Math 121 — Homework 04

Instructions Remember to show all of your work to get credit. Please do this assignment on a seperate sheet of paper. Remember to use your words when explaining something.

1. For a function $u = u(x,t)$ the wave equation is given by

$$u_{tt} - c^2 u_{xx} = 0.$$

where c is a constant. If f and g are functions of a single variable show that $u(x,t) = f(x-ct) + g(x+ct)$, gives a solution of the wave equation.

2. The ideal gas law states that

$$PV = nRT,$$

where

- P is the pressure of the gas,
- V is the volume of the gas,
- n is the amount of substance of gas (in moles),
- R is the ideal, or universal, gas constant, equal to the product of the Boltzmann constant and the Avogadro constant,
- T is the absolute temperature of the gas.

(a) Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

(b) Show that

$$T\frac{\partial P}{\partial T}\frac{\partial V}{\partial T} = nR.$$

3. If $u = e^{a_1x_1 + a_2x_3 + \cdots + a_nx_n}$, where $a_1^2 + a_2^2 + \cdots + a_n^2 = 1$ show that

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = u.$$