HOMEWORK SESSION 3-4

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1.2 Propositional Equivalences

4. Use truth tables to verify the associative laws

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

8. Use De Morgan's laws to find the negation for statements

a) Kwame will not take a job in industry and Kwame will not go to graduate school.

b) Yoshiko does not know java or Yoshiko does not know calculus.

c) James is not young or James is not strong.

d) Rita will not move to Oregon and Rita will not move to Washington.

14. Determine whether $(\neg p \land (p \to q)) \to \neg q$ is a tautology

p	q	$p \to q$	$\neg p$	$\neg p \land (p \to q)$	$\neg q$	$(\neg p \land (p \to q)) \to \neg q$
0	0	1	1	1	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	1
1	1	1	0	0	0	1

There is a false value when p is false and q is true. Therefore, it is not a tautology.

16. Show $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

p	q	$p \longleftrightarrow q$	$p \land q$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$(p \land q) \lor (\neg p \land \neg q)$
0	0	1	0	1	1	1	1
0	1	0	0	1	0	0	0
1	0	0	0	0	1	0	0
1	1	1	1	0	0	0	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

18. Show $p \to q \equiv \neg q \to \neg p$

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	0	0	1	0
1	1	1	0	0	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

24. Show that $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \lor (p \to r)$	$q \lor r$	$p \to (q \lor r)$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

26. Show that $\neg p \to (q \to r) \equiv q \to (p \vee r)$

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$	$p \vee r$	$q \to (p \vee r)$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	0	1	1	1
1	1	1	0	1	1	1	1

The 2 expressions have the same truth values. Therefore, they are equipvalent.

32. Show that $(p \wedge q) \to r$ is not equipvalent to $(p \to r) \wedge (q \to r)$

p	q	r	$p \wedge q$	$(p \wedge q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	0	0
0	1	1	0	1	1	1	1
1	0	0	0	1	0	1	0
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

The 2 expressions have different truth values. Therefore, they are not equipvalent.

1.3 Predicates and quantifiers

6.

a) $\exists x N(x)$: Some students in my school have visited North Dakota.

b) $\forall x N(x)$: All students in my school have visited North Dakota.

c) $\neg \exists x N(x)$: No student in my school has visited North Dakota.

d) $\exists x \neg N(x)$:There exists a student in my school who has not visited North Dakota.

e) $\neg \forall x N(x)$:Not all students in my school have visited North Dakota.

f) $\forall x \neg N(x)$:All students in my school have not visited North Dakota.

8.

a) $\forall x (R(x) \to H(x))$:If an animal is a rabbit, then it hops.

b) $\forall x (R(x) \wedge H(x))$: All animals are rabbits and hop.

c) $\exists x (R(x) \to H(x))$: There exists an animal that hops if it is a rabbit.

d) $\exists x(R(x) \land H(x))$: There exists an animal that is a rabbit and hops.

16.

a) $\exists x(x^2 = 2)$: True ($x = \pm\sqrt{2}$)

b) $\exists x(x^2 = -1)$: False ($i = \sqrt{-1}$ is the imaginary number)

c) $\forall x(x^2 + 2 \geq 1)$: True (For all real number, the left side is always at least 2)

d) $\forall x(x^2 \neq x)$: False ($x = 1$)

22.

a) Everyone speaks Hindi.

- True: all people in Hindi-speaking regions of India.
- False: all Vietnamese people.

b) There is someone older than 21 years.

- True: all Vietnamese people.
- False: all secondary students.

c) Every two people have the same first name.

- True: { Nguyen Tien Duc, Nguyen Van Duc, Tran Minh Duc, Le Hoang Duc }.
- False: all Vietnamese people.

d) Someone knows more than two other people.

- True: all Vietnamese people.
- False: newborn babies.

24.

$M(x)$: x is a student in my class
$C(x)$: x has a cellular phone
$S(x)$: x can swim
$Q(x)$: x can solve quadratic equation
$R(x)$: x want to be rich

Domain	Students in my class	All people
a)	$\forall x C(x)$	$\forall x(M(x) \rightarrow C(x))$
b)	$\exists x F(x)$	$\exists x(M(x) \wedge F(x))$
c)	$\exists x \neg S(x)$	$\exists x(M(x) \wedge \neg S(x))$
d)	$\forall x Q(x)$	$\exists x(M(x) \rightarrow Q(x))$
e)	$\exists x \neg R(x)$	$\exists x(M(x) \wedge \neg R(x))$

26.

a) Someone in your school has visited Uzbekistan.

