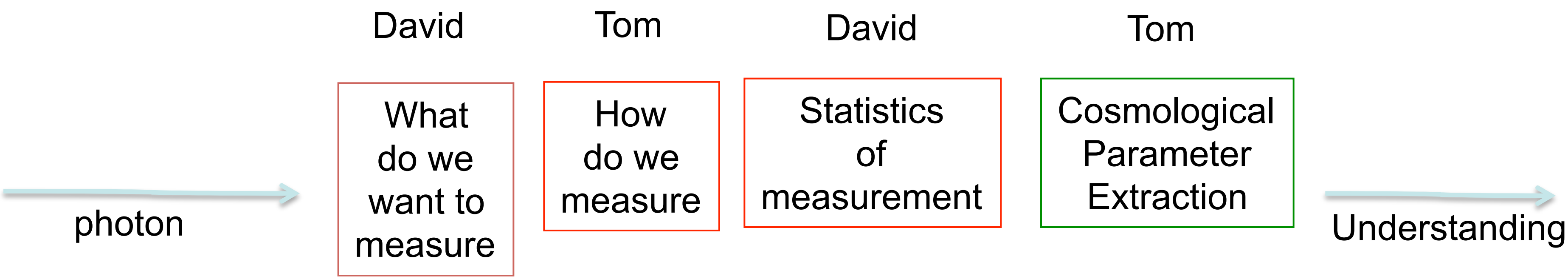
Statistics and Prediction

Tom Kitching
tdk@roe.ac.uk
@tom_kitching


David
Tom
David
Tom
photon
What do we want to measure
How do we measure
Statistics of measurement
Cosmological Parameter Extraction
Understanding

Recap

$$
\begin{aligned}
\beta &= \theta - \hat{\alpha}\frac{D_{ds}}{D_s} \\
\beta &= \theta - \alpha.
\end{aligned}
$$

- Lensing equation
- Local mapping
  - General Relativity relates this to the gravitational potential
  - Distortion matrix implies that distortion is elliptical : shear and convergence
  - Simple formalise that relates the shear and convergence (observable) to the underlying gravitational potential

Recap

- Observed galaxies have instrinsic ellipticity and shear
- Reviewed shape measurement methods
  - Moments - KSB
  - Model fitting - lensfit
- Still an unsolved problem for largest most ambitious surveys
- Simulations
  - STEP 1, 2
  - GREAT08, GREAT10

Recap

- Mass mapping
- Correlation functions
- Power Spectra

- What will we learn?

- Predictive parameter and model estimation
  - Fisher Matrix
  - Evidence

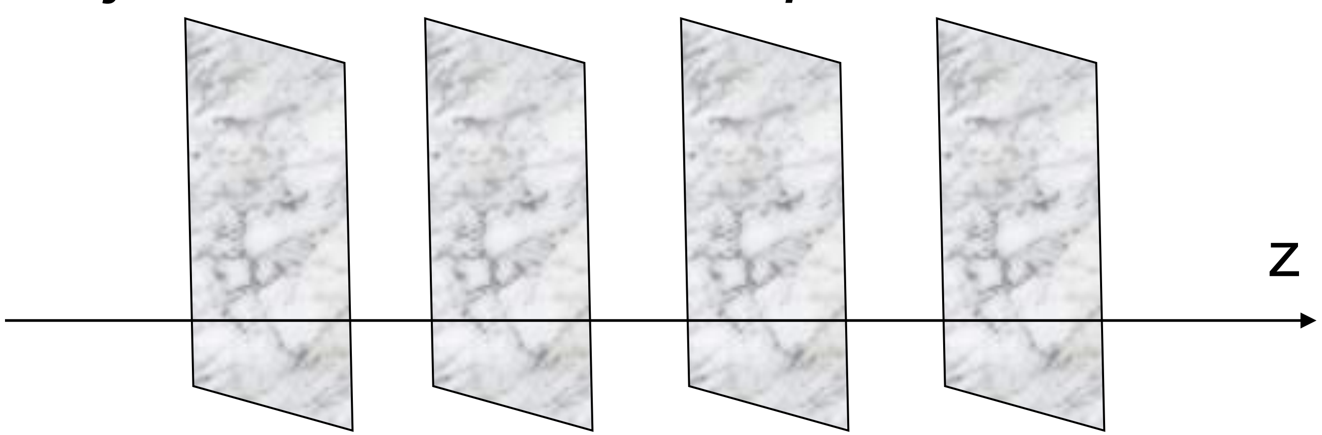
- Real Life parameter and model selection
  - MCMC

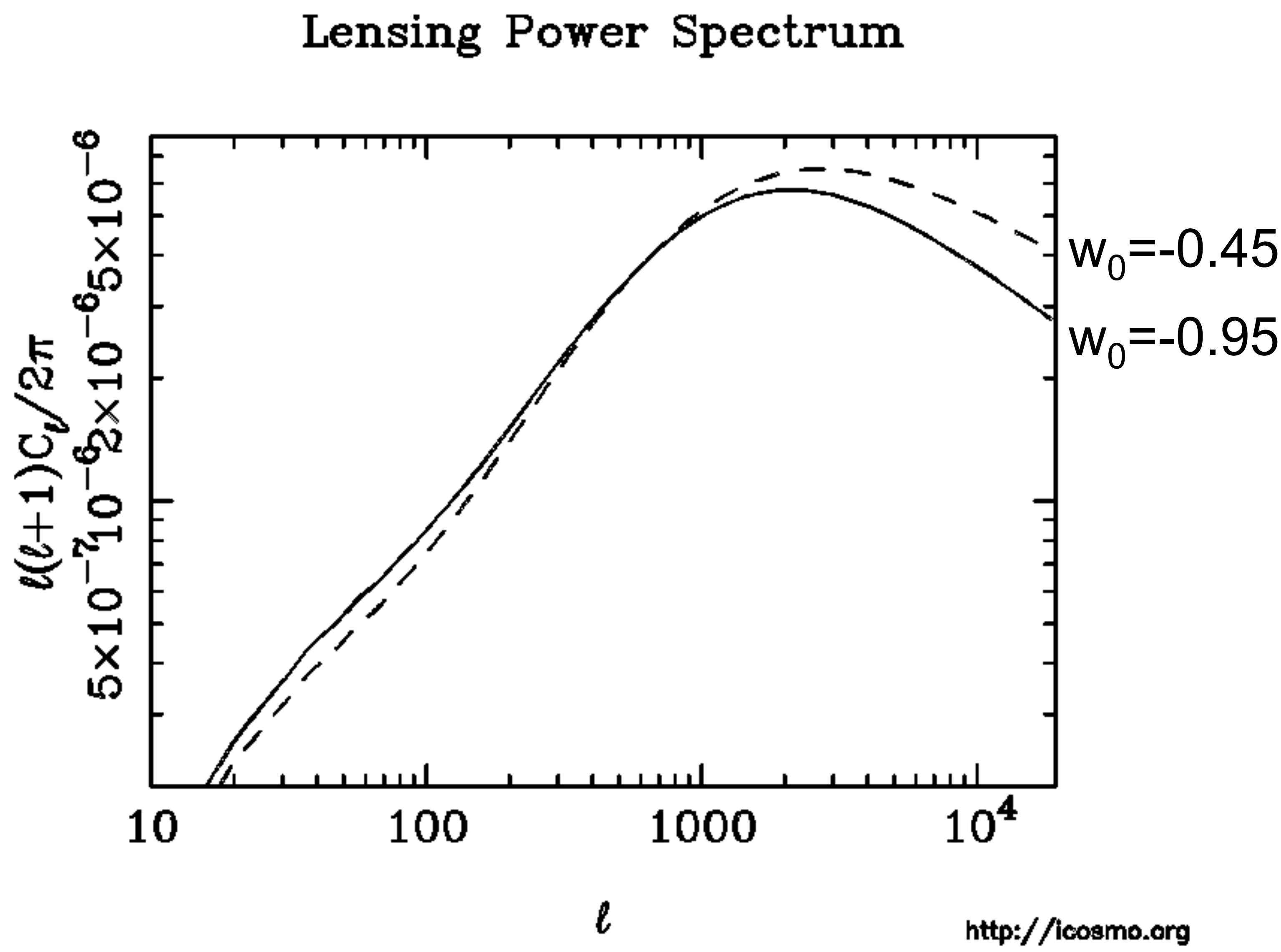
$$C_{ij}(\ell) = \int_0^{r_H} \mathrm{d}r\, W_{ij}^{\mathrm{GG}}(r) P_{\delta\delta}\left(\frac{\ell}{S_k(r)}; r\right)$$

$$W_{ij}^{\mathrm{GG}}(r) = \frac{q_i(r) q_j(r)}{S_k^2(r)}$$

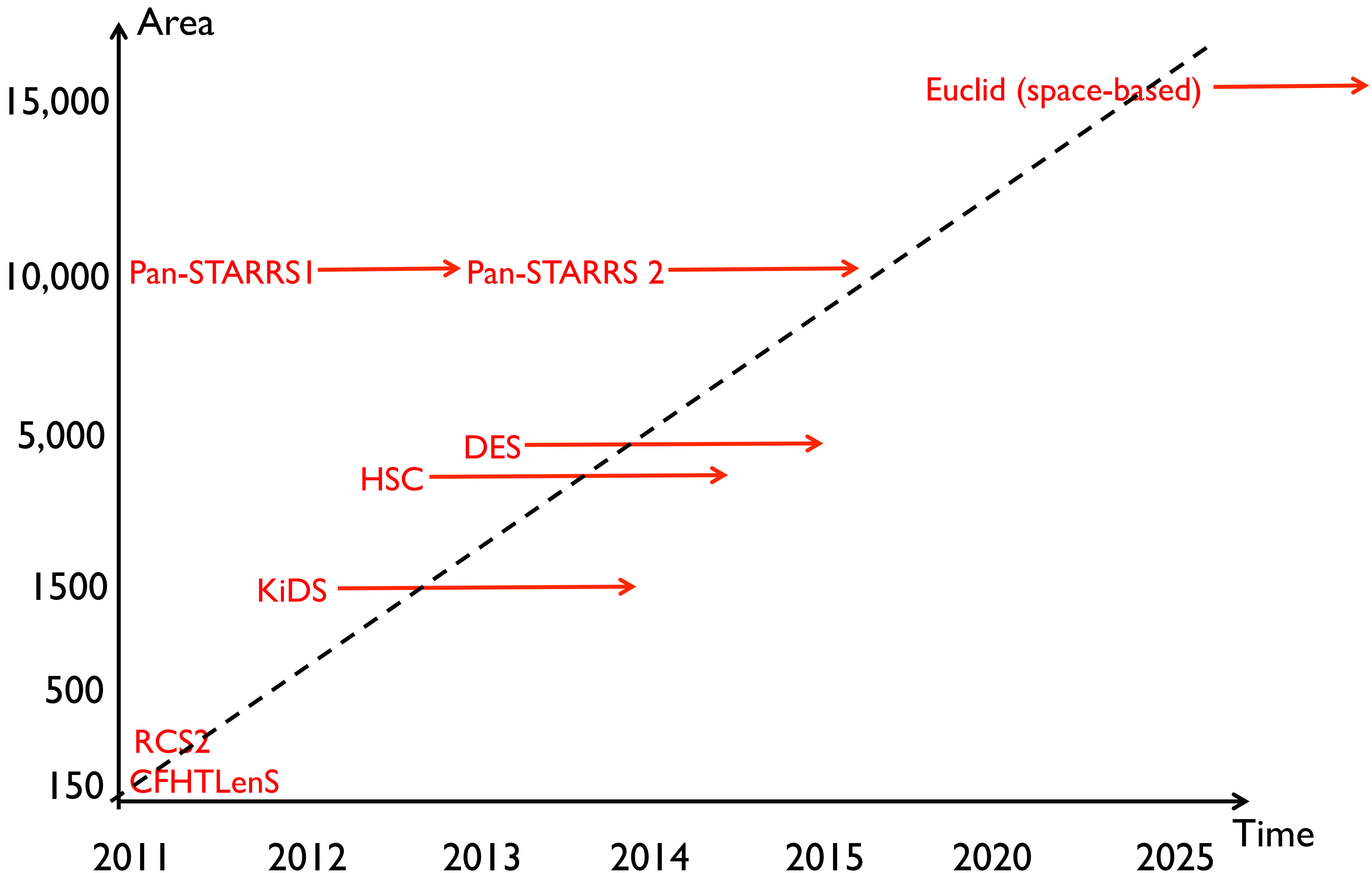
$$q_i(r) = \frac{3H_0^2 \Omega_m S_k(r)}{2a(r)} \int_r^{r_H} dr'\, p_i(r') \frac{S_k(r'-r)}{S_k(r')}$$

