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Perishable-asset Revenue Management: Generic and Multiple-price Yield Management with Diversion

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In any situation of fixed capacity and a perishable service or product, firms want to avoid spoilage of the service or product and receive the most revenue possible in the face of uncertain demand. Stimulation of demand from price-sensitive customers, however, through discount prices to customers that reserve early can help fill capacity. This is attractive if non-price-sensitive customers can be prevented from diverting to the lower rates. This paper provides generic results for deciding how many discount units to sell (the so-called yield-management problem) and extends the generic problem in several ways: (1) to situations with continuous, not discrete resources, (2) to treatment of yield management jointly with overbooking, and (3) to problems with diversion with more than two price classes. Simulation studies using airline data suggest that our decision rules perform better than previous rules now used in practice.

Key words—air transport, marketing, resource management, decision making/process, inventory control, Newsboy problem

1. INTRODUCTION

AMERICAN AIRLINES REPORTED that the benefits of yield management were $1.4 billion over 3 years, which exceeded the net profit of the company during that period (see [9]). Only slightly less dramatic success is found in other airlines and in hotel firms.

We prefer the term perishable-asset revenue management (PARM) for the optimal revenue management of perishable assets through price segmentation for reasons outlined in [10, 11], where detailed definitions and a review of the literature are provided. PARM can apply outside the lodging or travel industries. An electric utility might stimulate demand among users of other forms of energy by selling a guaranteed amount of wholesale power in advance. This approach, if managed well, has the potential to improve use of a fixed capacity to produce electricity without hindering the utility's ability to provide power on demand to 'core users' at regular rates.

In a dissimilar field, broadcast advertisers could use yield management to help fill available time slots with higher revenue advertisements. Advertising agencies act as brokers between the broadcasters and those who want to advertise on radio or television. Broadcasters seek to charge the right price to select the right customer in filling each slot, while achieving the highest revenue. By appropriate use of discounts

Table 1. Other examples

BOX OFFICE: theater, sports
how many group or season discount seats to sell
LODGING: hotel, apartment
how much discounting of contract, advanced purchase, and discretionary travel
SERVICE JOB SHOP: photo studio, barber, auto repair
use of promotional packages to fill up slack hours
SHIPPING: overnight, package
discount for low-priority cargo
TECHNOLOGY: new and critical high-performance components
amount of premium for high-demand components; reduction of premium with age and availability

on those slots that fill slowly and avoidance of discounts on popular slots, the broadcaster can obtain higher yields than otherwise.

The possibility of contracted, advance discount sales provide opportunities for PARM in many other applications, but applications of yield management go well beyond advance sales, as indicated by the situations in which it can be effectively used shown in Table 1.

This paper begins with a statement of the generic decision rule for when to curtail discount sales, which was originally developed by Littlewood [7]. Several extensions of the generic decision rule are offered to account for uncertain show-up of reservations. While the generic situation does not involve the diversion of full price to lower price categories, the rest of the paper concerns PARM with diversion. We review prior results for two price classes, then present new PARM results for three and then n price classes. There is no published optimal result for yield management with diversion that does not assume independent demands for the fare classes (although there are results [4] where independence is assumed). While our results are not dynamically optimal for the inventory control problem except under some restrictive conditions, the decision rules provide conceptual guidance with nested price classes, and may be used as an heuristic. Simulations have demonstrated economic gains relative to previous rules. Examinations of how well they perform relative to optimal solutions (based probably on stochastic dynamic programming models) must await the development of such solutions.

2. THE GENERIC TWO-CLASS PARM ALLOCATION PROBLEM WITHOUT DIVERSION

Of a fixed number, q_0, the capacity of units available at a certain date in the future to both price classes, we can choose to sell q_1 units at a discount price that provides contribution R_1. The units will be provided on the availability date; at that date, a random number of full-price customers, X, will present themselves and request our product or service. The generic PARM allocation problem is to choose q_1 to maximize our economic well-being. In this simplest of situations, all of the demand for discount units precedes the demand for full-price units; we choose to fill the discount demand until we have all that we want. If we choose q_1 discount units, we will receive discount revenue for exactly q_1 units, and the level of q_1 will not affect X. (These assumptions will be relaxed in the extensions that follow.) This is the problem faced by an electric utility deciding how much wholesale power to sell on contract, a hotel deciding how many rooms to sell on contract vs on demand, an airline determining the number of nonrefundable discount seats to sell, and many other situations where purchase contrasts are arranged in advance.

The question is: How many units should we protect for X: i.e. how large should $q_0 - q_1$ be to provide for the uncertain number of full-price customers? This problem is like the newsvendor problem. However, instead of choosing how many papers to order to meet demand, we are dividing the existing inventory into what we sell now, and what we save for full-price customers to come later.

2.1. The decision rule

We maximize **contribution to fixed cost**—i.e. the difference between revenue and costs that vary strictly on a per-unit basis, which can be written as

Total contribution =

$$\begin{cases} q_1 R_1 + X R_0 & \text{if } q_1 + X < q_0 \\ q_1 R_1 + (q_0 - q_1) R_0 & \text{if } q_1 + X \geqslant q_0 \end{cases}. \quad (1)$$

Analysis is simplified by looking at the problem incrementally. If we choose to make q_1 one unit higher, we will receive additional contribution R_1, with no other effect provided we do not overshoot and have too little capacity for the full-price customers, X. If we overshoot—i.e. if $q_1 + X$ exceeds capacity—we receive the contribution R_1 from accepting the additional discount customer but lose the contribution R_0 that

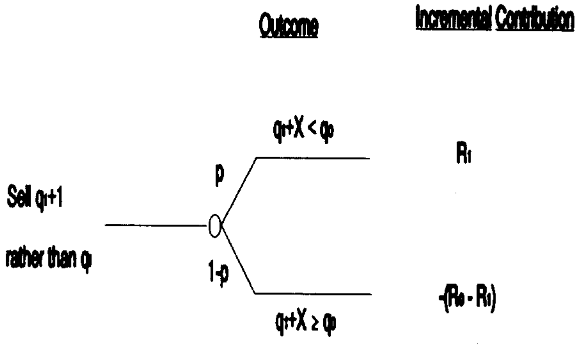

Fig. 1. Decision tree for generic PARM allocation problem.

could otherwise have been obtained from another full-price customer. Thus the outcomes from accepting an incremental discount customer, given we are already committed to q_1 of them, are as depicted in Fig. 1.

