Reflection Prompts for Applications of Finite Math

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This document contains all of the reflection questions for Math 17. There are no right or wrong answers to these questions.

WARNING: Prior to their being assigned a due date reflection questions are subject to change! This includes, but is not limited, to the order in which reflection questions are given and title of the reflections!

0.1 Flipping Coins (due Monday, Jan 23)

Produce a sheet of paper with 100 coin flips recording a sequential number of heads and tails. There are two ways you can do this. You can cheat and just write down a bunch of heads and tails or you actually flip a coin 100 times. All I ask is that you don't mix these two ways of completing this assignment. Either completely cheat and just write down random heads and tails or you actually flip coins and do this for the entire 100 flips. When you submit the assignment don't say which method you used.

0.2 Calculators and Really Small Numbers (due Friday, Jan 27)

We often take calculators for granted. Find you nearest calculator (google, iPhone, or laptop will work) and input $31 + 10^{-16}$, that is, try to get it to perform the addition $31 + 0.0000000000000001$. Explain what computer you used and what happened? Try it on another system. Why do you think this happens? What is the largest number such that 1 plus that number returns 1 on that device? [there are some devices where this won't work, I know this works on google.]

0.3 The Monty Hall Problem (due Friday, Feb 17)

1. Suppose you are going to buy a lottery tickey and the only prize is the jackpot. In the game you select six numbers 1-50. Which ticket is better the ticket with 1,1,1,1,1,1 or the ticket with 12,26,3,50,24,27? Why or why not?

2. Suppose you are flipping a coin and you flip 10 heads in a row. Is it more likely that you flip a tails the next time?

3. You flip 5 coins. Which outcome is more likely HHHHH or HTHTH?

4. You are the contestant an *Let's Make A Deal* with host Monty Hall. In this game there are three doors. Behind two of the doors there are goats and behind one of the doors is a brand new car. Your job is to guess which door has the car and if you get it right, you get the car. Monty asks you to select a door. After your selection he reveals goats behind one of the doors. You are allowed to stay with your current door or switch. What should you do? Should you stay with your current door? Should you switch to the new door? Does it matter?

0.4   What is Ranked Choice Voting? (due Wednesday, March 1)

In 2009 Burlington had an election for Mayor in which there were three candidates. At the time the city implemented "ranked choice voting" which is a system in which voters submit a ballot which ranks their perferred order of candidates. How do you suppose this worked? Make up some rules that would allow you to elect a mayor to take into account the voters will using these ranked ballots. How would you determine the winner if you were allowed to make up a voting system? Why?

0.5   Seven Bridges of Koenigsburg (due Wednesday March 29)

Euler lived in the city of Koenigsberg (modern day Kaliningrad). The city had a system of bridges as mapped out below in Figure1. Euler wanted to find a route through the city to walk across each one of the bridges exactly once in the city. How can Euler achieve such a route? Make a map and try making a route that achieves this.
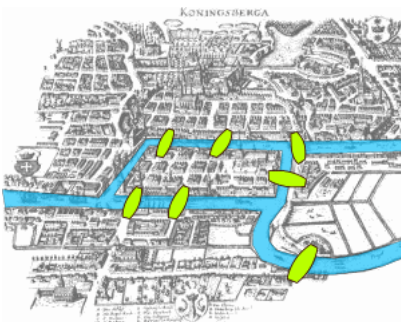

Figure 1: The city of Koenigsburg and its seven bridges.

0.6   Expected Values Are Everywhere (due Friday, March 31)

Expected values are everywhere. They can be used in games, psychology experiments, interpersonal relationships, business, biology, and elsewhere. Give an example of an expected value that you care about? This can be from a game you play or from something you like to collect or anything really.

0.7   Bernoulli's Game (due Friday, March 31)

[1] Consider the following five dollar carnival game. In this game, you are asked to repeatedly toss a coin and the game stops the first time you flip heads. The payout structure is as follows:

- if you flip heads (H) your first try you get $1.

- if you flip tails then heads (TH) you get $2.

- if you flip two tails then a heads (TTH) you get $4.

- if you flip TTTH you get $8.

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The pattern keeps going with the payout doubling for each tail you flip in a row. Should you play this game? How much should you pay to play this game?

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[1]Ellenberg "St Petersburg and Ellsburg" pg 242

0.8   3 Houses and 3 Utlilities (due Wednesday, April 5)

My 5th grade teacher math Mrs. Kelly gave me the following problem in 1995: There are three houses and three utilities as pictured below. Draw a line connecting each of the three houses to each of the three utilities without any of the lines cross each other. Explain what is going on in this problem. x
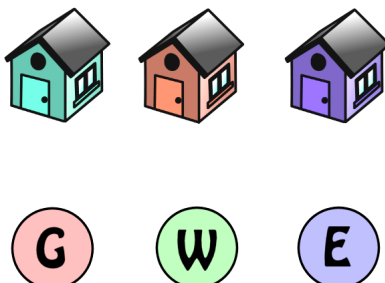

Figure 2: A image of the three houses and three utilities problem. Image taken from Nigel Coldwell's webpage.

0.9   Prisoner Problem

There are 50 prisoners numbered 1 through 50. There are 50 boxes numbered 1 through 50 in a room. Each of the numbers 1-50 are placed in the boxes at random with each number appearing in a box exactly once (so for example the number 7 could be placed in box 20). The Warden then plays a game for the prisoners freedom: each prisoner is led into the room one at a time and is asked to find their own number. They get two tries. After a prisoner leaves the room, the boxes and numbers are reset to their original configuration (so all of the prisoners get the same setup). If *all* of the prisoners find their number they will all be set free.

The prisoners are allowed to communicate beforehand to come up with a strategy but are not allowed to communicate after the game has started. What strategy should they use? (Spoiler: there exists a strategy that works about 30% of the time)

0.10

♠♠♠ Taylor: [TBD p-value]

0.11

♠♠♠ Taylor: [TBD regression to the mean]

0.12   Möbius Band

What do you think will happen if we take a Möbius band and cut it down the middle?