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Term Paper

Integrating Capacity Controls and Overbooking

Lilian Do Khac (342121)
lilian.do.khac@rwth-aachen.de
Norman Hofer (342523)
norman.r.hofer@gmail.com
Henriette Zudrop (302086)
henriette.zudrop@rwth-aachen.de

Supervisor: Univ.-Prof. Dr. rer. pol. Catherine Cleophas

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Contents

List of Figures

List of Tables

1 Business Context and Overview

With accordance to the German airline company *Deutsche Lufthansa AG*, in the following referred to as *Lufthansa*, almost every 10th flight passenger does not utilize his or her reservation. The total number of these non-utilizations at Lufthansa accumulates to approximately 5 million customers annually. Reasons for this non-utilizations are both, self-inflicted (e.g. not cancelling multiple bookings) or unintentionally (e.g. missed connection flights). Airline companies such as Lufthansa try to minimize the resulting opportunity costs by specifically offering more seats than are actually available. This practice is referred to as *Overbooking* (Talluri & Van Ryzin 2004, p.131). Through overbooking practices at Lufthansa the total number of non-utilization was reduced by approximately 900,000 seats in 2002 which is equal to approximately 18 percent[1]. On the other hand, on average one out of thousand flight passengers was denied service due to overbooking practices at Lufthansa[2](Lufthansa 2004, p.29).

The U.S. based airline company *American Airlines* estimates that roughly 15 percent of their sold-out jets (flights that are closed) are not utilized. American Airlines estimated a total revenue opportunity from overbooking in year 1990 of roughly 250 million USD. They actually transformed about 90 percent of overbooking opportunities to revenues amounting to approximately 210 million USD (Smith et al. 1992, p.24-25). In table 1.1 further performance outcomes are depicted. These numbers were computed with the American Airlines in-house *overbooking revenue opportunity model*. The performance levels ranging from 1988 to 1990 remained stable at a level around 90 percent.

Table 1.1: Overbooking Performance at American Airlines, 1988-1990 (Smith et al. 1992, p.25)

Year	Revenue Opportunity Earned (%)	Revenue Earned (Million $)
1988	92	210
1989	93	235
1990	90	225

[1]Dividing 900 tsd. seats by total number of non-utilizations (5 million seats): $\frac{0.9million}{5million} = 0.18$
[2]Coined as "denied service"

In both cases of Lufthansa and American Airlines, overbooking proofed itself to be successful in preventing opportunity costs against non-utilization. According to Curry (1990), overbooking can lead to additional revenues for airlines of around three to ten percent. However, overbooking has to be implemented with caution, because it can also result in negative effects as shown in figure 1.1. The maximum capacity is displayed by the black dashed line. The more seats are allocated or sold , the more revenues can be generated (dark blue line). This involves passing the maximum capacity. However, as a matter of denied service cost, net revenues would fall again, because passengers that are affected by denied service have to be compensated for their reservation. The green line depicts increasing costs due to oversales.

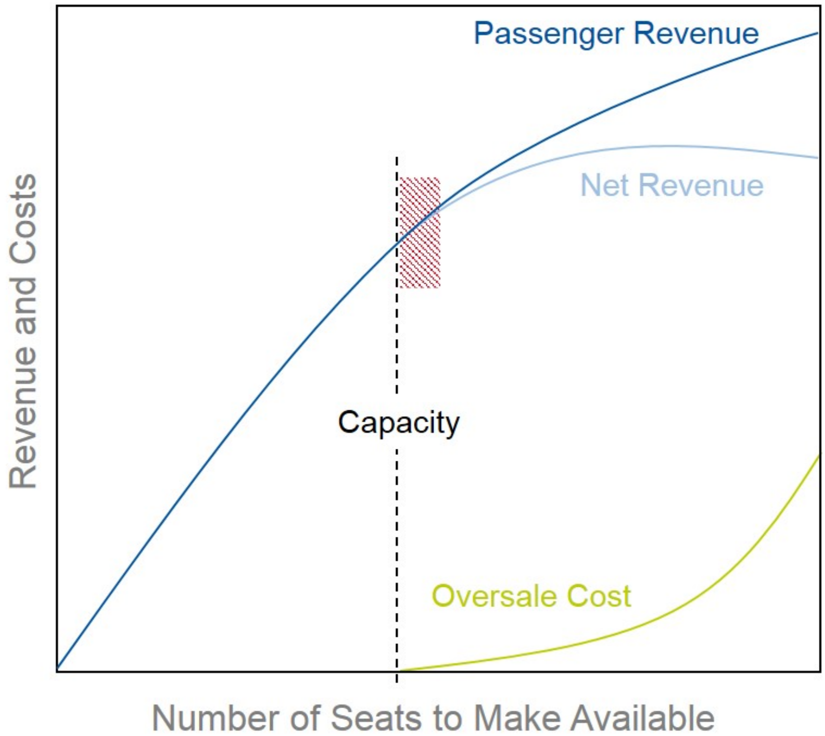

Figure 1.1: Optimal Overbooking Level in Accordance with Smith et al. (1992, p.12)

On the one hand, airlines have to bear in mind to find suitable compensations if passengers show up but are denied service due to overbooking. On the other hand, they have to manage the minimization of lost opportunities when they cannot utilize their total capacity (Talluri & Van Ryzin 2004, p. 130).

1.1 Historical Review of Overbooking

The practice of overbooking was accompanied by legal regulations. Up until the year 1961, overbooking was carried out clandestinely. However, when the Civil Aeronautics Board (CAB) in the U.S. conducted a study and found out about non-utilizations in the airline industry and the economic problems caused, they implemented penalty costs of 50 percent of the ticket prices for passengers who did not utilize the ticket and for airlines who denied bookings. Overbooking itself was not officially sanctioned though. In the year 1963 airline managements abandoned these penalties again, because they made traveling by airplane less attractive to customers. The CAB conducted another study regarding this issue from 1965 to 1966 and, as a result, officially sanctioned overbooking practices. Up until the year 1972 in the wake of the "Ralph Nadar" incident, regulations on overbooking practices were revised again. The resulting regulations as of 1974 are still valid today (Talluri & Van Ryzin 2004, p.131-134).