In the figure, p is the probability that $q_1 + X < q_0$, or the probability of spoilage. This probability of course varies with q_1. Based on expected contribution calculated from the decision diagram, we would wish to sell one more discount unit if the incremental expected contribution were greater than 0, which implies

$$pR_1 + (1-p)(R_1 - R_0) > 0. \quad (2)$$

Some algebra leads to the following rule for deciding when, given we have determined to sell at least one discount unit, to curtail discount sales:

Reserve an additional discount customer if

$$p > \frac{R_0 - R_1}{R_0}. \quad (3)$$

The value of p can be read from a probability distribution forecast of demand or a model may be used to forecast the pth fractile of the distribution directly. Note that demand for discount units may be such that we never cut off discount sales. In such a case, our analysis of when to cut off discount sales is still correct, even if not ever reached.

To understand what the rule means, consider an example. Suppose that the ratio of R_1 to R_0 is 50%; then continue to reserve discount customers until the probability of spoilage drops to $<50\%$. If the discount is smaller, say $R_1 = 0.8R_0$, then the optimal move is to reserve more discount customers so long as the probability of spoilage is >0.2.

Some assumptions of the generic problem may be varied with only a simple reinterpretation of the decision rule. We consider here the extension to (1) continuous resources and to (2) no denial of service—i.e. adding additional capacity, as needed, at a cost.

2.2. Continuous units of resource

Instead of *discrete* units of a resource, such as hotel rooms, the resource may be *continuously* divisible, as in the case of electric power. The continuous result [3] is

Sell the exact amount that makes

$$p = \frac{R_0 - R_1}{R_0}. \quad (4)$$

Consider an example of implementing the discrete rule (3), where $q_0 = 100$, full price is \$275, discount price is \$200, variable cost is \$25, and X is normally distributed (the rule, of

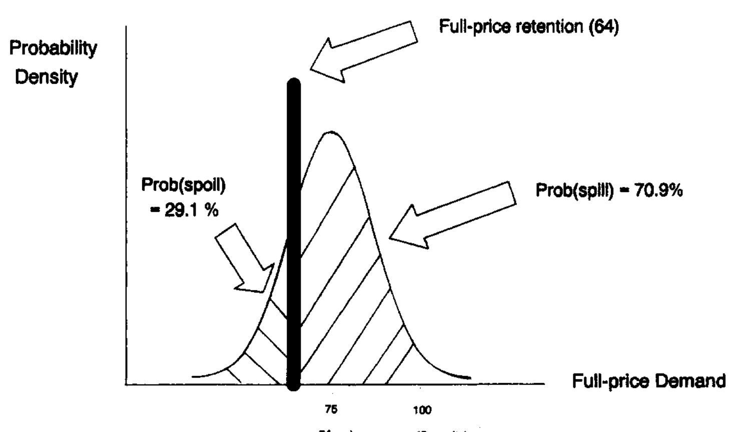

Fig. 2. Balancing spill and spoil.

Table 2. Expected contribution and percent change from optimal for varying allocations

Units protected for full-price	Discount units	Expected contribution	Percent change from optimal
100	0	\$18,497	13.5%
80	20	\$20,819	2.6%
64	36*	\$21,386	0%
40	60	\$20,419	4.5%
20	80	\$18,995	11.2%

*Denotes optimal choice.

course, works for any probability distribution) with a mean of 75 units and standard deviation of 20 units. For this example, our generic decision rule recommends selling discount units until the probability of spoilage equals (250 − 175)/250 or 0.30. Using a table of probabilities [6] for the given normal distribution of demand, we can determine that a 29.1% chance of spoilage would result if we retain approx. 64 units for sale at full price and 36 units for discount sale. Thus we should allow no more than this amount to be sold at discount.

Figure 2 illustrates the balancing of costs implicit in the generic decision rule. At issue is the placement of the retention level for full-price customers, indicated by the taller vertical bar. If that level is set too high—e.g. at 100—then the chance of spoilage will be high; if it is set too low—e.g. at 20—then there is a high chance of spill, i.e. turning away full-price customers when discount units have been sold. The decision rule places the level just right. Table 2 demonstrates the superiority of using the generic decision rule. It contributes, on average, \$21,386 vs \$18,497 if all 100 units are retained or \$18,995 if 20 units are retained.

2.3. No denial of service

So far we have assumed that when demand from full-price customers exceeds availability we lose the full-price customer. This bumping of the best customers is undesirable and many companies will go to great lengths to avoid it, to the point of having a company policy to never do it. This can be accomplished by temporarily augmenting capacity (at a premium) or by finding a volunteer customer that will agree to be bumped for some reward (such as a free coupon).

In some instances, for example, in the case of electric power, the company may be required to service all full-price customers. The utility may be forced, when demand exceeds capacity, to buy electricity on the spot market, generally at higher prices than its own production costs.

These situations can be handled simply by making a suitable reinterpretation of the penalty of exceeding capacity. Because all full-price demand will be met, when $q_1 + X > q_0$ the loss is not the full-price contribution, but the extraordinary (incremental) cost of either rewarding a volunteer to be bumped or buying excess capacity on the spot market. In fact the revenue of the full-price customer is not relevant to the decision about allowing another discount customer. The full-price customer will be served regardless. The relevant cost of running our of capacity is the extra bumping or spot purchase cost.

