Yield Management at American Airlines

BARRY C. SMITH

American Airlines Decision Technologies

PO Box 619616 MD 4462 CP4/HDQ

Dallas/Fort Worth Airport, Texas 75261-9616

JOHN F. LEIMKUHLER

American Airlines Decision Technologies

ROSS M. DARROW American Airlines Decision Technologies

Critical to an airline's operation is the effective use of its reservations inventory. American Airlines began research in the early 1960s in managing revenue from this inventory. Because of the problem's size and difficulty, American Airlines Decision Technologies has developed a series of OR models that effectively reduce the large problem to three much smaller and far more manageable subproblems: overbooking, discount allocation, and traffic management. The results of the subproblem solutions are combined to determine the final inventory levels. American Airlines estimates the quantifiable benefit at \$1.4 billion over the last three years and expects an annual revenue contribution of over \$500 million to continue into the future.

n its 1987 annual report, American Airlines broadly described the function of yield management as "selling the right seats to the right customers at the right prices." While this statement oversimplified yield management, it does capture the basic motivation behind the strategy. A better description of yield management as it applies to airlines is the control and

management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and fare structure.

The role of yield management at American is analogous to the inventory control function for a manufacturing company. Planning departments determine the airline's flight schedule and fares. The combi-

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nation of schedule and fares defines the products to be offered to the public. Yield management then determines how much of each product to put on the shelf (make available for sale). American's "store front" is the computerized reservations system, SABRE (semi-automated business research environment). All sale and cancellation transactions, whether from American Airlines reservations agents or travel agents, pass through SABRE, updating reservations inventory for all affected flights. New reservations are accepted only if yield management controls permit.

To increase the responsiveness and effectiveness of yield-management strategies and to coordinate reservations inventory decisions with SABRE, American has continually supported the development of automated decision tools. Because the yieldmanagement decision-making process is so large and complex at American, effective control of the inventory of seats can be accomplished only with automated models. The yield-management problem is best described as a nonlinear, stochastic, mixedinteger mathematical program that requires data, such as passenger demand, cancellations, and other estimates of passenger behavior, that are subject to frequent changes. To solve the system-wide yieldmanagement problem would require approximately 250 million decision variables. Because this mathematical programming formulation is intractable, American Airlines Decision Technologies (AADT) has developed a series of operations research models. These models effectively reduce the large problem to three much smaller and far more manageable subproblems, while still realistically modeling the realworld situation. The results of the subproblem solutions are then combined to determine the final inventory levels.

Although there is some interaction, the American Airlines yield management system is typically divided into three major functions:

(1) Overbooking is the practice of intentionally selling more reservations for a flight than there are actual seats on the aircraft. Airlines use overbooking to offset the effects of passenger cancellations and no-

Without overbooking controls, American estimates that 15 percent of seats would be spoiled on sold-out flights.

shows. Without overbooking, about 15 percent of seats would be unused on flights sold out at departure. Even more seats would be unused for flights sold out prior to departure.

- (2) Discount allocation is the process of determining the number of discount fares to offer on a flight. Airlines offer discount fares to stimulate demand and fill seats that would otherwise be empty. The availability of discount fares must be limited on popular flights to preserve space for latebooking, higher-revenue passengers. Therefore, airlines treat reservation space as a scarce resource that must be intelligently allocated to various discount-fares.
- (3) Traffic management is the process of controlling reservations by passenger origin and destination to provide the mix of markets (multiple-flight connecting markets versus single-flight markets) that maximizes revenue. A passenger flying into

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Dallas/Fort Worth can connect and continue on to any of several final destinations on a second flight. Therefore, to maximize revenue across the entire system of flights, reservation inventory controls for one flight must consider the passenger demand on connecting flights.

The nature of today's marketplace in the airline industry makes yield management absolutely essential to profitable operations. At large airlines, such as American, the number of inventory controls and the frequency of updates requires an automated decision-making system. American's current yield management system, DIN-AMO (dynamic inventory and maintenance optimizer), was fully implemented in 1988. DINAMO is the latest step of a

Using DINAMO, spoilage was only three percent.

development process that spans the last 25 years, responding to changes in the airline industry and taking advantage of innovations in computer technology. Three major changes motivated and shaped yield management development:

- The implementation of a computer reservations system (SABRE) in 1966,
 which had the capability of controlling reservations inventory;
- The introduction of super-saver discount fares in 1977; and
- The deregulation of airline schedules and prices in 1979.

Prior to the implementation of SABRE, American had no centralized methods for reviewing or controlling reservation activity. Reservation offices, established in major cities, acted independently in making inventory decisions. Without the data on passenger behavior needed to do more sophisticated yield management, reservations control was limited to rudimentary overbooking.

SABRE provided a central point from which to collect data. SABRE also provided the centralized control needed to coordinate and enforce these new decisions. In 1968, American Airlines implemented an automated overbooking process, the flight load predictor, which determined overbooking levels based on management specified service levels. In 1976 this system was replaced by a different approach to overbooking. The new system, RIPACS (reservations inventory planning and control system), attempted to maximize overbooking profitability by explicitly accounting for the revenue and costs associated with the decision. The current overbooking model, implemented in 1987, makes better use of information on costs and benefits. (For an interesting historical perspective on overbooking in the airline industry, see Rothstein [1985].)

With the introduction of super-saver discount fares in 1977, the role of yield management expanded to include allocating reservations inventory to different classes of passengers who compete for space on the same flights. Super-saver fares introduced new pricing controls to the industry and enabled airlines to effectively divide passengers into business travelers and personal and pleasure travelers. Turning super-saver availability on and off resulted in an early form of demand-responsive pricing. The objective of this type of pricing is to adjust the demand to match the supply

of seats. American offered cheaper fares to stimulate demand in a controlled manner on low demand flights while maintaining higher profits on popular flights by limiting the number of super-saver fares offered. Allowing the sale of too many discount fares causes the displacement of higherrevenue passengers, while allowing too few discount sales results in empty seats

Their inability to control availability of discount seats contributed to the demise of several carriers.

on the plane. In 1982, Decision Technologies developed an optimization model based on Littlewood's research [1972] to determine the appropriate number of reservations to allocate to each fare type.

Airline deregulation in 1979 led to additional complexity in the practice of yield management. Two major changes took place. First, the number and variety of discount fares increased. Second, airlines began offering connecting service, using centrally located airports as hubs, to serve more of the traveling public and provide national service. For example, a single American Airlines flight from Dallas/Fort Worth to Denver may serve passengers from over 40 different eastern cities who are connecting to the Denver flight. The resulting airline environment is very complex. Many different passenger itineraries are possible, all with many different fares. Also, prices change rapidly, with up to 50,000 fare changes made daily.

