

## SPECIAL ISSUE PAPERS

# Forecasting and control of passenger bookings

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### ABSTRACT

**KEYWORDS:** airline, passenger, forecasting, revenue, optimisation, control

*This paper won the 2004 Revenue Management and Pricing Section Historical Prize awarded by INFORMS. Written in 1972, it describes the early work on applying mathematical models to the development of revenue management in the airline industry. It describes methods of passenger forecasting and revenue control — and introduces the idea of maximising the revenue received on a particular flight, rather than maximising the number of passengers carried. It is considered a seminal work in the field of revenue management, and establishes the foundations for many of the yield control models which have been subsequently developed. It was presented at the 12th AGIFORS Symposium in October 1972.*

### INTRODUCTION

Several papers (Taylor, 1962; Deetman, 1964; Rothstein and Stone, 1967; Martinez and Sachez, 1970) have been presented at AGIFORS Symposia concerned with the determination of reservations control strategies which maximise the passengers carried by flight, subject to specified off-load risks. Recent moves in fares policies, however, now make it increasingly important for the airline objective to be to maximise revenue instead.

The introduction of sophisticated computer reservations systems provides an important source of data for various operational research studies. This paper will demonstrate how the Operational Research Branch in BOAC has used the information from the BOADICEA reservations system to help in maximising yield by flight. The models described fall into two categories. The first concerns the forecasting of departed loads, by day and by sector, based on a knowledge of forward bookings. Between two weeks and three months before departure, a forecasting model can highlight profitable last minute changes by consolidation or additional flights that can be made to the capacity offered on a route. The second category concerns the control of the acceptance and rejection of passengers' reservations at booking time. Apart from the determination of overbooking

policy, models have been developed to determine whether to accept or reject a reservation according to the fare paid.

The first section of this paper will describe the database which is used as input to all these models. After the forecasting and control models have been described, the final section discusses what contribution the work is likely to make in the future.

#### FLIGHT TRANSACTIONS HISTORY FILE

Every booking and cancellation transaction for every flight is fed through the BOADICEA reservations system. A complete history of passenger name records (PNRs) is stored by flight in the realtime system and,

on the day that each flight departs, this history is dumped onto computer tape. Before this information is destroyed, a program is run which produces a microfilm of the complete data and which extracts those parts of the PNR histories which are required for operational research studies. These extracts build up into the Flight Transactions History File.

Part of the Flight Transactions History File for a typical transatlantic flight is shown in Table 1. Each line represents a transaction associated with the flight and shows the number of days before departure and the time (in minutes past midnight) at which the transaction was made. The sector booked,

**Table 1: Extract from the flight transactions history file for a JFK-LHR flight**

<i>Days before departure</i>	<i>Time</i>	<i>Code</i>	<i>Board point</i>	<i>Off point</i>	<i>Class</i>	<i>Sold</i>	<i>PNR No.</i>	<i>Ticket</i>
170	1110	10	JFK	LHR	Y	2	161	
144	1093	10	JFK	LHR	Y	2	71	
115	1199	10	JFK	LHR	Y	1	59	
110	1219	20	JFK	LHR	Y	2	161	
95	894	20	JFK	LHR	Y	2	71	
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	
0	1140	20	JFK	LHR	Y	1	131	YTH
0	1158	10	JFK	LHR	Y	1	116	
0	1216	40	JFK	LHR	Y	1	61	
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
0	1250	12	JFK	LHR	Y	2	120	RED
—	—	70	JFK	LHR	Y	123	0	
—	—	70	JFK	LHR	F	11	0	

#### *Code*

10 = Booking  
20 = Cancellation  
40 = No show  
12 = Stand-by  
70 = Departed load

#### *Ticket*

YTH = Youth  
RED = Reduced Rate

the class and the number of seats sold are also recorded. The code column identifies the type of transaction and, apart from bookings (code 10) and cancellations (code 20), there are codes for standbys, no records, no shows, upgrades/downgrades and off-loads. The PNR number associated with each transaction is a unique number assigned to a PNR within a flight so that transactions associated with the same PNR can be matched. For example, in the extract PNR, 161 books 170 days before departure and cancels 110 days before departure. Finally, a ticket column gives the ticket type contained in their history. At the moment, very few fares are recognised at reservation time, but it is possible that more will be recognised in future.

The final information on any flight is fed in under code 70. This gives the details of actual departed loads by sector and by class as fed in by the flights' departure control.

This allows a check on the statistics that the recorded reservations minus the recorded cancellations are equal to the recorded departed load.