Define G to be the incremental cost of a bumped customer. In the case of finding a volunteer for bumping,

$$G \equiv \text{Goodwill cost} + \text{Volunteer customer contribution} + \text{Reward to induce volunteer}.$$

The volunteer may be either a discount or a full-price customer. If an auction is used to find the volunteer, it may be more likely to be a discount customer.

In the case of the electric utility,

$$G \equiv \text{Spot price} - \text{Normal variable cost}.$$

The decision rule to use when there is no denial of service would be the same as (3) and (4) with R_0 replaced by G. This can be demonstrated by changing the bottom endpoint of the decision tree in Fig. 1 to $R_1 - G$, and repeating the derivation.

2.4. Uncertain show-up of customers

Researchers [2] have derived formulas to find the optimal overbooking level under the assumption of a single price class. No one, so far as we know, has treated the joint overbooking/discount optimal-allocation decision, although there have been suggestions as to how to put together separate overbooking and optimal-allocation approaches [1].

In this section, we move beyond the assumption of the generic PARM allocation problem that a customer who requests service will definitely show up to receive the product or service at the availability date. Here we assume that the show-up of customers, both full-price and discount, is uncertain. We first decide how many discount customers to book. Then once discount sales are curtailed, we begin to book full-price customers that request service. If at the availability date, the number of discount and full-price customers which show up exceed capacity, then we find volunteers to be bumped by auction.

Let us define:

$S \equiv$ the number of survivals (a random variable) from the q_1 discount reservations (our decision variable),

$X \equiv$ the number of full-price customers that survive at the availability date (also a random variable), which may differ from the number of customers making a reservation for service,

$p \equiv$ the probability of spoilage, i.e. $\Pr(S + X < q_0)$ if we sell $q_1 + 1$ discount reservations,

$p_2 \equiv$ the probability that the q_1 + 1st reservation will survive (assumed independent of whether or not there is spoilage).

In addition, we use the first definition of G from the previous section, as the cost of bumping a customer.

We can describe the problem with the tree shown in Fig. 3. Note that R_0 does not appear anywhere in the decision diagram. This is because the relevant cost of spill (ending with more customers than capacity) is the cost of the bumped customer. Again, a volunteer is found for bumping, at a total cost, G. Using Fig. 3, we would take the additional discount customer provided that:

$$pp_2R_1 + (1 - p)p_2(R_1 - G) > 0. \tag{5}$$

After some rearranging, we would arrive at the decision rule:

Reserve an additional discount customer if

$$p > \frac{G - R_1}{G}. \tag{6}$$

Interestingly, the form of the decision rule is the same as before; i.e. (6) is the generic result when $G = R_0$. However, the optimal allocation will be different now since the probability distribution for $S + X$ is different from the probability distribution for $q_1 + X$ where q_1 is assumed certain. For example, if it is assumed that each discount reservation is an independent Bernoulli random variable with probability p_2 of surviving, then S has a binomial distribution with q_1 trials and probability p_2. For results in terms of the parameters of assumed forms of distributions of survivals, see [2].

To illustrate the above decision rule (6), we provide an example. The example assumes parameters similar to Section 2.2:

$$q_0 = 100, X \sim \text{Normal}(75, 20), R_0 = \$250,$$

$$R_1 = \$175, G = \$300, \text{ and } p_2 = 0.8.$$

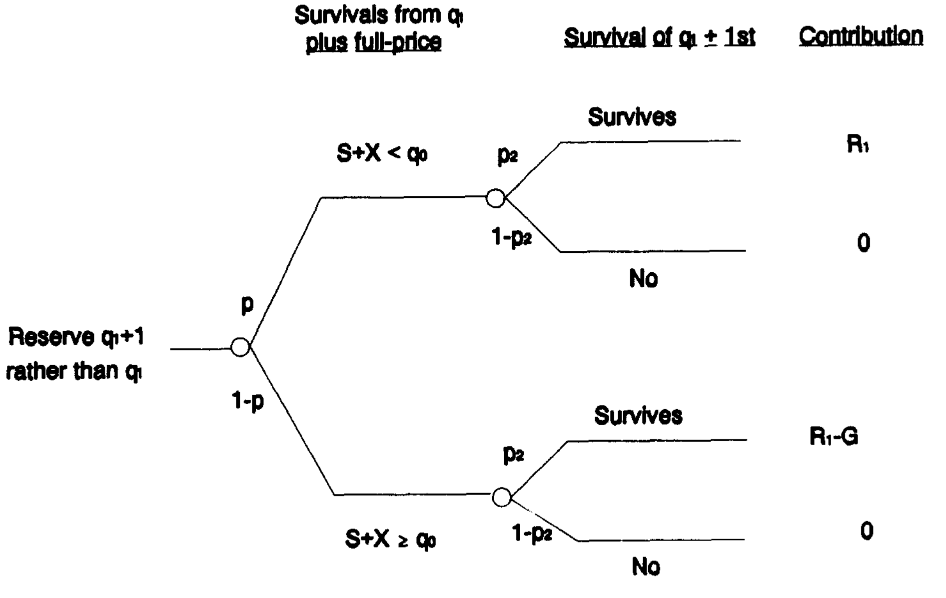

Fig. 3. Decision tree for allocation with uncertain survival.

Table 3. Expected contribution and percent change from optimal for varying allocations

Discount units	Expected contribution	Percent change from optimal
30	\$20,689	0.37%
32	\$20,730	0.18%
34	\$20,756	0.05%
36*	\$20,766.40	0%
37	\$20,766.12	0%
38	\$20,762	0.02%
40	\$20,743	0.11%

*Denotes optimal choice.