- Tomography
  - Generate 2D shear correlation in redshift bins
  - Can "auto" correlate *in a bin*
  - Or *"cross" correlate* between bin pairs
  - i and j refer to *redshift bin pairs*





Lensing Power Spectrum
w0=-0.45
w0=-0.95
10
100
1000
10^4
ℓ
ℓ(ℓ+1)C_ℓ/2π
5×10^-7
10^-6
2×10^-6
5×10^-6
http://icosmo.org


Area
15,000
10,000
5,000
1500
500
150
Time
2011
2012
2013
2014
2015
2020
2025
Euclid (space-based)
Pan-STARRS1
Pan-STARRS 2
DES
HSC
KiDS
RCS2
CFHTLenS

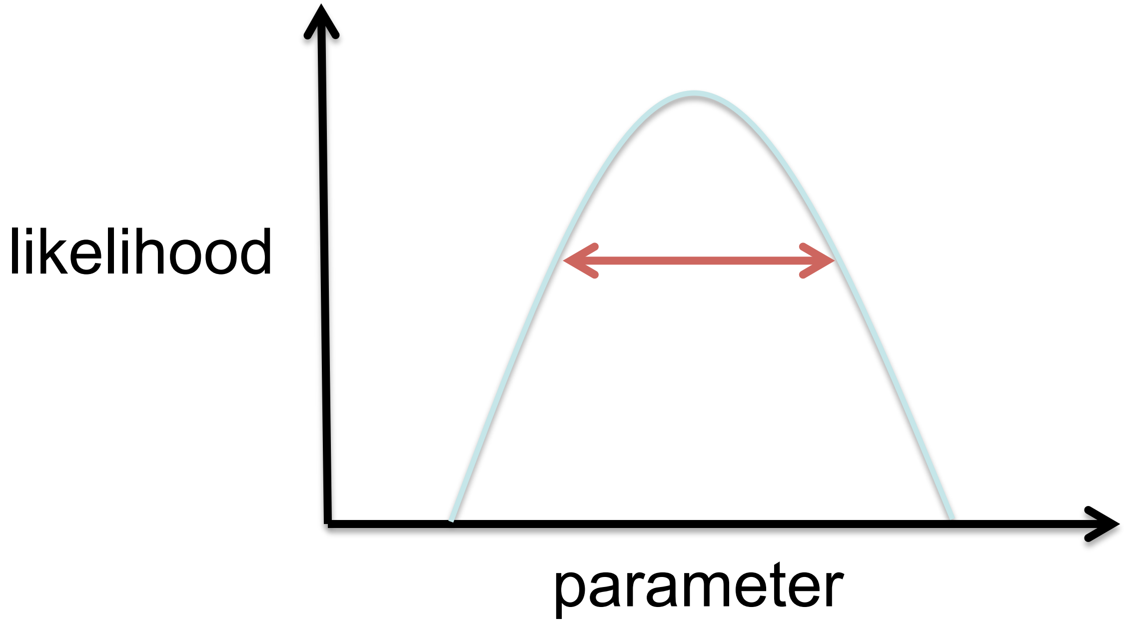
What do we want?

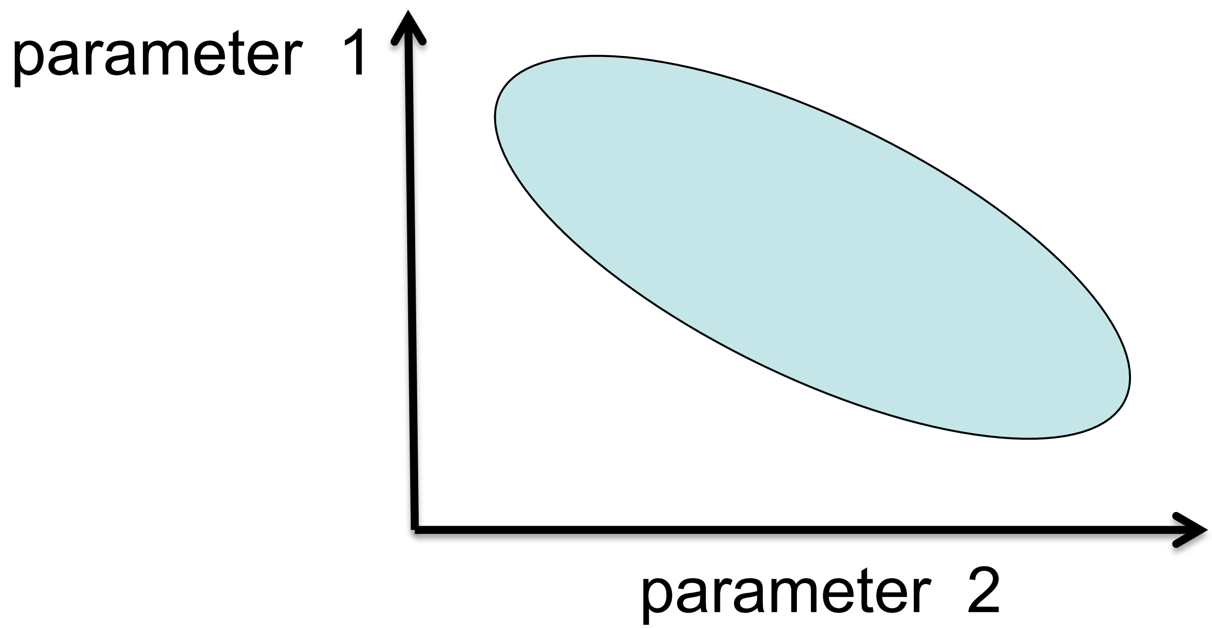
- How accurately can we estimate a model parameter from a given data set?

- Given a set of N data point $x_1,\ldots,x_N$
  - Want the estimator to be *unbiased* $\langle\theta\rangle = \theta_0$
  - Give small error bars as possible $\Delta\theta_\alpha \equiv \left(\langle\theta_\alpha^2\rangle - \langle\theta_\alpha\rangle^2\right)^{1/2}$

- The *Best Unbiased Estimator*
- A key Quantity in this is the Fisher (Information) Matrix

- Fisher 1935
- Tegmark, Taylor, Heavens 1997


likelihood
parameter


parameter 1
parameter 2

What is the (Fisher) Matrix?

- Lets expand a likelihood surface about the maximum likelihood point

$$\ln L(\mathrm{x};\boldsymbol{\theta}) = \ln L(\mathrm{x};\boldsymbol{\theta}_0) + \frac{1}{2}(\boldsymbol{\theta}_\alpha - \boldsymbol{\theta}_{0\alpha})\frac{\partial^2 \ln L}{\partial\boldsymbol{\theta}_\alpha \partial\boldsymbol{\theta}_\beta}(\boldsymbol{\theta}_\beta - \boldsymbol{\theta}_{0\beta}) + \dots$$

- Can write this as a Gaussian

$$L(\mathrm{x};\boldsymbol{\theta}) = L(\mathrm{x};\boldsymbol{\theta}_0)\exp\left[-\frac{1}{2}(\boldsymbol{\theta}_\alpha - \boldsymbol{\theta}_{0\alpha})H_{\alpha\beta}(\boldsymbol{\theta}_\beta - \boldsymbol{\theta}_{0\beta})\right]$$

- Where the Hessian (covariance) is

$$\mathrm{H}_{\alpha\beta} \equiv -\frac{\partial^2 \ln L}{\partial\theta_\alpha \partial\theta_\beta}$$

What is the Fisher Matrix?

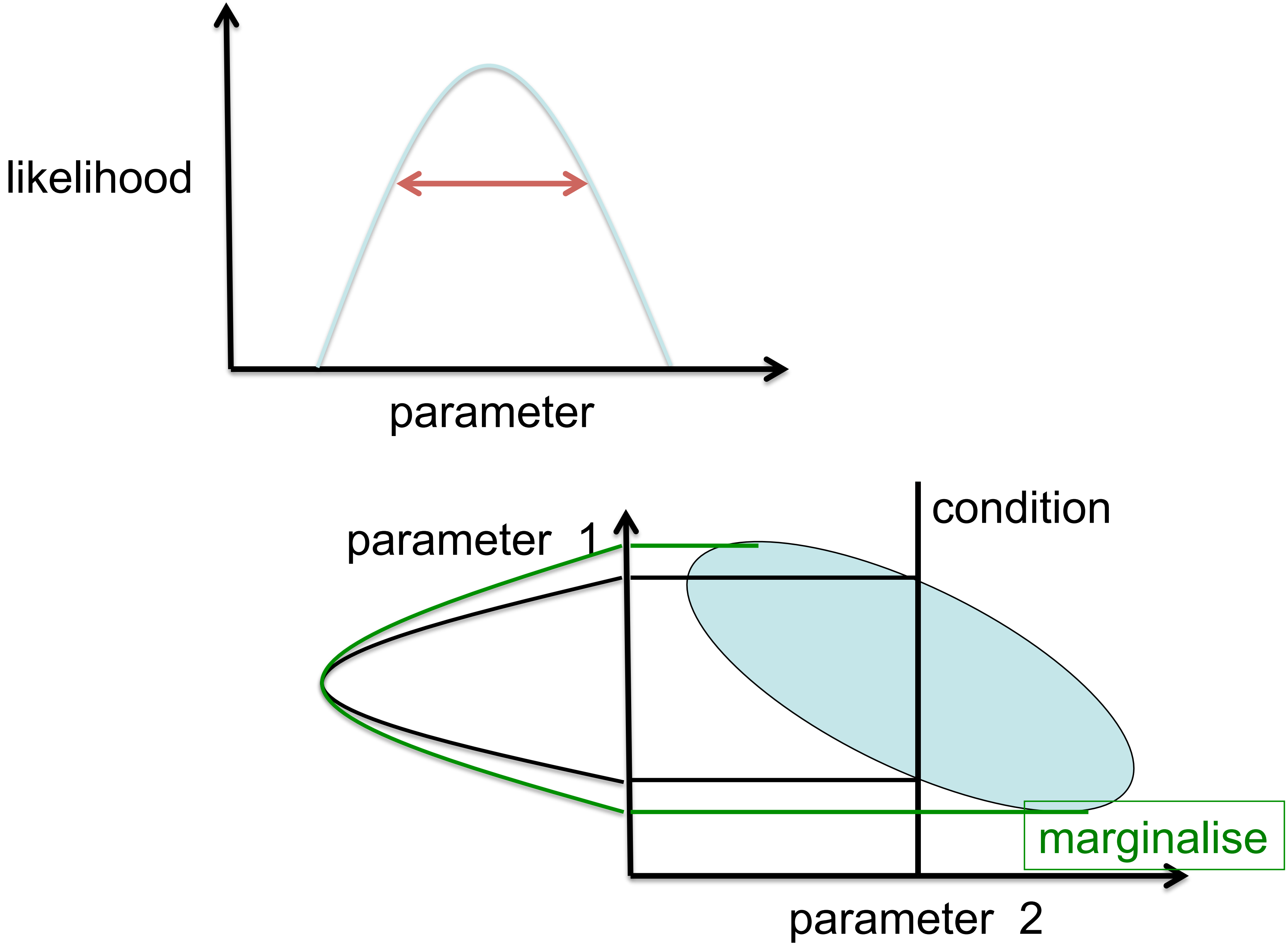
- The Hessian Matrix has some nice properties
- Conditional Error on α

$$\sigma_{\text{conditional},\alpha} = \frac{1}{\sqrt{\mathsf{H}_{\alpha\alpha}}}$$

- Marginal error on α

$$\sigma_\alpha = \sqrt{(\mathsf{H}^{-1})_{\alpha\alpha}}.$$

Matrix inversion performed


likelihood
parameter
condition
parameter 1
marginalise
parameter 2

What is the Fisher Matrix?