American Airlines has developed operations research techniques to address yield management problems in five major areas:

- Overbooking,
- Discount allocation,
- Traffic management,
- Modeling passenger preferences, and
- Determining yield management performance.

Overbooking

Airlines allow customers to cancel unpaid reservations with no penalty. Even after purchasing a ticket, many passengers may cancel or miss their flights and receive at least partial refunds. (A passenger who does not formally cancel a reservation and does not show-up for the flight is considered a no-show.) On average, about half of all reservations made for a flight are cancelled or become no-shows. American estimates that about 15 percent of seats on sold-out flights would be unused if reservation sales were limited to aircraft capacity.

By properly setting reservation levels higher than seating capacity, American is able to compensate for passenger cancellations and no-shows, resulting in greater seat utilization. Poor overbooking decisionmaking can be costly. If reservation levels are set too low (more passengers cancel or no-show than expected), then flights depart with empty seats that could have been filled by turned-away demand. Empty seats on sold-out flights are called spoiled because once a flight departs, an empty seat has no economic value. Spoilage therefore represents a lost-opportunity cost to the airline. Figure 1 shows an example of a typical reservations pattern with and without overbooking.

While overbooking generates additional revenue for the airline (because fewer

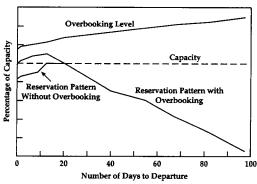


Figure 1: Overbooking allows more reservations to be accepted. The reservations pattern, defined as the number of reservations divided by capacity, is the same with and without overbooking for flights well in advance of departure. For flights close to departure, there are more reservations accepted with overbooking to compensate for cancellations and no-shows.

passengers are turned away), the practice of overbooking introduces a new cost. By overbooking, the airline takes the risk that more passengers may show up for a flight than there are seats on the aircraft. The airline must compensate such oversales (passengers not boarded because seats are not available) for their inconvenience and accommodate them on other flights.

The cost of oversales consists of compensation for the inconvenience (in the form of vouchers that can be redeemed on a future American Airlines flight), hotel and meal accommodations if necessary, and accommodation on a later flight, either on American or some other airline. If the passenger is put on another airline, American must pay the other carrier for transportation. The oversale cost is not constant. The more oversales that occur on a flight, the higher the voucher offer will be and the more likely that unaccommodated passengers will have to be transported on

another airline. Therefore, the total oversale cost is nonlinear (with a positive slope) as the number of oversales increases. If too few passengers volunteer to take a later flight in exchange for the airline's final compensation offer, then involuntary oversales occur. Involuntary oversales cause an additional cost through loss of customer goodwill.

Decision Technologies developed an optimization model that maximizes net revenue associated with overbooking decisions. In choosing the best overbooking level for a flight, this model balances the additional revenue that can be gained by selling a reservation against the cost of the additional oversale risk (Figure 2). As the overbooking level increases, net revenue (passenger revenues minus the cost associated with oversales) increases to a maximum value and then decreases as the incremental cost of an additional oversale exceeds the value of an additional reservation. The

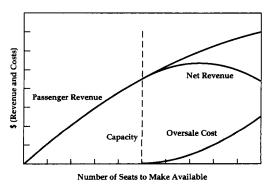


Figure 2: The optimal overbooking level occurs at the maximum of the net revenue curve, where net revenue is the difference between total passenger revenue and oversale cost. When there is little or no overbooking, total revenue equals net revenue. Oversale cost increases with higher overbooking levels, so there is a unique optimal overbooking level.

optimal overbooking level occurs at the point where the marginal revenue gained from allowing an additional reservation equals the marginal cost of an additional oversale.

The number of oversales allowed by using this unconstrained optimization model may sometimes degrade passenger service to an unacceptable level. To prevent this problem, we placed a constraint on the system to limit the expected number of oversales on each flight. We incorporated this constraint into the model using Lagrangian relaxation [Smith 1984]. The complete overbooking model accounts for

- The additional revenue generated by allowing more reservations,
- The probability distribution of passenger cancellations and no-shows,
- The expected number of oversales,
- The expected oversale costs,
- The maximum number of oversales allowed.
- The likelihood that a passenger who cannot secure a reservation on one flight will choose another American Airlines flight (called the recapture probability). High recapture probabilities imply that less oversale risk should be taken, so that the overbooking level will be lower.

Because these factors vary with the amount of time before departure, overbooking levels are recalculated several times before departure. Each update time is called a reading day, because information on a flight is read from SABRE. Four forecasts are needed for the overbooking model:

The probability that a passenger will cancel,

- The probability that a passenger with an active reservation will not show up on the departure day,
- The probability that a passenger who is turned away will choose another
 American flight (recapture probability), and
- The oversale cost.

Cancellation and no-show probabilities are estimated using exponential smoothing models (with multiplicative day-of-week adjustments). Because very few reservations are made well in advance of departure, the cancellation probability can be very unstable. We developed special default rules to handle these situations. Recapture probabilities are estimated using a passenger choice model.

Discount Seat Allocation

When all passengers on an airplane are paying the same rare, overbooking provides the complete yield management solution. Thus, in the 1960s and early 1970s, American concentrated its efforts on overbooking. In the mid-1970s, airlines began offering discount fares. Single-letter class codes are used to distinguish the different fare types: Y for full-fare coach, M and Q for discounts, V for deeper discounts, and so forth.

Because fares differ, airlines started to control availability by class code. To illustrate the problem, we will describe the situation when discount fares were first offered. Only two fares were available, full and discount. American's decision to accept or reject the next (or marginal) request for a discount seat can be illustrated using a decision tree (Figure 3). If it accepts a discount request, the revenue it earns is the discount fare. If it rejects the discount

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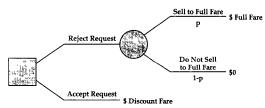


Figure 3: The decision to accept or reject a discount request is essentially a decision tree. The decision depends on p, the probability of getting full fare when a discount request is rejected. This estimate is updated several times before departure for any one flight. p depends on several factors, including the number of remaining available seats and the distribution of demand. If (p) (\$full fare) > \$discount fare, then the discount seat request is rejected.

request, two outcomes are possible. First, rejecting the discount request may result in an additional empty seat and no additional revenue. Second, all the remaining seats may be filled with full-fare passengers, because sufficient full-fare passenger demand exists (the flight sells out) or because discount-fare passengers choose to pay full-fare when told the discount fare is not available (sell-ups).

