Reflections

Course: Math 1077
Professor Dupuy

August 28, 2023

The following is a list of writing prompts for the course. Most of them are fun things to try or things to think about. There are no right or wrong answers. Turning them in gets you full credit. They are intended to get us thinking about certain topics before we discuss them in class.[1]

WARNING: Dates and problems are subject to change. Please check the webpage for the most up to date information on due dates.

[1]**ETIQUETTE**: We love our graders. We want to be nice to them. Generally it is considered polite to use a fresh 8.5 x 11 sheet of paper for our assignments. Sheets that are too small or too big mess up their stacks. It can be blank or lined but should not ripped from a spiral notebook. Frills from a spiral notebook also make the paper hard to stack and sort. This goes for every class, not just this one. If you don't do this you aren't wrong, you are just being a little rude. Also, you should also write your full name (or just your last name) at the top right of the page. This makes it easy for graders who are processing about a 100 of these things. It also lowers the risk that your assignment will get lost in the process.

1 Calculators and Really Small Numbers

We often take calculators for granted. Find you nearest calculator (google, iPhone, or laptop will work) and input $31 + 10^{-16}$, that is, try to get it to perform the addition $31 + 0.0000000000000001$. Explain what computer you used and what happened? If you find nothing strange try $31 + 10^{-32}$. Why do you think this happens? What is the largest number such that 1 plus that number returns 1 on your device?

2 Mark Twain Being Sassy

In *Life on the Mississippi* Mark Twain (a well known sarcastic ass) writes:

> The Mississippi between Cairo and New Orleans was twelve hundred and fifteen miles long one hundred and seventy-six years ago. It was eleven hundred and eighty after the cut-off of 1722. It was one thousand and forty after the American Bend cut-off. It has lost sixty-seven miles since. Consequently, its length is only nine hundred and seventy-three miles at present. Now, if I wanted to be one of those ponderous scientific people, and "let on" to prove what had occurred in the remote past by what had occurred in a given time in the recent past, or what will occur in the far future by what has occurred in late years, what an opportunity is here! ... Please observe: In the space of one hundred and seventy-six years the Lower Mississippi has shortened itself two hundred and forty-two miles. That is an average of a trifle over one mile and a third per year. Therefore, any calm person, who is not blind or idiotic, can see that in the Old Oolitic Silurian Period, just a million years ago next November, the Lower Mississippi River was upward of one million three hundred thousand miles long, and stuck out over the Gulf of Mexico like a fishing-rod. And by the same token any person can see that seven hundred and forty-two years from now the Lower Mississippi will be only a mile and three-quarters long, and Cairo and New Orleans will have joined their streets together, and be plodding comfortably along under a single mayor and a mutual board of aldermen. There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.

What is Mark Twain making fun of?

3 Expected Values Are Everywhere

Expected values are everywhere. They can be used in games, psychology experiments, interpersonal relationships, business, biology, and elsewhere. Give an example of an expected value that you care about? This can be from a game you play or from something you like to collect or anything really.

4 The Value of a Dollar

Who cares more about finding $100 on the street, a person who can't afford their monthly rent or a billionaire? Why?

5 Bernoulli's Game

[2] Consider the following five dollar carnival game. In this game, you are asked to repeatedly toss a coin and the game stops the first time you flip heads. The payout structure is as follows:

- if you flip heads (H) your first try you get $1.
- if you flip tails then heads (TH) you get $2.
- if you flip two tails then a heads (TTH) you get $4.
- if you flip TTTH you get $8.
- ...

The pattern keeps going with the payout doubling for each tail you flip in a row. Should you play this game? How much should you pay to play this game?

[2] Ellenberg "St Petersburg and Ellsburg" pg 242

6 The Monty Hall Problem

1. Suppose you are going to buy a lottery tickey and the only prize is the jackpot. In the game you select six numbers 1-50. Which ticket is better the ticket with 1,1,1,1,1,1 or the ticket with 12,26,3,50,24,27? Why or why not?