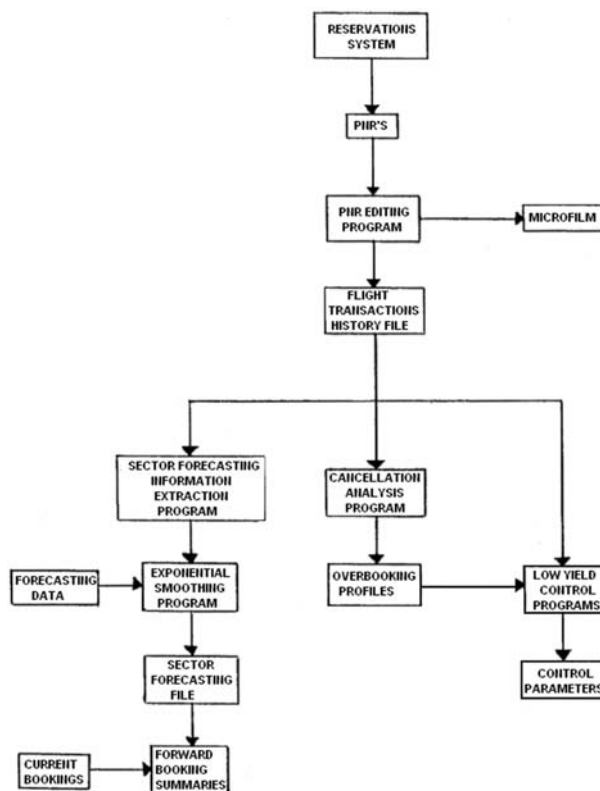
This file forms the basis of three areas of operational research work in BOAC. These areas are summarised in Figure 1, and they will now be described in turn.

### LOAD ESTIMATION FROM FORWARD BOOKINGS

It is possible to accumulate the information on the Flight Transactions History File in many different ways so as to provide information on passengers' booking habits. For example, cancellation rates by group size and time before departure and booking lead times by group size can be easily calculated.

There has been a requirement in BOAC for a considerable time for the statistics on forward bookings for flights to be pre-

Figure 1: The three areas of operational research work in BOAC



sented so that they can be easily interpreted. A simple presentation of the forward booking position at the same time on the same flight in the previous year is unsatisfactory for several reasons. First, a very large amount of past information would have to be stored. Secondly, there is evidence that passengers' booking lead times can vary considerably from year to year, and so year-old figures could be misleading. Thirdly, flight numbers and flight times can often be changed from year to year, so there is often no obvious flight to use for comparison.

### The model

For these reasons, it was decided to develop a system which would forecast demand by day and by sector, given a knowledge of forward bookings. If  $B$  passengers are booked on a given sector at a certain time before departure, an estimate  $\hat{D}$  of the demand for the sector is given by

$$\hat{D} = B(1 - \hat{c}) + \hat{S}$$

where  $\hat{c}$  and  $\hat{S}$  are estimates of the passengers' cancellation rate and the number of subsequent arriving passengers, respectively. (Subsequent arriving passengers are those passengers who book in the period between the time considered and departure, and fly.) The Flight Transactions History File provides an ideal database for observing the past values of  $c$  and  $S$ , and it was decided to smooth these values exponentially to provide future estimates of  $c$  and  $S$ .

The system is required to produce forecasts for a minimum of one week and a maximum of three months before departure. Obviously, it is not practicable or necessary to estimate the values  $\hat{c}$  and  $\hat{S}$  for each day in this one week–three month period, even though  $\hat{c}$  and  $\hat{S}$  should continually vary with time. It was decided to forecast the values,  $\hat{c}$  and  $\hat{S}$ , for five different pre-departure times for each flight sector. Values of  $\hat{c}$  and  $\hat{S}$  in

between these five times could be estimated by linear interpolation. The five pre-departure times selected were 15 weeks, 10 weeks, 6 weeks, 3 weeks and 1 week.

### Consolidated sectors

The amount of data to be held was further reduced by 'consolidating' certain flight sectors. For example, instead of holding separate estimates for each of the sectors LHR–NBO, FRA–NBO, ZRH–NBO, it was found that it was sufficient to hold one set of data for all three under a consolidated Europe–NBO sector. From the information on the Flight Transactions History File, it is now a simple task to extract the past values of  $c$  and  $S$  for the given pre-departure times and selected flight sectors and consolidated sectors.

