A Spatial Knowledge Economy

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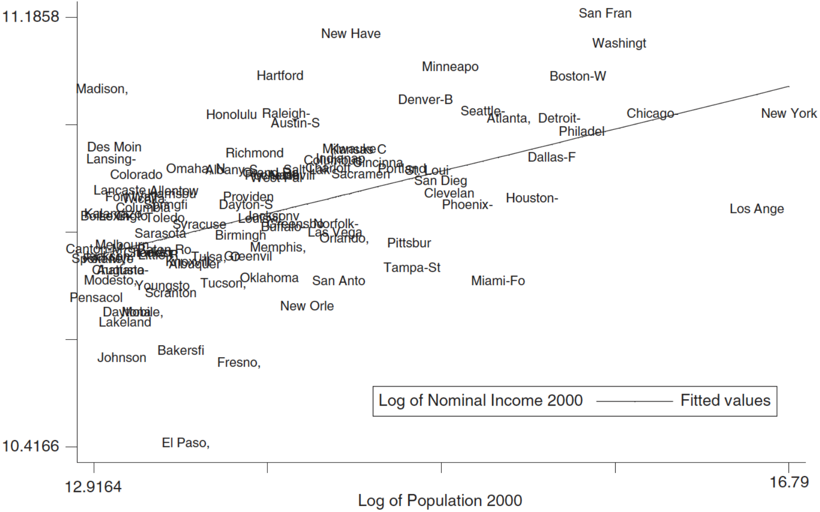
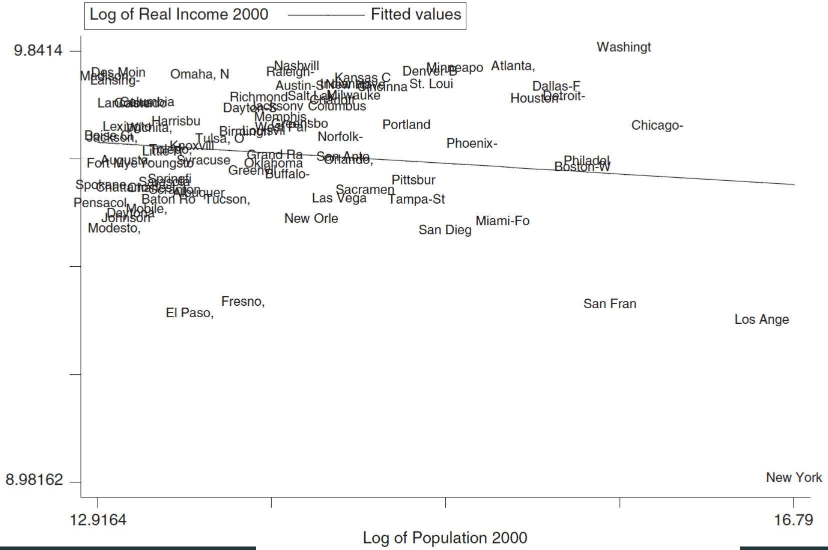
Some Background

Nominal wages, productivity and city size



Real wages and with city size

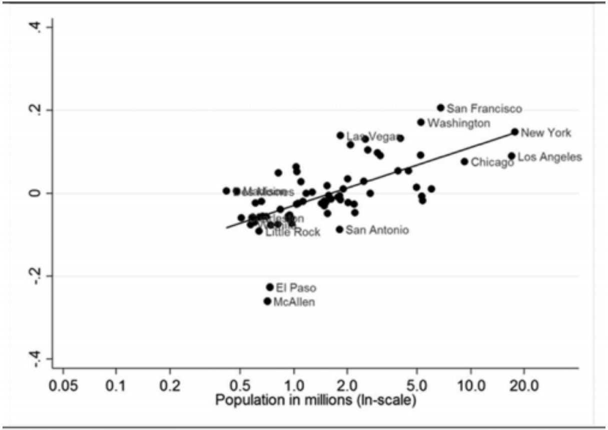


Implications: Glaeser and Mare (1994)

Why are wages 32% higher in cities than outside metropolitan areas? Higher costs of living and the inconveniences of cities may explain why labor does not immediately flock to those higher wages, but why do firms stay where the price of labor is so high?

If labor markets are perfectly competitive, this wage difference implies that the marginal product of labor is 30% greater in cities than in the hinterland.

