N,M);
I_N = eye(N);
I_M = eye(M);
P = 10;
Q = (P/M)*I_M;

L = []; % List containing the values of capacity
L_inf = []; % List containing the values of capacity when K is infinite

for k = 1:1:10000 % Computing the Monte Carlo simulation (10.000 iterations)

    % GENERATING A CHANNEL
    H_r = randn(N,M) + i*randn(N,M);
    H = sqrt(K/(K+1))*H_los + sqrt(1/(K+1))*H_r;
    H_H = ctranspose(H);
    % Sometimes, we need this code line to correctly compute the matrix product
    %C = log2(det(I_N+H.*Q.*H_H));
    C = log2(det(I_N+mtimes(H,mtimes(Q,H_H))));
    % We compute the capacity for K = infinite, H = H_los
    C_inf = log2(det(I_N+mtimes(H_los,mtimes(Q,ctranspose(H_los)))));
    L = [L C];
    L_inf = [L_inf C_inf];
end


cdfplot(L);
hold on;
cdfplot(L_inf);
title('SIMO fading case');
legend('Rayleigh fading channel', 'AWGN');
xlabel('C (bits/channel)');
ylabel('CDF');
```

```
% WITH CSIT: COMPUTING THE WATER FILLING ALGORITHM

% VARIABLES
M = 3;
N = 3;
K = 10;
H_los = ones(N,M);
I_N = eye(N);
I_M = eye(M);
P = 10;
Q = P/M*I_M;

L = []; % List containing the values of capacity with no CSIT
L_CSIT = []; % List containing the values of capacity with CSIT

for k = 1:1:10000 % Computing the Monte Carlo simulation (10.000 iterations)

    % GENERATING A CHANNEL
    H_r = randn(N,M) + i*randn(N,M);
    H = sqrt(K/(K+1))*H_los + sqrt(1/(K+1))*H_r;
    H_H = ctranspose(H);

    % WITH NO CSIT
    C = log2(det(I_N+mtimes(H,mtimes(Q,H_H))));
    % Sometimes, we need this code line to correctly compute the matrix product
    %C = log2(det(I_N+H.*Q.*H_H));
    L = [L C];

    % WITH CSIT
    epsilon = 0.01;
    Q_CSIT = zeros(M,M);
    u=0;
    A = zeros(M,M);
    [U,S,V]=svd(H);
    while trace(Q_CSIT)<P-epsilon % WATER FILLING ALGORITHM
        A(1,1)=max(0,u-1/S(1,1)^2);
        A(2,2)=max(0,u-1/S(2,2)^2);
        A(3,3)=max(0,u-1/S(2,2)^2);
        Q_CSIT=mtimes(V,mtimes(A,ctranspose(V)));
        u=u+0.01;
    end
    C_CSIT = log2(det(I_N+mtimes(H,mtimes(Q_CSIT,H_H))));
    %C_CSIT = log2(det(I_N+H.*Q_CSIT.*H_H);
    L_CSIT = [L_CSIT C_CSIT];
end

cdfplot(L_CSIT);
hold on;
cdfplot(L_CSIT);
title('MIMO Ricean case');
legend('Without CSIT', 'With CSIT');
xlabel('C (bits/channel)');
ylabel('CDF');
```

• • • END • • •