As shown in Figure 3, the decision to accept or reject a discount request depends on the value of *p* compared to the ratio of the discount fare to the full fare. The probability of receiving full fare for a seat after rejecting a discount request, *p*, is determined by three factors: future expected demand, the accuracy of the demand forecast, and the sell-up probability.

The future expected demand forecasts are computed using exponential smoothing time-series techniques. (Analysis is ongoing to develop new forecasting models that incorporate causal factors.) A time-series method is used to update a standard deviation estimate based on demand forecast

error. Currently the probability that passengers requesting discount seats are willing to pay full fare (the sell-up probability) is estimated subjectively, but we are conducting research to compute better estimates.

These three factors are used to determine the number of aircraft seats to withhold from sale to passengers requesting a discount fare. Because future expected demand depends on the length of time until departure, the discount allocation decision is updated on each reading day (along with the overbooking level).

Multiple-Fare Types

With multiple-fare types, the problem becomes more complicated. American's approach is still to weigh the marginal value of a fare request against the marginal value of all other fares. It considers the same variables discussed above, but must include values for each fare type.

As the number of reservations allocated for a fare type increases, the marginal expected value of an additional reservation in that fare type decreases. Although the fare is the same, the probability of an additional sale decreases with each additional seat offered. When the number of seats offered is much higher than future expected demand, the marginal revenue increases only slightly when an additional seat is offered. For example, if future expected demand is 10, then the marginal value of changing the number of seats offered from 10 to 11 is much greater than the value of changing from 50 to 51. Figure 4 shows the change in marginal revenue resulting from the allocation of additional reserva-

Figure 5 shows the marginal expected

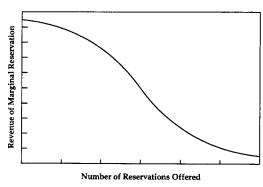


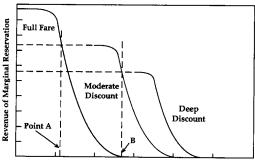
Figure 4: As the number of reservations offered increases, the incremental revenue from one additional seat decreases. When the number of seats offered is much higher than future expected demand, revenue increases only slightly when an additional seat is offered. This plot was first published by Belobaba [1987].

revenue curves for several passenger types. The point (A) at which the marginal value of the full fare (class code Y) is equal to the marginal value of the moderately discounted fare (class M) is the optimal number of reservations to protect for full-fare passengers and withhold from any discount passengers (M and Q). If there are only Y, M, and Q fare-types, then point B represents the total number of reservations to allocate to Y and M passengers and withhold from sale to Q passengers.

Nesting

The availability for each discount class is controlled through a process known as nesting. Nesting makes subsets of the seats available to various levels of discount fares. Smaller subsets are available to lower-valued discount classes than to full-fare or moderate-discount classes (Figure 6). If the fare classes are controlled independently, it would be possible to sell a low-revenue reservation and simultaneously turn away a high-revenue passen-

ger. (This situation would happen when all the seats allocated to the high-revenue class are sold, while some seats were still available to a lower-revenue class.) Nesting simplifies the maintenance of discount fares by automatically ensuring that a lowvalue seat is never available when a higher-valued fare is closed to additional sales. The use of nesting means that forecasting too much demand in a higher-revenue class causes poor discount allocation only for lower-revenue classes. If deep-discount seats are available for sale, then so are moderate and full-fare seats. If no moderate-discount seats are available, then neither are deep-discount seats. When fullfare seats are not available, no seats are



Number of Reservations Allocated

Figure 5: The marginal revenue analysis to determine discount allocation for several fare classes is treated as an extension of the twofare class situation. A separate marginal revenue curve exists for each fare class. The curve on the far left is for full fare (Y class code), in the middle is the curve for the moderate discount fare (M class), and on the far right is the marginal revenue curve for the deep discount fare (Q class). The curves would be continued to the right for more fare classes. Points A and B show the number of seats to protect for the current and all higher-revenue fare classes. Thus A seats are protected for full fare, and B seats are protected for fullfare and moderate-discount reservations combined.

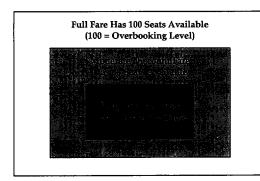


Figure 6: Nested fare classes can be represented as subsets of the next higher fare class. As seats are sold, the lowest-valued fare classes close first. Full fare is closed only if the flight has reached its overbooking level. In this example, all 100 seats are available to full-fare passengers, 60 seats are available to moderate-discount passengers, and 30 seats are available to deep-discount passengers. The difference in seat availability between full-fare and a discount-fare class is the number of seats to protect for all higher-revenue fare classes. Thus, 40 seats are protected exclusively for full fare (100-60) and 70 seats are protected for full-fare and moderate-discount fares (100-30). Full fare is closed only if the flight reaches its overbooking level.

available (the flight has reached its overbooking level).

Approaches to the Discount Allocation Problem

American's method for discount allocation was developed in 1980 [Swan]. The logic of the model is based on research by Littlewood [1972]. Decision Technologies modified Littlewood's method (also called the marginal revenue method) for determining discount allocation to account for

- Multiple fare types,
- Customer sell-up behavior,
- Timing of future demand (reservations protected for high-value customers can be made available to lower-value customers if all of the high-value demand

- does not occur),
- Cancellations (seats are protected for future demand minus expected cancellations of future demand).

American's current approach is not mathematically optimal, but the heuristic finds solutions that are near-optimal in terms of revenue. American uses the marginal revenue approach for determining discount allocation because other aspects of the real-world problem (for example, customer sell-up behavior, timing of future demand, and cancellations) can easily be incorporated while maintaining computational efficiency. Much academic research on solving the discount allocation problem is available in operations research and management science journals. For good overviews of published research, see Belobaba [1987] and Curry [1990]. Currently, Decision Technologies is researching the use of a mathematical programming approach to the discount allocation problem.

