

Part 1: Design a neural network

\* First of all, let us determine the linear equations separating red points from the others and then green points from the blue ones.

We separate red points from the others:

$$x_2 = -0,625x_1 + 80 \Leftrightarrow 0,625x_1 + x_2 - 80 = 0$$

Now we separate blue from green:

$$x_2 = 1,29x_1 - 18,6 \Leftrightarrow 1,29x_1 - x_2 - 18,6 = 0$$

So we have:

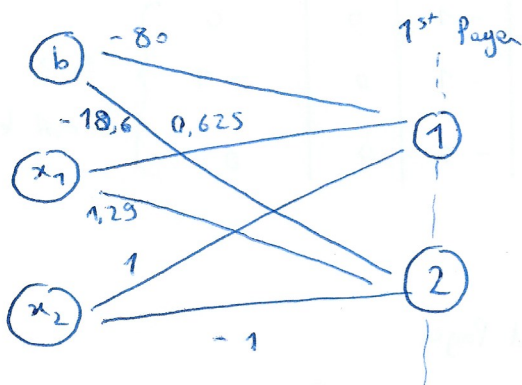
$$E: \begin{cases} 0,625x_1 + x_2 - 80 = 0 \\ 1,29x_1 - x_2 - 18,6 = 0 \end{cases}$$

Considering the transfer function  $f(t) = (t > 0)$ , we have:

$$f(0,625x_1 + x_2 - 80) = \begin{cases} 0 & \text{if red} \\ 1 & \text{if blue or green} \end{cases}$$

$$f(1,29x_1 - x_2 - 18,6) = \begin{cases} 0 & \text{if blue} \\ 1 & \text{if green} \end{cases}$$

We can build the first layer:



\* Now let's build the output layer. Recap what we have:

Value of neuron ①	Value of neuron ②	Color
1	1	green → neuron 2 in output
1	0	blue → neuron 3 in output
0	1	red
0	0	red → neuron 1 in output

Now we build the output neurons. Let's start by neuron ① for red color.

1st hidden layer									
	Neuron 1	Neuron 2	output before bias and weights	w11	w21	b1	output		
not red	1	0	1	-1	0	0,5	-0,5	} $\leq 0$	↳ not red
	1	1	2	-1	0	0,5	-0,5		
red	0	0	0	-1	0	0,5	0,5	} $> 0$	↳ red
	0	1	1	-1	0	0,5	0,5		

$\Rightarrow w_{11} = -1, w_{21} = 0, b_1 = 0,5$

Now neuron ② for green color.

	Neuron 1	Neuron 2	output before w and b	w12	w22	b2	new output		
green →	1	1	2	1	1	-1,5	0,5	} $> 0 \rightarrow$ green	
not green	1	0	1	1	1	-1,5	-0,5		} $\leq 0$
	0	1	1	1	1	-1,5	-0,5		
	0	0	0	1	1	-1,5	-1,5		

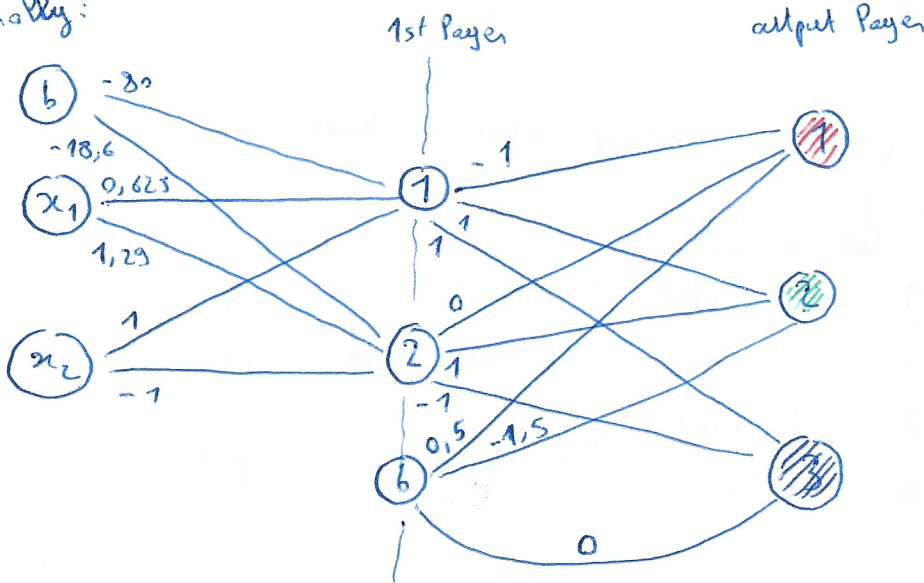
$\Rightarrow w_{12} = 1, w_{22} = 1, b_2 = -1,5$

Now neuron ③ for blue color.

	Neuron 1	Neuron 2	output before w and b	w13	w23	b3	new output		
blue →	1	0	1	1	-1	0	1	} $> 0 \rightarrow$ blue	
not blue	1	1	2	1	-1	0	0		} $\leq 0$
	0	1	1	1	-1	0	-1		
	0	0	0	1	-1	0	0		

$\Rightarrow w_{13} = 1, w_{23} = -1, b_3 = 0$

Finally:



Results

are recapped.  
in Notebook.