Collab: (See Code)

1) EC
2) Pics

a.

2A $P(x|y) = \theta_{x1c} \cdot \theta_{x2c} \cdots \theta_{xDc}$

Store 2^{j-1} for θ_{xjc}

$\theta_{xDc} = 2^0 + \dots + 2^{D-1} = 2^D - 1$ Params

c classes $\rightarrow (2^D - 1)c = O(c \cdot 2^D)$

2B) $O(c \cdot 2^D)$

$P(x|y)$ has D features and c class

feature $\in \{0,1\} \rightarrow 2^D \Rightarrow c \cdot 2^D$

2c Naive Bayes would work better. Naive Bayes assumes independence, and often generalizes better, when there is a small amount of data.
The full model is more likely to overfit

2d. In this case the full model is likely to perform better, assuming there is some dependency among the features. The larger dataset enables the full model to properly learn dependencies. The Naive Model never learns them since it assumes independence

2E Full $P(y|x) = \dfrac{P(y) \cdot P(x|y)}{\sum P(x|y=c)P(y=c)} \begin{array}{l} \rightarrow O(D) \cdot O(1) \\ \rightarrow O(C) \cdot O(D) \end{array}$

$$\begin{array}{cc} & C \text{ columns} \\ \begin{array}{c} 2D \\ \text{rows} \end{array} & \begin{pmatrix} \cdots & \cdots & \cdots \\ \vdots & & \vdots \\ \cdots & \cdots & \cdots \end{pmatrix} = M \end{array}$$

$O(C \cdot D)$ for full

Naive $P(y|x) = \dfrac{\overset{O(1)}{P(y)} \cdot P(x|y)}{\sum P(x|y=c)P(y=c)} = \dfrac{P(y) \cdot \overset{O(D)}{\prod_i^D (x_i|y)}}{\sum P(x|y=c)P(y=c)} \rightarrow O(C) \cdot O(C$

Naive is $O(C \cdot D)$ as well

b.

3) M

a.

```
File #0:
Emission Sequence             Max Probability State Sequence
######################################################################
25421                         31033
01232367534                   22222100310
5452674261527433              1031003103222222
7226213164512267255           1310331000033100310
0247120602352051010255241     2222222222222222222222103

File #1:
Emission Sequence             Max Probability State Sequence
######################################################################
77550                         22222
7224523677                    2222221000
505767442426747               222100003310031
72134131645536112267          10310310000310333100
4733667771450051060253041     2221000003222223103222223

File #2:
Emission Sequence             Max Probability State Sequence
######################################################################
60622                         11111
4687981156                    2100202111
815833657775062               021011111111111
2131022251596350501 5         0202011111111111021
6503199452571274006320025     1110202111111102021110211

File #3:
Emission Sequence             Max Probability State Sequence
######################################################################
13661                         00021
2102213421                    3131310213
166066262165133               133333133133100
5316466211216263415 6         2000002131313100213 3
1523541005123230226306256     1310021333133133313133133

File #4:
Emission Sequence             Max Probability State Sequence
######################################################################
23664                         01124
3630535602                    0111201112
350201162150142               011244012441112
0021400540201514636 2         1120111241244401111 2
2111266524665143562534450     2012012424124011112411124

File #5:
Emission Sequence             Max Probability State Sequence
######################################################################
68535                         10111
4546566636                    1111111111
638436858181213               110111010000011
1324033830844451468 8         0001000000011111110 0
0111664434441382533632626     2111111111111100111110101
```

```
File #0:
Emission Sequence             Probability of Emitting Sequence
######################################################################
25421                         4.537e-05
01232367534                   1.620e-11
5452674261527433              4.348e-15
7226213164512267255           4.739e-18
0247120602352051010255241     9.365e-24

File #1:
Emission Sequence             Probability of Emitting Sequence
######################################################################
77550                         1.181e-04
7224523677                    2.033e-09
505767442426747               2.477e-13
72134131645536112267          8.871e-20
4733667771450051060253041     3.740e-24

File #2:
Emission Sequence             Probability of Emitting Sequence
######################################################################
60622                         2.088e-05
4687981156                    5.181e-11
815833657775062               3.315e-15
21310222515963505015          5.126e-20
6503199452571274006320025     1.297e-25

File #3:
Emission Sequence             Probability of Emitting Sequence
######################################################################
13661                         1.732e-04
2102213421                    8.285e-09
166066262165133               1.642e-12
53164662112162634156          1.063e-16
1523541005123230226306256     4.535e-22

File #4:
Emission Sequence             Probability of Emitting Sequence
######################################################################
23664                         1.141e-04
3630535602                    4.326e-09
350201162150142               9.793e-14
00214005402015146362          4.740e-18
2111266524665143562534450     5.618e-22

File #5:
Emission Sequence             Probability of Emitting Sequence
######################################################################
68535                         1.322e-05
4546566636                    2.867e-09
638436858181213               4.323e-14
13240338308444514688          4.629e-18
0111664434441382533632626     1.440e-22
```

