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# Benefiting from fluctuations in markets

## Why mean reversion works, and when



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### Cross-Asset Quant

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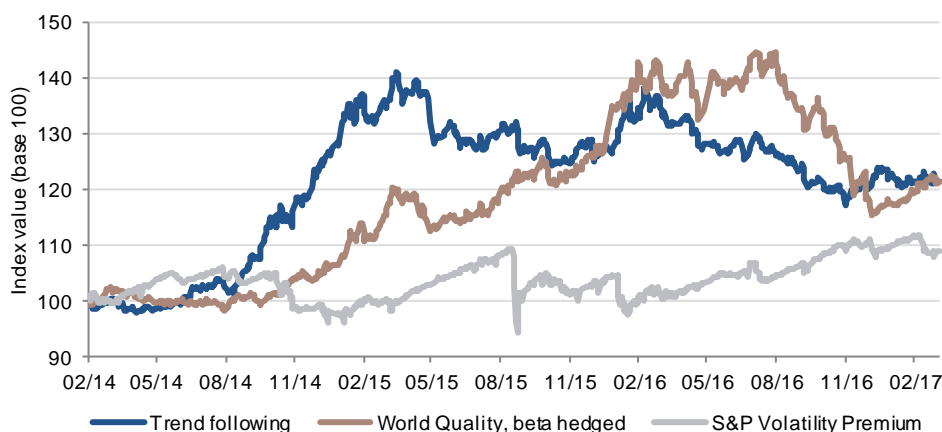
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## How to benefit from fluctuations in markets

Whether quantitative easing (QE) worked used to be a moot point<sup>1</sup>. The ECB president holds a sanguine view of this. As he put it in his latest press conference, “the pass-through of our monetary policy measures is supporting domestic demand and facilitates the ongoing deleveraging process”. For investors, QE brought substantial capital gains in risky assets and long-dated debt. But unconventional monetary policy cuts both ways. In the face of lofty valuations and low interest rates, investors have increasingly looked for alternative sources of returns elsewhere.

Trend systems benefited greatly from the US dollar rally in 2014, high-quality shares outperformed in 2015, and the volatility premium took off in 2016. However, with investors chasing any alternative opportunity, returns eventually leveled out.

### Winners take all<sup>2</sup>



Source: SG Cross Asset Research/Cross Asset Quant

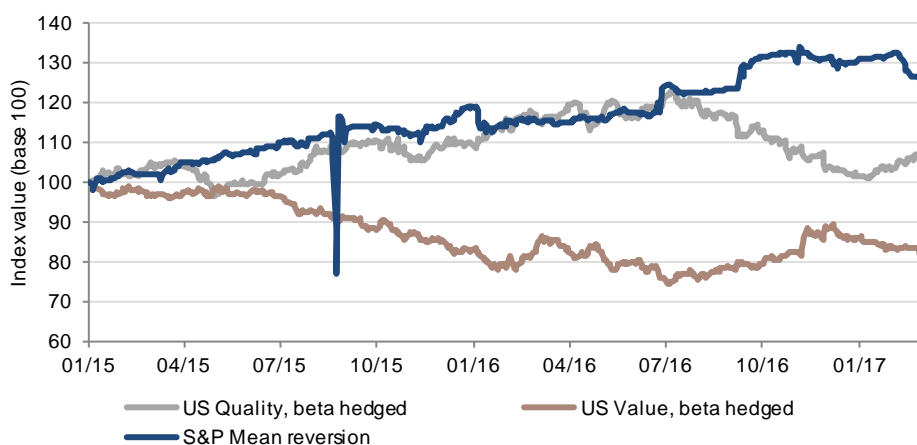
As long as central banks were pulling the strings, there was something obvious to do. Buying shares any time they went down was a simple way to benefit from the ‘Yellen put’. This approach was challenged during the China shock, but even there, central banks did not disappoint.

With the prospect of higher rates weighing on risky assets, there may even have been something like a ‘Yellen call’. The investor who bought the market when the price went down and sold when the price went up pocketed a hefty premium. Meanwhile, a combination of the two main equity factors, value and quality, struggled to beat the market.

<sup>1</sup> For a good overview, see [‘Did Quantitative Easing Work?’](#), Q1 2016, Edison Yu, Federal Reserve Bank of Philadelphia Research Department.

<sup>2</sup> We used the SGIXTXA Bloomberg index for cross-asset trends (see [our latest piece on the subject](#) [XQ9]), SGSLQAW Index for World Quality, and SGIXESPU for the volatility premium in the S&P. Indices were rescaled in order to achieve a volatility of 10%.

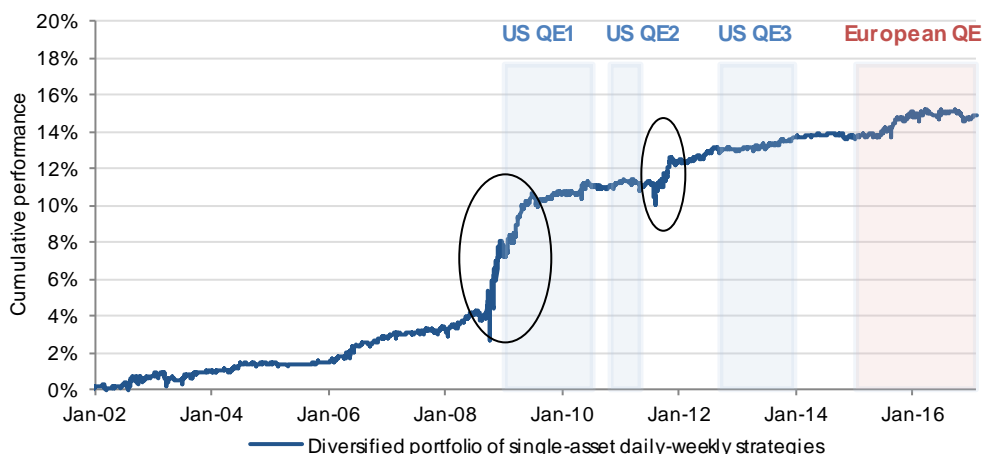
### When central banks are pulling the strings<sup>3</sup>



Source: SG Cross Asset Research

Just as the Federal Reserve is finally taking the plunge<sup>4</sup>, and as a new wave of politicians in the US and Europe look bent on redefining anything from haircuts and dress codes to the pillars of the Western liberal order, any talk of ‘mean reversion’ seems a bit out of sync. However, if we take a step back, mean reversion seems to have benefited greatly from such periods of stress as the subprime and Lehman financial crises, or the European debt crisis in 2010-11.

### Benefiting from market fluctuations



Source: SG Cross Asset Research/Cross Asset Quant

This may sound surprising, but contrarian investors can benefit greatly from market turmoil. Portfolio unwinding, hasty decisions and conflicting flows are all sources of opportunity. Discipline and careful risk management are the keys to harvesting them. Among the two

<sup>3</sup> We used the SGIXUSGR Bloomberg index for mean reversion in the S&P, SGEPQ2RU for US Quality, and SGEPVRU for US Value. Indices were rescaled in order to achieve a volatility of 10%, excluding the week of Aug 25, 2015.

<sup>4</sup> It has been arguably a slow and gentle plunge. See Albert Edward's latest piece, '[Can the pussycat Fed change its dovish spots to a leopard's?](#)'

graphs above, one strategy took a large hit in August 2015, the other did not. One trades the S&P only. The other involves a whole range of financial markets.

In this report, we investigate a particular strategy that is known as 'daily-weekly'. The strategy builds up contrarian positions and prudently unwinds the portfolio at the end of each trading week.

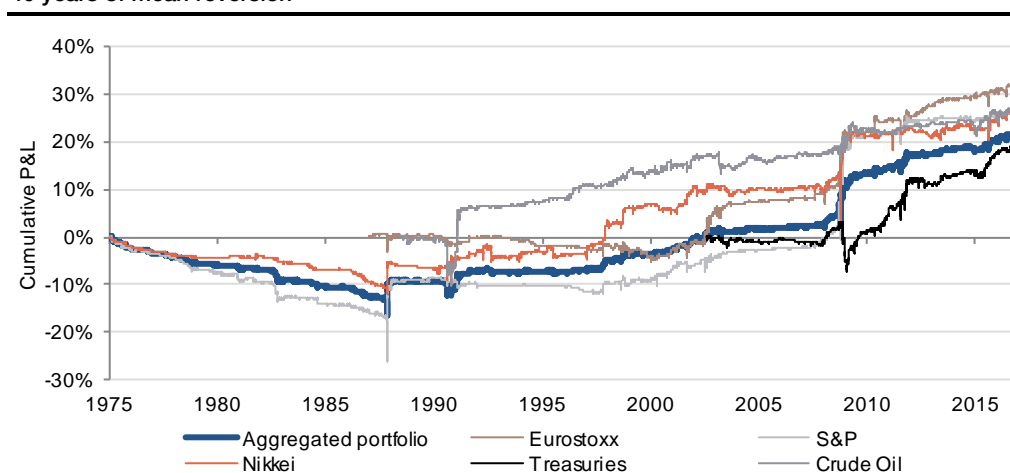
As long as prices are subject to a form of mean reversion, doing so enables investors to benefit from market fluctuations. Buying options or other products is usually the surest means to achieve this gain. The cost of a derivative product reflects its capacity to benefit in certain circumstances. The daily-weekly is a relatively simple strategy that does not trade options and can still benefit from market turmoil. In return, however, it is exposed to potential losses if the market does not revert to the mean.

Mean reversion can play a useful role in a global portfolio. It can come in addition to existing market exposure. For example, the manager of a fund that tracks the S&P can marginally adjust his market exposure using the daily-weekly. It is also an interesting addition to a portfolio of alternative risk premia.

Back in 2013 [RP0] we decided that a risk premium must have demonstrated an attractive positive historical return profile, possess a fundamental value that allows a judgement to be made on future expected returns, and bring diversification benefits when combined into a multi-asset portfolio.

Historical simulations suggest that mean reversion using the daily-weekly has worked consistently since the crash of 1987, for reasons that are probably related to changes in the microstructure of the market.

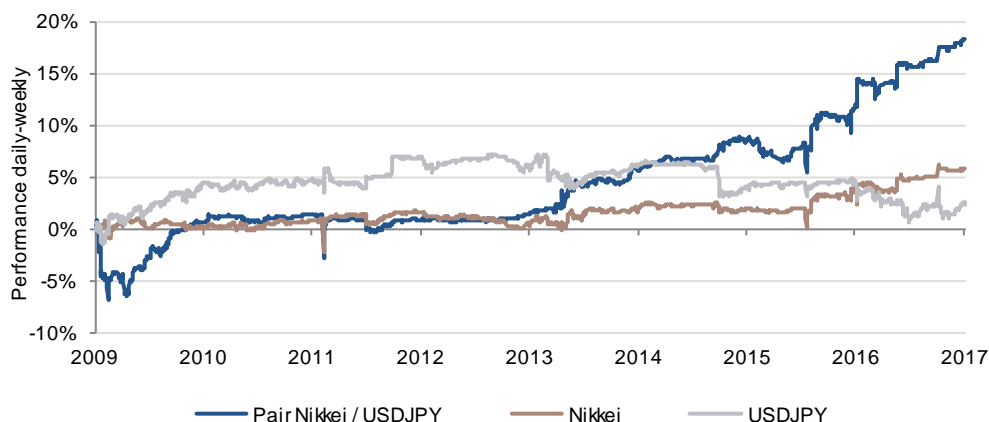
#### 40 years of mean reversion



Source: SG Cross Asset Research/Cross Asset Quant

Certain assets are exposed to macroeconomic factors that do not always revert to the mean. Once properly identified, those factors can be taken away through a pair trade.

### Removing the effect of USD/JPY from the Nikkei results in additional performance



Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bp for single-asset, 2bp for the pair)

The daily-weekly, in turn, is likely to perform well when applied to such carefully selected pairs.

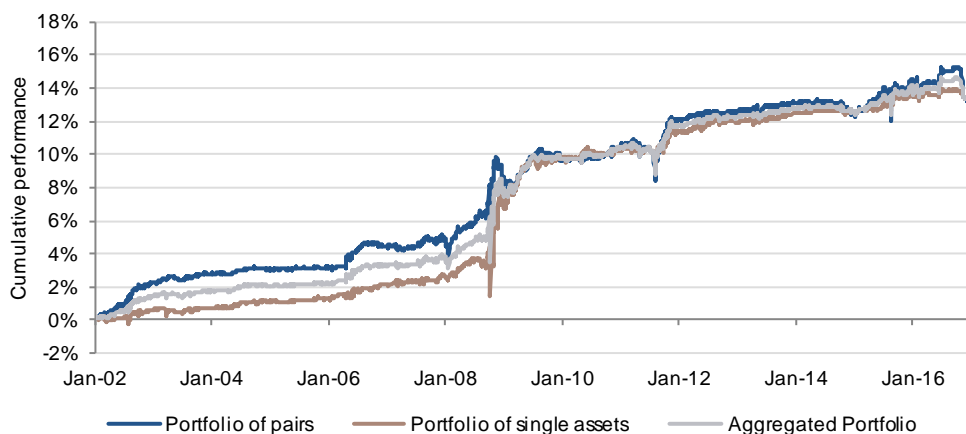
### Risk/return profile of the daily-weekly portfolios (since 2002)

	Return (p.a.)	St. dev. (p.a.)	Return / risk	MDD	MDD / st. dev.	Skewness	Kurtosis
Portfolio of single assets	0.9%	1.3%	0.65	3.1%	2.31	9.3	267.8
Portfolio of pairs	0.9%	1.3%	0.65	2.5%	1.91	5.1	147.1
Aggregated portfolio	0.9%	1.2%	0.72	2.8%	2.33	8.1	229.3

Source: SG Cross Asset Research/Cross Asset Quant

The flip side of the argument is that the link between both legs of a trade can loosen during certain periods of stress. For the contrarian investor, pair trades tend to perform better than single assets in normal times, and they can be more volatile in periods of stress.

### A well diversified portfolio of mean-reversion strategies

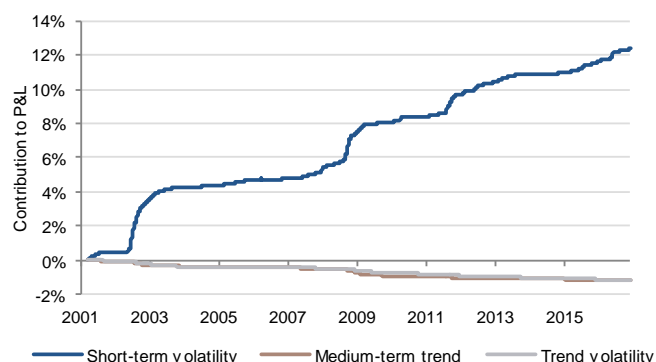


Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bp for single-asset, 2bp for the pair)

We show how to split the performance of the daily-weekly into three parts, one of which benefits from mean reversion and volatility, while the two other ones reflect trends that may emerge in the market. The strategy loses if the market rises or falls steadily, or reverses

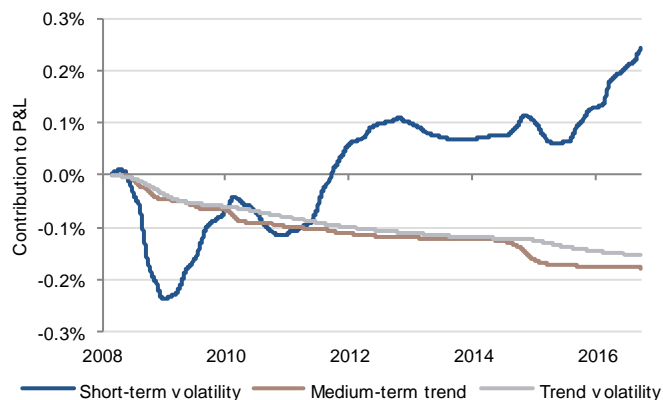
course every week. Mean reversion, daily and weekly volatility, and medium-term trends are the key parameters that determine the returns of the strategy.

#### Eurostoxx is volatile enough and reverts to the mean



Source: SG Cross Asset Research/Cross Asset Quant

#### Trends in the EURUSD offset any mean reversion gains



By monitoring these parameters, investors can identify those assets that are well suited for contrarian trading. By combining statistical measures with their own views about trends and volatility, they can form a view about fundamental value in the future.

Finally, mean reversion can bring diversification benefits to a portfolio.

#### Average correlations within and across factors (since 2006)

	Traditional asset classes	Risk premia	Mean reversion
<b>Market factors</b>	6%	8%	3%
<b>Alternative risk premia</b>	8%	5%	3%
<b>Mean reversion</b>	3%	3%	13%

Source: SG Cross Asset Research/Cross Asset Quant

Over the period and for the assets considered above, the correlation between market factors has been fairly low. This is due to the negative correlation between equities and fixed income. Over this decade, multi-asset mutual funds performed strongly. However, the correlation between stock and bonds may not remain as elevated if rates keep rising. Alternative risk premia and mean reversion can be an interesting source of diversification.

It is also worth noting that there is a high degree of confidence attached to the diversification benefits of alternative risk premia and mean reversion.

#### Standard-deviation of correlations between investment styles (since 2006)

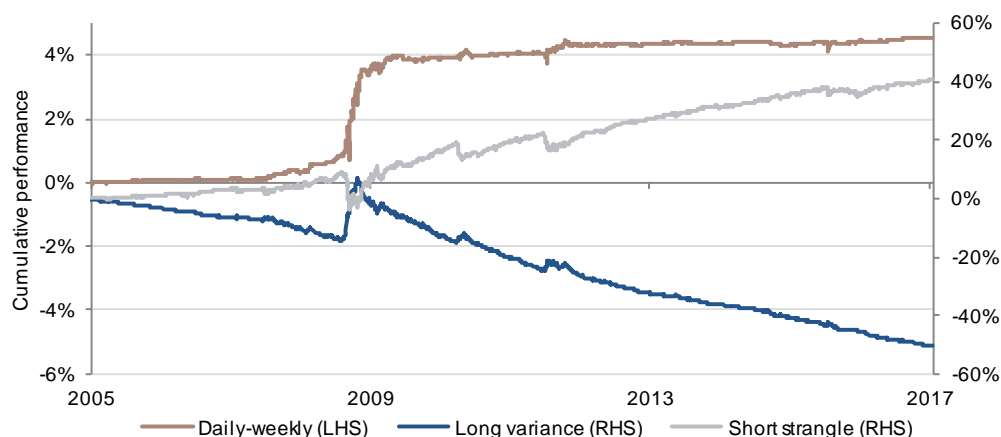
	Traditional asset classes	Risk premia	Mean reversion
<b>Traditional asset classes</b>	+/- 47%	+/- 31%	+/- 9%
<b>Risk premia</b>	+/- 31%	+/- 21%	+/- 10%
<b>Mean reversion</b>	+/- 9%	+/- 10%	+/- 16%

Source: SG Cross Asset Research/Cross Asset Quant

For all these reasons, we view mean reversion as a risk premium. We also show that it exhibits a dual risk profile, a situation which becomes clear once one views mean reversion as the combination of two important strategies. The first one systematically sells options, the second buys what we call synthetic variance.



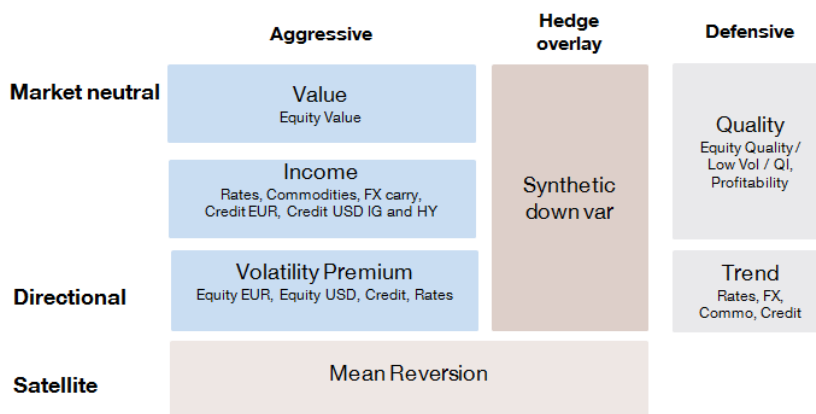
### The fabric of mean reversion



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are included.

We classify mean reversion as a satellite risk premium, which can bring an additional yield pick-up in normal times, and increase returns more substantially in periods of stress. It is both an aggressive premium and a hedge overlay.

### How mean reversion fits into a portfolio of risk premia

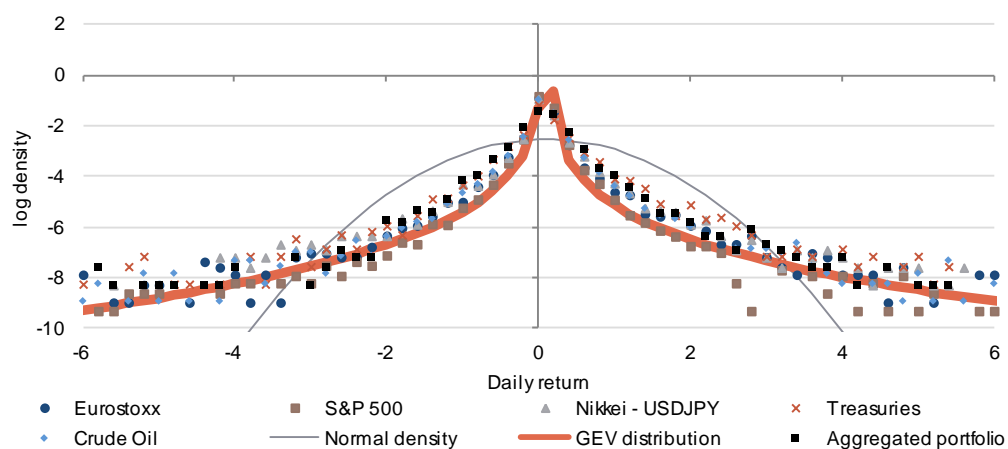


Source: SG Cross Asset Research/Cross Asset Quant

Diversification plays a key role in mitigating risks. By monitoring our key parameters, we show how to select assets and pairs for a well diversified portfolio. Given the level of correlations, a well-diversified portfolio should consist of 15 to 20 strategies.

By investigating past returns over a few decades and a wide range of assets and pairs, we estimate the distribution of risks in a mean reversion portfolio. Using a combination of extreme value theory and our standardised distribution, investors can estimate the level of risks in the portfolio. Conversely, they can also determine the size of the portfolio in order to achieve a level of risk.

### Standardized fat-tailed distribution (after volatility rescaling)



Source: SG Cross Asset Research/Cross Asset Quant

Volatility plays a key role in this risk measure. In our view, this observation justifies the use of classic volatility scaling when managing a portfolio. In this process, volatility ought to be measured on very long time windows, using practically as much data as possible.

The first part of this study details the mechanism of the daily-weekly strategy. We relate the return of the strategy to four key parameters, namely mean reversion, short-term volatility, the volatility and medium-term value of trends. The second part shows how mean reversion fits in a global portfolio, and especially in a portfolio of alternative risk premia. In this view, we delve further into the connection between mean reversion and volatility trading. The third part establishes the risk profile of the strategy, and shows the benefits of diversification. The fourth part shows how to build and manage a portfolio of mean reversion strategies, combining single assets and pair trades.

All mathematical formulas and assumptions regarding simulations are fully detailed in the addendum.

