Revenue Management and E-Commerce

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We trace the history of revenue management in an effort to illustrate a successful e-commerce model of dynamic, automated sales. Our discourse begins with a brief overview of electronic distribution as practiced in the airline industry, emphasizing the fundamental role of central reservation and revenue management systems. Methods for controlling the sale of inventory are then introduced along with related techniques for optimization and forecasting. Research contributions and areas of significant research potential are given special attention. We conclude by looking at how revenue management is practiced outside of the airline industry, its relationship to dynamic pricing, and future directions for the discipline.
(*Revenue Management; e-Commerce; Airline Industry*)

1. Introduction

E-commerce is frequently—and incorrectly—equated with the use of the Internet to transact business. While the Internet has unquestionably been at the forefront of the rapid expansion of e-commerce—providing a low-cost, ubiquitous tool for facilitating business transactions—the use of electronic media to engage in the purchase and sale of goods and services is far from new.

The most notable example can be found in the travel and hospitality industries and the airline industry in particular, which pioneered many groundbreaking e-commerce applications for more than three decades. Through the use of central reservation systems, travel and hospitality companies manage the sale of their inventory. Initially, central reservation systems were used simply to record transactions against the sale of physical inventory at a fixed price. Driven by market forces, it quickly became apparent that they could be used for better management of prices and inventory, leading to increased margins with minimal additional capital outlay. Overbooking, demand forecasting taking willingness to pay into account, optimizing the mix of fare products, and doing so dynamically evolved to the point where "rocket science" is now an apt descriptor. Collectively, these practices have come to be known as *revenue* or *yield management*, and they are considered essential in many industries. Testimonials from the business community abound (Sabre 1998, McCartney 2000, Cross 1997).

Here we trace the history of revenue management as practiced in the airline industry in an effort to illustrate a successful e-commerce model of dynamic, automated sales enabled by central reservation and revenue management systems. From our experience, the distribution model embodied in such systems provides a good case study for industries looking to develop an e-commerce strategy. The airline revenue management community has been extremely active over the years, with the first annual meeting of the Airline Group of the International Federation of Operational Research Societies (AGIFORS)—dating to the early 1960s. Since that time, there has been a vibrant intellectual exchange involving industry experts, academicians, and vendors, both through conferences and the inevitable flow of individuals among different companies. As a result, many industry best practices

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have arisen as the result of intense scrutiny. Throughout this paper, we seek to elucidate broadly applicable e-commerce lessons learned over many years. Special attention is given to the more significant research contributions that have helped shape revenue management, and we highlight areas where additional research could provide valuable contributions to the practice of revenue management. We conclude by taking a look at how revenue management is practiced outside of the airline industry, its relationship to dynamic pricing, and future directions for the discipline.

2. The Mechanics of Inventory Control

2.1. Distribution and Central Reservation Systems

Traditional revenue management is intimately related with distribution and central reservation systems. Distribution and central reservation systems represent a broad and fascinating topic in their own right. An excellent high-level account of airline planning, marketing, and distribution activities and their relation to operations research can be found in Smith et al. (2001). We provide only sufficient background information here to facilitate discussion of the main topic of this paper—revenue management.

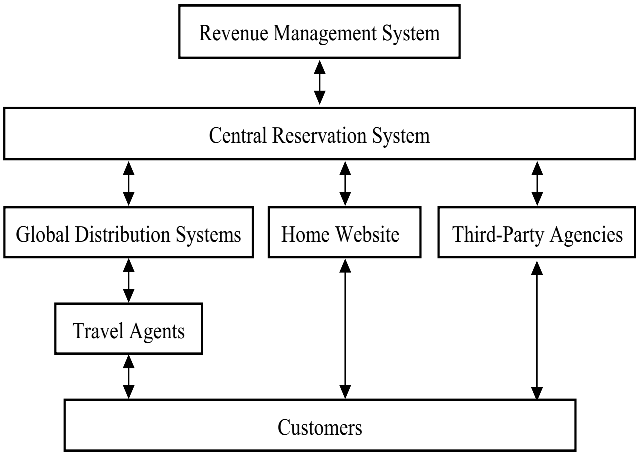
Conceptually, each airline has a central reservation system to record what seat inventory has been sold and what seat inventory is available for sale. If an individual desires to purchase a ticket from city AAA to city BBB on a given date, seat availability is provided by the central reservation system. As pioneering e-commerce efforts, central reservation systems were built using the mainframe technology predominant some 30 years ago. This technology never has been entirely replaced, and the functionality that has evolved on these platforms is expensive to change and maintain. Central reservation systems also need to be highly fault-tolerant, because down time means the sale of inventory effectively comes to a halt. For these reasons, central reservation systems largely remain unintelligent workhorses. The rules, or *inventory control mechanism*, by which inventory is sold are maintained in the central reservation system, but use numerical parameters normally supplied by a separate revenue management system. In §2, we discuss in detail examples of what these parameters are and how they are derived.

Airline tickets are an information good (see Geoffrion and Krishnan 2001), so they can be managed from a centralized source such as a reservation system without concern for their physical location. There is typically no process of distributing seat inventory, such as allocating 10 seats on a flight to one distribution channel and 20 seats to another. Instead, airlines create different products, establish prices for those products for different distribution channels, and then control product availability in the central reservation system.

For example, a carrier might establish a product on the flight leg from city AAA to city BBB called an "M-class ticket" that requires a Saturday night stay and carries a $100 change penalty (a form of *versioning* as discussed in Shapiro and Varian 1998). The price of this ticket nominally might be $300, but tickets sold through travel agency 1 might be worth only $250 after commissions, tickets bought on the carrier's home website might receive a flat 10% discount, and so on. Pricing departments are tasked with establishing the nominal price for each product, sales departments create discounts and commissions as incentives, and revenue management departments have responsibility for controlling the sale of inventory for each different product. If it is economically advantageous, the sale of M-class tickets on the AAA–BBB flight leg can be closed in favor of higher-value products.

The distribution environment faced by airlines is depicted in Figure 1. Among the major distribution channels linked to the central reservation system are global distribution systems, the carrier's home website, and various third-party agencies. The major global distribution systems—Amadeus, Galileo, Sabre, and Worldspan—maintain the product availability of many different airlines at one location and represent the single largest distribution channel through which airlines sell their inventory, with travel agents serving as yet an additional layer between the airlines and consumers. With decades of history, they are an integral, powerful part of the fabric of airline

Figure 1 Distribution Environment Faced by Airlines



and travel distribution. Global distribution systems leveraged themselves into a role as sales intermediary by actively courting travel agents, supplying them with desktop computer access in an era when this was not common. Global distribution systems charge service fees from both the airlines and travel agents. Travel agents are paid commissions by the airlines, charge service fees directly to clients, or both.

Home websites and third-party agencies were spawned by the Internet and offer a distinct cost advantage by predominantly selling directly to the consumer. Home websites bypass the commissions and fees associated with selling through global distribution systems, while third-party sites avoid travel agent commissions and charge competitive fees. Travel agent commissions historically have been the largest distribution cost borne by the airlines. Cost advantages are bringing about a change in the composition of ticket sales, with a migration away from global distribution systems and travel agents.

Third-party sites consist of a hodgepodge of arrangements that are still sorting themselves out in the marketplace, including such well-known entities as Expedia, Hotwire, Orbitz, and Priceline, among others. Travelocity, being owned and operated by Sabre, is best thought of as a Web front end to an existing global distribution system. This distinction is irrelevant to most users, but to Sabre, Travelocity represents an important effort to maintain dominance in the new world of distribution, where the long-term viability of legacy global distribution systems is in question. Orbitz is an interesting third-party site in that it was formed by a coalition of American, Continental, Delta, Northwest, and United in an effort to regain (and, perhaps, exercise) control over distribution by riding the growth of Internet sales.

2.2. Revenue Management Systems

Central reservation systems provided the impetus for the organic growth of revenue management. As a by-product of managing all purchase transactions, they systematically collected data that could be used to infer customer purchasing habits and provided a means to better control product sales once purchasing habits were understood. While the Internet has unquestionably brought about major changes to travel distribution that are still being sorted out in the marketplace, the proliferation of e-distribution channels has not unduly impacted the fundamentals of revenue management. At its core, revenue management tracks historical demand for products and establishes future product availability based on demand forecasts in an effort to maximize revenue. New distribution channels place increased demands on the many systems providing data for revenue management decision making. They may also change the quantity of observed demand for different products and, thus, have a short-term impact on forecasting. Revenue management ultimately focuses on demand for products, however. The net result is that traditional revenue management models are relatively robust to changing distribution environments and are broadly transferable to many e-commerce settings. This is borne out by the authors' experience working in multiple application domains where central reservation systems or their equivalent exist. One area where the changing distribution environment has impacted the practice of traditional revenue management is in the level of information available for decision-making purposes and the level of information that can be communicated to the purchaser, a topic we return to in §7.3.

