Minimum Size and Maximum Packing Density of Nonredundant Semiconductor Devices*

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***Summary*—It is shown that there exists an absolute lower limit to device size and an absolute upper limit to packing density of nonredundant semiconductor devices, whether integrated or non-integrated, based on fundamental physical phenomena such as statistical variations in impurity distribution, maximum resolution of semiconductor fabrication methods, power density and influence of cosmic rays. The influence of these phenomena falls in two categories, namely failures that appear during the fabrication of the devices (impurity distribution, dividing operation) and failures that appear during use. The latter may be temporary failures (cosmic ray ionization, carrier fluctuations) or permanent failures (atomic displacements by cosmic rays, heat generation).**

For a medium size computer (10^5 components) with a reasonable life expectancy (1 month mean time between failures), the minimum device size under reasonable conditions is approximately $(10\mu)^3$, which is not far from devices now in the planning stage and within reach with existing techniques. It is within a factor of 2–5 of the dimensions of the active region of many devices of today.

As microminiaturization by mere reduction in size appears headed for a not too distant limit it appears necessary from a device point of view to consider remedies which also have been suggested from a system point of view, namely redundancy, self-organizing systems, negative feedback, etc.

I. Introduction

Recently new methods for miniaturizing electronic equipment have been introduced which are capable of packing densities many orders of magnitude higher than used today [1]–[5]. While packing densities of the order of 10^2–10^3 components/cm^3 (10^3–10^4 components per cubic inch) are now commercially available it appears that 10^4 components/cm^3 (10^5 per cubic inch) is certainly attainable with the new techniques, and further improvements by many orders of magnitude have been predicted as methods and technology are developed so that smaller and smaller structures may be handled conveniently A review showing size and packing densities of various components and systems is given in Fig. 1. It becomes of considerable interest then to try to determine whether natural limits in one form or another will stop further progress, or whether limits will be set entirely by economical considerations of diminishing returns.

Limitations on packing density of semiconductor devices have been considered previously by Early [6], Wallmark [7] and Suran [18], although with less generality than intended in this article.

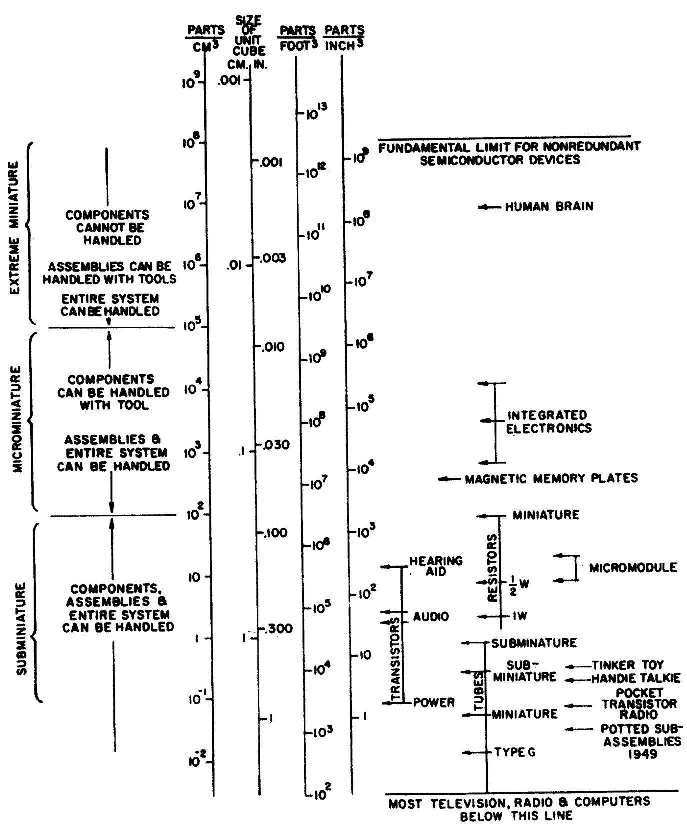

Fig. 1—Packing density and corresponding device size for various systems and components. Components are assumed to occupy circumscribed parallelepiped; leads, contact pins, etc., are neglected.

It will be shown in this article that for nonredundant semiconductor devices a limit on packing density, and therefore also on minimum device size, exists, set by fundamental laws of nature. This limit is within a factor of 2–5 of the dimensions for the active region of devices now being made. Therefore further work in miniaturization is necessarily limited to the package and, to a smaller extent, to non-essential parts of semiconductor devices, and little can be done to further miniaturize the active regions. However, it is also shown that new methods of a systems nature, such as redundancy techniques, may offer a way of partly circumventing the limitations investigated in this article.

II. Method and Basic Assumptions

Let us consider a typical medium size system, such as a computer, with some 10^5 components. Let us initially replace each component with a cube of semi-

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conductor acting as a resistor. This is the level at which the influence of several physical phenomena will be studied. Then let us identify these cubes with the active region of various semiconductor devices—the base region of transistors, the channel region of unipolar transistors, etc. Then let us relax the cubes into arbitrary shapes to fit each particular device. Then add contacts, emitters, insulation, etc., until the devices are complete. Finally add other components, connections, etc., until the circuit is complete.

In order for the analysis to be rigorous let us make the following assumptions which for clarity have been italicized and lettered as they appear in the text.

Assume a number of *cubes* of semiconductors. (A)

Geometries very different from a cube may be thought of as built up of a number of cubes.

Assume *10^5* such cubes. (B)

Assume *silicon as the device material.* (C)

Germanium would lead to quite similar results except when power density is considered, in which case germanium would give lower packing density. Compounds, such as GaAs, hold promise of higher packing density but are not yet sufficiently known for an exact analysis. For simplicity the analysis will be made on n-type silicon only. P-type silicon gives similar although slightly more restrictive results.

Assume that all the devices are *resistors.* (D)

This is a key assumption which allows a rigorous quantitative analysis. In the last sections this assumption will be abandoned and the results will be applied to various active and passive semiconductor devices.

Assume a *perfect fabrication method without acidents leading to shrinkage.* (E)

The justification for this seemingly unrealistic assumption is that shrinkage may be treated separately. An analysis of this problem has been published [7].

Assume an *ideal semiconductor surface*, with the band flat to the surface. (F)

This assumption is an unfortunate necessity because of our limited knowledge about the semiconductor surface, and means that surface problems are neglected.

Assume that the *connections and insulation between elements reduce the packing density by one order of magnitude.* (G)

This is a reasonable assumption which is being approached by the best methods today. It is unlikely to be in error by more than an order of magnitude.

These assumptions are general and apply to all cases considered. Special assumptions have to be added for each of the different physical phenomena examined and will be found below. The assumptions have been summarized in Table VII (Section VII). Fig. 2 shows

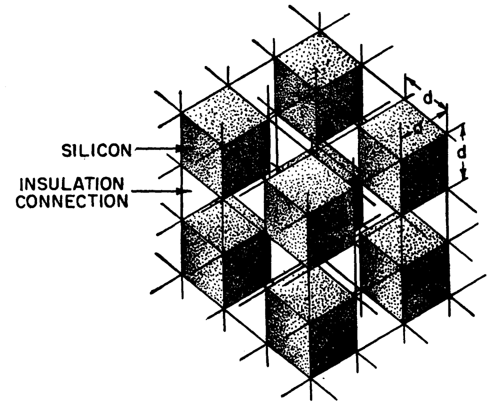

Fig. 2—Schematic picture of system to be considered.

schematically the structure to be considered.