- $S(x)$: x is in my school.
- $U(x)$: x has visited Uzbekistan.
- $V(x, y)$: x has visited country y.

1. Domain is all people in my school: $\exists x U(x)$
2. Domain is all people: $\exists x(S(x) \wedge U(x))$
3. Domain is all people: $\exists x(S(x) \wedge V(x, Uzbekistan))$

b) Everyone in your class has studied calculus and C++

- $C(x)$: x is in my class.
- $M(x)$: x has studied calculus.
- $P(x)$: x has studied C++.
- $S(x, y)$: x has studied y.

1. Domain is all peole in my class: $\forall x(M(x) \wedge P(x)$
2. Domain is all people: $\forall x(C(x) \rightarrow (M(x) \wedge P(x)))$
3. Domain is all people: $\forall x(C(x) \rightarrow (S(x, Calculus) \wedge S(x, \text{C++})))$

c) No one in your school owns both a bicycle and a motorcycle.

- $S(x)$: x are people in my school.
- $B(x)$: x owns a bicycle.
- $M(x)$: x owns a motorcycle.
- $S(x, y)$: x owns y.

1. Domain is all people in my school: $\forall x \neg(B(x) \wedge M(x))$
2. Domain is all people: $\forall x(S(x) \rightarrow \neg(B(x) \wedge M(x)))$
3. Domain is all people: $\forall x(S(x) \rightarrow \neg(S(x, bicycle) \wedge S(x, motorcycle)))$

d) There is a person in your school who is not happy.

- $S(x)$: x is a person in my school.

- $H(x)$: x is happy.
- $F(x, y)$: x feels y.

1. Domain is all people in my school: $\exists x \neg H(x)$
2. Domain is all people: $\exists x(S(x) \wedge \neg H(x))$
3. Domain is all people: $\exists x(S(x) \wedge \neg F(x, happy))$

e) Everyone in your school was born in the twentieth century.

- $S(x)$: x is all people in my school.
- $T(x)$: x was born in the twentieth century
- $B(x, y)$: x was born in century y^{th}.

1. Domain is all people in my school: $\forall x T(x)$
2. Domain is all people: $\forall x(S(x) \rightarrow T(x))$
3. Domain is all people: $\forall x(S(x) \rightarrow B(x, 20))$

28.

a) Some thing is not in the correct place.

- $C(x)$: x is in the correct place.
- Domain is everything.

$\Longrightarrow \exists x \neg C(x)$

b) All tools are in the correct place and are in excellent condition.

- $T(x)$: x is a tool.
- $C(x)$: x is in the correct place.
- $E(x)$: x is in excellent condition.
- Domain is everything

$\Longrightarrow \forall x(T(x) \rightarrow (C(x) \wedge E(x)))$

c) Everything is in the correct place and in excellent conditions.

- $C(x)$: x is in the correct place.
- $E(x)$: x is in excellent condition.
- Domain is everything

$\Longrightarrow \forall x(C(x) \wedge E(x))$

d) Nothing is in the correct place and is in excellent condittion.

- $C(x)$: x is in the correct place.
- $E(x)$: x is in excellent condition.
- Domain is everything

$\Longrightarrow \forall x \neg(C(x) \wedge E(x))$

e) One of your tools is not in the correct place, but it is in excellent condition.

- $T(x)$: x is a tool.
- $C(x)$: x is in the correct place.
- $E(x)$: x is in excellent condition.
- Domain is everything

$\implies \exists x(T(x) \wedge \neg C(x) \wedge E(x))$

1.4 Nested Quantifiers

10.

a) $\forall x F(x, Fred)$

b) $\forall x F(Evelyn, x)$

c) $\forall x \exists y F(x, y)$

d) $\neg \exists x \forall y F(x, y)$

e) $\exists x \forall y F(x, y)$

f) $\neg \exists x(F(x, Fred) \wedge F(x, Jerry))$

g) $\exists x_1 \exists x_2(\forall x(F(Nancy, x) \rightarrow (x = x_1 \vee x = x_2)) \wedge F(Nancy, x_1) \wedge F(Nancy, x_2) \wedge (x_1 \neq x_2))$

h) $\exists y(\forall x F(x, y) \wedge \forall z(\forall x F(x, z) \rightarrow z = y))$

i) $\neg \exists x F(x, x)$

j) $\exists x \exists y(x \neq y \wedge F(x, y) \wedge \forall z((F(x, z) \wedge z \neq x) \rightarrow z = y))$

