



# An empirical analysis of the optimal overbooking policies for US major airlines

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## Abstract

Overbooking in the airline industry has been studied intensively. However, these studies have paid little attention to the future revenue implications of rejecting (bumping) passengers. This paper seeks the optimal overbooking policies for US major airlines by considering how denied-boarding passengers would behave after they are bumped. The results imply that overbooking improves an airline's "current" revenue, but it also reduces the airline's future revenues. The results also imply that, although there is a significant negative overbooking effect, no airline should decrease overbooking levels because the positive side of overbooking is so strong that it more than offsets its negative side. © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Airline; Overbooking; Optimization; Econometric analysis

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## 1. Introduction

It is well known that airlines practice overbooking to increase their revenue. Without this practice, airline flights must frequently depart with unused seats, because a certain number of "reserved" passengers either cancel their reservations at the last minute or simply do not show up for the flights (no-shows). Smith et al. (1992) report that for flights whose seats are completely sold out airlines still experience approximately 15% of seats unfilled during the actual flights. This condition indicates that an airline that does not practice overbooking may suffer, relative to its competitors that practice overbooking, from the decreased passenger load factor and the loss of

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passenger revenue. The use of overbooking, however, generally results in rejecting (or “bumping”) passengers with confirmed reservation in some flights, and increases the compensation paid to the “bumped” passengers. An important question, therefore, is “what level of overbooking should airlines use to maximize revenue?”

Answers to this question can be found in the operations research literature, where the optimal overbooking has been studied for over 30 years.<sup>1</sup> Most of these studies, however, have focused on maximizing the passenger revenue for a particular flight. They either maximize the expected passenger revenue of a given flight by considering the trade-off between the marginal revenue and marginal cost (penalty) of overbooking an additional passenger, or control for the probability of bumped passengers for a given flight within a certain range. The studies have not considered explicitly the passengers’ behavioral patterns after they are bumped (i.e., after flight departures), and their implications on airline revenue.

Theoretically, bumped passengers can affect an airline’s revenue in two ways. First, they may positively affect the airline’s “current” revenue, because a certain proportion of bumped passengers wait for the later flights (with vacancies) of the same airline (which bumped them) to fly to their destinations. Thus, an increased number of bumped passengers can increase an airline’s current revenue by increasing the load factor of the airline’s less-demanded flights, while keeping the load factor of the more-demanded flights high (shifting demand). Second, the bumped passengers may negatively affect an airline’s future revenue, because a certain number of passengers with “bad” experiences (bump) may avoid using the same airline for future trips.

These “bumped-passenger effects” are not relevant to the traditional single-flight revenue maximization models, because these effects take place only after the flights have departed. If, however, an airline wants to maximize its *overall revenue*, rather than the single-flight revenue, the bumped-passenger effect must be considered. This paper examines the adequacy of US major airlines’ overbooking levels by comparing their “true” optimal overbooking levels, which maximize the overall revenues, with their actual overbooking levels. A unique aspect of this paper is that it “empirically” seeks the optimal levels; i.e., it uses the historical data of airlines to determine the “past” optimal overbooking levels that “would have maximized revenues if these levels had been used”.

## 2. Framework

To analyze the optimal overbooking level by airline, the data representing the actual number of overbooked passengers by airline and time period are needed. But since such data are unavailable, we instead use the data representing the number of denied-boarding passengers by carrier and time period. More specific, we solve an optimization model in which the objective function is the

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<sup>1</sup> See, for example, Beckmann (1958), Kosten (1960), Taylor (1962), Deetman (1964), Rothstein and Stone (1967), Rothstein (1968), Simon (1968), Falkson (1969), Rothstein (1971), Andersson (1972), Simon (1972), Vickrey (1972), Etschmaier and Rothstein (1974), Bierman and Thomas (1975), Rothstein (1975), Shlifer and Vardi (1975), Nagarajan (1979), Ruppenthal and Toh (1983), Rothstein (1985), Alstrup et al. (1986), Belobaba (1987), Alstrup (1989), Brumelle and McGill (1989), McGill (1989), Chatwin (1993), Dunleavy (1995), Karaesman and van Ryzin (1998), Chatwin (1999a,b), Subramanian et al. (1999).

passenger revenue and the decision (adjusting) variable is the number of bumped passengers to search for the optimal (revenue maximizing) number of bumped passengers.

The above approach of using bumped passengers in lieu of overbooked passengers can be validated if we assume that the relationship between the denied-boarding passengers and the overbooked passengers is given by the following formula:

$$DB_{it} = f(OB_{it}) + \varepsilon_{it}, \quad (1)$$

where  $DB_{it}$  is the number of passengers denied boarding by carrier  $i$  at time  $t$ ,  $OB_{it}$  is the number of overbooked passengers by carrier  $i$  at time  $t$ ,  $f(\cdot)$  is a monotonic function that maps  $OB_{it}$  onto  $DB_{it}$ , and  $\varepsilon_{it}$  is a stochastic error term.  $\varepsilon_{it}$  captures such effects as the random fluctuations of the number of no-show passengers. Eq. (1) indicates that, although  $OB_{it}$  and  $DB_{it}$  do not always move in the same direction due to the effect of stochastic factors (no-shows), the true underlying relationship between the two is represented by a monotonic (strictly increasing) function.