2. Suppose you are flipping a coin and you flip 10 heads in a row. Is it more likely that you flip a tails the next time?

3. You flip 5 coins. Which outcome is more likely HHHHH or HTHTH?

4. You are the contestant an *Let's Make A Deal* with host Monty Hall. In this game there are three doors. Behind two of the doors there are goats and behind one of the doors is a brand new car. Your job is to guess which door has the car and if you get it right, you get the car. Monty asks you to select a door. After your selection he reveals goats behind one of the doors. You are allowed to stay with your current door or switch. What should you do? Should you stay with your current door? Should you switch to the new door? Does it matter?

7 Flip Some Coins

Produce a sheet of paper with 100 coin flips recording a sequential number of heads and tails. There are two ways you can do this. You can cheat and just write down a bunch of heads and tails or you actually flip a coin 100 times. All I ask is that you don't mix these two ways of completing this assignment. Either completely cheat and just write down random heads and tails or you actually flip coins and do this for the entire 100 flips. When you submit the assignment don't say which method you used.

8 p-value reflection

♠♠♠ Taylor: [TBD p-value]

9 What is Ranked Choice Voting?

In 2009 Burlington had an election for Mayor in which there were three candidates. At the time the city implemented "ranked choice voting" which is a system in which voters submit a ballot which ranks their perferred order of candidates. How do you suppose this worked? Make up some rules that would allow you to elect a mayor to take into account the voters will using these ranked ballots. How would you determine the winner if you were allowed to make up a voting system? Why?

10 Seven Bridges of Koenigsburg

Euler lived in the city of Koenigsberg (modern day Kaliningrad). The city had a system of bridges as mapped out below in Figure1. Euler wanted to find a route through the city to walk across each one of the bridges exactly once in the city. How can Euler achieve such a route? Make a map and try making a route that achieves this.

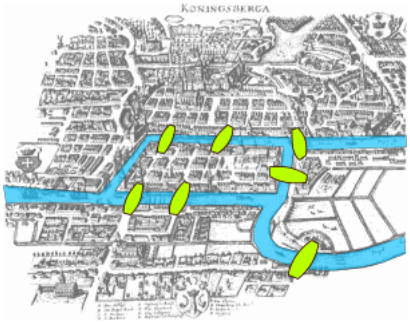

Figure 1: The city of Koenigsburg and its seven bridges.

11 3 Houses and 3 Utlilities

My 5th grade teacher math Mrs. Kelly gave me the following problem in 1995: There are three houses and three utilities as pictured below. Draw a line connecting each of the three houses to each of the three utilities without any of the lines cross each other. Explain what is going on in this problem. x

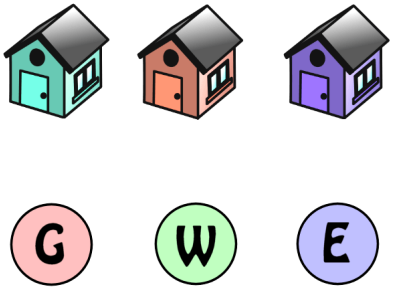

Figure 2: A image of the three houses and three utilities problem. Image taken from Nigel Coldwell's webpage.

12 Prisoner Problem

There are 50 prisoners numbered 1 through 50. There are 50 boxes numbered 1 through 50 in a room. Each of the numbers 1-50 are placed in the boxes at random with each number appearing in a box exactly once (so for example the number 7 could be placed in box 20). The Warden then plays a game for the prisoners freedom: each prisoner is led into the room one at a time and is asked to find their own number. They get to open 25 boxes. After a prisoner leaves the room, the boxes and numbers are reset to their original configuration (so all of the prisoners get the same setup). If *all* of the prisoners find their number they will all be set free.

The prisoners are allowed to communicate beforehand to come up with a strategy but are not allowed to communicate after the game has started. What strategy should they use? (Spoiler: there exists a strategy that works about 30% of the time)