Homework 3.1, Trig Summer 2009

June 26, 2009

In class we talked about the formulas

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)\\ \sin(\alpha+\beta) &= 2\sin(\alpha)\cos(\beta).\end{aligned}$$

The point of this homework is to show you how a lot other formulas come from this one.

1. In this problem we are going to derive the sum of angles formula for sine only using the sum of angles formula for cosine and some phase shift information.
   (a) Explain why $\cos(\alpha+\pi/2) = -\sin(\theta)$, (Do this either using the a unit circle or a graph of the function)
   (b) Prove that $\cos(\alpha+\pi/2) = -\sin(\alpha)$, using the sum of angles formula for cosine. (Hint: Let $\beta = \pi/2$ in the sum of angles formula for $\sin(\beta)$.)
   (c) Prove that $\sin(\theta+\pi/2) = \cos(\theta)$ using the formula in the previous problem (Hint: Let $\alpha = \theta+\pi/2$ and use the fact that $\cos(\theta+\pi) = -\cos(\theta)$.
   (d) Using the sum of angles formula for cosine derive the sum of angles formula for sine. (Hint: Let $\alpha = \theta+\pi/2$ in the sum of angles formula for cosine.)

2. I want you to prove the double angle formulas in this exercise using the formula for the sum of two angles.
   (a) Using the sum of angles formula sine prove the double angle formula for sine:
   $$\sin(2\theta) = 2\sin(\theta)\cos(\theta).$$
   (Hint: it's one line).
   (b) Using the sum of angles formula for cosine, prove the double angle formula for cosine:
   $$\cos(2\theta) = \cos(\theta)^2 - \sin(\theta)^2.$$

3. Using part (b) of the previous problem show that
   $$\cos(x) = 1 - 2\sin(x/2)^2.$$
   Here are some steps:
   (a) First make a substitution $\theta = x/2$.
   (b) Use the fact that $\cos(x/2)^2 + \sin(x/2)^2 = 1$ to finish.