Notice that when Eq. (1) is assumed to be true, an analysis of the optimal number of bumped passengers provides important policy-change implications on airline overbooking. For example, suppose that the revenue-maximizing  $DB_{it}$  of a particular carrier was found to be, on average, 20% lower than the actual levels. In this case, we know from Eq. (1) that, to maximize revenue, the airline should decrease its overbooking level such that its bumped passengers, on average, would be 20% lower than the current level. Although the model cannot predict by how much the airline's overbooking levels must be decreased to attain 20% reduction in bumped passengers, the model provides insights into the desired direction of policy change.

### 3. Optimization model

We seek the set of optimal overbooking levels that maximizes an airline's cumulative passenger revenue in the US over the planning horizon (for time periods  $t = 1, 2, \dots, T$ ). Let us define the US domestic passenger revenue of carrier  $i$  at time  $t$  by the following formula:

$$Rev_{it} = P_{it}MILE_{it}Q_{it}, \quad (2)$$

where  $P_{it}$  is the average per-mile airfare of carrier  $i$  at time  $t$  which is measured by the carrier's US domestic yield (revenue per passenger-mile) at time  $t$ ,  $MILE_{it}$  is the average trip length of carrier  $i$  passengers at time  $t$  in the US domestic market, and  $Q_{it}$  is the total passenger enplanement of carrier  $i$  at time  $t$  in the US. Observe that  $P_{it}MILE_{it}$  represents the average (or expected) per-passenger revenue of carrier  $i$  at time  $t$ .

Notice that Eq. (2) correctly measures the passenger revenue of an airline only if the airline does not practice overbooking. When overbooking is used, an airline's per-passenger revenue must be adjusted by the expected amount of compensation per bumped passenger. After making this adjustment, the expected per-passenger revenue of airline  $i$  at time  $t$  becomes:

$$P_{it}MILE_{it}(1 - (DB_{it}/Q_{it})) + (P_{it}MILE_{it} - COMP_i)(DB_{it}/Q_{it}), \quad (3)$$

where  $DB_{it}$  is defined the same as before, and  $COMP_i$  is the average compensation that airline  $i$  pays per bumped passenger. Observe that  $DB_{it}/Q_{it}$  measures the percentage of airline  $i$ 's passengers who are denied boarding at time  $t$ , and  $1 - (DB_{it}/Q_{it})$  measures the percentage of those

who are not bumped. After replacing  $P_{it}MILE_{it}$  of Eq. (2) by the functional form given in Eq. (3), and re-arranging the terms, we obtain the following formula:

$$Rev_{it} = P_{it}MILE_{it}Q_{it} - COMP_iDB_{it}. \quad (4)$$

Eq. (4) indicates that, when overbooking is used, an airline's expected passenger revenue equals the "gross" passenger revenue less the total compensation paid to the bumped passengers.

The passenger enplanement of carrier  $i$  at time  $t$  ( $Q_{it}$ ) is a function of variables representing the carrier's attractiveness at time  $t$ , such as price (airfare) (e.g., Suzuki et al. (2000)).  $Q_{it}$  can also be considered as a function of the "current" and "past" values of bumped passengers ( $DB_{it}$ ), because (as mentioned previously): (1) higher  $DB_{it}$  can increase the passenger load factor, and (2) higher  $DB_{it}$  can decrease the carrier's "future" passenger enplanement due to "bad" experience effects. If we let  $q(\cdot)$  be the function representing the relationship between  $Q_{it}$  and the carrier attractiveness variables, including  $DB_{it}$ , Eq. (1) can be re-written as follows:

$$Rev_{it} = P_{it}MILE_{it} \cdot q\left(\sum_{j=0}^J DB_{it-j}, \sum_{j=0}^J DB_{kt-j}, \sum_{m=1}^M X_{mit}, \sum_{m=1}^M X_{mkt}\right) - COMP_iDB_{it}, \quad (5)$$

where  $DB_{it-j}$  is the  $j$ -period lagged bumped passengers of carrier  $i$  ( $j = 0, 1, \dots, J$ ),  $DB_{kt-j}$  is the  $j$ -period lagged bumped passengers of carrier  $k$  (competitor of carrier  $i$ ,  $i \neq k$ ), and  $X_{mit}$  ( $X_{mkt}$ ) is the  $m$ th non-bumped-passenger variable representing the attractiveness of carrier  $i$  ( $k$ ) (e.g., price) at time  $t$  ( $m = 1, 2, \dots, M$ ). Observe that, since the values of  $j$  include zero,  $Q_{it}$  is a function of both the current and past values of  $DB_{it}$ . We include competitive variables such as  $DB_{kt-j}$  and  $X_{mkt}$  in Eq. (5), because a carrier's demand in a given market is determined by how attractive the carrier is in the market *relative to* other carriers (competitors).