- The Fisher Matrix defined as the *expectation of the Hessian matrix*

$$\mathsf{F}_{\alpha\beta} \equiv \langle \mathsf{H}_{\alpha\beta} \rangle = \left\langle -\frac{\partial^2 \ln L}{\partial \boldsymbol{\theta}_\alpha \partial \boldsymbol{\theta}_\beta} \right\rangle$$

- This allows us to make <u>predictions</u> about the performance of an experiment !

- The <u>expected</u> conditional error on α: $\sigma_\alpha = \sqrt{(1/F_{\alpha\alpha})}$
- The <u>expected</u> marginal error on α

$$\sigma_\alpha = \sqrt{(\mathsf{F}^{-1})_{\alpha\alpha}}.$$

Matrix inversion performed

The Gaussian Case

- How do we calculate Fisher Matrices in practice?

- Assume that the likelihood is Gaussian

$$2\mathcal{L} = \ln \det \mathsf{C} + (\mathbf{x} - \boldsymbol{\mu})\mathsf{C}^{-1}(\mathbf{x} - \boldsymbol{\mu})^T$$

The Gaussian Case

$$\mathsf{D} \equiv (\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^T$$

$$2\mathcal{L} = \mathrm{Tr}\left[\ln \mathsf{C} + \mathsf{C}^{-1}\mathsf{D}\right]$$

$$\boxed{\ln\det \mathsf{C} = \mathrm{Tr}\ln \mathsf{C}}$$

matrix identity

derivative

$$\boxed{(\ln \mathsf{C})_{,\alpha} = \mathsf{C}^{-1}\mathsf{C}_{,\alpha}}$$

$$\boxed{2\mathcal{L}_{,\alpha} = \mathrm{Tr}\left[\mathsf{C}^{-1}\mathsf{C}_{,\alpha} - \mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{D} + \mathsf{C}^{-1}\mathsf{D}_{,\alpha}\right]}$$

derivative

$$\boxed{\langle \mathcal{L}_{,\alpha} \rangle = 0.}$$

$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \mathsf{C}^{-1}[\boldsymbol{\mu}_{,\alpha}\boldsymbol{\mu}_{,\beta}^T + \boldsymbol{\mu}_{,\beta}\boldsymbol{\mu}_{,\alpha}^T]].$$

How to Calculate a Fisher Matrix

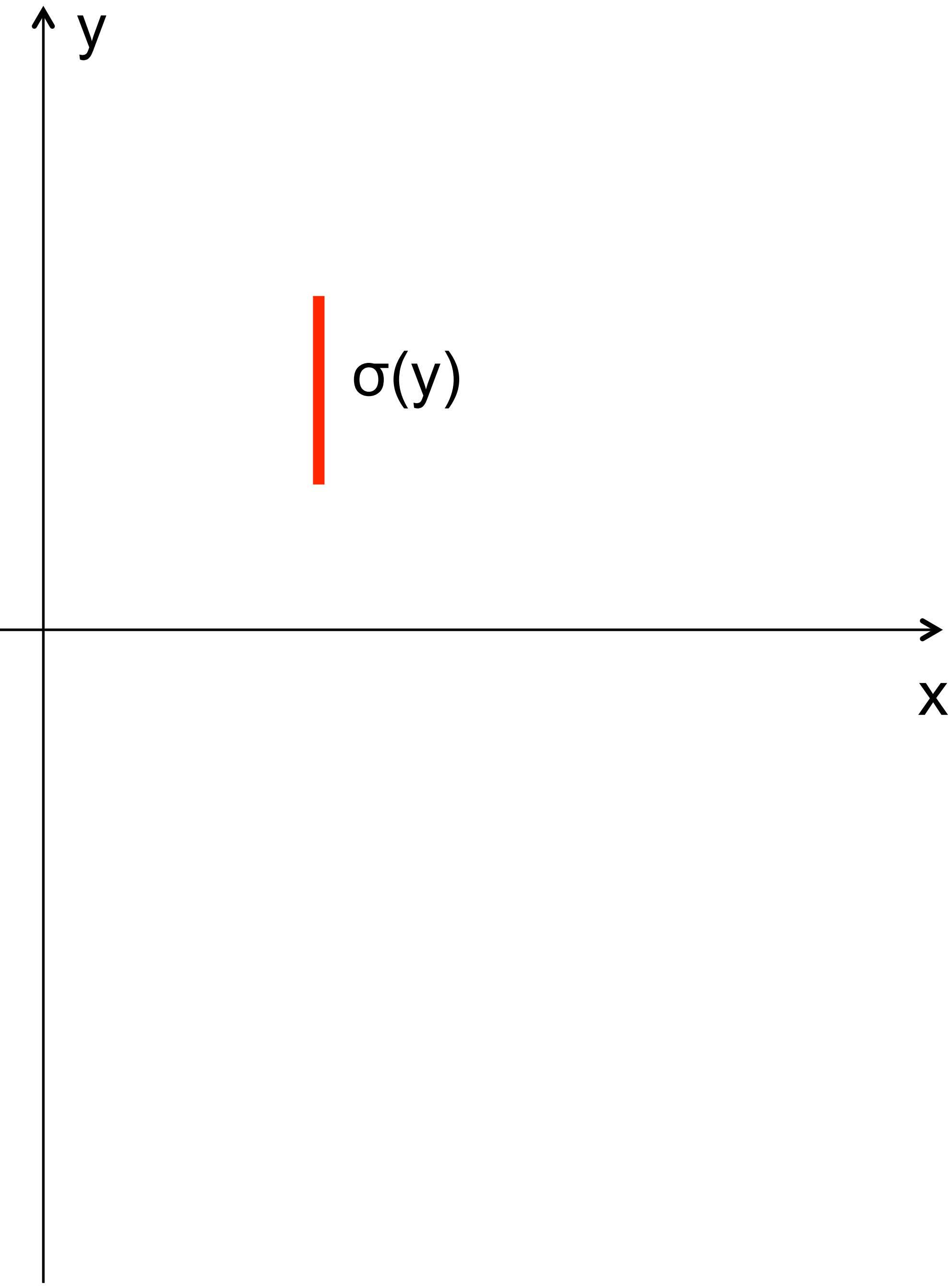
$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \mathsf{C}^{-1}[\,\boldsymbol{\mu}_{,\alpha}\,\boldsymbol{\mu}_{,\beta}^{T} + \boldsymbol{\mu}_{,\beta}\,\boldsymbol{\mu}_{,\alpha}^{T}\,]].$$

- We know the (expected) covariance and mean from theory
- Requires NO DATA!
- Worked example y=mx+c

- Theory: $y=mx$

- New Experiment can measure y with error $\sigma(y)$

- What value of x is best for measuring m?



$$F_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[C^{-1}C_{,\alpha}C^{-1}C_{,\beta} + C^{-1}[\mu_{,\alpha}\mu_{,\beta}^{T} + \mu_{,\beta}\mu_{,\alpha}^{T}]].$$

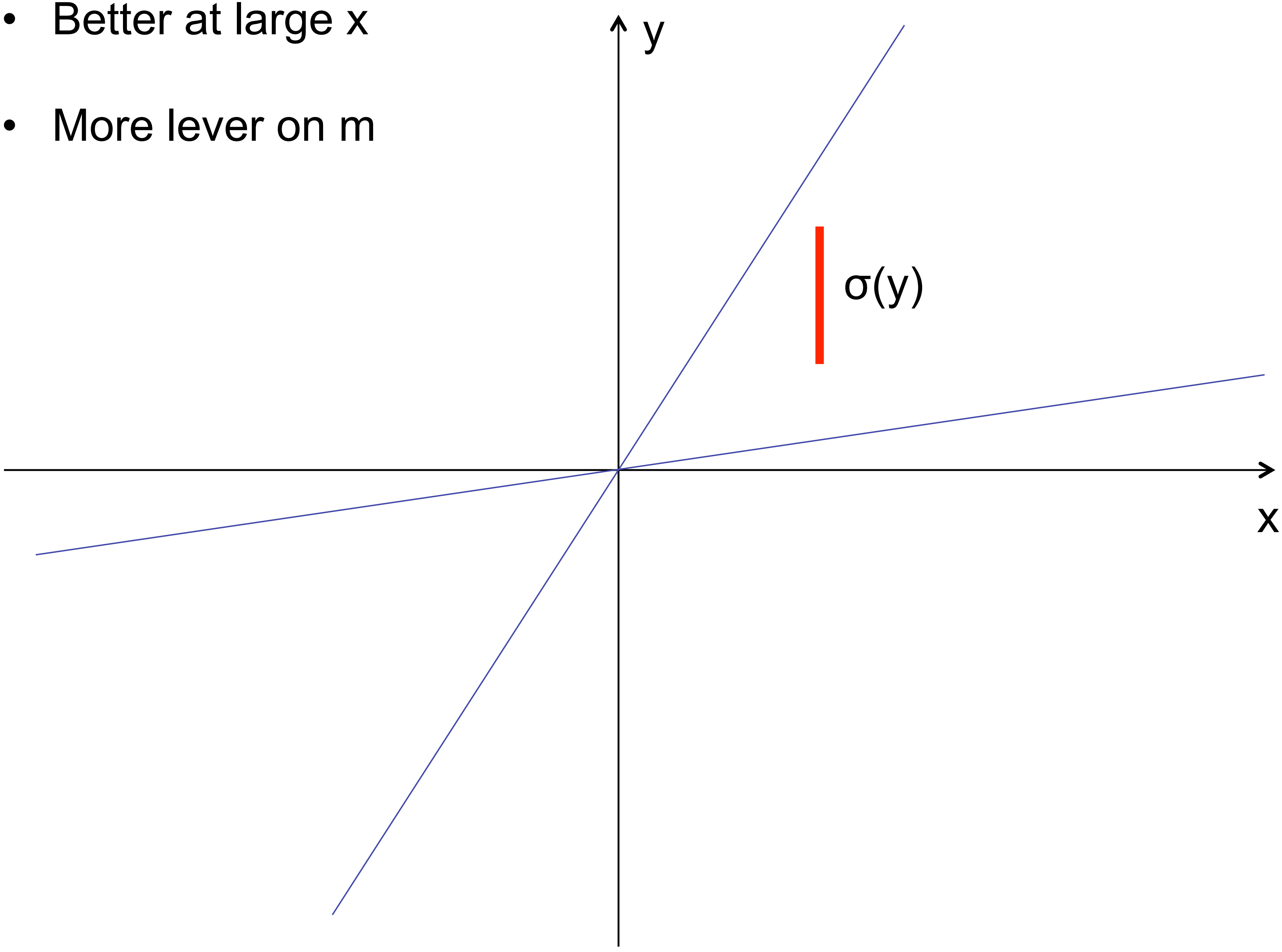
- The theory: y=mx
- Experiment can measure σ(y)
- Question: what x value is best?

- Covariance does not depend on m so first term zero
- dy/dm=x

- $F_{mm}=(1/\sigma^2(y))x^2$
- $\sigma(m)=\sqrt{[F^{-1}_{mm}]}=\sqrt{[\sigma^2(y)\ x^{-2}]}=\sigma(y)\ x^{-1}$

- Better measure of m at large x

- Better at large x
- More lever on m



- Some nomenclature

- $\sigma(m)=\sqrt{[F^{-1}{}_{mm}]}=\sqrt{[\sigma^2(y)\ x^{-2}]}=\sigma(y)\ x^{-1}$

  - Need a “fiducial” value to make quantitative predictions
  - If derivative is analytic then fairly straightforward (not always the case!)