1.2 Goal Setting and Methodology

The goal of this term paper is to get a better insight to this rather new practice of combining overbooking methods with capacity controls and its benefits. Therefore, a literature research was conducted to summarize current techniques as displayed in theory. First of all, substantial terms will be outlined in chapter 2. Following this, mathematical approaches for overbooking are explained in chapter 3. Moreover, some selected models combining overbooking and capacity controls will be evaluated in chapter 3. In chapter 4 two numerical examples of combined overbooking and capacity controls are being conducted. This paper is finalized in chapter 5 by a conclusion and an outlook.

2 Theoretical Framework

Since the economic background and importance of overbooking is outlined in chapter 1, the aim of this chapter is to introduce the appropriate terminology to approach the mathematical background in the following chapters.

2.1 Terminology

2.1.1 Cancellations and No-shows

As one asset for products suitable for revenue management, there needs to be a limited capacity of the product. As demand may exceed the supply of a product, one may choose to offer reservations to the customers to create certainty about the availability. A reservation is a contract between a customer and the firm which gives the customer 'the right to use the service in the future at a fixed price' (Talluri & Van Ryzin 2004, p.130). Reservations often include the option to be canceled in advance, however, this can be related to penalty costs which the customer needs to pay. Therefore, cancellations refer to customers rejecting the product prior to the day of service. Additionally, there are also customers which do not cancel their reservation and still do not show up for the service on the day itself. This is called a 'no-show' (Klein & Steinhardt 2008, p.150).

2.1.2 Overbooking

Overbooking refers to accepting more reservations for a product or service than there is capacity. Regarding the fact that some customers cancel their reservation in advance or are counted as no-shows on the day of service, there may occur unused resources. To reduce the amount of unused resources, companies tend to use overbooking methods to maximize the load factor of their resources. However, the level of overbooking is based on predictions so that it may occur that there are more customers than resources which are available. In this case, the firm has to deny the service for some of the customers which is also associated with costs to compensate the inconvenience caused.

2.1.3 Capacity Control

Capacity controls are one of the major instruments of revenue management and refers to the acceptance and rejection of booking requests in order to maximize the revenue of the firm offering the product or service. Additionally, it aims to support the implementation of price differentiation in the reservation process.

2.2 Overbooking and Capacity Controls in the Context of Revenue Management

The integration of overbooking and capacity controls has to be distinguished into two approaches: Successive and simultaneous planning. Successive planning can be divided into two phases: First, the overbooking limits for each resource are fixed. Afterwards, these overbooking limits are included in the calculations for the capacity control instead of the actual capacity of a resource (Klein & Steinhardt 2008, p.160-161).

However, these two phases can also be processed simultaneously. So far, research on the process of simultaneous processing focuses on single flights. Only few research was conducted on capacity controls and overbooking in networks.

3 Mathematical Approaches for Overbooking

This chapter is devoted to the depiction of fundamental decision models for overbooking. At first, mathematical approaches for the overbooking decision will be shown for single-resource problems. These can be distinguished into two types. These are on the one hand static models, presented in section 3.1, and on the other hand dynamic models, shown in section 3.2. Then in section 3.3 combined decision models for overbooking and capacity controls are depicted.

3.1 Static Models

Static overbooking models ignore the dynamics of cancellations and new reservations over time. They determine the maximum number of reservations to hold at the current time for given estimates of cancellation rates until the day of service. The maximum number of reservations, the so called overbooking limit, will be computed periodically prior to service to estimate cancellation probabilities over time. Because of their simplicity, flexibility and robustness static overbooking models are the most widely used mathematical approaches in practice. In chapter 3.2 it is shown that there are more sophisticated approaches which consider the dynamic factors of overbooking decisions. As already mentioned, there are two types of events effecting the overbooking decision - cancellations and no-shows. In static models there is no need to make a distinction between both events, because static models assume a static overbooking limit that does not need to be adjusted. The only thing that matters is the so called show demand, which is the probability that a customer reservation survives to the time of service. Changes in cancellation and no-show probabilities over time are taken into account through periodically re-computations. The current overbooking limit shows the maximum number of reservations that will be accepted at any time. Figure 3.1 represents this process along time (Talluri & Van Ryzin 2004, p.138-139).

