1 Problem

To Find. 5-digit numbers such that either all their digits are even or all their digits are odd.

Claim. The number of 5-digit numbers such that either all their digits are even or all their digits are odd is 5×5^4.

Proof. Let E be the set of 5-digit numbers such that all their digits are even and O be the set of 5-digit numbers such that all their digits are odd.

2 Problem

Given. A computer program takes as input binary codes of 5 bits and returns binary sequences of 4 bits. For every input, the program might either return an output or go into an infinite loop. We call a 5-bit input *valid* if the program does not hang. Two programs are regarded as the same if they have the same set of valid inputs and return the same output for each valid input.

2.1 Problem

To Find. The mathematical model of the above.

2.2 Problem

To Find. The number of different programs that can be implemented.

Claim. The number of different programs that can be implemented is 2^{16}.

Proof. Let P be the set of all programs that can be implemented. Let V be the set of all valid inputs. Let O be the set of all outputs. Let P_v be the set of all programs that return output for a valid input.

3 Problem

Given. In a class of 40, 18 are into chess, 23 are into crosswords, and unknown n are into quizzing. 7 are into both chess and quizzing. 9 are into both chess and quizzing. 4 students are into all the activities.

Claim. The number of students into quizzing is 12.

4 Problem

To Find. The number of ways in which 4 couples can be seated in a row of 8 seats if each couple wants to be seated together.

5 Problem

To Prove. The number of solutions of $x_1 + x_2 + \cdots + x_k = n$ in non-negative integers is C_n^{n+k1}.

6 Problem

Given. A computer network with 10 distinct computers is set up such that each computer is connected to another computer. I want to determine the optimal way of sending a packet of information through the network so that it reaches every computer exactly once and returns to the starting computer.

To Find. The number of ways in which the packet can be sent.

7 Problem

Done in Question 4.

8 Problem

To Prove. That 2 points located on the circumference of a circle with a diameter of 6 cm, there exist two points with a distance of less than 2 cm between them.

9 Problem

To Prove. That $\sqrt{2}$ is irrational following these steps.

1. Show that every rational number can be written as $\frac{p}{q}$, where p and q are relatively prime.
2. Assume $\sqrt{2} = \frac{p}{q}$, where p and q are relatively prime, and prove that p^2 is even.
3. Show that if n^2 is even, then n is even.
4. Use this to show that p and q are even, leading to a contradiction.

10 Problem

To Prove. Every prime number greater than 3 is congruent to either 1 *mod 6* or 5 *mod 6.*

11 Problem

To Prove. If $n + 1$ distinct numbers are randomly selected, there exist two numbers whose remainder when divided by n is the same.

12 Problem

To Prove. Using modular arithmetic that $a + b$ is even, if a and b are odd integers.

13 Problem

To Find. The digit at units place of 3^{101}.

Claim. The digit at units place of 3^{101} is 7.

Proof. Let n be the digit at units place of 3^{101}. We know that $3^4 = 81$, and $3^{4n} = 1$, so $3^{100} = 1$. Therefore, $3^{101} = 3$.

14 Problem

To Write. The PyRet code for the Euclidean algorithm and compute the greatest common divisor (GCD) of 124 and 284 using this algorithm by hand.