Math 121 — Homework 02

Instructions Remember to show all of your work to get credit. Please do this assignment on a seperate sheet of paper. The assignment is due at the beginning of class on Friday. If you need to explain something, use words.

1. Sketch the region in $\mathbf{R}^3$ bounded by the surfaces $z = \sqrt{x^2+y^2}$ and $x^2+y^2=1$ for $1 \leq z \leq 2$.
2. Find a vector function that represents the intersection of $x^2+y^2=4$ and $z=xy$.
3. Evaluate the integrals
   (a) $\int (2t\mathbf{i}+\mathbf{j}+\mathbf{k})dt$
   (b) $\int_0^3 (e^t, 0, te^t)dt$
4. Reduce the following quartic surfaces to one of the standard forms, classify the surface and sketch it.
   (a) $x^2+y^2-2x+6y-z+10=0$.
   (b) $x^2+2y-2z^2=0$.
   (c) $x^2-y^2+z^2-4x-2z=0$.
5. Consider the hyperbolic paraboloid X defined by
$$z = y^2 - x^2.$$
   (a) For a point (a,b,c) lying on X show that the line
$$\begin{cases} x = a+t, \\ y = b+t, \\ z = c+2(b-a)t \end{cases},$$
   lies on the surface of X.
   (b) Draw the hyperbolic paraboloid X and the line for the point $(1,1,0)$.

   (Surfaces where every point has a line through it completely contained in the surface is called "ruled". This exercise shows that this hyperbolic paraboloid is ruled.)
6. Prove the product rule for the cross product:
$$\frac{d}{dt}(\mathbf{u}(t)\times\mathbf{v}(t)) = \mathbf{u}'(t)\times\mathbf{v}(t)+\mathbf{u}(t)\times\mathbf{v}'(t).$$
   (Hint: Use the definition of $\mathbf{u}(t)\times\mathbf{v}(t)$ and the definition of the derivative on components. If it feels tedious you are doing it right.)
7. Find a parametrization of the line tangent to the curve at the indicated point:
   (a) $\mathbf{r}(t) = (e^t, \ln(t), t)$
   (b) $\mathbf{l}(s) = (s, s^2, s^3)$ at $(2,4,8)$.

   at the point $(e,0,1)$.

8. Prove that the derivative of angular momentum is torque:

$$\frac{d}{dt}[\vec{l}(t)] = \vec{\tau}(t).$$

Here we recall that for a particle of mass m and position $\vec{r}(t)$ we have the following definitions:

$$\text{(angular momentum)} = \vec{l}(t) = \vec{r}(t) \times \vec{p}(t)$$

$$\text{(torque)} = \vec{\tau}(t) = \vec{r}(t) \times \vec{F}(t).$$

$$\text{(momentum)} = \vec{p}(t) = m\dot{\vec{r}}(t)$$

$$\text{(force)} = \vec{F}(t) = m\ddot{\vec{r}}(t)$$

(Hint: The answer is one line.)

9. Two particles travel have position vectors given by

$$\begin{aligned} \vec{r}_1(t) &= (t, t^2, t^3), \\ \vec{r}_2(t) &= (1+2t, 1+6t, 1+14t). \end{aligned}$$

   (a) Do the particles collide?

   (b) Do the paths parametrized by these functions intersect?

10. Suppose that $\mathbf{r}(t)$ is a vector valued function such that $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$. Show that $\mathbf{r}(t)$ lives on a circle centered at the origin. (Hint: Write $\mathbf{r}(t) = (x(t), y(t), z(t))$ and take the derivative of the function $|\mathbf{r}(t)|^2$ after expanding everything out.)