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SCUOLA DI INGEGNERIA INDUSTRIALE
E DELL'INFORMAZIONE

Numbers, errors and computers.

Calcoli di Processo dell' Ingegneria Chimica

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Numbers representation.

*“In computing, **floating-point arithmetic (FP)** is arithmetic that represents subsets of real numbers using an integer with a fixed precision, called the significand, scaled by an integer exponent of a fixed base. Numbers of this form are called floating-point numbers. For example, 12.345 is a floating-point number in base ten with five digits of precision.”* [Wikipedia](#).

$$12.345 = \boxed{12345} \times \boxed{10^{-3}}$$

Mantissa

Exponent

Additional RECOMMENDED read.

Single or Double precision?

In any computer, the difference between single and double precision reflects on the number of bits used to represent a real number.

- ▶ Single precision floating point number 32 bits (4 bytes). Safe use with $\sim 7/8$ decimal digits.
- ▶ Double precision floating point number 64 bits (8 bytes). Safe use with $\sim 15/16$ decimal digits.

Elementary operations

Initialize a single precision variable in MATLAB (e.g. $x = 1e^{+25}$) using the function:
 $x = \text{single}(1.e+25)$.

Working in single precision predict and calculate from $x = 1.e+25$ and $y = 1.e+18$ the following values of z :

	Single Precision	Double Precision
$z = x * y$	inf	1.0000e+43
$z = x/y$	10000000	10000000
$z = y/x$	1.0000e-07	1.0000e-07
$z = x^2$	inf	1.0000e+50
$z = y^2$	1.0000e+36	1.0000e+36
$z = 1./(x * y)$	0	1.0000e-43
$z = 1./x/y$	9.9492e-44	1.0000e-43
$z = y + 1e10$	1.0000e+18	1.0000e+18
$z = x * y/(x * y + 1)$	NaN	1

Exercises

Compute the MACHEPS

Epsilon, or machine epsilon, is an important number in computing. Machine epsilon gives the distance between a number and the next largest floating point number on your computer. This is important to calculate, as the size of the floating point number may lead to round-off errors for certain calculations. Calculating machine epsilon can be done a number of ways, and many programming languages have built-in functions that can determine this value. However, it also can be determined algorithmically with a fairly simple routine. Write a function to determine the macheps of a generic number. Plot the results for numbers from ranging from 0 to 10.

- The strategy here is to iterate as long as the difference between n and $n + \text{epsilon}/2 > 0$ by halving at each iteration the value of the machine epsilon until convergence.

Sum of the inverse of numbers

Write a script which calculates:

$$\sum_{n=1}^{1000000} \frac{1}{n}$$

in **single** and **double** precision. Then compare with the results obtained inverting the order of the sum, so by computing:

$$\sum_{n=1000000}^1 \frac{1}{n}$$

The babylonian method

Write a function that implements the Babylonian method to compute the square root of a number with a precision of four decimal figures.

1. **Make an Initial guess.** Guess any positive number x_0 .
2. **Improve the first guess.** Apply the formula $x_1 = \frac{x_0 + \frac{S}{x_0}}{2}$. The number x_1 is a better approximation to \sqrt{S} .
3. **Iterate until convergence.** Apply the formula $x_{n+1} = \frac{x_n + \frac{S}{x_n}}{2}$ until the convergence is reached.

Convergence is reached when the digits of x_{n+1} and x_n agree to as many decimal places as you desire.

Vancouver: a nickel at a time

Analogously to what happened on the Vancouver stock market ([reference](#)), starting from a stock value of 1000, check what happens when a random variation of $\pm 1\%$ in its value is iterated for 10000 times. Try to round or approximate (truncate) to the lower or upper **second** decimal figure. Compare the results with the number obtained using the computer precision.

Please verify on the help page in Matlab how the functions *rand*, *floor*, *ceil*, *fix*, *round* work.

```
>> a = 1.2345
>> ceil(a * 100)/100 → ans = 1.2400
>> floor(a * 100)/100 → ans =
1.2300
>> round(a * 100)/100 → ans =
1.2300
```

```
>> a = -1.2345
>> ceil(a * 100)/100 → ans =
-1.2300
>> floor(a * 100)/100 → ans =
-1.2400
>> round(a * 100)/100 → ans =
-1.2300
```

Thank you for the attention!