Ps5 solutions

Question 1

preliminaries

```
# clear workspace
rm(list=ls())
# set random seed
set.seed(123)
library(mvtnorm)
```

Step 1

```
sigma_squared = 0

TargetDtbn <- function(theta) {
  theta1 = theta[1]
  theta2 = theta[2]
  if (theta1 > 1 || theta1 < 0 || theta2 < 0 || theta2 > 1) {
    return(0)
  }
  cond = -theta1 + 1
  target = 0
  # Bottom
  if (theta2 <= cond && theta2 <= theta1){
    target = 6 * theta2
  }
  # Left
  else if (theta2 <= cond && theta2 >= theta1) {
    target = 6 * theta1
  }
  # Right
  else if (theta2 >= cond && theta2 <= theta1) {
    target = 3 - 6 * (theta1 - 0.5)
  }
  # Top
  else {
    target = 3 - 6 * (theta2 - 0.5)
  }
  return(target)
}

SampleProposalDtbn <- function(theta) {
  covariance_matrix = diag(length(theta)) * sigma_squared
  return(rmvnorm(1, theta, covariance_matrix))
}

DensityProposalDtbn <- function(vec1, vec2) {
  covariance_matrix = diag(length(vec1)) * sigma_squared
  return((dmvnorm(vec1, vec2, covariance_matrix)))
}

MHsampling <- function(TargetDtbn, SampleProposalDtbn, DensityProposalDtbn, n, theta0) {
  sampleM = array(1, c(n, length(theta0)))
  stopifnot(TargetDtbn(theta0) > 0)
  t_prev = theta0
  for (t in 1:n) {
    #Draw candidate
    t_curr = SampleProposalDtbn(t_prev)

    #acceptance ratio
    target_prev = TargetDtbn(t_prev)
    target_curr = TargetDtbn(t_curr)
    dens_prev = DensityProposalDtbn(t_prev, t_curr)
    dens_curr = DensityProposalDtbn(t_curr, t_prev)
    r = ((target_curr / dens_curr) / (target_prev / dens_prev))

    #Set sample with some prob min{r, 1}
    prob = min(r, 1)
    if(sample(c(1, 0), 1, prob = c(prob, 1 - prob)) == 1) {
      sampleM[t, ] = t_curr
      t_prev = t_curr
    } else {
      sampleM[t, ] = t_prev
    }
    if (t %% 25000 == 0) {
      print(paste("Current t: ", t))
    }
  }
  return(sampleM)
}
```