The method to be used involves two steps. First, the limitations to fabrication of an operating circuit, namely statistical variations in doping concentration, and statistical variations in the cutting operation will be examined. Second, the limitations set by the use of an operating circuit once it has been built will be examined. These limitations are based on the influence of cosmic rays and heat generation.

The resistance R of a semiconductor cube with side d is

$$R = \frac{d}{q\mu(n_1 + n)d^2}, \tag{1}$$

where

n_1 is the density of ionized impurities in the cube,
n is the density of excess free carriers,
μ is the carrier mobility.

If the variables are independent and the sides vary independently we have by differentiating the logarithm of (1),

$$\frac{dR}{R} = -\frac{1}{q}\left[\frac{d(n_1 + n)}{n_1 + n} + \frac{3dd}{d} + \frac{d\mu}{\mu}\right]. \tag{2}$$

Of these terms the first takes into consideration variations in doping density and excess carriers created by cosmic rays. The second involves mechanical dimensions of the cubes. The third takes into account temperature variation of mobility. For any given maximum tolerable variation in R the maximum tolerable variation of each of these terms is only $\frac{1}{3}$ as large, as their effects are cumulative. Each of the terms will now be dealt with in turn.

III. Statistical Variations in the Doping Distribution

Assume that the silicon material used is *uniformly doped.* (H)

This represents the least limiting case as practical semiconductor crystals contain resistivity variations and crystal imperfections coupled to impurity concentration. However, improved technology may in the future remove these extra causes of doping variations. The assumption of uniform doping applies only on a macroscopic scale. On a microscopic scale the doping concentration will fluctuate statistically around a mean value. When the size of the element cubes decreases the total number of impurities in each cube will also decrease and therefore the statistical fluctuations will be proportionately larger. For sufficiently small cubes the probability of a deviation in impurity content above or below a certain limit can no longer be neglected.

If N is the mean number of impurities per cube, and ϵ is the maximum allowed fractional deviation from the mean, then the fraction of cubes S_1, in which ϵ is exceeded, may be obtained from

$$S_1 = 1 - \frac{2}{\sqrt{2\pi}} \int_o^{\epsilon N^{1/2}} \exp\left(-\frac{y^2}{2}\right) dy. \tag{3}$$

This expression is derived in Appendix I.

Table I gives some practical values.

TABLE I

Failure Rate S_1 for Various Total Number of Impurities per Device N, Assuming a Maximum Allowable Fractional Deviation ϵ of N

ϵ	N: 10^3	10^4	2×10^4	10^5
0.02	0.53	0.05		$<10^{-8}$
0.033	0.29	1×10^{-3}	5×10^{-6}	$<10^{-8}$
0.05	0.12	6×10^{-7}		$<10^{-8}$
0.10	2×10^{-3}	$<10^{-8}$		$<10^{-8}$

In large circuits, such as computers, the maximum deviation of resistance values is sometimes not more than one or a few per cent; 10 per cent may be a good average figure.

Let us allow for some progress in circuit design and assume a *± 10 per cent maximum allowed tolerance on resistance values.* (I)

This means a $3\frac{1}{3}$ per cent variation in resistance from doping alone according to (2). It is not likely that a large integrated circuit, consisting of 10^5 elements, could be made in practice if the shrinkage rate was much larger than 10^{-5} on individual elements. With $\epsilon = 0.033$ this gives a minimum number of impurities of 2×10^4 from Table I for each element.

The condition that the number of doping agents N in a volume d^3 of a material doped to ρ ohm m exceeds 2×10^4 may be expressed as

$$N = \frac{d^3}{\rho q \mu} \geqq 2 \times 10^4, \tag{4}$$

where

q is the electronic charge (1.6×10^{-19} coulombs),
μ is the carrier mobility (0.1350 m^2/v s for n-type Si),
d is the side of the cube in m.

Eq. (4) is shown as a dashed line marked "impurity fluctuations" in Fig. 9.

IV. Fluctuations in Element Size

In this section we will consider the limitation on packing density caused by fluctuations in element size which in their turn are caused by unavoidable inexactness in the element fabrication process.

The accepted method of fabricating large numbers of small semiconductor devices involves as a necessary step a photographic procedure involving exposure of photoresist either in the fabrication of the devices themselves or in the construction of jigs or masks for the fabrication of devices. The maximum resolution of a photographic procedure is limited to approximately a wavelength of light, *i.e.*, approximately 0.5 μ. This resolution is of course inadequate for fabrication of devices with dimensions of the order of a few microns. It is necessary to consider therefore more energetic particles than visual photons. Very energetic particles on the other hand such as gamma rays are not desirable even though the primary definition may be very high because scattering and particularly secondary particles reduce the definition. This question has been dealt with by Buck and Shoulders [8]. There is therefore an optimum energy of the particles for high definition which is more than 10 ev and less than 10^4 ev. Another requirement is that the total fabrication time be reasonably short, which would favor the higher limit. It seems probable therefore that the electron beam machining or exposure method which has recently been developed [9] is close to the theoretically best possible fabrication tool. The ultimate resolution of this method [8] has been given as approximately 100 Å although at present 10^4 Å is more typical. This figure includes the inexactness of locating the center of the beam, the variations in beam size including primaries as well as secondaries. An exact analysis of this figure has not yet been made.

We may now compute the distribution in resistance values of cubes cut with an inexactness of 100 Å. The fraction of unusable elements S_2 for which the resistance deviates from the mean value more than the tolerance ϵ is

$$S_2 = 1 - \left[\frac{2}{\sqrt{2\pi}} \int_0^y \exp\left(-\frac{y^2}{2}\right) dy\right]^3,$$

$$y = \frac{\epsilon d}{\sqrt{2}\sigma_1} \tag{5}$$

where

d is the side of the cube,
σ_1 is the standard deviation (100 Å).

This relation has been derived in Appendix II.

Inserting practical values in (5) gives the figures in Table II. If as in the previous section $\epsilon = 0.033$, and

$S_2 = 10^{-5}$, the smallest possible device has $d \approx 2\ \mu$. For not too small doping ($N \gg 10$) this is independent of the doping of the material and is indicated as a horizontal dashed line labeled "edge uncertainty" in Fig. 9.