### Conclusions

Before the model is discussed further, it is necessary to state three important conclusions resulting from the analysis of passengers' booking behaviour on a wide range of sectors. These are

- (1) For a given sector on a given day of the week, there is no evidence of any correlation between the value of  $B$  and the value of  $S$ . In other words, the subsequent arriving passengers can be regarded as independent of the booked load.
- (2) The day of week variation and trend in the value of  $\hat{c}$  are not significant enough to include them in the model. If this were to change, however, it would be a relatively simple task to introduce any variation into the model.
- (3) The value of  $S$  is not significantly biased by the inclusion of flights which hit a booking profile. It was feared that flights which were closed out at some stage during their history would tend to make an estimate of subsequent passengers too low. This was found not to be the case, because there is still consider-

able reservations activity (bookings and cancellations) on closed out flights and also because these flights tend to be only a small proportion of all the flights contributing to the value of S.

**Information extracted**

At the end of each period the following information is extracted from the Flight Transactions History File for each of the five pre-departure times:

- (1) *Mean subsequent cancellation rate.* This is simply the proportion of the passengers booked who did not subsequently fly.
- (2) *Mean subsequently arriving firm passengers.* This is the number of passengers who subsequently booked on the flight and flew.
- (3) *Day of week indices.* These are calculated by totalling the subsequently arriving passengers for the period by day of week and normalising to produce seven indices. These indices then show the relative popularity of particular days of the week for particular sectors.

**Sector forecasting file**

When this information has been extracted, exponential smoothing forecasting equations are used to project estimates of these parameters into the future. For the cancella-

tion rates and day of week indices, a simple exponential smoothing method is used incorporating a Trigg control system (Trigg, 1964; Trigg and Leach, 1967). For subsequent firm passengers, exponential smoothing with trend and seasonal indices is used with a delayed Trigg control system. The results of this analysis are stored on the Sector Forecasting File, and Table 2 shows the information which is kept there.

From this information and a knowledge of current booked loads, it is now possible to produce an estimate  $\hat{D}$  of the demand for a sector on a particular day  $i$  as

$$\hat{D} = B(1-\hat{c}) + \hat{S}\hat{d}_i$$

where  $\hat{c}$  is the estimated cancellation rate, calculated by interpolation if necessary,  $\hat{d}_i$  is the estimated day of week index for day  $i$ , and  $\hat{S}$  is an estimate of subsequent passengers, interpolated if necessary, based on the estimated trend and seasonal indices. It is expected that this system will soon be fully operational, producing management reports. It is not possible to report in detail yet how accurate the forecasts from this system will be, but a wide range of initial tests indicate that the results will be sufficiently good to provide guidelines as to which routes consolidated services or additional services could operate, as well as

**Table 2: Information on sector forecasting file**

For each sector and for each of the five pre-departure time periods.
Mean cancellation rate
Smoothed error of the cancellation rates
Smoothed absolute error of the cancellation rates
7 day of week indices
Smoothed error of each of the day of week indices
Smoothed absolute error of each of the day of week indices
Mean subsequent passengers
Trend in subsequent passengers
Smoothed error of subsequent passengers
Smoothed absolute error of subsequent passengers
12 period indices showing the seasonal variation in subsequent passengers
At each update of the Sector Forecasting File the day of week and seasonal indices are normalised.

aiding in short-term marketing decisions. In certain circumstances, weekly or period forecasts are sufficient and, when the daily figures are accumulated in this way, the accuracy can improve appreciably.

### Limitations

Of course it is always important to bear in mind the limitations of a system based on extrapolation rather than explanation of the variables. A manual intervention facility is being built into the system so that it can be made to respond quickly to major changes on a particular sector. The forecasts, however, are only intended to give a guideline to the interpretation of forward booking statistics. This system is more economic, and certainly more effective, than simply presenting the previous years' equivalent figures for comparison. Given that the PNR history database has been set up for a wide range of studies, the programs to extract the statistics and to produce the Sector Forecasting File are fairly simple and quick to run.

