QUIZ 04

1) Find $\frac{\partial w}{\partial s}$ & $\frac{\partial w}{\partial t}$ where $w = 2xy$, $x = s^2 + t^2$ & $y = s/t$.

Soln

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

$$= (2y)(2s) + \underbrace{(2x)}(1/t)$$

$$= (2(\tfrac{s}{t}))(2s) + 2\underbrace{}_{s^2+t^2}(1/t)$$

$$= \frac{4s^2}{t} + \frac{2(s^2+t^2)}{t}$$

$$= \frac{6s^2 + 2t^2}{t},$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= (2\tfrac{s}{t})(2t) + (2(s^2+t^2))\left(\frac{-s}{t^2}\right)$$

$$= 4s - \frac{2s^3 + 2t^2 s}{t^2}$$

$$= \frac{4st^2 - 2s^3 - 2t^2 s}{t^2} = \frac{2st^2 - 2s^3}{t^2}$$

Find the plane tangent to the graph of

2) $f(x,y) = e^x + 2xy + y^3$ at the point $(0,1,2)$.

soln.

$$z = f(x_0, y_0) + \nabla f(x_0, y_0) \cdot (x - x_0, y - y_0)$$

$$f_x = e^x + 2y \qquad f_x(0,1) = 1 + 2 = 3$$
$$f_y = 2x + 3y^2 \implies f_y(0,1) = 3$$

$$\therefore \quad z = 2 + (3,3) \cdot (x - 0, y - 1)$$
$$= 2 + 3x + 3(y - 1).$$