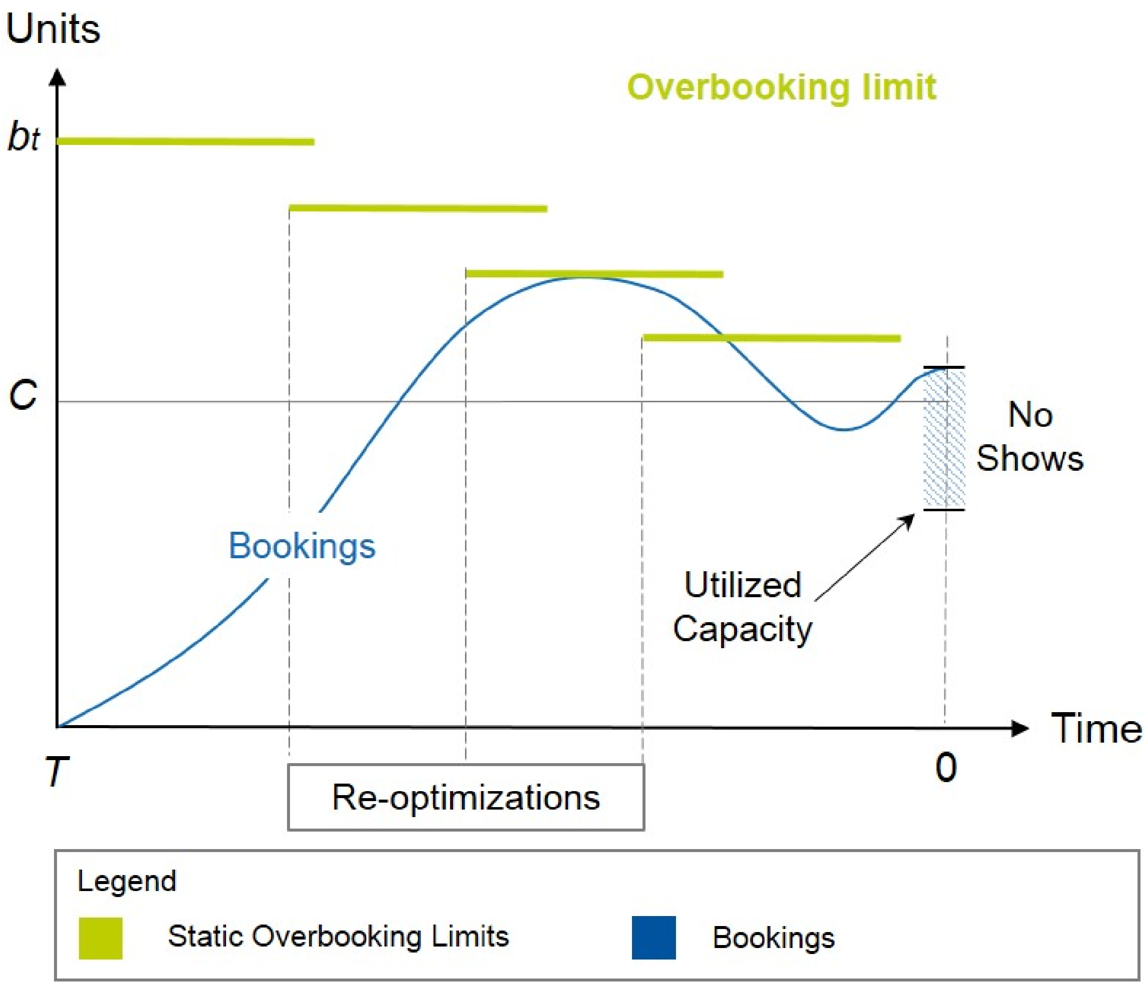

Figure 3.1: Illustration of Overbooking Limits and Reservations over Time in Accordance with Talluri & Van Ryzin (2004, p.140)

The upper curve represents the overbooking limit along time. Solving the static model at a certain point of time leads to a solution on this line. As the time of service T approaches, the overbooking limits decreases. With overbooking the accumulation of reservations in the system can exceed the existing capacity C. As the number of reservations reaches the overbooking level, new reservations will be rejected. Without overbooking, the show demand is noticeably lower than the available capacity, which would result in lost sales.

3.1.1 Binomial Model

The following description of the binomial model is based on Talluris' depiction of the model (Talluri & Van Ryzin 2004, p.139-141). Among the static models the binomial model is the simplest one. It makes no difference between no-shows and cancellations. No-shows are treated as cancellations done at the day of service. The binomial model is based on the following assumptions:

- Cancellations are independent of each other
- The probability of canceling a reservation is the same for each customer
- The cancellation probability is Markovian - That means the probability of canceling is only dependent on the time left to service.

Let y denote the number of reservations received until this point, C denote the available capacity, t the remaining time until service and q denote the probability that a customer who made a reservation shows up at the time of service. Hence, $1-q$ is the probability that a customer cancels his reservation prior to the time of service. For reasons of simplification in the notation the dependence of q on t is suppressed. Under the stated assumptions the number of customers showing up at the time of service (show demand) denoted $Z(y)$ is binomially distributed with p.d.f.:

$$P_y(z) = P(Z(y) = z)$$

$$= \binom{y}{z} q^z (1-q)^{y-z} \quad \text{for } z = 0, 1, ..., y, \tag{3.1}$$

with c.d.f.:

$$F_y(z) = P(Z(y) \leq z)$$

$$= \sum_{k=0}^{z} \binom{y}{k} q^k (1-q)^{y-k}, \tag{3.2}$$

with a mean of $E[Z(y)] = qy$ and a variance of $Var(Z(y)) = yq(1-q)$. As a convenience the use of the complement distribution of F_y is recommended, which is denoted by $\overline{F}_y$ and defined as follows:

$$\overline{F}_y(z) = P(Z(y) > z) = 1 - F_y(z). \tag{3.3}$$

For the determination of overbooking limits we will depict two fundamental types in the following. At first, we will see models that determine the overbooking limit based on service levels and secondly we will depict models that orientate on managerial aspects in the determination of the overbooking limit.

Overbooking based on Service Level Criteria

In general, the service level describes the ability to satisfy the show demand. More precisely, the so called Type 1 service level describes the probability of oversales at the point of service. The variable x denotes the overbooking limit. As long as the current number of reservations y does not exceed x, incoming reservations are accepted. When x is becoming larger than y, all new reservations will be rejected. The Type 1 service level, denoted by $s_1(x)$, is then defined by:

$$s_1(x) = \overline{F}_x(C) = P(Z(x) > C). \tag{3.4}$$

Choosing an overbooking limit x ensures that the probability of oversales will not be larger than $s_1(x)$. An essential disadvantage of the Type 1 service level is that it only captures the fact whether a denied boarding occurs or not. It gives no information about the number of denied boardings in relation to the reservations on hand y. This drawback is compensated when using the so called service level Type 2 (Klein & Steinhardt 2008, p.157).
The Type 2 service level is given by:

$$\begin{aligned} s_2(x) &= \frac{E[Z(x) - C)^+]}{E[Z(x)]} \\ &= \frac{\sum_{k=C+1}^{x}(k - C)P_x(k)}{x(1-q)}. \end{aligned} \tag{3.5}$$

By some simplifications the equation can be transformed into an easier form for computation:

$$s_2(x) = \overline{F}_{x-1}(C-1) - \frac{C}{qx}\overline{F}_x(c). \tag{3.6}$$

In practice, a firm at first defines a service level and then by numerical search identifies the largest overbooking level x^* satisfying the given service level. If the demand is lower than x^* the actual service level is higher than the specified $s_1(x^*)$ and $s_2(x^*)$. So effectively, a worst-case service level is considered, not an average service level. The calculation of the average service level would not be complicated as well, but is not recommended in practice. Customers who make reservations only on busy days will perceive a service level closer to $s_2(x)$ than to $\overline{s}_2(x)$. So an orientation on the worst-case service level guarantees that each customer perceives at least the defined service level (Talluri & Van Ryzin 2004, p.143-144).

Overbooking based on Economic Criteria

Another possibility to determine the overbooking limit is the orientation on economic criteria. The here shown model and its theoretical background is based on Talluri & Van Ryzin (2004, p.144-146). When the specification of the overbooking limit is based on economic criteria, estimates for the revenue loss through a denied reservation and the cost of a denied service are necessary.