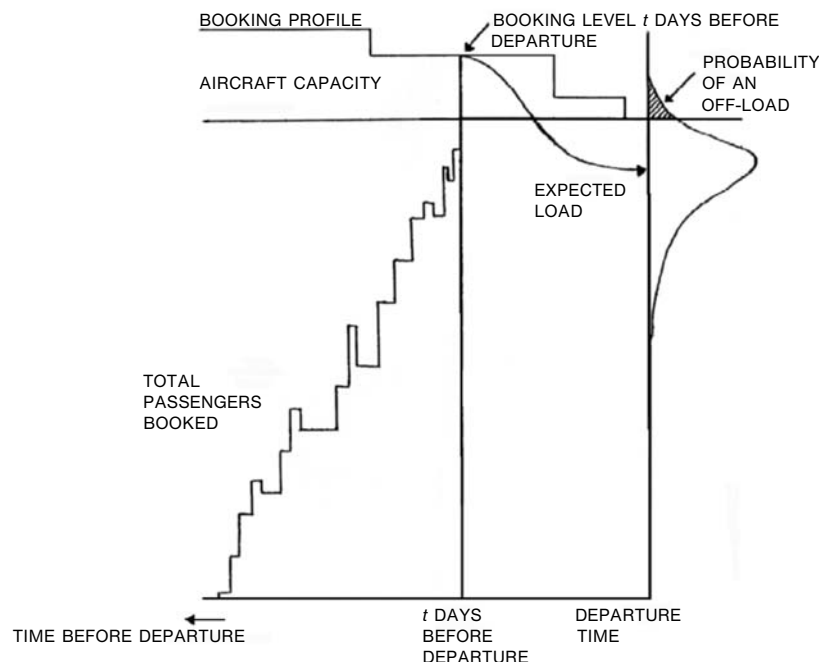
### DETERMINATION OF OVERBOOKING POLICIES

The initial purpose of setting up the Flight Transactions History File was to provide a database for a study into the setting of overbooking profiles. A system for doing this has been in operation for nearly 18 months. The BOAC model is based on the American Airlines model previously presented at AGIFORS (Rothstein and Stone, 1967). The PNR histories of several flights are read, and cancellation rates by group size and by time before departure are calculated. From these statistics, it is possible to evaluate the theoretical distribution of departed loads for various booking levels at time  $t$  before departure. From this distribution, it is possible to calculate the expected number of passengers carried and also the expected number of off-loads. The evaluation is shown diagrammatically in Figure 2.

### Outline of the model

The main assumption of the model is that the number of individuals who arrive at

Figure 2: Evaluation of the departed load distribution for a given booking profile

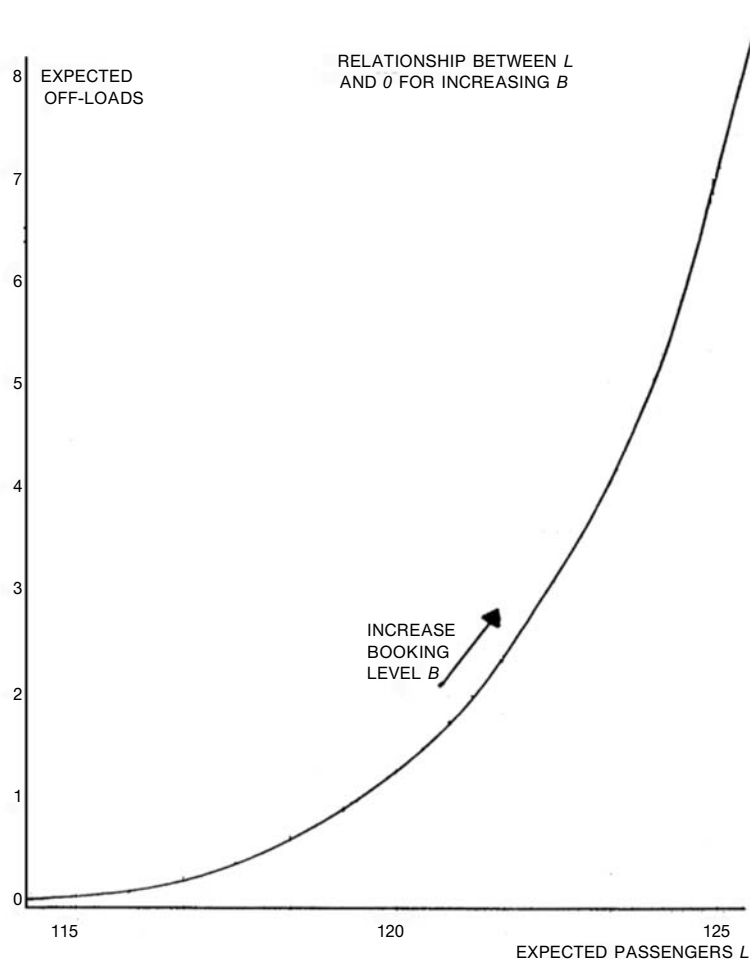


departure time for a flight which hits a booking profile at any given time  $t$  days before departure is made up of two independent random components — the number of individuals from those already booked who arrive for the flight, and the number of no-record passengers. The moments of these two distributions can be calculated from the information in the PNR histories and then, because of the assumption of independence, the moments of the combined distribution can be found by addition. The distribution of departed loads can then be approximated

by a Normal Distribution or, slightly more accurately, with a Gram–Charlier series of type A (Kendall and Stuart, 1969). From this distribution the mean load  $L$  and the expected number of off-loads  $O$  can be calculated for any booking level  $B$ .