An airline's objective is to maximize the right-hand-side of Eq. (5) by manipulating (or adjusting) its overbooking (bumped-passenger) levels over the planning horizon. This problem can be formulated as a non-linear mathematical programming model of the following form:

$$\text{Max}_{\delta_{it}} \left\{ \sum_{t=1}^T \left[ P_{it}MILE_{it} \cdot q\left(\sum_{j=0}^J [DB_{it-j}\delta_{it-j}], \sum_{j=0}^J DB_{kt-j}, \sum_{m=1}^M X_{mit}, \sum_{m=1}^M X_{mkt}\right) - COMP_iDB_{it}\delta_{it} \right] \right\}, \quad (6a)$$

subject to:

$$\delta_{it} \geq 0 \quad \forall i, t, \quad (6b)$$

where  $\delta_{it}$  is a policy-change indicator (% change in denied-boarding passengers) for airline  $i$  at time  $t$ , which can theoretically range from zero (no bumped-passenger policy) to positive infinity (infinitely many passengers should be bumped). Observe that the change-indicator value of 1 means no policy change, whereas the value larger than 1 and that less than 1 mean positive and negative policy changes, respectively.

It should be noted that all of the variable values in Eq. (6a), except for  $\delta_{it}$ , are the "actual" values (historical data). The model, therefore, maximizes an airline's past passenger revenues over the planning horizon by changing the past overbooking (bumped-passenger) levels, while holding other variable values constant. See Table 1 for a detailed discussion of how the model explains the effect of overbooking on airline passenger revenues.

Table 1  
Implied revenue effect of overbooking

Variable	Change direction	Direct effect	Implied revenue effect	Parameters
Overbooking ( $\delta_{it}$ )	Increase	Increase bumped passengers	*Current revenue increase <sup>a</sup>	$\lambda_0$
		Reduce unfilled seats	Current revenue increase	$\lambda_0$
		Decrease future demand	*Future revenue decrease <sup>b</sup>	$\lambda_j$
		Increase compensation to passengers	Current revenue decrease	$COMP_i$

\* These represent the “bumped passenger effects” which have not been considered in the past overbooking studies.

<sup>a</sup> Because a certain proportion of “bumped” passengers use later flights of the same airline (which “bumped” them).

<sup>b</sup> Due to possible negative future effect of denied boarding on passenger demand (market share). Note that a certain number of passengers may re-fly within the same time period (e.g., frequent flyers), which implies that the “future” bumped-passenger effect may appear within the same time period.

#### 4. Econometric model

To solve our optimization model we need to specify the functional form for  $q(\cdot)$ . To determine  $q(\cdot)$ , first note that the passenger demand of carrier  $i$  at time  $t$  can be written as:

$$Q_{it} = MKT_t S_{it}, \quad (7)$$

where  $MKT_t$  is the overall market size (total passenger enplanement of the US domestic market) at time  $t$ , and  $S_{it}$  is the market share of carrier  $i$  at time  $t$ . We model  $S_{it}$  rather than  $Q_{it}$  to estimate the passenger demand. By using this approach, we can eliminate the variables representing the time-specific effects from our model. Although the market share model does not directly provide the estimates of  $Q_{it}$ , the estimates can be easily obtained by multiplying the predicted  $S_{it}$  value and  $MKT_t$  (note that  $MKT_t$  values are available from the historical data).<sup>2</sup>

We use the following attraction function to model a carrier’s market share:

$$S_{it} = \frac{A_{it}}{\sum_{h=1}^n A_{ht}}, \quad (8)$$

where  $i, k \in h, n$  is the number of carriers in our data, and  $A_{it}$  is the attractiveness of airline  $i$  at time  $t$ , which is a function of the carrier attractiveness variables included in  $q(\cdot)$ . Eq. (8) requires that the predicted values of  $S_{it}$  be bounded between zero and one ( $\forall i, t$ ), and assures that the summed market shares of all carriers be always 1 ( $\forall t$ ). We specify  $A_{it}$  as follows:

$$A_{it} = \exp(\alpha_i + \varepsilon_{it}) \prod_{j=0}^J DB_{it-j}^{\lambda_j} \prod_{m=1}^M X_{mit}^{\beta_m}, \quad (9)$$

where  $\alpha, \beta$ , and  $\lambda$  are the model parameters, and  $\varepsilon_{it}$  is a stochastic error term. Eq. (9) is equivalent to the familiar Cobb–Douglas model, which captures the interaction effect among all of the right-hand side variables, and is capable of creating both the linear (when  $\lambda_j, \beta_m = 1$ ) and non-linear

<sup>2</sup> This statement implicitly assumes that, when we change overbooking levels (bumped passengers) in our optimization model (while holding other effects fixed), market size does not change. This assumption may be considered reasonable because, unlike airfares, overbooking level does not stimulate market demand.

forms. Note that our model captures the carrier-specific effects, including the effect of missing carrier variables, by the carrier-specific constants (i.e., fixed coefficient modeling method).