## Why mean reversion works, and when

### A strategy for the contrarian investor

The daily-weekly is a relatively simple strategy<sup>5</sup> that does not trade options and can still benefit from market turmoil. The investor who wishes to pursue this strategy must buy according to how much the market fell on the previous day or sell by how much it rose. One contrarian trade is added every day, and the portfolio is unwound at the end of each week.

In the most classic version of the strategy, the investor starts the business week with no positions in the portfolio. On Monday night, he or she compares the price of the market to the close on the previous Friday. If the market fell by 1% over the weekend, the investor buys the market for an amount of 1% of the nominal attributed to this strategy. If prices fall by another 1% from Monday to Tuesday night, the position is doubled. The investor carries on like this and unwinds the portfolio on Friday night, only to start again on the following Monday. Overall, the investor trades once per day and takes on risk from Monday night to Friday night only – that is, during four business days.

In practice, the investor must either trade at a price close but not exactly equal to the closing price or delegate trading to a counterpart that takes on execution risk. Returns can be measured on a percentage basis or as absolute changes. The strategy can start on any business day. In an enhanced version of the strategy, the investor can also keep a position open during the weekend and the following Monday<sup>6</sup>.

In the following, all our numerical simulations are based on this enhanced version, splitting the portfolio into five parts, each of which resettles at the end of a given day in the business week. But let us temporarily revert to the more classic version of the strategy, unwinding all positions on Friday night.

#### P&L decomposition for the standard daily-weekly strategy

Day	Daily P&L	Decomposition daily P&L			
		Order 1	Order 2	Order 3	Order 4
Day 1	0				
Day 2	$-\Delta R1 \cdot \Delta R2$	$-\Delta R1 \cdot \Delta R2$			
Day 3	$-(\Delta R1 + \Delta R2) \cdot \Delta R3$	$-\Delta R2 \cdot \Delta R3$	$-\Delta R1 \cdot \Delta R3$		
Day 4	$-(\Delta R1 + \Delta R2 + \Delta R3) \cdot \Delta R4$	$-\Delta R3 \cdot \Delta R4$	$-\Delta R2 \cdot \Delta R4$	$-\Delta R1 \cdot \Delta R4$	
Day 5	$-(\Delta R1 + \Delta R2 + \Delta R3 + \Delta R4) \cdot \Delta R5$	$-\Delta R4 \cdot \Delta R5$	$-\Delta R3 \cdot \Delta R5$	$-\Delta R2 \cdot \Delta R5$	$-\Delta R1 \cdot \Delta R5$
		4 terms	3 terms	2 terms	1 term

Source: SG Cross Asset Research/Cross Asset Quant

Starting with nothing in the portfolio on day 1, namely Monday, positions gradually build up in relation to daily returns. The value of the portfolio then fluctuates with the market. When the portfolio is unwound, the investor receives or pays an amount of money that represents the product between daily returns with various lags. In the long run, such products are directly related to the so-called autocovariance of daily returns and any trend that may arise in the market.

<sup>5</sup> See March 2016 publication [When pair trades revert to the Mean – A simple strategy](#) [XQ5].

<sup>6</sup> More details are given in pages 4-8 of [XQ5].

### Expected return of the daily-weekly strategy

If we assume that there are five business days within a week and 250 business days within a year, the annualised expected return of the daily-weekly is related to lagged covariances and the square of the trend in asset prices:

$$\Pi = - \sum_{i=1}^4 (\text{cov}(r_t, r_{t-i}) + \mu^2) \cdot \frac{5-i}{250} \cdot \frac{1}{5}$$

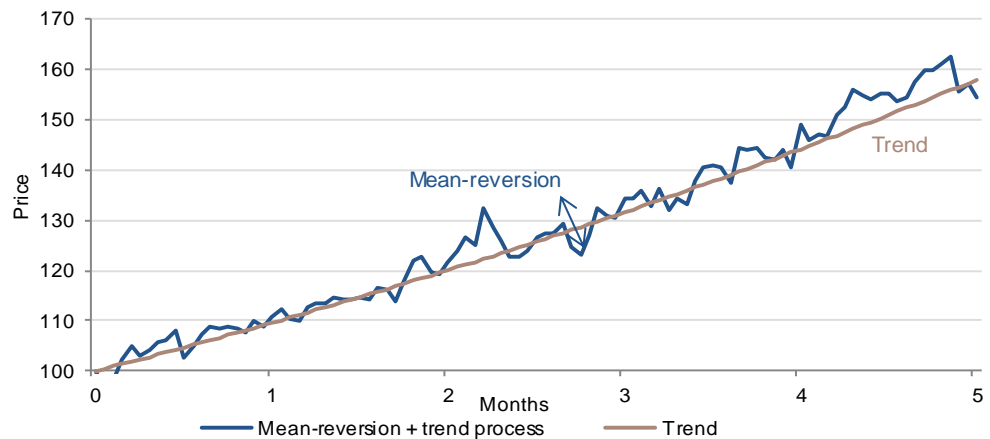
All calculation details are available in Addendum 2.

If the market reverts to the mean, autocovariances are negative and the strategy is likely to make a profit. However, the strategy is negatively impacted by any steady rise or fall in the market, in proportion of the square of the trend. Any gains or losses are thus related to the strength of the trend versus that of mean reversion. We investigate this point further in the next paragraphs.

### The key drivers of performance

Let us assume for a moment that the market fluctuates around a long-term trend – the market may temporarily stray off course, but sooner or later it returns to its long-term path.

#### Stock price simulation in our trend and mean-reversion model



Source: SG Cross Asset Research/Cross Asset Quant

### Mean reversion with a trend

We assume that the underlying asset follows a process composed of a mean-reverting process and a trend. The mean-reverting process is modelled as an Ornstein-Uhlenbeck process:

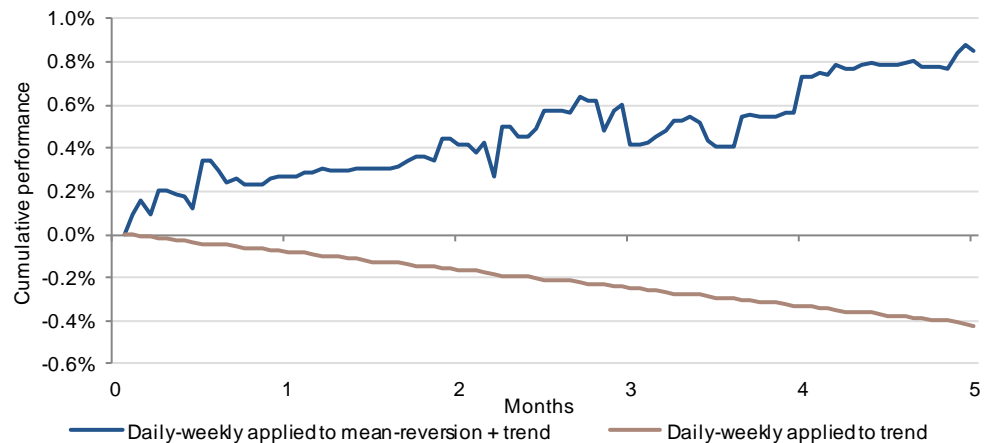
$$dY_t = -aY_t dt + \sigma dW_t$$

where  $a$  captures mean-reversion and  $\sigma$  volatility.  $Y$  represents the deviation between the market, in real or log terms, and the long-term trend  $\mu$ . The price of the market,  $X$ , evolves according to:

$$dX_t = dY_t + \mu dt$$

The model involves three parameters: mean reversion, volatility and trend. Let us simulate the performance of the daily-weekly strategy, once with the trend only and once including the mean-reversion component. As it turns out, the daily-weekly benefits from mean reversion but steadily loses due to the trend.

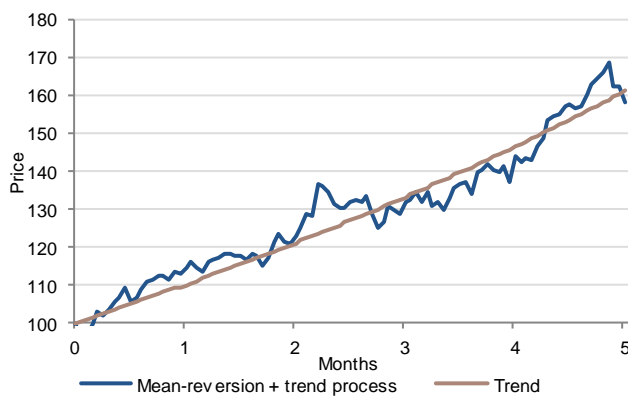
#### When mean reversion benefits outweigh the trends



Source: SG Cross Asset Research/Cross Asset Quant

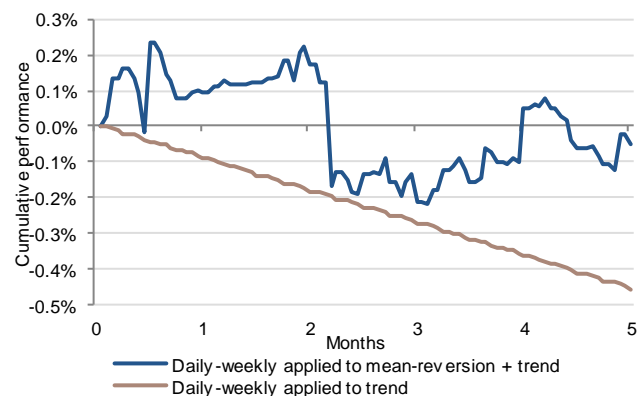
Mean reversion needs to be strong enough to compensate for the negative impact of trends. In the same simulation set up, if we decrease mean reversion by a factor of 10, the strategy no longer performs well.

#### If mean reversion occurs too slowly...



Source: SG Cross Asset Research/Cross Asset Quant

#### ... it is too soft to compensate for the negative impact of trend



This observation can be further refined if we use the model to derive an analytical formula. The expected return of the strategy appears as the sum of two terms, one for mean reversion and the other one for the trend.

### Taking a fresh look at the daily-weekly strategy

If we assume that the underlying process follows a mean-reverting process with a trend, the expected annualised P&L of the daily-weekly strategy is given by:

$$\Pi = \frac{1}{2} \frac{N}{250} [a \cdot \sigma^2 - \mu^2]$$

N is to the number of active trading days for the strategy within a week. It is in general equal to 4 for the standard daily-weekly strategy. Here we assume 250 trading days per years. The previous formula needs to be multiplied by a factor 1.5 for the enhanced daily-weekly strategy. All calculation details are available in the Addendum 3.

The strategy benefits from mean reversion, all the more so when the market fluctuates widely. The daily-weekly may appear to be a good way to buy volatility for free, but as science-fiction writer Robert Heinlein famously put in *The Moon is a Harsh Mistress*<sup>7</sup>, “one way or other, what you get, you pay for”. In our particular case, the capacity to benefit from volatility, though initially free, is ultimately paid for by exposure to trends. In the long run, the strategy makes sense if mean reversion holds over time and if trends are small enough compared to volatility.

In real life, asset prices do not follow constant trends, much to investors’ dismay. Our formula can be adjusted to incorporate the volatility of the trend. Mean-reversion gains are now offset by two negative forces, one related to the volatility of the trend and another to a trend that materialises in the long run.

### The key drivers of performance

If the trend is a random variable with mean  $\bar{\mu}$  and standard deviation  $\sigma_{\mu}$ , the expected annualised return of daily-weekly strategy becomes:

$$\Pi = \frac{1}{2} \frac{N}{250} (a \cdot \sigma^2 - [\bar{\mu}^2 + \sigma_{\mu}^2])$$

All calculations and estimation procedures are detailed in Addendum 3.

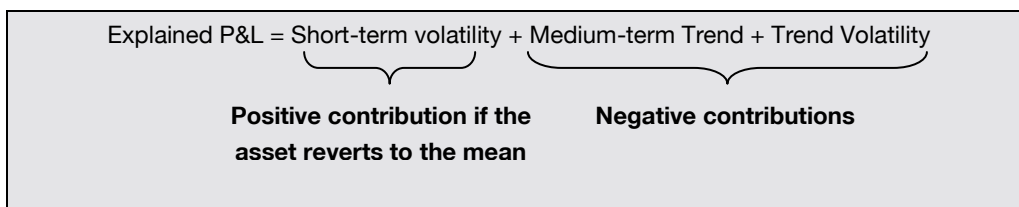
Let us point out in passing that, in both formulae, returns are weighted by the number of active trading days in a week. This factor is in the order of 6/250, that is, 2.4% on an annualised basis. Any gains or losses are likely to be small. This should come as no surprise. Each day, the strategy takes on positions in relation to past daily returns. Unless the market rises or falls by 100%, such positions are bound to remain small, leaving most of the nominal attributed to the strategy unused. The strategy is naturally underleveraged, and its nominal needs to be adjusted to achieve a meaningful return. This point is addressed in the fourth part of this study.

<sup>7</sup> 1966, G.P. Putnam’s Sons. The book is considered to have brought the expression ‘TANSTAAFL’, for ‘there ain’t no such thing as a free lunch’, into mainstream use.

Beyond mean reversion, this approach involves three key parameters, each of which is attached to a different time period. The volatility of the market is based on daily returns, while that of the trend involves weekly changes. The average trend is measured as a return over a period of six months. We refer to it as a 'medium-term trend'.

This formula sheds new light into the risk profile of the daily-weekly strategy<sup>8</sup>. The strategy benefits from the volatility of daily returns, through mean reversion. It is also short weekly volatility. The next part of this paper further investigates the connection between the daily-weekly and other volatility trades.

The interplay between the three parameters also determines the return of the strategy. Using this formula, investors can identify the markets that can be effectively traded in such a contrarian way. Asset prices need to revert to the mean quickly enough, and the product of mean reversion and daily volatility must offset any losses due to the two trend terms. In doing so, the investor can combine statistical analysis with his or her own views about future patterns.

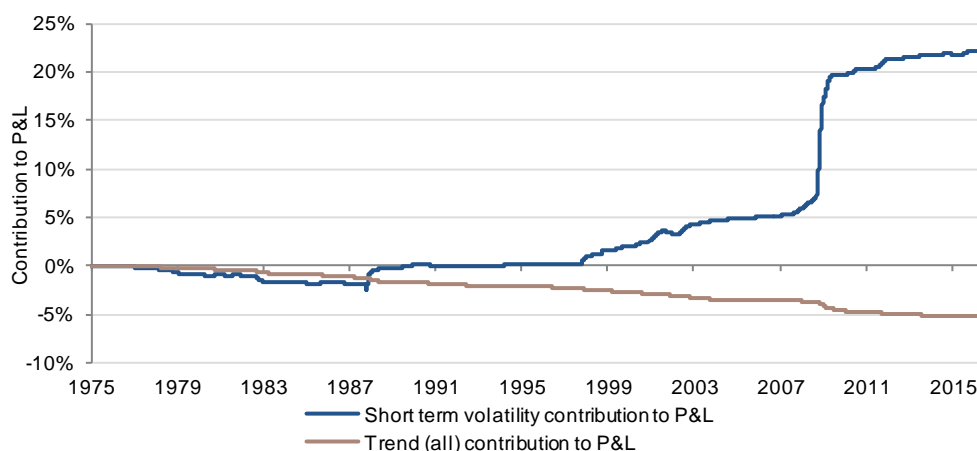


## David versus Goliath

The S&P is arguably the king of all market indices. How, then, can the contrarian investor confront this market giant? After all, the S&P has been subject to some powerful trends. The bull market in the 1990s and the downturn during the subprime crisis are cases in point. As it turns out, mean reversion has been consistently strong enough to leave the contrarian investor enough leeway to benefit from market fluctuations. And fluctuate the market did, handsomely.

<sup>8</sup> A relatively similar formula was published on page 8 of [XQ2]. At the time, we used a slightly different model, which was based on the correlation between successive daily returns. The connection between both approaches is the subject of Addendum 8.

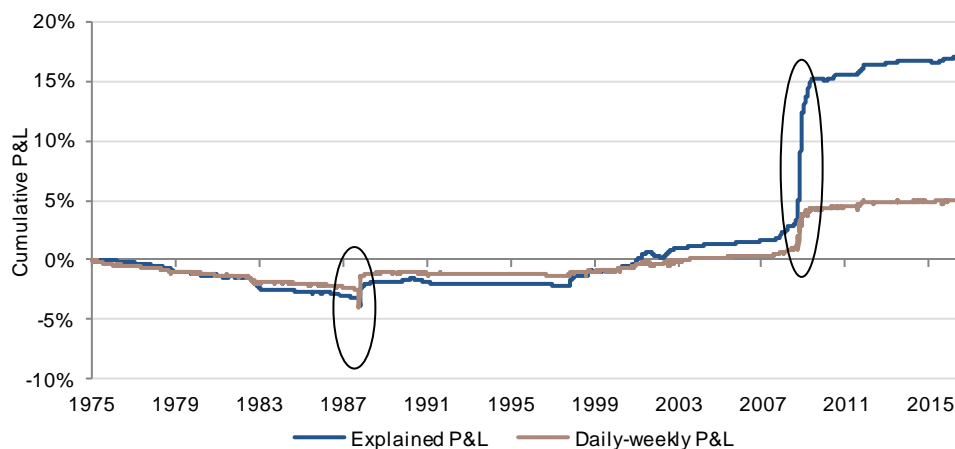
### Tossed by the waves but never sunk



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

Even during the mighty 1990s, mean reversion benefits were strong enough to offset the impact of trends, at least before transaction costs. Overall, the strategy would have achieved a return of 68% of volatility, before costs, since the beginning of the century. Much of this good performance is explained by our model.

### Explaining the performance of the daily-weekly



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

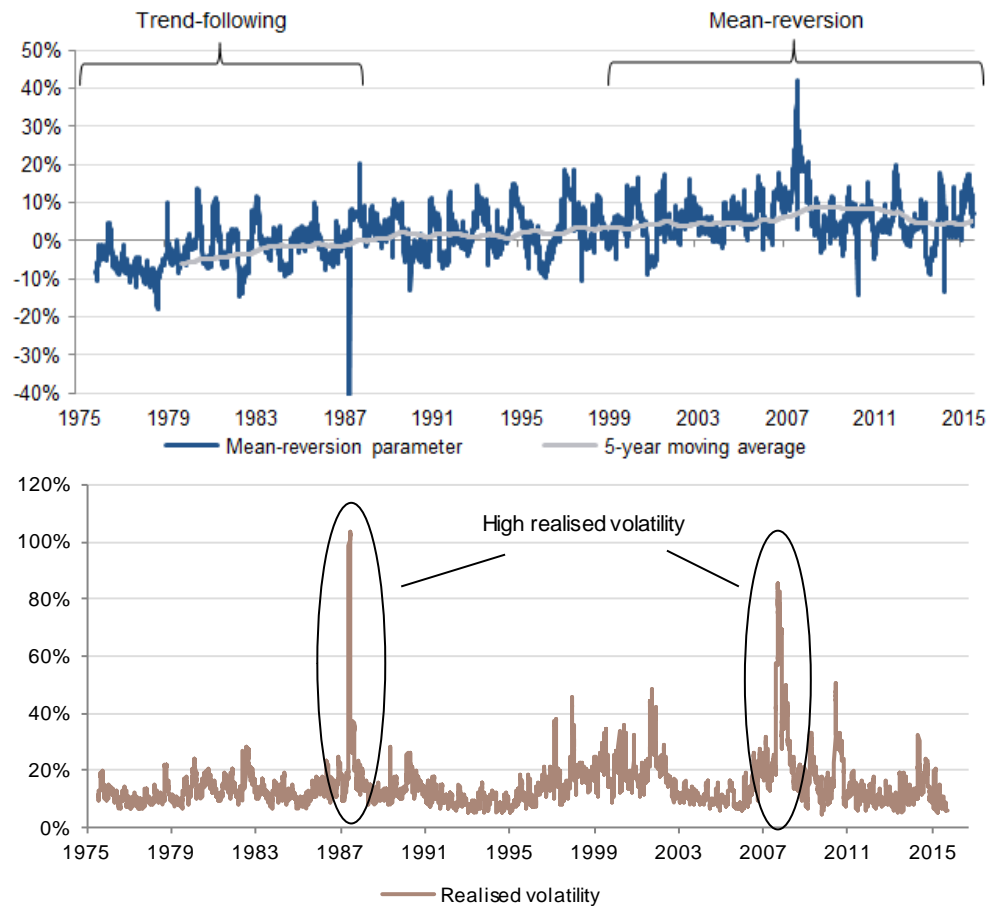
What happened during the crashes of 1987 and 2008 deserves further investigation. Our key parameters provided a poor description of the events of those two fateful months, October 1987 and October 2008. If the market had really followed a classic random walk with trend and mean reversion, the daily-weekly strategy would have lost much less on Black Monday and reaped huge profits in the wake of the subprime crisis.

Such a model is bound to fail during extraordinary events. In the third part of this study, we show how tail risks can be effectively diversified away. At this stage, it is enough to stress that our model accurately captures the performance of the strategy in all other circumstances. The model also highlights an interesting change of pattern, a situation that may be due to a change in the microstructure of the market.



The mean-reversion coefficient was negative until the crash of 1987 and turned positive afterwards. We can suggest two reasons for this change. Circuit breakers were introduced after Black Monday, limiting potential losses in contrarian trades. In another development, the options market grew steadily. Banks and arbitrageurs tend to hedge risks dynamically. If they are long convexity, they act as contrarian traders.

#### A structural shift

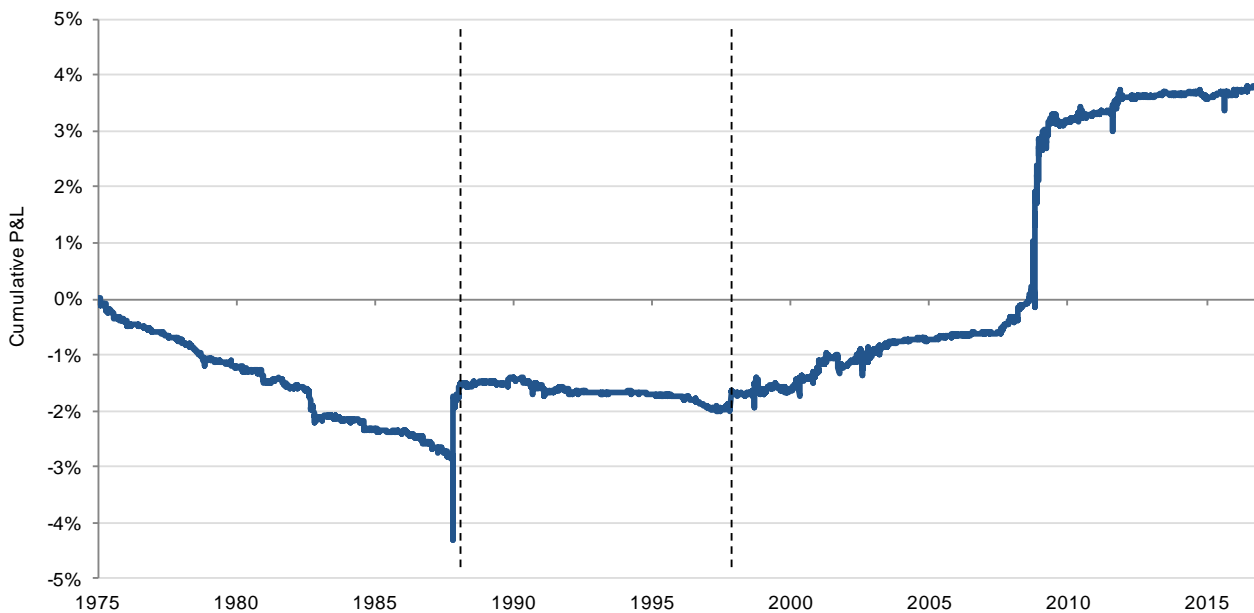


Source: SG Cross Asset Research/Cross Asset Quant

It is worth noting that trends and mean reversion tend to be relatively persistent over time. This observation will play a role when we discuss how to build and manage a portfolio of mean-reversion strategies.

