



## On Demonstration in the Sciences

Aristotle

### 1. Whether a Demonstrative Science exists

All communications of knowledge from teacher to pupil by way of reasoning pre-suppose some pre-existing knowledge. The truth of this statement may be seen from a complete enumeration of instances:—it is thus that the mathematical sciences are attained and every art also. The same is the case with dialectical arguments whether proceeding by means of the syllogism or of induction, for the former kind makes such assumptions as people who understand the meaning admit, the latter uses the recognized clearness of the particular as an indication of the universal, so that both convey their information by means of things already known. So too orators produce conviction in a like manner, using either Example, which is equivalent to induction, or Enthymeme, which corresponds to syllogism.

Pre-existing knowledge of two kinds is required: one must either assume beforehand that something exists, or one must understand what the word means, while sometimes both sorts of knowledge are required. As an example of the first case we may take the necessity for previously knowing the proposition ‘everything must be either affirmed or denied.’ Of the second case an instance would be the knowledge of the meaning conveyed by the word ‘triangle’; of the combination of both kinds, the knowledge both of what ‘Unit’ means, and of the fact that ‘Unit’ exists. The distinction is necessary, since the grounds of certainty differ in the two cases.

Some facts become known as a result of previously acquired knowledge, while others are learned at the moment of perceiving the object. This latter happens in the case of all things comprised under a universal, with which one is already acquainted. It is known to the pupil, before perceiving any particular triangle, that the interior angles of every triangle are equal to two right angles; but it is only at the moment of sense-perception that he learns that this figure inscribed in the semi-circle is a triangle.

In some cases knowledge is only acquired in this latter way, and the particular is not learned by means of a middle term: that is to say, in the cases where we touch the concrete particular, that is in the case of things which are not predicable of any subject. We ought to admit that, even before arriving at particulars, and so obtaining a syllogism, we do, from one point of view perhaps, possess knowledge, although from another we do not. For how, it may be asked, when he did not know whether the thing existed at all or not, could he have known absolutely that it contains two right angles? The answer is that he knows it from a particular point of view, in that he knows the universal, but he does not know it absolutely. On any other view we shall have the dilemma of the Meno—a man will either learn nothing at all or only what he knows before. This difficulty must not be solved as some try to do.

The question is asked, 'Do you or do you not know every dyad to be even?' On receiving an affirmative reply they bring forward some dyad of the existence of which the other was ignorant, and so could not have known it to be even. The solution suggested is to say that one does not know every dyad to be even, but only that which one knows to be a dyad. On the other hand one knows that of which one possesses or has received a demonstration, and no demonstration concerns merely (e.g.) every triangle, or number, one may happen to know, but every possible triangle or number. No demonstrative proposition is taken as referring to 'any number you may know of,' or 'any straight line you may know of,' but to the entire subject. Nothing, however, I should suppose, precludes our knowing already what we learn from one point of view and not knowing it from another. The absurdity would consist not in having some sort of knowledge of what one learns, but in having knowledge of it in a certain respect—I mean in the very same respect and manner in which one learns it.

## **2. What Knowing is, what Demonstration is, and of what it consists**

We suppose ourselves to know anything absolutely and not accidentally after the manner of the sophists, when we consider ourselves to know that the ground from which the thing arises *is* the ground of it, and that the fact cannot be otherwise. Science must clearly consist in this, for those who suppose themselves to have scientific knowledge of anything without really having it imagine that they are in the position described above, while those who do possess such knowledge are actually in that position in relation to the object.

Hence it follows that everything which admits of absolute knowledge is necessary. We will discuss later the question as to whether there is any other manner of knowing a thing, but at any rate we hold that that 'knowledge comes through demonstration.' By 'demonstration' I mean a scientific syllogism, and by 'scientific' a syllogism the mere possession of which makes us know.