When  $A_{it}$  of Eq. (8) is replaced by the functional form given in Eq. (9), the resulting function converges to the standard multiplicative competitive interaction (MCI) model (Cooper and Nakanishi, 1988), which is widely used in the marketing literature. By using the MCI model to measure  $Q_{it}$ , the objective function of our optimization model creates quite a flexible non-linear response surface with respect to  $\delta_{it}$ , depending on model parameter values such as  $MKT_t$ ,  $DB_{it}$ ,  $COMP_i$ , and  $\alpha, \beta, \lambda$  regression coefficients.

Cooper and Nakanishi (1988) show that the MCI model can be linearized as follows:

$$\ln \left( \frac{S_{it}}{S_t^*} \right) = \alpha_1 + \sum_{h=2}^n (\alpha_h - \alpha_1) C_h + \sum_{j=0}^J \lambda_j \ln \left( \frac{DB_{it-j}}{DB_{t-j}^*} \right) + \sum_{m=1}^M \beta_m \ln \left( \frac{X_{mit}}{X_{mt}^*} \right) + (\varepsilon_{it} - \bar{\varepsilon}_t), \quad (10)$$

where  $C_h$  is a dummy variable representing the  $h$ th airline ( $C_h = 1$  if  $i$ , 0 if not  $i$ ),  $S_t^*, DB_{t-j}^*, X_{mt}^*$  are the geometric means of  $S_{it}, DB_{it-j}, X_{mit}$ , respectively,  $\bar{\varepsilon}_t$  is the arithmetic mean of  $\varepsilon_{it}$ , and the subscript 1 represents an arbitrary “base” carrier. Since the model is linearizable, it is easily tractable in our optimization model and, assuming that the error terms are normally distributed, the model parameters can be estimated by a standard linear regression technique. While other specifications for the  $q(\cdot)$  function are also possible, we believe that the above formulation achieves the goal of this paper with less complexity and easier model interpretation.

## 5. Empirical analysis

### 5.1. Data

We use the data collected from the following publications: Air Travel Consumer Report; Air Carrier Traffic Statistics; Air Carrier Financial Statistics; Annual Review of Aircraft Accident Data; Monthly Labor Review and the US DOT web-site. The data set contains 40 consecutive quarterly observations (first quarter 1988 to fourth quarter 1997) of the 10 largest US airlines (Alaska, America West, American, Continental, Delta, Northwest, Southwest, Trans World, United, US Air). The total sample size is 400. These 10 carriers (major carriers) account for more than 90% of the domestic passenger revenue of the whole industry. Only the data for domestic passenger operations (scheduled services) are used. Table 2 shows the descriptive statistics of model variables.

### 5.2. Attractiveness variables

We use the following as  $X_{mit}$  variables: price ( $P_{it}$ ), average trip length ( $MILE_{it}$ ), number of airports served, frequency of flight services, and accident record. Except for  $MILE_{it}$ , these variables represent airline attractiveness and, as shown in previous airline demand studies, they all affect

Table 2  
Descriptive statistics (1988–1997 average figures)

	Market share	Passengers denied boarding	Price per passenger mile	Average flight miles	Airports served	Accidents per quarter	Departures per airport
Alaska	0.0172	3403	0.1471	813.8	33.0	0.000	885
America West	0.0386	11,485	0.1083	740.5	46.7	0.000	1076
American	0.1668	33,812	0.1286	1010.9	107.9	0.025	1753
Continental	0.0815	16,870	0.1211	932.7	85.5	0.000	1299
Delta	0.1874	27,850	0.1402	793.9	127.2	0.025	1746
Northwest	0.0947	21,895	0.1342	848.8	105.7	0.025	1209
Southwest	0.0855	14,938	0.1161	461.4	38.5	0.000	3185
Trans World	0.0499	9711	0.1226	947.4	73.6	0.025	915
United	0.1492	26,669	0.1231	1036.9	112.3	0.050	1504
US Air	0.1292	20,733	0.1687	606.6	108.8	0.100	1852

airline market share.<sup>3</sup> We include  $MILE_{it}$  in our market share model as a control variable to appropriately capture the price effect (this issue is discussed later). The number of airports served, the frequency of service, and the accident record reflect the availability of flights to a given origin–destination route, the relative convenience of flight schedule, and the safety reputation of a carrier, respectively (Proussaloglou and Koppelman, 1995).

### 5.3. Definition of measures

*Market share ( $S_{it}$ )*. The market share is computed by dividing the total number of passengers transported by carrier  $i$  at time  $t$  in the US domestic market by the total number of passengers transported by the 10 carriers included in our data set at time  $t$  in the US domestic market (all booking classes). Although carriers other than the largest 10 were also present in the US domestic market between 1988 and 1997, we do not use data for these smaller carriers to calculate market shares. This approach, which ignores market shares of non-major carriers, can be validated if we employ the “constant ratio assumption”, which has been used by several airline empirical studies. For a detailed discussion of this assumption, see e.g., Suzuki (2000).