Step 2

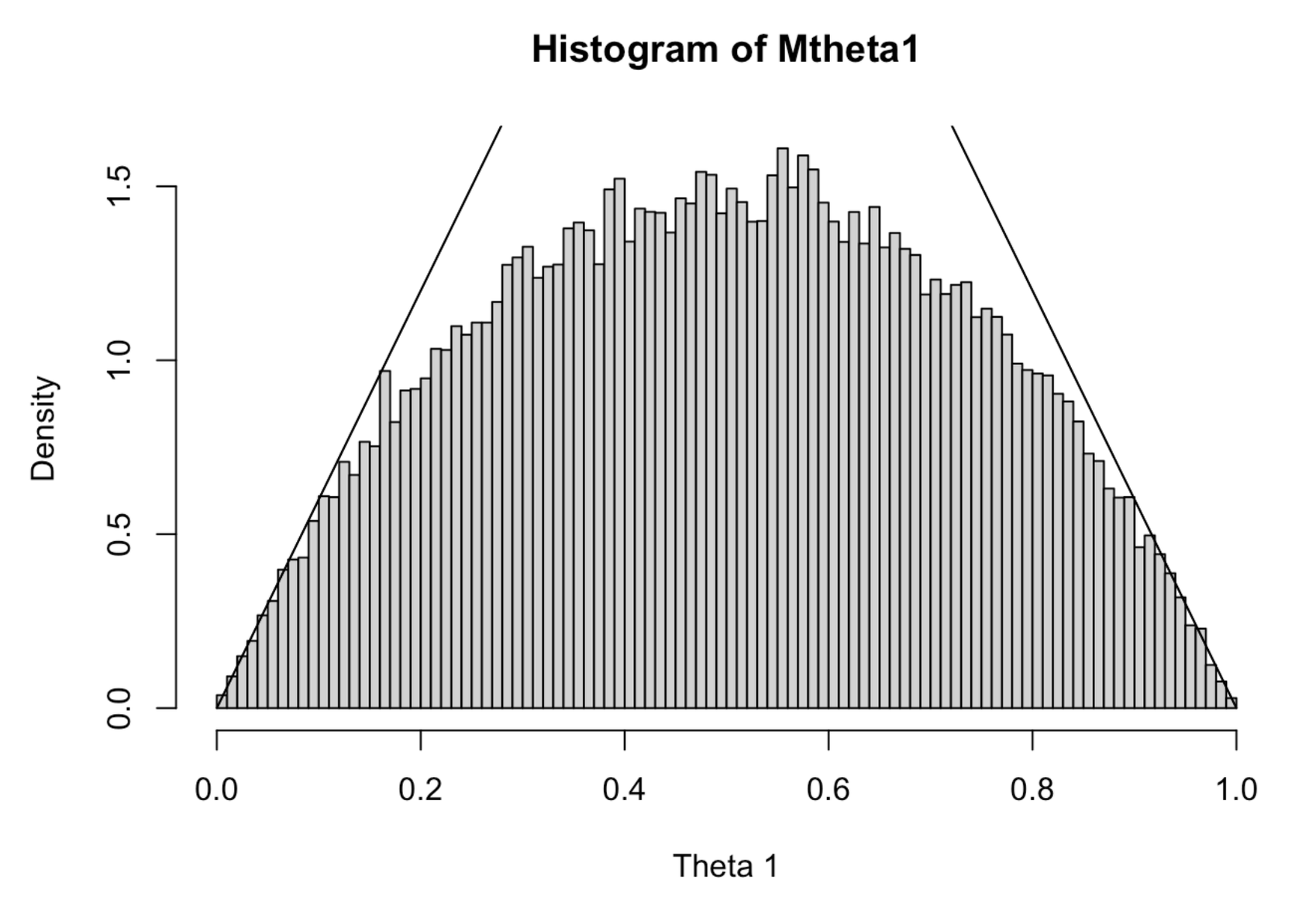
```
n = 250000
sigma_squared = 0.25
MHsamples =MHsampling(TargetDtbn, SampleProposalDtbn, DensityProposalDtbn, n, c(0.5, 0.5))
```

```
## [1] "Current t:  25000"
## [1] "Current t:  50000"
## [1] "Current t:  75000"
## [1] "Current t:  100000"
## [1] "Current t:  125000"
## [1] "Current t:  150000"
## [1] "Current t:  175000"
## [1] "Current t:  200000"
## [1] "Current t:  225000"
## [1] "Current t:  250000"
```

```
plot(main = "Theta 1 vs Theta 2 Plot", MHsamples[, 1], MHsamples[, 2],
     xlab = "Theta 1", ylab = "Theta 2",
     col=rgb(red=0.0, green=0.0, blue=1.0, alpha=0.0075) )
```

Step 3

```
Mtheta1 = MHsamples[, 1]
hist(Mtheta1, probability = TRUE, breaks = 100, xlab = "Theta 1")
theta1grid = seq(0, 1, length = length(Mtheta1))
lines(theta1grid, 3 - 6 * (theta1grid - 0.5))
lines(theta1grid, 6 * theta1grid)
```



Step 4

```
Mtheta2 = MHsamples[, 2]
paste("Mean of θ1: ", round(mean(Mtheta1), 5))
```

```
## [1] "Mean of θ1:  0.50084"
```

```
paste("Mean of θ2: ", round(mean(Mtheta2), 5))
```

```
## [1] "Mean of θ2:  0.50085"
```

```
paste("Variance of θ1: ", round(var(Mtheta1) , 5))
```

```
## [1] "Variance of θ1:  0.05046"
```

```
paste("Variance of θ2: ", round(var(Mtheta2) , 5))
```

```
## [1] "Variance of θ2:  0.0501"
```

```
paste("Correlation between θ1 and θ2: ", cor(Mtheta1, Mtheta2))
```

```
## [1] "Correlation between θ1 and θ2:  0.001545607604991"
```