If then the definition of knowledge be such as we have stated, the premises of demonstrative knowledge must needs be true, primary, immediate, better known than, anterior to, and the cause of, the conclusion, for under these conditions the principles will also be appropriate to the conclusion. One may, indeed, have a syllogism without these conditions, but not demonstration, for it will not produce scientific knowledge. The premises must be true, because it is impossible to know that which is not, e.g. that the diagonal of a square is commensurate with the side. The conclusion must proceed from primary premises that are indemonstrable premises, for one cannot know things of which one can give no demonstration, since to know demonstrable things in any real sense is just to have a demonstration of them. The premises must be Causal, Better known and Anterior; Causal, because we only know a thing when we have learned its cause, Anterior because anteriority is implied by causation, previously known not only in our second sense, viz. that their meaning is understood, but that one knows that they exist.

Now the expressions 'anterior' and 'better known' have each a double meaning; things which are naturally anterior are not the same as things anterior to us, nor yet are things naturally better known better known to us. I mean by things anterior, or better known, 'to us,' such as are nearer our sense-perception, while things which are absolutely anterior or better known are such as are more removed from it. Those things are the furthest removed from it which are most Universal, nearest to it stands the Particular, and these two are

diametrically opposed.

The phrase ‘the conclusion must result from primary principles’ means that it must come from elements appropriate to itself, (for I attach the same meaning to primary principle [πρωτον] and to element [αρχή]). Now the element of demonstration is an immediate proposition; ‘immediate’ meaning a proposition with no other proposition anterior to it. A premise is either of the two parts of a predication, wherein one predicate is asserted of one subject. A dialectical premise is one which offers an alternative between the two parts of the predication, a demonstrative premise is one which lays down definitely that one of them is true.

Predication is either part of a Contradiction. Contradiction is an opposition of propositions which excludes any intermediate proposition. That part of a Contradiction which affirms one thing of another is Affirmation, that which denies one thing of another is Negation.

I apply the name Thesis to an immediate syllogistic principle which cannot be proved, and the previous possession of which is a necessary condition for learning something, but not all. That which is an indispensable antecedent to the acquisition of any knowledge I call an Axiom; for there are some principles of this kind, and ‘axiom’ is the name generally applied to them.

A Thesis which embodies one or other part of a predication (that is that the subject does, or does not, exist) is a Hypothesis; one which makes no such assertion a Definition. Definition is really a kind of Thesis; e.g. the arithmetician ‘lays it down’ that Unity is indivisibility in respect to quantity, but this is not a Hypothesis, for the nature of unity and the fact of its existence are not one and the same question.

Since then belief and knowledge with regard to any subject result from the possession of a demonstrative syllogism, and since a syllogism is demonstrative when the principles from which it is drawn are true, we must not merely have a previous knowledge of some or all of these primary principles, but have a higher knowledge of them than of the conclusion.

The Cause always possesses the quality which it impresses on a subject in a higher degree than that subject; thus, that for which we love anything is dear in a higher degree than the actual object of our love. Hence if our knowledge and belief is due to its primary principles, we have a higher knowledge of these latter and believe more firmly in them, because the thing itself is a consequence of them. Now it is not possible to believe less in what one knows than in what one neither knows nor has attained to by some higher faculty than knowledge. But this will happen unless he whose belief is produced by demonstration has a previous knowledge of the primary principles, for it is more needful to believe in these principles, either all or some, than in the conclusion to which they lead.

Now in order to attain to that knowledge which comes by demonstration one must not only be better acquainted with and believe more firmly in the elementary principles than in the conclusion, but nothing must be better known nor more firmly believed in than the opposites of those principles from which a false conclusion contrary to the science itself can be educed; that is to say if he who possesses absolute knowledge is to be quite immovable in his opinions.

