Spatial Sorting

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Introduction

- It is well known that big cities pay higher wages
- But the source of this "urban wage premium" is little understood
  - Are wages higher simply because big cities are more productive?
  - Or are wages higher because more skilled workers locate in big cities? Why?
- Hypothesis: **Complementarities in production affect who works with whom and where**

- This paper:
  - Average skills are constant across city size
  - Fat tails: large cities have more of both **high** and **low** skilled workers
  - Consistent with a model of "extreme-skill complementarity":
    - High skilled worker productivity is disproportionately larger with low skilled workers
    - High skill workers have a high value of time, so benefit from low-skilled services
    - E.g. administration, housework, childcare

Model

- Model summary:
  - Static, spatial choice, skill heterogeneity, skill complementarity in production
- Setup:
  - *Citizens*: skills $i \in \{1, \cdots, I\}$ with productivity y_i and measure M_i
  - *Cities*: $j \in \{1, \cdots, J\}$ with TFP A_j, number of cities C_j, and land H_j
  - City size: $S_j = \sum_{i=1}^{I} C_j m_{ij}$, where m_{ij} is city j demand for skill i
- Market clearing:
  - Labor market: $\sum_{j=1}^{J} C_j m_{ij} = M_i \; \forall i$
  - Housing market: $\sum_{i=1}^{I} h_{ij} m_{ij} = H_j \; \forall j$

- In city j choose consumption c_{ij} and housing size h_{ij}:

$$U_{ij} = \max_{c_{ij},h_{ij}} c_{ij}^{1-\alpha} h_{ij}^{\alpha}$$

$$\text{s.t. } c_{ij} + p_j h_{ij} \leq w_{ij}$$

$$FOC \quad \Rightarrow \quad c_{ij}^* = (1-\alpha) w_{ij}, \quad h_{ij}^* = \alpha \frac{w_{ij}}{p_j}$$

- Free mobility $\Rightarrow$ indifferent between cities $\Rightarrow$ indirect utility:

$$U_i = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \frac{w_{ij}}{p_j^{\alpha}}, \quad \forall j$$

- And so for any two cities j and j' (using the indirect utility function):

$$\frac{w_{ij}}{w_{ij'}} = \left(\frac{p_j}{p_{j'}} \right)^{\alpha}$$

- In city j, representative firm chooses skills to maximize profits

$$\max_{m_{1j},m_{2j},m_{3j}} A_j F(m_{1j}, m_{2j}, m_{3j}) - (w_{1j}m_{1j} + w_{2j}m_{2j} + w_{3j}m_{3j})$$

- Possible production technologies:
  - **CES:** Constant substitution between all skills

$$F(m_{1j}, m_{2j}, m_{3j}) = \left[m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2 + m_{3j}^{\gamma} y_3\right]$$

  - **Extreme Skill Complementarity**: High-skill complementary with low-skill
  - E.g. Top lawyers need lots of low-skilled services e.g. cleaning, catering, admin

$$F(m_{1j}, m_{2j}, m_{3j}) = \left[\left(m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3\right)^{\lambda} + m_{2j}^{\gamma} y_2\right], \ \lambda > 1$$

  - **Top-Skill Complementarity**: High-skill complementary with mid-skill
  - E.g. Top surgeons need lots of nurses

$$F(m_{1j}, m_{2j}, m_{3j}) = \left[\left(m_{1j}^{\gamma} y_1 + m_{2j}^{\gamma} y_2\right)^{\lambda} + m_{3j}^{\gamma} y_3\right], \ \lambda > 1$$

- Firm's first order conditions under CES:

$$w_{1j} = A_j \gamma m_{1j}^{\gamma-1} y_1$$
$$w_{2j} = A_j \gamma m_{2j}^{\gamma-1} y_2$$
$$w_{3j} = A_j \gamma m_{3j}^{\gamma-1} y_3$$

- Firm's first order conditions under **Extreme Skill Complementarity**:

$$
\begin{aligned}
w_{1j} &= A_j \gamma m_{1j}^{\gamma-1} y_1 \lambda (m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3)^{\lambda-1} \\
w_{2j} &= A_j \gamma m_{2j}^{\gamma-1} y_2 \\
w_{3j} &= A_j \gamma m_{3j}^{\gamma-1} y_3 \lambda (m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3)^{\lambda-1}
\end{aligned}
$$