Some nice properties of Fisher matrices

- $F = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$

- Matrix Manipulation:
  - Inversion F^{-1}
  - Addition $F=G+H$
  - Rotation $G=RFR^T$
  - Schur Complement $F_A=A-BD^{-1}C$

Adding Extra Parameters

- To add parameters to a Fisher Matrix

- Simply extend the matrix

$$F = \begin{pmatrix} F^{\boldsymbol{\theta}\boldsymbol{\theta}} & F^{\boldsymbol{\theta}\boldsymbol{w}(\phi)} \\ F^{\boldsymbol{w}(\phi)\boldsymbol{\theta}} & F^{\boldsymbol{w}(\phi)\boldsymbol{w}(\phi)} \end{pmatrix}$$

Combining Experiments

- If two experiments are *independent* then the combined error is simply

$$F_{comb}=F_1+F_2$$

- Same for n experiments

- If not independent need to have a single Fisher matrix with a *joint covariance*

Re-Parameterising

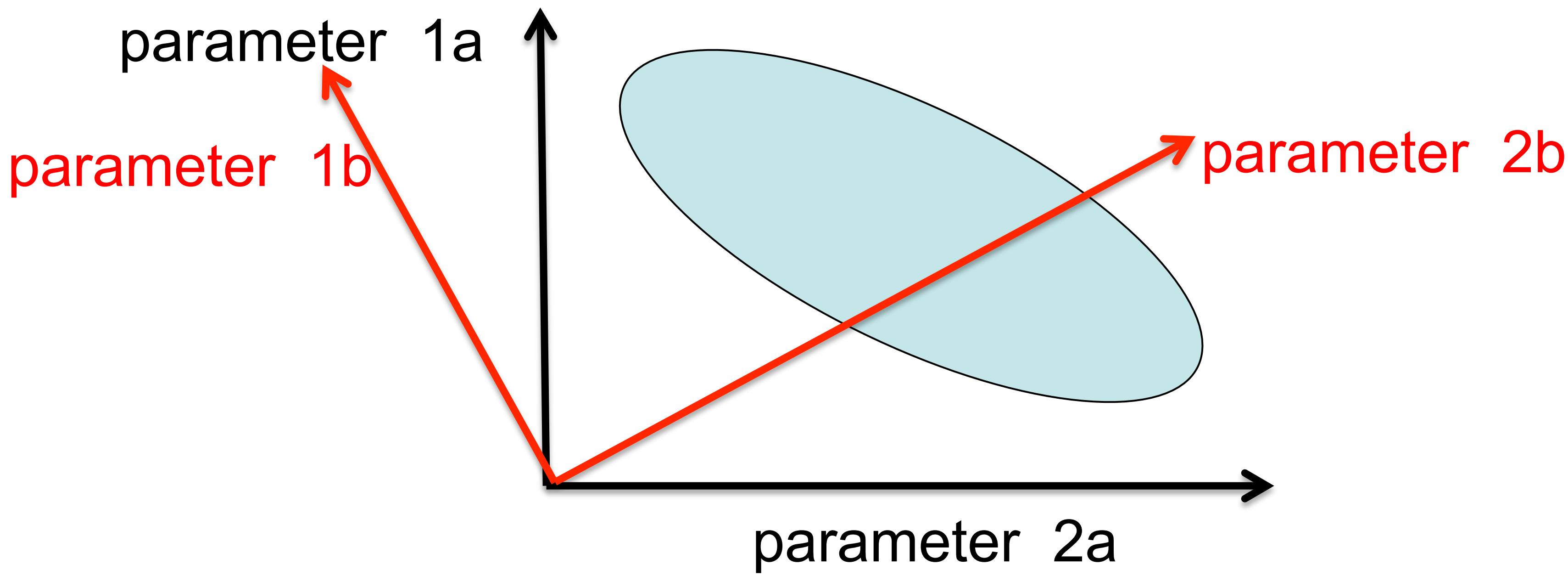
- Can change variables between two parameter sets through a Jacobian transform (rotation)

$$F(\text{new}) = J^T F(\text{old}) J \qquad J_{ij} = \frac{\partial b_j}{\partial a_i}$$

- Where J is a matrix of derivatives
  - NOTE can only do this if the basis sets are mutually complete (Kitching & Amara, 2009)
- Eigendecomposition is a special case of parameter rotation

$$J_{ij} = \frac{\partial b_j}{\partial a_i}$$

$$F(new) = J^T F(old) J$$



Schur Complement

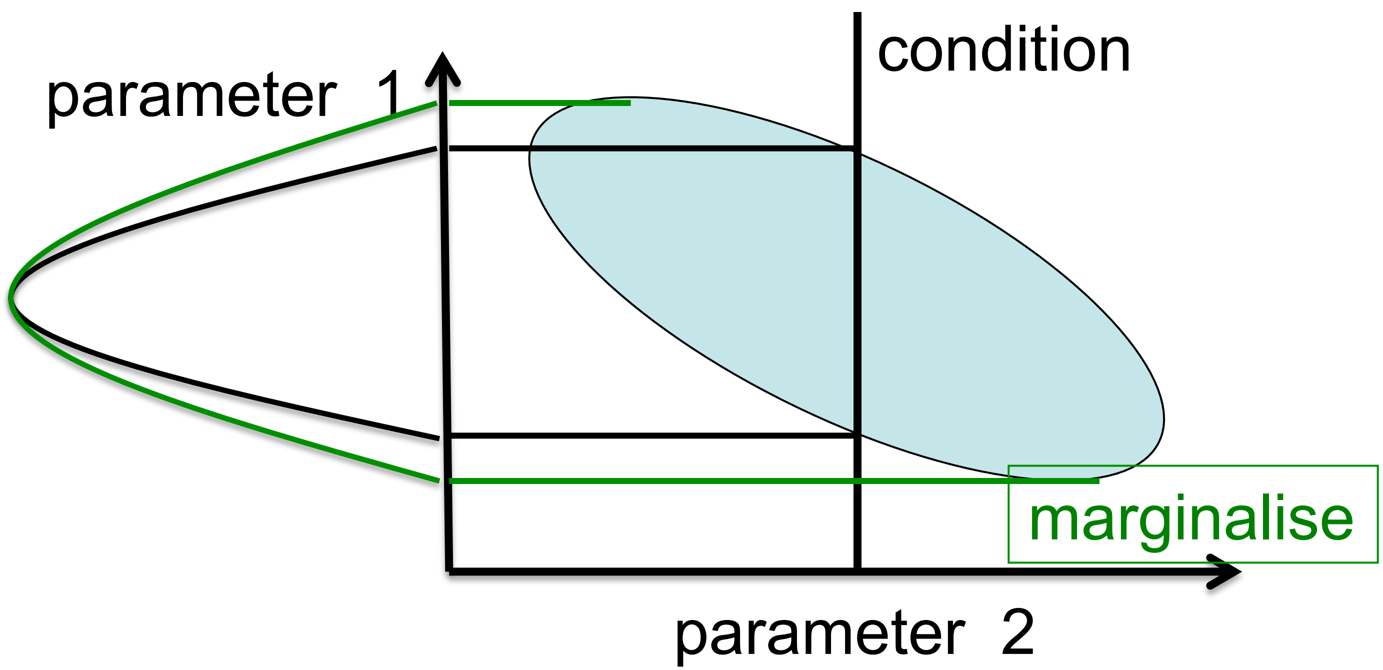
$$F = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

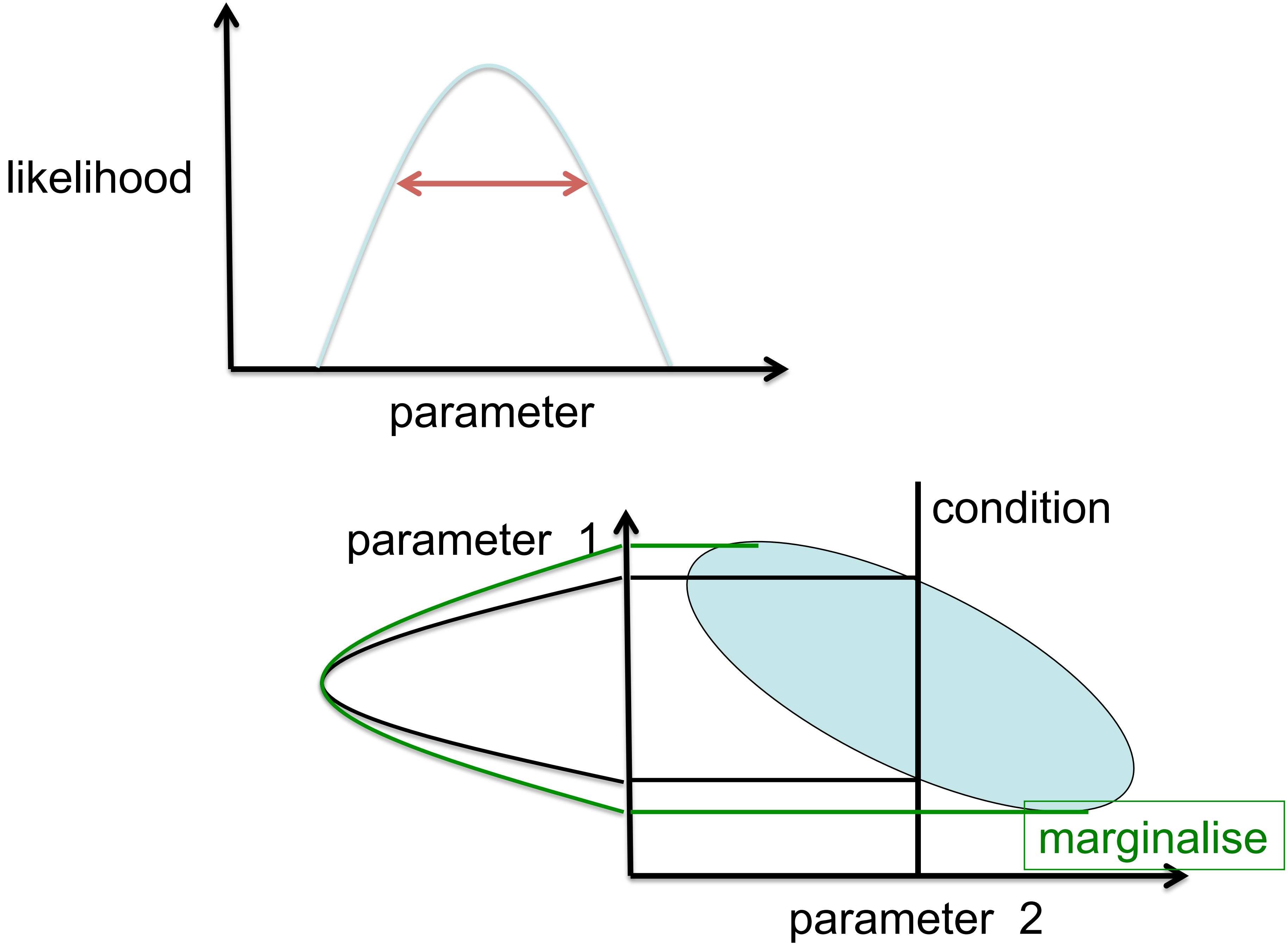
- Schur Complement $F_A = A - BD^{-1}C$
- This is equivalent of the following operation
  - Invert entire matrix F^{-1}
  - Select the A-part of the inverse $F^{-1}(A)$
  - Reinvert
  - What you have is a new “sub” Fisher matrix
- What is does this correspond to ?