Let z denote the number of customers showing up at the time of service (show demand), $c(z)$ denotes the denied-service costs, whereby we assume that each denied service costs a constant managerial amount h, so that,

$$c(z) = h(z-C)^+. \tag{3.7}$$

Let p denote the additional revenue generated through the acceptance of another reservation. For reasons of simplification, p is assumed to be constant. Then, the total expected revenue when given y reservations is

$$V(y) = py - E[c(Z(y))]. \tag{3.8}$$

For the binomial model, if c(.) is convex and thus V(.) is concave, it is optimal to accept reservations as long as the marginal profit is positive. As soon as it becomes negative, new reservations should be rejected. Thus, the optimal booking limit x^* is given by the largest value of x satisfying

$$\triangle V(x) = E[c(Z(x))] - E[c(Z(x-1))] \leq p,$$

what can be reduced for the binomial model to

$$hqP(Z(x-1) \geq C) \leq p. \tag{3.9}$$

Equation 3.9 can be interpreted as following: If the x^{th} reservation is accepted, penalty costs of h have to be payed if the existing demand on hand y consumes the whole available capacity C and the x^{th} customer shows up at time of service. Thereby the left part of equation 3.9 is the marginal penalty multiplied by the occurance probability of the described situation. x^* is the largest value for x where the marginal revenue is larger than the marginal costs.

3.1.2 Static Model Approximations

Often it is desirable to express the overbooking limit in a compact form equivalent to the simplicity of the binomial model. In the following approximations of the overbooking limit are shown that aim at such a simplification. The shown approximations orientate on Talluri & Van Ryzin (2004, p. 147-148).

Deterministic Approximation

The idea behind the deterministic approximation is that the overbooking limit x^* is set, so that the average show demand q equals the capacity C and thus is mathematically given by

$$x^* = \frac{C}{q}. \tag{3.10}$$

Normal Approximation

To simplify computations in practice often continuous approximations are made to the binomial model. Thereby, the normal approximation is a common option. Here, the normal distribution with mean μ_x and variance σ_x^2 replaces the distribution $F_x(.)$ with

$$\mu_x = xq$$

$$\sigma_x^2 = xq(1-q).$$

The appproximation for the Type 1 service level is then given by

$$s_1(x) \approx 1 - \Phi(z_x), \tag{3.11}$$

where $\Phi(.)$ is the c.d.f. of the standard normal distribution and

$$z_x = \frac{C - \mu_x}{\sigma_x}.$$

The approximation for the Type 2 service level is

$$s_2(x) \approx \frac{\sigma_x}{\mu_x}[\phi(z_x) - z_x(1 - \Phi(z_x))]. \tag{3.12}$$

The approximation of the overbooking limit is done by choosing x^* to satisfy

$$\Phi_{x*}(C) = 1 - \frac{p}{qh}. \tag{3.13}$$

Gram-Charlier Series Approximation

While the normal approximation tends to overestimate the fraction of denied boardings, the Gram-Charlier series approximation leads to better results. However, it still tends to be overestimated as well. The essential distinction is that the Gram-Charlier series approximation allows skewed distributions. The standardized density function is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} (1 + \frac{1}{6}\beta(z^3 - 3z)), \tag{3.14}$$

with the squared coefficient of skewness

$$\beta^2 = \frac{E^2[(Z - E[Z])^3]}{E^3[(Z - E[Z])^2]}. \tag{3.15}$$

For $\beta = 0$, equation 3.15 becomes the standard normal distribution. In case of the binomial model the coefficient is defined by

$$\beta = \frac{1 - q}{\sqrt{q(1 - q)x}}. \tag{3.16}$$

As above, let z_x denote the standardized booking level, $\phi(z)$ denotes the standard normal density and $\Phi(z)$ the standard normal distribution, the approximation of overbookings is given by

$$s_2(x) \approx \frac{\sigma_x}{\mu_x}[1 + \frac{1}{6} z_x \beta \phi(z_x) - z_x(1 - \Phi(z_x))]. \tag{3.17}$$

3.2 Dynamic Models

In contrast to the static models outlined in the previous section, dynamic models account for intertemporal effects such as arrivals, cancellations and decision making (Talluri & Van Ryzin 2004, p. 152). Therefore, the overbooking limit is adjusted on a regular basis due to changes by intertemporal effects. However, dynamic models are rarely used in practice as they are rather complex compared to the static approaches of overbooking.

3.2.1 Exact Approaches

The exact approach which is outlined below is based on a model of Richard E. Chatwin and was simplified by Talluri & Van Ryzin (2004). As in the static approaches, we consider the state variables time ($t = 1, ..., T$), the number of reservations y and the parameter of fixed capacity C. Also, some notation is added: Let $V_t(y)$ denote the value function which is shown below:

$$V_{T+1}(y) = \begin{cases} 0, & y \le C \\ -c(y-C), & y > C. \end{cases}$$

Further, let c denote the convex cost function which represents the penalization costs if customers have to be denied from service. If a reservation request is accepted, a revenue of $p(t) \ge 0$ is generated and if a cancellation in period t is placed, a cancellation refund of $r(t) \ge 0$ is paid. In each period, a random number of reservation requests is placed which is denoted by D_t. Finally, $Z_t(y)$ denotes the random number of surviving reservations into period $t+1$ where $Z_t(y)$ depends on the demand on hand from period t. It is assumed that $Z_t(y)$ is binomially distributed and q_t its survival probability. The following events are to occur in each period t:

- y reservations are on hand and D_t new reservation requests will be placed at the beginning of t
- In the next phase, booking decisions are made: Reservation requests are either accepted or rejected raising the current booking level x to $y \le x \le y - D_t$
- At the end of period t a certain number of cancellations is placed.

The problem is solved by using dynamic programming with the following two recursive formulas:

$$v_{t+1}(x) = E[V_{t+1}(Z_t(x)) - (x - Z_t(x))r(t)] \tag{3.18}$$

$$V_t(y) = E[\max_{y \le x \le y+D_t} v_{t+1}(x) + (x-y)p(t)] \tag{3.19}$$

For each period t we can determine the optimal booking limit $x^*(t)$: New reservation requests are accepted until a total number of accepted reservations of $x^*(t)$ is reached. By assuming that the survival probabilities, the revenues and refunds for denied service in each period t fulfill the following equation,

$$q_t(p(t) - p(t+1)) + (1-q_t)(p(t) - r(t)) \ge 0, \tag{3.20}$$

it is shown that $x^*(1) \ge x^*(2) \ge ... \ge x^*(T)$, saying that the optimal overbooking limit $x^*(t)$ declines over time.