In all traditional revenue management applications, four main elements dominate the design of a system: (1) the inventory control mechanism, (2) optimization, (3) demand model and forecasting, and

(4) interaction with users of the revenue management system. (Note that we depart from the classification provided by McGill and van Ryzin 1999, who sought to categorize areas of mathematical development.) All of the elements are important, and decisions related to one impact the others so that a joint or iterative design scheme is warranted. However, a natural hierarchy exists because the control mechanism drives what optimization algorithms can be employed, and the inputs to the optimization algorithm drive the demand model and what is forecast. User interaction usually is designed as a final step, though it has a high-level impact early in the design of a system. If optimization and/or demand models are so complicated and foreign to a user that she will not use the system, they spell failure from the outset.

The four elements of a traditional revenue management system are quite general in concept and share many characteristics from one application domain to another. We focus on the development of revenue management in the airline industry in an effort to provide concrete examples that illustrate more general e-commerce lessons. Airline revenue management is chosen as the illustrative vehicle because it has by far the most developed literature, thus, providing an opportunity to tie the discussion to a broad collection of publicly available references. Also, as a mature practice, airline revenue management continues to provide much of the market leadership in revenue management.

The remainder of this paper is organized as follows. Section 3 focuses on the first two elements of revenue management, inventory control mechanisms and optimization. Two different approaches to inventory control are introduced and serve to frame the discussion throughout this section. Section 4 addresses the third element of revenue management—forecasting—including unconstraining and demand models. The relationship of dynamic pricing to revenue management is briefly touched on in §5, while §6 discusses applications of revenue management outside of the airline industry. Section 7 comments on the future of revenue management, and concluding remarks are presented in §8.

3. Optimization and Control

3.1. Inventory Control Mechanisms

The inventory control mechanism is arguably the single most important element of revenue management. To better appreciate the role of the inventory control mechanism, consider the predominant method used within the airline industry—leg and class control. Under leg/class control, airlines establish a collection of products known as booking or fare classes that are designated by a single alphabetic character. A hypothetical airline with a particularly simple inventory structure might have four booking classes—Y, M, B, and Q—with Y-class tickets carrying no purchase or return restrictions and a high price, nonrefundable Q-class tickets requiring a Saturday night stay but a relatively low price, and M- and B-class tickets carrying intermediate restrictions and prices. For each flight leg, the revenue management system determines the number of tickets that can be sold in each class. A customer can then book a Q-class ticket on flight 111 connecting to flight 222 if there is Q-class availability on each flight leg. If sold, the number of available Q-class seats and also total seats are decremented on flights 111 and 222.

Although not universal, this basic inventory control structure is so prevalent that the entire language of the airline industry revolves around it. If, for example, an airline establishes a deal with a new distribution channel such as a third-party website, that site will be quoted a price for the different classes of inventory and different markets and then draw upon the available inventory associated with the class. In practice, deals can be far more complicated, but the basic concept of using booking classes as the basis for the deal is extremely widespread.

The shortcoming of class inventory control—and one that airlines themselves have been slow to recognize—is that it fails to correspond to the products that airlines actually sell. Looked at another way, it fails to distinguish between *resources* and *products*. A passenger booking a ticket that makes use of flights 111 and 222 is not purchasing two separate flight-leg products, but a single product that consists of the use of these two legs. A Y-class ticket on flight 111 has one fare, a Y-class ticket on flight 222 has another fare, a Y-class ticket using both flight legs has a third fare, and the sum of the first two fares rarely equals the third. The existence of three different fares arises from the fact that the airline offers three different products. Yet leg/class control does not recognize this distinction and, as a result, negatively impacts revenues by setting an upper bound on what even the best algorithms can achieve.

A simple example drives home this point. Consider an airline that offers two flight legs, one from city AAA to BBB and one from city BBB to CCC. Assume further that there is only one seat remaining on each flight leg and that, in both instances, it is allocated to fare class Y. Finally, assume there are two remaining passengers—one wanting to fly only the AAA–BBB leg at the $100 Y-fare, and the other wanting to fly both legs AAA–BBB and BBB–CCC at the $300 Y-fare. Clearly, the optimal strategy is to block the sale of inventory to the passenger desiring the AAA–BBB leg, while simultaneously making the legs available to the passenger desiring to use both legs. Leg/class control cannot achieve this. As a control mechanism, it is fundamentally flawed.

While this example is purposely oversimplified—ignoring such issues as variability in demand forecasts, the potential for cancellations, better alignment of network fare structures, and other issues—the essential message withstands analysis: The inventory control mechanism has a critical impact on revenues. Although airlines are reluctant to share the information, public domain industry simulations demonstrate that control mechanisms other than leg/class control commonly lead to revenue improvements of 1%–2% and higher, depending on the nature of the network and the demand (Feldman 1990, Weatherford 1991, Smith et al. 1992, Davis 1994, Wilson 1995, Belobaba and Wilson 1997, Zickus 1998). While many of the results are presented in the context of the best *algorithm* for controlling inventory (and the choice of algorithm is certainly an important factor), the control mechanism itself is more fundamental as it limits *which* algorithms can be used to control inventory.

Though the failings of leg/class control are not difficult to comprehend, there has been much debate surrounding the best way to overcome them. Half of the debate has focused on which control mechanism and algorithm should replace leg/class control, and goes under the broadly encompassing name of origin/destination control. The other half has focused on concerns about whether the proposed control mechanisms and algorithms will yield revenue improvements sufficient to warrant the time, cost, and effort required to overcome the practical issues involved in implementing them. Industry leaders have generally opted to move forward, believing that the revenues and competitive advantage associated with alternative control mechanisms are worth the investment.

It is difficult to overstate the importance of the control mechanism for managing inventory in e-commerce settings. On the one hand, the inventory control mechanism represents the means by which the results of forecasting and optimization algorithms are actually used to control inventory. On the other hand, it defines the business processes surrounding the sale and distribution of inventory. Changes to an established control mechanism may offer higher-revenue potential, but the business process changes cannot be ignored. From the perspective of operations research, understanding the existing inventory control mechanism and how it fits in the broader distribution environment is vital to designing forecasting and optimization algorithms that make economic sense.

Experience with inventory control mechanisms provides an important lesson for e-commerce. If next generation central reservation systems are not designed around a good inventory control mechanism when they are first deployed in a new industry, revenue that could otherwise be captured will be lost in the short term and, in the long term, tremendous time and expense will be devoted to working around the problems encountered with an entrenched mechanism. If, for example, leg/class control had been circumvented altogether in favor of one of the origin and destination control methods of §3.3 when central reservation systems were first designed, the revenue impact on airlines over the years would have been staggering. While this observation is not to suggest that it is necessary to foresee the future and design perfectly from the outset, it is to suggest that the control mechanism chosen by an industry expert and an industry expert with a background in operations research are likely to be quite different.

A final observation regarding inventory control mechanisms is their natural propensity to grow ever more complicated. Driven by anecdotal instances where a control mechanism fails to provide a sufficient level of control to make the revenue-maximizing set of decisions, central reservation systems quickly can become a repository for complicated decision rules. The more complicated the control mechanism, the more likely there are unforeseen interactions among the rules, which may, in turn, lead to an even more complicated control mechanism. While operations research practitioners are comfortable with decisions under uncertainty and maximizing expected revenue, business practitioners facing oversold or unsold inventory want to know why they occurred, and to fix the problem without fully comprehending additional problems that may be introduced. The essential e-commerce lesson for operations research is to remain diligent in keeping the control mechanism uncomplicated so that it can be effectively modeled and supported.

3.2. Leg/Class Control

Because leg/class control has long been the primary method for controlling inventory in the airline industry, it established the framework for virtually all of the early work in revenue management. Within the context of leg/class control are two well-known variants. Under *partitioned* control, the seats available in each leg/class are independently maintained, while under *nested* control, all inventory requests draw upon a common seat inventory, but with each class given access to increasingly greater inventory than the class of immediately lower value. Maintaining these nested *booking limits* has the advantage that it is not possible for a low-value class to be open while a high-value class is closed, something that can potentially occur with partitioned inventory.

Examples of partitioned and nested control for a flight leg are shown in Tables 1 and 2. For nested inventory control, the booking limit represents the total number of seats that may be booked in a given class. A table similar in concept to that shown in Table 2 is actually maintained on the central reservation system for each flight leg, with the revenue management system periodically updating the booking limits as a result of its forecasting and optimization activities. A detailed description of partitioned and nested control can be found in Curry (1990).

