Math017B –Problem Bank
last updated: January 17, 2023

This file contains the problem bank for Dupuy's Spring 2023 Math 17 course. All the problems for our quizzes are either going to be in this file or very similar to problems in this file.

WARNING: The problems in sections marked as "unstable" are subject to change. The order and the titles of the sections might even change. Problems need to be tested and sometimes this takes a couple iterations. I will try to provide enough stable material so that students who like to work ahead can. If you are finding that there is not enough stable material

Contents

1 Sets (unstable)

Here is a basic reference for set notation. Sets are just collections of things and there are a couple symbols we may use that help us count and talk about things. Here is a basic reference:

https://www.mathsisfun.com/sets/sets-introduction.html

We also use the following basic notation throughout this course:

$$\mathbb{N} = \{1, 2, 3, \cdots\} = \text{ (natural numbers)}$$
$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots, \} = \text{(integers)}$$
$$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\} = \text{ (rational numbers)}$$
$$\mathbb{R} = \text{(all numbers on the real number line)}$$

For the following problems let $A = \{a, b, c, 2\}$ and $B = \{a, 2, \pi\}$ where a, b and c are abstract letters.

1. Write out $A \cap B$ as a set. What is the cardinality of $A \cap B$?

2. Write out $A \cup B$ as a set. What is the cardinality of $A \cup B$?

3. Write out $A \setminus B$ as a set. What is the cardinality of $A \setminus B$?

4. Write out all of the subsets of A.

5. Is it the case that $2 \in A$?

6. Is it the case that $\pi \in A$?

7. Is it the case that $\{a, b\} \subset A$?

8. Write out the set of even numbers in set builder notation.

2 Counting (unstable)

This material is "officially" covered in section 16.2 of Tannenbaum. A slower paced introduction is provided in Chapters 1-5 of *The Magic Of Numbers* by Gross and Harris (GH).

1. How many numbers are there between 1776 and 2023? How many are even? How many are odd? (read GH Ch 1)

2. Suppose I have a textbook with a subsection of 5 problems that I want to rearrange for the next edition. How many ways can I do this for the next edition? (read GH Ch 2)

3. Consider the make your own sandwich option at Mill Market in South Burlington:

   https://themillmarket.com/deli-menu

   They have 9 proteins, 13 breads, 14 sauces, 6 cheeses, and 15 veggies. Supposing you only use one sauce and one protein (and one bread, let's not get crazy), how many sandwiches do they offer? (see GH Ch 2, or T *Permutations and Combinations* starting)

4. A license plate is three letters followed by a number between 100 and 999. How many license plates can you make following this rule? (see GH Ch3)

5. How many three digit numbers are there in base 10? (see GH Ch4)

6. Baskin Robbins has 31 flavors of ice cream. How many different three scoop ice cream servings are there? (see GH Ch4)

7. How many ways are there to rearrange the letters in the word MISSISSIPPI? (see GH Ch 4 and 5)

8. T: 16.28

3 Probability (unstable)

Here we follow Ch 16 of Tannenbaum. For additional help the reader should consult GH chapters Ch 4 and 5.

1. What does the sample space for a single coin flip look like? What does the sample space for two coin slips look like? What does the sample space for three coin flips look like? (see Tannenbaum examples 16.1 and 16.2).

2. Suppose you flip a coin 10 times. Which of the following sequences is more likely to occur?

   (a) $THHTHHTTHT$
   (b) $TTTTTTTTTH$

3. Suppose you flip a coin 10 times. Which of the following outcomes is more likely to occur? (see T: 17.74 for a similar problem)

   (a) Flipping 5 heads and 5 tails.
   (b) Flipping 2 heads and 8 tails.

4. Suppose you are dealt two cards from a standard 52 card deck.

   (a) What is the probability that the two cards are sequential (this includes $10J, JQ, QK, KA, A2$)?
   (b) What is the probability that the two cards have the same suit?
   (c) What is the probability that the two cards are a pair?

5. You meet two people randomly on the street. What is the probability that they share the same birthday?

6. (expected value) 16.59

7. (expected value) 16.57

8. (additivity of expected values) 15.67

9. Suppose you are in a poker hand and there is $75 in the pot and your are put into a situation where you need to pay $25 more to enter. What does the probability that you have a winning hand need to be in order for this to be a good bet?

4 Graphs and Networks (unstable)

1. (Graph terminology) 5.3, 5.4

2. (Adjacency) 5.7

3. (Turn a city map into a graph) 5.20.

4. Can the city graph from the previous problem be solved?

5. (Which graphs have Euler Circuits) 5.32

6. (Find Hamiltonian Paths) 6.2

7. (Count the TSP Paths) 6.55

8. (Ore's Condition) 6.68 and 6.69 — I would give these problems in a graph theory class and they are fun to try but I'm not going to do that here. Only try these if you want to dig into Hamiltonian Graphs more.

9. ♠♠♠ Taylor: [add graph minor problems].

5   Statistics (unstable)

The webpage https://www.explainxkcd.com may be helpful for these.