What is the value of q_1 that maximizes the expected contribution? Using the rule in (6) we get:

Reserve an additional discount customer if

$$\Phi(z) > \frac{300 - 175}{300} + \phi(z)\frac{\sqrt{1-0.8}}{2\sqrt{0.8q_1}},$$

or

$$\Phi\left(\frac{100 - 0.8q_1 - 75}{\sqrt{20^2 + q_1 * 0.8 * 0.2}}\right)$$
$$> 0.417 + \phi(z)\frac{\sqrt{0.2}}{2\sqrt{0.8q_1}}$$

(see [2] for details on the small correction term on the right-hand side of the decision rule which comes from the derivation for the normal approximation to a binomial survival process). A quick search for the optimal value for q_1 indicates that the optimum is to allow up to 36 discount customers. Table 3 demonstrates the superiority of using the optimal decision rule. It contributes, on average, \$20,766 vs \$20,689 if only 30 units are allocated for discount customers. Comparing the result to the example problem in Section 2.2. (with the same parameters except for $p_2 = 1$ and therefore $G = R_0$), we find that we reserve the same number of seats for discount customers. Interestingly enough, the effects of the bumping penalty ($G = \$300$) offset the amount which one would overbook due to the uncertain show up ($p_2 = 0.8$).

3. DIVERSION WITH MULTIPLE NESTED PRICE CLASSES

So far we have assumed that the customers who buy at full price are completely separate from the ones who buy at discount prices. In other words we have assumed there is no diversion, or that no individual who is willing to pay full price will take a discount unit. In the rest of this paper, we will consider cases of diversion. It will be necessary first to clarify some definitions and notation.

3.1. Definition and notation

All prices are organized into mutually exclusive and collectively exhaustive price classes ($i = 0, 1, 2, \ldots, I$). For each class, we define:

$\pi_i \equiv$ price,
$C_i \equiv$ variable cost,
$R_i = \pi_i - C_i \equiv$ the contribution from a unit sold in price class i.

(In PARM problems, a reasonable assumption is that the variable cost for all different units is the same.) Price classes are rank-ordered with class 0 being full fare: $R_0 > R_1 > R_2 > \ldots > R_I$.

'Buckets', q_i ($i = 0, 1, 2, \ldots, I$), of perishable asset are made available at price π_i; these buckets are nested as shown in Fig. 4. Thus total capacity is equivalent to q_0, which is available only to those willing to pay the highest price. An amount q_1 is available at the second highest price, with the difference, $q_0 - q_1$, protected for sale to full-price customers from those not willing to pay full price. The q_is will serve as our decision variables. When the firm begins taking reservations there are units available in all price categories. The q_I bucket sells out first, and at that point customers not willing to pay π_{I-1} will no longer purchase. Each bucket will sell out in order with q_0 being the last bucket with units available.

It is convenient to think of customers in nested classes as well, according to their willingness to pay. Those customers who will pay full price, class 0 customers, will also pay any lower

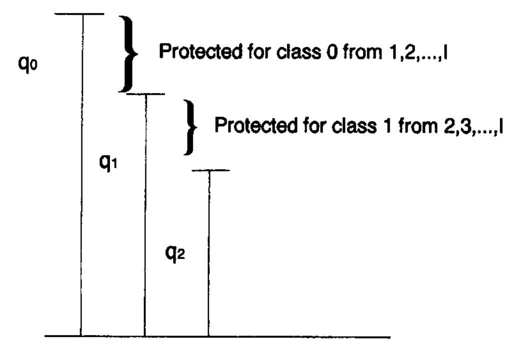

Fig. 4. Nested buckets of resource, q_i, made available at Price π_i.

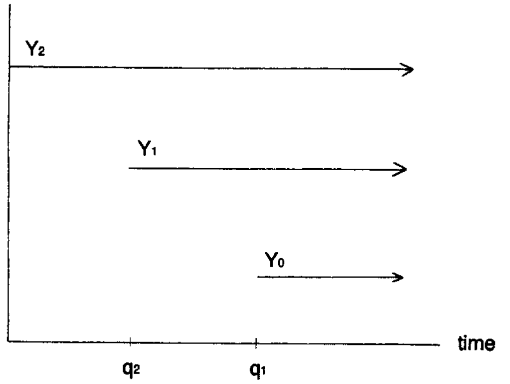

Fig. 5. Y_i over time for three price classes.

price and will therefore buy from all buckets. Of course, they will take the cheapest first if it is available. Class 1 is willing to pay the second highest price and any lower price, but no more, and so on for all classes down to class I which will buy only at the lowest price.

Diversion, the selling of a unit at a price less than the maximum the customer is willing to pay, is realistic since there is no guarantee that all class $i+1$ customers will arrive before all class i customers. Define:

> $\beta_i \equiv$ the probability that the next customer requesting a reservation is in class i
> (This probability may, of course, change through time or with the number of requests to date, since there is a tendency for customers in class i to arrive after customers in class $i+1$. In modern reservation systems, information on turndowns by customers of higher price classes is sometimes kept or sampled, which can be the basis for estimating the betas.),
> $X_i \equiv$ random variable for class i demand, $i = 1, \ldots, I$,
> $X \equiv$ random variable for full price demand.

We will need some additional notation for special conditional probabilities that will be needed in establishing decision rules for closing out buckets of inventory. Let:

> $Y_i \equiv$ random variable representing the demand for units in all price classes $\leqslant i$ (i.e. those customers willing to pay as much as π_i) subsequent to the arrival of the $q_{i+1}+1$st customer, $i = 0$ to I;
>
> $p_i \equiv$ probability that $Y_i \leqslant q_0 - (q_{i+1}+1)$, $i = 0$ to I. In other words, it is the probability that the subsequent demand will fit if we accept an additional reservation in class $i+1$.