We now factor in transaction costs. When applied to the S&P, the daily-weekly strategy would have performed well over the past 40 years and largely benefits from the high volatility during the financial crisis. We identify three distinct periods. Before 1987, the strategy performed negatively due to a lack of mean reversion. Between 1987 and 1997, performance was flat, as the trend components offset mean-reversion gains during the bull market. Since the 1997 Asian crisis, the strategy would have brought steady returns, with strong gains during the subprime crisis.

#### 40 years of daily-weekly, including estimated transaction costs



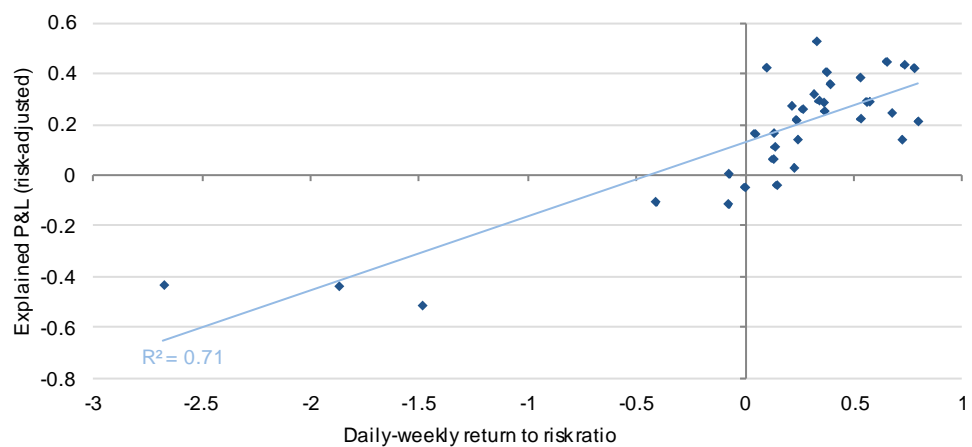
Source: SG Cross Asset Research/Cross Asset Quant – 1.5bps transaction costs are included.

In the third part of this study, we broaden this long-term simulation to include the Eurostoxx, Nikkei, US Treasuries and crude oil. But before we cross that bridge, let us use our model to explain the return of the daily-weekly in a broader investment universe.

### Explaining returns with the key drivers

We looked at the returns of the daily-weekly strategy over a range of 36 various financial instruments that are detailed on the following page. Transaction costs are not included at this stage because they are not considered in our formula.

#### Model versus reality



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

Our approach does a good job at explaining returns. In this universe, the average strategy achieves a return to risk ratio of 0.16, exactly in line with the model, which differentiates well positive from negative returns. As correlations are low, a diversified portfolio can achieve a much higher return for a level of risk. We elaborate further in the third part of this study.

### Explaining the return to risk ratio of the daily-weekly strategy

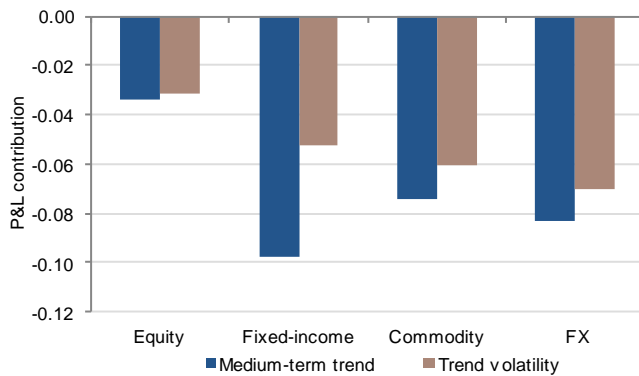
Asset class	Name	Ticker	Start Date	Daily-weekly Return/risk	Explained P&L	Risk-adjusted P&L attribution		
						Short-term volatility	Medium-term Trend	Trend volatility
Equity	S&P 500	SPX Index	Jan-00	0.68	0.25	0.27	-0.01	-0.01
	Russell 2000	RTY index	Jan-00	0.58	0.29	0.33	-0.02	-0.02
	Eurostoxx 50	SX5E Index	Jan-00	0.65	0.45	0.55	-0.05	-0.05
	Nikkei	NKY Index	Jan-00	0.38	0.41	0.47	-0.03	-0.03
	Eurostoxx	SXXE Index	Jan-00	0.39	0.36	0.51	-0.08	-0.07
	Japan Small Caps	SCJ US Index	Dec-07	0.77	0.42	0.49	-0.02	-0.05
	S&P Small Caps	SML Index	Jan-00	0.57	0.29	0.34	-0.02	-0.02
	Real Estate Equity	IYR US Index	Jun-00	0.79	0.21	0.23	-0.01	-0.01
	S&P Industrial	XLI US Index	Jan-00	0.34	0.29	0.37	-0.04	-0.04
	S&P Materials	XLB US Index	Jan-00	0.36	0.25	0.29	-0.02	-0.02
	Eurozone Equity	EZU US Index	Jul-00	0.71	0.14	0.17	-0.02	-0.01
	EuroStoxx small cap	SCXP Index	Jan-00	-0.42	-0.10	0.08	-0.11	-0.07
	Japan Equity	EWJ US Index	Jan-00	0.80	0.05	0.06	0.00	0.00
	S&P Metals and Mining	XME US Index	Jun-06	0.21	0.27	0.33	-0.03	-0.02
Fixed-income	Treasuries	TLT US Equity	Jul-02	0.54	0.22	0.36	-0.08	-0.06
	Bund SGI	SGIXBI Index	Jan-00	0.23	0.03	0.28	-0.15	-0.10
	JPY Bonds 10y SGI	SGIXBJ10 Index	Jan-00	0.15	-0.04	0.16	-0.11	-0.09
	iBoxx EUR IG	QW5A Index	Jan-00	-1.50	-0.51	-0.22	-0.23	-0.06
	iBoxx USD Asia ex-Japan	IBXXAX09 Index	Jan-06	-1.86	-0.44	-0.27	-0.11	-0.05
	iBoxx EUR HY	IBOXXMJA index	Jan-06	-2.66	-0.43	-0.27	-0.13	-0.03
	Corporate Bonds IG US	LQD US Equity	Jul-02	0.24	0.14	0.21	-0.03	-0.03
	US High Yield	HYG US Equity	Apr-07	0.13	0.17	0.22	-0.02	-0.03
	Real Estate Equity	IYR US Index	Jun-00	0.79	0.21	0.23	-0.01	-0.01
Commodity	Gold Future	GC1 Comdty	Jan-00	0.04	0.16	0.39	-0.12	-0.12
	Crude Oil - Brent	CO1 Comdty	Jan-00	0.36	0.29	0.34	-0.06	0.00
	Aluminium	LA1 Comdty	Jan-00	0.72	0.44	0.52	-0.05	-0.04
	Copper	LP1 Comdty	Jan-00	0.54	0.39	0.54	-0.09	-0.06
	Silver	SI1 Comdty	Jan-00	0.14	0.11	0.24	-0.06	-0.07
	US Oil Fund	USO US Equity	Apr-06	0.25	0.22	0.33	-0.07	-0.04
	Gold	GLD US Equity	Nov-04	0.09	0.42	0.53	-0.05	-0.05
	Gold Miners	GDX US Equity	May-06	0.32	0.53	0.73	-0.09	-0.11
FX	AUD/USD	AUDUSD F160 Index	Feb-07	0.25	0.26	0.40	-0.08	-0.05
	GBP/EUR	GBPEUR L163 Curncy	Feb-07	-0.06	0.01	0.14	-0.06	-0.07
	CAD/USD	CADUSD f160 Curncy	Aug-07	-0.09	-0.11	0.01	-0.07	-0.06
	USD/JPY	USDJPY T150 Curncy	May-07	0.35	0.32	0.50	-0.10	-0.08
	EUR/USD	EURUSD F113 Index	Feb-07	0.05	-0.05	0.14	-0.10	-0.09
Average				0.16	0.16	0.28	-0.07	-0.05

Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

In equities and commodities, trend effects are really small in comparison with the level of volatility and mean reversion. The trend is naturally higher in fixed income owing to the impact of carry. In spite of this, the daily-weekly still seems to perform well when applied to US Treasuries. In FX, some major currencies pairs just do not revert to the mean.

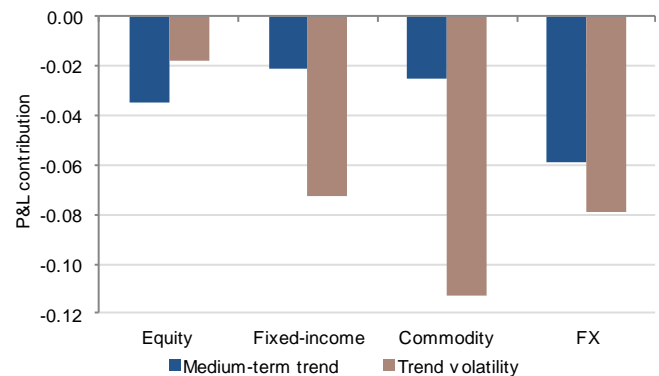
If we compare the volatility of the trend to its medium-term average, the latter usually prevails. This is especially clear in fixed income but is more nuanced in other asset classes. Last year, the volatility of the trend played a much more decisive role, reflecting the changing political and monetary landscape.

#### Comparing the two trend components



Source: SG Cross Asset Research/Cross Asset Quant

#### Changing times



We now turn to some case studies representing different possible configurations.

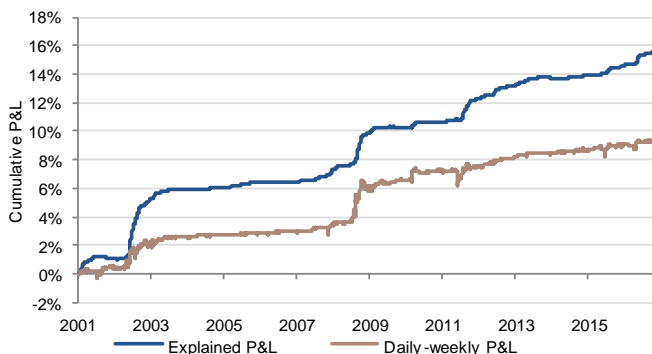
### A few case studies

By and large, the key parameters determine the return of the daily-weekly strategy. There are as many return profiles as there are combinations of those parameters.

#### A mean-reverting market

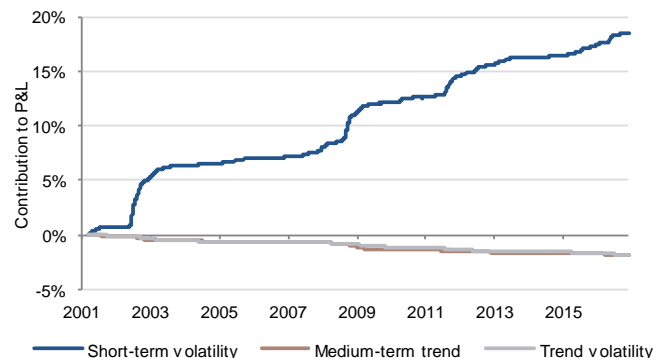
The EuroStoxx 50 has been strongly mean reverting since the beginning of the century. Volatility gains largely offset losses due to any of the trend components.

#### Trading the Eurostoxx 50 with the daily-weekly



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

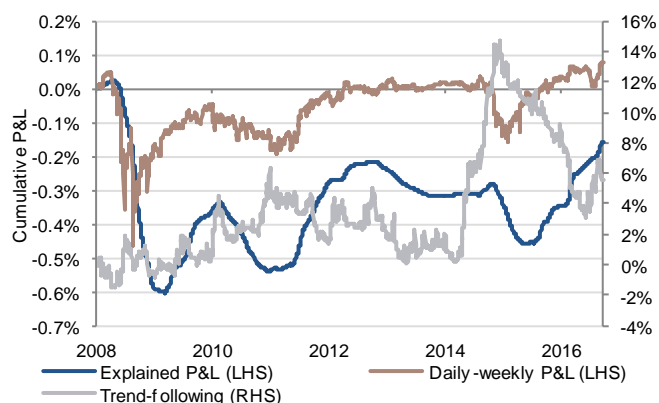
#### Volatility vs trends



### When trends erode trading gains

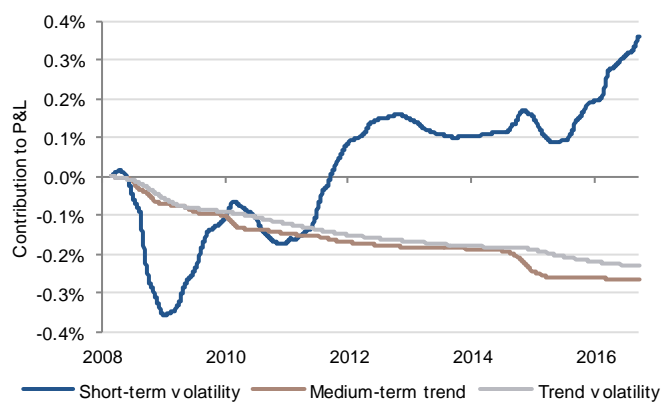
EUR/USD did not revert to the mean during the subprime crisis. The situation changed with the successive waves of quantitative easing. Just as trend-following strategies stopped performing on EUR/USD<sup>9</sup>, the daily-weekly picked up. However, trend effects remained strong, and the strategy lost during the dollar rally in 2014.

Trading EUR/USD with the daily-weekly



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

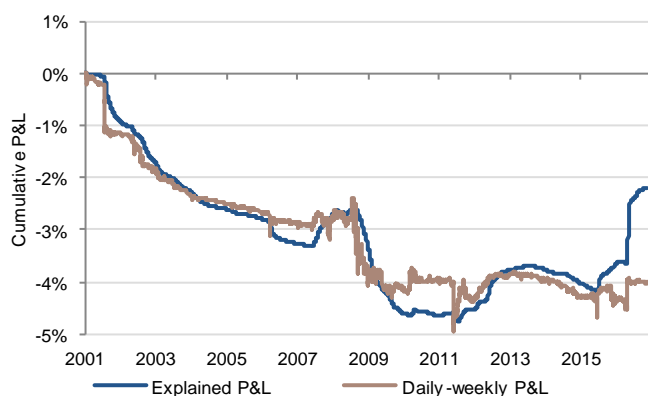
Volatility and trends offset each other



### Mean reversion can be more than offset by trends

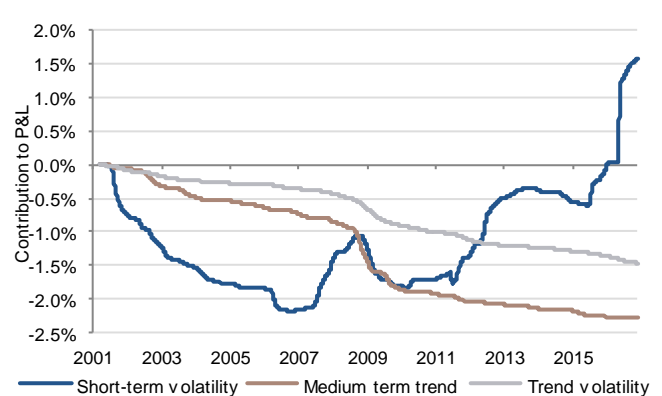
The EuroStoxx Small Cap did not revert to the mean between 2001 and 2007. Since then, the trend components have wiped out any mean-reversion gains. As statistical mean reversion turned positive after the subprime crisis, the daily-weekly may have been expected to turn a profit. However, market movements were rather chaotic, and expected gains have failed to materialise.

Below expectations



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

Mean reversion turned positive

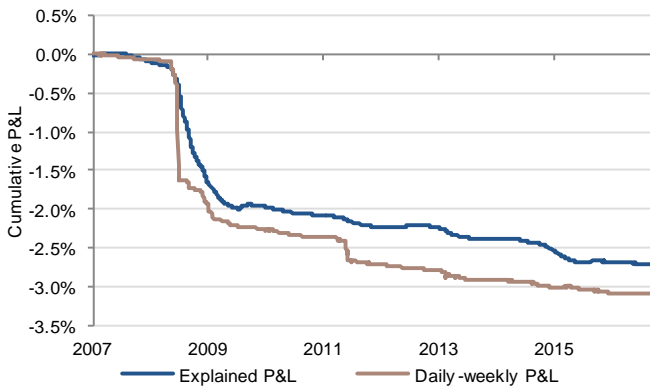


<sup>9</sup> Please refer to our February piece [Remember the last time rates were high?](#) [XQ9] for more details about our cross-asset trend-following strategies.

## Don't fight this trend

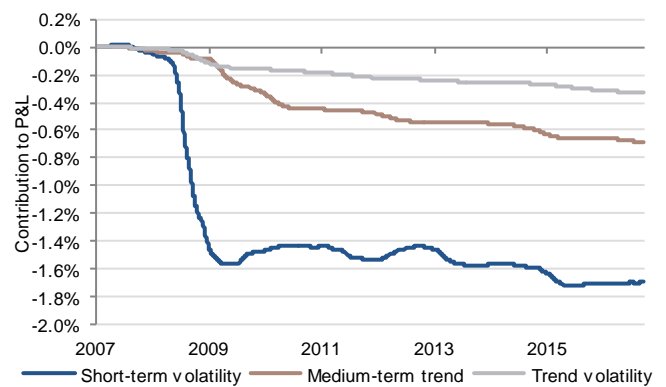
The daily-weekly strategy did not work well on the iBoxx Asia ex-Japan HY. The market did not revert to the mean and benefited from strong carry. The daily-weekly would have been a failure.

### A bad idea



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

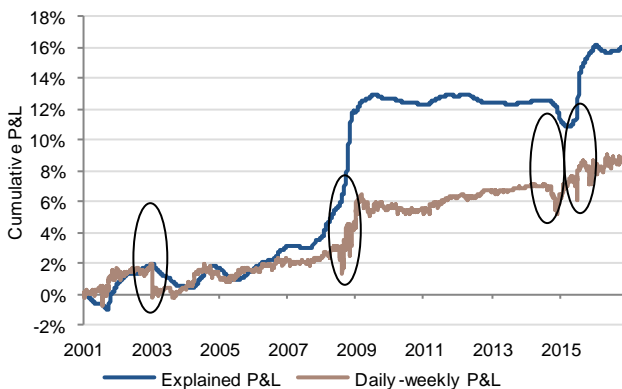
### All contributions are negative



## Mean reversion sometimes breaks down

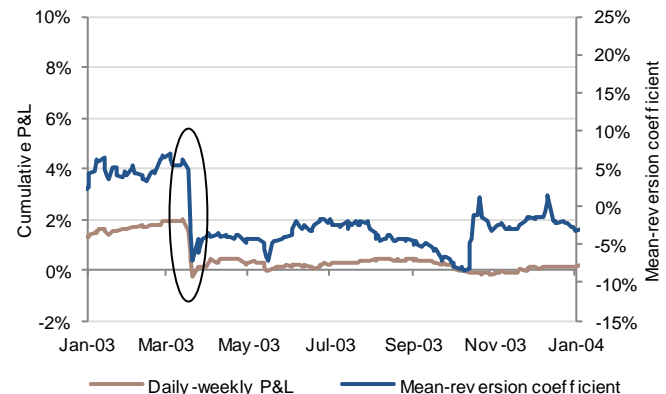
A good mean-reversion pattern does not prevent the strategy from being exposed to potentially high drawdown. The daily-weekly strategy would have performed well on the Brent since 2000 but would have suffered losses when oil prices jumped upwards or downwards. Such situations are difficult to anticipate, but the risks can be diversified away. In the following graph, on the right, the mean-reversion coefficient collapses after each drawdown.

### Trading Brent with the daily-weekly worked...



Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are not included.

### ... with a high level of risk



## How mean reversion fits into your portfolio

We view mean reversion as an alternative premium. It can come in addition to existing market exposure. For example, the manager of a fund that tracks the S&P can marginally adjust his or her market exposure using the daily-weekly.

Taking a step back, mean reversion belongs to a diversified portfolio of alternative risk premia. Back in 2013 [RP0] we decided that a risk premium must have demonstrated an attractive positive historical return profile, possess a fundamental value that allows a judgement on future expected returns to be made, and bring diversification benefits when combined into a multi-asset portfolio.

The next part of this study delves further into the historical return profile of mean reversion. In the previous part, we have seen how to relate future returns to a set of key parameters. Based on past measures and thoughts about future trends, the investor can assess the performance of mean reversion. It remains to see whether mean reversion brings in any diversification benefits. Looking at the following table, this seems to be the case:

Average correlations within and across factors<sup>10</sup> (since 2006)

	Traditional asset classes	Risk premia	Mean reversion
Market factors	6%	8%	3%
Alternative risk premia	8%	5%	3%
Mean reversion	3%	3%	13%

Source: SG Cross Asset Research/Cross Asset Quant

Over the period and for the assets considered above, the correlation between market factors is fairly low. This is due to the negative correlation between equities and fixed income. Over this decade, multi-asset mutual funds performed strongly. However, the correlation between stock and bonds may not remain as elevated if rates keep rising. Alternative risk premia and mean reversion can be an interesting source of diversification.

It is also worth noting that there is a high degree of confidence attached to the diversification benefits of alternative risk premia and mean reversion.

Standard deviation of correlations between investment styles (since 2006)

	Traditional asset classes	Risk premia	Mean reversion
Traditional asset classes	+/- 47%	+/- 31%	+/- 9%
Risk premia	+/- 31%	+/- 21%	+/- 10%
Mean reversion	+/- 9%	+/- 10%	+/- 16%

Source: SG Cross Asset Research/Cross Asset Quant

So, we can safely call mean reversion a risk premium. In order to better understand the nature of this premium and how to classify it, we need to delve further into its connection with volatility.

<sup>10</sup> Traditional asset classes are equities and fixed income indices in Europe and in the US, risk premia are multi-asset risk premium strategies (see [RP0] and [RP1]). Mean-reversion are daily-weekly strategies applied to single-asset and pairs, we use the universe of single-asset (19 strategies) and pairs (27 strategies) detailed in the fourth section 'How to manage a portfolio of mean-reversion strategies'.

## A simple way to trade variance

Simple algebra reveals that the payoff of the daily-weekly strategy is strictly equivalent to the differential between two hypothetical products that are linked to the variance of returns. The investor who trades using the daily-weekly receives the variance of all daily returns during the week, and pays the variance of weekly returns<sup>11</sup>.

