Part A: Writing a sequence of fibonacci numbers in terms of F_{n-1} and F_{n-2}

$F_n = F_{n-1} + F_{n-2}$

$F_{n+1} = (F_{n-1} + F_{n-2}) + F_{n-1}$ $\quad = 2F_{n-1} + F_{n-2}$

$F_{n+2} = (F_{n-1} + F_{n-2} + F_{n-1}) + (F_{n-1} + F_{n-2})$ $\quad = 3F_{n-1} + 2F_{n-2}$

$F_{n+3} = (F_{n-1} + F_{n-2} + F_{n-1} + F_{n-1} + F_{n-2}) + (F_{n-1} + F_{n-2} + F_{n-1}) = 5F_{n-1} + 3F_{n-2}$

$F_{n+4} = 5F_{n-1} + 3F_{n-2} + 3F_{n-1} + 2F_{n-2}$ $\quad = 8F_{n-1} + 5F_{n-2}$

$F_{n+5} = 8F_{n-1} + 5F_{n-2} + 5F_{n-1} + 3F_{n-2}$ $\quad = 13F_{n-1} + 8F_{n-2}$

$F_{n+6} = 21F_{n-1} + 13F_{n-2}$ $\quad = 21F_{n-1} + 13F_{n-2}$

$F_{n+7} = 34F_{n-1} + 21F_{n-2}$ $\quad = 34F_{n-1} + 21F_{n-2}$

Part B: To double n, go up n steps.

if $n=6$, $F_{2n} = F_{n+6}$

$F_{2n} = F_{n+6} = 21F_{n-1} + 13F_{n-2}$

$\leftarrow 21 = F_8$ and $13 = F_7$

$F_8 F_{n-1} + F_7 F_{n-2}$

$F_{n+2} F_{n-1} + F_{n+1} F_{n-2}$

$F_{n+2} = F_{n+1} + F_n \longrightarrow (F_{n+1} + F_n) F_{n-1} + F_{n+1}(F_n - F_{n-1})$ $\leftarrow$ We know $F_n = F_{n-1} + F_{n-2}$, so $F_{n-2} = F_n - F_{n-1}$

$F_{n+1}F_{n-1} + F_nF_{n-1} + F_{n+1}F_n - F_{n+1}F_{n-1}$ $\leftarrow$ First and last terms cancel out

$F_{2n} = F_{n+1}F_n + F_nF_{n-1}$