It is obvious that as  $B$  increases so do  $L$  and  $O$ . A typical relationship between  $L$  and  $O$  for increasing  $B$  a given number of days before departure is shown in Figure 3. This is an effective way of presenting a booking profile evaluation so that a suitable balance between  $L$  and  $O$  can be found.

Figure 3: Relationship between  $L$  and  $O$  for increasing  $B$



### Profile monitoring

Not only does the Flight Transactions History File provide a basis for studies into the setting of booking profiles, but it also provides the data for monitoring the performance of the profile. There are examples of passengers' cancellation probabilities changing significantly within a short period of time, sometimes necessitating a change of profile.

It is also hoped to produce in the near future a reporting system which will record by flight and season details of off-loads and spoilage (the average number of empty seats on flights which hit the booking profile during their history). This will allow management to keep a constant check on the performance of a booking policy.

### CONTROL OF LOW-YIELD FARES

The continuing introduction of new low-yield fares on BOAC services has highlighted the need for a more sophisticated control policy when accepting reservations for a flight. As stated earlier, all previous AGIFORS papers on booking control have been concerned with maximising passengers carried by flight, but it is now becoming increasingly important for the airlines' objective to be to maximise revenue instead. Airlines should now possibly be thinking in terms of a revenue load factor (revenue for a flight as a percentage of the maximum possible revenue for that flight) instead of a passenger load factor.

For simplicity in the models which follow, it will be assumed that all fare types fall into one of two categories: high yield and low yield. The models, however, can easily be extended to deal with any number of different fare categories. In the following discussion, 'standard of service' is defined as the probability that a passenger gets a reservation on the first flight of their choice.

Surveys indicate that there is a strong tendency for low-yield passengers to book earlier than high-yield passengers. If reservations are accepted on a 'first come first served' basis, the low-yield passenger will therefore get a higher standard of service than will the high-yield passenger. This situation is probably neither in the airlines' nor the passengers' interest. If the acceptance of a low-yield passenger results in the subsequent rejection of a high-yield passenger, the airline loses revenue. There is evidence also that standard of service is more important to the high-yield passenger. The low-yield passenger is probably much more prepared to be directed towards those flights on which the airline would prefer him to fly.

All these arguments demonstrate the desirability of having a method of controlling low-yield fares. The Operational Research Branch in BOAC has investigated several ways of controlling low-yield fares, two of which are described below. The difficulties associated with implementation are discussed at the end of this section.