TABLE II

FAILURE RATE S_2 FOR VARIOUS VALUES OF DEVICE DIMENSIONS d AND MAXIMUM ALLOWABLE FRACTIONAL DEVIATION ϵ WHEN THE UNCERTAINTY OF THE EDGE IS 100 Å

ϵ	d			
	Microns			
	0.5	1	2	4
0.02	0.48	0.16	0.02	$<10^{-8}$
0.033	0.24	0.02	7×10^{-6}	$<10^{-8}$
0.05	0.08	8×10^{-4}	$<10^{-8}$	$<10^{-8}$
0.10	8×10^{-4}	$<10^{-8}$	$<10^{-8}$	$<10^{-8}$

It may be argued that for junction devices one dimension, namely perpendicular to the junction, could be made smaller by diffusion, epitaxial growth, or the like, which may have smaller uncertainty than 100 Å. This would apply to thin film devices. On the other hand it may be argued that no thin film device can exist without support, the volume of which has to be included in the size analysis. Removing the support once the elements have been fabricated and interconnected again runs into edge uncertainty. The possibility of a substrate different from the device material and therefore easier to remove is not feasible at present. This question may have to be reexamined in the future.

V. Cosmic Radiation

Let us now consider the limitation on packing density of semiconductor devices set by cosmic radiation. It turns out that cosmic radiation is indeed the most severe limitation which will limit the packing density already at about 10^9 per cubic inch. This corresponds to elements about 10 μ on a side which is not far from what can be obtained with present semiconductor techniques. Shielding against cosmic radiation is of course incompatible with miniaturization techniques as the thickness of a lead shield reducing the radiation level 5–10 times would have to be of the order of 10 cm.

As cosmic rays are comparatively rare events, the average number of free carriers and displaced atoms in a volume of semiconductor is indeed very small and would be negligible in a conventional size component, such as an ordinary transistor or a diode. However, the fact that the disturbance in the trail of the cosmic ray is localized to essentially a mathematical line through otherwise undisturbed material becomes important when very small devices are considered. Then there is a definite probability that one element, or a line of elements, may be rendered inoperative through the passage of a cosmic ray while all the other elements may still be operative and in spite of the average density of the disturbance over the entire volume being extremely small.

Another complicating factor is that the number of created carriers and disturbed atoms in the wake of a cosmic ray is subject to statistical fluctuations much as the distribution of diffused impurities in the semiconductor. This means that in the wake of the cosmic ray there are occasional elements with larger than average concentration of carriers and/or displaced atoms.

The influence of cosmic rays on semiconductor devices is threefold. First the ionization of the cosmic ray will create a temporary excess of hole-electron pairs which will decay with the normal lifetime of minority carriers in the device. If this lifetime is of the order of, or larger than, the minimum operating time of the device, a false signal will result. In a nonredundant circuit such false signals can usually not be tolerated even though the circuit returns to normal operation after the error.

Although a possible alternative, it will be assumed that *speed cannot be sacrificed* to cancel such error signals. (J)

The second influence of cosmic radiation is atomic displacements in the lattice which are known to introduce additional levels in the forbidden band of the semiconductor. In the case of silicon these levels act as traps temporarily removing majority carriers and gradually making both *n*-type and *p*-type silicon more intrinsic. Although these displacements are gradually annealed out even at room temperature this is only partial and the time required is of course much larger than the operating time of the circuit. Therefore, annealing is of little help and may be neglected.

The third influence of cosmic rays arises from nuclear events in which silicon nuclei are hit with sufficient energy to break them up into fragments of various kinds. These fragments in their turn have large energies and ionize and displace many atoms. At the point of impact a "star" of convergent tracks will be formed raising the local density of ionization and displacement over that characterizing a single track.

A. Characteristics of Cosmic Radiation

Cosmic radiation [10]–[12] bombards the earth with a steady rain of charged particles. Only the most energetic ones reach sea level; the less energetic ones are reflected by the earth's magnetic field or absorbed in the atmosphere, creating in their turn a continuous rain of secondary, tertiary, etc., particles. For the purpose at hand the radiation may be considered constant in time as the variations are usually less than a few tenths of a per cent. Because of the influence of the magnetic field of the earth, the radiation increases towards the poles and with height in the atmosphere. It seems reasonable to carry out the main analysis for sea level and latitudes below 50° which would characterize most of the United States and make a rough estimation of conditions at airplane heights and in outer space. The constituents and intensity of cosmic radiation versus atmospheric depth are shown in Fig. 3. An idea of the variation with latitude and with height is given by Table III. At sea level the number of particles arriving is on the average 1–2 per cm^2 and minute.

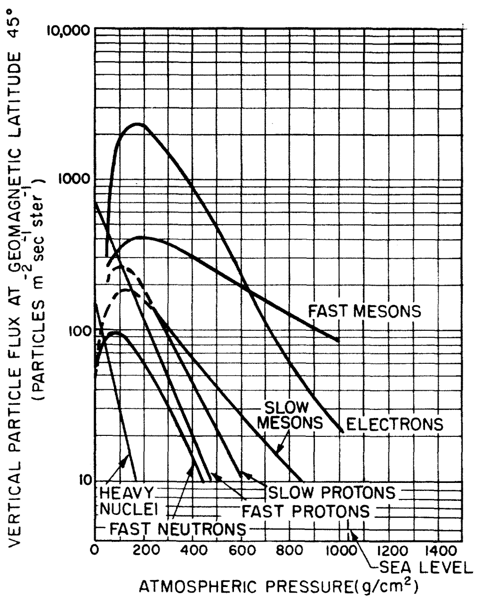

Fig. 3—Intensity of cosmic rays vs altitude (from Ref. 10).

TABLE III
DOSE RATES FROM COSMIC RAYS AT DIFFERENT ALTITUDES AND GEOGRAPHICAL LATITUDES*

Altitude, Feet	Dose Rate at Equator, mrads/year	Dost Rate at Latitude > 40° mrads/year
sea level	23	26
5,000	28	42
10,000	56	84
15,000	110	170

* From Ref. 15.

Because of the absorption of cosmic rays in the atmosphere their intensity increases with altitude as shown in Fig. 3. At the same time the character of the cosmic rays changes so that at higher altitudes the proportion of strongly ionizing particles and nuclear events increases. At airplane height, say 10 km (33,000 feet, 210 mm Hg) the over-all intensity has increased by a factor of 20, the strongly ionizing particles by a factor of 200 and the nuclear events by a factor of 400. Analogous numbers may be found for the top of the atmosphere and are shown in Table V.

In outer space three radiation hazards have to be considered, namely a general background of cosmic rays, trapped particles in the Van Allen belts and solar particle beams. In general the dose rates are very large compared to what is experienced on the ground as shown by Table IV, which shows some typical dose rates in satellites [14]. The increased dose rate would not markedly affect the number of created carriers per element, and therefore the limiting size of elements would not change appreciably. However, the probability of an element being hit by a ray would be considerably larger than on the ground and therefore the mean time between failures correspondingly smaller.

TABLE IV
TYPICAL DOSE RATES IN SATELLITES*

Location	Radiation	Dose Rate, r/Hour
Undisturbed interplanetary space	cosmic rays	0.0006–0.0014
Center of inner Van Allen belt	protons	24
Center of outer Van Allen belt	soft X rays	200
Solar proton beam	protons	$10–10^3$

* Note: 1 r (roentgen) ≈ 1 rad. (From Ref. 14.)