### **3. A refutation of the error into which some have fallen concerning Science and Demonstration**

Now some persons, because of the necessity of knowledge of the primary principles, infer

that knowledge does not exist, while others suppose that it does exist and that everything whatever is capable of demonstration. Neither of these views is either true or necessary. Those who assume that knowledge is not possible at all, think that it would involve an infinite regress, since one cannot know the later terms of a series by means of the earlier when such a series has no primary terms. In this they are right, for it is impossible to complete the infinite. But if there be a limit to the regress, and primaries do exist, they say that these must be unknowable, supposing that they admit of no demonstration, which is the only way of knowing they allow to exist. But if it be impossible to learn these primary principles, one cannot know their results either absolutely or in any proper sense, but only hypothetically, viz. on the assumption that such principles do exist.

The other party agrees with them in holding that knowledge can only be attained by demonstration, but considers that there is nothing to prevent a demonstration of everything being given, maintaining that demonstration may proceed in a circle, all things being proved reciprocally.

We, on the other hand, hold that not every form of knowledge is demonstrative, but that the knowledge of ultimate principles is indemonstrable. The necessity of this fact is obvious, for if one must needs know the antecedent principles and those on which the demonstration rests, and if in this process we at last reach ultimates, these ultimates must necessarily be indemonstrable. Our view then is not only that knowledge exists, but that there is something prior to science by means of which we acquire knowledge of these ultimates. On the other hand it is clear that absolute demonstration cannot proceed in a circle if it be admitted that the demonstration must be drawn from anterior and better known principles than itself; for it is impossible for the same things to be both anterior and posterior in relation to the same objects, except from a different point of view, e.g. some things may be anterior relatively to us and others absolutely anterior, a distinction which inductive proof illustrates. If this be so the definition of absolute knowledge might be considered defective, since it really has a double sense; or that second kind of demonstration drawn from principles better known in relation to us is ambiguous.

Those who hold that demonstration proceeds in a circle not only meet with the difficulty already mentioned, but really say that 'this is if this is,'—an easy method of proving anything whatsoever. This appears plainly when three terms are assumed (for it is immaterial whether one says that the proof passes through many or few terms before returning to the starting point, as also whether it be through a few or two only). For when: If A is, B must be; and if B is, C must be. Then, if A is, C will be. And when, if A is, B must be; and, if B is, A must be.

...Let A be placed in the position C held before. Then to say that 'If B is, A must be,' is equivalent to saying that C must be, and this proves that 'If A is, C must be'; and C is here identical with A.

Thus those who hold that the demonstration proceeds in a circle simply declare that if A is, A must be—an easy method of proving anything.

Nor is even this proof possible except in the case of reciprocals such as Properties. It has been already shewn (Prior An. II. 5) that it is never necessary that a conclusion should follow when only one thing is assumed (by 'one thing' I mean one term or one proposition); such can only happen when there are at least two antecedent propositions capable of producing a syllogism.

If then A be a consequence of B and of C, and these latter consequences of each other, and also of A, it is possible to prove reciprocally all the questions that can be raised, in the

first figure, as has been shewn in the treatise on the Syllogism (Prior An. II. 5). But it has also been shewn that in the other figures no circular demonstration can be effected, or none concerning the premises in question.

Circular demonstration is never admissible in the case of terms not reciprocal. Hence, as few such terms occur in demonstrations, it is clearly useless and untrue to maintain that demonstration consists in proving each term of a series by means of the others, and that consequently everything is demonstrable.

#### **4. The meaning of 'Distributive,' 'Essential,' 'Universal'**

Now since the object of absolute knowledge can never undergo change, the objects of demonstrative knowledge must be necessary. Knowledge becomes demonstrative when we possess a demonstration of it, and hence demonstration is a conclusion drawn from necessary premises. We must now then state from what premises and conclusions demonstrations may be drawn; and first let us define what we mean by 'Distributively true,' 'Essential' and 'Universal.'