Remember, q_{i+1} has been the decision variable for cutting off availability at the price class $i+1$. Note also that Y_I represents the total demand—i.e. the number of customers willing to pay at least the lowest price for the product. Figure 5 shows that Y_0 is demand after q_1 has been reached, etc. Figure 6 illustrates p_0 (note that q_0 is capacity). This probability is analogous to p, the probability of spoilage of resource (having left over capacity at the availability date) that was used in the applications of Section 2.

3.2. Diversion with two price classes

Relatively few researchers have dealt with the important issue of diversion, wherein full-price customers divert to discount rates. Belobaba [1], Brumelle *et al.* [5] and Pfeifer [8], each developed an equivalent decision rule for two price classes. We review here the results of Pfeifer, using our own notation.

Pfeifer makes the assumption that once a decision is made to curtail sales of units to discount-price customers, that price class will never be reopened. He uses a decision analysis approach incrementally to determine whether to accept an additional discount customer, i.e. move the class 1 bucket from q_1 to q_1+1. The decision depends on the probability that there is enough remaining capacity, $q_0 - (q_1+1)$, to accommodate full-price customers that will arrive later: p_0. It also depends on the probability, β_1,

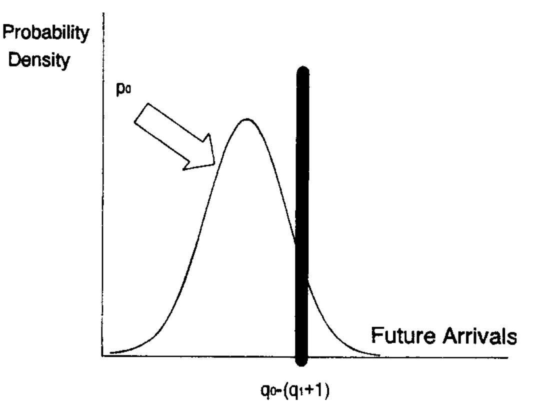

Fig. 6. p_0 Conditioned on q_1.

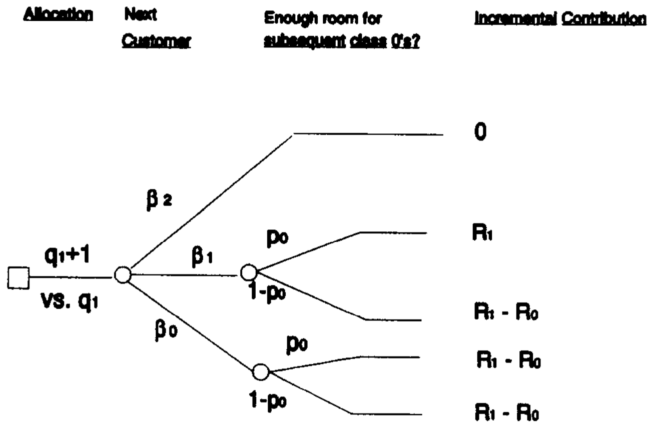

Fig. 7. Incremental decision tree for q_1.

that the next customer is one who would not pay full price. The decision rule he derives is

Reserve an additional discount customer if

$$\beta_1 p_0 > \frac{R_0 - R_1}{R_0}. \tag{7}$$

Pfeifer observes that both p_0 and β_1 will decrease with q_1 and therefore their product will decrease with q_1 to the point that the rule stops further sale of discount units.

3.3. Diversion with three price classes

This section extends the decision analysis to three price classes ($I = 2$). We assume, as did Pfeifer [8], that once a decision is made to curtail sales of units in a given price class leading to

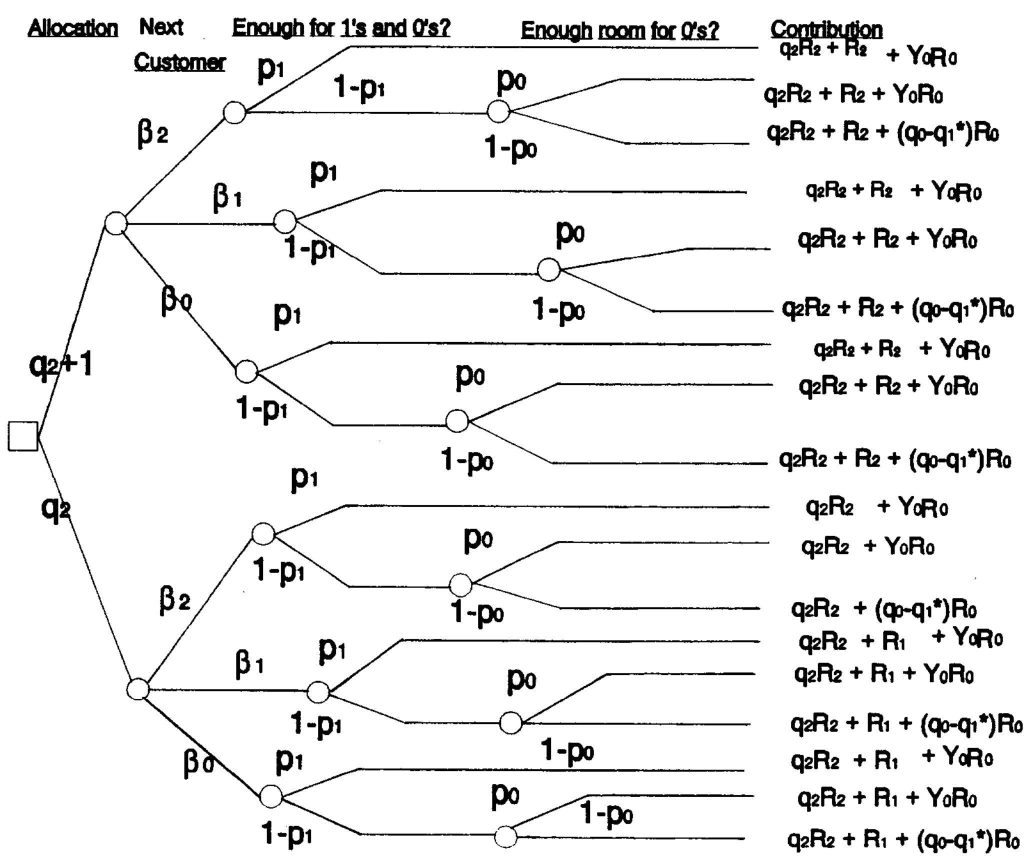

Fig. 8. CASE 1 decision tree for q_2 (three price classes with diversion).

