Test 1 — Math 264 — Spring 2010

March 4, 2010

Write your name at the top left corner of the first page. Remember that your work is what you are being graded on not your answers.

1. Find the angle between the vectors
   (a) $\mathbf{v} = (1, 1, 0)$ and $\mathbf{w} = (1, 0, 2)$
   (b) $\mathbf{v} = (1, 2, 0)$ and $\mathbf{w} = (-2, 1, \pi)$

2. Find a vector which is perpendicular to both
   (a) $\mathbf{v} = (2, 2, 0)$ and $\mathbf{u} = (0, 1, 2)$.
   (b) $\mathbf{v} = (-1, 0, 1)$ and $\mathbf{w} = (0, 1, 2)$

3. Find the area of the parallelogram spanned by the vectors $\mathbf{v} = (1, 2, 3)$ and $\mathbf{w} = (3, 2, 1)$.

4. (a) Find an equation of the plane that passes through the points $(1, 0, 0)$,$(2, 1, 0)$ and $(0, 0, 1)$.
   (b) Draw the plane $2x + y - z - 1 = 0$. Find its normal vector $\mathbf{n}$.

5. Find a parametrization of a line tangent to the curve $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ at the point $(0, 1, \pi/2)$.

6. Consider the curve parametrized by $\mathbf{r}(t) = (\cosh t, \sinh t, t)$. Find the arclength of the segment starting at $\mathbf{r}(0) = (1, 0, 0)$ and ending at $\mathbf{r}(1) = (\frac{e+e^{-1}}{2}, \frac{e-e^{-1}}{2}, 1)$. You might want to remember the following:

$$\cosh t = \frac{e^t + e^{-t}}{2}, \quad \sinh t = \frac{e^t - e^{-t}}{2}$$
$$(\cosh t)' = \sinh t \quad (\sinh t)' = \cosh t$$

and $(\cosh t)^2 - (\sinh t)^2 = 1$.

7. Find the distance from the point $P = (1, 0, 1)$ to the plane $2x + y = 0$.

8. The curves parametrized by $\mathbf{l}(t) = \cos(t-1)\mathbf{i} + t\mathbf{j} + (t^2 - 2t + 1)\mathbf{k}$ and $\mathbf{r}(t) = (t+1)\mathbf{i} + e^t\mathbf{j} + \sin t\mathbf{k}$ intersect at the point $(1, 1, 0)$. Find their angle of intersection. (Just give me something you could plug into a calculator... you don't need to compute the actual number).

9. Let $\mathbf{u} : \mathbb{R} \to \mathbb{R}^2$ and $\mathbf{v} : \mathbb{R} \to \mathbb{R}^2$ be vector valued functions which are differentiable. Prove that

$$\frac{d}{dt}[\mathbf{u} \cdot \mathbf{v}] = \mathbf{u}' \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{v}'.$$

10. Prove that the derivative of angular momentum $\mathbf{l}$ is torque $\mathbf{T}$. Recall that $\mathbf{l} := \mathbf{p} \times \mathbf{r}$ and $\mathbf{T} = \mathbf{F} \times \mathbf{r}$ where $\mathbf{F} = m\ddot{\mathbf{r}}$ is force and $\mathbf{p} = m\dot{\mathbf{r}}$ is momentum.

11. EXTRA CREDIT: Explain how the cross product and dot product are related to quaternionic multiplication.