- Schur Complement is equivalent of marginalisation over parameters

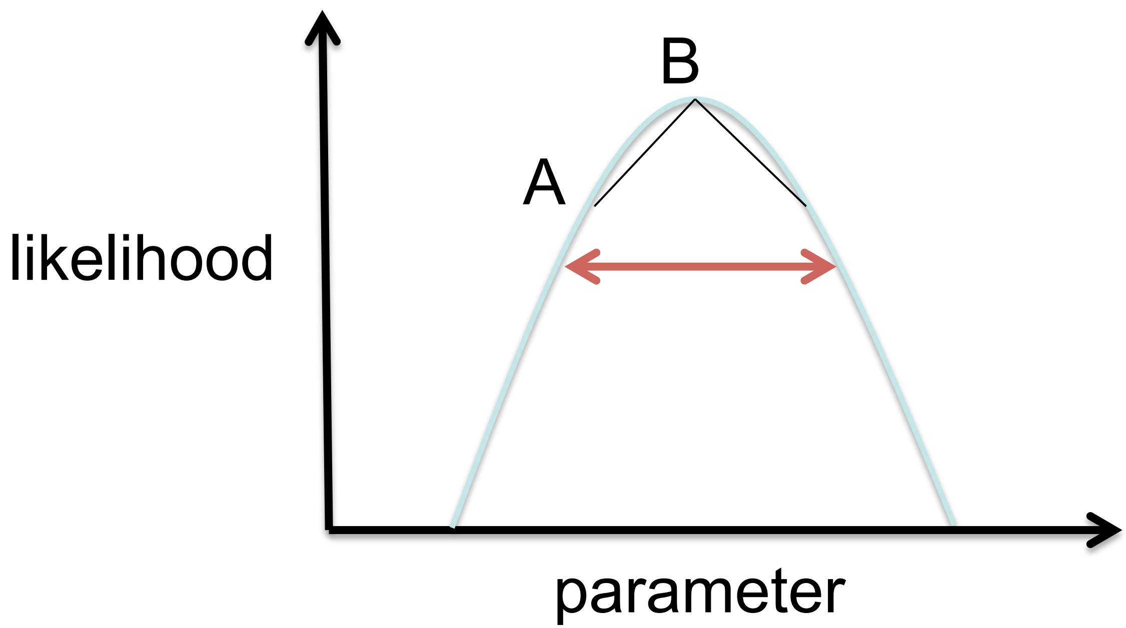
$$F = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

- Schur Complement
  - $F_A = A - BD^{-1}C < A$
- Marginalises over parameters not-A




likelihood
parameter
condition
parameter 1
marginalise
parameter 2

A Warning about derivatives

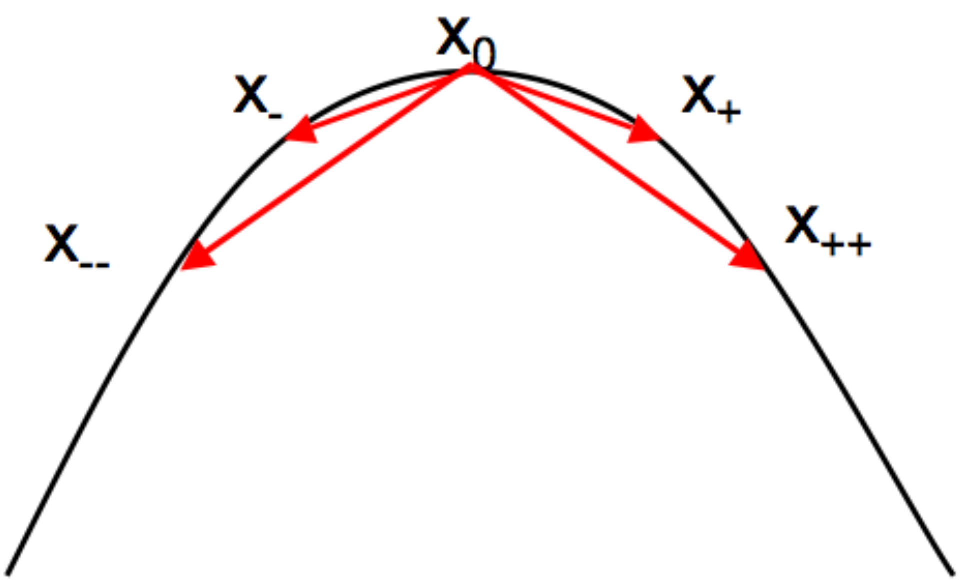


dL/dparameter= (B-A)/dp

Or can approximate using q parabola

Numerically must test if this is stable

- Quadractic approximation **Exercise Prove This**
- $df(x)/dx = [f(x_+)-f(x_-) -\{f(x_{++})- f(x_{--})-(2 f(x_+)+2 f(x_-))/6\}]/[x_+-x_-]$

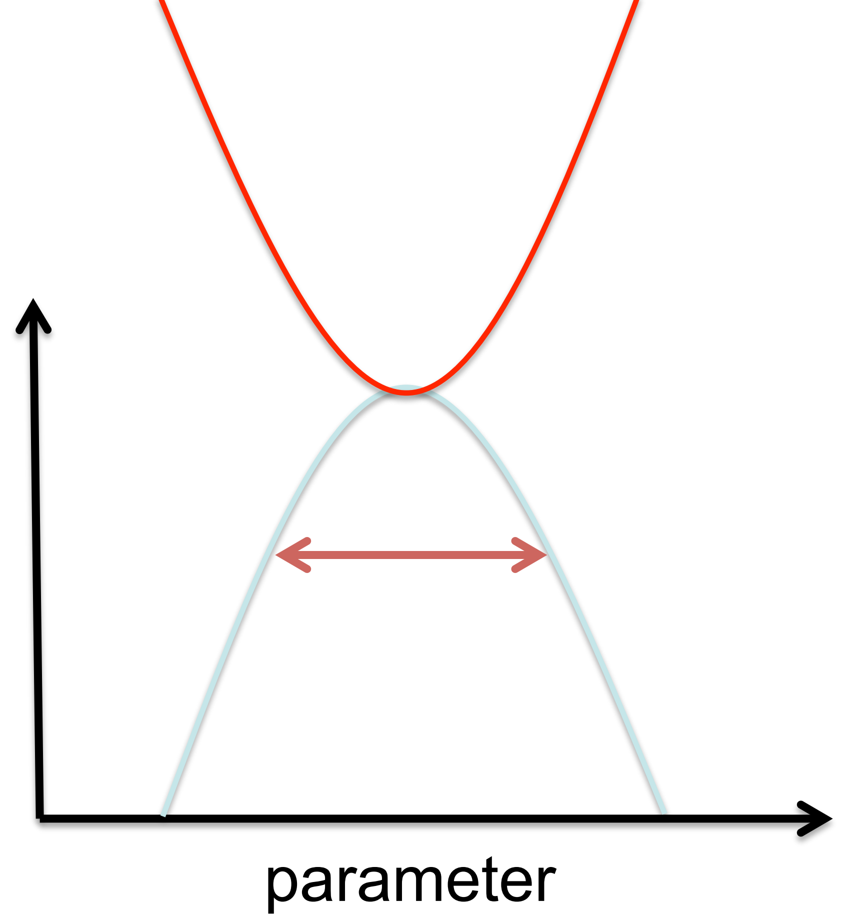


Warning about the two terms

$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \mathsf{C}^{-1}[\,\boldsymbol{\mu}_{,\alpha}\,\boldsymbol{\mu}_{,\beta}^{T} + \boldsymbol{\mu}_{,\beta}\,\boldsymbol{\mu}_{,\alpha}^{T}\,]].$$

- Can we use both terms and get twice the information?
- No not usually (almost never in cosmology)

- Fisher matrices must be positive definite
- A positive definite matrix has
  - ONLY POSITIVE EIGENVALUES
  - Because by definition the likelihood surface is assumed to be single peaked



- Can get non-positive definite due to numerical inaccuracies
  - Corresponds to a convex(negatively) curved surface

- Should always check matrices

Fisher Future Forecasting

- We now have a tool with which we can predict the accuracy of future experiments!

$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \mathsf{C}^{-1}[\,\boldsymbol{\mu}_{,\alpha}\,\boldsymbol{\mu}_{,\beta}^{T} + \boldsymbol{\mu}_{,\beta}\,\boldsymbol{\mu}_{,\alpha}^{T}\,]].$$

- Can easily
  - Calculate expected parameter errors
  - Combine experiments
  - Change variables
  - Add extra parameters

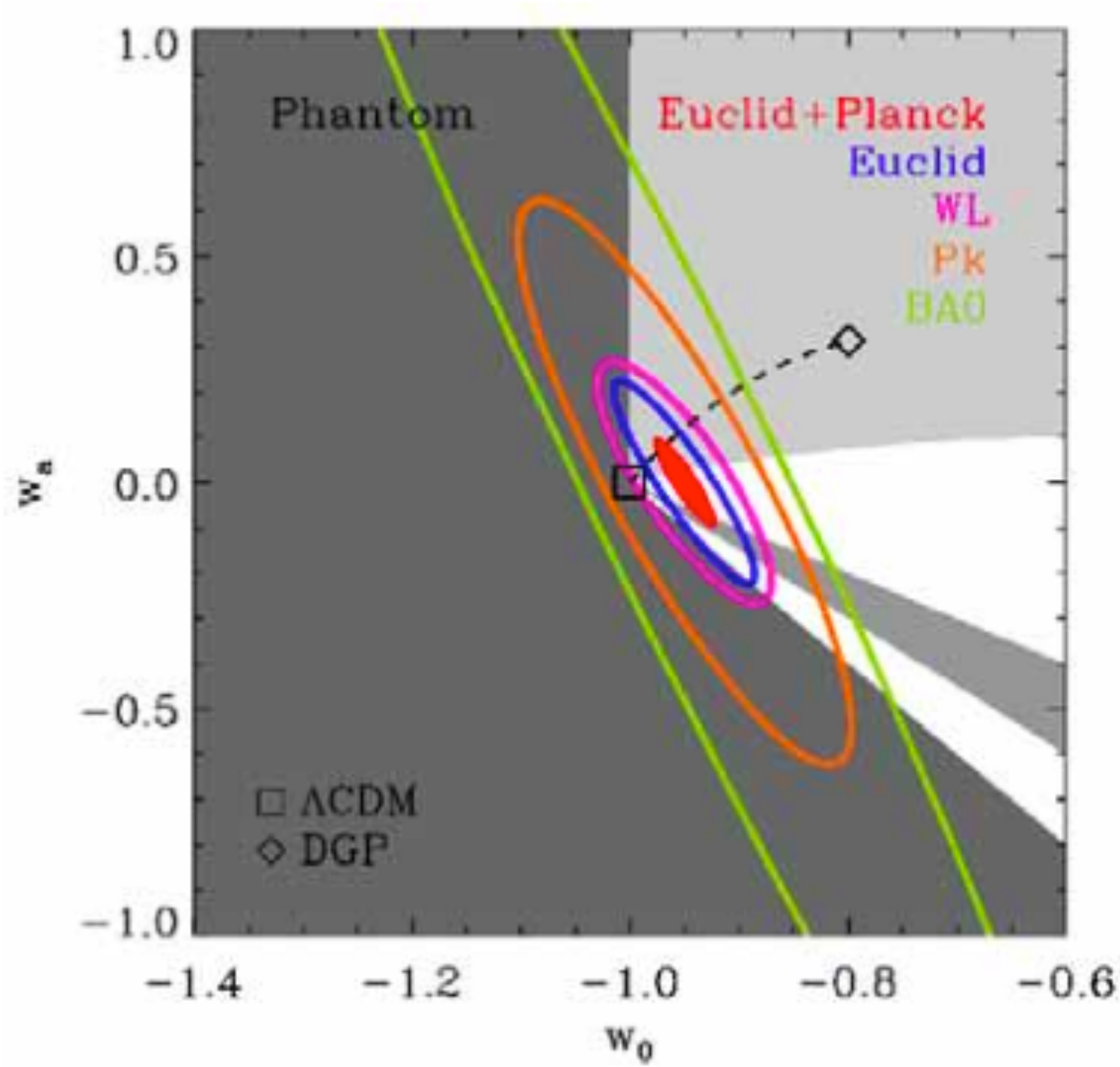
- For shear the mean shear is zero, the information is in the covariance so (Hu, 1999)

$$\mathbf{F}_{\alpha\beta} = \sum_{\ell=2}^{\ell_{\max}} (\ell + 1/2) f_{\rm sky} \, {\rm tr}[\mathbf{C}^{-1}\mathbf{C}_{,\alpha}\mathbf{C}^{-1}\mathbf{C}_{,\beta}]$$

- This is what is used to make predictions for cosmic shear and dark energy experiments

Dark Energy

- Expect constraints of 1% from Euclid



$$F_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[C^{-1}C_{,\alpha}C^{-1}C_{,\beta} + C^{-1}[\,\boldsymbol{\mu}_{,\alpha}\,\boldsymbol{\mu}_{,\beta}^{T} + \boldsymbol{\mu}_{,\beta}\,\boldsymbol{\mu}_{,\alpha}^{T}\,]].$$

- The theory: y=mx
- Experiment can measure σ(y)
- Question: what x value is best?