contribution R_i, we will never again open up that price class. In practice, as time progresses, sales of class 2 are closed out first, followed by class 1. Consistent with the fold-back procedures of decision analysis, however, we will first consider how to set q_1—i.e. decide at what point to curtail sales of class 1—then work our way to an earlier time to determine how to set q_2. In this approach, we ignore diversion that ripples across classes that are greater than one apart.

Consider the decision of how much to protect for full-price customers. If we assume that we are at a point in the arrival process at which the lowest price class has been closed out at q_2, and we are considering whether or not to allow one more unit to be sold in price class 1 ($q_1 \rightarrow q_1 + 1$), we have the incremental decision tree shown in Fig. 7. We would choose to raise q_1 to $q_1 + 1$ if:

$$\beta_2 * 0 + \beta_1 p_0 R_1 + \beta_1(1 - p_0)(R_1 - R_0) + \beta_0 p_0(R_1 - R_0) + \beta_0(1 - p_0)(R_1 - R_0) > 0 \quad (8)$$

which can be simplified to the decision rule:

Accept an additional class 1 *customer if*

$$\left(\frac{\beta_1}{\beta_1 + \beta_0}\right) p_0 > \frac{R_0 - R_1}{R_0}. \quad (9)$$

This rule can be seen to be the three-class equivalent of Pfeifer's rule [8], because his probability that the next customer is a shopper (i.e. from exactly class 1, conditioned on being either a class 1 or class 0 customer) is equal to $\beta_1/(\beta_1 + \beta_0)$.

Now consider how to make the incremental decision between q_2 and $q_2 + 1$. Note that we know the decision rule for determining the cutoff value for q_1; we may not know the exact cutoff, q_1^*, yet we may assume it will have the same value regardless of how we set q_2. In order to determine the rule for cutting off q_2 properly, we must consider two cases. First we must test whether the parameters and probabilities at the present time also meet the q_1 decision criteria just derived in (9). The result will be either

CASE 1, $q_1^* > q_2 + 1$, or

CASE 2, $q_1^* = q_2 + 1$,

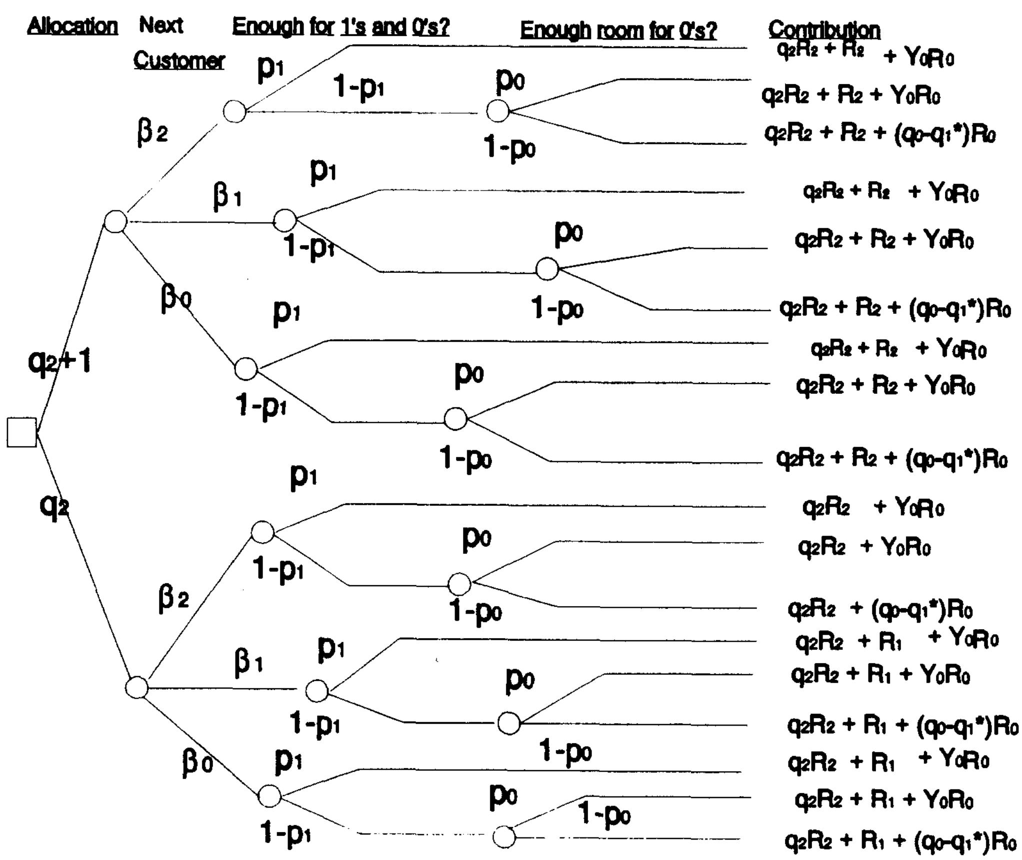

Fig. 9. CASE 2 decision tree for q_2 (three price classes with diversion).

