Problem Bank

Course: Math 1077
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This document contains the Problem Bank and Course Objectives for Math 1077. All the problems for our quizzes are either going to be in this file or very similar to problems in this file.

Textbook Abbreviations:

- T = Tannenbaum
- GH = Gross and Harris

WARNING: The problems in sections marked as "unstable" are subject to change. The order and the titles of the sections might even change. Problems need to be tested and sometimes this takes a couple iterations of the course to get this right. If you find any mistakes please email me and I will correct them as soon as possible.

Contents

1 Mathematical Thinking

The webpage https://www.explainxkcd.com may be helpful for some of the problems featuring xkcd cartoons.

1.1 Understanding Logarithms

This was talked about in class.

1. What is the approximate value of the logartihm of 12345689123456789 in base 10?[1]

1.2 Understanding How To Read Graphs

1. Explain the following xkcd cartoons.
   (a) https://xkcd.com/1007/
   (b) https://xkcd.com/1945/

1.3 Understanding How People Are Reckless With Extrapolation

1. Give one real life example of how people carelessly used linear regression or some other fitting technique to conclude something wrong and/or dangerous.
2. Explain the following xkcd comics related to extrapolation.
   (a) https://xkcd.com/476/
   (b) https://xkcd.com/605/
3. What is the Laffer Curve? (what was the original example)
4. Come up with an example of something that obeys the Laffer Curve rule.

[1] No calculators for this one, there is a super cheap-o way of doing this.

2 Counting

This material is "officially" covered in section 16.2 of Tannenbaum. A slower paced introduction is provided in Chapters 1-5 of *The Magic Of Numbers* by Gross and Harris (GH).

2.1 Understanding How To Count Numbers Between Numbers

This is chapter 1 of GH.

1. How many numbers are there between 1776 and 2023? How many are even? How many are odd?

2.2 Understanding Permutations and the Multiplication Principle

See GH Ch2 for this.

1. Suppose I have a textbook with a subsection of 5 problems that I want to rearrange for the next edition. How many ways can I do this for the next edition?

2. Consider the make your own sandwich option at Mill Market in South Burlington:

   https://themillmarket.com/deli-menu

   They have 9 proteins, 13 breads, 14 sauces, 6 cheeses, and 15 veggies. Supposing you only use one sauce, one protein, one bread, one cheese, and any combination of veggies, how many sandwiches do they offer (you don't need to multiply out your answer)? If we do get crazy and allow any combination of cheeses, sauces, and proteins, how many sandwiches do they offer? How does this compare to the number of seconds the universe has existed?[2][3]

2.3 Understanding the Subtraction Principle

See GH Ch3 for this section.

1. A license plate is three letters followed by a number between 100 and 999. How many license plates can you make following this rule?

2. How many three digit numbers are there in base 10? [there are two ways of doing this]

2.4 Understanding Collections

This is GH Ch4.

1. How many ways are there to choose 2 objects from a subset of 10 objects?

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[2] Do not attempt to multiply out this number.

[3] The universe has existed for approximately 4.4×10^{17} seconds.

2. In basketball, only 5 people can be on the court at the same time. If there is a team with 12 people on it, how many lineups can they make?
3. Baskin Robbins has 31 flavors of ice cream. How many different three scoop ice cream servings are there?
4. How many ways are there to rearrange the letters in the word MISSISSIPPI?
5. T: 16.28

3 More Counting: Coin Flipping and Decks of Cards

Here we follow Ch 16 of Tannenbaum. For additional help the reader should consult GH chapters Ch 4 and 5.

3.1 Understanding Sample Spaces

1. What does the sample space for a single coin flip look like? What does the sample space for two coin slips look like? What does the sample space for three coin flips look like? (see Tannenbaum examples 16.1 and 16.2).
2. Suppose you flip a coin 10 times. Which of the following sequences is more likely to occur?
   (a) $THHTHHTTHT$
   (b) $TTTTTTTTTH$
3. Suppose you flip a coin 10 times. Which of the following outcomes is more likely to occur? (see T: 17.74 for a similar problem)
   (a) Flipping 5 heads and 5 tails.
   (b) Flipping 2 heads and 8 tails.

