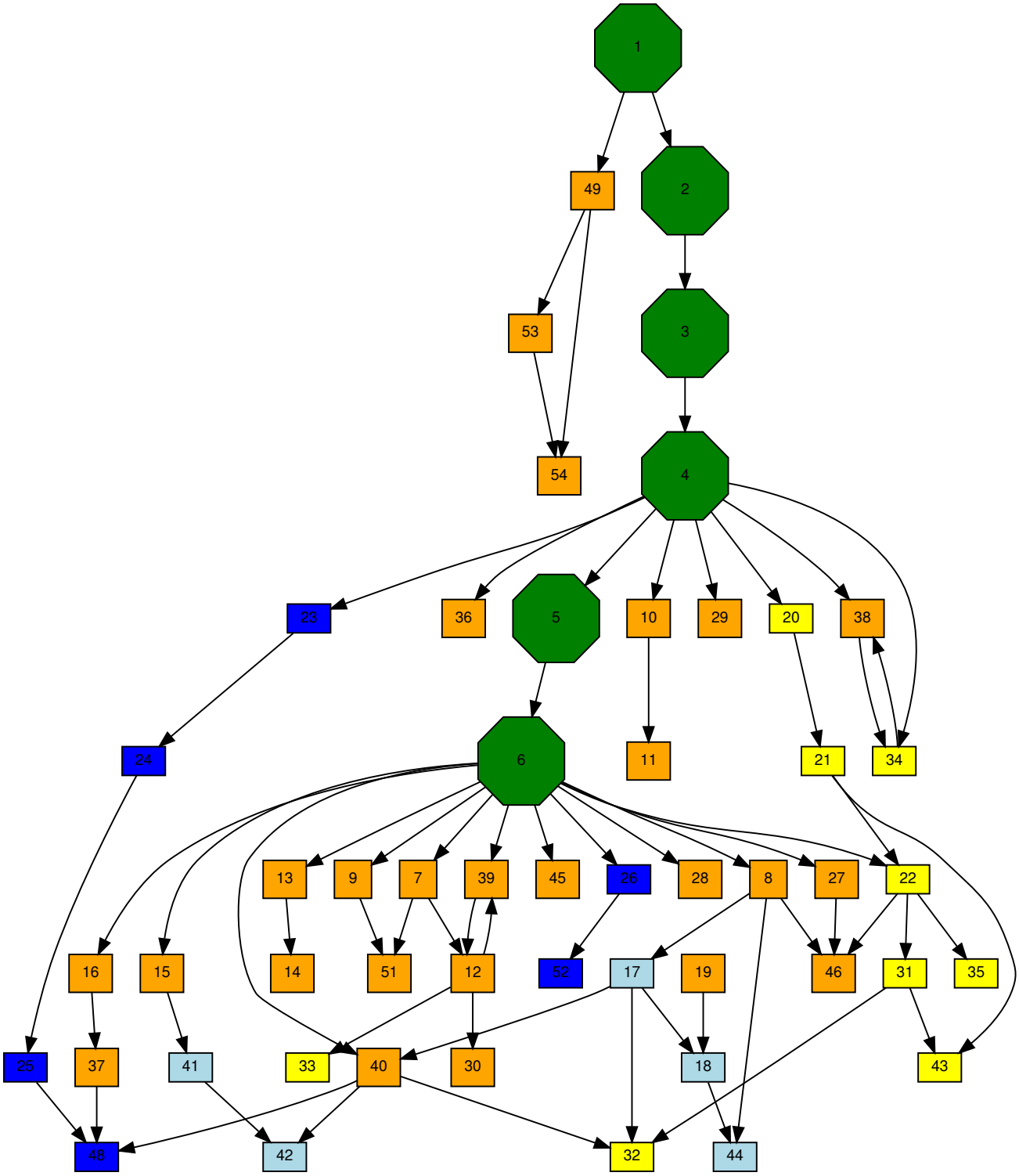
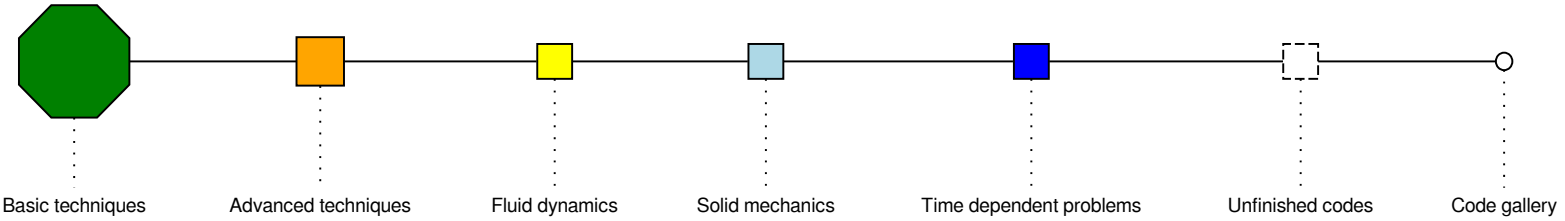


