Math 180 — Test 2.5 — Fall 2011

November 11, 2011

Instructions You are to work on all the problems BY YOURSELF and return the test at the beginning of the next class period. You may use your book and the internet. To get full credit you must show all of your work. Each problem is worth 10 points.

1. Consider the function

$$f(x) = \frac{x^5}{5} + \frac{x^4}{4} - 2x^3 + 1.$$

   (a) Find all the critical points of $f(x)$.

   (b) Find all of the relative extrema of $f(x)$ by applying the second derivative test to the critical points.

2. Sketch that graph of

$$f(x) = \frac{x^2 - 9}{x^2 - 4}.$$

You graph should include

(a) x and y intercepts.

(b) Relative extrema.

(c) Points of inflection.

(d) Vertical and horizontal asymptotes.

3. The velocity of blood (in centimeters/second) in a cylindrical artery can be given as a function of the distance r (in centimeters) to the central axis of an artery by

$$v(r) = k(R^2 - r^2)$$

where k is constant and R is the radius of the artery (also constant). Show that the velocity of blood is greatest along the central axis of the artery. What is the maximum velocity?

4. The quantity demanded each month of Sicard wristwatches is related to the unit price by the equation

$$p = \frac{50}{x^2/100 + 1}$$

where $0 \leq x \leq 20$, p is measured in the thousands of dollars and x in measures in units of a thousand. To yield a maximum revenue how many watches must be sold?

5. A Norman windox has the shape of a rectangle surmounted by a semicircle. If a Norman window is to have a perimeter of $4 + \pi$ ft what should the dimensions be in order to maximize the amount of light passing through the window. In your answer be sure to include the radius of the semicircle and the height of the rectangular portion of the window. (see page 320 problem 16 for a picture).