Productivity and city size (Ahrend et al. 2017)



Note: With the natural logarithm of population on the horizontal axis, the vertical axis plots city productivity, estimated by applying individual wage regressions to national microdata in order to control for workforce composition of cities. Log hourly wages/earnings are regressed on gender (dummy), age, age squared, education (dummies), occupation (dummies) and city-year dummies; the coefficients of the latter are taken to denote productivity differentials. The analysis is conducted at the Functional Urban Area level. Source: Own calculations based on microdata from national surveys.

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- Better employer-employee matches through denser labor markets
- Sorting on workers and/or selection of firms
- Reduction in transport costs: goods, people, and ideas

This paper

Spatial equilibrium model with costly idea exchange as the agglomeration force Preview of results:

- Symmetric fundamentals of locations, yet cities of heterogeneous sizes
- In equilibrium, larger cities have:
  - Higher nominal wages
  - Higher Housing prices
  - Higher Productivity
  - Better idea-exchange opportunities

The Model

Basics

- Individuals heterogeneous in productivity, z
- Individuals choose cities, c, of specific learning opportunities, Z_c
- Individuals choose occupations: tradables, t, and non-tradables, t
- Consumption of three goods: tradables, non-tradables, housing
- Housing and $\bar{n}$ units of non-tradables are necessities. Indirect utility function

$$V(p_{n,c}, p_{h,c}, y) = y - p_{n,c}\bar{n} - p_{h,c}$$

Prodcution of Googs

Individual of ability z

- Produces 1 unit of non-tradables or $\tilde{z}(z, Z_c)$ units of tradables
- Earns income equal to her productivity

$$y = \begin{cases} p_{n,c} & \text{if } \sigma = n \\ \tilde{z}(z, Z_c) & \text{if } \sigma = t \end{cases}$$

- For tradables, division of time between production, β, and idea exchange, $1 - \beta$

$$\tilde{z}(z, Z_c) = \max_{\beta \in [0,1]} B(1 - \beta, z, Z_c)$$

Exchange of ideas

Given $1 - \beta_{z,c}$ time devoted to idea exchange and $\mu(z, c)$ population of workers z living in c

- Value of learning opportunities: $Z_c = Z(\{1 - \beta_{z,c}\}, \{\mu(z, c)\})$
- Total time devoted to learning: $M_c = \int_{\mathcal{T}} (1 - \beta_{z,c})\mu(z, c)dz$

Assumptions

- Tradables output $\tilde{z}(z, Z_c)$ increasing in z and Z_c
- Individual ability, z, and learning opportunities, Z_c, are complements for productivity $\tilde{z}(z, Z_c)$
- If no time devoted to ideas, $M_c = 0$, then $Z_c = 0$
- If $M_c > M_{c'}$ and more mass of higher ability ability individuals in city c, then

$$Z_c > Z_{c'}$$

Some functional forms

Production function

$$\tilde{z}(z, Z_c) = \max_{\beta \in [0,1]} \beta z(1 + (1-\beta) A Z_c z)$$

Learning opportunities

$$Z_c = (1 - \exp(-\nu M_c))\bar{z}_c$$

Price of housing for city of size L_c

$$p_{h,c} = \theta L_c^{\gamma}$$

Equilibrium

Definition

An equilibrium for an ability distribution μz is a set of locations $\{c\}$, a set of prices $\{p_{h,c}, p_{n,c}\}$ and populations $\mu(z,c)$ such that workers maximize their utility V with their choices of city c, occupation σ, time devoted to ideas β, and markets clear

$$\beta_{z,c} = \beta(z, Z_c)$$
$$\mu(z) = \sum_c \mu(z,c) \forall z$$
$$L_c = \int \mu(z,c) dz \forall z$$
$$\bar{n} = \int_{\mathcal{N}} \mu(z,c)$$

Observe, equilibrium value of learning opportunities Z_c is a fixed point:

- Taken as given when individuals choose $\beta_{z,c}$
- But aggregation of $\beta_{z,c}$ has to add up to Z_c

Properties of the Equilibrium

Sorting on occupations

Lemma 1

If $\tilde{z}(z, Z_c)$ is strictly increasing in z, there is an ability level z_m such that individuals of greater ability produce tradables and individuals of lesser ability produce non-tradables

$$\sigma(z) = \begin{cases} t & \text{if } z > z_m \\ n & \text{if } z < z_m \end{cases}$$