Traffic Management

The airline yield management problem became more complicated after deregulation because airlines were allowed more flexibility in scheduling. To serve more markets, airlines developed the hub-andspoke system of scheduling flights. In this system, a large number of passengers connect at a hub airport to reach their destination. In 1980, approximately 10 percent of American's traffic consisted of connecting passengers. By the mid-1980s, about 66 percent of the passengers on a typical flight going to a hub airport were connecting to another flight to get to their destinations (Figure 7). For example, using Dallas/Fort Worth as a hub, some connecting

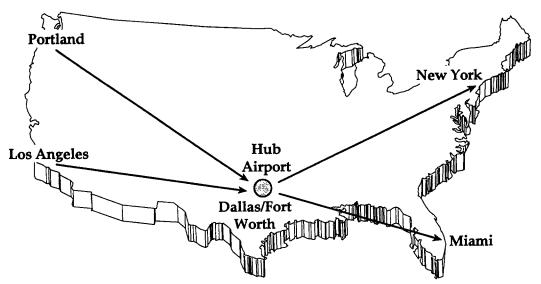


Figure 7: For this sample airline network, passengers travel from two western spoke cities (Portland, Oregon and Los Angeles, California) to Dallas/Fort Worth directly or connect at Dallas/Fort Worth to travel to two eastern cities (New York, New York and Miami, Florida). For American Airlines at Dallas/Fort Worth, passengers from one flight can connect to any of 30 or more other flights. Yield management must consider the many markets served by any one flight to or from a hub airport.

markets are Portland to Miami, Portland to New York, Los Angeles to Miami, Los Angeles to New York. Some local markets (not involving continuation on a connecting flight) are Portland to Dallas/Fort Worth, Los Angeles to Dallas/Fort Worth, Dallas/Fort Worth to New York, Dallas/Fort Worth to Miami.

From a yield-management perspective, flights that once served a single market now serve 30 or more markets. As a result, the diversity of customers on a single flight has increased dramatically, and inventory can no longer be controlled adequately by class code to support yield management. The fares for a single class code may be substantially different depending on market. For example, it is possible for the Portland to Dallas/Fort Worth full fare to be less than Portland to Miami (connecting over Dallas/Fort Worth) discount fare,

which is less than Portland to Miami (connecting over Dallas/Fort Worth) full fare.

The variability of revenue within a fare class can become large, especially when hundreds of market/fare classes are available on a single flight. In this example, the Portland to Dallas/Fort Worth full fare and Portland to Miami full fare are substantially different. If control was by class code alone (full fare versus discount), then American would accept Portland to Dallas/Fort Worth full-fare passengers while turning away Portland to Miami discount passengers. This could actually lower revenue, given the ordering of fares. In this situation, inventory control by fare class alone is ineffective because the full fare is more valuable than the discount fare in one case and less valuable in the other. By the early 1980s, American realized that it needed a new approach.

In 1983, American Airlines began to develop a new method of controlling reservation availability by market/fare class. This new method is called virtual nesting [Smith and Penn 1988]. Decision Technologies developed a series of models to control reservation availability in this environment.

Virtual Nesting

Ideally the airline would control its inventory of reservations at the market/fare class level, allowing it to control for each type of passenger individually. Unfortunately, the enormous number of controls required makes market/fare class control infeasible. American developed a method of clustering the market/fare classes into a small number of similarly valued groupings called buckets, reducing the inventory

controls to a manageable number. All the market/fare classes on each flight are clustered into eight buckets. First and business classes are each controlled in separate buckets. All coach market/fare classes are controlled in the remaining six or seven nested buckets, depending on whether a flight has a business-class cabin (Figure 8).

The clustering process is called indexing, and each market/fare class is said to be indexed into a bucket. American uses the term virtual nesting for this method of controlling availability without having to use separate controls for each of the thousands of market/fare-class combinations on each flight. The buckets are nested so that as sales increase, availability is restricted first to low value reservations, regardless of market/fare class.

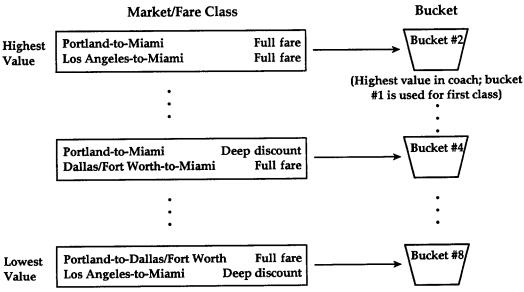


Figure 8: To control market/fare classes, American clusters them into buckets. For the Dallas/Fort Worth to Miami flight, American sorts market/fare classes by decreasing order of value. It groups similarly valued market/fare classes together (left side of figure) and indexes them into inventory control buckets (right side of figure). The buckets are nested so that as sales build, the airline restricts availability first to lower-revenue reservations, regardless of market/fare class. Some discount fares (on long haul markets) are valued higher than some short-haul full fares and are therefore indexed into a lower numbered bucket.

Virtual means that market/fare-class availability is not explicitly stored in the reservations system. Instead, market/fare-class availability is determined for each request by combining indexing with flight/bucket availability. A market/fare class is available for sale if, and only if, the bucket it is indexed into has seats available. For example, for a passenger requesting a discount fare from Portland to Miami via Dallas/Fort Worth, the bucket into which the Portland-to-Miami discount fare is indexed must have seats available on both the Portland-to-Dallas/Fort Worth flight and the Dallas/Fort Worth-to-Miami flight.

Indexing is a deceptively complex process, especially when considering the large number of passenger types for the system. (American currently serves 150,000 market/fare classes.) Decision Technologies developed a dynamic programming algorithm to index market/fare classes into buckets. The algorithm groups market/fare classes so that those with similar values are mapped into the same bucket and those with significantly higher (or lower) values are placed in higher (or lower) buckets. The objective is to minimize the variability of market/fare-class values within buckets and to maximize the variability between buckets. Constraints are applied to force a minimum amount of traffic into each bucket. These constraints are necessary to compute accurate bucket-level demand forecasts.

Reservation Value

An important consideration in indexing the market/fare classes is determining the value of each market/fare class to the airline. Simply using the total revenue provided from each market/fare class is not sufficient. For a passenger traveling from Portland to Miami, the trip involves two segments: Portland to Dallas/Fort Worth and Dallas/Fort Worth to Miami. If the revenue from Portland to Miami is \$800, Portland to Dallas/Fort Worth is \$500, and Dallas/Fort Worth to Miami is \$400, then it is better to sell two one-segment tickets (Portland to Dallas/Fort Worth and Dallas/Fort Worth to Miami) instead of one Portland to Miami ticket.

Of course, this higher revenue can be achieved only if there is sufficient demand on both segments. In this situation, the incremental value of a long-haul passenger is reduced because of the potential displacement of customers who do not want connecting flights (called local passengers). To solve this problem, American determines an effective revenue for each market/fare class based on the fare and the probability that a connecting passenger will displace local traffic on each segment of the routing. The result is that effective revenue from connecting passengers is less than the ticket price. These effective revenues are used in the clustering algorithm and in determining discount allocation.

Default Indexing

Every time a passenger requests a flight, a virtual nesting table is accessed by SABRE containing the bucket index for the requested market/fare class. Ideally, this table would include an entry for each market/fare-class in the American Airlines system. Because this table is accessed millions of times a day, SABRE processing is adversely affected if the table becomes too large. A sample virtual nesting table item is as follows, for the Portland to Miami moderate discount fare (fare class M): Origin:

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Portland, Connect Point: Dallas/Fort Worth, Destination: Miami, Fare: M, Bucket: 2.