12.

a) $\neg I(Jerry)$

b) $\neg C(Rachel, Jerry)$

c) $\neg C(Rachel, Jerry)$

d) $\neg \exists x C(x, Bob)$

e) $\forall x((x \neq Joseph) \leftrightarrow C(x, Sanjay))$

f) $\exists x \neg I(x)$

g) $\neg \forall x I(x)$

h) $\exists x(I(x) \wedge \forall y(I(y) \rightarrow y = x))$

i) $\forall x \exists y(x \neq y \leftrightarrow I(x))$

j) $\forall x(I(x) \rightarrow \exists y(C(x, y) \wedge y \neq x))$

k) $\exists x(I(x) \wedge \forall y(y \neq x \rightarrow \neg C(x, y)))$

l) $\exists x_1 \exists x_2(x_1 \neq x_2 \wedge \neg C(x_1 x_2))$

m) $\exists x \forall y C(x, y)$

n) $\exists x_1 \exists x_2(\forall y \neg(C(x_1, y) \wedge C(x_2, y)) \wedge x_1 \neq x_2)$

o) $\exists x_1 \exists x_2(\forall y(C(x_1, y) \vee C(x_2, y)) \wedge x_1 \neq x_2)$

16.

- $T(s, m, y)$: student s majors in m and is in year y.
- Abbreviations:
  - MA: mathematics
  - CS: computer science
- Domain:
  - s : all students in the class.
  - m : all possible majors (in this case only include MA and CS).
  - y : one in 4 years: freshman, sophomore, junior, senior.

a) $\exists s \exists m T(s, m, junior)$: True

b) $\forall s \forall y T(s, CS, y)$: False because there are also mathematics students.

c) $\exists s \exists m \exists y((m \neq MA) \wedge (y \neq junior) \wedge T(s, m, y))$: True because there are many CS sophomores.

d) $\forall s(\exists m T(s, m, sophomore) \vee \exists y T(s, CS, y))$: False because there is a MA freshman.

e) $\exists m \forall y \exists s T(s, m, y)$: False because there is no MA senior and there is no CS freshman.

22.

- Domain is all numbers.
- $I(x)$: x is an integer.

$\Longrightarrow \exists x(I(x) \wedge (x > 0) \wedge \forall a \forall b \forall c(x \neq (a^2 + b^2 + c^2))$

1.5 Rules of Inference

10.

a) h : I play hockey; w : I use the whirlpool; s : I am sore

1. $h \rightarrow s$
2. $s \rightarrow w$

3. $\neg w$

- Modus tollens using (2) and (3) => I am not sore (4)
- Modus tollens using (4) and (1) => I did not play hockey.

b) $W(d)$: I work on day d; $S(d)$: It is sunny or partly sunny on day d

1. $\forall d(W(d) \rightarrow S(d))$
2. $W(Monday) \vee W(Friday)$
3. $\neg S(Tuesday)$
4. $\neg S(Friday)$

- Modus tollens using (1) and (3) => I did not work on Tuesday.
- Modus tollens using (1) and (4) => I did not work on Friday. (5)
- Disjunctive syllogism using (2) and (5) => I worked on Monday.

c) $S(x)$: x has six legs; $I(x)$: x is an insect

1. $\forall x(I(x) \rightarrow S(x))$
2. $I(dragonflies)$
3. $\neg S(spiders)$

- Modus ponens using (1) and (2) => Dragon flies have 6 legs.
- Modus tollens using (1) and (3) => Spiders are not insects.
- We could say using existential generalization that, for example, there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.

d) $A(x)$: x has an Internet account; $S(x)$: x is a student

1. $\forall x(S(x) \rightarrow A(x))$
2. $\neg A(Homer)$
3. $A(Maggie)$

- Universal instantiation using (1) => If Homer/Maggie is a student, then she has an Internet account.
- Modus tollens using (1) and (2) => Homer is not a student.

e) $H(x)$: x is healthy; $T(x)$: x tastes good; $E(x)$: You eat x

1. $\forall x(H(x) \rightarrow \neg T(x))$
2. $H(Tofu)$
3. $\forall x(T(x) \leftrightarrow E(x))$
4. $\neg H(cheeseburgers)$

- Universal instantiation and modus ponens using (1) and (2) => Tofu does not taste good. (5)
- Universal instantiation and modus tollens using (3) and (5) => You will not eat tofu.
- Universal instantiation using (1) and (4) => Cheeseburgers taste good. (6)

- Universal instantiation and modus ponens using (3) and (6) => You will eat cheeseburgers.

f) D : I am dreaming; H : I am hallucinating; E : I see elephants running down the road.