It can also be assumed, that the current booking limits are affected by the amount of future reservation requests D_t. Therefore, the number of reservation requests in t can be defined as dependent of the distribution of the new arrivals. Mentioned distribution is denoted by Θ and D_t can now be denoted as $D_t(\Theta)$. It can be shown that if demand to come is increasing in time, booking limits are non-increasing. In general, it can be seen that the overbooking limits depend on the future demand. This is an important asset ignored by static models as in these models there is no opportunity to replace cancellations. Therefore, statically generated overbooking limits tend to be higher than dynamic overbooking limits.

3.2.2 Heuristic Approaches

Talluri & Van Ryzin (2004) also present a heuristic approach which is based on net bookings for considering the intertemporal effects. Net bookings are referred to as the 'relative changes in bookings on hand' (Talluri & Van Ryzin 2004, p.154). Therefore, net bookings reflect cancellations as well as new reservations. As experienced in practice, net bookings data is often the only data which is available for estimating overbooking parameters.

Compared to the exact approach outlined in the previous subsection, the estimation of overbooking limits is not based on the probability of customer-level cancellations. Again, a survival rate q_t is considered, however, in this approach it is defined as 'the average ratio of show demand to the number of bookings on hand in time t' (Talluri & Van Ryzin 2004, p.155). This alternative estimate can be used as an approximation to an exact dynamic model and, in fact, is well used in practice due to its simplicity.

3.3 Combining Overbooking and Capacity Controls

In the course of this chapter, a connection between overbooking and capacity controls will be drawn by illustrating different approaches for the single-resource case in particular and multiple-resource case in brief.

Single-resources, for example, refer to single flights, whereas multiple-resources[1] are the demand for a bundle of several connection flights (Ryzin & Talluri 2005, p.149). According to Ryzin & Talluri (2005, p.149), many of the real world quantity-based revenue management problems can be accounted to network problems. However, they are solved as if they are a collection of independent single-resource problems. Solving single-resource problems proofed to be useful as building blocks in heuristics for solving the network case. The application of network problems methods itself is stated to increase revenues by approximately one to two percent (Boyd & Bilegan 2003, p.1367).

The fundamental goal of capacity controls within quantity-based revenue management is to maximize revenues by managing the admission or denial of incoming booking requests for single-resource or multiple-resource demands (Talluri & Van Ryzin 2004, p.27). The capacity control for single-resources is managed with the rule of Littlewood, EMSR-a and EMSR-b methods. In the case of network problems, other methods are applied such as stochastic and dynamic models, approximative solutions, revenue oriented, and quantity oriented management methods Klein & Steinhardt (2008). According to Klein & Steinhardt (2008, pp.160-161), the integration of overbooking and capacity controls is generally done successively in practice (see figure 3.3). Therefore, overbooking limits are firstly evaluated and followed up by capacity control methods. This implies that capacity control calculations are based on the determined overbooking capacity. Another possibility to integrate both methods is a simultaneous approach. In the course of the following subsections a selection of approaches for combining overbooking and capacity controls taken from Klein & Steinhardt (2008) and Talluri & Van Ryzin (2004) will be illustrated. With regard to what has been said before and what can be found in current literature, simultaneous processing focuses on single flights and only few research has been done on network problems.

[1] Also referred to as "Network". Networks in the airline industry, for example, are represented by Hub & Spoke networks. A booking on one specific connection within this network influences capacities on another connection and therefore the total revenues. This correlation is called "Network Effect".(Klein & Steinhardt 2008, p.93)

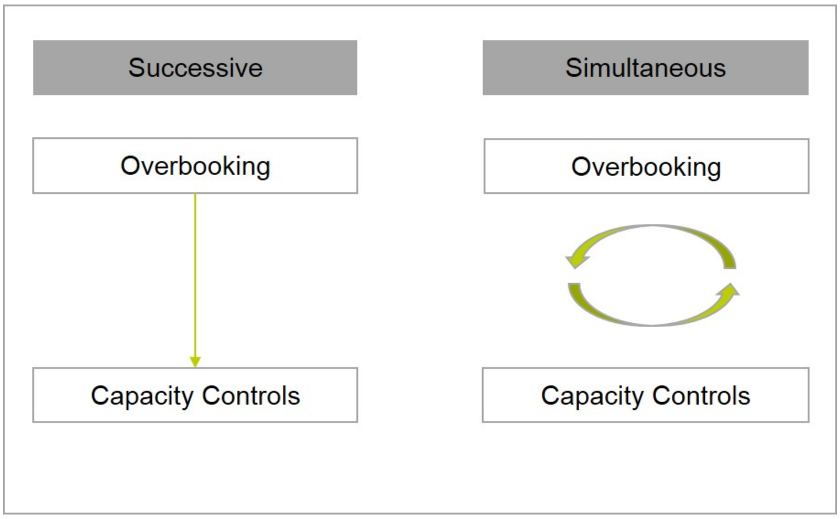

Figure 3.2: Solution Approaches for the Combination of Capacity Controls and Overbooking

3.3.1 Management of Single-Resource Problems

In the following, the approach for single-resource problems by Klein & Steinhardt (2008) will be illustrated. Klein & Steinhardt (2008) integrated the static capacity-based revenue management model "Littlewood's Rule" with overbooking methods. In the course of an extended formulation they implemented dynamic elements to Littlewood's Rule. Other approaches in Talluri & Van Ryzin (2004, Chapter 4) combine class-allocation of quantity-based revenue management with overbooking methods.By doing this, they increased the level of detail by distinguishing between no-shows and cancellations. In addition to that, they further evaluate the static and dynamic dimension for no-shows and cancellations. In this paper only the approach by Klein & Steinhardt (2008) will be explained due to its intuition.

Single-Resource Management with a Decision Tree Model in accordance with S. E. Bodily & Weatherford (1995)

In the following, model by S. E. Bodily & Weatherford (1995) will be described which is a modification of the rule of Littlewood. For this purpose, a short recap on the rule of Littlewood will be given at first. The following assumptions in accordance with Klein & Steinhardt (2008, p.86) shall hold for Littlewood's rule:

(i) The case of $n = 2$ products (P_1 and P_2) is considered.