By contradiction: assume there is an equilibrium such that $\exists z > z'$ such that $\sigma(z) = n$ and $\sigma(z') = t$. If they live in the same city with cost of living $p_c = \bar{n}p_{n,c} + p_{h,c}$, they get utility

$$V^z = p_{n,c} - p_c \qquad \text{and} \qquad V^{z'} = \tilde{z}(z', Z_c) - p_c$$

By assumption $\tilde{z}(z, Z_c) - p_c > \tilde{z}(z', Z_c) - p_c$. But because z produces n in equilibrium it follows

$$p_{n,c} - p_c > \tilde{z}(z, Z_c) - p_c > \tilde{z}(z', Z_c) - p_c$$

In that case low-ability individual would switch to producing $n \implies$ cannot be an equilibrium. Similar argument follows if they are in different cities.

Production of non tradables

By the previous lemma and the market clearing conditions, the total production of non tradables is:

$$\begin{aligned}
\int_0^{z_m} \mu(z)dz &= \sum_c \int_{\mathcal{N}} \mu(z,c)dz \\
&= \sum_c \bar{n} L_c \\
&= \bar{n} \sum_c \int \mu(z,c)dz \\
&= \bar{n} \int \sum_c \mu(z,c)dz \\
&= \bar{n} \int \sum_c \mu(z)dz = \bar{n}
\end{aligned}$$

Spatial Sorting of Tradable Producers

Lemma 2

Suppose that $\tilde{z}(z, Z_c)$ strictly increasing in z and z and Z_c are complements. For two tradable producers $z > z'$ living in cities $c > c'$ who spend some time in exchanging ideas $\beta(z, Z_c), \beta(z', Z_{c'})$, it follows

$$Z_c \geq Z_{c'}$$

Hence the marginal tradable producer z_m is located in the city with the lowest value $Z_1 = \min_c\{Z_c\}$.

From indifference between occupations

$$p_{n,1} = \tilde{z}(z_m, Z_1)$$

Moreover, in equilibrium there is a mass of non-tradable producers in each city (could not sustain the equilibrium otherwise). By indifference across cities it follows

$$(1-\bar{n})p_{n,c} - p_{h,c} = (1-\bar{n})p_{n,c'} - p_{h,c'} \implies (1-\bar{n})(p_{n,c} - p_{n,c'}) = p_{h,c} - p_{h,c'}$$

Equilibrium system of cities

Equilibrium System of cities: Equal-Sized Cities

- Given symmetric fundamentals equilibrium with equal-sized cities are possible, with equal local prices and equal learning opportunities for the marginal tradable producer z_m.
- There could be equilibria with no idea exchange $Z_c = 0$ in any city
- This type of equilibrium can only be stable if the marginal benefits of producing ideas are smaller than to cost of living
- Not empirically relevant

Proposition 1

Suppose that $\tilde{z}(z, Z_c)$ strictly increasing in z and z and Z_c are complements. In any equilibrium, a larger city has higher housing prices, higher non-tradables prices, a better idea-exchange environment, and higher-ability tradables producers

- Higher housing prices: $p_{h,c} = \theta L_c^{\gamma}$
- Higher non-tradable prices to sustain non-tradable producers in city c

$$(1-\bar{n})p_{n,c} - p_{h,c} = (1-\bar{n})p_{n,c'} - p_{h,c'} \implies (1-\bar{n})(p_{n,c} - p_{n,c'}) = p_{h,c} - p_{h,c'}$$

- Better idea-exchange environment to sustain tradables producers and compensate for higher costs of living $p_c = \bar{n}p_{n,c} + p_{h,c}$

$$\tilde{z}(z, Z_c) - p_c = \tilde{z}(z, Z_{c'}) - p_{c'}$$

- Higher-ability producers follows from complementarity between z and Z_c in $\tilde{z}(z, Z_c) \implies$ sorting on ability across cities

Conclusion

- Theory model of symmetric fundamentals with costly idea exchange
- Able to reproduce empirical patterns of larger cities having
    - Higher nominal wages
    - Higher Housing prices
    - Higher Productivity
    - Better idea-exchange opportunities
- Also shows sufficiency condition for skill premia increasing with city-size (not covered today)