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w_{3j} &= A_j \gamma m_{3j}^{\gamma-1} y_3 \lambda (m_{1j}^{\gamma} y_1 + m_{3j}^{\gamma} y_3)^{\lambda-1}
\end{aligned}
$$

- With market clearing, wage-price ratios, and two cities, then labor demands in city 1:

$$
\begin{aligned}
m_{11} &= \left(\left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\lambda\gamma-1)} \frac{M_1}{C_2} \right) \Bigg/ \left(1 + \frac{C_1}{C_2} \left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\lambda\gamma-1)} \right) \\
m_{21} &= \left(\left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\gamma-1)} \frac{M_2}{C_2} \right) \Bigg/ \left(1 + \frac{C_1}{C_2} \left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\gamma-1)} \right) \\
m_{31} &= \left(\left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\lambda\gamma-1)} \frac{M_3}{C_2} \right) \Bigg/ \left(1 + \frac{C_1}{C_2} \left[\left(\frac{p_1}{p_2} \right)^{\alpha} \frac{A_2}{A_1} \right]^{1/(\lambda\gamma-1)} \right)
\end{aligned}
$$

City size and TFP
Let $A_1 > A_2$, $\lambda\gamma < 1$, and $\gamma < 1$. Then the more productive city is larger: $S_1 > S_2$.

Extreme-skill complementarity (ESC) and thick tails
Given $A_1 > A_2$, $\lambda > 1$, and $\lambda\gamma < 1$, the skill distribution in the larger city has thicker tails.

- Under ESC, bigger cities have thick-tailed skills distributions
- Where do these thick-tails come from?
  - Under ESC: $\uparrow$ MPL of high and low skills $\Rightarrow$ $\uparrow$ demand for high and low skills
- This is despite higher house prices, since real wages are equalized for all skill levels
  - Higher productivity cities have greater demand for housing, and higher house prices
  - Higher MPL attracts high and low skilled citizens $\Rightarrow$ nominal wages fall $\Rightarrow$ real wages equalized

Top-Skill Complementarity (TSC) and first order stochastic dominance
Given $A_1 > A_2$, $\lambda > 1$, and $\lambda\gamma < 1$, the skill distribution in the larger city first-order stochastically dominates.

- Under TSC, top and middle skills are complements $\Rightarrow$ more productive cities $\Rightarrow$ disproportionately higher output
- This generates disproportionate demand for higher skills in more productive/larger cities

Empirical Evidence of Thick Tails

- Cross-city (CBSA) data in 2009:
  - Wage data: Current Population Survey, top-coded at $150,000
  - Housing rents: American Community Survey, rents + house characteristics, hedonic reg. $\rightarrow p_j$
- Skill distribution given by indirect utility: $U_i = \alpha^\alpha (1-\alpha)^{1-\alpha} \frac{w_{ij}}{p_j^\alpha}$

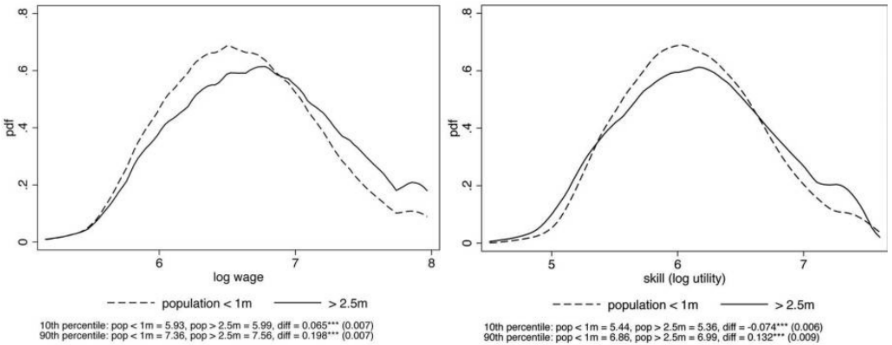


FIG. 1.—Wage and skill distribution for small and large cities, kernel density estimates (Epanechnikov kernel, bandwidth = 0.1); wage data from the 2009 CPS on 25,584 workers in 202 small CBSAs (population between 100,000 and 1 million) and 34,999 workers in 21 large CBSAs (larger than 2.5 million): *left*, wages; *right*, skills. Standard errors are in parentheses; $*p < .10$, $**p < .05$, $***p < .01$.