Table 1 Partitioned Inventory Control

	Booking limit	Accepted bookings	Still available
Class Y	20	20	0
Class M	35	25	10
Class B	20	20	0
Class Q	25	20	5

Although partitioned inventory control is almost nonexistent today, the differences between partitioned and nested control established the framework for driving some of the more important early results in optimization. Consider the following three assumptions made in the early development of revenue management:

Assumption 1. *Demand for class i is described by a random variable X_i.*

Assumption 2. *Demand is independent among fare classes.*

Assumption 3. *Class i has fare r_i for $i = 1, \ldots, M$, with $r_i > r_j$ when $i < j$.*

These assumptions underlie much of the research in revenue management and, in particular, all of the work described in §3.

It is easily demonstrated that the conditions for an optimal partitioning of inventory among classes is

$$r_i \cdot \Pr(X_i \geq A_i) = \text{constant} \quad \text{for } i = 1, \ldots, k \quad \text{and} \quad 0 \quad \text{for } i > k, \tag{1}$$

Table 2 Nested Inventory Control

	Booking limit	Accepted bookings	Still available
Class Y	100	20	15
Class M	80	25	15
Class B	45	20	5
Class Q	25	20	5

where A_i represents the number of seats allocated to class i, and k is the cutoff point where the price becomes inadequate for selling. Note that $r_i \cdot \Pr(X_i \geq A_i)$ is simply the *expected marginal revenue* associated with assigning an additional seat to class i, so that condition (1) is a statement about partitioning inventory among classes so as to balance marginal revenues. Forecasting under these assumptions consists of estimating the parameters defining the random variables X_i, which represent the *total demand to come* for each class.

When inventory is partitioned among classes and class demand is independent, there is no need to make any assumptions regarding the order of arrivals. Arriving demand takes inventory from its appropriate class, and this has no impact on the available inventory for other classes. This is not the case for nested control, because the sale of a seat in class i impacts the availability of seats in other classes. As a result, determining optimal booking levels in a nested control environment forces to the surface the issue of how the demand for different classes arrives to define and solve an optimization problem.

The earliest demand assumption was that customers arrive in the order of lowest- to highest-value class. Unrealistic as this assumption may seem, it reflects the prevailing belief that overall customer willingness to pay increases later in the booking cycle, as business travelers tend to have shorter planning cycles than leisure travelers. Also, this assumption respects the early forecasting archetype of estimating the total demand to come during the booking cycle. Analyses of the low-before-high arrival model, which are generally extensible to the case of class demand arriving in nonoverlapping intervals, can be found in Brumelle and McGill (1993), Curry (1990), Robinson (1995), and Wollmer (1992). With the exception of Robinson (1995), all of these published references were preceded by presentations or technical reports dating from 1988 (Brumelle and McGill 1988, Curry 1988, Wollmer 1988). Curry (1988) referred to the approach as a method for generating optimal booking limits, though no name has been universally accepted.

Using a highly-accessible graphical presentation, Belobaba (1987) proposed an alternative approach for establishing booking limits for nested class control that he named the *expected marginal seat revenue* (EMSR) method. In practice, the EMSR method does not protect higher-valued classes as aggressively as the optimal booking limit method, though Brumelle and McGill (1993) provided a counterexample to a specific conjecture that the two highest-value classes will always be underprotected. Both Wollmer (1988) and Robinson (1995) concurred that while the booking limits generated by the EMSR and optimal booking limit methods could significantly differ, the optimal booking limit method only marginally outperforms the EMSR method. Robinson (1995) specifically concluded that "the additional revenues generated by the [optimal booking limit method] may only rarely justify using the substantial numerical integrations required. But by ignoring the fact that future fare classes are also nested, [the EMSR] heuristic can give rise to policies which are arbitrarily bad" (p. 258). In keeping with Robinson's (1995) conclusions, the optimal booking limit method was never broadly adopted and put into practice. Belobaba (1992) introduced a new heuristic called EMSRb that emulates the optimal booking limit method, tending to provide greater protection for higher-valued fare classes than its predecessor. It has also proven to work well in practice and has become an industry standard for establishing leg/class control limits.

The works by Brumelle and McGill (1993), Curry (1990), Robinson (1995), and Wollmer (1992) were important in that they resolved the basic issue of finding optimal nested booking limits under the existing forecasting archetype of estimating the total demand to come. However, these works were equally important in bringing to the forefront limitations of the predominant models for demand arrivals. Further progress required additional, more realistic assumptions regarding the underlying arrival processes.

A natural next step is to assume that arrivals for each class are generated according to independent Markov processes. Innocuous as this alternative demand model may appear, it carries with it fundamental changes to the way the revenue management problem is conceived. These conceptual differences, in turn, manifest themselves in disagreement about the way revenue management should be practiced.

The import of the alternative demand model can best be understood in the context of the accompanying optimization model—most commonly, dynamic programming. In the most basic dynamic programming model, time until departure is discretized to form stages for the dynamic program, and the states within each stage correspond to the number of bookings for a given flight. State transitions correspond to the arrival of demand from a class (or no arrival), with transition probabilities determined from the forecast arrival rates. Dynamic programming models have the added advantage of easily incorporating cancellations, no-shows (passengers that fail to arrive on the day of departure), go-shows (passengers that arrive on the day of departure without prior reservations), and other phenomena within the optimization model itself. The optimal booking limit and EMSR methods rely on overbooking to first establish a virtual capacity that accounts for such phenomena, and then calculating booking class limits for the virtual capacity.

McGill and van Ryzin (1999) provide an extensive chronological list of dynamic programming research related to leg/class optimization (Stone and Diamond 1992; Sun 1992; Lee and Hersh 1993; Shaykevich 1994; Young and van Slyke 1994; Brumelle and Walczak 1997; Zhao and Zheng 1998a, b; Lautenbacher and Stidham 1999; Subramanian et al. 1999; Zhao 1999). While a handful of conference presentations (Mayer 1976) and some indirectly related work (Rothstein 1971, Ladany and Bedi 1977) reveal an awareness of the applicability of dynamic programming methods prior to 1992, the lack of research publications prior to this date demonstrates that dynamic programming simply was not part of the mindset of the revenue management community. Even today, dynamic programming has not been widely embraced by revenue management practitioners for leg/class control.

The computational aspects of dynamic programming bring to the surface a mathematically natural alternative to leg/class control. Arising directly from the dynamic programming recursion, if at a given time there are k seats in inventory, a seat should be sold if the immediate revenue received from selling the seat plus the expected revenue generated from selling the remaining $k-1$ seats in the future exceeds the revenue from holding the seat and selling k seats in the future. The immediate implication is that there is a price point referred to as the *bid price* such that any class with a sales price above the bid price should be sold, and any class with a sales price below the bid price should not. By the nature of dynamic programming, a bid price is generated for every inventory level and point in time, providing a dynamic control mechanism. Though there are a number of excellent ways to generate nested booking class limits from the output of a dynamic programming algorithm, the optimization process is not nearly as transparent for users as, for example, EMSR heuristics.

Beyond user transparency, dynamic programming has likely received little attention in practice because the potential revenue benefits are largely negated by the limitations of leg/class control to begin with. There is no escaping the fact that under leg/class control, a huge variety of itineraries and fares draw upon the same inventory, and detailed improvements on class inventory levels can only go so far toward improving revenues. Origin/destination control largely alleviates this fare and inventory problem, making dynamic programming a more relevant alternative.

An indirect objection to dynamic programming comes with the associated shift in demand modeling from total demand to a (memoryless) stochastic process framework. Business practitioners conceptualize demand in terms of the number of units of inventory sold and its variability, not in terms of arrival rates and interarrival distributions, and this can lead to confusion. Consider the following example. A plane has 100 seats and there are 50 high-value Y-class passengers and 50 lower-value Q-class passengers. How does algorithm A and its associated control mechanism handle this situation? Any reasonable method for calculating nested booking class limits will set aside 50 Y-class seats and 50 Q-class seats. However, for dynamic programming, the problem is ill-posed because total demand is specified rather than arrival rates; and making the assumption that the arrival rates over the remaining time interval of duration T are both $50/T$ does not guarantee that the bid price (or set of bid prices) will accept exactly 50 Y- and

50 M-class passengers. This behavior is absolutely correct when modeling demand as a stochastic process. Yet, it still may not satisfactorily answer the straightforward question posed by the business practitioner.