Question 2

preliminaries

```
data = read.csv("Wages1.csv")
schooling = data$school
exper = data$exper
log_wages = log(data$wage)
```

Step 1

```
TargetDtbn2 <- function(theta) {
  b0  = theta[1]
  bsc = theta[2]
  bec = theta[3]
  sig = theta[4]
  logLike = sum(log(dnorm(log_wages - (b0 + (bsc * schooling) + (bec * exper)), mean = 0, sd = sig)))
  prior = 1 / (sig ^ 2)
  posterior = prior * logLike
  return(posterior)
}

SampleProposalDtbn2 <- function(theta) {
  b0  = theta[1]
  bsc = theta[2]
  bec = theta[3]
  sig = theta[4]

  sigma = rnorm(1, sig, sigma_squared)

  #sd > 0 by definition
  while (sigma < 0) {
    sigma = rnorm(1, sig, sigma_squared)
  }
  candidate = c(rnorm(1, b0, sigma_squared),
                rnorm(1, bsc, sigma_squared),
                rnorm(1, bec, sigma_squared),
                sigma)
  return(candidate)
}

DensityProposalDtbn2 <- function(vec1, vec2) {
  covariance_matrix = diag(length(vec1)) * sigma_squared
  return(dmvnorm(vec1, vec2, covariance_matrix))
}

MHsampling2 <- function(TargetDtbn2, SampleProposalDtbn2, DensityProposalDtbn2, n, theta0) {
  sampleM = array(1, c(n, length(theta0)))

  t_prev = theta0
  for (t in 1:n) {
    #Draw candidate
    t_curr = SampleProposalDtbn2(t_prev)

    #acceptance ratio
    target_prev = TargetDtbn2(t_prev)
    target_curr = TargetDtbn2(t_curr)
    dens_prev = DensityProposalDtbn2(t_prev, t_curr)
    dens_curr = DensityProposalDtbn2(t_curr, t_prev)
    r = ((target_curr / dens_curr) / (target_prev / dens_prev))

    #Set sample with some prob min{r, 1}
    if (r == Inf) {prob = 0} else {prob = min(r, 1)}
    if(sample(c(1, 0), 1, prob = c(prob, 1 - prob)) == 1) {
      sampleM[t, ] = t_curr
      t_prev = t_curr
    } else {
      sampleM[t, ] = t_prev
    }
    if (t %% 100000 == 0) {
      print(paste("Current t: ", t))
    }
  }
  return(sampleM)
}
```