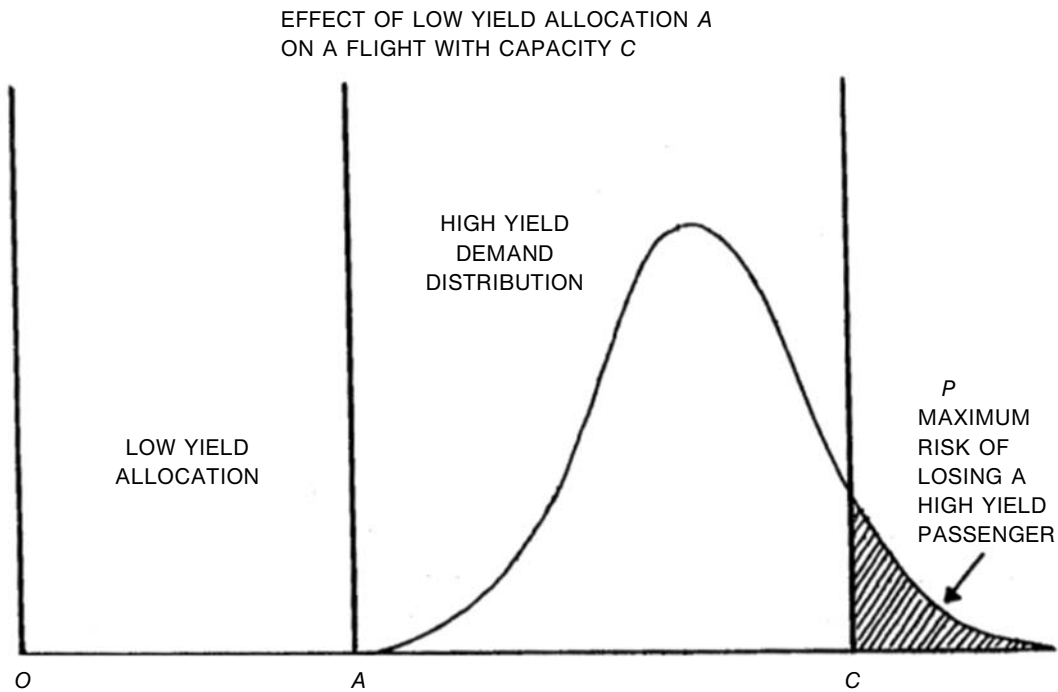
### Low-yield fare allocation

The distribution of high-yield loads on a series of flights over a particular sector was studied after all flights hitting a booking profile had been excluded. Seasonal variation was removed, and each day of the week was studied separately. From this, it was possible to deduce a relationship between the mean  $\mu_D$  and the variance  $\sigma_D^2$  of the high-yield demand distribution.

From this demand distribution, it is possible to calculate the probability  $P$  that the demand for high-yield seats exceeds  $C-A$ , where  $C$  is the aircraft capacity, and  $A$  is the low-yield allocation (ie  $P$  is the maximum risk that the acceptance of a low-yield passenger will result in the subsequent rejection of a high-yield passenger). This is shown in Figure 4.



Figure 4: Effect of low-yield allocation A on a flight with capacity C



A low-yield allocation table can now be produced which, for given  $P$ ,  $C$  and forecast  $\mu_D$ , shows the value of  $A$ . An extract from a typical table with  $P=0.20$  is shown in Table 3. The level at which  $P$  should be set depends on the control objective. If the sole objective is to maximise revenue by flight, and the mean revenue obtained from a high-yield passenger is  $R$  and from

a low-yield passengers is  $r$ , low-yield passengers should continue to be accepted as long as

$$r \geq (1-P)R$$

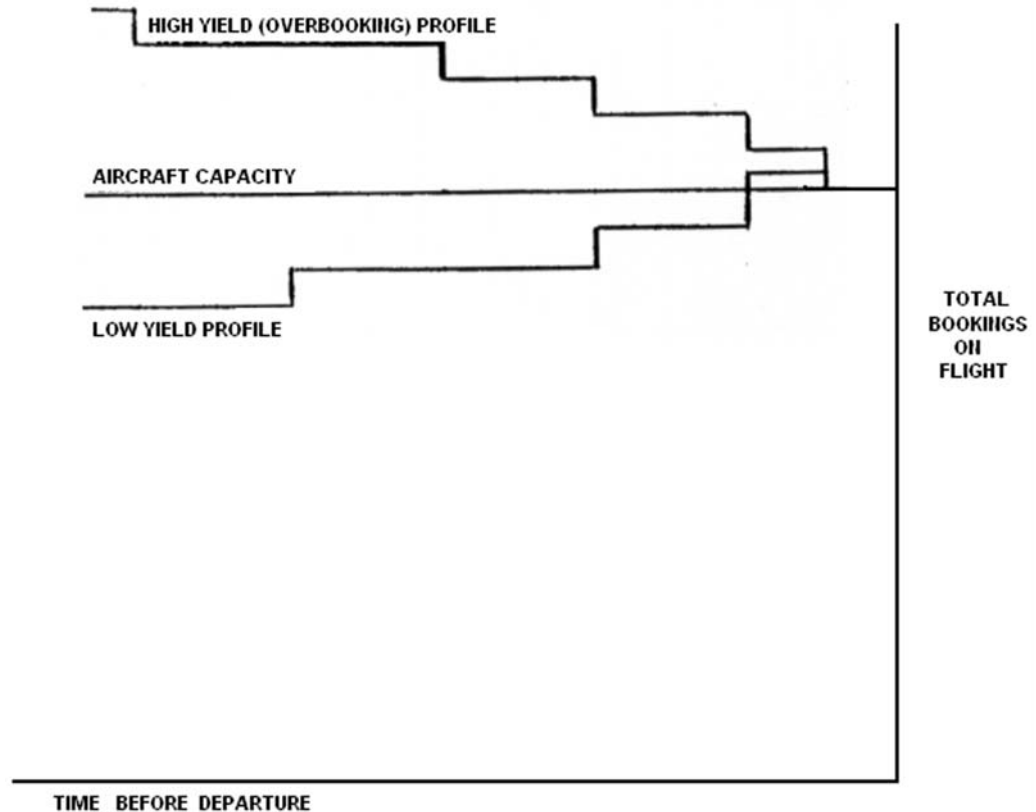
ie  $(1-P) \leq r/R$

In other words, to maximise revenue, low-yield passengers should continue to be accepted until  $(1-P)$  reaches the value of