The two different perspectives on demand modeling pose a dilemma for e-commerce applications. As evidenced by the evolution of leg/class revenue management, defining the underlying optimization problem requires that assumptions be made regarding arrival processes. Yet, business practitioners tend to conceptualize their businesses in terms of units of inventory sold. While there is no uniform solution to this dilemma, awareness on the part of operations research practitioners can help alleviate confusion and disagreement as it arises.

3.3. Origin/Destination Control

While origin/destination control can be viewed as an effort to overcome the limitations imposed by leg/class control, the driving force behind the move to origin/destination control can be found in revenue improvements demonstrated by operations research models. Like leg/class control, a gap exists between academic journal publications and industry practice, with conference proceedings and technical reports filling part of the gap (see McGill and van Ryzin 1999). As evidenced by these works, two control methods have come to dominate the discussion of origin/destination control: (1) *virtual nesting*, and (2) *bid price* methods.

Virtual nesting is a creative solution to origin/destination control that closely mimics leg/class control. It evolved as a response to the primary deficiency of leg/class control, namely, using the same inventory availability to control the sale of products with entirely different values.

To illustrate this point, consider once again the example of §3.1 in which there are two flight legs AAA–BBB and BBB–CCC with one remaining seat each, one passenger willing to pay the $100 Y-fare on leg AAA–BBB, and one passenger willing to pay the $300 Y-fare using both legs AAA–BBB and BBB–CCC. Under leg/class control, availability can only be established by leg/class, and it is impossible to block sale to the $100 Y-fare while leaving availability open to the $300 Y-fare. The optimal leg/class booking limits are shown in Table 3.

Table 3 **Nested Class Booking Limits for Example**

	Leg AAA–BBB booking limit	Leg BBB–CCC booking limit
Class Y	1	1
Class M	0	0

Virtual nesting avoids this problem by establishing "virtual buckets" of inventory that are based on value rather than class. Each leg in an itinerary and class is then assigned to a virtual bucket from which it draws inventory, where an *itinerary* is any connecting collection of flight legs. In this particular example, two virtual buckets might be established as in Table 4—a high-value bucket 1 and a low-value bucket 2—with inventory made available only in bucket 1 on both legs. If the $100 Y-fare is assigned to draw inventory from bucket 2 on leg AAA–BBB, and the $300 Y-fare is assigned to draw inventory from bucket 1 on leg AAA–BBB and bucket 1 on leg BBB–CCC, the $300 Y-fare can be sold while the $100 Y-fare cannot.

Virtual nesting requires that virtual buckets be established on each leg, and that each leg in each itinerary and class be mapped to a virtual bucket. While bucket assignments are made on value, the process is not as simple as assigning high-priced classes to high-value buckets. Suppose there is a third passenger in our example who is willing to purchase a $250 Y-fare that uses only the leg BBB–CCC. In this case, the optimal solution is to block the $300 Y-fare in favor of the $100 and $250 Y-fares. Again, this cannot be achieved with fare class control. It can be achieved with virtual nesting using the inventory availability shown in Table 4, but requires that the $100 Y-fare be assigned to bucket 1 on leg AAA–BBB, the $250 Y-fare to be assigned to bucket 1 on leg BBB–CCC, and the

Table 4 **Nested Virtual Bucket Booking Limits for Example**

	Leg AAA–BBB booking limit	Leg BBB–CCC booking limit
Bucket 1	1	1
Bucket 2	0	0

\$300 Y-fare to have at least one of its legs assigned to bucket 2.

Different methods have been proposed for mapping itinerary/class/legs to virtual leg/buckets. One of the most popular methods operates by first breaking down itinerary and class fares to leg-adjusted fares, and then assigning the itinerary/class/leg to a bucket based on its leg-adjusted fare. For example, the \$300 Y-fare using legs AAA–BBB and BBB–CCC might have a leg-adjusted fare of \$50 for the AAA–BBB leg and \$150 for the BBB–CCC leg. With leg-adjusted fares established, virtual buckets can be based on leg-adjusted fare ranges, and each leg of an itinerary mapped to a leg/bucket based on the leg-adjusted fare range into which it falls.

Continuing with our example, suppose that the virtual buckets and their fare ranges are as shown in Table 5, and suppose that fare adjustment for the \$100 and \$250 Y-fares leaves their leg-adjusted fares unchanged. In this case, the \$100 Y-fare using leg AAA–BBB would map to bucket 1 on leg AAA–BBB, the \$250 Y-fare using leg BBB–CCC would map to bucket 1 on leg BBB–CCC, the \$50 AAA–BBB leg-adjusted fare for the \$300 Y-fare would map to bucket 2 on leg AAA–BBB, and the \$150 BBB–CCC leg-adjusted fare for the \$300 Y-fare would map to bucket 1 on leg BBB–CCC. Note that different fare ranges or different leg-adjusted fares could lead to different itinerary/class/leg mappings, and that nested inventory levels can be calculated for the different buckets using the leg/class (leg/bucket) optimization algorithm of choice.

Many different methods for calculating leg-adjusted fares have been proposed and discussed, and this calculation is critical to realizing revenue gains from virtual nesting. The most successful methods use dual information from a mathematical program on the flight network to estimate expected leg displacement costs and, hence, to adjust fares. Smith and Penn (1988), Williamson (1992), and Wei (1997) represent three publicly available references that discuss such methods.

Table 5 Leg-Adjusted Fare Ranges for Virtual Nesting Example

	Leg-adjusted fare range	Leg AAA–BBB booking limit	Leg BBB–CCC booking limit
Bucket 1	\$75–\$300	1	1
Bucket 2	\$0–\$75	0	0

Although the origins of virtual nesting are rooted in a heritage of leg/class control, properly implemented, it has a sound mathematical foundation. To understand this foundation as well as to provide an example of how leg-adjusted fares might be calculated, consider the following linear program that has been studied and used in many different contexts within the realm of revenue management, including, but not limited to, Glover et al. (1982), Alstrup et al. (1986), Wollmer (1986), and Talluri and van Ryzin (1999):

$$
\begin{aligned}
\max\ & \sum_{i\in I} r_i x_i \\
\text{s.t.}\ & \sum_{i\in I(l)} x_i \le c_l, \qquad l\in L \quad (\lambda_l), \\
& x_i \le d_i, \qquad i\in I \quad (\omega_i), \\
& x_i \ge 0, \qquad i\in I.
\end{aligned} \tag{P}
$$

Here, I represents the set of itinerary and class combinations, and the variables x_i represent the amount of demand that should be accepted from this itinerary and class when sold at fare r_i. The problem is to choose the demand that maximizes revenue while not exceeding the capacity of any flight leg or accepting more demand than exists. L represents the set of flight legs in the network, $I(l)$ denotes the set of itinerary and classes that include flight leg l, and c_l and d_i, respectively, denote the capacity of flight leg l and the expected demand for itinerary and class i. The problem (P) is a resource allocation model, or more appropriately, a *demand* allocation model.

Introducing auxiliary variables $x_{il} = x_i$ representing the amount of demand allocated to itinerary and class i on leg l, (P) can be written in the equivalent form

$$
\begin{aligned}
\max\ & \sum_{i\in I} r_i x_i \\
\text{s.t.}\ & \sum_{i\in I(l)} x_{il} \le c_l, \qquad l\in L \quad (\lambda_l), \\
& x_{il} \le d_i, \qquad i\in I,\ l\in L(i) \quad (\omega_{il}), \\
& x_{il} - x_i = 0, \qquad i\in I,\ l\in L(i) \quad (\pi_{il}), \\
& x_{il} \ge 0, \qquad i\in I,\ l\in L(i),
\end{aligned} \tag{P'}
$$

where $L(i)$ is the set of flight legs comprising itinerary i. The Lagrangian obtained by relaxing the constraints $x_{il} - x_i$ and reducing using Lagrange multipliers π_{il} is

$$\begin{aligned} v(\pi) = \max \ & \sum_{i\in I} \sum_{l\in L(i)} \pi_{il} x_{il} \\ \text{s.t.} \ & \sum_{i\in I(l)} x_{il} \le c_l, \qquad l \in L \\ & x_{il} \le d_i, \qquad i \in I,\ l \in L(i), \\ & x_{il} \ge 0, \qquad i \in I,\ l \in L(i), \end{aligned} \tag{L}$$

while the Lagrangian dual is

$$\begin{aligned} \min_{\pi} \ & v(\pi) \\ \text{s.t.} \ & r_i - \sum_{l\in L(i)} \pi_{il} = 0, \qquad i \in I, \end{aligned} \tag{D}$$

where the constraints in (D) come from necessary conditions arising in the Lagrangian for it to be bounded (Boyd 2000).