- Larger cities have fatter tails:
  - The average skill does not change with city size
  - But there are more workers at the bottom and the top of the skill distribution
- A quantile regression shows this result more formally

simply

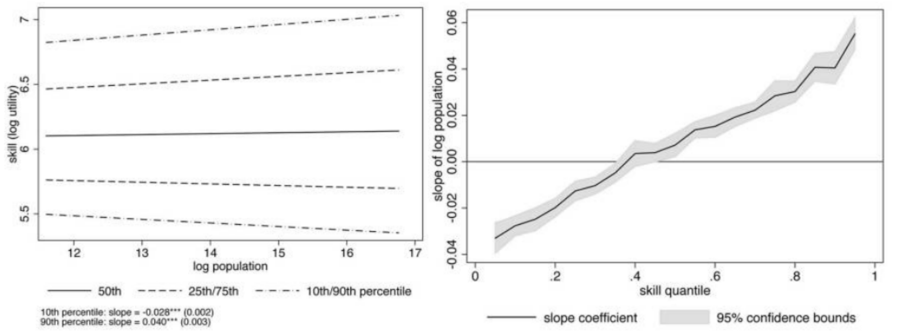


Fig. 3.—Quantile regression of skills (utility) on population: *left*, five selected quantiles; *right*, estimated slope for all quantiles

- Use the model and data to back out:
  - TFP across large vs. small cities, high vs. low skill productivity, extreme-skill complementarity
- To map data to the model:
  - Three skill types *i*: bottom 20th, middle 60th, and top 20th percentiles
  - Two city sizes: population $\geq$ 2.5 million, population $\leq$ 1 million

TABLE 2
QUANTIFYING THE PRODUCTION TECHNOLOGY

	Observed Model Outcomes						
City j	w_{1j}	w_{2j}	w_{3j}	m_{1j}	m_{2j}	m_{3j}	C_j
1	416	844	1,923	730,509	1,953,303	730,509	21
2	354	717	1,634	30,900	105,516	30,900	204
	Implied Production Technology for Different Values of γ						
γ	λ	A_1	A_2	y_1	y_2	y_3	
.655	1.0407	190,228	59,107	.2329	1	1.0762	
.8	1.0193	19,118	9,065	.3189	1	1.4733	
.9	1.0086	3,992	2,534	.3964	1	1.8317	

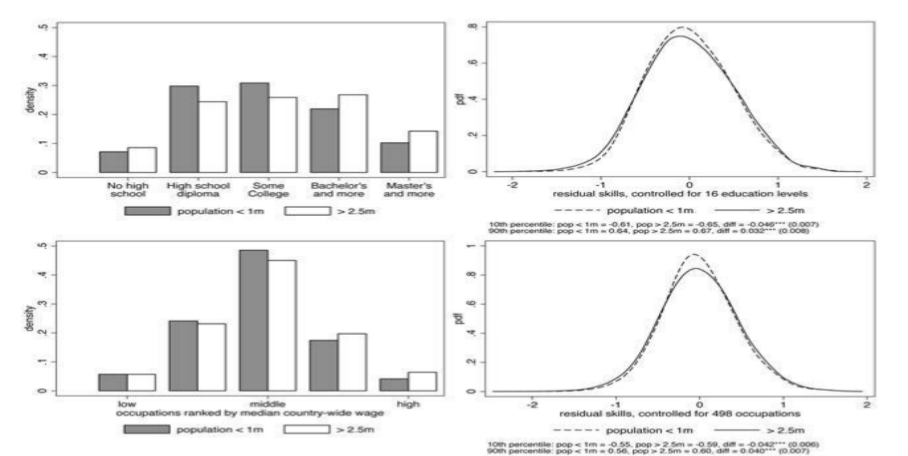
NOTE.—The term w_{ij} is the weekly median wage (in US dollars); m_{ij} is the number of workers of skill type i in cities of type j (in units); C_j is the number of cities of type j (in units); γ is the exogenously chosen technology parameter; and λ, A_j, and y_i are the estimated technology parameters.