*Market Size ( $MKT_t$ )*. The overall size of the US domestic airline passenger market at time  $t$  is defined as the summed passenger enplanement (US domestic passenger enplanement) of the 10 major carriers at time  $t$ .

*Passengers denied boarding ( $DB_{it}$ )*. This variable represents the total number of passengers denied boarding by airline  $i$  at time  $t$  in the US domestic market. Assuming that airline passengers

<sup>3</sup> Airline demand studies that used these variables as right-hand-side variables include Corsi et al. (1997), Borenstein (1991), Dresner and Windle (1992), Nako (1992), Proussaloglou and Koppelman (1995), Yoo and Ashford (1996), Suzuki (1998), Suzuki (2000), and Suzuki et al. (2000).

have the inter-travel times of less than a year, we use four lagged  $DB_{it}$  variables in the market share model to capture the future bumped-passenger effect ( $J = 4$ ).<sup>4</sup>

*Price ( $P_{it}$ )*. This variable represents the US domestic yield (revenue per passenger-mile) of carrier  $i$  at time  $t$ . We measure this variable by the constant dollar (i.e., CPI adjusted).

*Average trip length ( $MILE_{it}$ )*. We include average trip length in the market share model as a control variable. As discussed previously, we measure a carrier's price of transportation service by the revenue per passenger-mile. Although used in several empirical studies, this price measurement may not represent the true level of airfares, because the per-mile airfare tends to decrease as the overall flight length increases. This problem, however, can be resolved by including the average trip length ( $MILE_{it}$ ) into the right-hand side of our market share model. Notice that if  $MILE_{it}$  is included, the interpretation of the price coefficient in our model becomes "the effect of price on market share when the average trip length is held fixed." Thus, by including  $MILE_{it}$ , the effect of price on market share can be captured appropriately.

*Number of airports served ( $PORT_{it}$ )*. An important component of airline demand model is a measure of network size. We measure the network size of an airline at time  $t$  by the number of airports served by the airline in the US domestic market at time  $t$ .

*Frequency of service ( $FREQ_{it}$ )*. We approximate a carrier's service frequency by the average flight departures per airport. This variable is obtained by dividing the total number of departures performed by an airline at time  $t$  by the number of airports served by the airline at time  $t$ .

*Accident record ( $CRASH_{it}$ )*. This is a dichotomous (0/1) measure indicating whether carrier  $i$  had "notable" air accident(s) at time period  $t$ . An accident is considered "notable", if the accident involves fatalities and the aircraft damage is classified as "destroyed." We use only the one-period lagged accident record in our market share model because the "current" accident record may be endogenous. Often, the accident record of a carrier is viewed as a function of the carrier's size (market share), because the frequency of accidents depends on the amount of risk exposures (e.g., number of flight departures and average flight miles) (Rose, 1990).<sup>5</sup> The lagged accident record, on the other hand, shall be viewed as exogenous, for the "current" value of the dependent variable can have no effect on the "past" values of the right-hand side variables.

*Average compensation per bumped passenger ( $COMP_t$ )*. Since data for this variable are unavailable, we contacted all of the 10 major airlines included in the data and asked if they knew the average compensation. Although none of the airlines knew the exact average compensation, many gave us the range of compensations they pay for a bumped passenger, which ranged from \$25 to \$1000. (Amount of compensation varies from passenger to passenger depending on such factors as flight miles, booking classes, and the availability of substitution flights.) In this study, we

<sup>4</sup> Note that  $DB_{it}$  has lagged effects on market share because the passengers with "bad" experiences may avoid using the same airline for subsequent trips. This condition suggests that, on average, the duration of the negative (future) bumped-passenger effect on market share should be less than or equal to the average inter-travel time of airline passengers, which is typically one to three months (see Suzuki, 2000). Thus, the use of four lagged  $DB_{it}$  variables, which allows the negative bumped-passenger effect to last for up to a year, may be considered reasonable.

<sup>5</sup> Initially, we tried to use the predicted values of the accident record to include the "current" accident record into our model (i.e., two-stage least squares). We employed the Poisson regression model (e.g., Rose, 1990) to predict the accident record, but the model fit was poor. For this reason, we chose to simply eliminate the "current" accident record from our econometric model rather than to use the poorly predicted values of the accident record.



calculate the optimal overbooking (bumping) levels of all airlines for a wide range of compensation values that encompass the “observed” range of \$25–\$1000 (in increment of \$10). By examining how an airline’s optimal  $\delta_{it}$  values react to the changes of average compensation per bumped passenger, we attempt to draw implications on how each airline should change its overbooking policy.

#### 5.4. Method

As discussed previously, we calibrate the market share model by linearizing the model via Cooper and Nakanishi transformation and applying the standard least-squares method. Assuming that all of the right-hand side variables are exogenous, we estimate model parameters by the ordinary least squares (OLS). We solve our optimization model by the quasi-Newton’s non-linear search algorithm. Since Eq. (6a) does not always produce a strictly concave function with respect to  $\delta_{it}$ , we use a variety of starting values to search for the global maximum.