- dy/dm=x

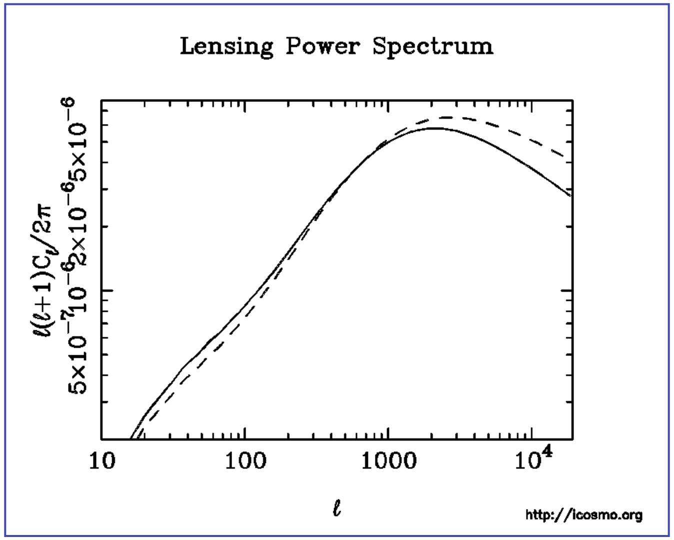
- $F_{mm}=(1/\sigma^2(y))x^2$
- $\sigma(m)=\sqrt{[F^{-1}_{mm}]}=\sqrt{[\sigma^2(y)\, x^{-2}]}=\sigma(y)\, x^{-1}$

- Better measure of m at large x

$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \mathsf{C}^{-1}[\,\boldsymbol{\mu}_{,\alpha}\,\boldsymbol{\mu}_{,\beta}^{T} + \boldsymbol{\mu}_{,\beta}\,\boldsymbol{\mu}_{,\alpha}^{T}\,]].$$

- The theory: $w(z)=w_0+w_a(1-a)$
- Experiment can measure $\sigma(C_l)$
- Question what redshift/area?

$$C_{ij}(\ell) = \int_0^{r_H} \mathrm{d}r\, W_{ij}^{\mathrm{GG}}(r) P_{\delta\delta}\left(\frac{\ell}{S_k(r)}; r\right)$$



Know from Theory/Simulations

$$\mathsf{F}_{\alpha\beta} = \langle \mathcal{L}_{,\alpha\beta} \rangle = \frac{1}{2}\mathrm{Tr}[\mathsf{C}^{-1}\mathsf{C}_{,\alpha}\mathsf{C}^{-1}\mathsf{C}_{,\beta} + \cancel{\mathsf{C}^{-1}[\boldsymbol{\mu}_{,\alpha}\boldsymbol{\mu}_{,\beta}^T + \boldsymbol{\mu}_{,\beta}\boldsymbol{\mu}_{,\alpha}^T]}].$$

✓ ✓

Mean is shear is zero

Hu 1999

POWER SPECTRUM TOMOGRAPHY WITH WEAK LENSING

WAYNE HU
Institute for Advanced Study, Princeton, NJ 08540

Submitted June 16, 2012

Generalizing the results of Hu & Tegmark (1998) to multiple correlated power spectra, we obtain[1]

$$\mathbf{F}_{\alpha\beta} = \sum_{\ell=2}^{\ell_{\rm max}} (\ell + 1/2) f_{\rm sky} \, {\rm tr}[\mathbf{C}^{-1}\mathbf{C}_{,\alpha}\mathbf{C}^{-1}\mathbf{C}_{,\beta}] \,, \qquad (8)$$

under the assumption of Gaussian signal and noise, where $f_{\rm sky}$ is fraction of sky covered by the survey, the covari-

l+1/2 sums over m-modes
f_{sky} scales the covariance with the survey area

Note also need to include noise on covariance **C**=C(l)+N
N=σ(e)/n_{galaxy}

- Question we have address is:
  - Given an experiment how accurate can I measure parameter values?
- Alternative/additional question is
  - How accurately can I determine a model (set of parameters)

- Bayes' Theorem

Prior

Likelihood

$$p_i(\boldsymbol{e}|\boldsymbol{y}_i) = \frac{\mathcal{P}(\boldsymbol{e})\,\mathcal{L}(\boldsymbol{y}_i|\boldsymbol{e})}{\int \mathcal{P}(\boldsymbol{e})\,\mathcal{L}(\boldsymbol{y}_i|\boldsymbol{e})\,d\boldsymbol{e}}$$

Evidence

Posterior

- Measure likelihood of data given parameters
- Assume prior on parameters
- Evidence for a Model (set of parameters)

How to compute expected evidence?

- Evidence $$p(D|M) = \int d\theta\, p(D|\theta, M) p(\theta|M).$$

- Bayes Factor=Ratio of Evidences

$$\frac{p(M'|D)}{p(M|D)} = \frac{p(M')}{p(M)} \frac{\int d\theta'\, p(D|\theta', M') p(\theta'|M')}{\int d\theta\, p(D|\theta, M) p(\theta|M)},$$

$$B \equiv \frac{\int d\theta'\, p(D|\theta', M') p(\theta'|M')}{\int d\theta\, p(D|\theta, M) p(\theta|M)}$$

- What does the Bayes factor mean?

$$B \equiv \frac{\int d\theta' \, p(D|\theta', M')p(\theta'|M')}{\int d\theta \, p(D|\theta, M)p(\theta|M)}$$

- Odds: how much would you gamble?

- Jeffereys Scale (take with a pinch of salt)
  - LnB $\leq$ 1 "inconclusive"
  - 1< LnB $\leq$ 2.5 "significant" odds ~1:12
  - 2.5 < LnB < 5.0 "strong" odds ~1:150
  - LnB > 5.0 "decisive" odds better than 1:150

- Can assume Gaussian likelihoods and perform the integration (Heavens, Kitching, Verde, 2008)

- Can compute *from the Fisher matrix* the expected evidence for *nested* models

$$\langle B \rangle = (2\pi)^{-p/2} \frac{\sqrt{\det F}}{\sqrt{\det F'}} \exp\left(-\frac{1}{2}\delta\theta_\alpha F_{\alpha\beta}\delta\theta_\beta\right) \prod_{q=1}^{p} \Delta\theta_{n'+q}$$

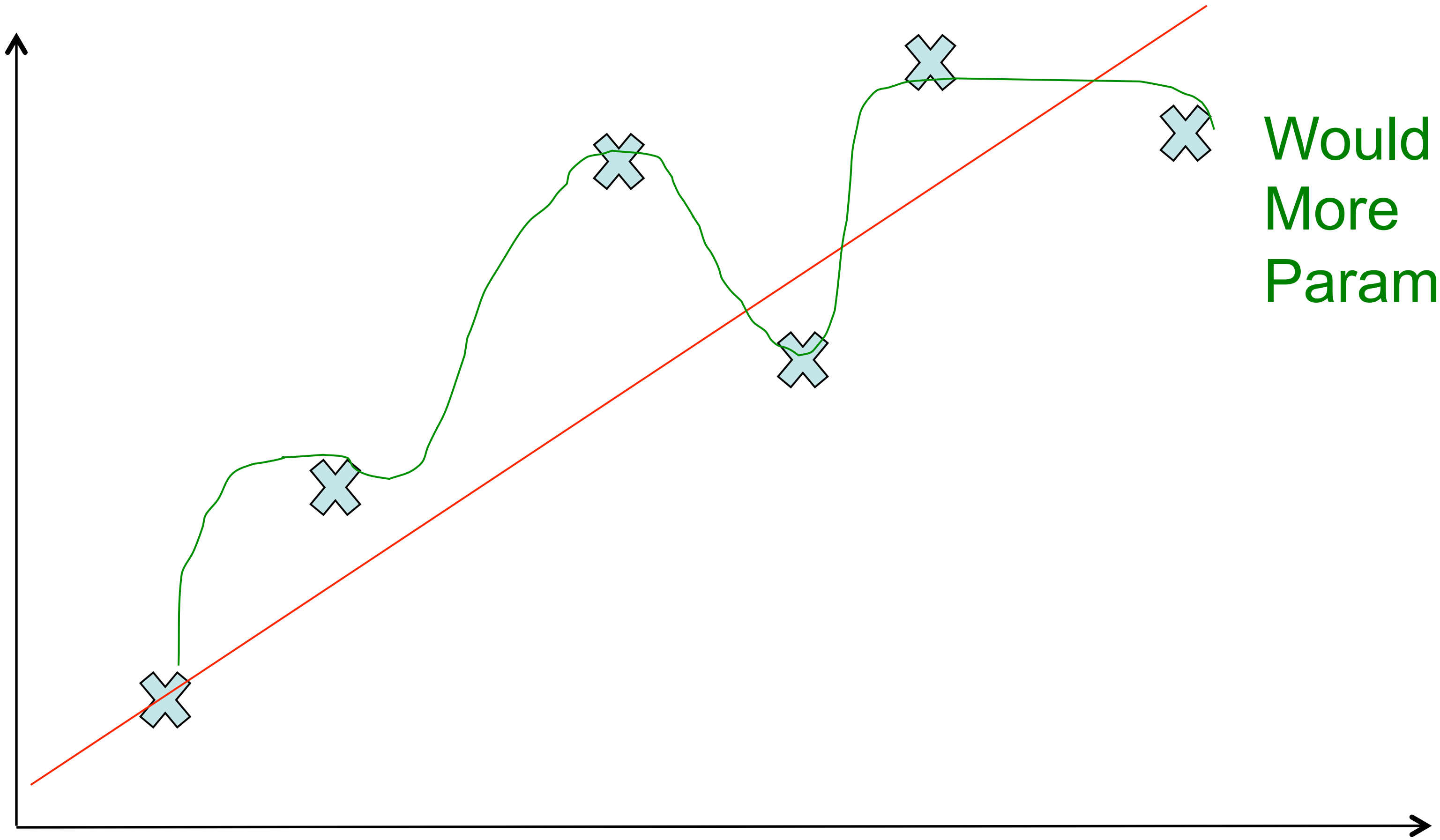
- Other similar approaches in Trotta (2008)