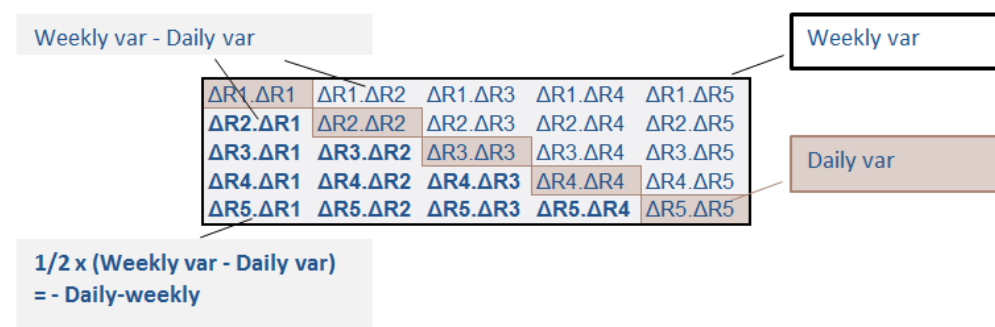
### Equivalence between the daily-weekly and variance payments

$$2 \text{ Standard daily/weekly} = \text{Daily var} - \text{Weekly var}$$

Calculation details are available in the Addendum 4.

This can be illustrated by the following chart. The weekly variance is the sum of all the terms in the matrix. The investor who buys variance every day receives all the terms in the diagonal. The differential between weekly variance and the sum of all day-to-day variances is given by all the elements that do not lie on the diagonal. We have seen these terms before, on page 11, when discussing the return of the daily-weekly strategy.

### The daily-weekly buys weekly vs daily variance



Source: SG Cross Asset Research/Cross Asset Quant

Variance can normally be bought or sold using financial derivatives<sup>12</sup>. What makes the daily-weekly so special is its lack of recourse to such derivatives. Squared returns cannot be simply traded on markets, but the product of lagged returns can. Following the rules of the daily-weekly is just enough.

This connection between mean reversion and variance sheds new light on our key parameters. The daily-weekly is naturally long the volatility of daily returns, and short that of the trend, the latter term being measured using weekly changes.

Why the medium-term trend should also play a role requires further explanation. In this view, assume that the market moves up and down by one unit during the four first days of the week

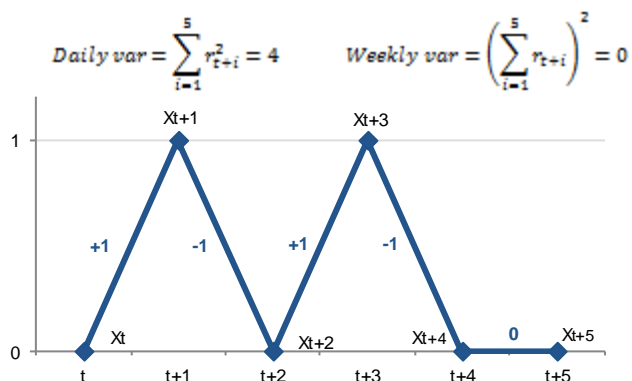
<sup>11</sup> This strategy has been the subject of a previous research paper in November 2013 [XQ2].

<sup>12</sup> Much of [XQ7] is devoted to the task of buying and tailoring variance, using either standard options or variance swap.



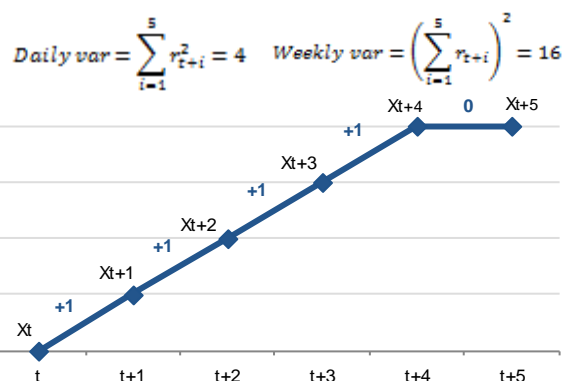
and remains unchanged the last day. Compare this situation with another one, where the market just rises by one unit during four consecutive days. In both cases, buying daily variance brings in a total of four price units. But selling weekly variance costs nothing in the former case and as much as 16 units in the latter.

#### What happens when the market goes up and down...



Source: SG Cross Asset Research/Cross Asset Quant

#### ... or rises steadily



The daily-weekly strategy is exposed to the risk of the market heading for a losing or a winning streak, hence its negative exposure to trends in the medium term.

## The fabric of mean reversion

Mean reversion can be split into two very familiar factors. This becomes clear once we view mean reversion as the dynamic hedge of a portfolio of options.

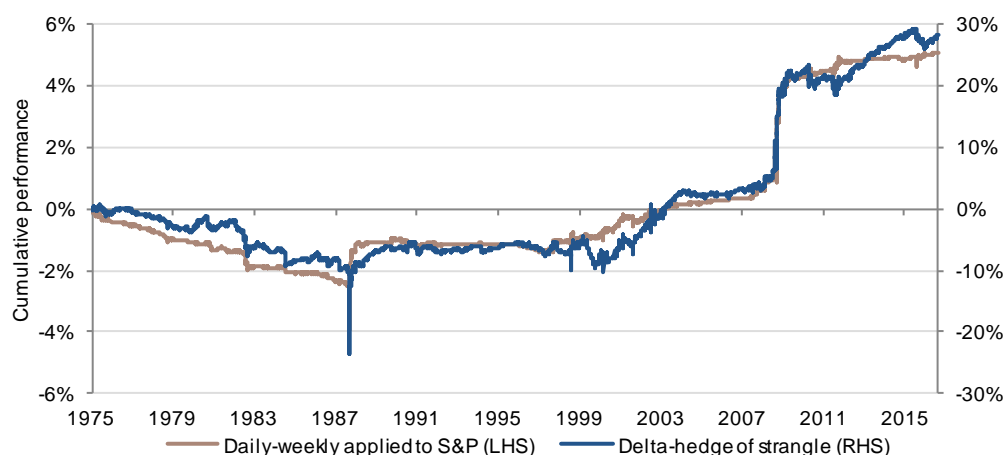
A portfolio of strangles with constant convexity, or gamma<sup>13</sup>, and rolled at the end of each week, can be effectively hedged by dynamically trading the underlying market. This dynamic hedge is nothing other than the daily-weekly strategy, up to a scaling factor<sup>14</sup>. The following chart compares the return of the daily-weekly with the dynamic hedge of a strangle<sup>15</sup> on the S&P since 1975.

<sup>13</sup> As illustrated in "Hedges are not a luxury" [XQ7] (p15), a portfolio of calls and puts across the whole range of possible strikes has a constant gamma when weighting the options by the inverse of the square of their strike. Such portfolio actually replicates a variance swap when delta-hedged dynamically.

<sup>14</sup> More details are given in Addendum 5.

<sup>15</sup> We buy a strangle with an expiry of 1m and strikes of 95% and 105%, and unwind the position at the end of each week. When no implied volatility is available for options, we use the VIX index (since 1990) combined with regression factors as a proxy for implied volatilities. When the VIX is not available (before 1990) we rely on the realised volatility of the S&P with regression factors.

**Similar P&L profile between the daily-weekly and the delta-hedge of a long strangle option**



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are not included.

It takes a bit of imagination to hedge an option that we do not possess. Let us set things in order, and add the portfolio of strangles into the equation. By following the daily-weekly strategy, the contrarian investor is effectively holding a portfolio of strangles and hedging it dynamically. All else being equal, the investor must also sell the aforementioned portfolio.

$$\text{Daily} - \text{weekly} \propto \text{Long Synthetic Variance} + \text{Short portfolio of strangles}$$

As we argue in '[Hedges are not a luxury](#)' [XQ7], buying a portfolio of strangles, weighting and hedging them properly is just one way of buying variance. By entering such a trade, the investor benefits from realised volatility, and pays for this a certain amount that stands in relation to the premia of the puts and calls in the portfolio. We refer to this as 'synthetic variance'. The synthetic variance monetises the differential that may arise between realised volatility and option premia.

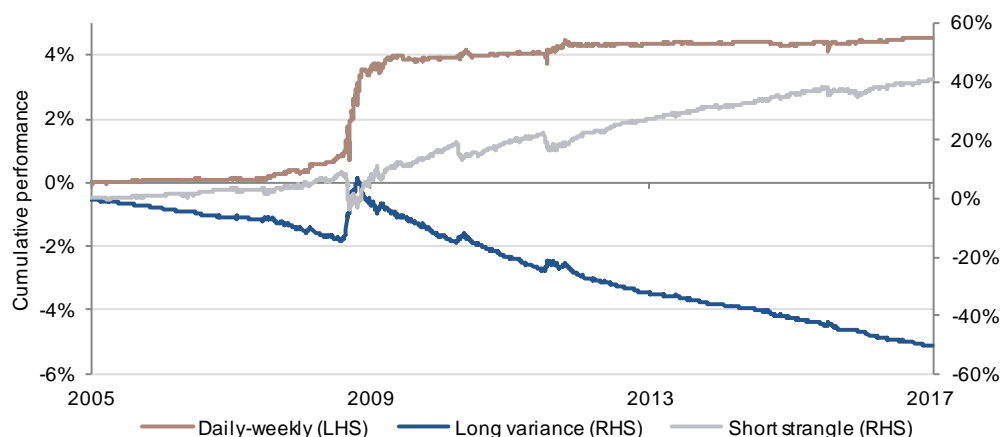
Systematically selling naked strangles is a classic income strategy, which we refer to as the 'volatility premium'<sup>16</sup>. In the long run, it harvests the differential between options premia and any losses due to do the exercise of the options.

Therefore, mean reversion is the combination of the volatility premium and synthetic variance. This remarkable property is well illustrated by the following graph<sup>17</sup>.

<sup>16</sup> See page 13 of [RP]

<sup>17</sup> The short strangle option strategy consists in purchasing a one month 95%/105% strangle every week and unwinding the position on a weekly basis. The long variance strategy is a long strangle option, with similar characteristics to the short one, delta-hedged. Transaction costs are included here.

### The fabric of mean reversion



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are included.

This approach provides yet another look at the dual nature of the link between mean reversion and volatility. Like a strangle, the daily-weekly benefits when markets are range bound, and loses when they rise or fall steadily. But once the market is in a funk, synthetic variance kicks in. The strategy benefits from fluctuations, just like any tail hedge.

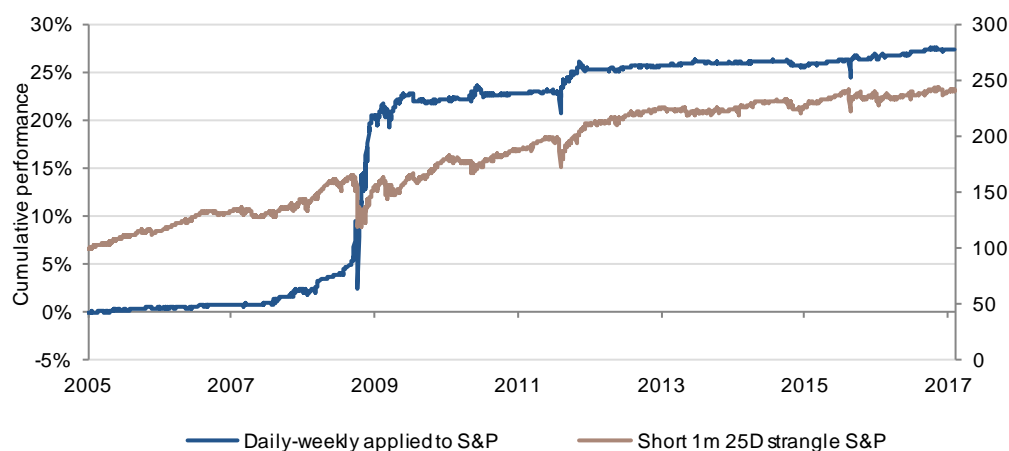
### The dual nature of mean reversion

	Synthetic variance	Volatility premium	Daily-weekly mean-reversion
<b>Realised volatility</b>	Benefits from realised volatility		Benefits from short-term realised volatility
<b>Mean reversion</b>		Options are likely to expire worthless	Benefits from mean reversion
<b>Trend</b>		Options are likely to be exercised	Loses due to the trend components
<b>Premium</b>	Pays options (or variance swap) premia	Receives strangles premia	No premium

Source: SG Cross Asset Research/Cross Asset Quant

If we zoom in on the daily-weekly and a strategy that sells naked strangles, both are quite similar in normal times. As long as mean reversion holds, which was the case for the S&P during the subprime crisis, the daily-weekly benefits from market turmoil. Both strategies are exposed to potential drawdown in periods of stress. However, the volatility premium tends to recover much more slowly.

### A tale of two premia<sup>18</sup>



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are included.

Hedging the risks of the volatility premium using synthetic variance is exactly what we did in our flagship portfolio of risk premia [RP1], except that we used the 'synthetic down variance' strategy presented in [XQ7]. The Synthetic Down Var only benefits from fluctuations when the market is declining, and adjusts the hedge in order to reduce costs in the long run.

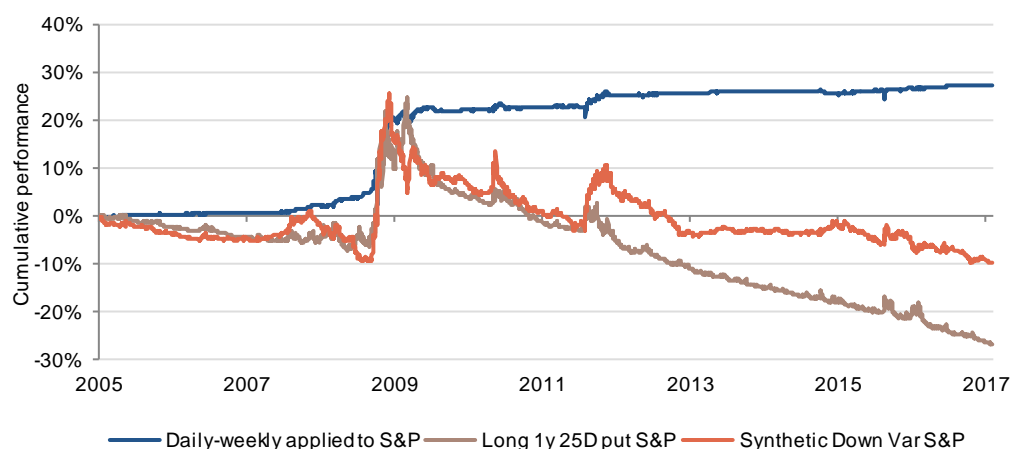
Anyway, we like the combination of the volatility premium and synthetic variance. The daily-weekly strategy allows investors to access this combination in markets where variance is not easily traded. This includes the pair trades that are discussed in the third part of this study.

Let us now compare the daily-weekly with what we view as two key systematic hedges – one strategy that rolls put options<sup>19</sup>, and the Synthetic Down Var.

<sup>18</sup> We simulate the selling of 1m 25-delta strangle, smoothed on a weekly basis and kept until expiry.

<sup>19</sup> See the section on page 9 of '[The Right Hedge for You](#)' [XQ6].

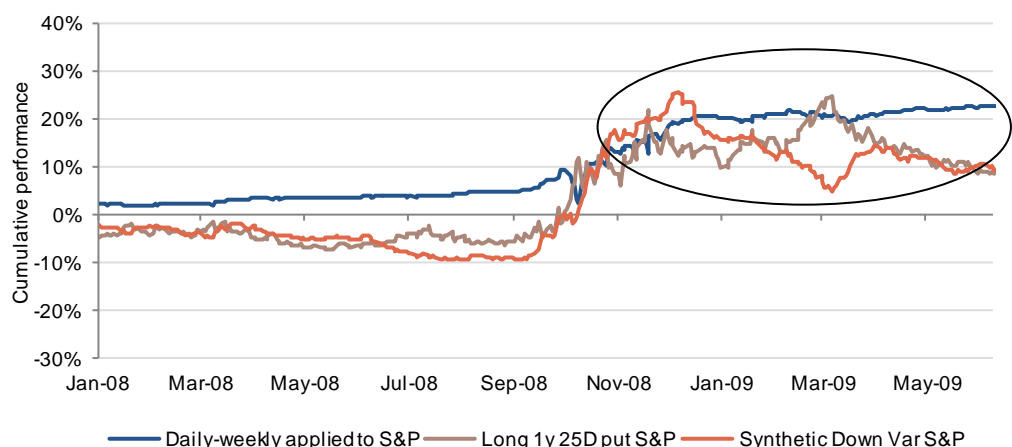
## Daily-weekly compared to systematic hedges<sup>20</sup>



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are included.

Although the daily-weekly benefits comparatively less in periods of stress, it does so for no cost in normal times. Mean reversion is particularly attractive in the first stage of a crisis, when markets fluctuate widely but not yet wildly. In such circumstances, the cost of options is usually already high, putting a drag on the performance of systematic hedges. Of course, this situation can be easily handled by buying put spreads rather than puts<sup>21</sup>. Contrarian trading is another alternative, which has the advantage of not giving gains back once the crisis is over.

## Maintain your gains



Source: SG Cross Asset Research/Cross Asset Quant - Transaction costs are included.

<sup>20</sup> We simulate the purchase of 1y 25-delta put, smoothed on a monthly basis and kept until expiry. For the Synthetic Down Var we simulate the purchase of delta-hedged basket of 1y puts 60-100% smoothed on a monthly basis and kept until expiry. Options are delta-hedged with a skew adjusted hedge ratio. The strategy is sized by a factor of 8 to have a vega notional of 1% on average.

<sup>21</sup> See 'where have the put spreads gone?' on pages 35-36 of 'hedges are not a luxury' [XQ7].

## The patterns of history

We now investigate the performance of these three strategies in various market regimes, along the lines that were set out in ‘[Hedges are not a luxury](#)’ [XQ7]<sup>22</sup>. All strategies are applied to the S&P, and we use data that starts in 1980.

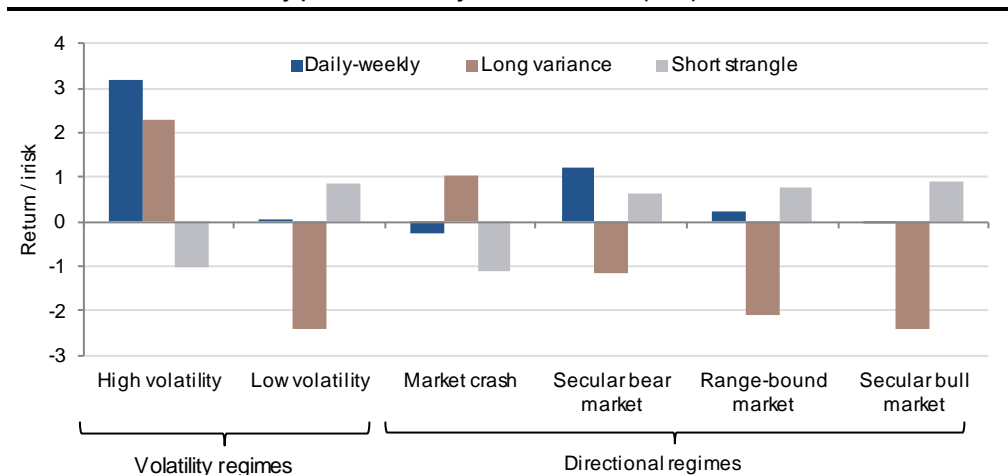
Similarly to a long variance position, the daily-weekly tends to gain in a high volatility regime. When volatility is low, or in a bull market, gains are small but positive.

Mean reversion is also likely to gain in what we call a secular bear market, when markets fall gradually. In such conditions, mean reversion usually holds, and the daily-weekly benefits from fluctuations. In contrast, synthetic variance tends to be negatively impacted by the higher cost of buying options.

In a range-bound market, the daily-weekly strategy posts a slightly positive performance, just like the strategy that sells strangles. Fluctuations, albeit small in scale, outweigh the trends.

Finally, both mean reversion and the strategy that sells options are exposed to possibly a large drawdown in a market crash. The former tends to bounce back very quickly, but the latter recovers more slowly due to the persistence of implied volatility. Diversification is discussed in the next part of this study.

### Mean reversion vs volatility premium and synthetic variance (S&P)



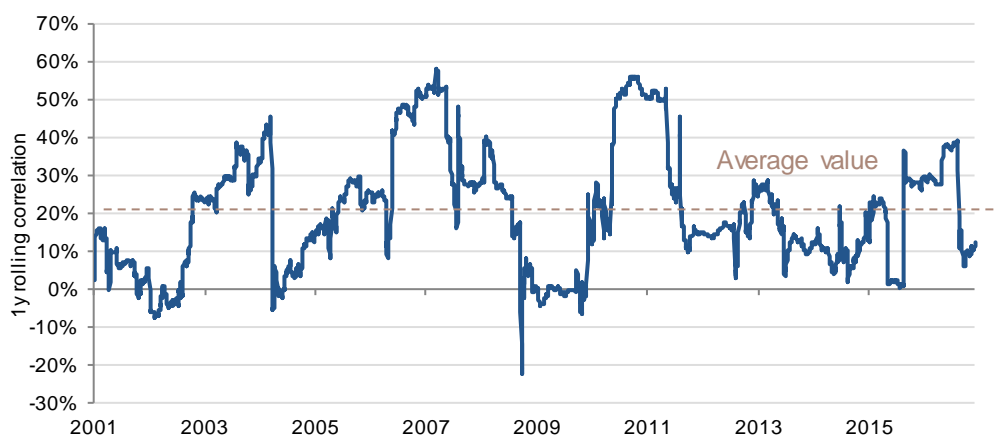
Source: SG Cross Asset Research/Cross Asset Quant

## Adding mean reversion in a risk premia portfolio

Trading the S&P with the daily-weekly would add diversification to a portfolio of alternative risk premia. We consider here the flagship portfolio described in our latest Risk Premia Outlook [RP1]. During the height of the financial crisis in 2008, correlation even fell to 0%.

<sup>22</sup> See the section on page 32 of [XQ7]. It shall be noted that returns and regimes are measured on the same periods of time. In order to define a regime, we use information that was not available at past points in time. Our measures accurately depicts past patterns, but cannot be used as trading signals. For example, although the volatility premium does outperform when realised volatility is low, it is impossible to know whether volatility will stay low in the near future. An investor who sells options at the end of a period of low volatility can end up suffering losses if the situation changes.

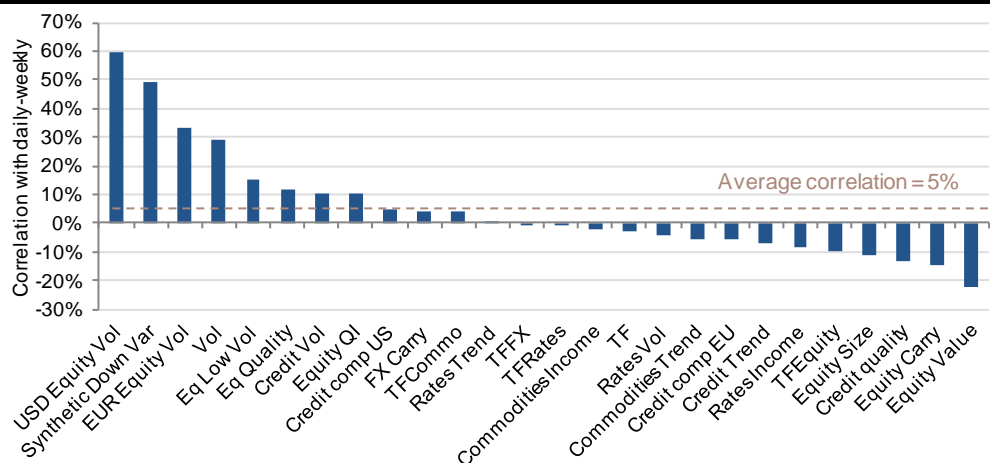
### Adding diversification to a risk premia portfolio



Source: SG Cross Asset Research/Cross Asset Quant

It comes as no surprise that mean reversion on the S&P is positively correlated with the equity volatility premia and our hedge-overlay, the Synthetic Down Var. Mean reversion is negatively correlated with Equity Value. In periods of stress, mean reversion tends to benefit from market fluctuations, while Equity Value underperforms.