### 6. Results

#### 6.1. Econometric estimation

Table 3 shows the estimation results of the market share model. All of the estimated coefficients, except  $\lambda_4$ , have the expected signs and are generally statistically significant. The fit of the model is good, with more than 99% of the data variance explained by the model. In terms of  $t$ -values, the best predictors of airline market shares, aside from carrier constants, are the number of airports served, frequency of flight services, and price. The  $\lambda_0$  coefficient is positive and significant, and  $\lambda_1$  is negative and significant. This pattern implies that increasing the overbooking level improves an airline’s “current” market share, but it also reduces the airline’s future market share. Therefore, a significant negative bumped-passenger effect was detected.

#### 6.2. Optimization results

Optimization results are shown in Fig. 1 and Table 4.<sup>6</sup> Due to space limitation, we report only the average optimal  $\delta_{it}$  values of airlines (average % change in denied-boarding passengers that airline  $i$  must make to maximize revenue). Fig. 1 shows how optimal  $\delta_{it}$  values react to different  $COMP_i$  values by carrier. Table 4 shows, for each carrier, the level of  $COMP_i$  where a carrier’s optimal  $\delta_{it}$  becomes 1 (the point where an airline’s curve intersects  $\delta_{it} = 1$  line in Fig. 1; i.e., optimal  $\delta_{it} \geq 1$  if compensation is less than this value, but optimal  $\delta_{it} < 1$  if compensation is more than this value). We refer to this level of  $COMP_i$  value (where optimal  $\delta_{it} = 1$ ) as the “threshold level of compensation” from now on.

The results indicate that, as expected, the higher the average compensation per bumped passenger, the lower the optimal overbooking levels (see Fig. 1). The results also indicate that, for all

<sup>6</sup> Since  $\lambda_2$ ,  $\lambda_3$ , and  $\lambda_4$  are not-at-all insignificant (see Table 3), they are set to zeros during model optimization.

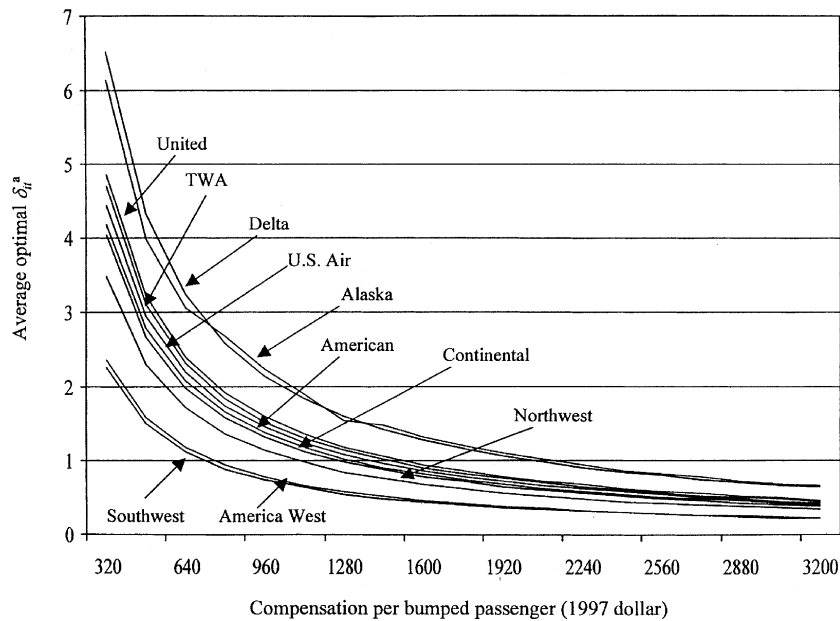
Table 3  
Estimation results (market share model)

	Coefficient	S.E.	<i>t</i> -statistics
Model intercept ( $\alpha_1$ )	−0.2594	0.028	9.19 <sup>a</sup>
Carrier constants ( $\alpha_h - \alpha_1$ )			
Alaska	Base	—	—
America West	0.1423	0.022	6.59 <sup>a</sup>
American	0.4487	0.042	10.65 <sup>a</sup>
Continental	0.1895	0.032	6.00 <sup>a</sup>
Delta	0.4555	0.043	10.68 <sup>a</sup>
Northwest	0.2039	0.036	5.70 <sup>a</sup>
Southwest	0.3416	0.049	6.90 <sup>a</sup>
Trans World	0.1256	0.026	4.84 <sup>a</sup>
United	0.4113	0.040	10.6 <sup>a</sup>
US Air	0.2753	0.044	6.33 <sup>a</sup>
Denied boarding coefficients ( $\lambda_j$ )			
Current effect ( $\lambda_0$ )	0.0430	0.009	4.88 <sup>a</sup>
1-period lagged ( $\lambda_1$ )	−0.0232	0.009	2.69 <sup>a</sup>
2-period lagged ( $\lambda_2$ )	−0.0033	0.009	0.38
3-period lagged ( $\lambda_3$ )	−0.0057	0.008	0.67
4-period lagged ( $\lambda_4$ )	0.0090	0.009	1.05
Carrier attractiveness coefficients ( $\beta_m$ )			
Price (yield)	−0.4288	0.039	10.94 <sup>a</sup>
Passenger flight miles	−0.0416	0.053	0.79
Number of airports served	0.9820	0.031	31.49 <sup>a</sup>
Crash record (1-period lagged)	−0.0186	0.018	1.01
Frequency of carrier service	0.7026	0.032	22.10 <sup>a</sup>
Sample size	400		
Fit statistic ( $R^2$ )	0.995		