Table 3: Extract from a low yield allocation table with  $P=0.20$ 

High yield forecast demand	Capacity											
	99	114	117	120	123	127	130	144	157	162	311	331
6-15	80	95	100	105	105	110	115	125	140	145	295	316
16-25	70	85	90	90	95	100	100	115	130	135	285	300
26-35	60	75	75	80	80	85	90	110	115	120	270	290
36-45	45	60	65	70	70	75	80	90	105	110	260	280
46-55	35	50	55	55	60	65	65	80	95	100	250	270
56-65	25	40	40	45	50	50	55	70	80	85	235	255
66-75	15	30	30	35	35	40	45	60	70	75	225	245

Figure 5: Graphical representation of high-yield and low-yield profiles. The number of seats between the profiles represents the capacity left exclusively for high-yield passengers at various times before departure



the ratio of the mean revenues from low-yield and high-yield passengers. If the acceptance of low-yield passengers is stopped sooner, a higher standard of service will be offered to the high-yield passenger or, if the acceptance is stopped later, a lower standard of service will be offered to the high-yield passenger. It is probably better to state one's objective in terms of standard of service to high-yield passengers, and, once this has been defined, the relevant value of  $P$  can be calculated.

This type of control is mainly useful for controlling fares with a long booking lead time. In particular, it has been used successfully in determining allocation levels for BOAC's Earlybird fares (advance purchase fare on cabotage routes) and is being

extended to deal with group acceptance levels.

#### Low-yield profile

An alternative method of low-yield fare control that has been developed in BOAC is the use of a low-yield profile. This profile works in a similar way to the over-booking (or high-yield profile) except that it cuts off only the low-yield bookings for a flight instead of all bookings. Figure 5 shows a possible form that this profile could take.

The setting of this profile is achieved by analysing the historical patterns of bookings and cancellations by fare type. This information is again available on the Flight Transactions History File. The year before departure is divided up into a suitable

number of pre-departure periods of varying lengths. Consider the setting of the low-yield profile  $i$  periods before departure. If the total reservations at this time are  $n$ , the distribution of reservations held  $(i-1)$  periods before departure is made up of two components: first, the distribution of those remaining of the  $n$  booked, which can be deduced from a knowledge of the inter-period cancellation rate (ie the probability that a passenger booked at the end of period  $i$  will cancel in period  $(i-1)$ ), and secondly, the distribution of new passenger arrivals in period  $(i-1)$ , which can be deduced by studying historical information. The mean of the distribution of reservations held  $(i-1)$  periods before departure is reduced after evaluating the expected number of reservations above the overbooking profile. In other words, the distribution is truncated at the overbooking control level.

This method is continued through periods  $i-2, i-3, \dots, 1$  and, at each stage, the probability  $P_j$  of hitting the overbooking profile in period  $j$  (ie the probability of turning away a high-yield passenger) can be calculated. Hence the total probability  $P = \sum_{j=1}^{i-1} P_j$  of turning away a high-yield passenger can be found.

So, if a maximum value of  $P$  is stated, the maximum value of  $n$  can be found by successive iterations. By continuing this evaluation for each time period, the low-yield profile is produced.

At the moment, there are very few occasions when a control system like this can be used, for the reasons discussed below. A system of this kind, however, is being used to control staff rebated fare firm reservations.

### Limitations

The successful implementation of any method of low-yield control is restricted by the fact that very few fare types are recognised at reservation time. It is possible