B. Ionization by Cosmic Rays

From the appearance of the tracks in the photographic emulsions used in cosmic-ray studies, the ionizing particles are usually divided into two groups, namely lightly ionizing particles, and heavily ionizing particles. The lightly ionizing particles, mainly fast electons and mesons, arrive with a frequency of $2–3 \times 10^2$ m^{-2} sec^{-1} and create on the average about 4000 ion pairs per m path in air at sea level. However, there is also a number of heavier ionizing particles, mainly protons, constituting about 0.2 per cent of the lightly ionizing particles, but ionizing from 3 to 50 times more, say on the average, 10 times more.

The interaction of high-energy particles is directly proportional to the encountered mass and is therefore increased in a semiconductor over that in air by the ratio of the specific weights. Then the number of electron-hole pairs n created by a cosmic ray in an element of silicon, d meter on a side, is

$$n = Md \frac{E_1}{E_2} \frac{W_2}{W_1}, \tag{6}$$

where M is the number of ionized pairs per m path in air (≈4000 for lightly ionizing particles, ≈40,000 for heavily ionizing particles).

E_1 is the average energy loss of the primary particle per ion pair in air (≈32 ev).
E_2 is the average energy loss per hole-electron pair in silicon (or germanium) (≈3 ev).
W_1 is the specific density of air at sea level and 20°C (1.19 kg/m^3).
W_2 is the specific density of silicon at 20°C (2330 kg/m^3).

Insertion of these values in (6) gives

$$n_l = 8.4 \times 10^7 d \text{ for lightly ionizing particles,}$$
$$n_h = 8.4 \times 10^8 d \text{ for heavily ionizing particles.} \tag{7}$$

The probability f that a surface element d^2 be hit by a cosmic ray particle during the time t seconds is obtained from

$$f = \nu t d^2, \tag{8}$$

where ν is the number of cosmic particles hitting one unit of surface area from all directions in one sec ($2-3\times10^2$ per m² and sec). For a volume in the form of a cube with a side d, the probability of being traversed by a cosmic particle is

$$f_1 = 3\nu t d^2. \tag{9}$$

Assuming N cubes distributed at random in a space $10Nd^3$ so that the probability of traversing the system without hitting a cube can be neglected, the probability of at least one cube being traversed is

$$f_2 = 3(10N)^{2/3}\nu t d^2. \tag{10}$$

In this equation, t has the significance of mean time between failures.

Let us assume a *mean time between failures of 1 month.* (K)

This may seem long compared to present-day practice. However, with decreased physical size of systems the replacement of faulty parts has to encompass larger and larger parts of the system. In the extreme case the entire system may have to be replaced. This is illustrated in the left part of Fig. 1. Consequently economic considerations force a longer mean time between failures than current practice allows.

Insertion of practical values in (10) for lightly ionizing particles gives

$$f_l = 2.0\times10^{13}d^2. \tag{11}$$

For heavily ionizing particles,

$$f_h = 4.0\times10^{10}d^2. \tag{12}$$

C. Atomic Displacements and Nuclear Events

The atomic displacements, or radiation damage, by cosmic radiation is not well known as radiation damage studies have mostly been made in reactors and accelerators with particles and energies quite different from what is common in cosmic rays. However, a reasonable evaluation is possible. The principal action of radiation damage is that each atomic displacement causes the appearance of one or more localized levels in the forbidden band of the semiconductor. In silicon these levels act as traps for majority carriers so that both n-type and p-type silicon become gradually intrinsic with bombardment. The number of carriers lost per displacement is as an average two in silicon. The average energy loss for atomic displacements per unit length of the path of a particle in silicon is 30 ev. Therefore, the number of displacements is 1/10 of the corresponding number for ionization. Then the number of removed carriers is from (6),

$$n = 1.7\times10^7 d. \tag{13}$$

The nuclear events, transmutation of an atom into various fragments by the impact of a sufficiently high-energy particle, are comparatively rare, amounting to approximately 1 per gram of material per day at ground level. The number of secondary particles from such an event range from 3 and up with an average number of 4, most of them (say 75 per cent) heavily ionizing. Using (6), the number of carriers at the origin of the secondary tracks is then

$$n_n = 2.5\times10^9 d. \tag{14}$$

The probability of occurrence, according to (10), in a system of 10^5 cubes, each of volume d^3, is

$$f_n = 7.1\times10^{12}d^3. \tag{15}$$

The results have been summarized in Table V.

TABLE V

Excess (or Deficit) of Carriers, n, and the Probability of its Occurrence, f, at Various Alitudes

Altitude, km	0		0	10	100
	Sea Level		Natural Radio-activity	Air-plane Height	Top of Atmos-phere
	n	f	f increases by a factor of		
Ionization					
Lightly ionizing particles	8×10^7d	$2\times10^{13}d^2$	3	20	<1
Heavily ionizing particles	8×10^8d	$4\times10^{10}d^2$	3	200	1000
Atomic displacements	2×10^7d	$2\times10^{13}d^2$	3	20	2.5
Nuclear events	3×10^9d	$7\times10^{12}d^3$	3	400	4000

D. Radioactive Background

In addition to cosmic rays, the natural radioactive background has to be considered, particularly for equipment on the ground. This background originates to a large part in radioactive minerals and therefore varies considerably from place to place. Because the natural shielding of the air practically eliminates the alpha and beta particles, the radiation to consider consists of gamma rays. Some typical values of dose rates are given in Table VI with cosmic rays as a comparison [15]. Very few places on earth deviate more than a factor of two from the range given in Table VI.

TABLE VI

Dose Rates from Natural Radioactivity in Soil (mrads/Year)*

Rock	mrads/Year
Igneous	66
Shale	53
Sandstone	28
Limestone	11

* From Ref. 15.

In buildings, the dose rate from local gamma emitters will be reduced, by a negligible amount in the case of frame houses but considerably in the case of brick or strone structures. However, the radioactivity of the latter materials will more than compensate for the shielding. Assuming that they are of the same composition as the ground, the dose rate inside will be approximately double that outside from purely geometrical considerations.

The dose rate from natural radioactivity in a sedimentary rock area, assuming that buildings elevate the dose rate by an average factor of 1.5, but that strong radioactive sources such as nuclear material, wristwatch dials, high voltage equipment, etc., are absent, is approximately 45 mrads/year. It is further assumed that the circuit does not contain material that is radioactive such as ordinary glass. In order to take the radioactivity into account, the figures for ground level in Table V should then be multiplied by approximately 3.

E. Fluctuations in Ionization

Because the ionization rate of a cosmic ray particle is not constant but varies along a track, and varies considerably between different tracks, it may happen that even when the average ionization may be neglected, particularly dense parts of the track may not. Therefore a measure of these fluctuations is necessary. Two causes of such fluctuations will be considered, namely energy distribution of the ionizing particles and random statistical variations in ionization. For simplicity only mesons will be dealt with, other particles behaving analogously.

The energy distribution of μ-mesons at sea level is shown in Fig. 4. The average ionization per particle per m path vs momentum is shown in Fig. 5. Combining the data in these figures gives the distribution in ionization rates over all μ-mesons as shown in Fig. 6. It is particularly the low energy mesons that contribute to the spread in ionization rates. From measurements it is known that only 0.05 per cent of the mesons are slow enough to ionize ten times more than the average. This fact determines the slope of the falling lines in Figs. 7 and 8.