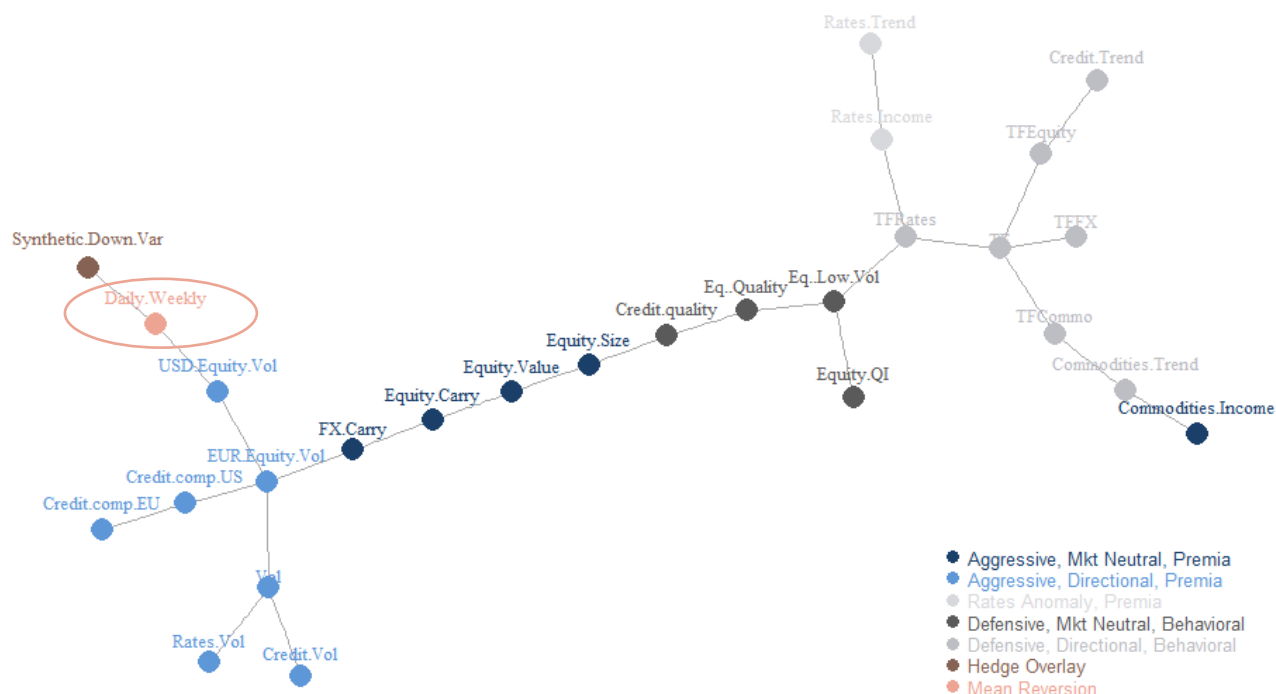
### Ranking risk premia by their correlation with S&P mean reversion (since 2006)



Source: SG Cross Asset Research/Cross Asset Quant

At this stage, our regular readers must be wondering when we will pull out one of our landmark spanning trees. The time has come for us to add mean reversion to our beloved graphical representation of risk premia.

The time has come



Source: SG Cross Asset Research/Cross Asset Quant

A minimum spanning tree helps to visualise interactions between the strategies. It identifies the strategy that is most correlated with all the others and puts it in the middle of the graph. It then proceeds to build a number of branches, so that the highly correlated strategies appear next to each other, in order of decreasing correlation<sup>23</sup>.

Aggressive factors are clustered on the branch on the left side, and defensive ones tend to appear on the right side. The Synthetic Down Var is connected with the equity volatility premium, not because of its risk profile, which is of a rather defensive nature, but because both strategies are closely related with the dynamics of implied volatility.

Mean reversion appears right in the middle between the volatility premium and synthetic variance. This observation once again highlights the dual nature of the volatility profile of the daily-weekly.

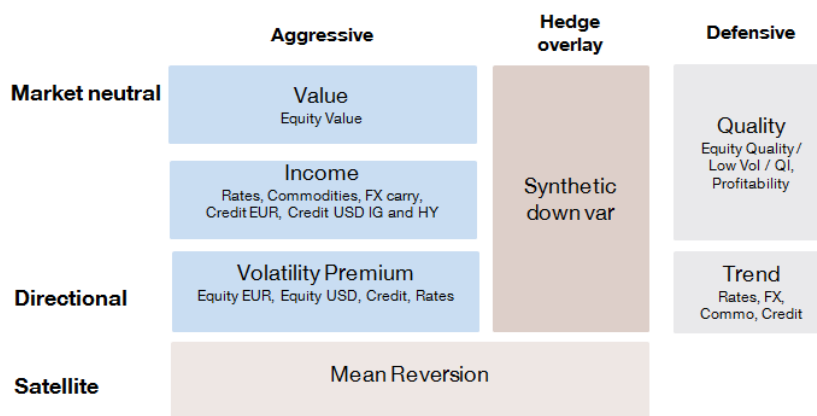
When all is said and done, we view mean reversion as a risk premium. As we argue in the next part of this study, mean reversion has exhibited persistent positive returns. Mean reversion itself is a behavioural pattern, and we relate it to changes in the microstructure of the market. As long as this statistical pattern holds, such strategies as the daily-weekly monetise the volatility of the market against two trend components. Based on statistical patterns and one's own views, the investor can ascribe a fundamental value to mean reversion. Mean reversion also seems to add diversification to a portfolio.

<sup>23</sup> For the distance measure, we choose an anti-symmetric measure of correlation in order to differentiate positive from negative correlations – the distance will be shorter for assets that are more positively correlated while it will be wider for assets that are more negatively correlated.



With a mixed risk profile, mean-reversion is both an aggressive premium and a hedge overlay<sup>24</sup>. Diversification plays a key role in mitigating risks. As tail risks cannot be fully diversified away, the last part of this study shows how to determine the size of a mean reversion portfolio for a given level of risk. Overall, we view mean reversion as a satellite strategy, which can bring an additional yield pick-up in normal times and increase returns more substantially in periods of stress.

#### How mean reversion fits into a portfolio of risk premia



Source: SG Cross Asset Research/Cross Asset Quant

<sup>24</sup> Please refer to our latest risk premia outlook [RP1] for more details about our classification of risk premia strategies

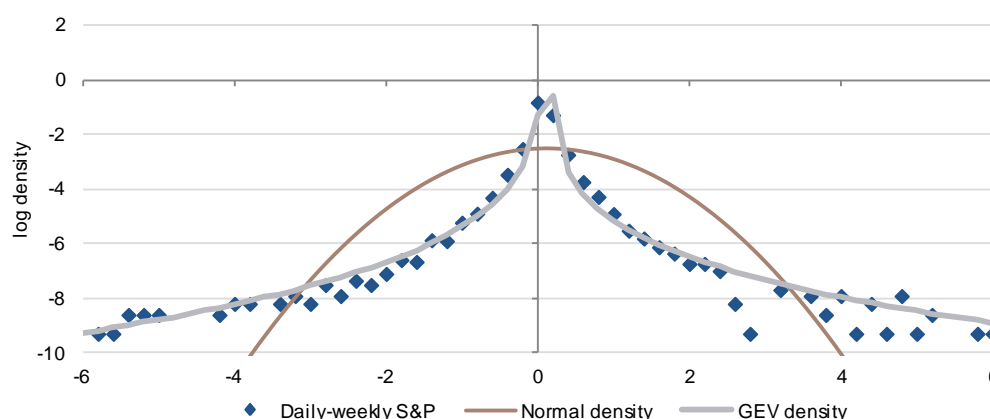
## Diversifying the mean-reversion premia

### The distribution of risks

The returns of the daily-weekly strategy are heavy-tailed both on the downside and on the upside. The probability of seeing extreme moves – positive or negative – is much higher than with a normal distribution. The strategy could benefit from high realised volatility in crisis periods. It is also exposed to potentially high drawdown when the mean-reversion breakdowns and the asset price moves in one direction for a period of several consecutive days.

The graph and table below depict the distribution of the strategy daily returns (in logarithm) applied to the S&P since 1975 compared to those of the normal distribution and those of a typical fat-tailed distribution. The daily returns of the S&P have been re-normalised by their long-term average return and their long-term standard deviation.

**A fat-tailed distribution (daily-weekly applied to the S&P since 1975)**



Source: SG Cross Asset Research/Cross Asset Quant

**Quantile of the S&P daily-weekly return distribution**

Percentile value	0.1%	0.5%	1.0%	5.0%	10.0%	20.0%	30.0%	50.0%	70.0%	80.0%	90.0%	95.0%	99.0%	99.5%	99.9%
S&P	-5.40	-2.21	-1.50	-0.53	-0.30	-0.14	-0.07	-0.01	0.04	0.11	0.26	0.50	1.57	2.38	10.45
Fitted GEV distribution	-5.68	-2.38	-1.50	-0.36	-0.15	-0.04	-0.01	0.01	0.01	0.04	0.16	0.43	2.14	3.61	9.71
Gaussian distribution	-3.09	-2.58	-2.33	-1.64	-1.28	-0.84	-0.52	0.00	0.52	0.84	1.28	1.64	2.33	2.58	3.09

Source: SG Cross Asset Research/Cross Asset Quant

For extreme moves, the returns of the daily-weekly strategy are much higher than those that would have been obtained under a normal assumption. Conversely, the number of very small moves is very high compared to the normal distribution. And the probability of large positive moves is higher than the probability of negative moves.

This sort of statistical behaviour is well described by a generalised extreme value (GEV) distribution. We fitted the historical distribution of the S&P daily-weekly strategy with such a GEV distribution with good accuracy. In the final part of this document, we return to this result and explain how to use it when constructing a portfolio of daily-weekly strategies.

### Generalised extreme value (GEV) distribution

The GEV law is characterised by the following cumulative distribution function:

$$F_{\xi, \mu, \sigma}(x) = \exp \left( - \left( 1 + \xi \frac{x - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right)$$

As the strategy is characterised by a fat tail distribution both on the left side and on the right side, we set a GEV distribution on both sides as following:

$$\text{If } x < \mu \quad F(x) = F_{\xi_d, \mu_d, \sigma_d}(x)$$

$$\text{If } x \geq \mu \quad F(x) = 1 - F_{\xi_u, \mu_u, \sigma_u}(-x)$$

We fix  $\mu_d = \mu_u = 0$  and estimate  $\sigma_d = 0.047$ ,  $\xi_d = -3.20$ ,  $\sigma_u = 0.030$ ,  $\xi_u = -3.66$  to fit the historical return distribution.

We use different parameters for the distribution on the downside and on the upside as the daily-weekly distribution tends to be positively skewed with a higher probability of having large positive movements than large negative ones. We estimate the parameters  $(\sigma_d, \xi_d)$  and  $(\sigma_u, \xi_u)$  separately, to fit the return distribution on the downside and the upside as the daily-weekly distribution tends to be positively skewed.

More information about extreme value theory is available in [XQ1].

Traditional statistics confirm that the daily-weekly strategy is very fat-tailed and positively skewed. Positive skewness means that positive returns have been more frequent than negative returns in the period considered here (since 1975). A large kurtosis attests to the presence of fat tails.

### Statistics of the distribution (since 1975)

	Return (p.a.)	Stdev	Return / stdev	MDD	Skewness	Kurtosis
Daily-weekly applied to the S&P	0.6%	3.0%	0.20	26%	6.7	769

Source: SG Cross Asset Research/Cross Asset Quant

This short analysis on the S&P sums up pretty well the behaviour of mean-reversion strategies: small returns are very often obtained, and from time-to-time a very large move to the downside or upside can happen.

To benefit from the attractive premium offered by mean reversion without being hurt too much by drawdowns, it is necessary to diversify away the risks. There is no scarcity of daily-weekly strategies applied to single assets readily available to the investor, as seen below.

### Single-asset mean reversion strategies

S&P daily-weekly mean reversion	Captures the spread between daily and biweekly variance of the S&P 500	SGIXUSGR Index
Eurostoxx daily-weekly mean reversion	Captures the spread between daily and biweekly variance of the Eurostoxx	SGIXEUGR Index
VIX futures daily-weekly mean reversion	Captures the mean-reversion in the VIX futures curve	SGIXVIGR Index

Source: SG Cross Asset Research/Cross Asset Quant

In addition, investors can extend the universe of daily-weekly strategies by using pairs of assets, whose spread is mean-reverting within a week<sup>25</sup>. Here below is a selection of indices available on Bloomberg.

#### Daily-weekly strategy applied to pairs

US Treasuries (TLT US) vs US Equities (IWM US)	Daily-weekly applied to the spread between US Treasuries and US Equities	SGIXMRIT Index
Gold (GLD US) vs Gold Miners (GDX US)	Daily-weekly applied to the spread between Gold and Gold Miners	SGIXMRGD Index
US Materials (XLB US) vs US Industrials (XLI US)	Daily-weekly applied to the spread between US Materials vs US Industrials	SGIXMRXL Index
Japanese Equities (NKY) vs USDJPY (USDJPY T150)	Daily-weekly applied to the spread between Nikkei and USDJPY	SGSLMRJP Index
US Treasuries (TLT US) vs US Real Estate (IYR US)	Daily-weekly applied to the spread between US Treasuries and US Real Estate	SGIXMRTY Index
Japanese Equities (EWT US) vs Eurozone Equities (EZU US)	Daily-weekly applied to the spread between Japanese and Eurozone Equities	SGIXMRREE Index
US Equities (IWM US) vs Eurozone Equities (EZU US)	Daily-weekly applied to the spread between US and Eurozone Equities	SGIXMRIE Index
US Miners (XME US) vs AUDUSD (AUDUSD F160)	Daily-weekly applied to the spread between US Miners and AUDUSD	SGIXMRXA Index

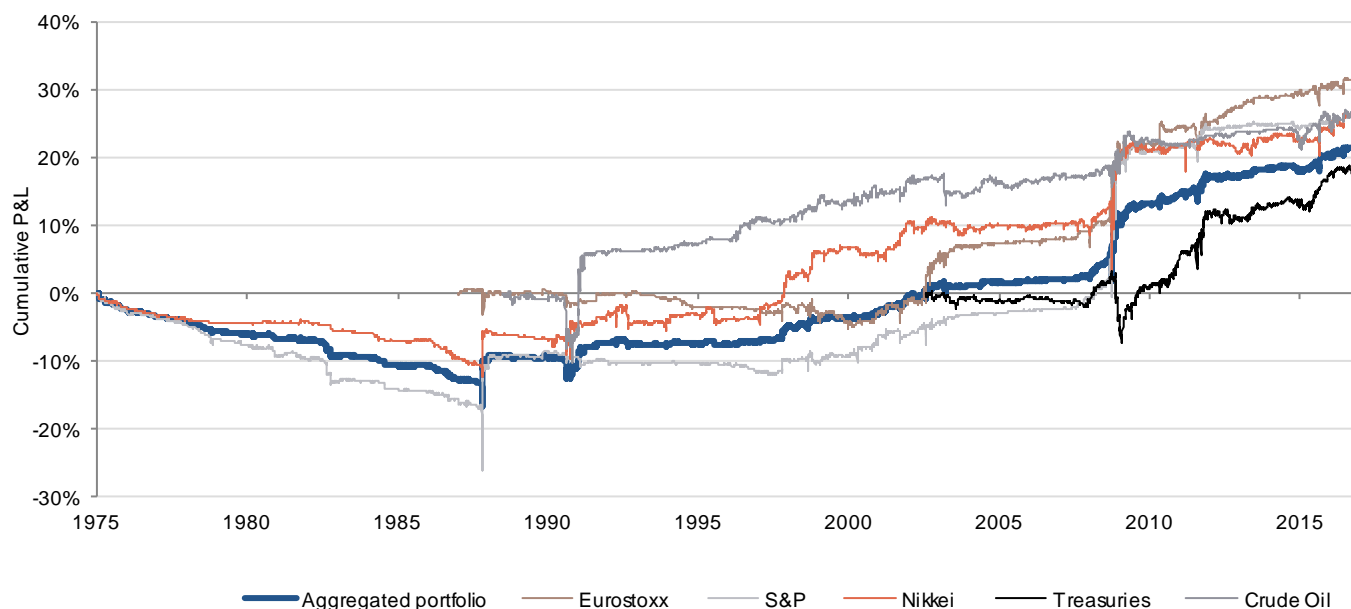
Source: SG Cross Asset Research/Cross Asset Quant

For the sake of completeness, we extend the scope of our study to a larger set of assets and pairs. In the following, we use our own simulations of past performances for single-asset and pairs strategies and our own assumptions for transaction costs<sup>26</sup>.

### 40 years of mean reversion

Whenever possible, we estimated the long-term performance of daily-weekly strategies across asset classes: equities in Europe, the US and Japan, Treasuries and crude oil. This work mimics the returns of a contrarian investor over half a century.

#### 40 years of mean reversion



Source: SG Cross Asset Research/Cross Asset Quant – 2bp transaction costs are included.

<sup>25</sup> Please refer to our March 2016 publication “[When pair trades revert to the Mean – A simple strategy](#)” [XQ5] for details on how to apply the daily-weekly strategy to pairs of assets

<sup>26</sup> Please refer to Addendum 1 for details on our simulation assumptions

Having a long-term view is important. It brings into picture a few important crises such as the 1987 crash, the 1990 oil-price shock, and the Asian crisis of 1997. As discussed before in our case study on the S&P, the daily-weekly strategies did not perform well before 1987. Since then, they have done relatively well. And in the long run, the contrarian investor would have posted positive gains.

But the daily-weekly strategy can experience severe drawdown: 1987 for equity indices, 1990 for crude oil or Treasuries are all cases in point. If drawdowns are potentially high, they do not necessarily happen at the same time. When combining strategies, an investor would access a better risk-adjusted return for much less tail risk. And this is true even over the past 42 years.

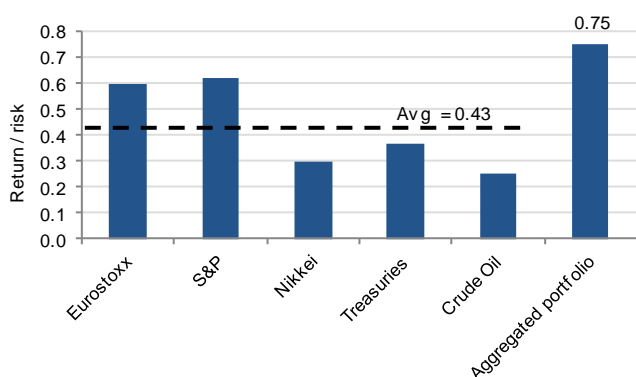
#### A global positive performance for the long-term portfolio<sup>27</sup>

	Eurostoxx	S&P	Nikkei	Treasuries	Crude Oil	Aggregated portfolio
Start date	1/1/1987	1/3/1975	1/3/1975	7/29/2002	6/24/1988	1/3/1975
Return (p.a.)	1.0%	0.6%	0.6%	1.1%	0.9%	0.5%
Std. dev. (p.a.)	3.0%	3.0%	3.0%	3.0%	3.0%	1.7%
Return / risk	0.34	0.20	0.21	0.37	0.30	0.29
Maximum drawdown	6%	26%	12%	10%	11%	17%
Skewness	8.5	6.7	9.8	0.2	6.2	11.5
Kurtosis	284.9	769.2	414.9	65.6	273.2	669.0
<b>Since 2002</b>						
Return (p.a.)	2.1%	2.1%	1.2%	1.1%	0.6%	1.4%
Std. dev. (p.a.)	3.6%	3.4%	4.0%	3.0%	2.4%	1.9%
Return / risk	0.60	0.62	0.30	0.37	0.25	0.75
Maximum drawdown	5%	8%	10%	10%	5%	5%
Skewness	9.3	9.8	5.9	0.2	2.3	3.8
Kurtosis	269.4	281.3	245.5	65.6	60.9	124.1

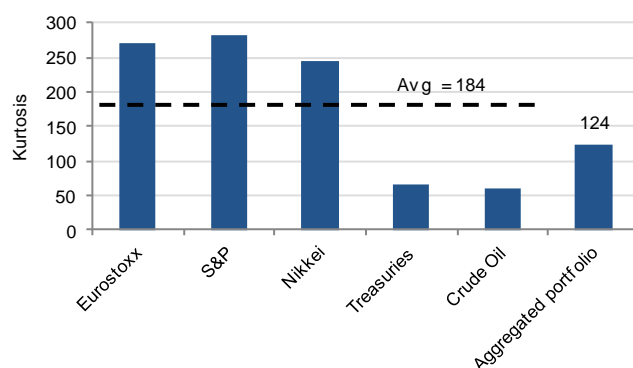
Source: SG Cross Asset Research/Cross Asset Quant. Transaction costs are included.

The two charts below illustrate the return-to-risk ratio and the kurtosis of the return distributions on the five single-asset strategies<sup>28</sup> vs the long-term portfolio.

#### Diversification benefits with a higher return-to-risk ratio...



#### ... and lower tail risk for the aggregated portfolio



Source: SG Cross Asset Research/Cross Asset Quant

<sup>27</sup> To make the comparison easier, the strategies have been sized so that their long-term volatility corresponds to 3%

<sup>28</sup> Statistics are calculated from 2002 to cope with strategies which do not start at the same time

## The benefits of diversification

We now consider two different baskets<sup>29</sup>: a large basket of single-assets made of 19 strategies split into four different asset-classes, and a large basket of 27 daily-weekly strategies applied to pairs of assets<sup>30</sup>.

### A low correlation

Daily-weekly strategies show a low correlation with each other. The average correlation is close to 15% in the two baskets of single-asset and pairs. Compared to volatility strategies, they offer a much less correlated universe.

We showed in [Pick your battles – Finding the best opportunities in variance swaps](#) [XQ8] that correlations between volatilities are on average higher than correlations between assets. The correlation in the universe of cross-asset variance swap (12m) considered in the piece was close to 60% on average since 2006. This is a good thing for pair trading between variance swaps but it increases the systemic risk when selling cross-asset volatilities.