Taken together, (P′), (L), and (D) show how Lagrangian techniques can be used to decompose the network problem (P) into leg-level subproblems, with leg-adjusted itinerary and class fares arising as a result. The Lagrangian (L) is separable in each leg l, with each subproblem having the same form as (P), but with a single flight leg instead of the entire network. The fare for each itinerary and class is a leg-adjusted fare π_{il} that differs for each leg subproblem. Interestingly, the constraints in (D) ensure that for each itinerary and class i, the set of leg-adjusted fares π_{il} for each leg sum to r_i; that is, the leg-adjusted fares represent a true proration of the itinerary and class fares to the legs comprising the itinerary. As previously mentioned, in the context of virtual nesting, the leg-adjusted fares are used as input to leg-level optimization algorithms other than the linear programs found in (L).

Observe that the linear program (P) and the subsequent problems (P′), (L), and (D) are all dependent upon the capacity of each flight leg and the expected demand for each itinerary and class i. This implies that the leg-adjusted fares change as the values and c_l and d_i change. As a result, in the most dynamic form of virtual nesting, itinerary/class/legs are remapped to different leg/buckets on an almost continuous basis. Less dynamic forms of virtual nesting are often used in practice.

The major competing origin/destination methodology to virtual nesting is based on bid prices and provides an equally creative, but entirely different perspective on inventory control. Like its leg cousin, bid price control establishes a price point for each leg. A class is available for sale if the fare exceeds the sum of the bid prices on the legs comprising the itinerary; otherwise it is not. The most effective forms of bid price control come from the solution of mathematical models involving the entire flight network. Exercising the same reasoning employed in virtual nesting, the dual variables λ from (P) are frequently proposed as bid prices (Simpson 1989, Williamson 1992, Wei 1997, Talluri and van Ryzin 1998), though there are many alternatives. A particularly notable method due to its similarity with virtual nesting is the use of dynamic programming to solve leg-level subproblems arising from the decomposition of a network-level mathematical program.

Both virtual nesting and bid price control show clear revenue improvements over leg/class control when properly implemented. Which method generates the greatest revenue improvement is dependent on the algorithm, the assumptions made in simulating the different methods, and the specific problem under consideration. While not supplying definitive answers, experimental simulations by Belobaba and Hopperstad, and contributing airline participants (Belobaba 1996, 1997, 1998, 1999, 2000; Hopperstad 1996a, b, 1997a, b) have provided extensive insight into the general behavior of these methods. As a rule, both show similar revenue improvements and behave well under adverse conditions. The authors of this paper are of the opinion that the best bid price methods demonstrate a consistent advantage over virtual nesting methods, but it is unlikely that universal consensus will ever be reached on the issue.

The debate between advocates of virtual nesting and advocates of bid price control is often heated and provides many valuable insights into the strengths and weaknesses of the two different approaches that extend beyond revenue improvement. It also elucidates many key issues that will likely arise in scores of different e-commerce settings.

Virtual nesting is an attractive control mechanism because of its similarity to leg/class control. Users continue to work with nested inventory levels, and the number of virtual buckets is typically on the order of the number of classes so that the quantity of inventory data maintained by the central reservation system does not significantly increase.

Nonetheless, virtual nesting has significant drawbacks. Users must grapple with the fact that the products, which continue to be priced by itinerary and class, map to different virtual inventory buckets. In the most effective form of virtual nesting, a product such as an M-class ticket from AAA to CCC through city BBB may draw on different inventory buckets on the different legs, adding to user confusion. A table must be maintained by the central reservation system that maps each itinerary/fare class to its appropriate legs and buckets, and this may require a large amount of data depending on how the mapping is handled. (According to Smith and Penn 1988, the name *virtual nesting* derives from the fact that the mapping can be handled virtually.) To reap the full benefit of virtual nesting, the mapping of itinerary/fare classes to legs and buckets must change at least periodically, and preferably dynamically. Changing buckets can be a source of confusion to users. Changing buckets can also be a source of confusion to forecasting systems that use virtual bucket booking levels as their primary source of historical data, because what constitutes bucket demand changes as buckets are redefined. Virtual nesting is difficult to translate to inventory classes with multiple attributes such as cargo, a problem not shared by bid price methods (Karaesmen 2001).

The attractiveness of bid price control stems from its simplicity. Most of the complications associated with virtual nesting derive from the mapping of itinerary/class/legs to virtual buckets. Bid price control avoids these complications by doing away with buckets and replacing them with a bid price.

The single biggest drawback to bid price methods is user acceptance. Nested booking limits for legs and classes have embedded themselves in the culture of airline inventory control over a period of decades. Bid prices are a radical departure in that they do away with physical nested booking limits in favor of a yes or no price—trading a primal control for a dual control.

Revenue management systems operate by periodically running forecasting and optimization algorithms to refresh control parameters in the central reservation system, and concerns have been voiced on the best way to update bid prices in the interim period between refresh points. Virtual nesting provides interim updates by decrementing the number of units of inventory in a particular bucket when a seat is sold. Early bid price methods used a range of bookings to trigger refreshing the bid price, e.g., while the number of bookings on leg AAA–BBB is in the range $[72, 84]$, use a bid price of \$150, otherwise request a new bid price from the revenue management system (Williamson 1992). If the dual variables λ in (P) are used as bid prices, for example, such ranges might come from linear programming sensitivity analysis. As borne out by experience, the difficulties with this approach are the amount of communication between the central reservation and revenue management systems and the number of mathematical programs that may need to be solved. More recent bid price methods alleviate these difficulties by providing a vector of bid prices to the central reservation system with a different bid price for each level of bookings.

4. Forecasting

Forecasting for revenue management purposes has not achieved a degree of industry consensus allowing for a detailed synthesis of existing research and practice. In this section, we touch on forecasting methods found in the literature before turning to the topics of unconstraining, demand modeling, and some of the more interesting general questions that present themselves in the context of origin/destination forecasting. For a more extensive overview of the forecasting literature, readers are strongly encouraged to see McGill and van Ryzin (1999).

4.1. Methods

One of the first published works dealing with revenue management forecasting was the paper by Littlewood

(1972). Littlewood studied models to forecast demand on a daily basis using bookings data, thus showing some of the important uses that can be made of reservation system data to contribute to the "short-term control of the production and distribution" of aircraft capacity. Duncanson (1974) sought to provide more accurate forecasts through a more thorough statistical analysis and providing a consistent, uniform base forecast on which subjective evaluations could be made. The paper describes then-current work at Scandinavian Airlines on short-term forecasting using seasonal analysis and exponential smoothing, the latter being considered at that time to yield more accurate forecasts than methods in place at Scandinavian Airlines. Adams and Vodicka (1987) presented their work on forecasting methodologies for the period within seven days of departure being investigated at Qantas. Various applications are described and performance is discussed in terms of both predictive accuracy and practical usefulness.

Ben-Akiva (1987) presented results for three flight-specific, class-specific forecasting models: a regression model for advanced bookings, a time series model for historical bookings, and a combined advanced bookings and historical bookings model. His research did not account for the effects of booking limits, and monthly data were used. At a lower level of aggregation, Sa (1987) addressed time series and regression models using advanced bookings, a seasonal index, a day of week index, and an historical average of bookings to come as explanatory variables. As a result, he found that regression models performed better than time series models. A particularly notable work in airline reservations forecasting is the Ph.D. thesis of Lee (1990), which proposes a classification that clearly delineates forecasting models with respect to their level of aggregation: macrolevel forecasting at the most highly-aggregated level; microlevel forecasting at the intermediate level of predicting demand by flight, date, and fare class at a given point in time (the level typically used by leg/class revenue management algorithms); and choice modeling for the study of a single individual's behavior based on socioeconomic factors, characteristics of alternative travel options, and so on.

In his master's thesis, Svrecek (1991) discussed the issues involved with trying to characterize the stochastic nature of airline group passenger demand, and identified the primary elements of variability associated with it. These primary elements of demand were used to develop a mathematical model for the distribution of group passengers on a given flight. Another master's thesis, that of Wickham (1995), conducted an evaluation of the relative performance of selected forecasting techniques used to predict demand within eight weeks of the date of departure. The selected techniques were representative of practices in the airline industry at that time, including simple time series, linear regression, and booking pickup models. Wickham's (1995) studies showed that booking pickup models consistently outperformed the time series and regression models, and that advanced pickup models produced the best results.

With the objective of examining the predictive ability of air travel demand models, Karlaftis et al. (1996) proposed an analytical framework for developing econometric models. They also examined the effect of external factors such as the deregulation of the air transport industry. Their results suggest that simple models with few independent variables perform as well as more complicated and costly models, and that external factors have a pronounced effect on air travel demand.