To limit the size of the table, we do not include unique entries for many market/ fare classes; instead, we use a default table. We use one table item to index many market/fare classes, thus eliminating the need for separate entries for each of the market/ fare-classes covered by the default item. For example, for the many Portland moderate discount fare-class (class code M) markets connecting over Dallas/Fort Worth that should be indexed into bucket #2, we use one entry (called a default entry), like the following, to replace several hundred detailed table entries: Origin: Portland, Connect Point: Dallas/Fort Worth, Destination: anywhere, Fare: M, Bucket: 2.

During an availability request, SABRE scans the virtual nesting table for the first pattern that matches the request. Some market/fare classes are replaced by default entries with incorrect indices. For example, a low-demand market such as Portland, Oregon to Jacksonville, Florida on a moderate discount fare does not warrant a specific detailed item for just that market/fare class. Its index may be incorrectly reflected in the default item (perhaps its bucket should be 3 instead of 2). Thus, passenger requests may be accepted when they should not be or vice-versa. There is a trade-off between the number of items in the virtual nesting table and the number of market/fare classes correctly indexed by defaults.

Decision Technologies developed a mixed-integer linear-programming algorithm in 1990 to optimally determine the default table items. The objective is to maximize the number of market/fare classes indexed correctly. Correct indexing occurs when the default bucket matches the bucket determined by the dynamic-programming clustering algorithm. With effective default indexing, even high-demand market/fare classes do not always need individual table entries.

Passenger-Choice Modeling

While time series models provide reliable, stable forecasts of passenger demand, their results tend to lag behind changes in the marketplace. This shortcoming has become more serious because of the frequency of schedule and price changes in the airline industry. American needed a way to evaluate the impact of fare and schedule changes on customer demand. To solve this problem, Decision Technologies developed a passenger-choice model. This model estimates a passenger utility function using the following variables:

- Departure time,
- Type of service (nonstop or connecting),
- Time between departure and arrival,
- Airline.
- Price, and
- Restrictions (such as cancellation penalties, advance purchase requirements, and so forth).

A utility function is estimated for each possible option offered to a potential customer. We then use these functions in conjunction with a logit-choice model to estimate customer preference and market share (see Ben-Akiva and Lerman [1985] for statistical details).

American is devoting considerable research to more fully incorporate the pas-

senger-choice model into yield-management decisions. Recent research provides estimates of how demand will change when major schedule changes are made, and an effort to better estimate sell-up probabilities is also underway. The long-term plan is to use passenger-choice modeling (or some other causal-modeling technique) to forecast demand and cancellations in response to price changes, sales, schedule changes, and so forth made by American and by its competitors. These forecasts would supplement the time-series forecasts now in use.

Monitoring and Performance

As American Airlines grew and Decision Technologies developed more sophisticated operations research models for yield management, American needed to develop formal procedures to monitor the system and measure performance. We developed most of these procedures since 1987. We use four tools primarily:

- Demand forecast monitoring to flag biased forecasts,
- Critical flight identification to identify flights requiring manual review,
- Index monitoring to report on virtual nesting and to flag market/fare classes that should be reindexed, and
- Revenue opportunity modeling to measure overall yield-management performance.

Demand Forecast Monitoring

Decision Technologies uses quality control methods to generate reports identifying passenger demand forecasts that have become biased. Forecasts can become biased because of permanent changes in passenger behavior that the time-series models do not rapidly incorporate. Cur-

rently, we use a cumulative sum tracking signal. We are pursuing research in other methods, including statistical runs tests. In addition, similar techniques will be used to monitor other forecasts, such as cancellation rates.

Critical Flight Identification

Some flights require manual review, either because the input data are in error or because of special circumstances (such as a major sporting event or Mardi Gras). The critical flight identifier computes a "criticality index" between zero and 100. High indices indicate a high probability of oversales or spoilage or situations in which oversales and spoilage are particularly costly. The critical flight identifier eliminates the need for yield management analysts to identify problems manually.

Yield management analysts review a critical flight, using the critical flight analyzer recently developed by American's data processing department. This sophisticated system permits them to perform flight-specific revenue analysis and re-optimization and to change inventory levels in SABRE.

Index Monitoring

Because fares change, indexing that is correct today may become incorrect tomorrow. Such fluctuations mean that the fixed indexing stored in the virtual nesting table will no longer match the output of the dynamic programming clustering algorithm. In such cases, market/fare classes need to be re-indexed. Decision Technologies has developed a reporting system to identify market/fare classes that are indexed incorrectly. Each week it computes statistics on indexing errors and recommends corrective action.

Overbooking Revenue Opportunity Model

Traditional industry overbooking statistics, such as oversales and spoilage, are not the best measure of overbooking performance. These statistics are affected by factors external to the airline and do not reflect the revenue impact of yield management decisions. American measures the impact of overbooking by comparing the actual net revenue (net revenue equals total revenue minus any oversale costs) to the maximum net revenue that could have been achieved with perfect overbooking controls. This method is referred to as measuring the revenue opportunity.

It measures the amount of revenue resulting from overbooking controls on a flight-by-flight basis as the difference between actual net revenue earned for the flight minus the estimated net revenue that would result from the same flight if no overbooking controls were applied. In addition, it estimates the maximum net revenue that could have been achieved by perfect overbooking controls. The total revenue opportunity available from overbooking is the difference between the "perfect controls" net revenue and the "no controls" net revenue. It measures overbooking performance as the percent of revenue opportunity earned (actual net revenue divided by the revenue opportunity). The system-wide overbooking performance measure is the average of the individual flight performance measures.

American estimates the net revenue based on the "no controls" scenario by evaluating the reservation process as if the overbooking level were set at aircraft capacity. It does not count any reservations above capacity, because those reservations would have been turned away with no overbooking. When the number of reservations equals capacity, new reservations are allowed only when other reservations have been cancelled. The estimate of net revenue is the sum of the fares of those reservations that were accepted and actually boarded a flight and does not include cancellations or no-shows.

American estimates the net revenue of the "perfect controls" scenario by eliminating all oversales and spoilage. If the airline has oversold a flight being evaluated, then it adds back the actual oversale costs into the actual revenue, and subtracts the revenue from the lowest n fares (where n is the number of actual oversales) from the actual revenue. If spoilage occurs on a flight being evaluated, then it adds the m largest fare reservations (where m is the number of spoiled seats) that were turned away because of overbooking controls to the actual

Yield management has provided quantifiable benefits of over \$1.4 billion for the last three years.

revenue of the flight. This method is summarized as follows:

Perfect controls case 1: *n* oversales occurred on flight;

Perfect controls net revenue = actual revenue + actual oversale costs — revenue from n lowest fare passengers.