(ii) Revenues of P_1 are equal or higher then revenues of P_2: $r_1 \geq r_2$.

(iii) The amount of demand for products P_1 and P_2 is described via the random variables D_1 and D_2.

(iv) Demand for P_2 arrives completely before demand for P_1.

(v) The total capacity is expressed by C in capacity units.

It is assumed that x_2 requests for P_2 have already arrived. The object of the decision-making situation results in the next incoming requests of P_2 $(x_2 + 1)$. The value of p (see figure 3.3.1) describes the probability that the sum of accepted requests for P_2 and additional upcoming requests for P_1 is smaller than the current available capacity C:

$$p = P(x_2 + D_1 < C). \tag{3.21}$$

A decision has to be made whether to accept the additional request for P_2 and therefore earning additional revenues of r_2 (upper half of figure 3.3.1) or to deny the additional request for P_2 (bottom half of figure 3.3.1) and to earn a revenue of r_1 with the probability $(1-p)$. The incoming request for P_2 will be accepted when: $p \geq \frac{r_1 - r_2}{r_1}$.

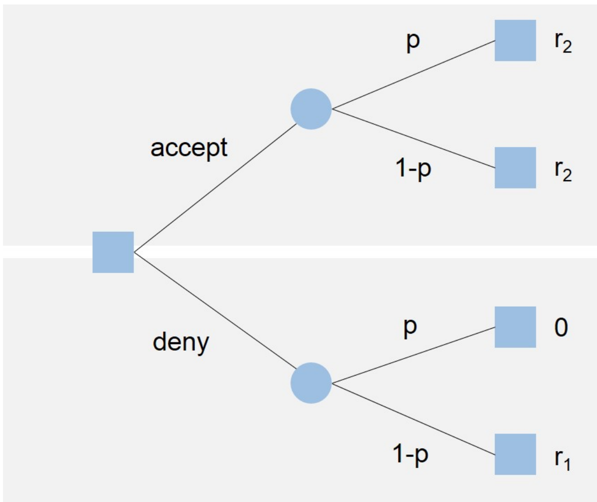

Figure 3.3: Capacity Control for Single-Flights in Accordance with Klein & Steinhardt (2008, p.162)

Littlewood's rule was extended by S. E. Bodily & Weatherford (1995, p.176 ff.) for solving simultaneously combined overcapacity and capacity control problems for single-resources (see figure 3.3.1). The following assumptions shall be added:

(vi) The random variables S_1 and S_2 define how many of the accepted requests of P_1 and P_2, respectively, will show-up until point of service.

(vii) All additional requests for P_1 will be accepted.

(viii) The variable q defines the probability that the additional request will turn up upon point of service.

(viiii) The variable g defines the costs of denied service.

In contrast to the previous formulation, the value of p describes the probability that the sum of S_1 and S_2 will be smaller than the total capacity C:

$$p = P(S_1 + S_2 < C). \tag{3.22}$$

In this formulation, marginal revenues are illustrated. In the upper half of figure 3.3.1, assuming that x_2 requests for P_2 have already arrived and under condition (vii), the acceptance of an additional request for P_2 would generate additional marginal revenues r_2 because the capacity is not exhausted. On the bottom half, however, an acceptance of an additional demand of P_2 with no free capacity would result in increased marginal revenues minus the costs of denied service. The incoming request for P_2 will be accepted when: $p \geq \frac{g-r_2}{g}$W. L. Bodily SE (1994).

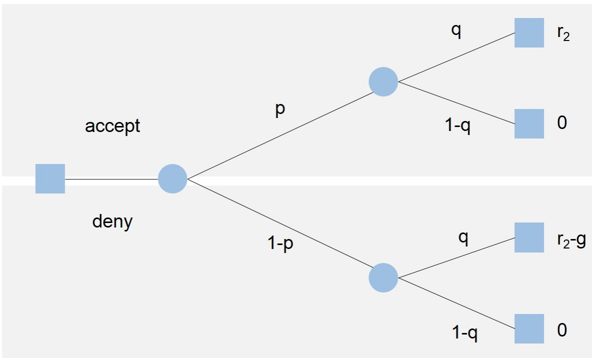

Figure 3.4: Simultaneous Overbooking and Capacity Control Management for Single-Flights in Accordance with Klein & Steinhardt (2008, p.163)

3.3.2 Network Resource Management

The following subsection is outlined in Accordance with Klein & Steinhardt (pp.165-168 2008, referring to Karaesmen & van Ryzin (2004)). The following model represents an extension of a *Deterministic Linear Programming (DLP)* model. Within the framework of an DLP uncertain parameters are exchanged with the parameters expected values (Klein & Steinhardt 2008, p.109). The extended DLP is described by a notation hereinafter known as:

Decision variables:

x_i	Amount of contingent of product P_i, $i\epsilon A^h$
y_h	Expected number of denied boardings on a flight h
z_h	Overbooking limit on a flight h

Parameters:

d_h	Average costs for denied boardings on a flight h
r_i	Price (gross margin) from P_i in money units, $i\epsilon A^h$
p_h	Probability that on a flight h a product P_i will be used, $i\epsilon A^h$
A^h	Amount of products, which use resource h
C_h	Capacity flight h
$\overline{D}_i$	Expected demand, $i\epsilon A^h$

$$max \sum_{i\epsilon I} r_i \cdot x_i - \sum_{h\epsilon H} d_h \cdot y_h \tag{3.23}$$

$$\begin{array}{llrlll}
s.t. & \sum_{i\epsilon A^h} x_i \leq z_h & & \forall & h\epsilon H & (1) \\
& y_h \geq p_h \cdot z_h - c_h & & \forall & h\epsilon H & (2) \\
& x_i \leq \overline{D}_i & & \forall & i\epsilon I & (3) \\
& z_h \geq c_h & & \forall & h\epsilon H & (4) \\
& y_h \geq 0 & & \forall & h\epsilon H & (5) \\
& x_i \geq 0 & \text{integer} & \forall & i\epsilon I & (6)
\end{array}$$

The objective function 3.23 as stated above, maximizes total revenues under the condition of restrictions (1) to (6). The total revenue is determined by the difference of revenues, which are the sum of revenues times contingents, and denied service costs, which are the sum of absolute denied boardings times costs. In order to solve this model one must relax the integer assumption at first and then apply the simplex-algorithm for example. Following this, an approximation of bid-prices for the products can be found.