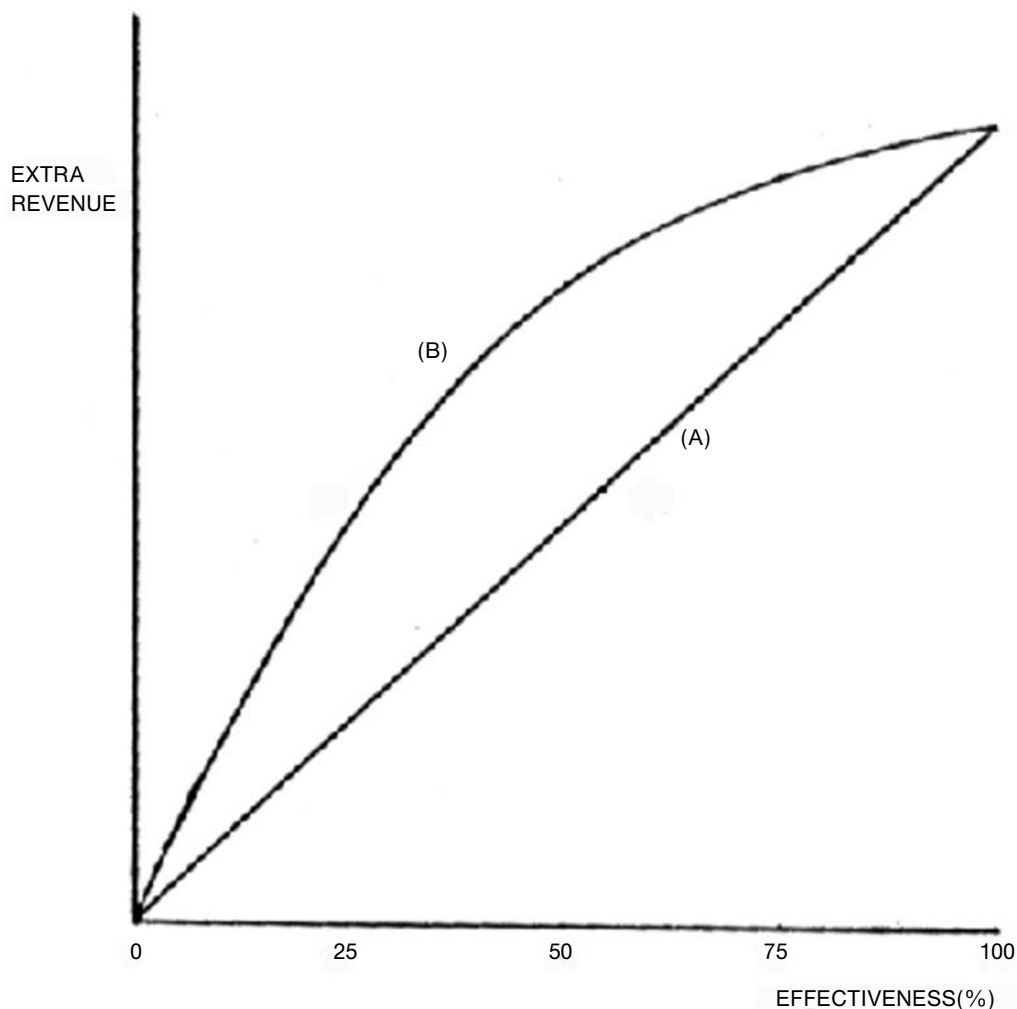
with the introduction of BOAC's automatic ticketing system that this situation will be improved. For this reason, the two models discussed above are presented very much as initial ideas rather than as results which can have an immediate wide application.

Even in ideal conditions, however, it is unlikely that any control system can be 100 per cent effective. A detailed study using a computer simulation has been done on one of BOAC's routes to see how revenue improvement is altered by the effectiveness of the control system. The results of this study are summarised in Figure 6. Curve A shows how revenue improvement changes with the control effectiveness if a low-yield profile is constructed in the way described above. If an estimate of the expected effectiveness can be made, however, the profile can be altered in the light of this knowledge. For example, as the effectiveness drops, if the low-yield profile drops as well (so that those low-yield fares that are recognised are cut off earlier), the revenue loss is not so great. Curve B in Figure 6 shows the effect of using profiles adjusted corresponding to the effectiveness of the control. From curve A, it can be seen that only 50 per cent of the extra revenue can be gained for 50 per cent effectiveness if no adjustment is made. However, from curve B, if the profile is set in the light of expected 50 per cent effectiveness, 80 per cent of the extra revenue possible from low-yield control can still be gained.

### CONCLUSION

BOAC's realtime reservations system has been a very valuable source of data both for a wide range of operational research projects and for basic management information. The forecasting system would have been impossible and the overbooking profile evaluation would have been very difficult without it.

Figure 6: Extra revenue plotted against the effectiveness of the profile control: curve A gives the results expected if the optimal profile for 100 per cent effective control were used; curve B gives the results using the adjusted profiles accounting for less than for 100 per cent effectiveness



The work on the control of low-yield fares is still in its infancy and, with the future fares policies of IATA airlines uncertain, it is difficult to predict how large a contribution operational research can make to this subject. Operational research in BOAC has in recent months become increasingly concerned with advising on the control of particular fares after they have been agreed. It is possible that, in future, the effectiveness of a booking control system will dictate the fare condi-

tions, and even the fare levels as well. I believe the airline operational research groups, particularly through AGIFORS, can start to make a significant contribution in this field.

The models described in this paper are the result of work and ideas of many people in BOAC's Operational Research Branch. I hope they have demonstrated some of the uses that can be made of a reservations system database and have suggested the ways in which operational

research, through very short-term forecasting or the evaluation of booking control systems, can contribute to the short-term control of the 'production and distribution' of aircraft capacity.

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