1. Explain the following xkcd cartoons related to graphing.

   (a) https://xkcd.com/1007/
   (b) https://xkcd.com/1945/

2. Explain the following xkcd comics related to extrapolation.

   (a) https://xkcd.com/476/
   (b) https://xkcd.com/605/

3. Explain the following xkcd cartoons related to p-values.

   (a) https://xkcd.com/1478/
   (b) https://xkcd.com/2533/
   (c) https://xkcd.com/882/

4. The following is from *Currents of Fear* on Frontline.[1]

   > ... [I]n 1992, a landmark study appeared from Sweden. A huge investigation, it enrolled everyone living within 300 meters of Sweden's high-voltage transmission line system over a 25-year period. They went far beyond all previous studies in their efforts to measure magnetic fields, calculating the fields that the children were exposed to at the time of their cancer diagnosis and before. This study reported an apparently clear association between magnetic field exposure and childhood leukemia, with a risk ratio for the most highly exposed of nearly 4.
   >
   > ... Surely, here was the proof that power lines were dangerous, the proof that even the physicists and biological naysayers would have to accept. But three years after the study was published, the Swedish research no longer looks so unassailable.
   >
   > ... [T]he original contractor's report... reveals the remarkable thoroughness of the Swedish team. Unlike the published article, which just summarizes part of the data, the report shows everything they did in great detail, all the things they measured and all the comparisons they made. ... [N]early 800 risk ratios are in the report....1

   To be clear, the Swedish team took 800 thinks like leukemia and looked for associations between leukemia and living near power lines. Why is it not surprising that one of these came up? With a p value threshold of 0.05, on average, how many "risk ratios" would we need to be computed to expect "find some link"?

5. (Capture Recapture Example) 14.61

---

[1]I found this on https://www.fallacyfiles.org/multcomp.html

6. In *Life on the Mississippi* Mark Twain writes

> The Mississippi between Cairo and New Orleans was twelve hundred and fifteen miles long one hundred and seventy-six years ago. It was eleven hundred and eighty after the cut-off of 1722. It was one thousand and forty after the American Bend cut-off. It has lost sixty-seven miles since. Consequently, its length is only nine hundred and seventy-three miles at present. Now, if I wanted to be one of those ponderous scientific people, and "let on" to prove what had occurred in the remote past by what had occurred in a given time in the recent past, or what will occur in the far future by what has occurred in late years, what an opportunity is here! ... Please observe: In the space of one hundred and seventy-six years the Lower Mississippi has shortened itself two hundred and forty-two miles. That is an average of a trifle over one mile and a third per year. Therefore, any calm person, who is not blind or idiotic, can see that in the Old Oolitic Silurian Period, just a million years ago next November, the Lower Mississippi River was upward of one million three hundred thousand miles long, and stuck out over the Gulf of Mexico like a fishing-rod. And by the same token any person can see that seven hundred and forty-two years from now the Lower Mississippi will be only a mile and three-quarters long, and Cairo and New Orleans will have joined their streets together, and be plodding comfortably along under a single mayor and a mutual board of aldermen. There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.

7. As of Monday, Jan 16, 2023: basketball-reference.com shows currently shows that your cousin, Trevor Hudgins, as the league leader in unqualified three-point percentage (3P%):

   https://www.basketball-reference.com/leagues/NBA_2023_totals.html#totals_stats::fg3_pct

   Your family is very proud. How do you explain to explain to your grandma at Christmas that cousin Trevor is not as good as Steph Curry at three pointers?

8. Your friend Taylor loves to flip coins and owns a fair coin he flips all the time.

   (a) After an amazing 15 streak of heads, he is insists that the next flip will more likely be tails in order to make up for the number of heads that just occured. What mistakes is he making?

   (b) After flipping 100 coins what is the expected disparity between the number of heads and number of tails?

9. Jersey girls are three times more likely to have a gel manicure (this is totally made up). Your friend sees a girl with a gel manicure and concludes that she must be from Jersey. What mistake is your friend making?

10. Taylor and Gloria are close friends. Gloria shares the terrible news with Taylor that her husband Albert has lung cancer. Knowing that the CDC says smokers are 15-30 more likely to get lung cancer than non-smokers Taylor suspects that Albert was probably a smoker. This is false and only about 65% of lung cancer patients are not smokers.[2] What mistake did Taylor make?

11. Some people on the internet argue that men are smarter than women in the following way: they point out that brain size is correlated with IQ scores and that IQ scores are correlated with gender. What is wrong with this argument?

---

[2]https://www.lungevity.org/for-supporters-advocates/lung-cancer-awareness/lung-cancer-statistics

6 Decision Theory (unstable)

1. ♠♠♠ Taylor: [add problems on decision theory]

2.

7 Mathematics of Elections (unstable)

A general reference for this material is Chapter 1 of Tannenbaum. Note that the terminology that Tannenbaum uses slightly different terminology from class.

Problems 1–5 concern Table 1-31 of the book.

1. (Most Votes Wins) T: 1.11

2. (Borda Method) T:1.21

3. (Instant Runoff) T:1.31

4. (Round Robin) T:1.41

5. (Borda Violates Pairwise Comparison) T:1.51 [this example is telling us the Borda method is not cool]

6. Come up with a simple example of a ranked choice election with three candidates where the Border, Instant-Runoff, and

8 Symmetry and the Golden Ratio (unstable)