- “Occam Factor” can be seen

Occam Factor

$$\frac{\sqrt{\det F}}{\sqrt{\det F'}}$$

- Occam's Razor: Simpler models are prefered
- Stops you over fitting your data

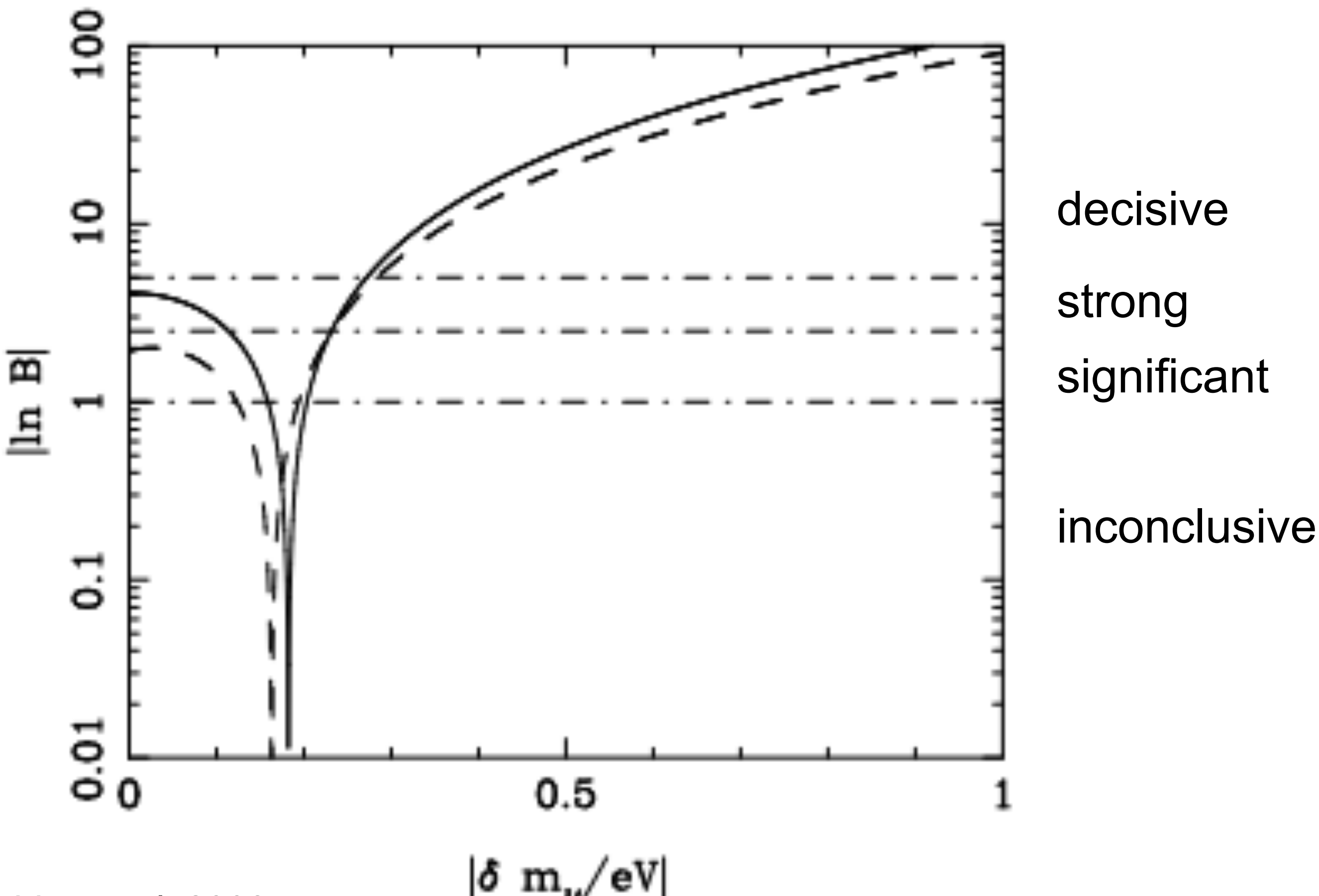


Would need More Parameters

- Example from neutrino mass from weak lensing

- Neutrinos have mass = model A
- Neutrinos do not have mass = model B

- Example from neutrino mass from weak lensing



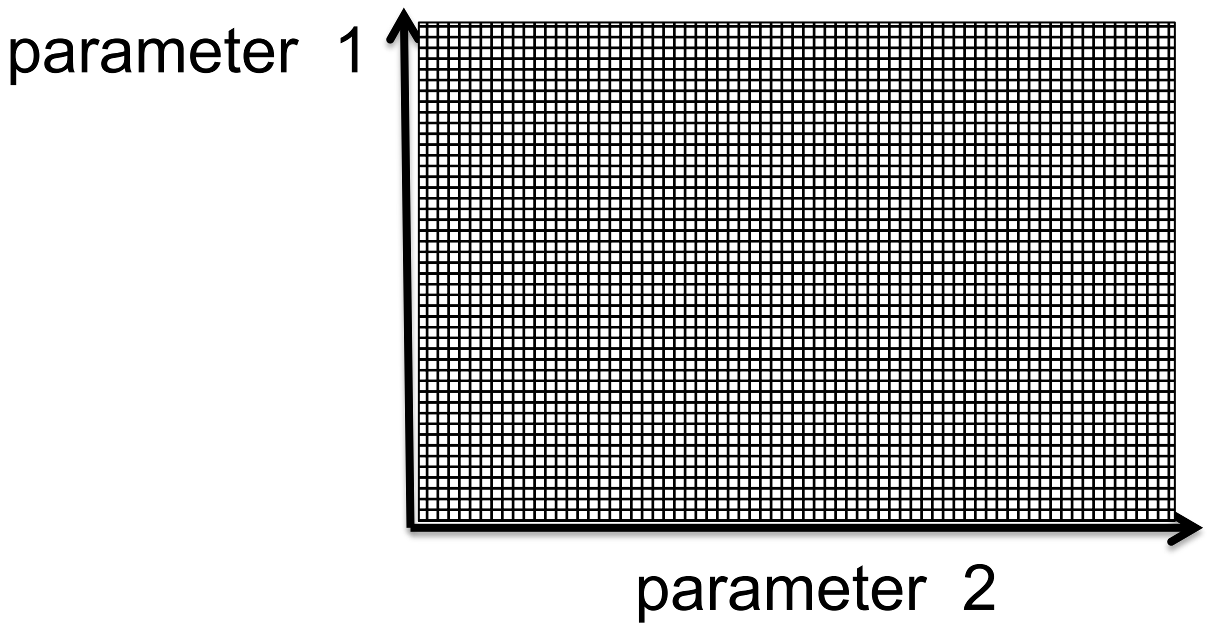
Euclid-like survey: Kitching et al. 2008

- What will we learn?
  - Fisher matrices
  - Likelihood sampling

Likelihood Sampling

- We have *at least* 10 cosmological parameters
- Others may be
  - non-zero
  - functions of scale and/or redshift
  - w(z)+1
  - b(z,k)-1

Parameter	Symbol
Hubble parameter	h
Total matter density	Ω_{m}
Baryon density	Ω_{b}
Cosmological constant	Ω_{Λ}
Radiation density	Ω_{r}
Neutrino density	Ω_{ν}
Density perturbation amplitude	$\Delta^2_{\mathcal{R}}(k_*)$
Density perturbation spectral index	n
Tensor to scalar ratio	r
Ionization optical depth	τ



Grid. Evaluate likelihood function at a grid of points in parameter space

What is wrong with this approach? N^D

MCMC Methods

- Monte Carlo Markov Chain

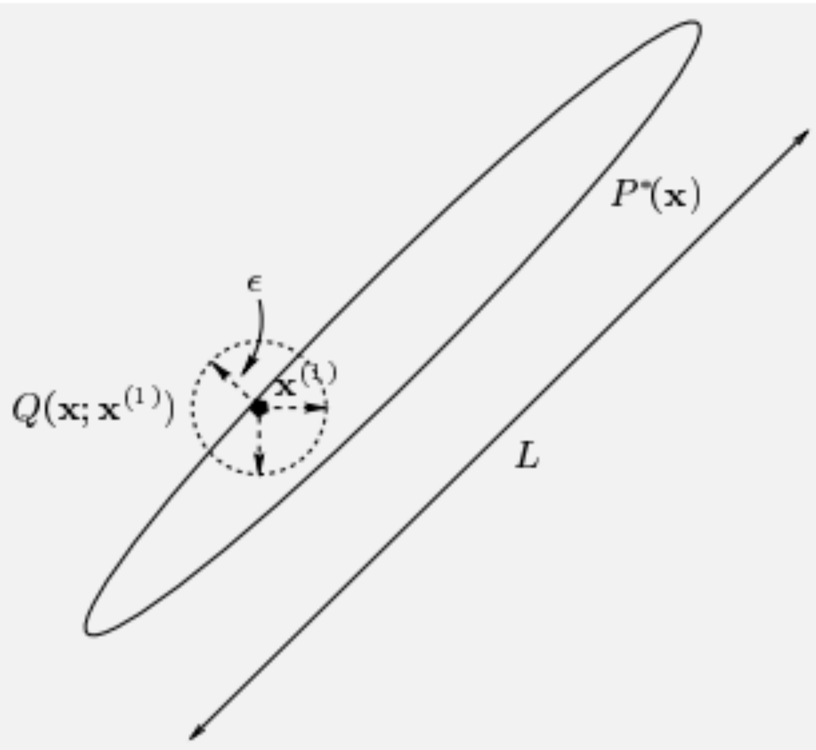
Randomly Sample Likelihood Space

A chain of likelihood (or other) evaluations that is "memoryless"

Metropolis-Hastings

- Sample likelihood space with a random walk

1) Pick a point in parameter space x_i
2) Evaluate likelihood $L(x_i)$
3) Pick a new random point x_{i+1} from a *proposal distribution*
4) Evaluate likelihood $L(x_{i+1})$
5) If $a=L(x_{i+1})/L(x_i)\geq 1$ ACCEPT (and goto 3)
6) Accept with probability a
   - Draw a uniform random number b and if $b<a$ ACCEPT else reject

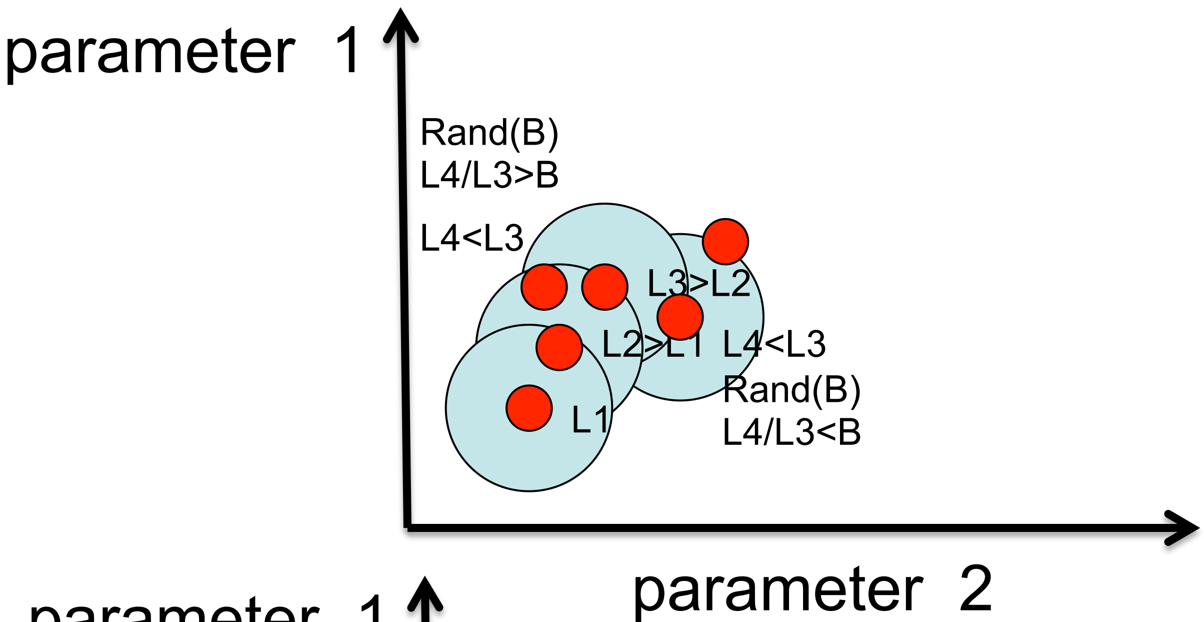


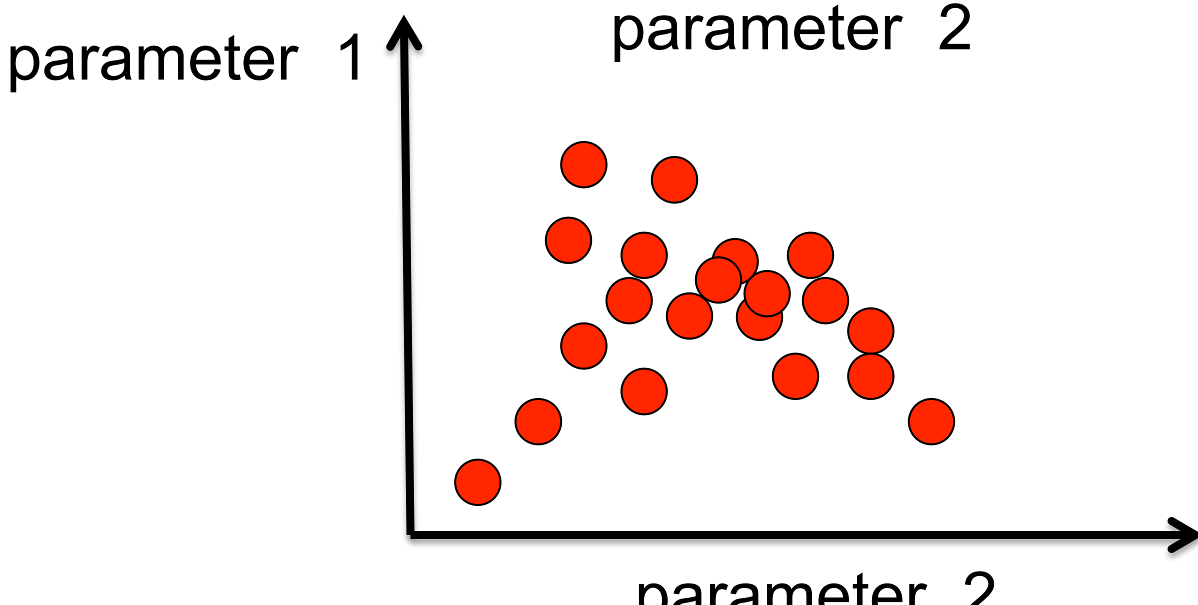
What choice of proposal?