Perfect controls case 2: *m* seats spoiled on flight;

Perfect controls revenue = actual net revenue + revenue from *m* largest fare

passengers that were denied reserva-

Appendix A contains a detailed example of the calculation of overbooking revenue opportunity.

Discount Allocation Revenue Opportunity Model

Because of the dynamic nature of the marketplace, the effectiveness of discount allocation controls is difficult to measure. Traditional airline industry statistics, such as load factor (percent of seats filled) and yield (revenue per passenger per mile), are affected more by external factors (schedule and price competition, and general economic conditions) than by yield-management decisions. American Airlines developed a method of measuring performance, the discount allocation revenue opportunity model, which is less sensitive to these other factors.

It uses revenue opportunity statistics in conjunction with traditional measures to track yield-management performance. The model measures revenue performance as the percent of revenue opportunity achieved through discount controls. It measures revenue performance by estimating the revenue (on a flight-by-flight basis) that American would have earned with no discount controls. This amount is the minimum revenue expected for the flight.

Next, it estimates the revenue that could have been earned with perfect discount controls. This amount is the most revenue that could have been earned for the flight. It is possible to estimate this "perfect control" figure because of a model Decision Technologies developed in 1987 to statistically estimate the number of passengers turned away (unsatisfied demand).

The difference between the minimum and maximum revenues equals the total revenue opportunity available through discount controls. American determines the revenue opportunity it achieved by subtracting the minimum revenue (no discount controls) from the actual revenue earned from the flight (Figure 9). It measures performance as the percentage of revenue opportunity earned divided by the total opportunity.

Appendix B provides a detailed example

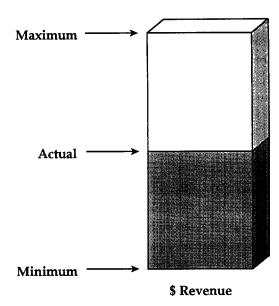


Figure 9: The discount allocation revenue opportunity model determines the percentage of the total revenue opportunity American actually earned with its discount allocation controls. Total revenue opportunity is the difference between maximum revenue possible and minimum revenue possible. Maximum revenue is computed assuming perfect yield management controls. This computation is possible after the flight has departed (perfect hindsight) using estimates of how much demand was turned away in each bucket. The minimum revenue is computed assuming no discount allocation yield management controls. In the figure, the shaded area represents the revenue opportunity earned.

of the calculations. As the example shows, the minimum and maximum revenue calculations take the overbooking level into account. In the "perfect controls" case, high revenue passengers are turned away only if the overbooking level has been reached. Because the minimum and maximum figures do not count the revenue from passengers turned away because the overbooking level had been reached and do not count oversale costs, the results of the overbooking and discount allocation revenue opportunity models do not double count the same benefits.

Avoiding Double Counting

In measuring yield-management performance, it is important to avoid double counting benefits. While the methods used to measure overbooking and revenue-mix performance are similar, they do not double count benefits because they isolate different aspects of the yield-management process.

In measuring overbooking performance, one looks at the revenue associated with the total number of passengers on flights. Overbooking errors result in oversales or spoilage.

In measuring discount allocation performance, one examines the revenue (market and fare class) of the passengers on board. When demand is sufficiently high, yield-management controls can increase the average value of each seat occupied. In this case, the concern is with the quality (or value) of the bookings, rather than the quantity. The discount allocation revenue opportunity model measures performance by reviewing the mix of sales among nested buckets, rather than the total sales count.

To some extent, overbooking and revenue mix can offset each other. Good overbooking levels increase available capacity and increase the supply of reservations relative to demand. Lower demand for a flight reduces the revenue-mix opportunity. Similarly, revenue-mix controls reduce the total number of reservations accepted for a flight and in turn reduce overbooking opportunity.

Thus, the revenue opportunity performance measures isolate the effects of different reservation controls and in some cases may undervalue the benefits of yield management.

Benefits

Prior to implementing DINAMO, American's yield-management department relied on a series of data-processing tools to assist in manually controlling seat inventory. In 1988, it brought decision support tools into daily use, greatly improving overall yield-management revenues. The benefits of yield management can be divided into four categories: overbooking, revenue mix, productivity, and pricing flexibility.

Overbooking

The financial benefit provided by overbooking is measured as the increased revenue resulting from a reduction in the number of empty (or spoiled) seats on sold-out flights, minus the cost of oversales. Without overbooking controls, American estimates that 15 percent of seats would be spoiled on sold-out flights. In 1980, using an earlier version of the overbooking model, spoilage on sold-out trips was reduced to seven percent. In 1990, using the current DINAMO overbooking model, spoilage was only three percent.

The benefit from reduced spoilage has

been realized at the cost of additional oversales. In 1980, American averaged eight oversales per 10,000 passengers boarded. This number rose to 16 oversales per 10,000 boarded by 1990. However, DINAMO explicitly accounts for oversale costs and reduces the overbooking level on flights where oversale costs are highest. This adjustment has resulted in a 62 percent reduction in involuntary oversales over the past 10 years.

In 1990, American estimated its total revenue opportunity resulting from overbooking at \$250 million, using the overbooking revenue opportunity model. This estimate assumes that spoilage and oversales are completely eliminated and therefore, represents the maximum revenue that could be earned through overbooking controls. American estimates that it actually achieved 90 percent, or \$225 million, of this total revenue opportunity. Table 1 lists the performance and revenue-benefit statistics for 1988, 1989, and 1990. Estimates are not available for earlier years. The financial benefit resulting from overbooking will continue through future years, and should increase as enhancements are implemented in the next few years.

Revenue Mix (Discount Allocation and Traffic Management)

American uses the discount allocation revenue opportunity model to measure the revenue performance of discount-fare controls. In 1988, before it started using the current system of models, it realized approximately 30 percent of the revenue opportunity from discount controls. This percentage provides a baseline estimate of the revenue from discount allocation. Since DINAMO was implemented, the percent

Year	Revenue Opportunity Earned (%)	Revenue Earned (\$)	
1988	92	210 million	
1989	93	235	
1990	90	225	

Table 1: Percent of revenue opportunity earned by overbooking has remained fairly constant. Estimates in this table are computed by the overbooking revenue opportunity model.

of revenue opportunity earned has increased, and in 1990, the percentage had increased to 49 percent (Table 2).

American Airlines attributes the increased revenue in this area to greater use of operations research models and to improved model interfaces, which make DINAMO results more accessible and easier to understand. Planned enhancements to DINAMO should increase the percentage of revenue opportunity earned in the future.