4 Numerical Example

In the following subsections two numerical examples will be given in order to evaluate the benefits from implementing overbooking methods in revenue management. First of all, the successive approach for single-resources will be shown via an Excel implementation. Then, a simultaneous approach will be shown which has been implemented in the 'General Algebraic Modeling System' (GAMS).

4.1 Successive and Single-Resource Implementation

For the implementation of a successive approach for a single-resource a Microsoft Excel tool was built. The tool mask is shown in the following figure 4.1.

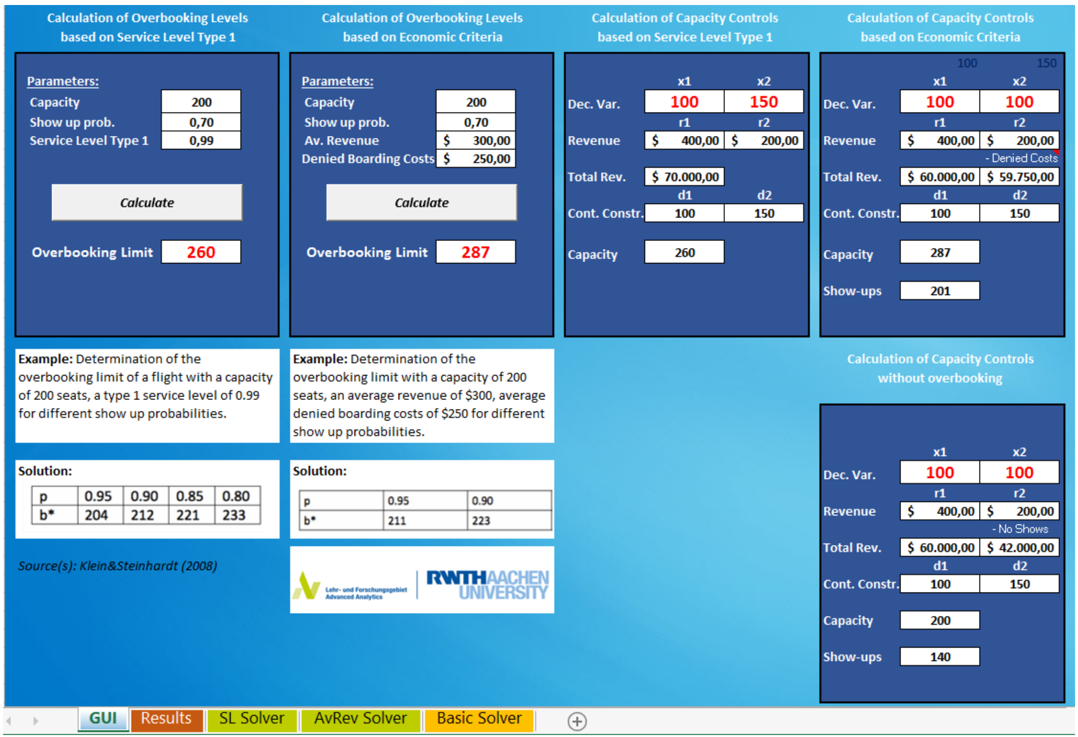

Figure 4.1: Successive Approach - Excel Tool Mask

The tool proceeds in two steps as illustrated in figure 4.2. In the first step it calculates the optimal overbooking limit. Therefore, we implemented two out of the three options for the determination of the overbooking limit shown in figure 4.2 - a calculation based on Type 1 service level and a calculation based on economic criteria. The mathematical approaches for both methods are outlined in section 3.1.1. In the second step the tool calculates the capacity controls based on the mathematical model shown in figure 4.2. But instead of choosing the physically available capacity for the determination of the parameter C, the optimal overbooking limit b that has been calculated in step one is chosen.

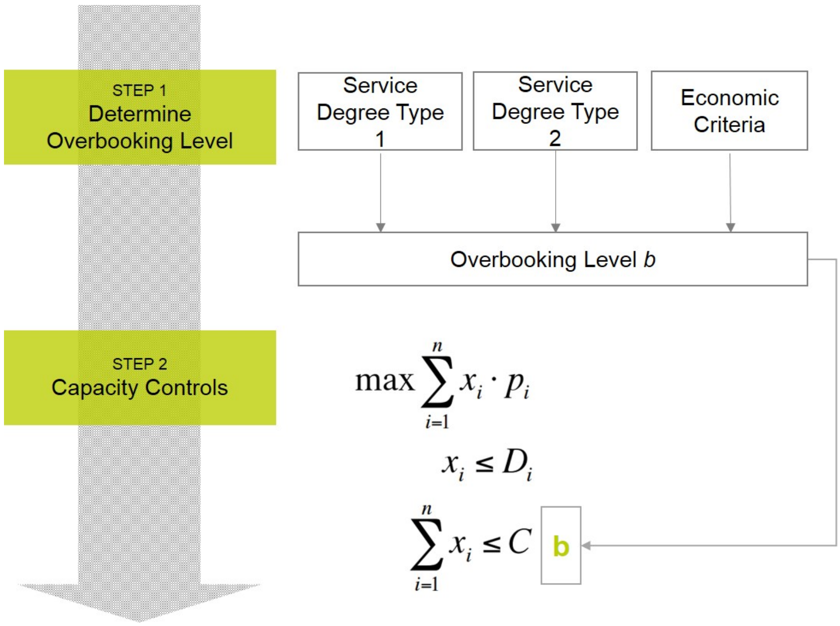

Figure 4.2: Successive Approach in Accordance with Klein & Steinhardt (2008)

To validate the tool, example exercises from Klein & Steinhardt (2008, p. 156 - 160) were implemented. For these exercises, the Excel tool calculates the correct results, so that the correctness of the tool can be seen as proven. Sales with and without the application of overbooking and capacity controls were calculated by using the Excel tool to illustrate the positive impact of overbooking. The calculations were done based on a determination of the overbooking limit through economic criteria and the following model parametrization: Capacity $C = 200$, average revenue $= 300$, denied boarding costs $= 250$, two booking classes with a demand of $d_1 = 100$, $d_2 = 150$ and the prices $p_1 = 400$ and $p_2 = 200$ for different show up probabilities. The results are summarized in figure 4.3 and show that higher revenues are achieved with the use of overbooking and capacity controls. With decreasing show up probabilities the use of these methods is leading to significantly higher revenues.

