

Last Homework - Discrete Mathematics

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10.1

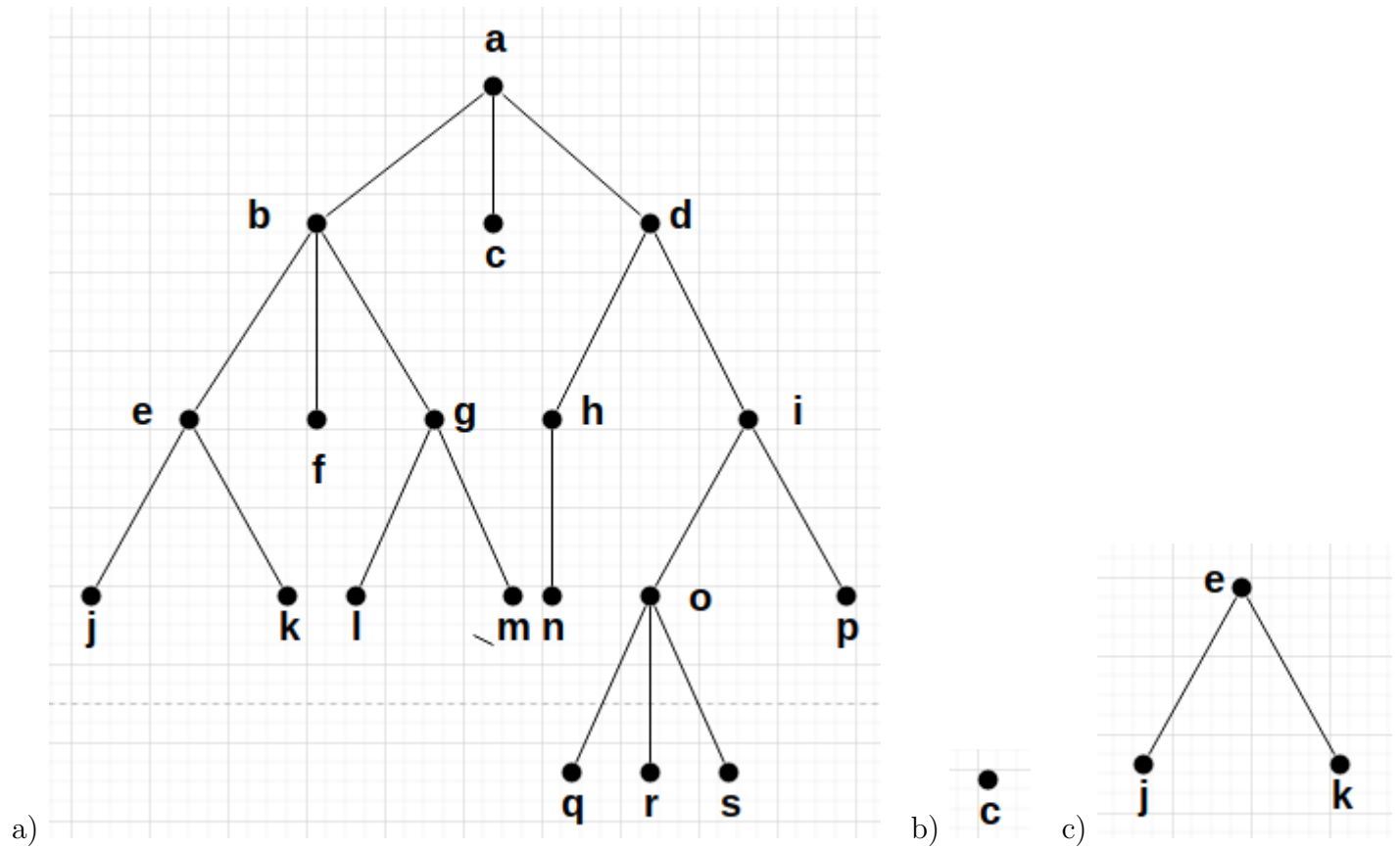
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- a) Vertice a is the root
- b) Internal vertices: a, b, d, e, g, h, i, o
- c) Leaves vertices: c, f, j, k, l, m, n, q, r, s, p
- d) j has no children vertice.
- e) Sibling of o: p
- f) Ancestor of m: g, b, a
- g) Descendants of b: e, f, g, j, k, l, m

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Some vertices have 3 children, while some only have 1 or 2 children. Therefore, that is not a full m-ary tree.