b.

```
File #0:
Emission Sequence             Probability of Emitting Sequence
######################################################################
25421                         4.537e-05
01232367534                   1.620e-11
5452674261527433              4.348e-15
72262131645122672 55          4.739e-18
0247120602352051010255241     9.365e-24

File #1:
Emission Sequence             Probability of Emitting Sequence
######################################################################
77550                         1.181e-04
7224523677                    2.033e-09
505767442426747               2.477e-13
72134131645536112267          8.871e-20
4733667771450051060253041     3.740e-24

File #2:
Emission Sequence             Probability of Emitting Sequence
######################################################################
60622                         2.088e-05
4687981156                    5.181e-11
815833657775062               3.315e-15
21310222515963505015          5.126e-20
6503199452571274006320025     1.297e-25

File #3:
Emission Sequence             Probability of Emitting Sequence
######################################################################
13661                         1.732e-04
2102213421                    8.285e-09
166066262165133               1.642e-12
53164662112162634156          1.063e-16
1523541005123230226306256     4.535e-22

File #4:
Emission Sequence             Probability of Emitting Sequence
######################################################################
23664                         1.141e-04
3630535602                    4.326e-09
350201162150142               9.793e-14
00214005402015146362          4.740e-18
2111266524665143562534450     5.618e-22

File #5:
Emission Sequence             Probability of Emitting Sequence
######################################################################
68535                         1.322e-05
4546566636                    2.867e-09
638436858181213               4.323e-14
13240338308444514688          4.629e-18
0111664434441382533632626     1.440e-22
```

```
Transition Matrix:
######################################################################
2.833e-01   4.714e-01   1.310e-01   1.143e-01
2.321e-01   3.810e-01   2.940e-01   9.284e-02
1.040e-01   9.760e-02   3.696e-01   4.288e-01
1.883e-01   9.903e-02   3.052e-01   4.075e-01


Observation Matrix:
######################################################################
1.486e-01   2.288e-01   1.533e-01   1.179e-01   4.717e-02   5.189e-02   2.830e-02   1.297e-01   9.198e-02   2.358e-03
1.062e-01   9.653e-03   1.931e-02   3.089e-02   1.699e-01   4.633e-02   1.409e-01   2.394e-01   1.371e-01   1.004e-01
1.194e-01   4.299e-02   6.529e-02   9.076e-02   1.768e-01   2.022e-01   4.618e-02   5.096e-02   7.803e-02   1.274e-01
1.694e-01   3.871e-02   1.468e-01   1.823e-01   4.839e-02   6.290e-02   9.032e-02   2.581e-02   2.161e-01   1.935e-02
```

c.

```
Transition Matrix:
######################################################################
4.345e-01   1.559e-01   9.612e-02   3.134e-01
5.904e-15   2.996e-01   7.004e-01   1.328e-11
5.219e-01   3.828e-18   2.259e-01   2.522e-01
5.016e-03   3.679e-01   7.971e-03   6.191e-01


Observation Matrix:
######################################################################
2.235e-01   8.353e-09   7.603e-02   9.263e-02   2.315e-02   1.023e-02   1.273e-01   3.294e-01   1.288e-10   1.177e-01
2.260e-01   3.339e-05   7.427e-14   2.039e-01   3.226e-02   1.151e-01   1.575e-01   3.954e-30   8.256e-02   1.828e-01
3.403e-02   1.351e-01   1.900e-01   5.006e-02   2.247e-01   2.150e-05   6.462e-02   2.926e-02   2.723e-01   3.988e-65
8.931e-02   1.173e-01   1.035e-01   9.344e-02   1.505e-01   2.098e-01   9.588e-07   7.679e-02   1.594e-01   1.312e-07
```

d.

```
Transition Matrix:
######################################################################
1.887e-01   3.750e-01   5.354e-02   3.828e-01
1.516e-01   2.204e-01   4.083e-01   2.196e-01
2.832e-01   1.556e-02   3.924e-01   3.089e-01
1.647e-01   4.254e-01   1.559e-01   2.540e-01


Observation Matrix:
######################################################################
6.272e-02   5.705e-02   8.345e-02   5.738e-02   1.380e-01   5.191e-02   8.730e-02   1.572e-01   2.127e-01   9.231e-02
2.152e-01   3.537e-02   5.388e-02   1.786e-01   8.065e-02   1.888e-02   8.992e-02   1.053e-01   1.168e-01   1.054e-01
1.988e-02   8.040e-02   1.846e-01   1.378e-02   1.243e-01   1.626e-01   1.081e-01   4.324e-02   1.937e-01   6.944e-02
2.222e-01   1.001e-01   5.614e-02   1.652e-01   1.169e-01   1.376e-01   3.193e-02   1.205e-01   3.943e-02   9.930e-03
```