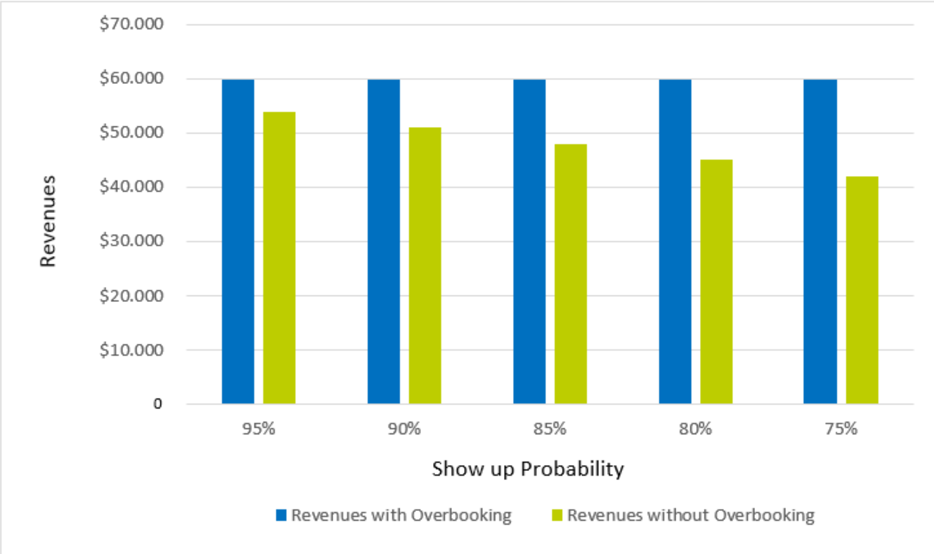

Figure 4.3: Successive Approach - Revenues with and without Overbooking

4.2 Simultaneous and Network Implementation

For the implementation of a simultaneous and network approach, the model by Klein and Steinhardt, stated in chapter 3.3.2, is now considered again. First, one slight adaption is made to the model: The subset A^h is substituted by an allocation matrix $a_{h,i}$ due to the fact that it eases up the implementation. GAMS is used for the implementation of the model. A small example with the following conditions is considered: There are five products which can be offered on five flights. The allocation of the products to the flights is determined by the allocation matrix $a_{h,i}$ (see table 4.1).

Table 4.1: Allocation Matrix

Flight h/Product i	A1	A2	A3	A4	A5
F1	1	1	0	0	0
F2	1	0	0	0	1
F3	0	1	1	1	0
F4	0	0	1	1	1
F5	0	0	1	0	1

All other parameters such as denied boarding costs on flight h (k_h), show up probability for flight h (p_h), capacity of flight h (c_h), demand for product i (d_i) and revenue for product (r_i) are set in the table 4.2.

Table 4.2: Parameter Matrix

Parameter	F1	F2	F3	F4	F5
k_h in Euro	380	300	310	305	340
p_h	1	0.89	1	0.94	0.98
c_h	200	200	116	116	116
d_i	210	221	132	112	123
r_i in Euro	250	280	300	350	400

The decision variables, contingent of product i (x_i), expected denied boardings on flight h (y_h) and overbooking limit on flight h (z_h) are then determined by solving the model with the data from the example. We get the following sizes of the product contingents:

A1: 106.352 A2: 93.648 A4: 22.352 A5: 118.367

It is shown that product 3 is not offered in the optimal solution. Furthermore, booking limits for each flight are determined:

F1: 200 F2: 224.719 F3: 116.000 F4: 140.719 F5: 118.367

Since the relaxed model is solved, an algorithm (e.g. Branch-and-Bound-Algorithm) needs to be applied to find an integer solution for the optimal contingent size of each product as only whole products can be sold. As a simplification, the results for both, contingent size and overbooking limit can also be rounded to an integer number. However, this might not lead to the revenue maximizing objective function value but to a very close result. Finally, also expected denied boardings for one flight are determined by the variable y_h: The expected denied boardings on flight four account for 16.276 units.

5 Conclusion and Outlook

From theory, it is known now that combining overbooking with capacity controls can be beneficial in terms of revenues. The successive approach is widely applied in practice whereas simultaneous approaches are neither further explored in theory as well as seldomly conducted in practice. Especially network problems are rather approached as if they are a collection of single-resources due to the complexity of solving network problems. With accordance to the numerical examples, benefits of doing the combination can be proven. For future observations an integration of overbooking models with price-based revenue management methods could be taken into closer consideration. To name an example: A successive approach for the price differentiation optimization model could be conducted such as in the first numerical example for the successive combination approach in chapter 4. However, since overbooking practices are closely related to customer dissatisfaction due to possible denied service, for example, service level criteria as a capacity-based approach are a good strategy to integrate this factor. By choosing a high service level in the process of setting the capacity on a certain product, the expected number of denied services can be taken into account and narrowed down if wished.

Overbooking methods are also applied in other industry areas than the airline industry such as the iron and steel industry (see Rehkopf (2006, p.51ff.)), as well as in Spa and Hotel industry. Even in terms of course bookings at universities overbooking could be used to make the course registration process more efficient. In this case, overbooking could be applied as well, if the number of course registrations exceeds the number of available places. Students usually try out more lectures in the beginning of a semester in order to find out which ones turn out to be their favorite ones. Consequently, they just do not show up in a lot of lectures after a couple of weeks. Possibly, by the use of these methods the course registration process at universities could be made more efficient, so that a larger number of students could attend the courses they would like to attend.

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Documentation of Work Distribution within the Group

Documentation			
Part 1	Do Khac	Hofer	Zudrop
Part 2	Do Khac	Hofer	Zudrop
Part 3	Do Khac	Hofer	Zudrop
Part 4	Do Khac	Hofer	Zudrop
Part 5	Do Khac	Hofer	Zudrop
Formatting	Do Khac	Hofer	Zudrop
Presentation			
Content	Do Khac	Hofer	Zudrop
Formatting	Do Khac	Hofer	Zudrop

We enjoyed working together. The work was evenly distributed and cross feedbacked by all group members at all parts.