Legend:



Tutorial programs listed by number

step-1 Creating a grid. A simple way to write it to a file.

step-2 Associate degrees of freedom to each vertex and compute the resulting sparsity pattern of matrices. Show that renumbering reduces the bandwidth of matrices significantly, i.e. clusters nonzero entries around the diagonal.

step-3	Actually solve Laplace's problem. Object-orientation. Assembling matrices and vectors. Boundary values.
step-4	This example is programmed in a way that it is independent of the dimension for which we want to solve Laplace's equation; we will solve the equation in 2D and 3D, although the program is exactly the same. Non-constant right hand side function. Non-homogeneous boundary values.
step-5	Computations on successively refined grids. Reading a grid from disk. Some optimizations. Using assertions. Non-constant coefficient in the elliptic operator (yielding the extended Poisson equation). Preconditioning the CG solver for the linear system of equations.
step-6	Adaptive local refinement. Handling of hanging nodes. Higher order elements. Catching exceptions in the `main`; function.
step-7	Helmholtz equation. Non-homogeneous Neumann boundary conditions and boundary integrals. Verification of correctness of computed solutions. Computing the error between exact and numerical solution and output of the data in tables. Using counted pointers.
step-8	The elasticity equations will be solved instead of Laplace's equation. The solution is vector-valued and the equations form a system with as many equations as the dimension of the space in which it is posed.
step-9	Linear advection equation, assembling the system of equations in parallel using multi-threading, implementing a refinement criterion based on a finite difference approximation of the gradient.
step-10	Higher order mappings. Do not solve equations, but rather compute the value of pi to high accuracy.
step-11	Solving a Laplace problem with higher order mappings. Using mean value constraints and intermediate representations of sparsity patterns.
step-12	Discontinuous Galerkin methods for linear advection problems.
step-13	Software design questions and how to write a modular, extensible finite element program.
step-14	Duality based error estimators, more strategies to write a modular, extensible finite element program.
step-15	A nonlinear elliptic problem: The minimal surface equation. Newton's method. Transferring a solution across mesh refinement.
step-16	**Multigrid** preconditioning of the Laplace equation on adaptive meshes.
step-17	Using PETSc for linear algebra; running in parallel on clusters of computers linked together by MPI.
step-18	A time dependent problem; using a much simplified version of implementing elasticity; moving meshes; handling large scale output of parallel programs.
step-19	Input parameter file handling. Merging output of a parallel program.
step-20	Mixed finite elements. Using block matrices and block vectors to define more complicated solvers and preconditioners working on the Schur complement.
step-21	The time dependent two-phase flow in porous media. Extensions of mixed Laplace discretizations. More complicated block solvers. Simple time stepping.
step-22	Solving the Stokes equations of slow fluid flow on adaptive meshes. More on Schur complement solvers. Advanced use of the **ConstraintMatrix** class.
step-23	Finally a "real" time dependent problem, the wave equation.
step-24	A variant of **step-23** with absorbing boundary conditions, and extracting practically useful data.
step-25	The sine-Gordon soliton equation, which is a nonlinear variant of the time dependent wave equation covered in **step-23** and **step-24**.
step-26	The heat equation, solved on a mesh that is adapted every few time steps.
step-27	hp finite element methods
step-28	Multiple grids for solving a multigroup diffusion equation in nuclear physics simulating a nuclear reactor core
step-29	Solving a complex-valued Helmholtz equation. Sparse direct solvers. Dealing with parameter files.
step-30	Anisotropic refinement for DG finite element methods.
step-31	Time-dependent Stokes flow driven by temperature differences in a fluid. Adaptive meshes that change between time steps.
step-32	A massively parallel solver for time-dependent Stokes flow driven by temperature differences in a fluid. Adapting methods for real-world

