Math 180 — Practice Test 3 — Fall 2011

November 16, 2011

1. Compute the following derivatives of the following functions
   (a) e^{2x}
   (b) $\log_2(x)$
   (c) xe^{x^2}
   (d) 3^x
   (e) x^3
2. Find all the real solutions of $e^{2x^2-2} - 5e^{x^2-1} + 6 = 0$.
3. (a) Express $\log_{10}(x)$ in terms of $\ln(x)$.
   (b) Show that
   $$\frac{d}{dx}\left[\log_{10}(x)\right] = \frac{1}{\ln(10)x}.$$
4. Graph the function $f(x) = x^2e^{-2x^2}$.
5. If we assume that every child born has 2^n ancestors n-generations back (e.g. 2 parents, 4 grandparents, 8 great grandparents, 16 great great grandparents etc.) and that every generation has children when 30 years old (e.g. parents are born 30 years before a child, grandparents were born 30 before the parents, great grandparents are born 30 year before the grand parents etc).
   (a) How many years ago where a childs great great great great grandparents born?
   (b) How many generations does one need to go back in order to find that a single child born today has more than 7 billion unique ancestors? How many years ago was this?

   (Remark: The current population of the Earth is 7 billion. This exercise shows that everyone in inbred.)
6. Find the horizontal asymptotes of the function
   $$f(x) = \frac{e^{2x} - 3e^{-2x}}{e^{2x+1} + e^{-2x}}.$$
7. If you invest one penny at an interest rate of 10% annually which is compounded monthly, how many years will it take for your penny to turn into one dollar?
8. At the end of 2008 a single share of Google stock was worth 300 dollars a share. Now, a share of Google stock is worth 600 dollars. If we assume that an investment into google stock is continuously compounding then what is interest rate over this two year period?
9. Newton's law of cooling says that the temperature of an object placed in a medium constant temperature is
   $$T(t) = S + (T_0 - S)e^{-kt},$$

Where

$$\begin{aligned} T(t) &= \text{temperature of the object at time } t \\ S &= \text{outside temperature} \\ T_0 &= \text{initial temperature of the object} \\ t &= \text{time passed since object was introduced into the environment} \\ k &= \text{a constant} \end{aligned}$$

Suppose that a corpse was discovered in a motel room at midnight and its temperature was 70 degrees Farenheit. The temperature of the room is kept constant at 60 degrees. Two hours later the temperature of the corpse dropped to 75 degrees.

(a) Find the value of the constant k that appears in Newton's law of cooling.

(b) Find the time of death.

(Hints: The initial temperature was 98.6° Farenheit.)

10. Simplify the expression

$$\log_2\left(\frac{2^x(x^2-1)}{\sqrt{x^2+1}}\right).$$