#### Intra-universe correlation

	X-asset volatility	Single-asset	Pairs	Single-asset vs pairs
<b>Average correlation</b>	60%	19%	13%	13%

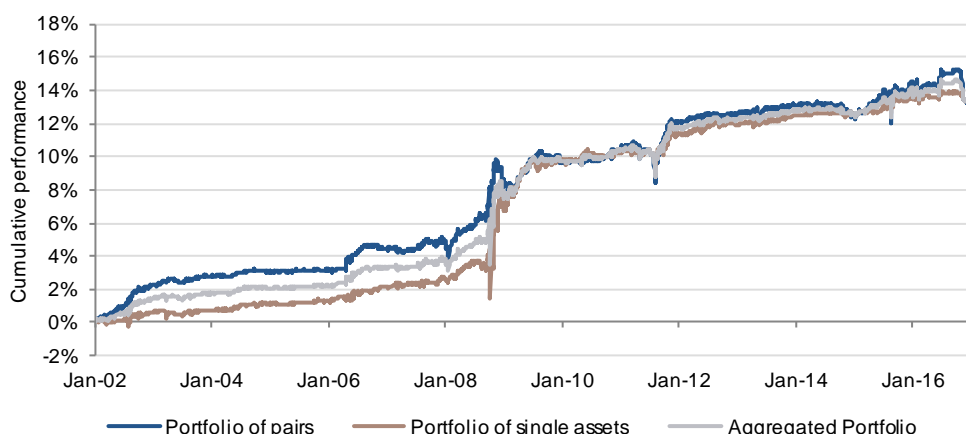
Source: SG Cross Asset Research/Cross Asset Quant

Daily-weekly strategies are exposed to realised volatility of the underlying asset. But in contrast with volatility strategies, they are not exposed to the systemic risk induced by implied volatilities. This is why the daily-weekly strategies are less correlated with each other. And a low correlation is the first pre-requisite for good diversification<sup>31</sup>.

### A well diversified portfolio

We compare below the performances of our single-asset portfolios, our portfolio of pairs, and a combination of the two. The latter is equally split between the portfolio of single assets and the portfolio of pairs.

#### A well diversified portfolio of mean-reversion strategies



Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bps for single-asset, 2bps for the pair)

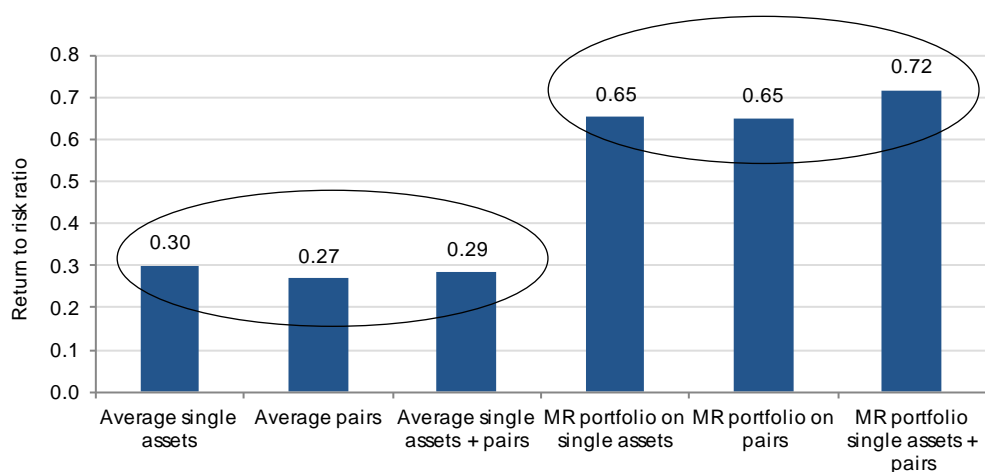
<sup>29</sup> The construction process of the two baskets is detailed in the next section 'How to manage a portfolio of mean-reversion strategies'. The basket of pairs is the one presented in the next section, the basket of single-asset is built on a like-for-like basis for the sake of consistency.

<sup>30</sup> See [XQ5] for details on how to apply the daily-weekly strategy to pairs of assets

<sup>31</sup> We quantify in Addendum 9 how the correlation level impacts the diversification benefit

Compared to the average standard deviation of the stand-alone strategies, the volatility of a portfolio of daily-weekly strategies is reduced by 60%. As a consequence, the return-to-risk ratios of the daily-weekly portfolios are close to 2.5 times higher than the average stand-alone ratios.

#### Diversification benefits (since 2002)



Source: SG Cross Asset Research/Cross Asset Quant

#### Risk/return profile of the daily-weekly portfolios (since 2002)

	Return (p.a)	St. dev. (p.a.)	Return / risk	MDD	MDD / st. dev.	Skewness	Kurtosis
Portfolio of single assets	0.9%	1.3%	0.65	3.1%	2.31	9.3	267.8
Portfolio of pairs	0.9%	1.3%	0.65	2.5%	1.91	5.1	147.1
Aggregated portfolio	0.9%	1.2%	0.72	2.8%	2.33	8.1	229.3

Source: SG Cross Asset Research/Cross Asset Quant

At first sight, the added value of the portfolio of pairs seems limited. But there is more to it than just the risk-adjusted return.

First the portfolio of single assets contains 30 strategies. And, as explained below, the return-to-risk ratio cannot be increased further beyond 15 to 20 strategies. Secondly, the portfolio of pairs outperforms in relatively calmer periods, such as 2004-2007 and 2015-2016.

Certain assets are exposed to macroeconomic factors that do not always revert to the mean. Once properly identified, those factors can be taken away through a pair trade. The daily-weekly, in turn, is likely to perform better when applied to such carefully selected pairs. The flip side of the argument is that the link between both assets can loosen during certain periods of stress.

For the contrarian investor looking to enhance the returns of their portfolio, while mitigating a downside risk and controlling the number of strategies in his portfolio, pairs are worth a try.

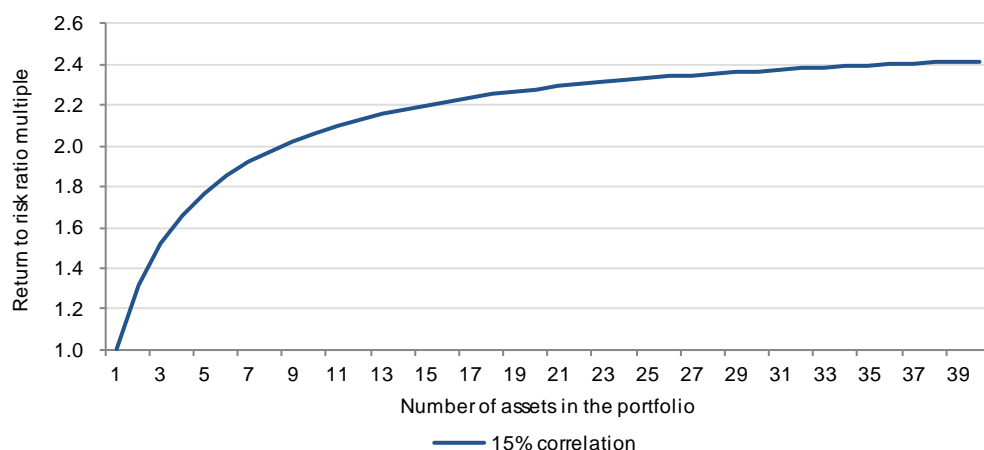
### The limits of diversification

The following chart depicts the expected improvement in the return-to-risk ratio when mixing several strategies. Here we assume that strategies are identically and normally distributed<sup>32</sup>. We also assume that the correlation between the strategies is constant at 15%, close to the average correlation between the stand-alone daily-weekly.

The marginal impact in adding one stand-alone strategy in the portfolio is high when the size of the portfolio is low. It reduces quickly when the number of strategies in the portfolio increases. We typically find that 15-20 assets in the portfolio are enough to maximise the return-to-risk ratio. The marginal impact from adding more assets in the portfolio is limited.

We could expect a return to risk ratio 2.3 times higher than stand-alone strategies for an equal-weight portfolio of 20 strategies. This number is in line with the multiples observed above on the daily-weekly diversified portfolios.

### Benefits and limits of diversification



Source: SG Cross Asset Research/Cross Asset Quant

The contrarian investor wishing to boost the performance of the portfolio needs to select the strategies likely to perform the best in the near future. We discuss this in the following section of the report.

<sup>32</sup> Calculations are detailed in Addendum 9



## How to manage a portfolio of daily-weekly strategies

Having a framework is important. Our process consists of three main steps: strategy selection, risk measurement and portfolio allocation.

### Picking winners

#### Single-asset strategies

We have shown above that periods of trend-following and mean-reverting regimes have been consistent across the decades: an asset that mean-reverts today is likely to mean-revert in the future.

We also introduced a formula-based estimation of the P&L of the daily-weekly strategy: the strategy is negatively impacted by the volatility of the trend and the medium-term trend, while it benefits from high realised volatility when the asset price reverts well to its mean.

We use these results to identify potential winning strategies. Namely, we select only those which have a strong long-term mean-reversion and a high short-term volatility. We also check that the medium-term trend and its volatility won't jeopardise the expected performance. Here is our selection below.

A basket of single assets<sup>33</sup>

Asset class	Name	Ticker	Start Date	Long term performance			Risk-adjusted P&L attribution							
				Daily-weekly strategy			Long term				Last year			
				Average Return	Std Dev	Return/risk	Explained P&L	Short term volatility	Medium-term trend	Trend volatility	Explained P&L	Short term volatility	Medium-term trend	Trend volatility
Equity	S&P 500	SPX Index	Jan-00	1.93%	3%	0.64	0.25	0.27	-0.01	-0.01	0.02	0.03	-0.01	0.00
	Russell 2000	RTY index	Jan-00	1.64%	3%	0.55	0.29	0.33	-0.02	-0.02	-0.08	-0.04	-0.04	0.00
	Eurostoxx 50	SX5E Index	Jan-00	1.84%	3%	0.61	0.45	0.55	-0.05	-0.05	0.43	0.51	-0.03	-0.05
	Eurostoxx	SXXE Index	Jan-00	1.05%	3%	0.35	0.36	0.51	-0.08	-0.07	0.71	0.81	-0.05	-0.06
	Nikkei	NKY Index	Jan-00	1.03%	3%	0.34	0.41	0.47	-0.03	-0.03	0.24	0.28	-0.02	-0.02
Fixed income	Treasuries	TLT US Equity	Jul-02	1.38%	3%	0.46	0.22	0.36	-0.08	-0.06	1.11	1.17	-0.01	-0.04
	Bund SGI	SGIXBI Index	Jan-00	-0.03%	3%	-0.01	0.03	0.28	-0.15	-0.10	0.84	1.06	0.00	-0.22
	JPY Bonds 10y SGI	SGIXBJ10 Index	Jan-00	0.06%	3%	0.02	-0.04	0.16	-0.11	-0.09	0.35	0.55	-0.03	-0.17
	Corporate Bonds IG US	LQD US Equity	Jul-02	0.64%	3%	0.21	0.14	0.21	-0.03	-0.03	0.16	0.20	0.00	-0.03
	US High Yield	HYG US Equity	Apr-07	0.32%	3%	0.11	0.17	0.22	-0.02	-0.03	-0.07	-0.03	-0.02	-0.02
	Real Estate Equity	IYR US Index	Jun-00	2.32%	3%	0.77	0.21	0.23	-0.01	-0.01	-0.01	0.00	0.00	0.00
Commodity	Gold Future	GC1 Comdty	Jan-00	-0.04%	3%	-0.01	0.16	0.39	-0.12	-0.12	0.33	0.50	0.00	-0.17
	Crude Oil - Brent	CO1 Comdty	Jan-00	1.00%	3%	0.33	0.29	0.34	-0.06	0.00	0.33	0.44	-0.02	-0.09
	Aluminium	LA1 Comdty	Jan-00	2.02%	3%	0.67	0.44	0.52	-0.05	-0.04	0.01	0.07	-0.05	-0.01
	Copper	LP1 Comdty	Jan-00	1.53%	3%	0.51	0.39	0.54	-0.09	-0.06	-0.02	0.08	-0.12	0.03
	Silver	SI1 Comdty	Jan-00	0.34%	3%	0.11	0.11	0.24	-0.06	-0.07	0.06	0.15	0.00	-0.09
FX	AUDUSD	AUDUSD F160 Index	Feb-07	0.63%	3%	0.21	0.26	0.40	-0.08	-0.05	0.14	0.19	0.00	-0.05
	USDJPY	USDJPY T150 Curncy	May-07	0.75%	3%	0.25	0.32	0.50	-0.10	-0.08	-0.11	0.11	-0.11	-0.11
	EURUSD	EURUSD F113 Index	Feb-07	-0.14%	3%	-0.05	-0.05	0.14	-0.10	-0.09	0.64	0.73	-0.04	-0.05

Source: SG Cross Asset Research/Cross Asset Quant – 1.5bp transaction costs included

<sup>33</sup> The single strategies have been sized so that their long-term volatility corresponds to 3%

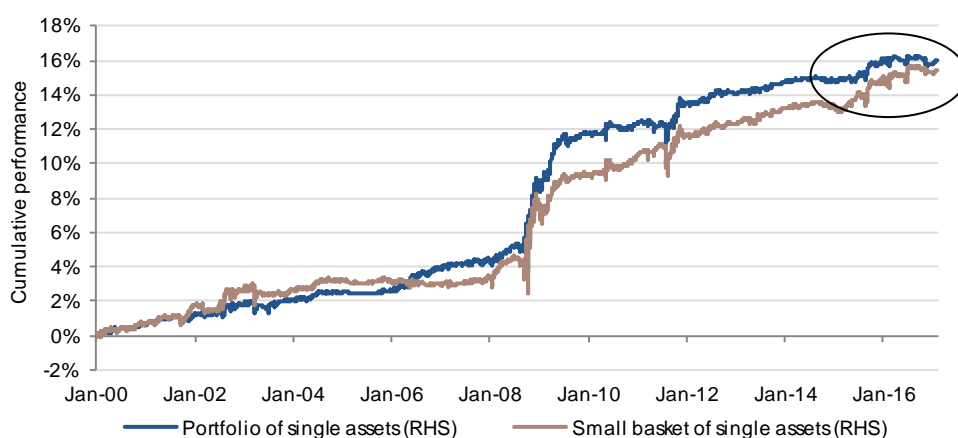
Our starting point is the large basket of 36 single-assets presented on page 17. From this basket, we removed assets that did not revert to the mean. In order to avoid a bias toward equity risk, we also removed the smaller equity indices and kept only the major ones. Our final selection counts 19 strategies and is presented on the table on the previous page.

Among those 19 strategies, eight are particularly attractive and are highlighted in blue. Conceptually, these are the ones that mean revert well, both in the long term and over the past year. In practice, this means that these are the ones that have a short-term volatility contribution at least 0.1 points higher than the trend contributions both in the long run and over the past year.

As a side note, investors may wish to pick strategies that are consistent with their macroeconomic analysis. In particular, investors might have a forward-looking view on the future realised volatility of some assets. But this point goes beyond the scope of this paper. We only use past information and its consistency to estimate the future P&L of the strategies.

Reducing the number of assets in the portfolio increases its risk, the portfolio being more sensitive to the potential drawdown of one individual strategy. But it may also be a good source of additional returns if we pick up the good candidates. We compare the eight single-asset strategies with the extended portfolio of 19 single assets<sup>34</sup>.

#### Reducing the number of assets may be a good source of additional returns



Source: SG Cross Asset Research/Cross Asset Quant – 1.5bp transaction costs included

When looking at past performances in the long run, the small basket would have underperformed the diversified portfolio. Over the past two years, it would have well outperformed with a risk-adjusted performance more than 35% higher than the diversified portfolio.

#### Diversification benefits of the mean-reversion portfolio (since 2000)

	Return (p.a.)	St. dev. (p.a.)	Return to risk ratio	MDD	MDD / st. dev.	Skewness	Kurtosis
Portfolio of single assets	0.9%	1.2%	<b>0.78</b>	2.7%	2.22	6.5	179.3
Small basket of single assets	0.9%	1.5%	0.58	3.0%	2.03	6.1	210.5

Source: SG Cross Asset Research/Cross Asset Quant – 1.5bp transaction costs included

<sup>34</sup> Portfolios are equally weighted across all daily-weekly strategies. See Addendum 1 for more.

### Enhancing mean-reversion strategies with pairs

The universe of potential candidates for the daily-weekly strategy remains small when considering only single-asset strategies. One way to extend this universe is to apply the daily-weekly strategy to pairs of assets whose spread is mean-reverting within a week.

The construction process for pairs is two-fold [XQ5]. First we identify the pairs of assets. The daily-weekly on a pair should be able to monetise the mean reversion in the first asset, while removing macro-economic biases with the second asset. Thus the two assets should belong to the same asset class or the same region.

Second, the two assets should be correctly sized to capture that macro-economic bias. We use a regression analysis, to dimension one asset versus the other one and to construct the spread. The daily-weekly trading rule is then applied on that spread<sup>35</sup>.

The table on the following page shows a basket of 27 pairs of assets. We consider a large number of pairs covering four broad cross-asset portfolios (equity-FX, equity-rates, equity-credit). We analyse European pairs, US pairs and Japanese pairs. We also consider intra-commodity strategies, commo-FX strategies and intra-equity strategies.

The model that we used for selecting single assets can also be used to identify candidates for the daily-weekly strategy applied to pairs<sup>36</sup>. We highlight below 12 pairs in **blue**, using the same filters as before for single assets: the daily-weekly strategies have a short-term volatility contribution 0.1 points higher than the trend contributions both in the long run and over the past year.

<sup>35</sup> The strategy is therefore applied to a spread that can be positive or negative. As simple or compounded returns are no longer defined for spreads oscillating around zero, we would rather use absolute returns to compute our positions and performance. The beta is estimated by a linear regression over a 2y rolling window and is updated on a monthly basis

<sup>36</sup> We use a long-term linear regression on log prices, with a fix beta to apply our simplified model on the residual

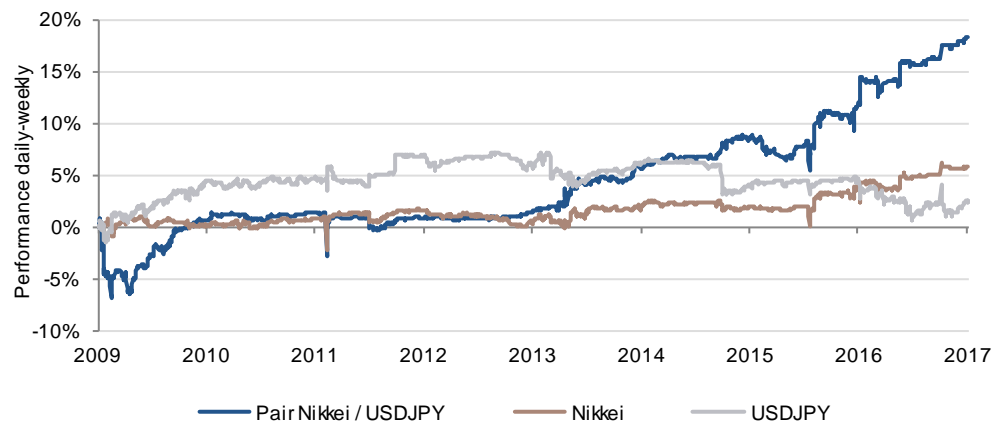
## A basket of pairs

	Pairs	Start date	Return/risk	Long term risk-adjusted P&L attribution				Last year Explained P&L
				Explained P&L	Short-term volatility	Medium-term Trend	Trend volatility	
Equity/FX	S&P 500 (SPX) / EURUSD (EURUSD F163)	Feb-07	0.23	0.32	0.35	-0.02	-0.02	0.14
	Eurostoxx 50 (SX5E) / EURUSD (EURUSD F113)	Feb-07	0.27	0.51	0.64	-0.06	-0.07	0.50
	Nikkei (NKY) / USDJPY (USDJPY t150)	Feb-07	0.59	0.47	0.51	-0.01	-0.02	0.14
Equity/Rates-Credit	Eurostoxx 50 (SX5E) / Bund SGI (SGIXBI)	Jan-00	0.28	0.43	0.53	-0.05	-0.05	0.42
	Nikkei (NKY) / JPY Bonds 10y SGI (SGIXBJ10)	Jan-00	0.08	0.33	0.40	-0.03	-0.03	0.24
	Russell 2000 ETF (IWM US) / Corporate Bonds IG US (LQD US)	Jul-02	0.35	0.35	0.44	-0.04	-0.04	0.23
	S&P 500 (SPX) / US High Yield (HYG US)	Apr-07	-0.3	0.21	0.25	-0.01	-0.03	-0.08
	S&P 500 (SPX) / Corporate Bonds IG US (LQD US)	Jul-02	0.66	0.33	0.37	-0.02	-0.02	0.21
	Real Estate Equity (IYR US) / Treasuries (TLT US)	Jul-02	0.07	0.22	0.24	-0.01	-0.01	0.00
	Russell 2000 ETF (IWM US) / Treasuries (TLT US)	Jul-02	0.01	0.27	0.33	-0.03	-0.03	0.03
	S&P 500 (SPX) / Treasuries (TLT US)	Jul-02	0.33	0.25	0.29	-0.02	-0.02	0.13
Commo/FX	Aluminium (LA1) / AUDUSD (AUDUSD F130)	Feb-07	0.57	0.38	0.54	-0.07	-0.09	-0.02
	Copper (LP1) / AUDUSD (AUDUSD F130)	Feb-07	0.38	0.16	0.31	-0.06	-0.09	-0.10
	S&P Metals and Mining (XME US) / AUDUSD (AUDUSD F160)	Feb-07	0.3	0.38	0.45	-0.04	-0.04	0.33
Commodity	Copper (HG1) / Silver (SI1)	Jan-00	-0.04	0.25	0.38	-0.06	-0.06	-0.20
	Gold Future (GC1) / Silver (SI1)	Jan-00	-0.06	0.08	0.18	-0.05	-0.05	0.06
	Gold (GLD US) / Gold Miners (GDX US)	May-06	0.32	0.42	0.51	-0.05	-0.05	0.32
	Copper (HG1) / Gold Future (GC1)	Jan-00	0.17	0.24	0.38	-0.08	-0.06	-0.08
	Crude Oil - Brent (CO1) / Gold Future (GC1)	Jan-00	0.21	0.20	0.32	-0.06	-0.06	0.37
	Crude Oil - Brent (CO1) / Silver (SI1)	Jan-00	0.38	0.21	0.31	-0.05	-0.06	0.03
Equity	S&P 500 (SPX) / Russell 2000 ETF (IWM US)	May-00	0.17	0.10	0.13	-0.01	-0.01	0.03
	S&P Industrial (XLI US) / S&P Materials (XLB US)	Jan-00	0.41	0.42	0.48	-0.02	-0.03	0.08
	Russell 2000 ETF (IWM US) / Eurozone Equity (EZU US)	Jul-00	-0.12	0.19	0.25	-0.03	-0.03	-0.10
	Eurozone Equity (EZU US) / Japan Equity (EWJ US)	Jul-00	0.29	0.07	0.09	-0.01	-0.01	0.19
	Eurostoxx 50 (SX5E) / Eurostoxx Small Caps (SCXP)	Jan-00	0.47	0.46	0.53	-0.03	-0.04	0.23
	S&P 500 (SPX) / Eurozone Equity (EZU US)	Jul-00	0.14	0.26	0.29	-0.01	-0.01	0.02
	Japan Small Caps (SCJ US) / Japan Equity (EWJ US)	Jan-00	0.6	0.00	0.01	0.00	0.00	-0.15

Source: SG Cross Asset Research/Cross Asset Quant – 2bp transaction costs included

An illustrative example is the case of the Nikkei and USD/JPY. The mean-reversion strategy applied solely to the Nikkei would have largely been improved by removing the macro bias induced by USD/JPY. The strategy exhibits a much better absolute return for the same risk. The return-to-risk ratio is improved by 25% from 0.47 to 0.59.