where q_1^* is the cutoff for class 1 obtained using (9). In case 2, if we move to $q_2 + 1$, then there would be no sales of units at price π_1. We can test which case applies and use the decision rule appropriate for that case. If $q_1^* > q_2 + 1$ (i.e. at least one unit will be sold in price class 1), we have the decision tree shown in Fig. 8. The first event fork in that tree is about whether the next customer is a class 2, 1, or 0. The second event fork has to do with whether, after accepting another customer in price class 2, there are enough units allocated for the subsequent class 1 and class 0 customers. If there is not, we have an additional event on whether there is enough room for class 0 customers in its protection. If one studies the contribution endpoints, it will be apparent that we assume that allowing another class 2 customer has no effect on the subsequent cutoff level q_1^*. In our notation for the endpoints of Figs 8 and 9, we use Y_i for brevity, but it should be clear that this represents $E[Y_i]$. We would allocate the incremental unit ($q_2 \rightarrow q_2 + 1$) as long as:

$$\beta_2 p_1 R_2 + [\beta_2(1 - p_1) + \beta_1 + \beta_0](R_2 - R_1) > 0 \tag{10}$$

which can be simplified to the following rule for CASE 1 ($q_1^* > q_2 + 1$):

Accept another class 2 customer if

$$\beta_2 p_1 > \frac{R_1 - R_2}{R_1}. \tag{11}$$

If, on the other hand, we test the condition and find that $q_1^* = q_2 + 1$, then we have the decision tree shown in Fig. 9 and can derive the following rule for CASE 2:

Accept an additional class 2 customer if

$$\beta_1 + \beta_0 < \frac{R_2}{R_1}. \tag{12}$$

The difficulty in applying these rules is in estimating p_1, in equation (11), which is the probability that remaining class 1 and class 0 demand fits, given that the next customer is accepted at contribution amount R_2. Even granting the difficulty of estimating such a probability, expressions (9), (11), and (12) can be excellent heuristics for the PARM PROBLEM of two discount-price classes.

3.4. The general case (more than three price classes)

If we draw the decision tree for the general case, it has the form shown in Fig. 10 (to save

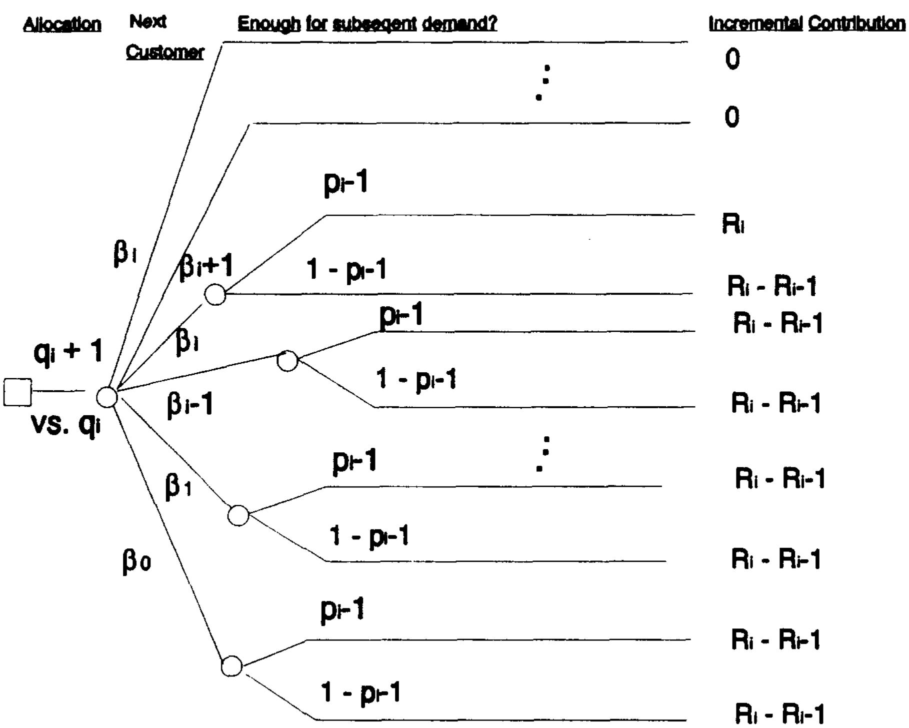

Fig. 10. Incremental decision tree for q_i (I discount price classes with diversion).

space, we have just looked at the incremental difference in contributions). To simplify, let us assume that we will sell at least one unit in each bucket. Consider first the q_1 decision; using the decision tree, we can write a condition that reduces to a decision rule:

Accept an additional class 1 customer if

$$\left(\frac{\beta_1}{\beta_1+\beta_0}\right)p_0 > \frac{R_0-R_1}{R_0}. \tag{13}$$

Continuing with the general decision rule for q_i, $i = 2, 3, \ldots, I-1$, we have, after simplification,

Accept an additional class i customer if

$$\left(\frac{\beta_i}{\beta_i+\beta_{i-1}+\cdots+\beta_0}\right)p_{i-1} > \frac{R_{i-1}-R_i}{R_{i-1}}. \tag{14}$$

For the last decision, q_I, we have

Accept an additional class I customer if

$$\beta_I p_{I-1} > \frac{R_{I-1}-R_I}{R_{I-1}}. \tag{15}$$

These new decision rules in (13), (14) and (15) provide, for the first time, an approach for dealing with diversion with any number of price classes. If we had allowed for re-opening of a bucket once it had been closed, the analysis would have been considerably (hopelessly?) more complex.

3.5. Comparison of new results with previous decision rules

Previous decision rules used by airlines [1] ignore diversion. Of course, it is useful to know how much could be gained by using our new decision rules when there are three or more price classes. We carried out simulations using actual airline data for demand in order to provide some preliminary indications. Bookings were simulated as Poisson arrivals for 18 separate booking periods, using arrival rates for each fare class/booking period combination. We considered two models, one for 3 price classes and one for 7 price classes, and in the 3-price model, two cases for the level of diversion.