e. The transition and emission matrices in 2C and 2D have significantly different values because in 2C, the smallest value is of order 10^-3, whereas in 2D, the observation matrix contains values much closer to 0, with the largest being 10^-65. Moreover, the transition matrix in 2D contains extremely small values such as 10^-18 suggesting highly unlikely mood transitions. These differences suggest that the supervised learning model provides a more realistic and accurate representation of Ron's moods and how they influence his music choices as the sparseness in the 2D matrices hint at unrealistic mood changes. We improve the unsupervised model by adding more training points to provide more information and better model examine the relationships in the dataset hence avoiding underfitting which might be the issue here therefore avoiding low probability emissions and transitions.

f. Sequence Generation

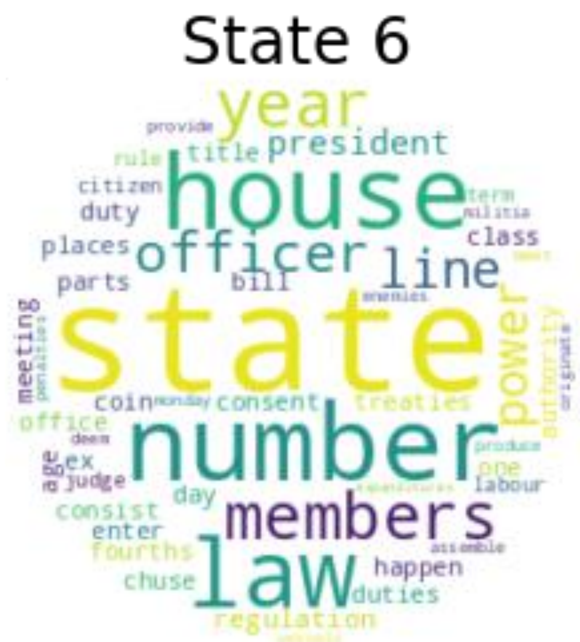
```
File #0:
Generated Emission
######################################################################
62750745470675742541
74714654044574040224
22766741574276554477
34205504451554545425
77464574027665260545

File #1:
Generated Emission
######################################################################
41777564104657450444
01077667425254150771
74762240076424574700
25564747712040112151
24510272500145276720

File #2:
Generated Emission
######################################################################
80506005230359674441
02359433052920645517
08776355331420701633
45776626557522429932
75070123556535321722

File #3:
Generated Emission
######################################################################
26316611420315540641
61006161162312621641
10562315501112641045
61236611265126061126
44601466135422611164

File #4:
Generated Emission
######################################################################
30006165146606152663
04526616114452234254
42662024632014341531
00112323164606066021
06266641256014041164

File #5:
Generated Emission
######################################################################
65036182654343085343
65026805468884463411
81163126484446368428
06845328841844340452
53334638618630816881
```

g. Both the O and A matrix are very sparse. This means that many states do not transition to each other and that it may be possible to get stuck in a loop. This makes sense since noun to noun transitions should likely be rare. Additionally the sparsity of the O matrix seems to suggest that only some states and behaviors are linked very closely.

h. As the number of hidden states increase, the sentences get closer to actual English sentence structure. With one hidden state, we get a random scramble of incoherent words, however, with other states there exists a pattern of word placement and selection. Therefore, allowing more hidden states is similar to increasing model complexity hence reducing underfitting in our training set by allowing more underlying patterns to be explored.

i.

```
[25] hmm1 = unsupervised_HMM(obs, 1, 100, seed=1)
     print('\nSample Sentence:\n====================')
     print(sample_sentence(hmm1, obs_map, n_words=25))

     Sample Sentence:
     ====================
     Vice peace the congress no invasions the of article resignation its several shall greatest ambassadors to constitution of in manner to amendments excises against than...

[20]
     hmm2 = unsupervised_HMM(obs, 2, 100, seed=1)
     print('\nSample Sentence:\n====================')
     print(sample_sentence(hmm2, obs_map, n_words=25))

     Sample Sentence:
     ====================
     Any in which duty under when of a courts greatest of or the state prejudice of constitution thereof number each without until states united marque...

[21] hmm4 = unsupervised_HMM(obs, 4, 100, seed=1)
     print('\nSample Sentence:\n====================')
     print(sample_sentence(hmm4, obs_map, n_words=25))

     Sample Sentence:
     ====================
     Without shall office shall union and shall state states from with to on by the have prescribed power fill law letters foreign same forfeiture two...

[22] hmm16 = unsupervised_HMM(obs, 16, 100, seed=1)
     print('\nSample Sentence:\n====================')
     print(sample_sentence(hmm16, obs_map, n_words=25))

     Sample Sentence:
     ====================
     Parts of adjournment and the age of thousand president of the president of a judicial congress shall be power word any states according the majority...
```



j.