Decomposing Observable and Unobservable Skills

- Want to understand how much of the sorting by skill is due to observable skills
- Investigate sorting by:
  - Education, occupation, and industry
  - Mobility, migration, and nationality
  - Life stage

- Econometric decomposition of observable vs. unobserable skills across cities by quantile
  - Asymmetry in explanation of sorting of low vs. high skilled workers
  - 50-60% of difference in high skills in small vs. large cities explained by observables
  - 25-30% of difference in low skills in small vs. large cities explained by observables

- Larger cities have fatter tails in observable skills, too
- Larger cities have a fatter lower tail in unobservable skills



- More immigrants in large cities possibly due to strong networks effects
    - If immigrants are disproportionately low-skilled, might account for fat lower tail in skills
- But larger cities have thick upper and lower tails for both migrants and locals

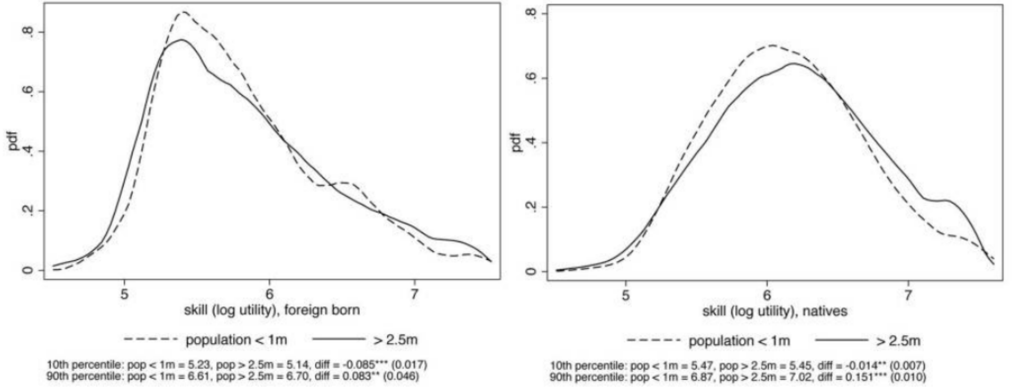
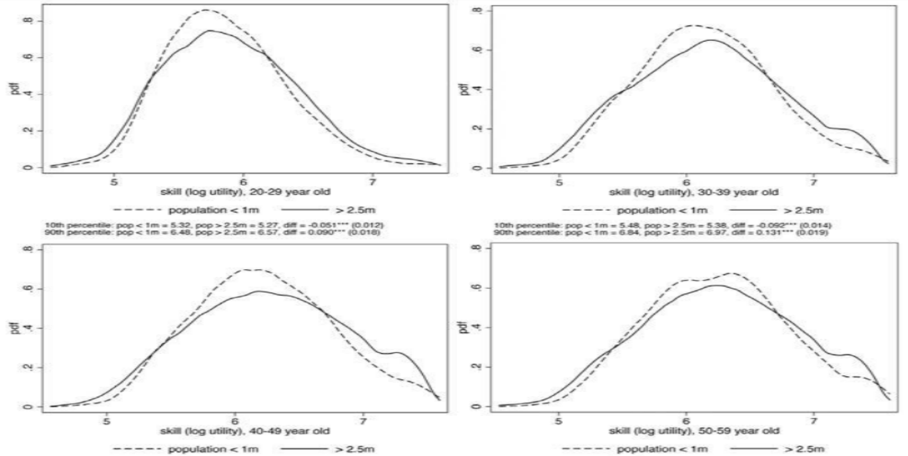


FIG. 12.—Skill distribution for small and large cities: *left*, foreign-born workers; *right*, natives

- Young people may be disproportionately attracted to large cities:
  - Entertainment, thick marriage markets, better initial labor market opportunities
- But, again, larger cities have thick upper and lower tails for all age groups



Other Sources of Cross-City Heterogeneity

- Assumed that all locations within a city are uniformly attractive
    - Allowed to deflate wages by a constant per-unit house price in each city
- But there is strong sorting within cities: location choices differ by income