The influence of random statistical variations may be analyzed as in Appendixes I and II. However, the influence is small and accomplishes only a slight rounding of the sharp top in Fig. 6.

F. The Fabrication Phase

In considering the total radiation falling on the circuit it may be necessary to consider also the fabrication phase. The reason for this is that the best method known at the present time for fabricating minute details of a size corresponding to 10 μ on a side or smaller, and with a resolution approaching 100 Å, involves the use of a high-energy electron beam as mentioned in Section IV. Although the total integrated radiation dose in fabrication could be considerable, its influence may be neglected here for two reasons. First, the electron beam could be used only for the most critical operation, using other tools for less critical fabrication phases. Second, the radiation dose would only influence the frequency of failures (f in Table V) but not the minimum element size.

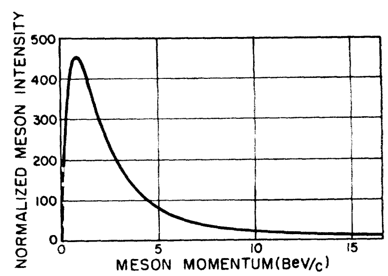

Fig. 4—Differential meson spectrum with respect to momentum, at sea level. The spectrum has been normalized to give 1000 particles between 1 and 10 bev/c (from Ref. 11).

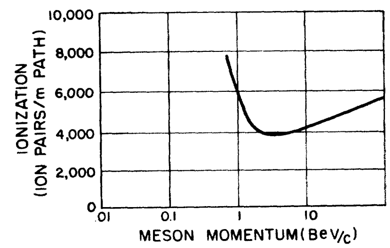

Fig. 5—Ionization rate vs momentum for mesons in air (from Ref. 12)

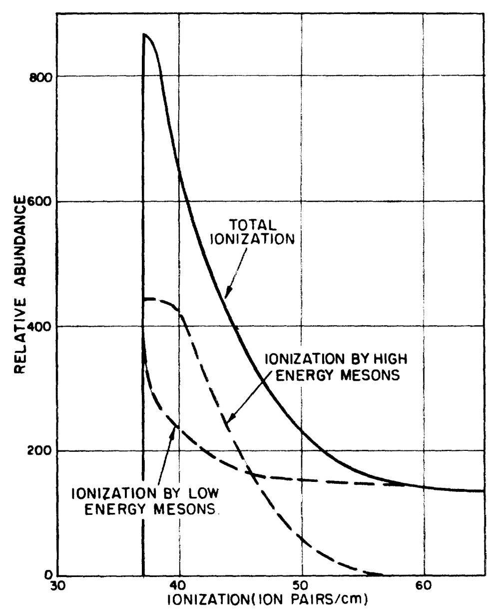

Fig. 6—The distribution in ionization per path length for μ-mesons.

G. Limitations by Cosmic Rays

For each category in Table V, lightly ionizing particles, heavily ionizing particles, etc., two straight lines are obtained which have been plotted in Fig. 7.

One is given by setting the probability equal to one that at least one element of 10^5 is traversed by a cosmic ray in one month, *i.e.*, $f=1$. This condition gives lines rising with increasing d in Fig. 7. To the left of these lines the element size is so small that less than one element of 10^5 will be hit in one month. To the right of these lines more than one element will be hit. For $d=1$ μ, for example, the probability of hit may be found by ex-

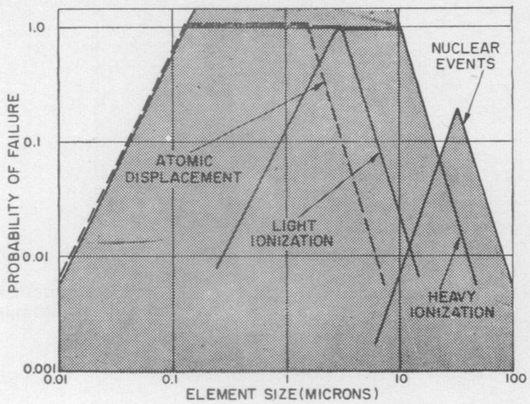

Fig. 7—Probability of failure through cosmic rays at ground level for 20 ohm cm material vs device size.

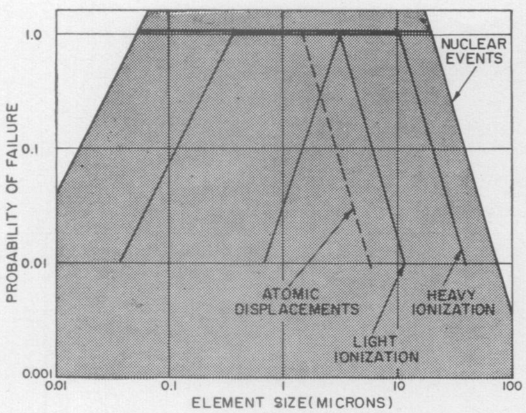

Fig. 8—Probability of failure through cosmic rays at airplane altitude (10 km, 33,000 feet) for 20 ohm cm material vs device size.

tension of the line to be 50, meaning that 50 elements of 10^5 will be hit in one month.

The second line is obtained by the condition that the excess, or deficit, carriers not exceed $3\frac{1}{3}$ per cent of the equilibrium carrier density, or

$$n \leq \frac{0.033d^3}{\rho q \mu}\cdot \tag{16}$$

For reasons given in the next section, ρ may be assumed 0.2 ohm m. Then (16) with results from Table V substituted gives four values for d. In addition the slopes of the lines going through the points defined by these values are given by considerations in Section V-E. The lines are shown in Figs. 7 and 8. To the left of these lines the influence of cosmic rays exceeds the tolerance limit while to the right the influence may be neglected. Thus, the region between the lines shown gray in Figs. 7 and 8 represents element sizes excluded by cosmic ray failure.

For heavily ionizing rays (16) gives

$$d^2 \geq 5.2 \times 10^{-10}\rho. \tag{17}$$

Eq. (17) is shown in Fig. 9 marked "cosmic rays" and represents the most limiting condition at high resistivities.

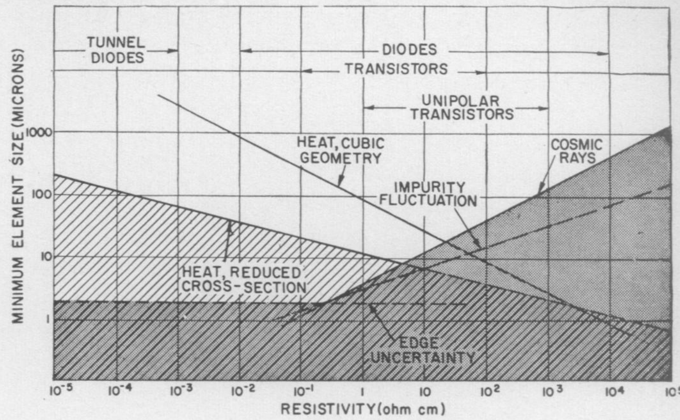

Fig. 9—Minimum device size vs resistivity of the material. Common ranges for some devices are indicated. Dotted area is excluded. When devices are close together, cross-hatched area is also excluded.