Step 2

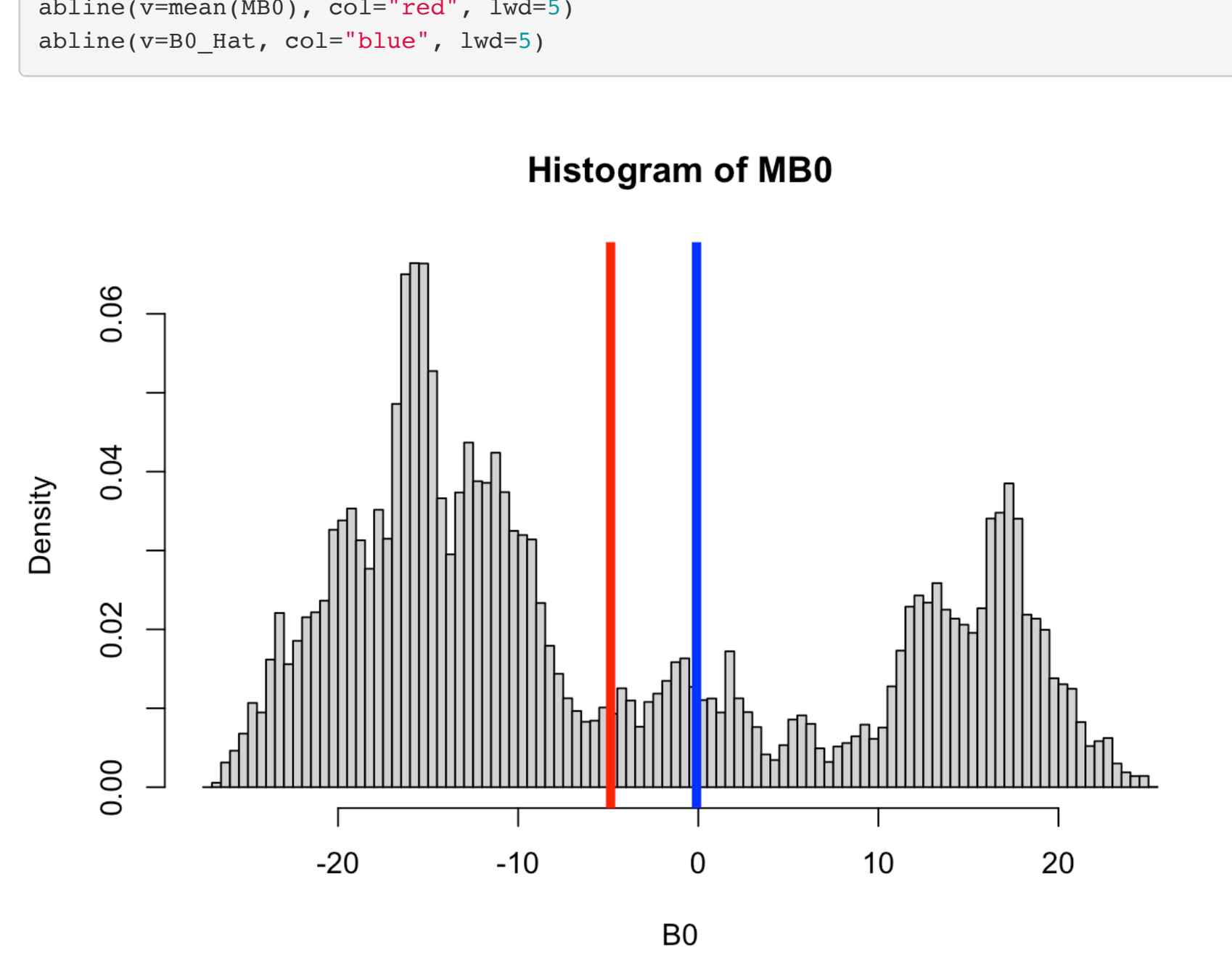
```
n = 510000
sigma_squared = 0.05
MHsamples2 = MHsampling2(TargetDtbn2, SampleProposalDtbn2,
                         DensityProposalDtbn2, n, c(0.5, 0.5, 0.5, 0.5))
```

```
## [1] "Current t:  100000"
## [1] "Current t:  200000"
## [1] "Current t:  300000"
## [1] "Current t:  400000"
## [1] "Current t:  500000"
```