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The model is a full 5-ary tree with 10,000 internal vertices.
Total number of vertices: $n = m \cdot i + 1 = 5 \cdot 10,000 + 1 = 50,001$ vertices
All vertices except the root received the letter. Thus, there are 50,000 people received the letter.
Leaves vertices represent people who received but did not send out the letter. Thus, the number of people in this group is: $50,001 - 10,000 = 40,001$ people.

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A full m -ary tree of height h has:

m^0 vertices at level 0.

m^1 vertices at level 1.

m^2 vertices at level 2.

...

m^h vertices at level h .

Therefore, total number of vertices is a geometric series:

$$n = m^0 + m^1 + m^2 + \dots + m^h = \frac{m^{h+1} - 1}{m - 1}$$

Number of leaves is the number of vertices at level h : m^h

The root of the tree represents the entire book.

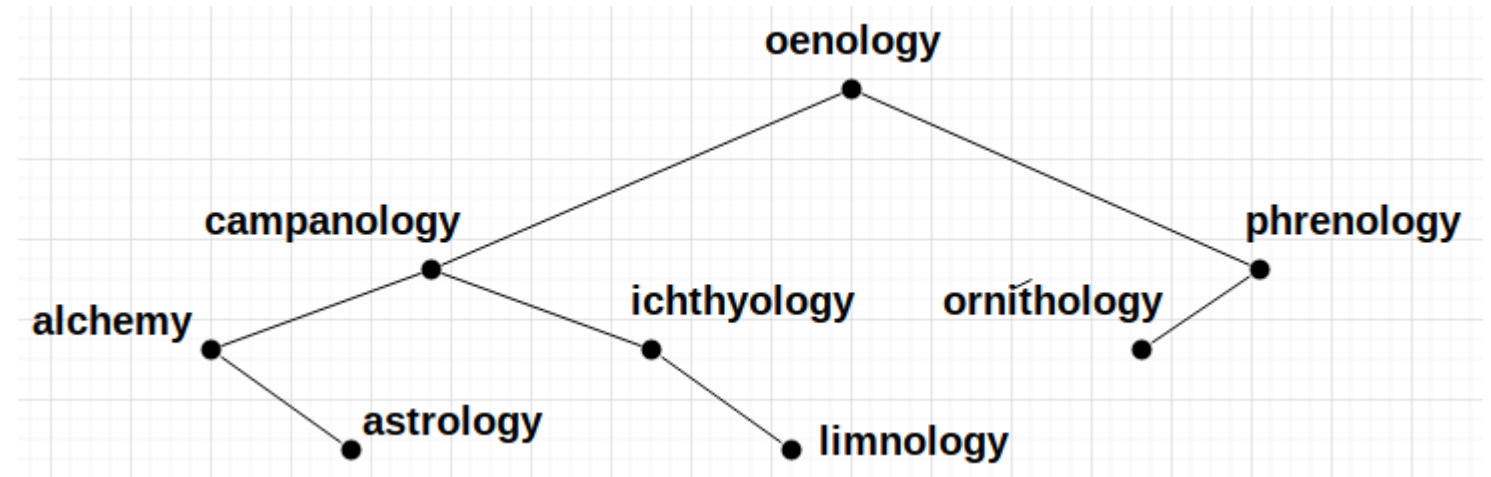
The vertices at level 1 represents chapters of the book.

The vertices at level 2 represents sections of the book.

The vertices at level 3 represents subsections of the book.

10.2

2



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1 counterfeit coin in 4 coins. Thus, there are 4 outcomes corresponding to 4 leaves.

Each comparison has 3 outcomes, corresponding to a ternary tree.