Common to choose multivariate Gaussian (inefficient for degenerate parameters)

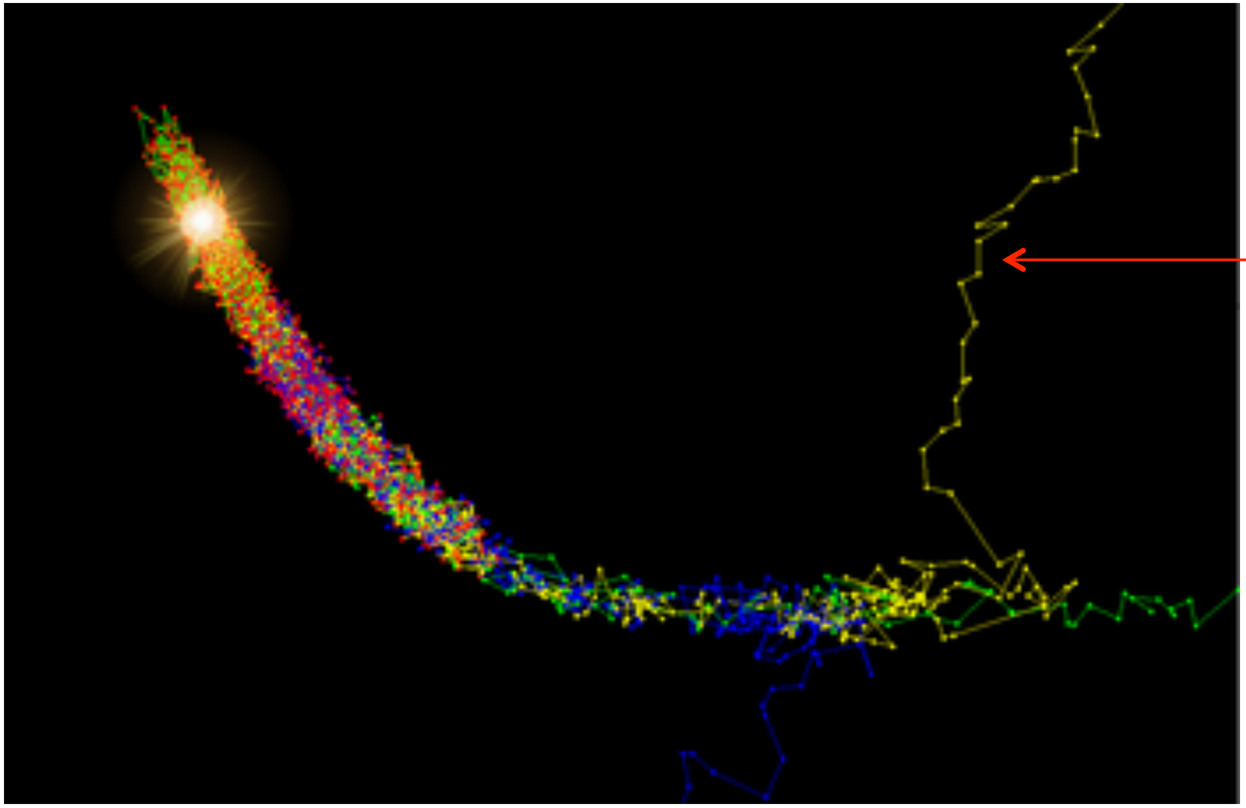
Could also choose the Fisher matrix

1) Pick a point in parameter space x_i
2) Evaluate likelihood $L(x_i)$
3) Pick a new random point x_{i+1} from a *proposal distribution*
4) Evaluate likelihood $L(x_{i+1})$
5) If $a=L(x_{i+1})/L(x_i)\geq 1$ ACCEPT (and goto 3)
6) Accept with probability a
   - Draw a uniform random number b and if b<a ACCEPT else reject





- The **density** of points is proportional to the likelihood



“run in”

Convergence

- When to stop (never!)
- But chain will "converge"
- Gelman-Rubin
  - "variance between chains consistent with variance within a chain"
  - Run multiple chains; m group of n chains n chains

$$\hat{\sigma}^2 = \frac{n-1}{n} W + \frac{1}{n} B,$$

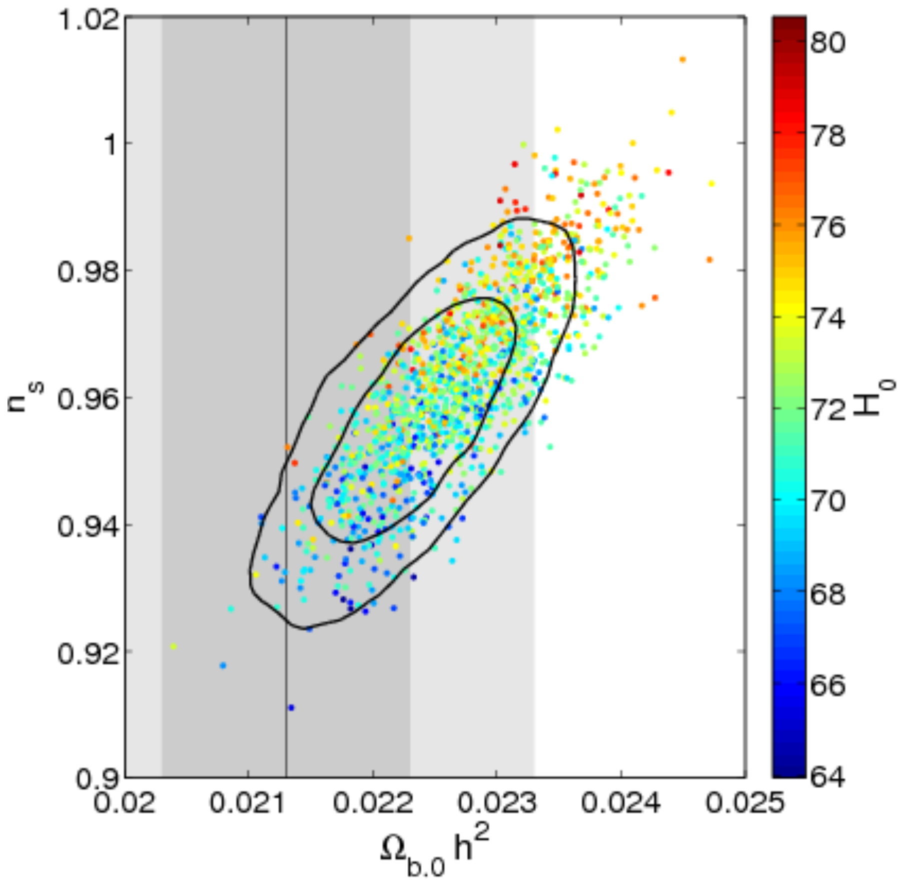
  - W=mean variance over chains
  - B=variance between chains
  - σ unbiased estimate of target
  - m groups of n chains

$$\sqrt{\hat{V}} = \sqrt{\hat{\sigma}^2 + B/mn}$$

  - V=1 means convergence

MCMC for Cosmology

- Most commonly used package is
  - cosmomc http://cosmologist.info/cosmomc/
  - Coupled with CAMB
  - Used for WMAP analysis (and many others)
  - Can download WMAP MCMC chains to play around with
  - http://lambda.gsfc.nasa.gov/toolbox/

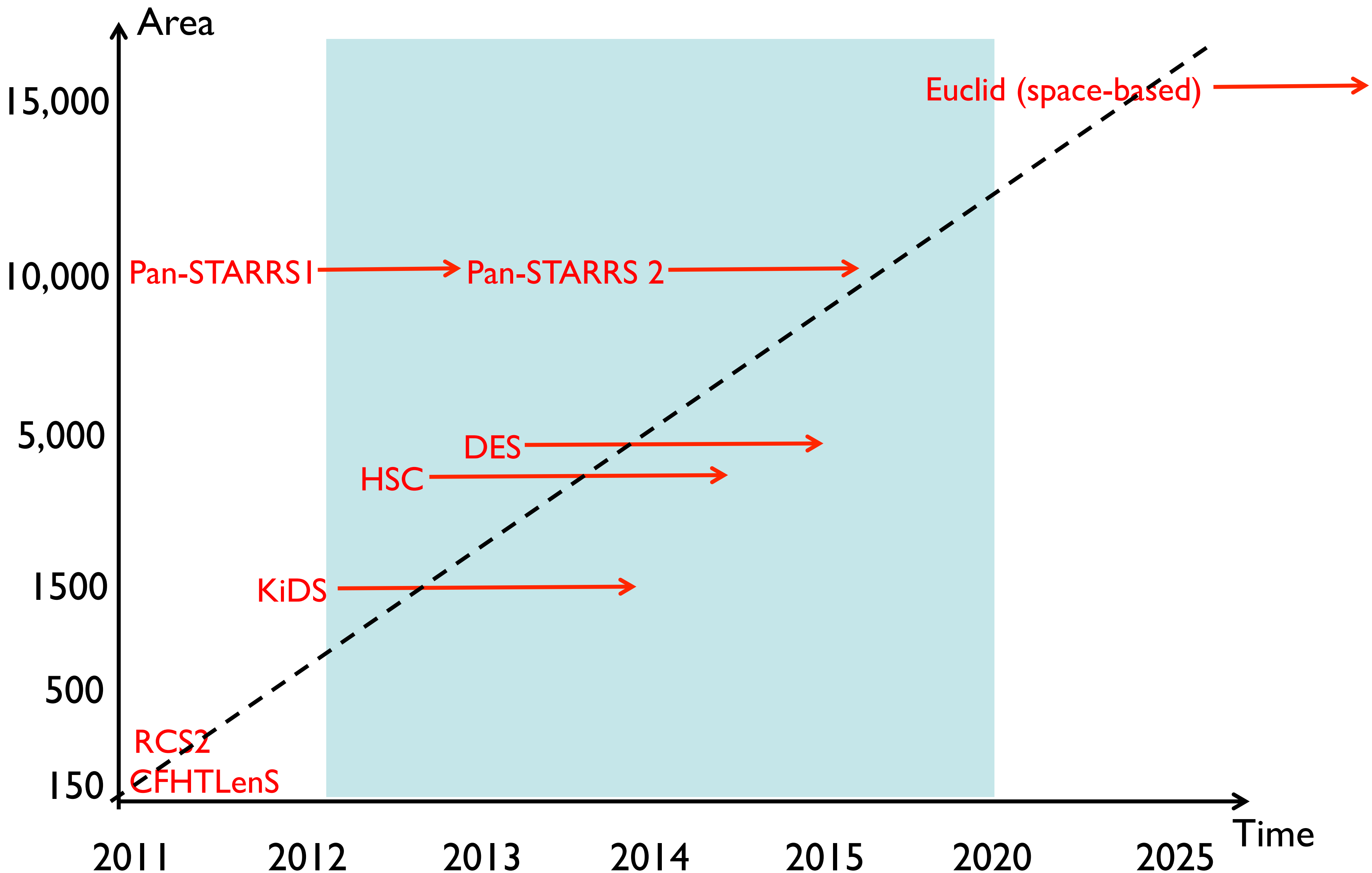


Other MCMC methods

- What are the problems with MCMC?
  - Multiple peaks
  - Evidence?
- Have some different types/flavours
  - Gibbs sampling
  - Simulated annealing
- What alternatives?
  - Nested Sampling
    - Feroz & Hobson (2008), refined by Feroz, Hobson & Bridges (2008)
  - Population Monte Carlo
    - Kilbinger et al. 2009

Recap

- Predictive parameter and model estimation
  - Fisher Matrix
  - Evidence
- Real Life parameter and model selection
  - MCMC



Area
15,000
10,000
5,000
1500
500
150
Euclid (space-based)
Pan-STARRS1
Pan-STARRS 2
DES
HSC
KiDS
RCS2
CFHTLenS
Time
2011
2012
2013
2014
2015
2020
2025

Conclusion

- Lensing is a simple cosmological probe
  - Directly related to General Relativity
  - Simple linear image distortions

- Measurement from data is challenging
  - Need lots of galaxies and very sophisticated experiments

- Lensing is a powerful probe of dark energy and dark matter