Revenue Benefit from Automated Systems

Clearly, using simple yield-management controls can provide greater revenues than the theoretical minimum values. Setting default overbooking levels and discount controls is better than allowing the market-place to dictate the number and mix of customers on each flight. However, the following two questions need to be addressed:

- (1) Why use a theoretical minimum instead of a heuristic?
- (2) How much revenue should be attributed to the automated systems?

An alternative to using the theoretical minimum is to use the revenue that can be attributed to a "good" solution. A good solution should produce a relatively stable

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Year	Revenue Opportunity Earned (%)	Revenue Earned (\$)	
1988	30	198 million	
1989	46	256	
1990	49	313	

Table 2: DINAMO has increased the revenue earned from discount allocation by over \$100 million per year. Percent revenue opportunity and revenue earned figures are computed by the discount allocation revenue opportunity model.

percentage of earned revenue opportunity. If the good solution is used as the lower bound for measuring performance, the DINAMO performance will reflect lower percentages, but the relative values will be unchanged. Many rules can produce good solutions, but which rule should be used for which situation is debatable. The theoretical minimum can be determined without debate.

We measured the benefits of the yield-management process from the theoretical minimum. We measured the benefits of DINAMO relative to performance prior to implementation rather than against the theoretical minimums. For example, the entire revenue-mix process generates approximately \$300 million per year. The improvement in revenue-mix controls (from 30 percent of revenue opportunity earned to 49 percent) is attributable to DINAMO. **Productivity**

Before the development of DINAMO, yield-management specialists manually reviewed every flight departing on specified future dates. Since the implementation of DINAMO, all flights have been reviewed automatically by the critical flight identifier. A critical flight does not meet yield-management control parameters. Only crit-

ical flights require manual attention and are reviewed by yield-management analysts. In 1990, five percent of all flights were identified as critical. Noncritical flights do not require further processing or adjustment. This reduction in work load has allowed yield-management analysts to spend more time reviewing only critical flights, thus making better revenue decisions. The capability to identify critical flights has also increased productivity in American's yield-management department. Between 1988 and 1990, productivity per analyst increased by over 30 percent. This increased productivity represents a savings of \$1 million annually.

Pricing Flexibility

The discount allocation process adjusts the minimum available price based on demand for each flight. In addition to this revenue-mix benefit, yield management controls allow much greater flexibility in offering a variety of products for sale. This flexibility is an important competitive tool. In the years following deregulation, their inability to control the availability of discount seats effectively contributed to the demise of several carriers.

Following airline deregulation, competition from new-entrant carriers resulted in fare discounts in many major markets. Carriers in these markets often had to choose between revenue dilution if they matched the lowest fare or loss of market share if they did not match the lowest fare. In 1984, American introduced a pricing strategy made possible by yield management, a very low and highly restrictive fare, called Ultimate Super Savers. Using yield-management controls, American can maintain market share by offering a low

fare that stimulates additional traffic. It limits revenue dilution by restricting the availability of these fares on flights that have substantial high-value demand.

Other Applications (Benefits for Other Industries)

Yield management also provides substantial benefits to a variety of industries other than airlines and applies to many businesses that

- Allow advance reservations,
- Offer a range of customer values (different prices available),
- Experience cancellations and no-shows, and
- Stock perishable inventory.

American Airlines Decision Technologies has applied yield-management techniques to other industries since 1987. The first nonairline applications were with passenger railroads. The greatest difference is that passenger railroads operate linear networks, instead of the hub-and-spoke networks airlines use. The linear network approach allows many stops along the routing of one train, the number of possible markets is very large, and effective traffic management is very important.

Decision Technologies has also applied yield-management techniques to the lodging industry. While many airline yield-management techniques can be applied directly, there are two differences. First, hotel customers arrive continuously throughout the day. While airlines control arrivals and departures at specific flight departure times, hotel controls must respond to an evolving process of arrivals and departures. There is no single point, such as the closing of the aircraft door, when yield management is complete. Second, hotels

have prices based on the length of stay, which is analogous to the airline traffic flow problem. Hotels must control the flow of customers in a network spanning length of stay instead of origin and destination.

Decision Technologies is also applying yield-management techniques to cruise lines, tour operators, car rental companies, and the broadcasting industry. All these businesses need to control the availability of fixed inventory in a scientific, profit-enhancing way. Although we have not completed long-term benefit studies, we anticipate that these industries will increase their revenues by percentages similar to those achieved at American Airlines.

Benefit Summary

Yield management has played a key role in allowing American Airlines to compete and succeed in an environment of stiff price competition. Some of the benefits are difficult to measure, such as increasing American Airlines' ability to survive price wars. Overall, yield management has provided quantifiable benefits of over \$1.4 billion for the last three years. To put these increased revenues in perspective, AMR (the holding company for American Airlines) had net profit (after all expenses and taxes) of \$892 million for the same threeyear period. As the airline grows and yield-management models become more sophisticated, the annual revenue benefit will increase.

New Applications of Management Science

Our research in yield management has led to the development of several applications of management science techniques that are unique to American Airlines. While some of these processes have been implemented at other airlines, they were conceived and developed at American.

Decision Technologies developed the first closed-form solution to the constrained overbooking problem. When this model was developed, most alternative approaches required a search for potential solutions.

Decision Technologies developed an extension of Littlewood's [1972] marginal revenue approach to handle multiple fareclasses, customer sell-up behavior, cancellations, and timing of demand. Also, we have recently completed research (using censored regression methods) to estimate the probability that a passenger will purchase a more expensive ticket when a lower fare is not available (sell-up probability).

In 1983, American realized that the development of hub-and-spoke networks would require a method for controlling the flow of traffic across connections. We developed virtual nesting to address this issue. (Since then United Airlines has also implemented virtual nesting.) In the process of implementing virtual nesting at American, we realized that several new types of yield-management-related models were necessary. The use of dynamic programming for optimal indexing of market/ fare classes is unique to American. We believe that the use of mathematical programming to control the size of the virtual nesting table is also unique to American.

Reliable recapture estimation is very difficult using traditional statistical methods because of the sometimes small effects being measured in a very dynamic environment. Recapture has been accurately measured using passenger choice modeling. We believe that this is the first successful recapture measurement in the airline industry.

Yield-management performance is difficult to measure because of the dynamic nature of the marketplace. Decision Technologies developed a reliable and credible method of measuring performance that we believe is unique in the airline industry.

APPENDIX A

Examples of the Calculations for the Overbooking Revenue Opportunity Model

The overbooking revenue opportunity model reviews each flight to determine actual revenue, revenue that would have been achieved with perfect controls, and revenue that would have been achieved without overbooking. Total revenue opportunity is defined as the difference between revenue with perfect controls and revenue without overbooking, and actual revenue earned is expressed as a percentage of this total revenue opportunity. Two examples are shown here.