In the context of forecasting hotel demand, Baker and Collier (1999) looked at data availability, data accuracy, forecast ability, computer capability, and user understanding as important criteria in the selection process for the heuristics they studied. They also suggested the necessity of comparing alternative demand forecasting methods within a simulation environment. Concerning group forecasting accuracy, Kimes (1999) gathered forecast data from approximately 90 hotels of a large North American hotel chain and used it to determine the accuracy of group forecasts, and to identify factors associated with accurate forecasts. The results of her research state that larger hotels, hotels with a higher dependence on group business, and hotels that frequently updated their forecasts during the month before arrival have more accurate forecasts. Weatherford et al. (2001)

looked at what level of aggregation to perform forecasts. Their study showed that purely disaggregate forecasts strongly outperform even the best aggregate forecasts, concluding that "even though forecasting larger numbers may be more accurate in itself, the process required to [disaggregate these forecasts] resulted in lower accuracy than just forecasting at the more detailed level in the first place." This latter result touches on questions discussed in §4.4.

4.2. Unconstraining

A particular problem faced in revenue management applications is that of censored data. Central reservation and revenue management systems record the sale of inventory, so that if a product is closed for sale, no sales will be observed. If sales history is used as the basis for forecasting future demand, the forecasts will underestimate actual demand if unobserved demand is not accounted for. Unconstraining is the process of generating true demand history from sales history. Though it is possible to develop forecasting algorithms that account for censored data without explicitly unconstraining historical observations, unconstraining has proven by far the most popular technique in revenue management applications.

Unconstraining is not unique to revenue management, and many papers and books have been written on the topic, including Tobin (1958), Hartley and Hocking (1971), Dempster et al. (1977), Little (1982), Little and Rubin (1987), and Zeni (2001). The significance of unconstraining is well appreciated by revenue management practitioners, because underestimating demand can lead to an undervaluation of inventory.

Common to almost all unconstraining approaches proposed for airline revenue management is the assumption that demand for different fare classes is independent. A notable exception is the work of McGill (1995) who discusses unconstraining when correlation exists. In his paper, McGill introduces the expectation maximization method of Dempster et al. (1977) to the revenue management community. McGill's (1995) work also demonstrates how quickly models that incorporate correlated demand can become computationally intractable.

The assumption of demand independence first discussed in §3.2 makes the unconstraining process relatively straightforward. Zeni (2001) provides a categorization of unconstraining techniques and presents extensive computational results for airline revenue management applications. A considerable difficulty when evaluating unconstraining techniques in practice is that the values being estimated are unknown—even for dates in the past. If method A generates a value x and method B generates a value y, which is a better estimate of true demand? Using randomized methods to constrain known true demand is equally thorny, because the best unconstraining methods are those designed to undo the process by which the demand was constrained in the first place. Thus, to a large extent, unconstraining consists of adopting a demand model and choosing an unconstraining method that is theoretically appropriate for the chosen demand model.

4.3. Demand Models

Unconstraining explicitly surfaces the much larger question of what represents the "best" demand model. Certainly, the demand in different fare classes is not independent. The important question is if more sophisticated models actually yield better forecasts and, if so, can they be effectively implemented in a real-time operational environment. Equally important is whether users will accept more sophisticated models.

The interdependence of demand among fare classes has long been recognized by practitioners and dealt with in a variety of ways. Among the more significant published research efforts are those that have focused on modeling consumer choice behavior, including Andersson (1998), Algers and Besser (2001), and Talluri and van Ryzin (2002). In particular, Talluri and van Ryzin (2002) establish a general theoretical framework for addressing revenue management from a consumer choice perspective.

Boyd et al. (2001) analyzed a simple consumer choice model based on consumers purchasing the lowest available fare class independent of their actual willingness to pay. One of their key findings was that good demand estimates are difficult to generate without setting nonoptimal inventory controls for the sole purpose of gathering information about demand and its willingness to pay, raising the interesting theoretical and practical question of how to best balance the needs of forecasting and optimization algorithms. As revenue management is now practiced, forecasting algorithms simply make use of whatever bookings are generated as a result of using optimal controls—optimal, of course, given the forecasts provided to the optimization algorithm. While consumer choice models offer tremendous promise, many practical questions remain to be addressed before they are broadly adopted as the primary forecasting method in revenue management applications.

4.4. Origin/Destination Forecasting

Both virtual nesting and bid price methods face a common problem of origin/destination demand forecasting. To fully appreciate the problem, consider once again the linear program (P). The value of d_i are forecasts that represent the expected demand for each itinerary and class i in the flight network under consideration. Conceptually, this includes all itinerary and classes on which demand may be realized. For a medium-sized carrier, this can easily translate into millions of demand forecasts; and because the frequency of travel may be less than once per flight, many d_i are less than one.

The volume and magnitude of the numbers that must be forecast provides unlimited fuel for the debate on what to forecast and how to forecast it. The debate surrounding what to forecast arises from the observation that with so many small numbers, the best solution may be to rethink whether variables in the optimization model can be aggregated in some way without adversely impacting the model. If the dual variables λ are the primary output required from (P), how important is it that all of the primal variables x_i be included in the formulation? Given the variability in the demand forecasts d_i, would better values of λ be generated from more aggregate forecasts? Beyond aggregating by class to the itinerary level, what levels of aggregation actually make sense? On the other hand, as variables are aggregated, there is a loss in resolution of the fare paid for a ticket. If, for example, different classes on the same itinerary are grouped into a single variable, the objective function coefficient must represent some average fare. In point of fact, it is not even the case that all itinerary and classes are charged the same fare, but may substantially vary by point of sale and distribution channel, implying that the formulation (P) might benefit from having the variables x_i further *dis*aggregated.

Collectively, these questions constitute the *level of detail* problem, and they reveal a pervasive quandary not only in revenue management but in the practice of e-commerce and operations research in general: What is the appropriate compromise between the needs of optimization models—which frequently call for data at a fine level of detail—and forecasting models—which become increasingly difficult to implement at finer levels of detail?

While no consensus exists as to the best level of detail at which to forecast, some general conclusions can be drawn from the many years of effort devoted to studying the problem. First and foremost, some form of origin/destination forecasts are required to reap the revenue benefits associated with origin and destination control. As obvious as this may seem at a mathematical level, it is not at all clear at a practical level, and has long been a subject of debate among practitioners. It is quite conceivable that the noise in the many small demand forecasts could overwhelm the potential benefit of solving a network optimization problem, and that leg/class control would continue to provide better performance. Yet, extensive empirical evidence has shown that the best virtual nesting and bid price algorithms require input from a network-level mathematical program. A second conclusion is that itinerary and classes with small demand cannot simply be ignored. In this context, small demand means numbers on the order of 10 bookings *per year*. While such numbers are inconsequential in and of themselves, in aggregate, they can represent sufficient demand to impact the results of optimization algorithms.

Closely related to the question of what to forecast is the question of how to forecast. Here, we focus on two major approaches to origin/destination forecasting and how they are influenced by practical issues related to data availability and processing time.

Information on the itineraries traveled by passengers is not as easily available as the number of passengers booked in legs and classes. When a passenger makes a booking, a data entity known as a passenger name record is created for the purpose of tracking and modifying the transaction. Passenger name records contain similar information from one carrier and central reservation system to another, but vary in format and in the details of the information they contain. From the standpoint of revenue management, passenger name records contain itinerary and class information that can be used for origin/destination forecasting, but require that the transactions contained in the passenger name records be processed and loaded into an appropriately structured database for this purpose. Complications include the sheer volume of passenger name records and the numerous business rules used to break up the passenger name record information. However, once processing and loading have been addressed, the data source is the best available for forecasting purposes.

An alternative source of itinerary and class data can be found in revenue accounting databases. Revenue accounting systems are used to reconcile ticket purchases with the actual money received. As this process typically takes place weeks to months after the purchase transaction, and because the data is not of the same quality as passenger name record data for a variety of business reasons, it is not adequate for revenue management forecasting purposes. It is, however, much easier to access than processed passenger name records, thus, inspiring a pragmatic alternative for origin/destination forecasting in which leg and class forecasts continue to be made using inventory levels from the central reservation system, after which revenue accounting data is used to disaggregate the forecasts into itinerary and class forecasts. For example, if a particular leg AAA–BBB was forecast to have 50 M-class passengers arrive, and if historically 80% were AAA–BBB passengers and 20% were AAA–BBB–CCC passengers, then the AAA–BBB M-class forecast would be 40 and the AAA–BBB–CCC M-class forecast would be 10. There are many variations on this basic theme of leg-disaggregate forecasting using revenue accounting data.

