Problem 4: (Errata)

There is a missing factor of 4 in the computation of m_y.

Correct Answer:

$$m_y = \frac{R}{15}$$

$$\Rightarrow COM = \left(0, \frac{R}{15}, \frac{3R}{4}\right).$$

Wikipedia:

$$\begin{cases} I_x = I_y = \frac{3h}{20}\left(R^2 + 4h^2\right)m \\ I_z = \frac{3hR^2}{10}m \end{cases}$$

Our Answer:

$$\begin{cases} I_x = I_y = \frac{\pi R^4 h k}{20} + \pi \frac{R^2 h^3}{5} k \\ I_z = \frac{\pi R^4 h k}{10} \end{cases}$$

Relationship:

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi R^2 \frac{h}{3}} = \frac{3m}{\pi R^2 h} = k.$$

$$(\text{our } I_x) = \frac{\pi R^2 h}{20} k \left(R^2 + 4h^2\right)$$

$$= \frac{\pi R^2 h}{20} \frac{3m}{\pi R^2 h}\left(R^2 + 4h^2\right)$$

$$= \frac{3m}{20}\left(R^2 + 4h^2\right) = (\text{wikipedia's } I_x)$$