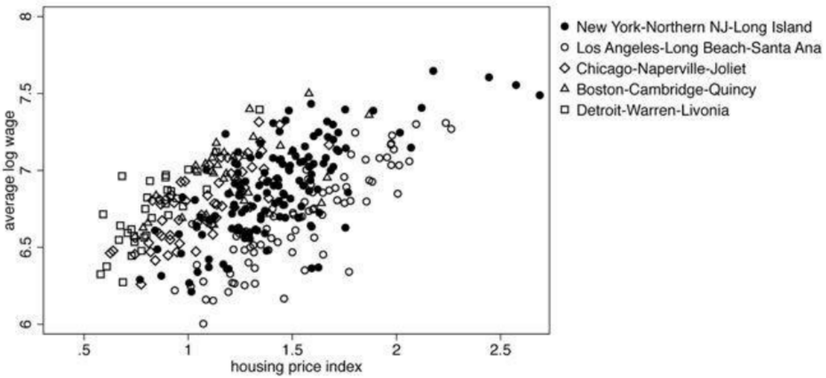


Fig. 7.—Average rental housing prices and average log wages across PUMAs in five selected CBSAs.

- Highest value houses are in the most desirable locations
- Other neighborhood prices are distorted by relative disamenities (e.g. commuter distance)
- So use median house price from top 10% of (undistorted) neighborhoods in each city

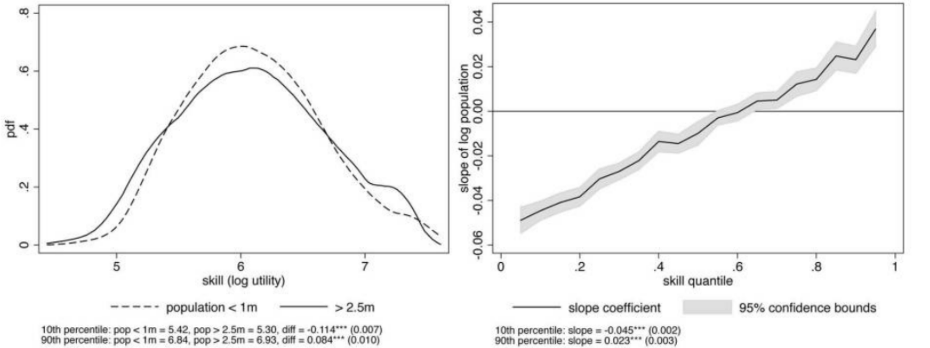


Fig. 4.—Skills based on the average housing price index in the top 10 percent neighborhoods (PUMA) of a city; wage data from the 2009 CPS: *left*, skill distribution for small and large cities; *right*, slopes in quantile regressions of log utility on log population.

- Now suppose heterogeneous preferences: loc. attractiveness complementary with income
- Then each level of income has a different valuation of each house type
- Relevant house price is then the income-dependent valuation of most attractive location
- But each income type does not live in all locations
- Use house price in own location as lower bound on (undistorted) city price for that type

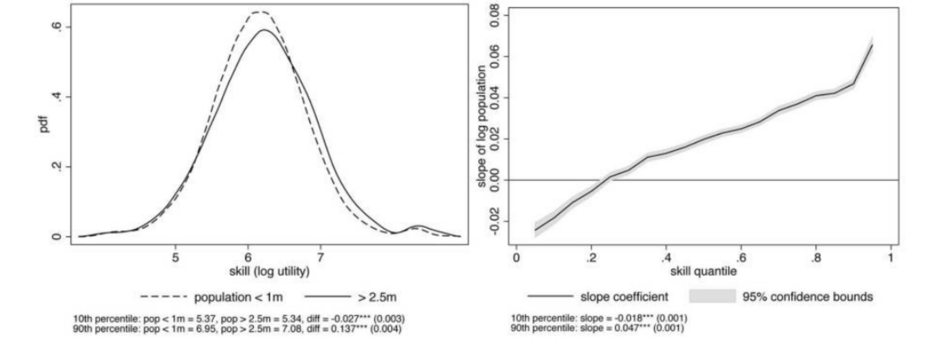


Fig. 6.—Skills based on the housing price in the neighborhood of residence; wage data from the 2009 ACS: *left*, skill distribution for small and large cities; *right*, slopes in quantile regressions of log utility on log population.