equations.

step-33	A nonlinear hyperbolic conservation law: The Euler equations of compressible gas dynamics.
step-34	Boundary element methods (BEM) of low order: Exterior irrotational flow. The ParsedFunction class.
step-35	A projection solver for the Navier–Stokes equations.
step-36	Using SLEPc for linear algebra; solving an eigenspectrum problem. The Schrödinger wave equation.
step-37	Solving a Poisson problem with a multilevel preconditioner without explicitly storing the matrix (a matrix-free method).
step-38	Solving the Laplace-Beltrami equation on curved manifolds embedded in higher dimensional spaces.
step-39	Solving Poisson's equation once more, this time with the interior penalty method, one of the discontinous Galerkin methods developed for this problem. Error estimator, adaptive meshes, and multigrid preconditioner, all using the **MeshWorker** framework.
step-40	Techniques for the massively parallel solution of the Laplace equation (up to 10,000s of processors).
step-41	Solving the obstacle problem, a variational inequality.
step-42	A solver for an elasto-plastic contact problem, running on parallel machines.
step-43	Advanced techniques for the simulation of porous media flow.
step-44	Finite strain hyperelasticity based on a three-field formulation.
step-45	Periodic boundary conditions.
step-46	Coupling different kinds of equations in different parts of the domain.
step-48	Explicit time stepping for the Sine–Gordon equation based on a diagonal mass matrix. Efficient implementation of (nonlinear) finite element operators.
step-49	Advanced mesh creation and manipulation techniques.
step-51	Solving the convection-diffusion equation with a hybridizable discontinuous Galerkin method using face elements.
step-52	Solving the time dependent neutron diffusion equation using Runge-Kutta methods.
step-53	Describing the geometry of complex domains and curved boundaries.
step-54	Using CAD files to describe the boundary of your domain.

Tutorial programs grouped by topics

Basic techniques

Creating a grid. A simple way to write it to a file	**step-1**
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Dimension independent programming, non-zero data	**step-4**
Computing on uniformly refined meshes	**step-5**
Adaptivity	**step-6**, **step-26**
Evaluating errors	**step-7**
Nonlinear problems, Newton's method	**step-15**

Advanced techniques

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Using Trilinos	**step-31**, **step-32**, **step-33**, **step-41**, **step-42**, **step-43**
Parallelization via PETSc and MPI	**step-17**, **step-18**, **step-19**, **step-40**
Parallelization via Trilinos and MPI	**step-32**, **step-42**
Parallelization on very large numbers of processors	**step-32**, **step-40**, **step-42**
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Matrix-free methods	**step-37**, **step-48**
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Preconditioned CG solver	**step-5**
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Elasticity and elasto-plasticity equations	**step-8**, **step-42**, **step-46**
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Quasi-static elasticity equations	**step-18**, **step-44**
Transport (advection) equations	**step-9**, **step-21**, **step-31**, **step-32**, **step-43**, **step-51**

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The wave equation, in linear and nonlinear variants	step-23, step-24, step-25, step-48
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