By 'Distributively true,' I mean a quality which is not merely present in some instances and absent in others, or present at some times and absent at others; e.g. if the quality 'Animal' be distributively predicable of man, if it be true to say 'this is a man,' it must also be true to say 'this is an animal'; and if he be the one now, then he must be the other now; so too if 'Point' be true of every line. An empirical proof of this is the fact that when the question is raised whether one thing is true of another distributively, our objections take the form of asserting that it is not true of some particular instance or at some particular time.

I. 'Essential' qualities are all those which enter into the essence of a thing, (as 'line' does into that of 'triangle,' and 'point' into that of 'line'; for 'line' and 'point' belong to the essence of 'triangle' and line respectively), and are mentioned in their definition.

II. Essential qualities are, further, attributes of subjects in the definition of which the subject is mentioned, thus 'Straight' or 'Curved' are essential attributes of 'Line'; 'Odd' or 'Even' of 'Number'; as also 'Prime' and 'Compound,' 'Equilateral' (as 3) and 'Scalene' (as 6); in all these cases the things form part of the definition of the real nature of the attributes mentioned, these things being in the first instance 'Line,' in the second 'Number.' So too in other instances I call attributes which inhere in either of these ways 'essential,' while attributes which do not belong to the subject in either of these ways I call 'accidental'; e.g. 'Musical' or 'White' as applied to 'Animal.'

III. Thirdly, essential is that which is not predicated of anything other than itself as attribute of subject; thus if I say, 'the walking thing,' some other independent thing is 'walking' or is 'white.' On the other hand substances and everything which denotes a particular object are not what they are in virtue of being anything else but what they are. Things then which are not predicable of any subject I call 'essential,' those which are so predicable 'accidental' [in the sense of dependent].

IV. In a fourth sense the attribute which exists in a subject as a result of itself is essential, while that which is not self-caused is accidental. E.g. Suppose lightning to appear while a person is walking. This is accidental, for the lightning is not caused by his walking, but, as we say, 'it was a coincidence.' If, however, the attribute be self-caused it is essential: e.g. if someone is wounded and dies, his death is an essential consequence of the wound, since it has been caused by it:—the wound and death are not an accidental coincidence. In



the case then of the objects of absolute knowledge, that which is called 'essential' in the sense of inhering in the attributes or of having the latter inhering in it is self-caused and necessary; for it must inhere either unconditionally or as one of a pair of contraries, e.g. as either straight or curved inhere in line, odd or even in number. Contrariety consists in either the privation or the contradiction of a quality in the case of homogeneous subjects: e.g. in the case of numbers 'even' is that which is not 'odd,' in so far as one of these qualities is necessarily present in a subject. Hence, if one of these qualities must be either affirmed or denied, essential attributes are necessary. This then may suffice for the definition of Distributive and Essential.

By 'Universal' I mean that which is true of every case of the subject and of the subject essentially and as such. It is clear then that all universal attributes inhere in things necessarily. Now 'essentially' and 'as such' are identical expressions: e.g. Point and Straight are essential attributes of line, in that they are attributes of it as such. Or again the possession of two right angles is an attribute of triangle as such, for the angles of a triangle are essentially equal to two right angles. The condition of universality is satisfied only when it is proved to be predicable of any member that may be taken at random of the class in question, but of no higher class; e.g. the possession of two right angles is not a universal attribute of figure, for though one may demonstrate of a particular figure that it has two right angles, it cannot be done of any and every figure, nor does the demonstrator make use of any and every figure, for a square is a figure, but its angles are not equal to two right angles. Any and every isosceles triangle has its angles equal to two right angles, but it is not a primary, 'triangle' standing yet higher. Thus any primary taken at random which is shewn to have its angles equal to two right angles, or to possess any other quality, is the primary subject of the universal predicate, and it is to that demonstration primarily and essentially applies; to everything else it applies only in a sense. Nor is this quality of having its angles equal to two right angles a universal attribute of isosceles triangle, but is of a wider application.