1. $D \lor H$
2. $\neg D$
3. $H \to E$

- Disjunctive syllogism using (1) and (2) => I am hallucinating. (4)
- Modus ponens using (4) and (3) => I am seeing elephants running down the road.

14.

a) $R(x)$: x owns a red convertible; $T(x)$: x has gotten a speeding ticket; $C(x)$: x is in this class

$C(Linda)$
$R(Linda)$
$\forall x(R(x) \to T(x))$
$\therefore \exists x(C(x) \land T(x))$

$\Longrightarrow$

Step	Reason
1. $\forall x(R(x) \to T(x))$	Hypothesis
2. $R(Linda) \to T(Linda)$	Universal instantiation using (1)
3. $R(Linda)$	Hypothesis
4. $T(Linda)$	Modus ponens using (2) and (3)
5. $C(Linda)$	Hypothesis
6. $C(Linda) \land T(Linda)$	Conjunction using (4) and (5)
7. $\exists x(C(x) \land T(x))$	Existential generalization using (6)

b) $R(x)$: x is one of the five roommates listed; $D(x)$: x has taken a course in discrete math; $A(x)$: x can take a course in algorithms

$\forall x(R(x) \to D(x))$
$\forall x(D(x) \to A(x))$
$\therefore \forall x(R(x) \to A(x))$

$\Longrightarrow$

Step	Reason
1. $\forall x(R(x) \to D(x))$	Hypothesis
2. $R(y) \to D(y)$	Universal instantiation using (1)
3. $\forall x(D(x) \to A(x))$	Hypothesis
4. $D(y) \to A(y)$	Universal instantiation using (3)
5. $R(y) \to A(y)$	Hypothetical syllogism using (2) and (4)
6. $\forall x(R(x) \to A(x))$	Universal generalization using (5)

c) $S(x)$: x is a movie produced by Sayles; $C(x)$: x is a movie about coal miners; $W(x)$: x is a wonderful movie

$\forall x(S(x) \to W(x))$
$\exists x(S(x) \land C(x))$
$\therefore \exists x(C(x) \land W(x))$

$\Longrightarrow$

Step	Reason
1. $\exists x(S(x) \land C(x))$	Hypothesis
2. $S(y) \land C(y)$	Existential instantiation using (1)
3. $S(y)$	Simplification using (2)
4. $\forall x(S(x) \to W(x))$	Hypothesis
5. $S(y) \to W(y)$	Universal instantiation using (4)
6. $W(y)$	Modus ponens using (3) and (5)
7. $C(y)$	Simplification using (2)
8. $W(y) \land C(y)$	Conjunction using (6) and (7)
9. $\exists x(C(x) \land W(x))$	Existential generalization using (8)

d) $C(x)$: x is in this class; $F(x)$: x has been to France; $L(x)$: x has visited the Louvre

$\exists x(C(x) \wedge F(x))$
$\forall x(F(x) \rightarrow L(x))$
$\therefore \exists x(C(x) \wedge L(x))$

$\Longrightarrow$

Step	**Reason**
1. $\exists x(C(x) \wedge F(x))$	Hypothesis
2. $C(y) \wedge F(y)$	Existential instantiation using (1)
3. $F(y)$	Simplification using (2)
4. $C(y)$	Simplification using (2)
5. $\forall x(F(x) \rightarrow L(x))$	Hypothesis
6. $F(y) \rightarrow L(y)$	Universal instantiation using (5)
7. $L(y)$	Modus ponens using (3) and (6)
8. $C(y) \wedge L(y)$	Conjunction using (4) and (7)
9. $\exists x(C(x) \wedge L(x))$	Existential generalization using (8)

18.

There exists a person s so that s is shorter than Max is valid. However, it does not follow that s can also be Max because a person cannot compare his height with himself. The argument is invalid from this step.