- Poorer households tend to spend more on housing
- Stone-Geary preferences: $u(c,h) = c^{1-\alpha}(h - \underline{h})^{\alpha}$
- Indirect utility is then:

$$U_i = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{w_{ij}}{p_j^{\alpha}} - \underline{h}\right)$$

- Non-homothetic preferences do not generate thick tails on their own (e.g. with CES tech.)
- Housing expenditure share: $\frac{ph}{w} = \alpha + (1-\alpha)\underline{h}\frac{p}{w}$
- Using data on wages and local prices, run the regression:

$$\frac{p_j h_i}{w_i} = \alpha + \beta\frac{p_j}{w_i} + \varepsilon_i, \quad \text{where } \underline{h} = \frac{\beta}{1-\alpha}$$

Non-Homothetic Preferences: Minimum House Size

- Using the estimates: $\hat{\alpha} = 0.224$ and $\hat{\underline{h}} = 27.7$
- Again find thick tails in the distribution of skills in large cities

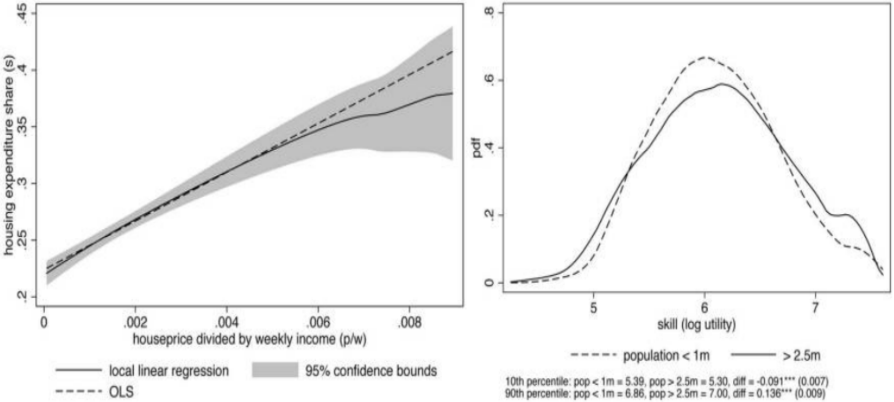


FIG. 9.—Expenditure share (*left*) and skill distribution (*right*) using Stone-Geary preferences, $\hat{\alpha} = 0.224$, $\hat{\underline{h}} = 27.7$

- Non-housing goods prices may differ systematically in large and small cities
- Deflate wages by all prices: grocery + housing + utilities + transport + health + services

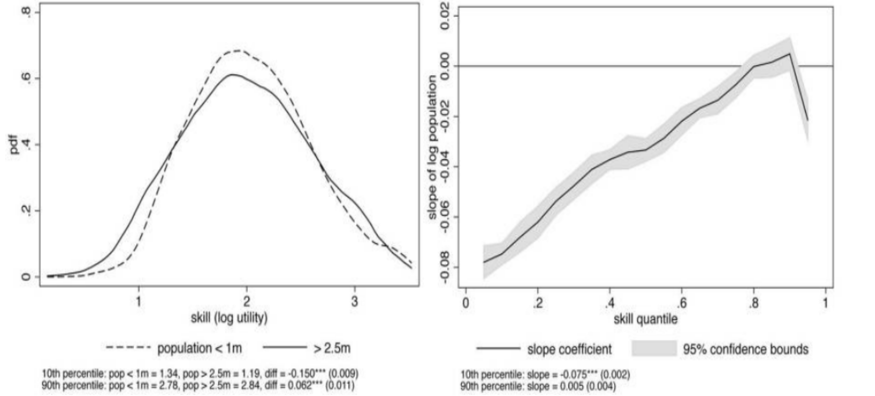


Fig. 10.—Skill distribution using ACCRA Cost of Living Index, adjusting for variation in prices for all goods: *left*, skill distribution for small and large cities; *right*, slopes in quantile regressions of log utility on log population.