## **5. From what causes mistakes arise with regard to the discovery of the Universal. How they may be avoided**

We must not fail to notice that mistakes frequently arise from the primary universal not being really demonstrated in the way in which it is thought to be demonstrated. We fall into this mistake firstly when no universal can be found above the particular or particulars: secondly, when such a universal is found applicable to specifically different subjects, but yet has no name; thirdly, when the universal to be demonstrated stands to the true universal in the relation of part to whole.

In this last case the demonstration is indeed applicable to all the particular parts, but will not contain a primary universal. I consider the demonstration to be primary and essential when it is a demonstration of a primary universal. If then it were to be proved that perpendiculars to the same line are parallel, it might be thought that this was the primary subject of the demonstration because it is true in the case of all right angles so formed. This, however, is not the whole truth. The lines are parallel not because each of the angles at their base is a right angle, and consequently equal to the other, but because such angles are in all cases equal to two right angles.

So, too, if there were no other kind of triangle than the isosceles it might be supposed that the quality of possessing angles equal to two right angles was true of the subject as isosceles.

Again, the law that proportionals, whether numbers, lines, solids, or periods of time, may be permuted, would be a case, as it used to be proved, viz., of each case separately, though it may really be proved of all together by means of a single demonstration; but since no single designation included magnitudes, times and solids, and since these differ specifically, they were treated of separately. The law is now, however, proved universally. It does not apply to numbers or lines as such, but only because it belongs to the universal conception as such in which all are supposed to be. Hence even if it be proved of equilateral, scalene and isosceles triangles separately, whether by means of the same or by different proofs that every one has its angles equal to two right angles, one will not know except accidentally, that triangle possesses this quality nor will one know it of the universal triangle, even though there is no other sort of triangle than those mentioned. One does not in fact know it of triangle as such, nor yet of every individual triangle, except distributively, nor does one know it of every triangle ideally, even if there is no triangle of which one does not know it.

When, we may ask, is our knowledge not universal and when is it absolute? It is clear that our knowledge of the law would be universal if triangularity and equilateral triangularity were identical in conception. If, however, the two concepts be not identical but diverse, and if the quality in question belong to triangle as such, then a knowledge of the law as relating merely to a particular form of triangle is not universal. Now does this quality belong to triangle as such, or to isosceles triangle as such? Further, what is its essential primary subject? Also, when does the demonstration of this establish anything universal? Clearly when, after the elimination of accidental qualities, the quality to be demonstrated is found to belong to the subject and to no higher subject. For example, the quality of having its angles equal to two right angles will be found to belong to bronze isosceles triangle, but will still be present when the qualities 'bronze' and 'isosceles' are eliminated; so too, it may be said they will cease to be present when Form or Limit are eliminated. But they are not the first conditions of such disappearance. What then will first produce this result? If it is triangle, the quality of having two right angles belongs to the particular kinds of triangles as a result of its belonging essentially to triangle, and the demonstration in regard to triangle is a universal demonstration.

## **6. Demonstration is founded on Necessary and Essential Principles**

If then demonstrative knowledge be derived from necessary principles (and that which one knows is never contingent), and if the essential attributes of a subject be necessary (and essential attributes either inhere in the definition of the subject, or, in cases where one of a pair of opposites must necessarily be true, have the subjects inhering in their definition), then it is clear that the demonstrative syllogism must proceed from necessary premises. Every attribute is predicable either in the way mentioned or accidentally, but accidental attributes are not necessary. We should then either express ourselves as above or lay it down as an elementary principle that demonstration is something necessary, and that if a thing has been demonstrated it can never be other than it is; and consequently that the demonstrative syllogism must proceed from necessary premises. It is indeed possible to syllogize from true premises without demonstrating anything, but not so if the premises be also necessary, for this very necessity is the characteristic of demonstration.