Example 1: Consider a flight with the following passenger and revenue results:

Aircraft capacity = 150,

Number of oversales = 5,

Actual revenue = \$31,000,

Actual oversale costs = \$1,500,

Actual net revenue = \$31,000 - \$1,500 = \$29,500,

"No controls" net revenue = \$24,000 (esti-

"No controls" net revenue = \$24,000 (estimated by simulating the reservation process with the overbooking level set at capacity = 150),

"Perfect controls" revenue = \$29,500 + \$1,500 - 5 * \$80 = \$30,600 (\$80 was the cheapest fare on the flight, and there were at least five passengers at this fare), Overbooking revenue opportunity = \$30,600 - \$24,000 = \$6,600, Revenue attributed to overbooking controls = \$29,500 - \$24,000 = \$5,500, and Performance measure = \$5,500/\$6,600

Bucket	Passengers			Revenue	
	Boarded	Spilled	Total	Average	Total
Y0	12	0	12	\$313	\$3,756
Y1	6	0	6	258	1,548
Y2	10	0	10	224	2,240
Y3	3	0	3	183	549
Y4	30	29	59	164	4,920
Y5	16	5	21	140	2,240
Y6	32	32	64	68	2,176
Total	109	66	175		\$17,429

Table 1B: Actual passenger and revenue information for a sample flight. The number of turned away passengers ("spilled" column) is estimated by a statistical model. Overbooking level is 138. The numbers in this table will be compared to theoretical minimum and maximum revenues available from discount allocation controls.

= 83.3% (% of revenue opportunity earned).

Example 2: Consider a flight with the following passenger and revenue results: Aircraft capacity = 150,

Number of spoiled seats = 2,

Actual revenue = \$28,400,

Actual net revenue = \$28,400 (no oversale costs),

"No controls" net revenue = \$24,000 (estimated by simulating the reservation process with the overbooking level set at capacity = 150),

"Perfect controls" revenue = \$28,400 + 2*\$500 = \$29,400 (\$500 was the largest spilled fare, and there were at least two potential passengers at this fare), Overbooking revenue opportunity

= \$29,400 - \$24,000 = \$5,400,

Revenue attributed to overbooking controls = \$28,400 - \$24,000 = \$4,400, and Performance measure = \$4,400/\$5,400 = 81.5%.

APPENDIX B

An Example of a Discount Allocation Revenue Opportunity Model Calculation

Discount allocation revenue performance is measured on a flight-by-flight basis using the discount allocation revenue opportunity model, with system-wide performance computed as the average performance of the individual flights. An example will clarify the calculations.

The actual revenue performance: Consider a flight with the passenger and revenue results shown in Table 1B.

The revenue performance with no discount controls: If discount controls are not used then reservations are accepted until the overbooking level is reached. Because lesser-price reservations are typically purchased first, this scenario will result in

	Total	Passengers	Revenue	
Bucket	Demand	Boarded	Average	Total
Y0	12	0	\$313	\$0
Y1	6	0	258	0
Y2	10	0	224	0
Y3	3	0	183	0
Y4	59	53	164	8,692
Y5	21	21	140	2,940
Y6	64	64	68	4,352
Total	175	138		\$15,984

Table 2B: Passenger and revenue results that would have been achieved with no controls is computed by assuming demand comes from the lowest valued buckets first, so that high revenue demand is turned away.

	Total	Passengers	Revenue	
Bucket	Demand	Boarded	Average	Total
Y0	12	12	\$313	\$3,756
Y1	6	6	258	1,548
Y2	10	10	224	2,240
Y3	3	3	183	549
Y4	59	59	164	9,676
Y5	21	21	140	2,940
Y6	64	27	68	1,836
Total	175	138	\$22,545	

Table 3B: Passenger and revenue results that would have been achieved with perfect controls is computed by assuming that demand was known exactly prior to departure, so that the ideal discount allocation controls could have been used.

some higher-price passengers being displaced by lower-price demand. The revenue earnings for this case are estimated as the least possible revenue that can be obtained by filling the flight to capacity. Revenue earned for the example flight is shown in Table 2B.

The revenue performance with perfect discount allocation controls: If perfect discount controls are used then the maximum possible revenue is realized for each flight. In this case, the number of seats preserved for the higher-price passengers would exactly match the actual demand. Only the least-price passengers would be turned away (only when no more space is available). The revenue earned for the example flight in this scenario is shown in Table 3B.

Measuring the revenue earned from discount allocation controls: Total revenue opportunity is defined as the difference between the "perfect controls" and the "no controls" scenarios. This difference is the amount of revenue that could possibly be obtained through discount allocations controls. The amount of revenue attributed to the use of discount allocations controls is measured as the difference between the actual revenue and the "no controls" revenue.

Total revenue opportunity through discount controls = Revenue earned in "perfect controls" scenario — Revenue earned in "no controls" scenario = \$22,545 — \$17,429 = \$5,116 for the example flight.

Revenue earned through discount controls

= Actual revenue — Revenue earned in

= Actual revenue - Revenue earned in "no controls" scenario = \$17,429

- \$15,984 = \$1,445 for the example flight. Thus, the percentage of discount allocation revenue opportunity earned is 1,445 divided by 5,116 = 28%.

The system-wide performance is measured as the average performance of the individual flights.

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R. L. Crandall, Chairman, President, and CEO of AMR and American Airlines attested to the value of the work: "I believe

that yield management is the single most important technical development in transportation management since we entered the era of airline deregulation in 1979. . . . The development of yield management was a key to American Airline's survival in the post-deregulation environment. Without yield management we were often faced with two unsatisfactory responses in a price competitive marketplace. We could match deeply discounted fares and risk diluting our entire inventory, or we could not match and certainly lose marketshare. Yield management gave us a third alternative—match deeply discounted fares on a portion of our inventory and close deeply discounted inventory when it is profitable to save space for later booking higher value customers. By adjusting the number of reservations which are available to these discounts, we can adjust our minimum available fare to account for differences in demand. This creates a pricing structure which responds to demand on a flight-byflight basis. As a result, we can more effectively match our demand to supply. . . .

"The development of the American Airline's yield-management system has been long and sometimes difficult, but this investment has paid off. We estimate that yield management has generated \$1.4 billion in incremental revenue in the last three years alone. This is not a one-time benefit. We expect yield management to generate at least \$500 million annually for the foreseeable future. As we continue to invest in the enhancement of DINAMO we expect to capture an even larger revenue premium."