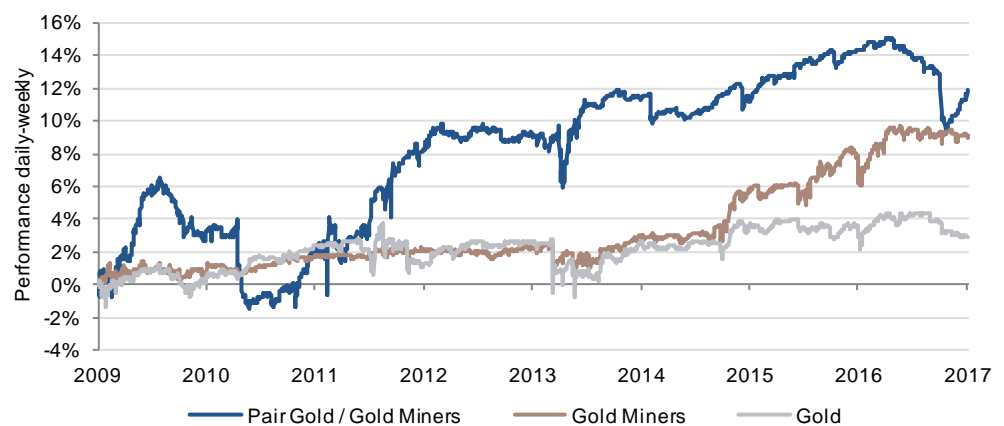
#### Removing the effect of USD/JPY from the Nikkei results in additional performance



Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bp for single-asset, 2bp for the pair)

The Gold/Gold Miners pair is another good example. The daily-weekly strategy would not have worked well when applied solely to Gold. It would work better when applied to the residual of the regression Gold against Gold Miners. The return-to-risk ratio rises from 0.06 to 0.32.

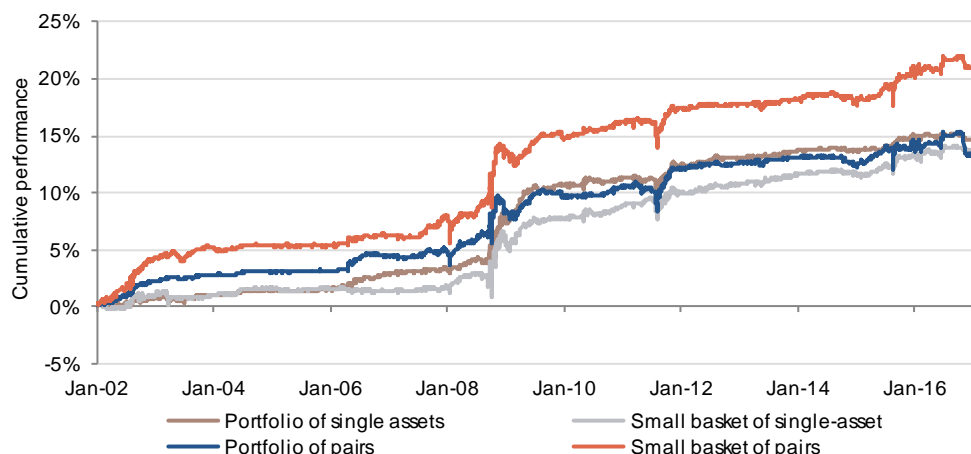
#### Good mean reversion on the Gold/Gold Miners pair



Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bp for single-asset, 2bp for the pair)

We put the 12 strategies together in a small basket of pairs and compare this small basket of pairs with the diversified portfolios and the small basket of single-assets.

#### Enhancing daily-weekly with pairs



Source: SG Cross Asset Research/Cross Asset Quant – Transaction costs included (1.5bp for single-asset, 2bp for the pair)

The small basket of pairs would have outperformed the diversified portfolios of pairs and single-assets in the past with a return-to-risk ratio of 0.84 compared to 0.78 for the diversified portfolio of single-assets and 0.65 for the diversified portfolio of pairs. It would have outperformed the small basket of single-assets as well (0.84 vs 0.58), benefiting from better individual performances (0.34 vs 0.31 for the average return-to-risk ratio) and a greater granularity (12 pairs compared to eight single-assets).

#### Performance analysis

	Return (p.a)	St. dev. (p.a.)	Return/risk	MDD	MDD / st. dev.	Skewness	Kurtosis
Portfolio of single-assets	0.9%	1.2%	0.78	2.7%	2.22	6.5	179.3
Small basket of single-assets	0.9%	1.5%	0.58	3.0%	2.03	6.1	210.5
Portfolio of pairs	0.9%	1.3%	0.65	2.5%	1.91	5.1	147.1
<b>Small basket of pairs</b>	<b>1.3%</b>	<b>1.6%</b>	<b>0.84</b>	<b>2.9%</b>	<b>1.82</b>	<b>5.1</b>	<b>154.9</b>

Source: SG Cross Asset Research/Cross Asset Quant

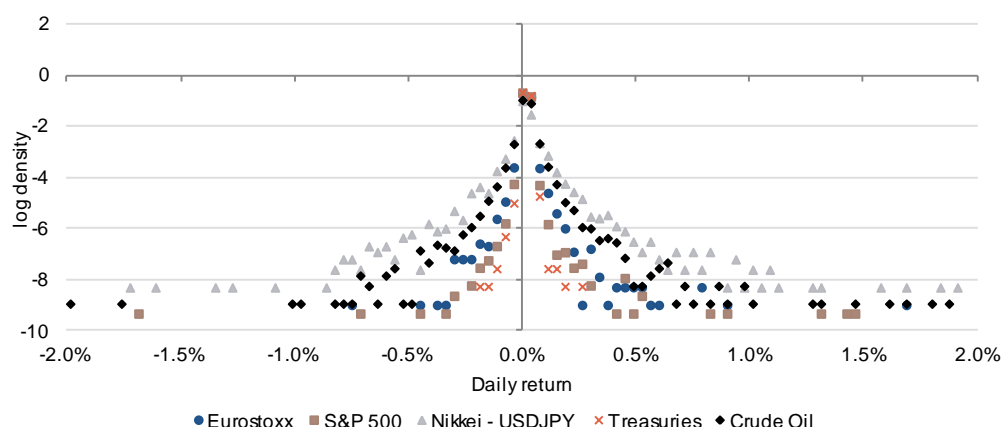
## Measuring the risks of a portfolio

As explained earlier, the distribution of the daily-weekly strategy has fat-tailed both on the downside and on the upside. It is also positively skewed, with a higher probability of observing important movements on the upside than on the downside. We show that the strategies have similar risk profiles when re-normalised by their long-term volatility and propose a standardised distribution using extreme value theory.

We already highlighted that the strategy's returns, when applied to the S&P, are fat-tailed and distributed both on the upside and on the downside. They are also positively skewed, with a higher probability of observing important movements on the upside than on the downside.

What is true for the S&P is also true for other assets and pairs. We compare below the strategy's return distribution on the EuroStoxx, S&P, Treasuries, Crude Oil and on the Nikkei-USDJPY spread. We find similar fat-tailed profiles in all cases, but the distributions differ in terms of deviation around the mean.

### Returns distribution before volatility rescaling

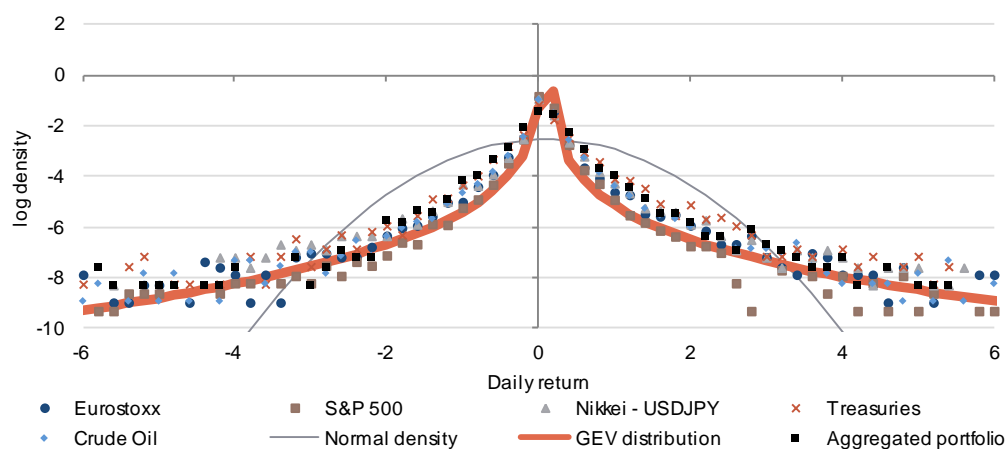


Source: SG Cross Asset Research/Cross Asset Quant

We find more similar fat-tailed distributions after re-normalizing the daily returns by their long-term average and long-term standard deviation. As highlighted below, the normalized distributions are well proxy by a GEV distribution<sup>37</sup>. Interestingly, we also found a similar distribution profile for the diversified portfolio as well (the aggregation of the portfolio of 19 single-asset strategies and the 27 pair strategies).

<sup>37</sup> The GEV distribution was detailed in a Quant box earlier in the document. Here we use  $\mu_d = 0$ ,  $\sigma_d = 0.047$ ,  $\xi_d = -3.20$ ,  $\mu_u = 0$ ,  $\sigma_u = 0.030$  and  $\xi_u = -3.66$  for the parameters. We have calibrated the parameters  $\sigma_d$ ,  $\xi_d$ ,  $\sigma_u$  and  $\xi_u$ , to fit the return distributions of the five underlying assets (EuroStoxx, S&P, Nikkei-USDJPY, Treasuries and Crude Oil).

### Standardised fat-tailed distribution (after volatility rescaling)



Source: SG Cross Asset Research/Cross Asset Quant

The following table shows the quantiles of the standardised GEV distribution and compares them with the quantiles of the historical distributions of the different daily-weekly strategies. It confirms the similar risk profile when returns are re-normalised. This is an important point to consider for risk management and risk budgeting purposes.

### Quantiles of the daily-weekly standardised daily return distribution<sup>38</sup>

Percentile value	0.1%	0.5%	1.0%	5.0%	10.0%	20.0%	30.0%	50.0%	70.0%	80.0%	90.0%	95.0%	99.0%	99.5%	99.9%
Eurostoxx	-6.78	-3.42	-2.25	-0.76	-0.42	-0.18	-0.09	-0.02	0.05	0.14	0.37	0.78	2.67	3.91	10.29
S&P 500	-5.40	-2.21	-1.50	-0.53	-0.30	-0.14	-0.07	-0.01	0.04	0.11	0.26	0.50	1.57	2.38	10.45
Nikkei - USDJPY	-10.13	-3.66	-2.73	-0.97	-0.50	-0.20	-0.09	-0.01	0.06	0.19	0.49	0.94	2.93	4.54	11.08
Treasuries	-7.02	-3.81	-2.71	-1.02	-0.64	-0.31	-0.15	-0.03	0.12	0.29	0.68	1.16	2.76	3.93	7.06
Crude Oil	-6.41	-3.43	-2.35	-0.80	-0.46	-0.20	-0.10	-0.02	0.06	0.18	0.45	0.82	2.61	3.59	9.71
Aggregated portfolio	-6.55	-2.96	-2.05	-0.95	-0.61	-0.33	-0.19	-0.03	0.15	0.29	0.56	0.96	2.71	3.93	10.31
<b>Standardised GEV distribution</b>	<b>-7.12</b>	<b>-3.04</b>	<b>-1.93</b>	<b>-0.48</b>	<b>-0.20</b>	<b>-0.05</b>	<b>-0.01</b>	<b>0.01</b>	<b>0.01</b>	<b>0.04</b>	<b>0.17</b>	<b>0.45</b>	<b>2.20</b>	<b>3.68</b>	<b>9.74</b>
Gaussian distribution	-3.09	-2.58	-2.33	-1.64	-1.28	-0.84	-0.52	0.00	0.52	0.84	1.28	1.64	2.33	2.58	3.09
Amplification factor	2.3x	1.2x	0.8x	0.3x	0.2x	0.1x	0.0x	0.0x	0.0x	0.0x	0.1x	0.3x	0.9x	1.4x	3.2x

Source: SG Cross Asset Research/Cross Asset Quant

The table above can be used to directly estimate the risk of a daily-weekly strategy knowing its average return and volatility. Compared to a standard Gaussian model, the Gaussian volatility has to be multiplied by an amplification factor, to take into account fat tails (2.3x for the quantile 0.1%).

### Value-at-risk calculation of a daily-weekly strategy

$$\text{VaR}_{\text{DW}}(x) = \mu_{\text{DW}} + \sigma_{\text{DW}} \cdot Q_{\text{GEV}}(1 - x)$$

$\text{VaR}_{\text{daily-weekly}}(x)$  is the Value-at-Risk of the daily-weekly strategy. It corresponds to the maximum daily loss of the strategy with a probability  $1-x$ .

<sup>38</sup> Quantiles are estimated from the GEV distribution detailed earlier in the document. We use  $\mu_d = 0$ ,  $\sigma_d = 0.047$ ,  $\xi_d = -3.20$ ,  $\mu_u = 0$ ,  $\sigma_u = 0.030$ , and  $\xi_u = -3.66$  for the parameters.



$\mu_{\text{daily-weekly}}$  and  $\sigma_{\text{daily-weekly}}$  are the expected average and standard deviation of the strategy's returns.

$Q_{\text{GEV}}$  is the quantile of the standardised GEV distribution.

Let's take the S&P as an example. The annualised average return and the annualised volatility of the daily-weekly strategy applied to the S&P are 0.41% and 0.69% respectively, estimated using data since 2000. Using the standardised GEV distribution we find that the maximum daily loss with a 99.9% probability, or 99.9% Value at Risk (VaR), is -0.18%.

$$\text{VaR}(99.9\%) = -7.12 * 0.41\% \sqrt{\frac{1}{252}} + \frac{0.69\%}{252} = -0.18\%$$

## Allocating across strategies

In '[Risk parity vs minimum variance – A comparative study of portfolio optimisation techniques](#)' [XQ4], we compared various portfolio optimisation techniques on different type of portfolios: x-asset portfolio, risk-premia portfolio and a pure equity portfolio.

We globally found best performances applying a risk budgeting approach for portfolio construction. We also showed that on a low-correlated risk-premia portfolio an equal risk contribution (ERC) approach is almost equivalent to the "naïve" risk parity approach (inverse-volatility).

In the same way, the mean-reversion strategies are low correlated with each other. So we don't see particular added-value in using advanced allocation techniques to optimise the correlation structure.

However, in mean-reversion strategies characterised by fat-tailed distribution, small returns are very often obtained, and there are very large moves to the downside or the upside from time to time. There is the risk to over-allocate using some strategies that have not experienced a major drawdown for a long time by using the volatility of the strategy over a recent period. So it is important to take into account extreme risks in order to determine the risk budget of one particular strategy.

Allocating the strategies by the inverse of their long-term volatility seems a good approach. It has a longer memory and takes into account potentially high movements that happened in the past. And we just showed that, when rescaled by their long-term volatility, the daily-weekly strategies have very similar risk profiles. So, there is no need in using extreme risk measures in the allocation process as the results are likely to be very similar.

## Addendum

### Addendum 1: Simulation assumptions

In applications, our implementation of mean-reversion is based on a slightly modified version of this standard daily-weekly – we dubbed it the enhanced daily-weekly in [XQ5]. It takes the latest observed return as a new position instead of resetting it to zero, and we average out all the possible choices for the reset day to avoid any cherry-picking bias.

For single-assets, the strategy has been computed on log-returns assuming a 1.5bp transaction cost. To compute the final portfolio of single-asset strategies, we have first vol-targeted each individual strategy to get a volatility of 3%, and then computed an equal-weight portfolio of these strategies.

For pairs of asset, the beta has been estimated by a linear regression over a rolling 2y time window. The parameter beta is updated each month. The strategy is computed using variations of the resulting spread instead of classical returns as the spread can become negative. The final strategy has been resized in-sample to achieve a volatility of 3%. We assume a transaction cost of 2bp. The portfolio of pairs of assets has been computed as an equal weight portfolio of these individual vol-targeted strategies.

On the currency side, we use daily fixing rates for a specific time zone in order to have good synchronization with other indices. The EURUSD F113 Index is typically for the European time zone while the EURUSD F163 Index is for the US one. It explains why our FX time series simulations start in 2007.

We did not include execution costs or additional costs such as the remuneration of the manager or operational risk in the strategies.

## Addendum 2: P&L of a daily-weekly strategy

The weekly P&L of the daily-weekly strategy is the sum-of-the-expectations of these cross-products.

$$\Pi = - \sum_{0 \leq i < j \leq N} E(r_{t+i} r_{t+j}) \delta^2$$

Expectations are annualised.

We note  $\delta$  is the shortest time horizon (1/250 if we assume 250 business days within a year),  $\Delta$  the longest one (1/50 if we assume 50 weeks within a year), and  $= \frac{\Delta}{\delta}$ .

Expressed with the lagged-terms representation, the P&L becomes:

$$\Pi = - \sum_{i=1}^{N-1} E(r_t, r_{t-i}) (\Delta - \delta i) \delta$$

The weighting term  $(\Delta - \delta i)$  simply counts the number of terms in each diagonal (four terms for the lag one cross-product, three terms for the lag two...).

By definition, the auto co-variance function of a process at order  $i$  is:

$$\text{cov}(r_t, r_{t-i}) = E([r_t - E(r_t)] \cdot [r_{t-i} - E(r_{t-i})])$$

So,

$$E([r_t, r_{t-i}]) = \text{cov}(r_t, r_{t-i}) + E(r_t)E(r_{t-i})$$

The previous equation can be simplified if we assume that the expected returns are constant in time:  $E(r_t) = E(r_{t-i}) = \mu$ .

This leads to:

$$\Pi = - \sum_{i=1}^{N-1} (\text{cov}(r_t, r_{t-i}) + \mu^2) (\Delta - \delta i) \delta$$

If we annualised the P&L it becomes:

$$\Pi_{\text{annualised}} = - \sum_{i=1}^{N-1} (\text{cov}(r_t, r_{t-i}) + \mu^2) \frac{\Delta - \delta i}{N}$$

The expected P&L can be expressed as minus the weighted sum of the lagged co-variance plus the squared trend of the asset.

### Addendum 3: Daily-weekly strategy in an Ornstein-Uhlenbeck model with trend

#### Simplified payoff in an Ornstein-Uhlenbeck model

We try to recover a closed-form formula in a simple risk model for the stock price. Our model for the stock prices (log) consists in two equations:

- First, the stock price  $X$  has a positive trend  $\mu$
- Second, the de-trended process  $Y$  mean-reverts around zero. This is given by an Ornstein-Uhlenbeck process with a mean-reversion parameter  $a$ , and a volatility  $\sigma$ .

$$\begin{aligned} dX_t &= dY_t + \mu dt \\ dY_t &= -aY_t dt + \sigma dW_t \end{aligned}$$

We remind readers again that the integrated form of the Ornstein-Uhlenbeck process is given by:

$$Y_t = Y_0 e^{-at} + \int_0^t \sigma e^{-a(t-s)} dW_s$$

Integrating the equation in the stock price  $X$ .

$$X_t = X_0 + (Y_t - Y_0) + \mu t$$

We are interested in the difference between two log-prices with a lag:

$$\begin{aligned} X_{t+\delta} - X_t &= (Y_{t+\delta} - Y_t) + \mu\delta \\ &= \mu\delta - Y_0 e^{-at}(1 - e^{-a\delta}) + \sigma \left[ \int_0^{t+\delta} e^{-a(t+\delta-s)} dW_s - \int_0^t e^{-a(t-s)} dW_s \right] \\ &= \mu\delta - Y_0 e^{-at}(1 - e^{-a\delta}) + \sigma e^{-at} \left[ e^{-a\delta} \int_0^{t+\delta} e^{as} dW_s - \int_0^t e^{as} dW_s \right] \\ &= \mu\delta - Y_0 e^{-at}(1 - e^{-a\delta}) + \sigma e^{-at} \left[ (e^{-a\delta} - 1) \int_0^t e^{as} dW_s + e^{-a\delta} \int_t^{t+\delta} e^{as} dW_s \right] \end{aligned}$$

Then the expectation of the squared value is:

$$\begin{aligned} E[(X_{t+\delta} - X_t)^2] &= [\mu\delta - Y_0 e^{-at}(1 - e^{-a\delta})]^2 \\ &\quad + \sigma^2 e^{-2at} \left[ (e^{-a\delta} - 1)^2 \int_0^t e^{2as} dt + e^{-2a\delta} \int_t^{t+\delta} e^{2as} dt \right] \\ &= [\mu\delta - Y_0 e^{-at}(1 - e^{-a\delta})]^2 + \sigma^2 \left[ (1 - e^{-a\delta})^2 \frac{1 - e^{-2at}}{2a} + \frac{1 - e^{-2a\delta}}{2a} \right] \end{aligned}$$

To simplify, we assume that the initial value of the detrended process is null:  $Y_0 = 0$ .

$$E[(X_{t+\delta} - X_t)^2] = [\mu\delta]^2 + \sigma^2 \left[ (1 - e^{-a\delta})^2 \frac{1 - e^{-2at}}{2a} + \frac{1 - e^{-2a\delta}}{2a} \right]$$

The final P&L of the standard daily-weekly strategy is half the difference between:

- the expected daily variance over a week:  $\sum_{k=0}^{N-1} E[(X_{t+\delta(k+1)} - X_{t+\delta k})^2]$

- and the expected weekly variance:  $E[(X_{t+\Delta} - X_t)]^2$

Expected daily variance over a week:

$$\begin{aligned} & \sum_{k=0}^{N-1} E[(X_{k\delta+\delta} - X_{k\delta})^2] \\ &= \mu^2(N\delta^2) + \frac{\sigma^2}{2a} \left[ (1 - e^{-a\delta})^2 \sum_{k=0}^{N-1} (1 - e^{-2a(k\delta)}) \right] + N\sigma^2 \frac{1 - e^{-2a\delta}}{2a} \\ &= \mu^2(N\delta^2) + \frac{\sigma^2}{2a} \left[ (1 - e^{-a\delta})^2 \left( N - \frac{1 - e^{-2a\Delta}}{1 - e^{-2a\delta}} \right) \right] + N\sigma^2 \frac{1 - e^{-2a\delta}}{2a} \\ &= \mu^2(N\delta^2) + \frac{\sigma^2}{2a} \left[ (1 - e^{-a\delta})^2 \left( \frac{\Delta}{\delta} - \frac{1 - e^{-2a\Delta}}{1 - e^{-2a\delta}} \right) + \frac{\Delta}{\delta} (1 - e^{-2a\delta}) \right] \end{aligned}$$

Expected weekly variance:

$$\begin{aligned} E[(X_{t+\Delta} - X_t)^2] &= E\left[\left(\mu\Delta - Y_t e^{-at}(1 - e^{-a\Delta})\right)^2\right] + E\left[\left(\sigma \int_t^{t+\Delta} e^{-a(t-s)} dW_s\right)^2\right] \\ E[(X_{t+\Delta} - X_t)^2] &= \left(\mu\Delta - Y_0 e^{-at}(1 - e^{-a\Delta})\right)^2 + \frac{\sigma^2}{2a} (1 - e^{-2a\Delta}) \end{aligned}$$

$$\begin{aligned} \Pi &= \frac{1}{2} \left( \sum_{k=0}^{N-1} E[(X_{t+\delta(k+1)} - X_{t+\delta k})^2] - E[(X_{t+\Delta} - X_t)^2] \right) \\ \Pi &= \frac{1}{2} (\mu^2(N\delta^2 - \Delta^2) + \frac{\sigma^2}{2a} \left[ (1 - e^{-a\delta})^2 \left( \frac{\Delta}{\delta} - \frac{1 - e^{-2a\Delta}}{1 - e^{-2a\delta}} \right) + \frac{\Delta}{\delta} (1 - e^{-2a\delta}) - (1 - e^{-2a\Delta}) \right]) \end{aligned}$$

