Math 121 — Homework 03

Instructions Remember to show all of your work to get credit. Please do this assignment on a seperate sheet of paper. The assignment is due at the beginning of class on Friday.

1. Setup but don't compute the arclength of $\vec{\gamma}(t) = (t^2, t^3, t^4)$ from $0 \leq t \leq 1$.

2. On page 864 of Stewart, he gives a formula for the curvature of a parameteric curve $\vec{r}(t)$:

$$\kappa(t) = \frac{|\dot{\vec{r}}(t) \times \ddot{\vec{r}}(t)|}{|\ddot{\vec{r}}(t)|^3}.$$

   Using this formula, show that the curvature of a plane parametric curve $(x(t), y(t))$ is given by

$$\kappa(t) = \frac{|\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)|}{[\ddot{x}(t)^2 + \ddot{y}(t)^2]^{3/2}}$$

3. Consider the curve

$$\vec{\gamma}(t) = \left(\frac{2}{t^2+1} - 1, \frac{2t}{t^2+1} \right).$$

   (a) Reparametrize this curve in terms of arclength measured from the point $\vec{\gamma}(0) = (1, 0)$.

   (b) What is the curve parametrized by $\vec{\gamma}(t)$? (You can say the name of it, or give its algebraic equation)

4. Suppose that $\mathbf{r}(t)$ is a vector valued function such that $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$. Show that $\mathbf{r}(t)$ lives on a sphere centered at the origin. (Hint: Write $\mathbf{r}(t) = (x(t), y(t), z(t))$ and take the derivative of the function $|\mathbf{r}(t)|^2$ after expanding everything out.)