3.2 Understanding How to Compute Basic Probabilities

1. Suppose you are dealt two cards from a standard 52 card deck.
   (a) What is the probability that the two cards are sequential (this includes $10J, JQ, QK, KA, A2$)?
   (b) What is the probability that the two cards have the same suit?
   (c) What is the probability that the two cards are a pair?
2. You meet two people randomly on the street. What is the probability that they share the same birthday?

3.3 Coinflipping

1. What is the plinko game?
2. What does coinflipping a coin 10 times in a row have to do with a single ball going through plinko with 10 levels? Why are these things similar? (draw a picture)
3. If I flip 100 coins, how far away from a 50/50 split are we expected to be? (how many extra heads or tails are expected on average?)
4. Your friend Taylor loves to flip coins.
   (a) After an amazing 15 streak of heads, he is insists that the next flip will more likely be tails in order to make up for the number of heads that just occured. What mistakes is he making?

(b) After flipping 100 coins what is the expected disparity between the number of heads and number of tails?

4 Expected Values

4.1 Understanding What to Do With Expected Values

1. Suppose you are in a poker hand and there is $75 in the pot. You need to put in $25 to play. What does the probability that you have a winning hand need to be in order for this to be a good bet?

4.2 Understanding Expected Values

1. T:16.59

2. T:16.57

3. T:15.67

4.3 Understanding How To Estimate The Number of German Tanks

1. What ideas went into the estimation of German Tanks by Statisticians in World War 2?

2. (Capture Recapture Example) 14.61

5 p-Values

5.1 Understanding How Messed-Up and Misleading p-Values Are

1. What is a p-value and how is one computed? (give an executive summary of the process without worrying about technical details).

2. Explain the following xkcd cartoons related to p-values.

   (a) https://xkcd.com/1478/

   (b) https://xkcd.com/2533/

   (c) https://xkcd.com/882/

3. The following is from *Currents of Fear* on Frontline.[4]

   > ...[I]n 1992, a landmark study appeared from Sweden. A huge investigation, it enrolled everyone living within 300 meters of Sweden's high-voltage transmission line system over a 25-year period. They went far beyond all previous studies in their efforts to measure magnetic fields, calculating the fields that the children were exposed to at the time of their cancer diagnosis and before. This study reported an apparently clear association between magnetic field exposure and childhood leukemia, with a risk ratio for the most highly exposed of nearly 4.
   >
   > ...Surely, here was the proof that power lines were dangerous, the proof that even the physicists and biological naysayers would have to accept. But three years after the study was published, the Swedish research no longer looks so unassailable.
   >
   > ...[T]he original contractor's report...reveals the remarkable thoroughness of the Swedish team. Unlike the published article, which just summarizes part of the data, the report shows everything they did in great detail, all the things they measured and all the comparisons they made. ...[N]early 800 risk ratios are in the report....1

   To be clear, the Swedish team took 800 things like leukemia and looked for associations between leukemia and living near power lines. Why is it not surprising that one of these came up? With a p value threshold of 0.05, on average, how many "risk ratios" would we need to be computed to expect "find some link"?

4. In Ellenberg's *How Not To Be Wrong* he proves that scientists are regularly abusing p-values. How does he do this? (You just need to say what goes into this roughly)

[4] I found this on https://www.fallacyfiles.org/multcomp.html

6 Mathematics of Elections

A general reference for this material is Chapter 1 of Tannenbaum. Note that the terminology that Tannenbaum uses slightly different terminology from class.

6.1 Understanding Burlington's 2009 Mayoral Race

Problems 2–5 concern Table 1-31 of the book.