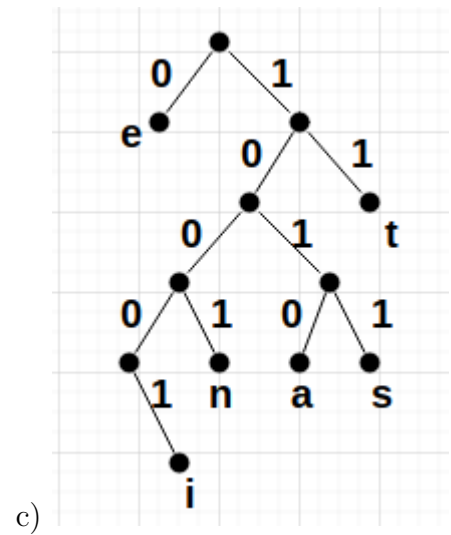
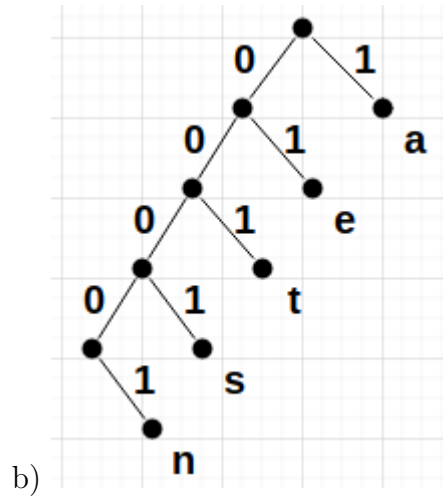
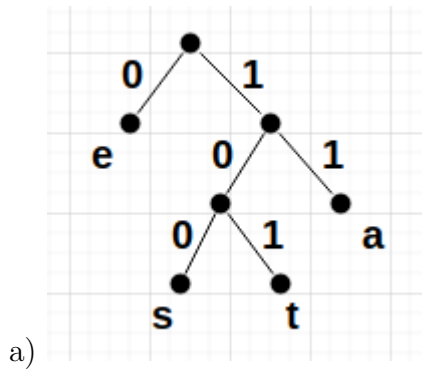
Therefore, $h \geq \lceil \log_3 4 \rceil = 2$

\Rightarrow At least 2 weighings are needed.

Algorithm using 2 comparisons:

- Divide 4 coins into 2 groups and compare the total weight of 2 groups. The light group contains the counterfeit coin.
- Compare 2 coins in the group that contains the counterfeit coin. The lighter coin is the counterfeit one.

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a) test b) beer c) sex d) tax

10.3

4

a) Level 5

b) Address of parent of v: 3.4.5.2

c) v has at least 3 siblings

d) T has at least 1 root node and $3 + 4 + 5 + 2 + 4$ internal nodes, totally 19 vertices.

e) Other addresses that must occur: o, 1, 2, 3, 3.1, 3.2, 3.3, 3.4, 3.4.1, 3.4.2, 3.4.3, 3.4.4, 3.4.5, 3.4.5.1, 3.4.5.2, 3.4.5.2.1, 3.4.5.2.2, 3.4.5.2.3

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Preorder traversal: a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p

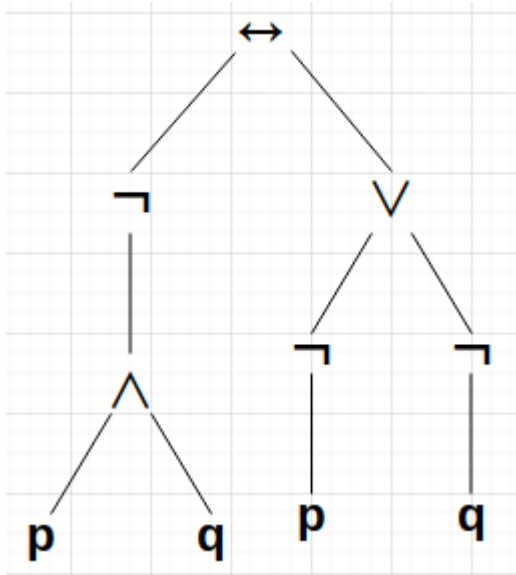
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Inorder traversal: k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

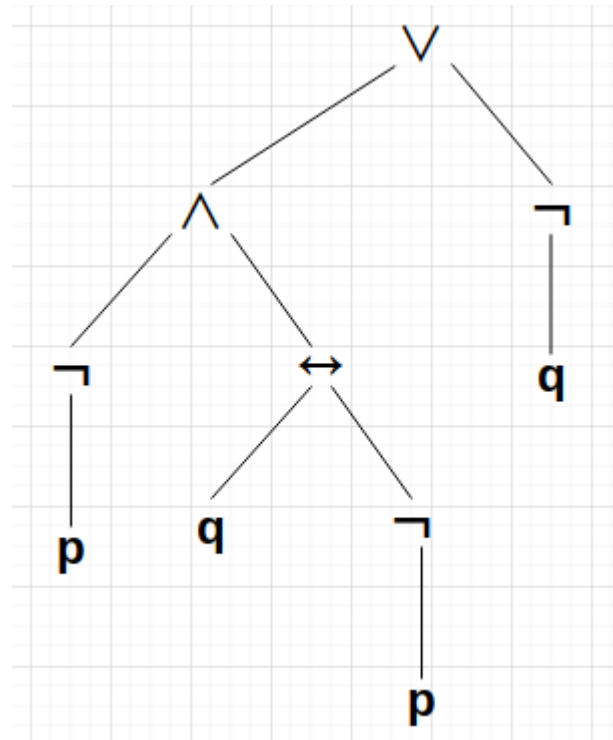
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Postorder traversal: d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a