An empirical confirmation of the view that demonstration results from necessary premises

is that when we bring forward objections against persons who imagine themselves to be producing a demonstration, we bring our objections in the form ‘There is no necessity.’ Whether we hold that the things in question are really contingent or only considered to be so for the sake of a particular argument. It is clear from this that it is folly to suppose oneself to have made a good choice of scientific principles so long as the premise be generally accepted and also true, after the manner of the sophists who assume that ‘Knowing is identical with possessing knowledge.’ It is not in fact that which is generally accepted or rejected which constitutes a principle, but the primary properties of the genus with which the demonstration deals; nor is everything which is true also appropriate to the conclusion to be demonstrated.

It is also clear from the following considerations that the syllogism can proceed from necessary premises only. If one who, in a case where demonstration is possible, is not acquainted with the cause, can have no real knowledge of the demonstration, then one who knows that A is necessarily predicable of C, whilst B, the middle term by means of which the demonstration has been effected, is not necessary, must be ignorant of the cause of the thing, for in this case the conclusion is not rendered necessary by the middle term; in fact the middle, since it is not necessary, may not exist at all, but the conclusion is necessary.

Moreover if one who now knows (accidentally) the cause of a necessary conclusion remain unchanged while the thing itself remains unchanged, and if, though he has not forgotten it, yet he has no real knowledge of it, then he can never have had any real knowledge of it before. When the middle term is not anything necessary, it may pass away. In such a case, if the man remain unchanged while the thing remains unchanged, he may hold fast the cause of the thing, but he has no real knowledge of the thing itself, nor has he ever had such knowledge. But if the thing denoted by the middle term has not passed away, but yet is capable of doing so, that which results from it is only the possible, not the necessary; and when one’s inference is derived only from the possible one cannot be said to have knowledge in the true sense of the word. When the conclusion is necessary there is nothing to prevent the middle term, by means of which the conclusion was proved from being necessary, for it is possible to infer the necessary from the not necessary, just as one may infer the true from the untrue.

But when the middle term is necessary the conclusion also is necessary, just as true premises always produce a true conclusion. Thus, suppose A to be necessarily predicable of B, and B of C; A then must be necessarily predicable of C. But when the conclusion is not necessary, it is impossible that the middle should be necessary.

Suppose that, Some C is A, and again that All B is A, and that All C is B. But then All C will be A, which is contrary to our original hypothesis.

Since then that which one knows demonstratively must be necessary, it is clear that one ought to obtain the demonstration by means of a necessary middle term. Otherwise one will neither know the cause of the thing demonstrated nor the necessity of its being what it is, but one will either think one knows it without doing so (that is if one suppose to be necessary that which is not necessary), or one will think one knows it in a different way if one knows the fact of the conclusion with the help of middle terms, and when one knows its cause without the help of middle terms. Now there is no demonstrative science of accidents (attributes) which are not essential according to our definition of ‘essential.’ It is not in this connection possible to prove that the conclusion is necessarily true, for the accidental *may* not be true; (it is of accidents of this kind that I am speaking).



A difficulty might perhaps be raised as to why accidental premises are asked for for the purposes of a conclusion, if the conclusion drawn from them be not necessary; for it might be maintained that it would make no difference if any sort of premise were brought forward and then the conclusion were subjoined. Premises should however be laid down not because the conclusion is necessarily true because of them, but because the person who admits the premises must necessarily admit the conclusion, and his admission will be correct if the premises are true.

Now since only the essential attributes of any genus and those belonging to it as such are necessary, it is clear that scientific demonstrations both deal with and are drawn from essential attributes. As accidental attributes are not necessary one does not require to know the cause of the conclusion, not even if this be an eternal attribute without being essential, as in the case of syllogisms based on universal concomitance. In this latter connection the essential will be known, but not the fact that it is essential, nor yet why it is so. (By 'knowledge of why it is essential' I mean 'knowing its cause.') In order then to possess knowledge of this sort the middle term must result from the nature of the minor, and the major from the nature of the middle.

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Aristotle. *Posterior Analytics*. Book 1, sect 1-6. Trans. E.S. Bouchier. Oxford: Blackwell, 1901.

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