1. Why is the Burlington 2009 Mayoral Election such an important example?
2. (Most Votes Wins) T: 1.11
3. (Borda Method) T:1.21
4. (Instant Runoff) T:1.31
5. (Round Robin) T:1.41

6.2 Understanding Slime Molds

1. What does the slime mold experiment have to do with ranked choice voting?

6.3 Understanding that Elections Are Fundamentally Flawed

1. What does the Arrow's Impossibility Theorem say?
2. (Borda Violates Pairwise Comparison) T:1.51 [this example is telling us the Borda method is not cool]
3. Come up with a simple example of a ranked choice election with three candidates where the Border, Instant-Runoff, and Pairwise comparison are all give different election results.

7 Graphs and Networks

7.1 Understanding the Basic Terminology of Graphs and Networks

1. T:5.3, 5.4

2. T: 5.7

7.2 Understanding what the Koenigsberg Bridges Problem

The material for this section is 5.1 and 5.2 of Tannenbaum in *The Mathematics of Getting Around*.

We started this section with the *Koenigsberg Bridges Puzzle*. A map of the city of Koenigsberg is pictured in Figure 1. The *Koenigsberg Bridges Puzzle* asks the following: is

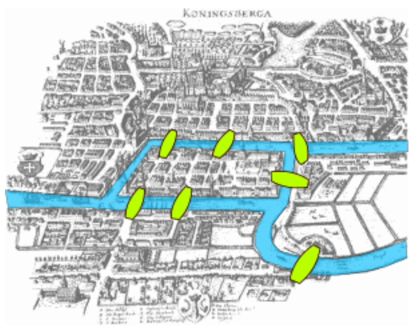

Figure 1: The city of Koenigsburg and its seven bridges.

there a way to take a walk in the city where you cross each bridge exactly once?

1. (Turn a city map into a graph) T:5.20.

2. Consider the city in the previous problem. Can we walk across all of the bridges exactly once without repeating? Why or why not?

3. (Which graphs have Euler Circuits) 5.32

4. Consider the appartment with a floor plan pictured in Figure 2. Explain why it is impossible to make a tour of the appartment using all of the doors exactly once. [5]

7.3 Understanding the Traveling Salesman Problem

1. Why is the Traveling Salesman Problem so hard?

2. (Find Hamiltonian Paths) 6.2

[5] This puzzle was given to a collaborator of mine in 5th grade as an extra credit problem. It is impossible.

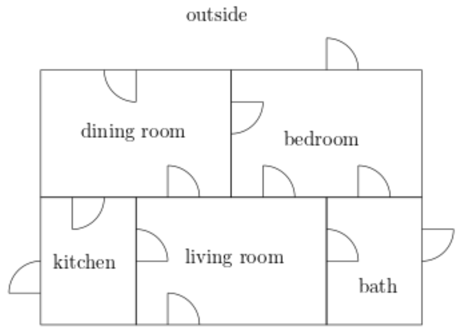

Figure 2: The floor plan of a ground floor appartment, its rooms, and the various doors connecting each of the rooms.

3. (Count the TSP Paths) 6.55 [6]

4. (Ore's Condition) 6.68 and 6.69 [7]

7.4 Understanding the Three Houses Three Utilities Problem

This section is about planar graphs which isn't covered in the book, so here is a great video lecture series on the topic:

Sarada Herke's Lectures on Planar Graphs

I motivated the section with the *Three Houses Three Utilities* puzzle. The *Three Houses and Three Utilities* puzzle says that there are three houses that need to be connected to three utilities but none of the lines are allowed to cross. How do you configure your lines so that each of the houses can have all three ultilities? The initial setup for the *Three Houses and Three Utilities* puzzle is pictured in Figure 3. The following questions are about explaining why the *Three Houses and Three Utilities* puzzle is impossible.

1. What is a planar graph?

2. Draw a graph and a minor of the graph.

3. Draw $K_{3,3}$ and K_5.

4. Give an example of a planar graph.

5. Give an example of a non-planar graph that is not $K_{3,3}$ or K_5.

6. Consider the graph pictured in Figure 4. Can this graph be made planar or not? Why? (Hint: contract de, hf, and fg.)

7. Why is the *Three Houses Three Utilities* puzzle impossible? [8]

[6] I will not quiz on this since it is longer.
[7] I will not quiz on this problem since it is harder
[8] My 5th grade Mathematics teacher gave this problem to us as students as extra credit.