a)



$$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$$



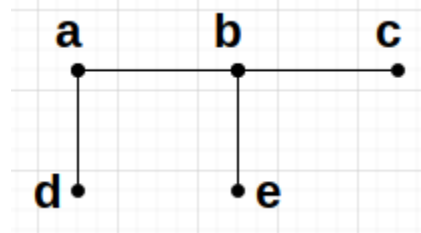
$$(\neg p \wedge (q \leftrightarrow \neg p)) \vee \neg q$$

b) Prefix: $\leftrightarrow \neg \wedge p q \vee \neg p \neg q$ and $\vee \wedge \neg p \leftrightarrow q \neg p \neg q$ c) Postfix: $p q \wedge \neg p \neg q \vee \leftrightarrow$ and $p \neg q p \neg \leftrightarrow \wedge q \neg \vee$ d) Infix: $((\neg(p \wedge q)) \leftrightarrow ((\neg p) \vee (\neg q)))$ and $((((\neg p) \wedge (q \leftrightarrow (\neg p)))) \vee (\neg q))$

10.4

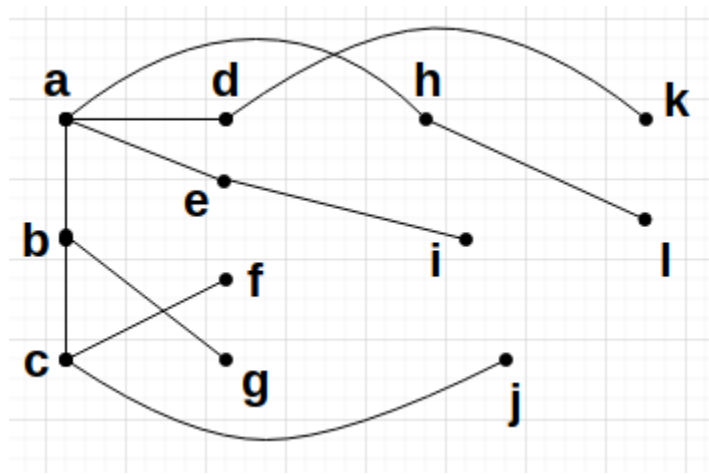
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Remove edges b-d and c-e:



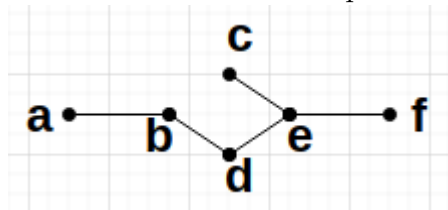
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Remove edges d-e, e-f, f-g, i-j, i-h, i-k, g-h, g-l, k-l, j-l:

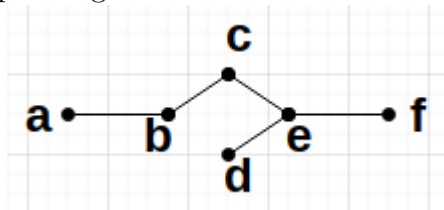


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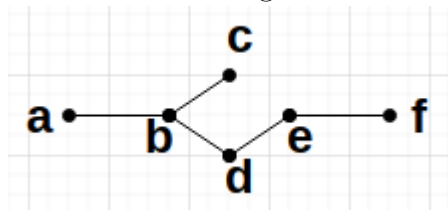
Four possible spanning trees:



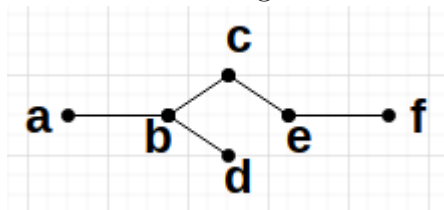
Remove edge: b-c



Remove edge: b-d



Remove edge: c-e



Remove edge: d-e