At airplane altitudes nuclear events have to be considered and give similarly

$$d^2 \geq 1.9 \times 10^{-9}\rho. \tag{18}$$

VI. Heat Generation

The temperature limitation on packing density of electronic systems has been treated by several authors [7], [18]. The point taken in this article is a more general one, namely that the temperature limitation sets a limit to how highly the material in the active region of semiconductor devices with high packing density may be doped. For material doped below this critical density the limitations caused by cosmic rays and natural radioactivity are more severe; for material doped above this critical density, temperature is the limiting factor. The optimum doping density is just equal to the critical density.

The temperature θ at a radius r in a homogeneous sphere with radius R and a surface temperature θ_o is

$$\theta = \theta_o + \frac{p}{6\lambda}(R^2 - r^2), \tag{19}$$

where λ is the heat conductivity and p is the power developed per unit volume.

The power density p with one resistor in each volume $10d^3$ is

$$p = \frac{V^2}{10\rho d^2}, \tag{20}$$

where V is the applied voltage. From (19) and (20) we obtain

$$R = d\left[\frac{60\lambda\rho(\theta_i - \theta_o)}{V^2}\right]^{1/2}, \tag{21}$$

where θ_i is the temperature in the center of the sphere

The total number of elements in the sphere N, considering assumption G above, is

$$N = \frac{4\pi R^3}{30d^3}\,. \tag{22}$$

Combination of (21) and (22) gives

$$N = \frac{4\pi}{30}\left[\frac{60\lambda\rho(\theta_i - \theta_o)}{V^2}\right]^{3/2} \tag{23}$$

Now let us insert practical values.

In the case of pure silicon, a temperature difference between the hottest and the coolest element, $\theta_i - \theta_0$, may not exceed approximately 4°C in view of assumption (I) and the known temperature dependence of the carrier mobility. However, the temperature variation of mobility decreases with doping and is also smaller for germanium than for silicon. Therefore a temperature difference of 20°C may be possible and will be assumed here.

Assuming a construction with semiconductor elements mounted on printed circuits on ceramic wafers, the heat conductivity would be a compromise between that of gas like air (0.025 W/m C°), that of semiconductor (Si 148 W/m C°, Ge 58 W/m C°), that of ceramic (≈ 2 W/m C°) and that of metal (Ni 88, Au 300, Al 218 W/m C°). A fair estimate of the resulting heat conductivity may be 1 W/m C°. It is possible that this figure could be increased by special cooling. However, this would unavoidably add to the volume offsetting, at least partly, the advantage gained.

In this analysis it will be assumed that *no special cooling* is employed. (L)

The minumum voltage for operation of junction-type devices is, as assumed earlier, 0.25 v. Then (23) gives

$$N = 1.1 \times 10^6 \rho^{3/2}. \tag{24}$$

With $N = 10^5$ we obtain $\rho = 0.2$ ohm m.

When ρ is reduced below 0.2 ohm m, the packing density also has to be reduced. This means that although the devices may be made smaller, the space they occupy must be enlarged. The volume occupied per device may be obtained from

$$d^3 = d_0^3 \left(\frac{0.2}{\rho}\right)^{3/2}, \tag{25}$$

where d_0 is the dimension at 0.2 ohm m. From cosmic-ray considerations $d_0 = 20\ \mu$. Eq. (25) is indicated in Fig. 9 and marked "heat, cubic geometry."

VII. Re-Examination of Assumptions

At this point it is interesting to go back and reexamine the basic assumptions to see whether the results are sensitive to variations of these assumptions. Table VII summarizes this review. Two of the assumptions (A and D) will be abandoned in Section VIII. The abandoning of three more (E, F and H) would only make things worse and raise the minimum size of elements. The largest group (B, C, G, J and K) are not very critical and little can be gained by abandoning them. The remaining two (I and L) are more influential and changes in them could conceivably reduce the minimum size somewhat.

TABLE VII
Basic Assumptions and Their Influence on the Final Results

	Assumptions	Abandoned in Section VIII	No Improvement Possible	Little Improvement Possible	Improvement Possible
A	Cubic shape	X			
B	10^5 components			X	
C	Silicon, *n*-type			X	
D	Resistors	X			
E	Perfect fabrication		X		
F	Ideal surface		X		
G	Packing factor 1/10			X	
H	Uniform doping		X		
I	10 per cent tolerances				X
J	No speed sacrifice			X	
K	1 month mean time to failure			X	
L	No special cooling				X

For example, an increase in the heat conductivity by a factor of 100, bringing it close to that of copper (395 W/C° m) would move the heat curve in Fig. 9 down by one order of magnitude. However, the necessary copper cooling ducts would probably bring the packing density back near the starting point. A change of geometry from a sphere to a plane would similarly allow better cooling but would represent little improvement in packing density as other planes could not be packed close to the first. Another alternative subject to the same objection is the division of the sphere into several smaller spheres.

The other assumption, namely 10 per cent tolerances, could probably be relaxed in some systems. However, even allowing 100 per cent tolerances, a quite liberal change, would only reduce the minimum size by a factor of 3.

VIII. Application to Practical Devices

A. Influence of Device Geometry

In order to be general and rigorous the analysis this far has been made in terms of semiconductor resistors of cubic dimensions. It is now time to extend the analysis to various semiconductor devices of arbitrary geometry.

In this section let us retain the assumption of resistors but relax the assumption of cubic geometry and allow any arbitrary shape. Let us consider any arbitrary shape as synthesized from a number of cubes. For example, a filamentary transistor may be thought of as a string of cubes, or a narrow-base junction transistor as a layer of cubes. Two extreme cases may be considered, namely a cosmic ray transit through the "short" dimension affecting only one cube, or a transit through the "long" dimension affecting a maximum number of cubes.

Let us consider the worst case, namely a cosmic ray transit through the maximum number of elements. Let us consider the extreme cases of Q elements in a cube, a sheet or a row as shown in Fig. 10. Then the maximum number of carriers in a sheet n_s, compared to that in a cube n_c, is

$$n_s = Q^{1/6} n_c. \tag{26}$$

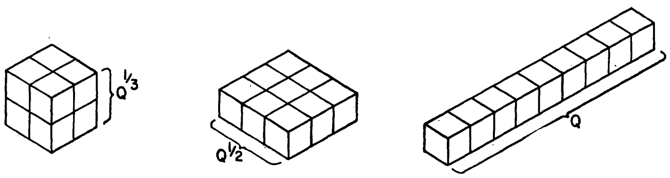

Fig. 10—Q cubes arranged in cube, sheet and row.

Similarly in a row,

$$n_r = Q^{2/3} n_c. \tag{27}$$

The probability of transit in the long direction compared to a transit through the cube is proportional to the solid angle. Therefore the probability of transit in the long direction of a sheet f_s, compared to the probability of transit in a cube f_c, is given by

$$f_s \approx Q^{-1/2} \cdot f_c, \tag{28}$$

and similarly for a row

$$f_r \approx Q^{-2} \cdot f_c. \tag{29}$$

Some practical values are given in Table VIII.