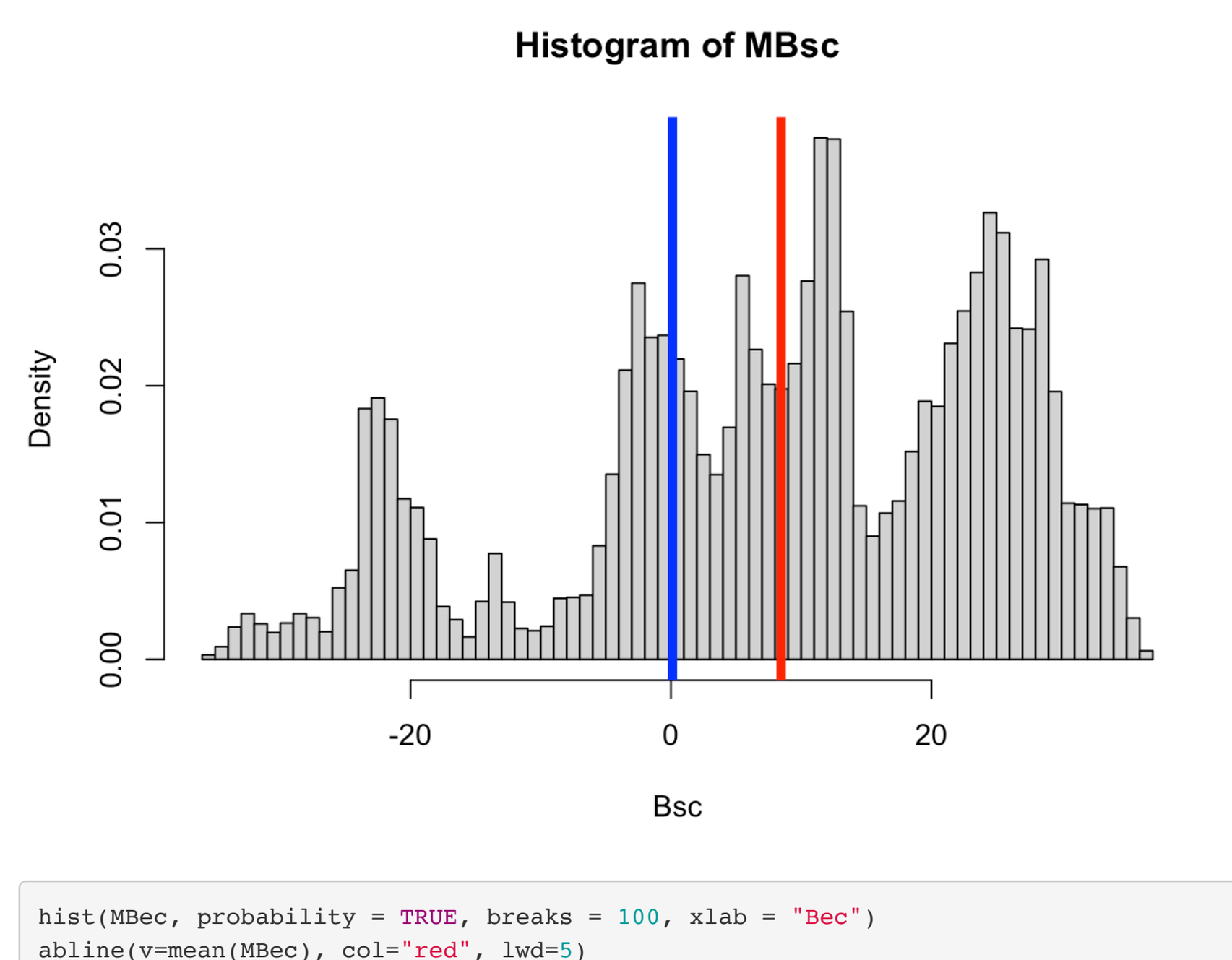
```
MB0 = MHsamples2[10001:n, 1]
MBsc = MHsamples2[10001:n, 2]
MBec = MHsamples2[10001:n, 3]
MSig = MHsamples2[10001:n, 4]

model = lm(log_wages ~ schooling + exper)
ole = coef(model)
B0_Hat = ole[1]
Bsc_Hat = ole[2]
Bec_Hat = ole[3]
```

