Math 180 — Practice Test 1/2 — Fall 2011

August 31, 2011

1. Simplify the following rational expressions

   (a) $\frac{2x^3-6x^2+4x}{x^2-4}$.

   (b) $\frac{x^4-27x}{2x^2-5x-3}$.

2. (a) Find the x and y intercepts of the line $3x+2y+1=0$.

   (b) Find the slope of the line $x+2y+3=0$.

   (c) Find the equation of the line passing through $(1,1)$ and $(2,1)$.

   (d) Find an equation of the line parallel to the y-axis passing through $(1,-1)$.

3. (a) Factor the polynomial x^3+27 into a product of monomial terms. (some of the roots may be complex.)

   (b) Factor the polynomial $2x^3-12x^2+10x$ completely.

4. Add the fractions and rationalize the denominator

$$\frac{1}{x-1}+\frac{x}{2x-2}$$

5. (a) Find the values of x where $|x+3|\le 4$ and graph them on a number line.

   (b) Find the values of x so that $x^2-1\le 0$

   (c) Find the values of x so that $(x-1)(x+3)\le 0$

   (d) Find the values of x such that $\frac{1}{x+1}\le 2$.

Solution to (4d) Whenever you have an inequality with two a fraction involved we should split it up into two cases. The case where the denominator is positive and the case where the denominator is positive. We will be applying the rule

$$a\le b \text{ and } c\le 0 \implies ca\le cb.$$

Note that we have two cases when the denominator is negative and when the denominator is positive

Suppose $x+1>0$**:**

$$\begin{aligned}\frac{1}{x+1}\le 2 \text{ and } x+1\ge 0 &\implies 1\le 2x+2\\ &\implies \frac{1}{2}-1\le x\\ &\implies -1/2\le x.\end{aligned}$$

So we have

$$x>-1 \text{ and } -1/2\le x.$$

which is equivalent to just $x\ge -1/2$

Suppose $x+1<0$: The algebra is exactly the same only now inequality flips since

$$\begin{aligned} \frac{1}{x+1} \le 2 \text{ and } x+1 \le 0 \quad &\Longrightarrow \quad 1 \ge 2x+2 \\ &\Longrightarrow \quad \frac{1}{2} - 1 \ge x \\ &\Longrightarrow \quad -1/2 \ge x \end{aligned}$$

So we have

$$x < -1 \text{ and } -1/2 \ge x.$$

which is the same as $x < -1$.

Putting the two cases together we have

$$x \ge -1/2 \text{ and } x < -1$$

which means solutions lie in $[-1/2, \infty) \cup (-\infty, -1)$.

Another solution of (4d) If you stop and *think* to get other ways of solving the problem. For example the whole right hand side of the inequality

$$\frac{1}{x+1} \le 2$$

becomes negative when $x+1<0$. This means $\frac{1}{x+1} < 0$ which is certainly less than 2. Now we need to deal with what happens when $x+1>0$. Well, very near $x=1$ but with $x>-1$ ever so slightly (think $x = -1 + 0.00000000000000001$) the fraction $\frac{1}{x+1}$ is HUGE! It will be much much bigger than 2. It is clear now that if we can find where $\frac{1}{x+1} = 2$ then the values larger than that number will be also be solutions.

$$\begin{aligned} \frac{1}{x+1} = 2 \quad &\Longrightarrow \quad 1 = 2x+2 \\ &\Longrightarrow \quad x = -1/2. \end{aligned}$$

So when $x \ge -1/2$ we will have $\frac{1}{x+1} \le 2$. Note that these numbers also make sense: This makes sense because this includes very very very large numbers and

$$\frac{1}{x+1} = \frac{1}{\text{Something Very Very Huge}} = \text{Something Very Very Close to Zero}$$

when $x+1 = $ (Something Very Very Huge).

We conclude that for x such that

$$x \ge -1/2 \text{ or } x < -1$$

our inequality is satisfied.