Much of the debate surrounding these two alternative methods of origin/destination forecasting centers on the relative complexity of the two approaches versus the revenue benefits. Leg-disaggregate forecasting builds on historical leg and class forecasting and uses an existing source of data not intended for forecasting, while processed passenger name record forecasting requires relatively sophisticated new systems, but provides a much better source of data. The debate is universal to e-commerce and operations research applications, pitting a higher quality, but more complex solution against a solution of lower quality that is simpler to implement. Empirical tests can be developed to evaluate the different approaches, but surprisingly fail to definitively resolve the argument as easily as might be expected. An extremely important side benefit of processed passenger name records is that they can be used as a platform for expanded revenue management activities other than forecasting.

The origin/destination forecasting debate is not limited to complexity, as there are advocates of the two different approaches from a mathematical standpoint. Proponents of processed passenger name record forecasting, who, as a rule, promote some form of bottom-up forecasting at or near the itinerary and class level, argue that the underlying random processes driving demand are at the origin/destination level and are, therefore, best modeled at this level. They also point to issues such as the ability to better manage pervasive problems with schedule changes. Proponents of leg-disaggregate forecasting argue that itinerary and class data are too sparse to generate reliable forecasts and, therefore, a top-down approach is warranted. In reality, there are many intermediate possibilities, including formal hierarchical techniques that can be found in the statistics literature. The fact that most, if not all, of these methods require information unavailable in leg and booking class data makes a strong mathematical case for using processed passenger name records. It also provides an essential e-commerce lesson by revealing the fundamental importance of transaction-level processing for purposes of providing unfettered analytical decision support.

5. Dynamic Pricing

The relationship between revenue management and dynamic pricing is often misunderstood. When a prospective passenger finds that she can purchase a Houston to Boston ticket today for $300, but tomorrow the price has jumped to $600, for all practical purposes, "the ticket price has doubled." In actuality, it is quite likely that no ticket prices have changed at all, but that the fare class selling for $300 has simply been closed for sale.

The distinction between dynamically adjusting the price of a single product and managing the availability of different products using the same underlying resource is important because it impacts how the problem is modeled. The optimization models discussed in §3 assume demand is estimated by class, and that a price for each class is supplied exogenously—price is not a variable. These models implicitly assume that if the fare for a particular class changes, it will be reflected in the demand forecasts. Dynamic pricing models treat price as a variable, and typically focus directly on the price and demand relationship (see Elmaghraby and Keskinocak 2002).

The revenue management and dynamic pricing problems are certainly related, and if the underlying products are identical, the problems are fundamentally equivalent. This relationship has led to ambiguities in the way revenue management is practiced, as is demonstrated by the dichotomy in the way forecasting and optimization and control are treated by central reservation and revenue management systems. Revenue management forecasting embraces a product archetype with a focus on estimating demand by class. On the other hand, nested inventory control—whether for classes or virtual buckets—is explicitly based on the premise of ordering products from high to low value with no explicit regard for their defining characteristics. Bid price methods also ignore defining characteristics, generating a price threshold to determine whether or not a given class should be available for sale.

Organizationally, revenue management is equated with inventory control. The classes developed by airline marketing departments are intended to be true products. Classes are priced with strategic objectives in mind, while revenue management departments observe demand for these classes and are tasked with controlling their sale. Beyond the fact that the distribution process is designed around classes, it is frequently difficult for revenue management departments to gain access to price information, much less to model how changes in price impact demand. Airlines may file tens of thousands of price changes daily, special deals negotiated by sales departments may lead to only a fraction of tickets being sold at the published fare, and revenue-sharing arrangements for travel involving multiple carriers may skew realized revenue. Pricing departments operate using their own distinct practices that often lack any direct connection with the revenue management department, though this is changing. In short, there is a schism between pricing and inventory control, though this is not unique to the airline industry.

6. Other Industries

Revenue management can be usefully categorized into *traditional* and *nontraditional* applications. Traditional applications are similar to the airline model at a mathematical level, while nontraditional applications use models that are sufficiently different to warrant a separate categorization. Models for nontraditional applications share some similarities with each other and with traditional applications, but have not matured to the point where broadly recognized standard practices have evolved, as is the case with traditional applications.

6.1. Traditional Applications

The hotel and car rental industries represent two applications of traditional revenue management that have evolved apart from the airline industry. These two applications nicely illustrate the similarity of the mathematical models, and how the application of these models is impacted by differences in the distribution environment. Other traditional application areas with which the authors have familiarity include cargo, passenger rail, cruise, and tour operators, though there are certainly others.

6.1.1. Hotel and Hospitality. At a modeling level, airline fare classes correspond to hotel room rates. For example, a hotel might offer "deluxe" and "standard" room rates, with deluxe rooms commanding a premium price. Room rates may exist solely as a way to charge different prices for undifferentiated products, or differentiation might consist of a free breakfast, bathrobes, or other amenities not attached to the physical characteristics of the room itself. Rooms can also be separated into clusters based on physical characteristics in the same way that airlines have separate cabins for first, business, and coach passengers. These physical characteristics provide opportunities for managing inventory through upgrade—providing customers with a higher-value product than they paid for—as opposed to substitution.

The leg/class and origin/destination distinction is mathematically identical for airlines and hotels, but appears in a different context for hotels. Rather than a route network comprising individual flights, the hotel problem is that of a time network comprised of individual nights. To see the problem equivalence, consider the linear program (P) introduced in §3.3. Let L represent the set of date/room rate combinations in a fixed time horizon—say, tonight for the next 200 nights—and let I represent the set of arrival date/length of stay/room rate combinations. $I(l)$ denotes the set of arrival date/length of stay/room rate combinations that use date/room rate combination l. For example, an arrival on June 1 for a stay of three nights in a deluxe room would correspond to an index $i \in I$, and if $l \in L$ corresponds to a date/room rate combination of a deluxe room on June 2, then $i \in I(l)$. The parameter c_l represents the number of rooms in date/room rate combination l, which typically does not vary with date, while d_i denotes the expected demand for arrival date/length of stay/room rate combination i. The variables x_i represent the amount of demand of type i that should be accepted when sold for total charge r_i.

The equivalent of leg/class control in the hotel industry is establishing room rate availability for each date, and making a room rate available for sale if there is room rate availability for each night of the purchase request. Date/room rate control is extremely natural from a user perspective, but suffers from the same revenue limitations as leg/class control in the airline industry. By not accounting for length of stay, short duration bookings may "checkerboard" inventory, blocking long duration stays and leaving rooms needlessly empty. Virtual nesting and bid price control also have their counterparts. Virtual nesting assigns different arrival date/length of stay/room rate combinations to different inventory buckets on different nights, and a room class can be sold if there is availability in each of the appropriate buckets during the duration of the stay. Bid price methods establish a bid price for each night, and a room rate can be sold if the total charge for the room exceeds the sum of the bid prices for the nights of the stay. Variants of these techniques, often simplified, are used in practice.

The hotel and airline problems are similar, but not identical from a distribution perspective. The primary difference is found in the degree of autonomy exercised by individual properties within a chain. Chains operate central reservation systems and sell inventory through global distribution systems, their home websites, and various third-party agencies just as airlines do. Not surprisingly, in adhering to the one-stop shopping principle, the global distribution systems used by hotels frequently overlap with those of the airlines.

However, inventory control normally resides at the property level, so that the central reservation system can be likened to a distribution channel for the individual property. Of course, this analogy only goes so far, because hotel chains and their individual properties are more tightly coupled than a typical supplier and distributor relationship. The details by which properties and central reservation systems coordinate the sale of inventory may significantly vary, and depends upon the business model employed by the chain: Are properties independent profit centers or do they serve a common balance sheet?

6.1.2. Car Rental. Like hotels, the car rental industry shares many similarities with the airline industry at a modeling level. Cars are first stratified into classes based on the physical characteristics of a car, for example, "economy," "midsize," and "full size." The concept of car class is analogous to cabins in the airline industry. Within each class, cars are separated into tiers, which are the equivalent of airline fare classes. Product differentiation between tiers within a class is negligible, so that tiers are fundamentally a way to charge different prices for the same resource.

The leg/class and origin/destination distinction may be modeled in a way that is mathematically identical to the problem faced by airlines and hotels. In this case, length of keep in the car rental industry corresponds to hotel length of stay, and tier to room rate. The car rental industry also faces the same distribution issues as the hotel industry, with regions and rental sites exercising a reasonably high degree of autonomy. The balance between centralization and decentralization depends upon the specific practices of a given company.