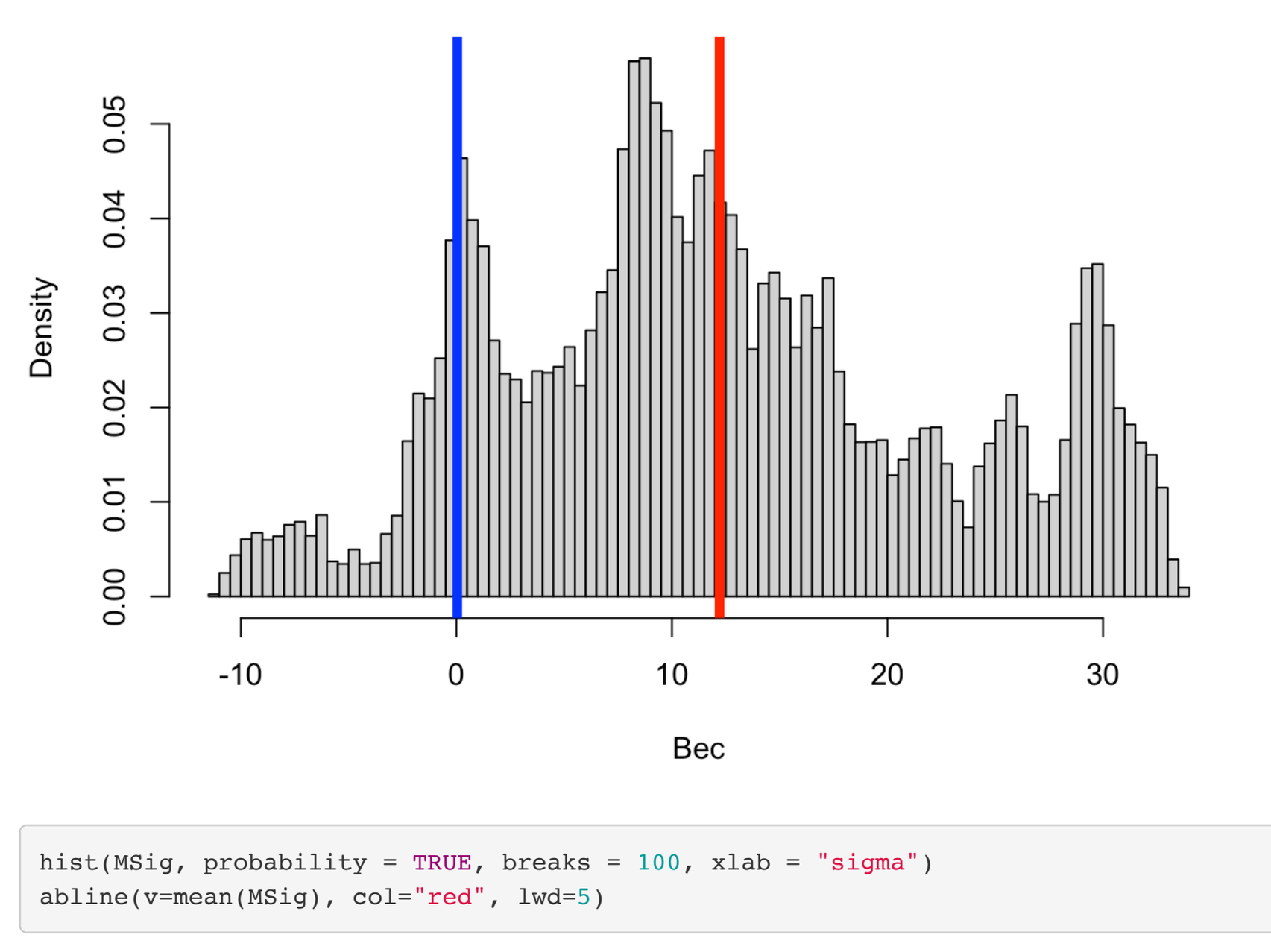
```
hist(MB0, probability = TRUE, breaks = 100, xlab = "B0")
abline(v=mean(MB0), col="red", lwd=5)
abline(v=B0_Hat, col="blue", lwd=5)
```