Considering that  $a\delta \ll 1$  and  $a\Delta \ll 1$ , we have:

$$1 - e^{-a\delta} = a\delta - \frac{(a\delta)^2}{2} \text{ and so } (1 - e^{-a\delta})^2 = (a\delta)^2$$

$$\frac{1 - e^{-a\delta}}{a\delta} = 1 - \frac{a\delta}{2} + \frac{(a\delta)^2}{6} ; \text{ which leads to}$$

$$\frac{1 - e^{-2a\Delta}}{1 - e^{-2a\delta}} = \frac{\Delta}{\delta} \cdot \frac{1 - e^{-2a\Delta}}{2a\Delta} \cdot \frac{1}{(1 - e^{-2a\delta})/2a\delta}$$

$$\frac{1 - e^{-2a\Delta}}{1 - e^{-2a\delta}} = \frac{\Delta}{\delta} \cdot \left( 1 - a\Delta + \frac{2(a\Delta)^2}{3} \right) \cdot \left( 1 + a\delta - \frac{2(a\delta)^2}{3} \right) = \frac{\Delta}{\delta} (1 - a(\Delta - \delta))$$

$$\begin{aligned} \Pi &= \frac{1}{2} (\mu^2\Delta(\delta - \Delta) + \sigma^2 \left[ \frac{(a\delta)^2}{2a} \left( \frac{\Delta}{\delta} - \frac{\Delta}{\delta} (1 - a(\Delta - \delta)) \right) + \Delta(1 - a\delta) - \Delta(1 - a\Delta) \right]) \\ \Pi &= \frac{1}{2} (\mu^2\Delta(\delta - \Delta) + \sigma^2 \left[ \frac{a\delta\Delta}{2} (1 - (1 - a(\Delta - \delta))) + a\Delta(\Delta - \delta) \right]) \\ \Pi &= \frac{1}{2} (-\mu^2\Delta(\Delta - \delta) + \sigma^2 [a\Delta(\Delta - \delta)]) \end{aligned}$$

After simplification, the formula simplifies as:

$$\begin{aligned} \Pi &= \frac{1}{2} (-\mu^2\Delta(\Delta - \delta) + a\sigma^2\Delta(\Delta - \delta)) \\ \Pi &= \frac{1}{2} \Delta(\Delta - \delta) (-\mu^2 + a\sigma^2) \end{aligned}$$

This P&L is calculated for one week. It needs to be divided by  $\Delta$  to be annualised:

$$\Pi_{\text{annualised}} = \frac{1}{2}(\Delta - \delta)(-\mu^2 + \sigma^2)$$

If we assume that the trend  $\theta$  is a random variable following a Gaussian distribution with an average  $\mu$  and a constant standard deviation  $\sigma_\theta$  in a stationary state, we have:

$$E(\theta^2) = (\mu^2 + \sigma_\theta^2)$$

The expected P&L of the strategy becomes:

$$\Pi_{\text{annualised}} = \frac{1}{2}(\Delta - \delta)(-\mu^2 - \sigma_\theta^2 + \sigma^2)$$

The model can be as well applied to the enhanced daily-weekly strategy. The number of active trading days for the strategy is five instead of four within a week and the previous formula needs to be multiplied by a factor 1.5. It corresponds to the ratio between the number of elements below the diagonal in a 6x6 matrix (15) for the enhanced daily-weekly and the number of elements below the diagonal in a 5x5 matrix (10) for the standard daily-weekly. There are overlapping of returns in the enhanced strategy that may slightly change the trend estimation, we do not take it into account.

### Accuracy vs reactivity trade-off

From a practical standpoint, the choice of the time windows used to estimate the parameters runs into the well-known accuracy vs reactivity trade-off.

Obviously, if we want the estimate of a regression coefficient to be meaningful, we need a sufficiently large number of observations. However, from a reactivity standpoint, we would like our model to adapt very quickly to the new environment. Our parameters therefore need to be estimated on a small time window. This is especially true as our daily-weekly strategy is reset every week and we do not necessarily need for the parameters to be constant or even close from one week to the other.

We use weekly change for the trend and estimate its average and standard deviation using a six-month time window.

We de-trend the asset price (in log) time series with a six-month moving average and get the time series  $Y$ .

The mean-reversion parameters are estimated by linear regression. The daily change of  $Y$  is regressed against the difference between  $Y$  lagged by one day and its five-day moving average. In practice, we subtract the five-day moving average to have a good measure of the intra-week mean reversion, as the daily-weekly strategy is reset on a weekly basis. The parameter is estimated dynamically with a linear regression using a six-month rolling window.

$$Y_{t+\delta} - Y_t = \alpha(Y_t - \text{avg}(Y_{t-\delta} - Y_{t-\delta-\Delta})) + \varepsilon_t$$

We chose a one-month time window to estimate the volatility of the mean-reversion process which needs to be more reactive.

## Addendum 4: equivalence between the daily-weekly and a variance differential payoffs

If we note the weekly variance term as the squared value of the return measured over a week:

$$\text{Weekly var} = Y_{t:t+5}^2$$

With  $Y_{t:t+5} = \ln\left(\frac{S_{t+5}}{S_t}\right)$  when possible, or in the case of spreads that can be negative  $Y_{t:t+5} = S_{t+5} - S_t$ .

Detailing the weekly-return into daily components leads to:

$$\text{Weekly var} = \left\{ \sum_{i=1}^5 Y_{t+i:t+i+1} \right\}^2$$

For convenience, the lag-one return:  $Y_{t+i:t+i+1}$  is now noted  $Y_{t+i+1}$ .

Finally, expanding this last expression leads us to the standard daily-weekly strategy:

$$\begin{aligned} \text{Weekly var} = & \sum_{i=1}^5 Y_{t+i+1}^2 + 2Y_{t+2} \cdot \{Y_{t+1}\} + 2Y_{t+3} \cdot \{Y_{t+2} + Y_{t+1}\} \\ & + 2Y_{t+4} \cdot \{Y_{t+3} + Y_{t+2} + Y_{t+1}\} \\ & + 2Y_{t+5} \cdot \{Y_{t+4} + Y_{t+3} + Y_{t+2} + Y_{t+1}\} \end{aligned}$$

We notice that  $\sum_{i=1}^5 Y_{t+i+1}^2$  – the sum of squared daily returns (in brown) – is the daily variance measured over a week. The terms in blue define a strategy. Repeated week after week, they boil down to the standard daily-weekly strategy that we introduced in [XQ5].

The previous equation can be summed-up as:

$$2 \text{ Standard daily/weekly} = \text{Daily var} - \text{Weekly var}$$

With

$$\begin{aligned} & \text{Standard daily weekly} \\ = & -Y_{t+2} \cdot \{Y_{t+1}\} - Y_{t+3} \cdot \{Y_{t+2} + Y_{t+1}\} \\ & - Y_{t+4} \cdot \{Y_{t+3} + Y_{t+2} + Y_{t+1}\} \\ & - Y_{t+5} \cdot \{Y_{t+4} + Y_{t+3} + Y_{t+2} + Y_{t+1}\} \end{aligned}$$

## Addendum 5: link between option strategies and mean-reversion

We can write the P&L of the delta-hedge of an option portfolio between t-1 and t as:

$$\Delta \Pi_t = -\Delta_{\text{option}}(S_{t-1}, t-1) \cdot \Delta S_t$$

We assume that the option portfolio as a constant gamma. We demonstrated in “Hedges are not a luxury” [XQ7] (p15) that a portfolio of calls and puts across the whole range of possible strikes has a constant gamma when weighting the options by the inverse of the square of their strike. Such portfolio actually replicates a variance swap when delta-hedged dynamically.

If we neglect the time effect on the option delta calculation, with a first order Taylor development we have:

$$\Delta_{\text{option}}(S_{t-1}) \sim \Delta_{\text{option}}(S_{t-2}) + \frac{\partial \Delta_{\text{option}}}{\partial x}(S_{t-2}) \cdot \Delta S_{t-1}$$

With a simple recursion, and assuming  $\Delta_{\text{option}}(S_{t_0})=0$ , which is in general the case with strangle options we have:

$$\Delta_{\text{option}}(S_{t-1}) \sim \sum_{i=1}^{t-t_0-1} \Gamma_{\text{option}}(S_{t-i-1}) \cdot \Delta S_{t-i}$$

If we assume the gamma of the strangle option constant we get:

$$\Delta_{\text{option}}(S_{t-1}) \sim \Gamma \cdot \sum_{i=1}^{t-t_0-1} \Delta S_{t-i}$$

And

$$\Delta \Pi_t \sim -\Gamma \cdot \sum_{i=1}^{t-t_0-1} \Delta S_{t-i} \cdot \Delta S_t$$

We finally get the P&L of the delta hedge between  $t_0$  and t proxy as:

$$\Pi_t \sim -\Gamma \cdot \sum_{k=1}^{t-t_0} \sum_{i=1}^{k-t_0-1} \Delta S_{t_0+k-i} \cdot \Delta S_{t_0+k}$$

We again find the lower diagonal terms and the P&L of the daily-weekly strategy.

$$\text{Delta hedge of Long Strangle Portfolio} \propto \text{Daily/weekly}$$

This result is strong and it shows the direct link between options and the daily-weekly strategies. We can re-write the delta-hedge of a long strangle portfolio as the sum of a short position on the strangle portfolio and a long position on the delta-hedged strangle portfolio. The delta-hedged strangle portfolio, with constant gamma (with weights proportional to the inverse of the square of the strikes), is a synthetic replication of variance swap (see [XQ7]). This way we show that the daily weekly is a mix of two styles. It is the sum of a short position on a strangle portfolio and a long position on synthetic variance.

**Daily-weekly = Short Strangle Ptf + long delta-hedged Strangle Ptf**

**Daily-weekly = Short Strangle Ptf + long Synthetic Variance**

**Short option**

**Long realised volatility**



## Addendum 6: Augmented Dickey-Füller Test for stationarity

A standard augmented Dickey-Fuller test can be used for measuring the stationarity of an asset. It tests the null hypothesis of the presence of a unit root. If the statistic is below the critical value, the null hypothesis is rejected. The time series tends to revert well to its mean.

The ADF test is run on the increment of the price series relative to the price itself. At the order 1, we can write the underlying model as:

$$\Delta Y_t = \alpha Y_{t-1} + \mu + \theta t + \varepsilon_t$$

The null hypothesis is :  $\alpha = 0$  against  $\alpha < 0$

The test statistic is

$$ADF = \frac{\hat{\alpha}}{SE(\hat{\alpha})}$$

Where  $\hat{\alpha}$  is estimated by ordinary least-square regression and  $SE(\hat{\alpha})$  is its standard error. The critical value is -3.5 at 5% level.

As such, the ADF test is implemented on the discretisation scheme of an Ornstein Uhlenbeck process on price series with trend.

## Addendum 7: Estimating the Ornstein-Uhlenbeck parameters

We present here two ways for estimating the parameters  $a$  and  $\sigma$  in an Ornstein-Uhlenbeck process.

$$dY_t = -aY_t dt + \mu + \sigma dW_t$$

The first method is based on a statistical estimation of the parameters via linear regression whereas the second one is based on the integrated formula for volatility. Interestingly, this latter method also leads to another test for mean-reversion – the Hurst exponent method.

### Regression method

The first step is to discretise the Ornstein-Uhlenbeck process.

$$\begin{aligned}\Delta Y_{t+\delta} &= -aY_t\delta + \mu + \sigma(W_{t+\delta} - W_t) \\ Y_{t+\delta} &= (1 - a\delta)Y_t + \mu + \sigma\sqrt{\delta}\varepsilon\end{aligned}$$

With  $\varepsilon = N(0,1)$  a white noise.

Then, the idea is simply to run a regression between the process and itself lagged by one day.

$$Y_{t+\delta} = \alpha Y_t + \mu_r + \varepsilon_r$$

And estimate the co-efficients  $(\hat{\alpha}, \hat{\mu}_r, \hat{\sigma}_{\varepsilon_r})$ . We then identify the parameters of the original Ornstein-Uhlenbeck with the estimated co-efficients of the linear regression:

$$\begin{cases} \hat{\alpha} = (1 - a\delta) \\ \hat{\mu}_r = \mu \\ \hat{\sigma}_{\varepsilon_r} = \sigma\sqrt{\delta} \end{cases}$$

### Parametric volatility method

The Ornstein-Uhlenbeck process leads to closed form-formulas for the underlying process. Indeed, the price  $X_t$  is given by the formula:

$$X_t = X_0 e^{-at} + \int_0^t \sigma e^{a(s-t)} dW_s$$

This equation states that the price follows a normal distribution with mean and variance:

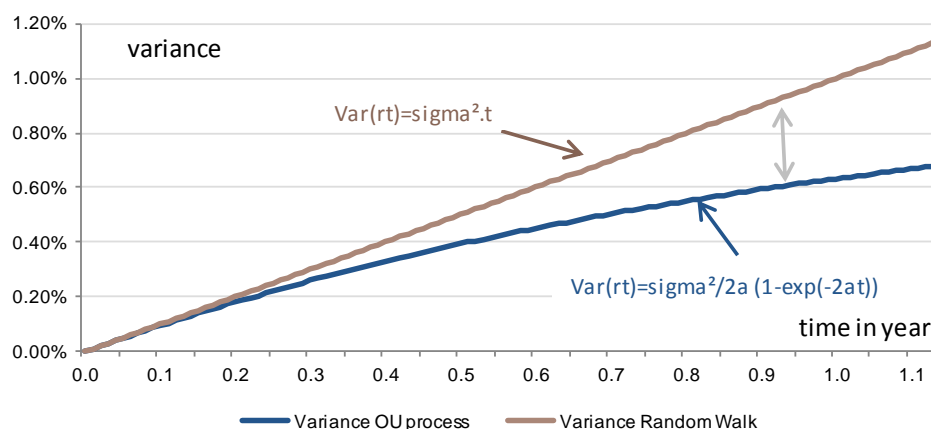
$$E(X_t) = X_0 e^{-at}$$

$$\text{Var}(X_t) = \frac{\sigma^2}{2a} (1 - e^{-2at})$$

Therefore, the volatility measured between two times is given by:

$$\sigma(X_{t+h} - X_t) = \sigma \sqrt{\frac{(1 - e^{-2ah})}{2a}}$$

### Parametric form of the variance in an Ornstein-Uhlenbeck process



Source: SG Cross Asset Research/Cross Asset Quant

If we sample the time series with different frequencies (daily, weekly, biweekly, monthly for instance) and compute the volatility, we can then estimate directly our co-efficients  $(a, \sigma, \mu)$ . This is done by minimising an objective function that measures the discrepancy between the empirical estimates and the analytical result under the Ornstein-Uhlenbeck diffusion.

Interestingly, this latest method illustrates very well that in case of mean-reversion, the variance measured over long sampling frequency does not scale linearly with the variance measure at higher frequencies. Indeed, the blue curve representing the variance of the process lies below the brown line. Linearity for the variance would be obtained in the case of a random-walk. Expressed in terms of volatility instead of variance, it simply means that the volatility of a mean-reverting process measured at a given sampling frequency is expected to be lower than the square root of the sampling frequency (in the case of a random-walk). Besides the augmented Dickey-Fuller statistic, another test commonly used for the mean-reversion is the Hurst exponent method. As described in the box below, this method simply states that as long as the exponent driving the volatility of the process is below 0.5, then the asset reverts to its mean.

#### Maths corner: the Hurst exponent method for mean-reversion

To test if an asset reverts to its mean, a commonly used method is to compute the volatility for different sampling frequencies.

Three cases can happen:

- 1) If this volatility is scaled in the squared-root of time for different sampling frequency then the asset follows a random-walk.
- 2) On the other hand, if the volatility measured for long-frequencies stands below this line, then the asset is stationary and supposed to mean-revert.
- 3) Finally, if the volatility stands above the line defined by random-walk, then the process is non-stationary and the price can diverge.

## Addendum 8: AR(1) process on the returns

### From an AR(1) process to an Ornstein Uhlenbeck process

An AR(1) model is written:

$$Y_t = \alpha Y_{t-1} + \mu + \sigma \varepsilon_t$$

For a mean-reverting Ornstein-Uhlenbeck process with parameter  $a$ , volatility  $\sigma$  is written

$$dY_t = -aY_t dt + \mu + \sigma dW_t$$

where  $a$  is the (positive) mean-reversion parameter and  $\sigma$  is the volatility.

With the Euler-Maruyama discretisation method, we can write the Ornstein-Uhlenbeck process as an AR(1) process:

$$Y_t = (1 - a\delta)Y_{t-\delta} + \mu + \sigma(W_{t+\delta} - W_t)$$

With the equivalence between the parameters

$$\alpha = (1 - a\delta)$$

Very often in the literature, the Ornstein-Uhlenbeck process is presented with a long-term mean  $Y_\infty$ . If this modelling can make sense in the case of interest-rates, we see no reason here to postulate such an equilibrium level either for prices or returns.

### An auto-regressive model on asset returns

In [XQ2] we used an AR(1) model for the asset returns, which expressed in terms of the underlying asset is equivalent to

$$\Delta X_t = \alpha \Delta X_{t-1} + \mu + \sigma \varepsilon$$

With  $\Delta X_t = \ln(S_t) - \ln(S_{t-1}) = r_t$ .

With the mean-reverting Ornstein-Uhlenbeck parameter  $a$ , we have – as demonstrated in the appendix – the equivalence with the model:

$$\Delta X_t = (1 - a)\Delta X_{t-1} + \mu + \sigma \varepsilon$$

where  $a$  is the mean-reversion parameter and  $\sigma$  is the volatility.

If we develop this equation we obtain the following equation for the price series:

$$X_t = -aX_{t-1} + (1 - a)X_{t-2} + \mu + \sigma \cdot \varepsilon$$

This process is an auto-regressive process AR(2) on the time series. As the sum of the lagged co-efficients is equal to 1, this process is a random-walk and is non-stationary, as the variance is not constant over time.

To explicit this non-stationarity, we can integrate the equation and obtain:

If we compute the terms from the first value:

$$\begin{aligned}\Delta X_1 &= \mu + \sigma \cdot \varepsilon_1 \\ X_1 &= X_0 + \mu + \sigma \cdot \varepsilon_1\end{aligned}$$

For the second term:

$$\Delta X_2 = (1 - a)\Delta X_1 + \mu + \sigma \cdot \varepsilon_2$$

Which leads to:

$$\begin{aligned}X_2 &= X_1 + (1 - a)\Delta X_1 + \mu + \sigma \cdot \varepsilon_2 \\ X_2 &= X_0 + \mu + \sigma \cdot \varepsilon_1 + (1 - a)\Delta X_1 + \mu + \sigma \cdot \varepsilon_2\end{aligned}$$

The third term is then:

$$\begin{aligned}X_3 &= X_2 + (1 - a)\Delta X_2 + \mu + \sigma \cdot \varepsilon_3 \\ X_3 &= X_0 + \mu + \sigma \cdot \varepsilon_1 + (1 - a)\Delta X_1 + \mu + \sigma \cdot \varepsilon_2 + (1 - a)\Delta X_2 + \mu + \sigma \cdot \varepsilon_3 \\ X_3 &= X_0 + 2 \cdot \mu + \sigma \cdot (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + (1 - a)(\Delta X_1 + \Delta X_2)\end{aligned}$$

With  $\Delta X_1 + \Delta X_2 = X_2 - X_0$ , it simplifies in:

$$X_3 = X_0 + 3 \cdot \mu + \sigma \cdot (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + (1 - a)(X_2 - X_0)$$

By recursion, one finds that:

$$X_t = X_0 + \mu \cdot t + \sigma \cdot \left( \sum_{i=1}^t \varepsilon_i \right) + (1 - a)(X_{t-1} - X_0)$$

So:

$$\Delta X_t = -aX_{t-1} + \mu \cdot t + \sigma \cdot \left( \sum_{i=1}^t \varepsilon_i \right) - aX_0$$

Compared with the previous modelling directly on the price we see that the variance actually grows with time. Indeed, each innovation on the returns brings in a new stochastic term. These terms simply adds up for the resulting price time series. As a consequence, the effects are blurred. The volatility term in this model will result in an uncertainty on the trend and real short-term volatility.

From a modelling standpoint and in our particular case where we want to explain the mean-reversion in terms of variance and trend, it seems therefore more appropriate to use a stationary model in prices such as the trend plus mean-reversion model presented in appendix 1. In this latest model, the unconditional volatility of the price series is constant and allows for more direct interpretation.

## Addendum 9: Diversification benefits

We assume that the returns of the stand-alone strategies are identically distributed, following a Gaussian distribution with average  $\mu$  and standard deviation  $\sigma$  and are uniformly correlated with each other with a correlation  $\rho$ .

The return-to-risk ratio of each stand-alone strategy is  $\mu/\sigma$ .

We can easily show that the standard deviation of an equally-weighted portfolio of  $N$  strategies is:

$$\sigma_{\text{ptf}} = \sigma \sqrt{\frac{(N-1)\rho + 1}{N}}$$

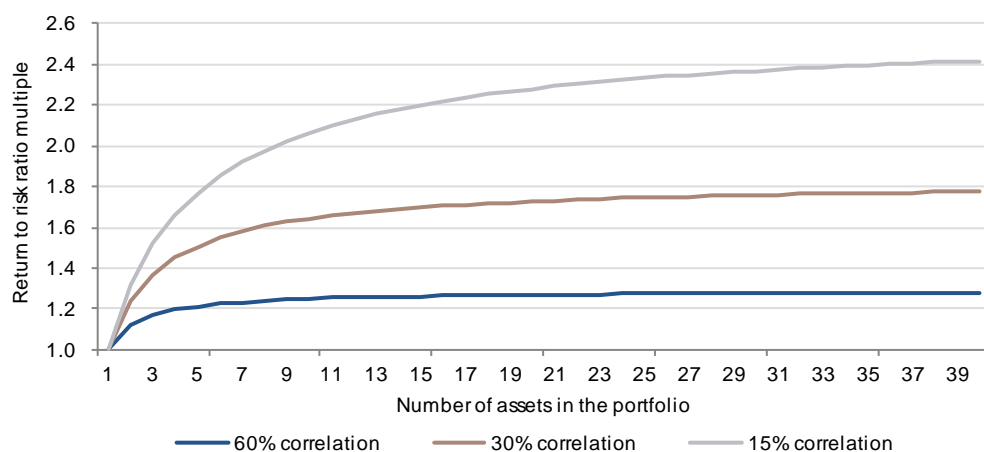
The ratio between the return-to-risk ratio of the portfolio and the return-to-risk ratio of the stand-alone strategy is then:

$$\text{Return to risk ratio multiple} = \sqrt{\frac{N}{(N-1)\rho + 1}}$$

It shows that the maximum gain in multiple you can expect is  $\frac{1}{\sqrt{\rho}}$  when  $N$  is big.

The marginal impact in adding one stand-alone strategy in the portfolio is high when the size of the portfolio is low, but it reduces quickly when the number of strategies in the portfolio increases. The correlation level strongly impacts the diversification benefit in combining the strategies together.

### Benefits and limits of diversification



Source: SG Cross Asset Research/Cross Asset Quant

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