TABLE VIII

Relative Change in Excess Carriers n and Probability of Occurrence f for Sheet and Row Geometry Synthesized by Q Cubes

Q	Sheet		Row	
	n_s/n_c	f_s/f_c	n_r/n_c	f_r/f_c
10	1.5	0.3	4.6	0.01
100	2.2	0.1	22	0.0001
1000	3.2	0.03	—	—
10000	4.6	0.01	—	—

As geometries corresponding to $Q = 100$–10000 are common in transistors, and $Q = 10$–20 in unipolar transistors, the minimum element size shown in Fig. 7 may have to be raised by a factor of 1–5 depending on the geometry of the devices. For more extreme geometries such as thin film devices, the probability of transit may drop below the critical level. From Fig. 7 this may be seen to happen at a ratio $f/f_c \approx 0.001$.

The second most limiting condition, heat generation, depends on element geometry in the following way. If the resistor is made with reduced cross section in the form of a stick or a sheet rather than in the form of a cube the power density is reduced. Let us as a most favorable case consider a stick with cross section d_1^2, as before in a volume $10d^3$. Then the power density is

$$p = \frac{V^2 d_1^2}{10 \rho d^4} \cdot \tag{30}$$

Maximum power density is given by the number of elements through (19). Then a reduction in d_1 by a factor k reduces d by a factor $k^{1/2}$. For semiconductor resistors with cross section reduced to $(2\mu)^2$ we obtain the curve indicated in Fig. 9. This curve also satisfies the condition for stick geometry, namely $d \gg d_1$.

Sheet geometry is intermediate between cubic and stick geometry.

The combinations of element size and doping excluded by heat generation are shown by the cross-hatched area in Fig. 9.

B. Influence of Device Type

In this section the influence of device type will be considered. At the same time results of previous sections will be applied to typical devices. In considering various devices a start will be made with the *unipolar transistor* as a particularly simple case. In the unipolar transistor the volume of the source, drain and gate regions may be neglected compared to that of the channel region as they can if desired be made smaller than the channel region. The channel region is obviously a semiconductor resistor with the added restriction that the resistor dimensions may be altered by the gate depletion layer. Therefore the deductions for semiconductor resistors apply directly. The geometry of (the active region of) many unipolar transistors is a particularly unfavorable one corresponding to the row considered above. For medium frequency units made in the laboratory today, such as shown in Fig. 11, the factor Q (the number of cubes in the row) is approximately 20. This means an increase of the minimum device size of 7 times from $(10\mu)^3$ to $(70\mu)^3$.

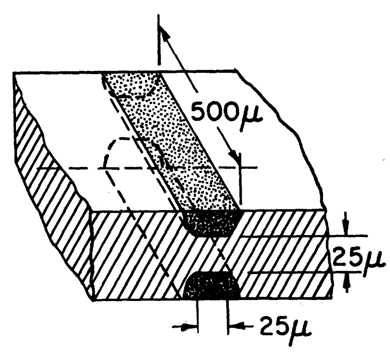

Fig. 11—Active region of medium speed unipolar transistor.

The volume of the active region in Fig. 11 is 3×10^5 cubic microns, or an element size of about $(70\mu)^3$ which is marginal. Higher frequency units may violate the minimum size requirement while the Tecnetron [16] which has more nearly cubic geometry, has a margin of approximately a factor 3.

The next simplest device to consider is the *junction diode*. The volume of the highly doped side of the junction may be neglected and only the side with the lowest doping considered, and only the part within one or a few diffusion lengths from the junction or, for a reverse biased diode, the depletion layer. This then constitutes the "active region" of the diode. The geometry of the active region of diodes is usually somewhere between cubic and thin layer dependent upon doping, area, signal amplitude, etc. The maximum allowed tolerance on diode characteristics, say on current for a given voltage, is not easy to agree on, as many different circuits with different requirements exist. A figure somewhere between 10 and 100 per cent seems reasonable. The choice is not critical as the difference between 10 and 100 per cent corresponds to only a factor of 3 in element size. As intermediate values of tolerance combined with intermediate geometry (or 100 per cent tolerance and a moderately thin layer) is equivalent to 10 per cent tolerance and cubic geometry we will choose the latter for simplicity. As 10 per cent tolerance in diode characteristics corresponds to 10 per cent tolerance in carrier density, the curves in Fig. 9 for $\rho > 20$ ohm cm apply.

Most diodes, particularly those for high speed, have a resistivity lower than 20 ohm cm in the active region. For these heat generation has to be considered. The heat dissipation in a diode may be lower than that in a semiconductor resistor because of the nonlinearity of the former, particularly at low voltages.

The power density is given by

$$p = \frac{V_i}{10d} \cdot \tag{31}$$

The maximum power density for a given number of elements is given by (20). Setting these equal gives

$$i = \frac{V}{\rho d} \cdot \tag{32}$$

For typical values

$$\rho = 10^{-3} \text{ ohm m}$$

$$d = 10^{-4} \text{ m}$$

$$i = 2.5\ 10^{6} \text{ A/m}^2.$$

However, a typical current density in a diode at 0.25 volt is only about 10^4 amperes/m^2 representing a reduction in power density of about $100\times$ and a corresponding reduction in minimum size of about $10\times$, or from $(100\ \mu)^3$ to $(10\ \mu)^3$. Also the diode area may be further reduced to the limit set by edge uncertainty, $(2\mu)^3$. This represents another reduction in element size by $10^{2/3}$ or about 5. The diode element size may therefore come close to the limit set by edge uncertainty, two or a few microns.

The situation for the *tunnel diode* is somewhat analogous to that of the diode. Tunnel diodes require a doping corresponding to approximately 10^{-4} ohm cm. Then a tunnel diode with

$$\rho = 10^{-6} \text{ ohm m}$$

$$d = 5\ 10^{-3} \text{ m}$$

would have a current density according to (32) of

$$i = 5\ 10^{7} \text{ amperes/m}^2,$$

which is just typical of tunnel diodes of germanium with maximum peak-to-valley current ratio. Therefore no reduction in power density comes from nonlinearity compared to a semiconductor resistor. However, the area of the tunnel diode may be reduced by nearly four orders of magnitude to the limit set by edge uncertainty $(2\ \mu)^3$. Then from (30) the minimum size of the element may be reduced by nearly two orders of magnitude, or to just below $(100\ \mu)^3$. Tunnel diodes then, although fabricated as small as $(2\ \mu)^3$, still require an element size of about $(100\ \mu)^3$ for proper cooling.