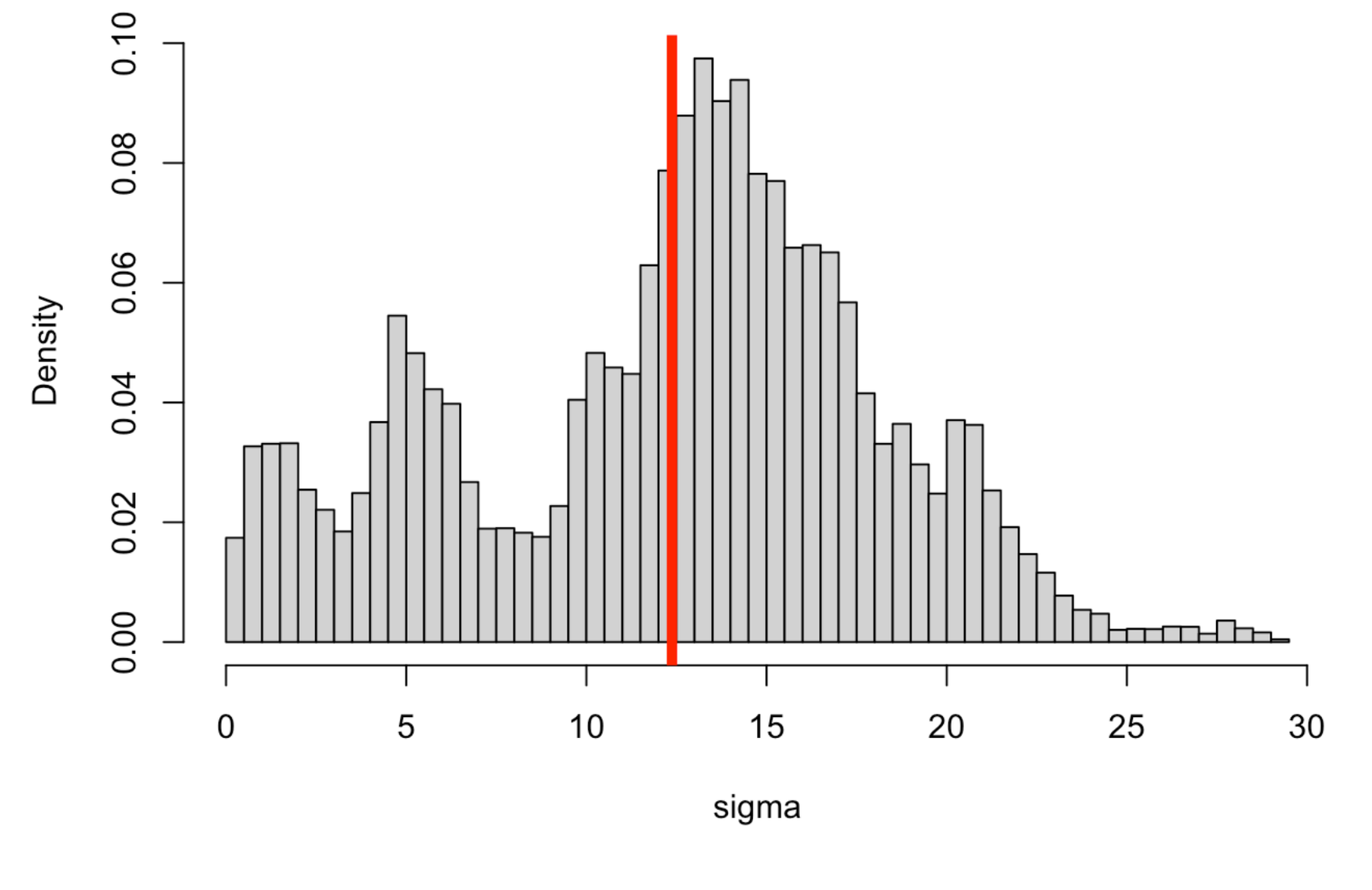

```
hist(MBsc, probability = TRUE, breaks = 100, xlab = "Bsc")
abline(v=mean(MBsc), col="red", lwd=5)
abline(v=Bsc_Hat, col="blue", lwd=5)
```




```
hist(MBec, probability = TRUE, breaks = 100, xlab = "Bec")
abline(v=mean(MBec), col="red", lwd=5)
abline(v=Bec_Hat, col="blue", lwd=5)
```




```
hist(MSig, probability = TRUE, breaks = 100, xlab = "sigma")
abline(v=mean(MSig), col="red", lwd=5)
```



Step 3

```
paste("P(β0 |data): ", round(sd(MB0), 5))
```

```
## [1] "P(β0 |data):  14.21684"
```

```
paste("P(βSC |data: ", round(sd(MBsc), 5))
```

```
## [1] "P(βSC |data:  16.57395"
```

```
paste(" P(βEC |data): ", round(sd(MBec), 5))
```

```
## [1] " P(βEC |data):  10.24665"
```

```
paste("Posterior correlation between βSC andβEC: ", round(cor(MBsc, MBec)))
```

```
## [1] "Posterior correlation between βSC andβEC:  -1"
```