<sup>a</sup> Significant at the 99% level.

carriers, the “true” optimal overbooking levels are higher than the actual levels, if the average compensation is less than \$720 (notice that \$720 is the smallest “threshold” value in Table 4). This condition implies that, even when the negative “bumped-passenger effect” is considered, the marginal revenue of overbooking an additional passenger is always higher than its marginal cost within the reasonable range of average compensations per bumped passenger (\$0–\$720).<sup>7</sup> Our results, therefore, suggest that airlines may benefit by increasing overbooking. Note, however, that the above implication is valid only if an airline can increase its overbooking level *while*

<sup>7</sup> Although our interviews indicated that the compensation can range from \$25 to \$1,000, these figures represent the compensation for “compensated” passengers, not the average compensation per bumped passenger. (Not all bumped passengers receive compensation – for example, if an airline puts a bumped passenger to another flight that arrives within one hour of the original arrival time, the airline does not have to pay compensations.) The average compensation per bumped passenger (which include both the compensated and non-compensated passengers) is much lower than the figures that we obtained from airlines. The study by Alstrup et al. (1986), which analyzed the actual airline data, estimates that the average compensation per bumped passenger is less than \$50 in 1997 dollar.



<sup>a</sup> Average % change in denied-boarding passengers that airline  $i$  must make to maximize revenue.

Fig. 1. Optimization results (average optimal  $\delta_{it}$  by carrier).

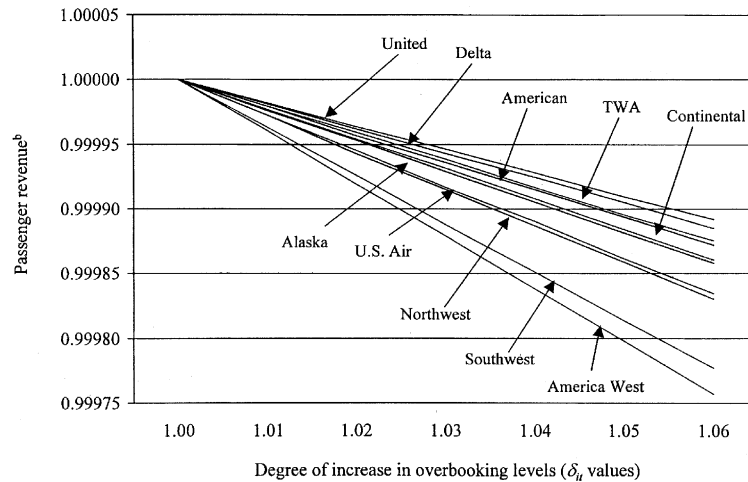
Table 4  
Optimization results

	Threshold level of compensation <sup>a</sup> by airline (1997 dollar)
Alaska	2112
America West	752
American	1312
Continental	1264
Delta	2048
Northwest	1104
Southwest	720
Trans World	1456
United	1520
US Air	1392

<sup>a</sup> The level of  $COMP_i$  where a carrier's optimal  $\delta_{it}$  becomes 1. If average compensation per bumped passenger is lower than this value the "true" optimal overbooking level is higher than the actual level, whereas if average compensation is higher than this value the "true" optimal overbooking level is lower than the actual level.

holding the overbooking levels of other carriers constant. This is an unlikely scenario, because if a carrier increases overbooking and improves its passenger revenue, other carriers will also increase overbooking levels to re-gain the lost passenger revenues. Thus, an increased overbooking level may improve an airline's passenger revenue only in the short run.

To understand the long-run effect of increased overbooking levels, we perform a simulation analysis and investigate how passenger revenues of airlines would change if all the airlines increase



<sup>a</sup> During the simulation we assumed that the average compensations of airlines are proportional to their per-mile airfares (e.g., if Delta's per-mile airfare is 5% higher than that of US Air, the average compensation of Delta is also 5% higher than that of US Air). See Table 2 for average per-mile airfares of airlines. The above figure shows the simulation results for the scenario where the overall average of average compensations across airlines is equal to \$100.

<sup>b</sup> Value of eqn (9) which is normalized such that the "observed" revenue (when  $\delta_u = 1$ ) is 1 for all airlines.