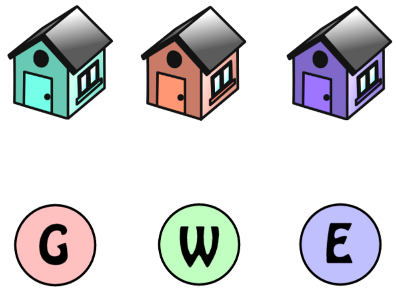

Figure 3: A image of the three houses and three utilities problem. Image taken from Nigel Coldwell's webpage.

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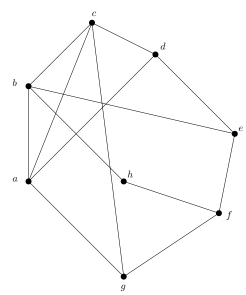

Figure 4: A graph for a homework problem.

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8 Mistakes with Sampling

8.1 Understanding Sample Sizes

1. As of Monday, Jan 16, 2023: basketball-reference.com shows currently shows that your cousin, Trevor Hudgins, as the league leader in unqualified three-point percentage (3P%):

   https://www.basketball-reference.com/leagues/NBA_2023_totals.html#totals_stats::fg3_pct

   Your family is very proud. How do you explain to explain to your grandma at Christmas that cousin Trevor is not as good as Steph Curry at three pointers?

[9]

[9] There is a common fallacy that because someone has disease X and people from group Y have disease

8.2 Understanding Base Rate Fallacies

1. Jersey girls are three times more likely to have a gel manicure (this is totally made up). Your friend sees a girl with a gel manicure and concludes that she must be from Jersey. What mistake is your friend making?

2. Alice and Bob are close friends. Bob shares the terrible news with Alice that his husband Charlie has lung cancer. Knowing that the CDC says smokers are 15-30 times more likely to get lung cancer than non-smokers Alice suspects that Charile was probably a smoker. This is false and only about 65% of lung cancer patients are not smokers.[10] What mistake did Alice make? [11]

[10] https://www.lungevity.org/for-supporters-advocates/lung-cancer-awareness/lung-cancer-statistics

[11] There is a common fallacy that because someone has disease X and people from group Y have a higher incidence of disease X that someone with disease X is more likely to come from group Y. Humans want to make this mistake all the time. Try to think of a couple examples.

A Sets

Here is a basic reference for set notation. Sets are just collections of things and there are a couple symbols we may use that help us count and talk about things. Here is a basic reference:

https://www.mathsisfun.com/sets/sets-introduction.html

We also use the following basic notation throughout this course:

$$
\begin{aligned}
&\mathbb{N} = \{1, 2, 3, \cdots\} = \text{ (natural numbers)}\\
&\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots, \} = \text{(integers)}\\
&\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\} = \text{ (rational numbers)}\\
&\mathbb{R} = \text{(all numbers on the real number line)}
\end{aligned}
$$

The way $\mathbb{Q}$ is defined is an example of "set builder notation".

A.1 Understanding Union, Intersection, Subsets, Elements, Membership, and Cardinality

For the following problems let $A = \{a, b, c, 2\}$ and $B = \{a, 2, \pi\}$ where a, b and c are abstract letters.

1. Write out $A \cap B$ as a set. What is the cardinality of $A \cap B$?
2. Write out $A \cup B$ as a set. What is the cardinality of $A \cup B$?
3. Write out $A \times B$ as a set. What the cardinality of $A \times B$?
4. Write out $A \setminus B$ as a set. What is the cardinality of $A \setminus B$?
5. Write out all of the subsets of A.
6. Is it the case that $2 \in A$?
7. Is it the case that $\pi \in A$?
8. Is it the case that $\{a, b\} \subset A$?

A.2 Understanding Set Builder Notation

1. Write out the set of even numbers in set builder notation.

B Decision Theory (unstable)

B.1 Understanding Utils (unstable)

C Symmetry and the Golden Ratio (unstable)