In conventional (bipolar) *transistors* the active region is the base region and the volumes of the emitter and collector may be neglected. For transistors even more than for diodes a maximum allowed tolerance in carrier density in the base region is not easy to agree upon because of the large variety of circuits with different requirements. However, a figure somewhere between 10 and 100 per cent seems reasonable. The geometry of most transistors is much like the sheet considered above. Values of the factor Q (the number of cubes in the sheet) are 10^2–10^4 for most transistors with the higher number for modern mesa transistors like the one shown in Fig. 12. The net result is that the influence of a large tolerance like 100 per cent and the geometry largely cancel and the values in Fig. 9 apply for $\rho > 20$ ohm cm. For $\rho < 20$ ohm cm the same considerations as for diodes apply and the smallest possible element size will be somewhere between $(2\ \mu)^3$ and $(10\ \mu)^3$.

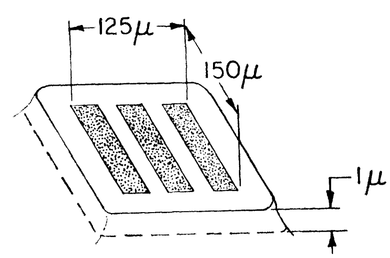

Fig. 12—Active region of mesa transistor.

The volume of the active region in Fig. 12 is 2 10^4 cubic microns corresponding to an element size of about $(30\ \mu)^3$ or a margin of a factor of 3 over the minimum device size. In the same manner, the results of the analysis may be applied to any arbitrary semiconductor device.

IX. Conclusions

The conclusion to these results is that already at the present time the essential part of semiconductor devices, the active region, is close to the minimum size possible. Further miniaturization, therefore, must come entirely from reduction of nonessential parts, particularly the package. To a smaller extent some gain could be obtained from reduction of parts of the device outside the active region, emitter, collector, contacts, etc.

At about 5 ohm cm silicon in the active region an optimum exists, corresponding to elements just below $(10\ \mu)^3$ and allowing a packing density of approximately 10^8 components/cm^3 (10^9 components/inch^3) of most semiconductor devices such as transistors, unipolar transistors, diodes, semiconductor resistors, etc. When the resistivity is higher than 5 ohm cm, the volume of the active region has to be larger, or else the devices fail from cosmic-ray bombardment. When the resistivity is lower than 5 ohm cm, the device can be made smaller but the total volume occupied by the device must be larger than the minimum or else the power density becomes excessive.

There are, however, ways in which these limitations may be circumvented. One of the most attractive alternatives has already been suggested by systems considerations, namely redundancy. Redundant circuits, in which one or more elements are allowed to fail without violating the proper functioning of the circuit, remove one of the assumptions on which the analysis of maximum packing density was based, and would allow the limit to be exceeded. The most important gain of redundancy is of course that it also remedies a situation where devices fail for other reasons than those considered here, *e.g.*, surface changes, mechanical imperfections, thermal fatigue, etc. There is a price to be paid for redundancy, namely an increase in number of components, which partly offsets the advantage gained.

Another possibility is the use of negative feedback in a general sense. By the use of negative feedback in circuits the device tolerances may be wider. Also negative feedback could be applied in the fabrication of devices, as is now being done with tunnel diodes and some narrow-base transistors, where units are in effect measured during the fabrication and treated until they fall within tolerances. The price paid for closer tolerances is reduced performance, *e.g.*, amplification or speed of operation or speed of fabrication. Another way in which negative feedback in a general sense could be applied is through the use of self-organizing systems which could in principle self-heal faulty circuits. Again the price to be paid is an increase in the number of devices and decreased speed.

It is interesting that the minimum size of semiconductor components $10\ \mu$ is comparable to the size of the building block of biological "computers," namely the neuron. If we were to build a computer with 10^{10} components, each $10\ \mu$ on a side, with an over-all heat conductivity of 0.6 W/m C° and a maximum temperature difference of 2C°, (19) and (22) would give a total power of 1 W and total volume of 0.1 dm^3. Increasing the heat conductivity 10 times by liquid cooling would give 10 W but at the same time larger volume, say 1 dm^3, values that are typical of the human brain. When speed is considered, however, the semiconductor devices are faster by a factor of 10^8–10^9. Undoubtedly, though, some of this speed advantage would have to be given up in order to reach the reliability of the brain.

Appendix I

Shrinkage from Impurity Fluctuations

Assume that the distribution of impurities in the semiconductor is equivalent to the distribution of particles in an ideal gas, *i.e.*, a Poisson distribution. Then the standard deviation σ of the number of impurities in a volume from the mean number of impurities N is

$$\sigma = N^{1/2}.$$

When N is large the distribution can be approximated with sufficient accuracy by a normal distribution. Then the probability P_1 that the number of impurities in a cube is between x and N, where x may be larger or smaller than N, may be expressed as

$$P_1 = \frac{2}{\sqrt{2\pi}} \int_0^{(x-N)/\sigma} \exp\left(-\frac{y^2}{2}\right) dy,$$

where y is the normalized deviation

$$y = \frac{x-N}{\sigma}\cdot$$

With the condition that $x-N$ should not exceed ϵN, where ϵ is the tolerance,

$$P_1 = \frac{2}{\sqrt{2\pi}} \int_0^{\epsilon N^{1/2}} \exp\left(-\frac{y^2}{2}\right) dy.$$

The fraction of cubes S_1 in which the number of impurities exceeds the tolerance ϵ is

$$S_1 = 1 - \frac{2}{\sqrt{2\pi}} \int_0^{\epsilon N^{1/2}} \exp\left(-\frac{y^2}{2}\right) dy.$$

Appendix II

Shrinkage from the Cutting Operation

Assume that in forming the cubes the location of the cube edges follows a normal distribution with a standard deviation σ_1. From the discussion in Section IV, $\sigma_1 = 100$ Å. Each side d of the cube is defined by two cuts, one at each end, each with a standard deviation σ_1. Then d also follows a normal distribution but with a standard deviation σ_d which is [19]

$$\begin{aligned}\sigma_d &= (\sigma_1^2 + \sigma_1^2)^{1/2} \\ &= \sqrt{2}\sigma_1\end{aligned}$$

Let us first assume that two sides of the cube are exact and only one side varies. Then the resistance R also follows a normal distribution with standard deviation σ_R, where

$$\frac{\sigma_R}{R} = \frac{\sigma_d}{d} \cdot$$

The probability P_2' that the resistance through the cube between x and R, where R is the mean value of the resistances of all cubes, may be expressed as

$$P_2' = \frac{2}{\sqrt{2\pi}} \int_0^y \exp\left(-\frac{y^2}{2}\right) dy,$$

where

$$y = \frac{x - R}{\sigma_R} \cdot$$

With the condition that $x - R$ should not exceed ϵR, where ϵ is the tolerance,

$$y = \frac{\epsilon R}{\sigma_R} = \frac{\epsilon d}{\sigma_d} \cdot$$

When all three sides vary randomly, and independently so that there is no correlation in their variation, the probability P_2 that the resistance be within ϵR is

$$P_2 = (P_2')^3.$$

The fraction of cubes S_2, for which the resistance deviates more than the tolerance ϵ, is

$$S_2 = 1 - \left[\frac{2}{\sqrt{2\pi}} \int_0^y \exp\left(-\frac{y^2}{2}\right) dy\right]^3$$

with

$$y = \frac{\epsilon d}{\sqrt{2}\sigma_1} \cdot$$

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