Fig. 2. Changes of passenger revenue by airline when all airlines increase overbooking simultaneously<sup>a</sup>.

overbooking levels simultaneously by the same percentage. The simulation is performed for a variety of average compensation levels, and all of them show very similar results. Fig. 2 shows a typical result of our simulation runs. The figure indicates that, if all of the airlines increase overbooking levels simultaneously, every carrier is worse off.<sup>8</sup> This pattern suggests that, while airlines may be tempted to increase overbooking levels to attain short-run revenue improvements, they may not wish to do so because it may result in the outbreak of an "overbooking war", the end result of which is a decreased revenue for every carrier.

In sum, our optimization results should not be interpreted as "airlines should increase overbooking levels". Rather, they should be interpreted as "an airline should *not* decrease its overbooking level as long as the average compensation per bumped passenger is less than the threshold level for the airline". Notice that if an airline decreases its overbooking level and loses revenue, no other carrier will follow this practice. Thus, in both the short and the long runs, an airline's revenue will always decrease if the airline decreases its overbooking level.

It is worth noting that both the optimization and simulation results imply that high-fare carriers may receive higher benefit from the use of overbooking than other types of carriers. Our optimization results (Table 4, Fig. 1) indicate that high-fare carriers are more tolerant to higher

<sup>8</sup> This pattern may reflect the condition that, if all airlines increase overbooking levels by the same percentage, neither the market size nor the carriers' market shares change (recall that overbooking does not stimulate market demand), but the only change is that airlines are giving reservations of the highly demanded (conveniently scheduled) flights to those passengers who would have reserved the less-demanded flights if overbooking levels were not increased. If this is the case, the "gross" passenger revenues of airlines do not increase, while the total amount of compensation paid to the bumped passengers does.

Table 5  
Sensitivity analysis (1997 dollar)

	Smallest “threshold” compensation observed when $\lambda_0$ is adjusted	Smallest “threshold” compensation observed when $\lambda_1$ is adjusted
Alaska	832	1376
America West	336	528
American	608	928
Continental	544	864
Delta	880	1392
Northwest	464	736
Southwest	320	512
Trans World	624	992
United	640	1024
US Air	608	960

compensations than low-fare carriers. Table 4 shows that Southwest, a low-fare carrier, must reduce its overbooking level once the average compensation reaches the level as low as \$720, whereas Delta, a high-fare carrier, does not have to reduce overbooking until the average compensation exceeds \$2000 (see Table 2 for average per-mile airfares of airlines). This pattern makes intuitive sense, because the marginal revenue of overbooking an additional passenger for low-fare carriers is small and likely to be more quickly offset by an increased compensation per bumped passenger than high-fare carriers. Further, our simulation results indicate that, if all airlines increase overbooking levels, passenger revenues of low-fare carriers decrease more quickly than those of high-fare carriers (see Fig. 2). These results imply that low-fare carriers are more likely to suffer from the aggressive use of overbooking strategy than high-fare carriers.

### 6.3. Sensitivity analysis

We examine the sensitivity of model solutions to changes in parameter values that represent the effects of denied-boarding passengers in our optimization model, namely,  $\lambda_0$  and  $\lambda_1$ , both of which are estimated by an econometric method. The range of parameters that we examine is from  $-25\%$  to  $+25\%$  for both parameters.

Table 5 shows, for each airline, the smallest “threshold” compensation (where the optimal  $\delta_{it}$  equals 1) that is observed when  $\lambda_0$  and  $\lambda_1$  are adjusted within the range of our sensitivity analyses. We see that the lowest threshold compensation in the table is \$320, which happens for Southwest Airlines when  $\lambda_0$  is adjusted. This condition implies that, for a reasonable range of parameter deviations from the estimated values ( $\pm 25\%$ ), the optimal  $\delta_{it}$  value is still higher than 1 for all airlines within the realistic range of average compensations (\$0–\$320: see Footnote 7). Therefore, for the entire range of sensitivity analyses, we consistently obtain same managerial implications suggesting that no airline should decrease overbooking levels.

## 7. Conclusion

We have empirically examined the overbooking by airlines in the US domestic market by comparing their optimal overbooking levels, which maximize the overall (not just a single flight)

passenger revenues of airlines, with their actual levels. As expected, we found a significant “negative bumped-passenger effect” (the more passengers are bumped in the current period, the lower the market share of the airline in the future periods), which has been ignored by past overbooking studies. The optimization model produced fairly robust solutions, suggesting that no airline should decrease overbooking levels. This condition implies that, although there is a significant negative overbooking effect, the positive side of overbooking is so strong that it more than offsets its negative side. Our results, however, also suggest that, while airlines may be tempted to increase overbooking levels to attain higher revenues, they may not wish to do so. Our simulation results imply that, if an airline increases overbooking, it may trigger an “overbooking war” (all carriers increase overbooking levels), the end result of which is a decreased revenue for every airline. Given these results we conclude that, to date, airlines appear to have used appropriate levels of overbooking.

One limitation of the study is its aggregate modeling approach. The study results, in particular the econometric estimation results, may be subject to data aggregation biases. Although we have performed intensive sensitivity analyses to address this issue, the aggregate modeling approach provides rather limited insights. Future research could test the robustness of our study results by re-examining the optimal overbooking levels for airlines by using the disaggregate data, if such data are available.

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