The car rental industry differs from the airline and hotel industries in its ability to move inventory. Airlines have some ability to do so, but the operational difficulties involved in coordinating capacity changes within three months of departure largely render capacity fixed for the purposes of revenue management. Inventory movement within the car rental industry incorporates many components, including daily operational activities involving early and late returns, one way rentals, weekly shifting among rental sites, regional seasonal movement, and the purchase and sale of inventory at a national or international level. The freedom to move capacity poses challenges not faced in other industries that practice revenue management, but also provides many new opportunities for revenue enhancement.

6.2. Nontraditional Applications

Nontraditional applications require different models than traditional applications, primarily because of different distribution processes and methods of order fulfillment. We here touch upon two of the industries discussed in Secomandi et al. (2002). We make no claim that these industries represent the entirety of nontraditional revenue management modeling, only that they provide insight into how distribution and fulfillment alter revenue management models.

6.2.1. Broadcast. The broadcast industry generates revenue by selling advertising time. The inventory of advertising time is sold differently in different parts of the world, but sales typically consist of both an up-front market where contractual negotiations are undertaken to establish the price and type of advertising time that will be used by a client, and a scatter market for handling all other transactions. Under existing business practices, the up-front market dominates inventory sales and occurs once a year rather than on a rolling horizon. An excellent description of how the U.S. market operates can be found in Bollapragada et al. (2002).

While the sale of inventory is quite different than for traditional revenue management applications, the conceptual problem remains the same: Estimate demand and its willingness to pay so that inventory may be properly managed to maximize profit. However, with so much inventory sold as a result of negotiations, the focus shifts from short-term variable price to contractual terms.

6.2.2. Hospitals. Hospitals sell a variety of services, including access to operating room facilities, medical equipment, beds, and nursing care, among others. In the United States, payment for patient services comes almost entirely from insurers and their governmental equivalents, Medicare and Medicaid. All of these agencies establish contracted terms of payment with hospitals for rendered services for the constituencies they represent. For example, constituency A might be an employer of 1,000 people that purchases health coverage for its employees from insurer B. Insurer B then negotiates a contract with hospital C to provide services for constituency A and any other constituencies represented by insurer B.

From the standpoint of hospitals, revenue generation is dependent upon contract terms. Contracts typically involve only the terms of payment by the insurer for rendered services, with no minimum or maximum commitment on the number of patients who will use the services. Thus, the revenue generated by the contract may be quite variable (capitated contracts, in which a fixed amount is paid to the hospital for every covered patient, are an exception). The sale and distribution of inventory and services clearly does not support a short-term variable price model, nor is it easy to imagine that such a model will ever be relevant in the health-care industry.

7. The Future of Revenue Management

The success of revenue management in industry has had a tremendous impact on the landscape for future e-commerce applications. By proving itself to executives as an essential tool for profitability, it has paved the way for broad industry adoption. By being employed in the highly-visible application of establishing the effective price of airline tickets, it has helped popularize dynamic pricing. Yet, despite its connection with dynamic pricing, revenue management began and remains the practice of managing inventory against market demand to generate maximum profit. It is within this context that many of the future opportunities for revenue management can be found.

7.1. Contracts

"Revenue managing" the contractual terms under which inventory is sold, remains almost untouched in the revenue management literature in spite of the vast majority of business that is transacted under negotiated contracts. According to *The Economist* (2000), "an estimated 80%–90% of all business goods and services are actually traded through extended term contracts." This is unlikely to change in the near future, because firms perceive tremendous security from working with customers to lock in sales at the expense of higher expected revenues—a fact that many proposed e-commerce applications fail to take into consideration. The nontraditional applications discussed in §6.2 help testify to the significance of contract-based pricing.

7.2. Alliances and Collaboration

In an effort to develop a global market presence, many firms have joined forces to form alliances. Part of forming an alliance involves establishing agreements by which alliance partners have access to each other's inventory, raising an entirely new set of questions for revenue management. These questions range from how inventory should be valued and controlled in a decentralized environment to the potential for competitive gaming among partners (Boyd 1998). The growth of alliances and collaborative business initiatives (Schachtman 2000) has received little attention in the revenue management literature.

7.3. Customers

In its most basic form, revenue management focuses on establishing general product availability without accounting for the source of a purchase request. Product availability or a bid price is posted on the central reservation system and all requests draw upon the same information. Conceptually, it is possible to differentiate product availability based on information related to the purchase request, and many carriers incorporate some form of this information. The most rudimentary method from a mathematical perspective is to account for the cost of sale through a particular distribution channel, charging more to cover the cost of more expensive means of distribution. Although it is a benign observation from the viewpoint of operations research, an important e-commerce lesson is that the low cost of selling through the Internet is a strong economic factor driving the growth of e-commerce.

A major outstanding question confronting the future of revenue management is how to make better use of information contained in purchase requests. The ultimate goal is to charge each and every customer exactly what she is willing to pay. The challenges are enormous, spanning both distribution and modeling.

From the distribution perspective, inventory requests arrive at the central reservation system through many different channels, and different channels provide diverse levels of information about the purchaser (see Figure 1). Many of the third-party channels are entirely opaque, giving no information about the customer and, therefore, providing no opportunity for customer differentiation. Global distribution systems are capable of providing various levels of customer information. At one extreme, no customer information is supplied to the central reservation system because the central reservation system is not directly involved in the transaction. Instead, inventory availability from the central reservation system is "mirrored" on the global distribution system to reduce the volume of communication between these two systems. At the other extreme, individual requests can be passed through the global distribution system to the central reservation system, providing more information about the customer and allowing some degree of customer-level control, but at a higher cost per transaction. Home websites have no intermediary involved in the transaction, allowing virtually unlimited freedom to incorporate information about the customer to design tailored responses. Whatever degree of customer-level control is ultimately adopted by revenue management practitioners will ultimately have to account for the variety of different channels found in existing distribution environments.

Equally many challenges can be found at the modeling level. How should customer segments be defined? How can cannibalization associated with alternative distribution channels and different levels of information about the customer be avoided? Given the quantities of data that must be processed and the small time windows, what are the data-processing implications? How can products be dynamically bundled and packaged so as to promote up-sell and cross-sell? How can analytical models be implemented within the framework provided by existing Customer Relationship Management (CRM) software? Many of the modeling questions are actually addressed in the marketing science literature. The unique and exciting questions faced by operations research and revenue management practitioners are related to making the models work in a dynamic operational environment.

7.4. Exchanges

One special form of third-party distribution channel can be found in electronic exchanges—virtual marketplaces where buyers and sellers come together to trade. Exchanges offer an expansive e-commerce opportunity for entrepreneurs and researchers alike. However, experience in the airline industry provides a cautionary tale nicely punctuated by *The Economist* (2000): "Most [exchanges] have been guilty of thinking that business transactions are a lot simpler than they really are."

Firms frequently shy away from exchanges and related distribution channels because they tend to commoditize inventory. For example, websites listing dozens of options for travel from city AAA to BBB ordered by price or schedule encourage price competition and do not promote brand loyalty. Airlines diligently work to distinguish their products through frequent flier programs, service, and advertising in an effort to command a premium price. Commoditization is a frightening prospect because it leaves price as the primary competitive lever. Recent efforts to avoid commoditization by Southwest Airlines included legal action to keep the Orbitz online travel site from listing Southwest Airlines fares alongside those of its competitors (Wolverton 2001, *CityBusiness* 2001). Smith et al. (2001) provide a fascinating account of how the order in which ticket purchase alternatives are presented to a consumer can significantly impact decision making.

Exchanges also meet with resistance because they have the potential to exercise tremendous power. If an exchange becomes successful enough for sellers to find they have no alternative but to transact business through the exchange, it can assess high fees. Global distribution systems became a dominant force in the travel and hospitality industry by actively courting travel agents, thus, leveraging themselves into a highly-profitable business niche as a sales intermediary. Interestingly, the rise of the Internet brought with it the first real challenge to global distribution systems in the form of home websites and the wave of new third-party distribution channels.

8. Conclusions

The history of revenue management illustrates a successful e-commerce model of dynamic, automated sales enabled by central reservation and revenue management systems. Scrutinized by academicians and practitioners alike, it has evolved to a discipline with its own language and a recognized set of best practices, providing a wide-ranging model for managing inventory against market demand. Even firmly established users like the airline industry can benefit from existing technologies—such as origin/destination control—that are well understood but not yet widely implemented. Nonetheless, there remain research questions with the potential to provide great benefit to revenue management practitioners.

Looking beyond traditional applications, a rich collection of opportunities present themselves. Contracts, alliances, collaboration, distribution, and the use of customer-level information present sizable challenges for revenue management and the operations research community. Taking these challenges and making them profitably work in operational e-commerce settings will provide research opportunities for many years to come.

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