In the simulation of 3 price classes, mean demand was 45 for class 0, 48 for class 1, and 57 for class 2, for a total of 150. In Case A, 40% of the customers demanding class 1 would be willing to pay the class 0 fare, and 30% of the customers demanding class 2 would be willing to pay the class 1 price. In Case B, 30% of the customers demanding class 1 would be willing to pay the class 0 fare, and 20% of the customers demanding class 2 would be willing to pay the class 1 price. Our decision rules produced a gain of 0.15–0.30% of revenues for case A with three prices, depending on the level of demand (the ratio of total demand to capacity varied from 1.0 to 1.5). Again for three price classes, our decision rule provided gains of 0.23–3.09% (as the demand factor varies from 1.0 to 1.5) for case B. These may seem to be small percentage gains, yet they are on a large base of billions of dollars; airlines have pursued elaborate schemes to gain smaller revenue increases than even the lower ends of these ranges.

When there are more than three fare classes, there are indications that even greater improvements are possible. In the simulation of 7 price classes, mean demand for classes 0 through 6 were (45, 12, 23, 15, 48, 57, 38), respectively, for a total of 238. The percent of customers demanding a class i fare that would be willing to pay a class $i-1$ fare for classes $1, 2, \ldots, 6$ was (0.4, 0.3, 0.2, 0.3, 0.3, 0.3), respectively (again, actual airline data was used to set the simulation). With the model for seven fare classes, the revenue gains are between 3.6 and 5.35% (as the demand factor varies from 1.0 to 1.5) for case A. This is a tremendous potential gain.

Our new rules for multiple fare classes capture the 'neighboring' diversion effect (from one nested class to the next) and provide substantial improvement over not anticipating diversion at all, which is the common practice typical at airlines and within many other services. Given that airlines typically forecast the parameters needed by our model, and that the decision rules would be straightforward to implement, there is great potential for improved revenue. Even greater improvements may be possible if one considers diversion not only from one class to its immediate neighbor, but also to all possible higher fare classes.

4. SUMMARY AND FUTURE RESEARCH OPPORTUNITIES

This paper has developed the generic PARM allocation problem and derived a simple solution that is applicable to a wide class of

Table 4. Comprehensive taxonomy

Elements	Descriptors
A. RESOURCE	Discrete/**Continuous**
B. CAPACITY	Fixed/Nonfixed
C. PRICES	Predetermined/Set optimally/Set jointly
D. WILLINGNESS TO PAY	Buildup/Drawdown
E. DISCOUNT PRICE CLASSES	1/2/3/. . ./*I*
F. RESERVATION DEMAND	Deterministic/Mixed/Random-independent/Random-correlated
G. SHOW UP OF DISCOUNT RESERVATION	Certain/Uncertain without cancellation/Uncertain with cancellation
H. SHOW UP OF FULL-PRICE RESERVATION	Certain/Uncertain without cancellation/Uncertain with cancellation
I. GROUP RESERVATIONS	No/Yes
I. DIVERSION	No/Yes
J. DISPLACEMENT	No/Yes
K. BUMPING PROCEDURE	None/Full-price/Discount/FCFS/Auction
L. ASSET CONTROL MECHANISM	Distinct/Nested
M. DECISION RULE	Simple Static/Advanced Static/Dynamic

Key to problems discussed in this paper
Generic Two-Class Without Diversion: Section 2
Continuous Units of Resource: Section 2.2
Uncertain Show-up of Customers: Section 2.4
Diversion With 2/3/. . ./*I* Price Classes: Section 3

industries. These results should be of interest to any firm with fixed capacity and a perishable service or product. Simple extensions of this generic problem to continuous-resource and unlimited-capacity situations and several more complicated PARM problems have also been given. The generic problem and the extensions can be placed into perspective using the taxonomy of PARM problems that is depicted in Table 4, which was developed in [11]. The shaded descriptors in that table identify the generic problem of Section 2.[1] Detailed explanations of these descriptors and a survey of problems that have been treated appear in [11].

We extended the generic results in several ways: (1) from discrete resource to continuous resource [designated by **boldface** in Table 4] and to cases where there is no bumping (2) from assumed certainty that customers will show up to uncertain show-up [underlined in the table], treating overbooking, (3) to problems with diversion with three or more price classes [double underline in the table]. The joint treatment of yield-management/overbooking [item (2) in this list] moves beyond simply putting together solutions to the two yield management and overbooking problems that have been separately derived. A second major extension is the consideration of PARM with three or more price classes with diversion.

The taxonomy itself indicates that the number of possible avenues for future research is huge; let us suggest a few of the more promising directions from this work. A difficult but rewarding future research effort could be devoted to each of the extensions covered here **without** the assumption that once a price category is closed it will never be re-opened. Under this revised assumption, one would need to deal explicitly with cancellations of reservations, not just the number of survivals at the availability date, which would be a welcome contribution. In addition, a solution to the *I* discount-price allocation decision without the difficult probability forecast needed in this paper would be

[1]The generic PARM allocation decision problem has *discrete* units of RESOURCE, *fixed* CAPACITY, *predetermined* PRICES, *buildup* WILLINGNESS TO PAY, *one* DISCOUNT PRICE CLASS, *mixed* RESERVATION DEMAND, *certain* SHOW-UP of the discount reservation, *certain* SHOW-UP of the full-price reservation, *no* GROUP RESERVATIONS, *no* DIVERSION effects, *no* DISPLACEMENT effects, *no* BUMPING PROCEDURE, a *nested* ASSET CONTROL MECHANISM, and a *simple static* DECISION RULE.

beneficial. Finally, significant work remains to be done on integrated forecast/decision systems with diversion.

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