Discrete Mathematics
Professor T. V. H. Prathamesh — Week 1
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Collaborators: none

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1 question

Given. Murder occured involving father, mother, son and daughter. M_1 murdered M_2, M_3 was witness, M_4 was an accomplice.

- M_2 and M_4 are opposite sex
- Oldest member and M_4 are opposite sex
- Youngest member and M_3 are opposite sex
- M_4 age $>$ M_2 age
- Father was the oldest
- M_1 is not the youngest

To Find. The murderer.

Solution.

2 question

Given. A pack of two-sided cards, claiming where if one side is vowel $\rightarrow$ other side is an even number. 4 cards are shown where we see **W, U, 4, 9**.

To Find. The cards to turn to check if the conjecture is true or not.

Solution.

3 question

To Negate.

1. $\forall n, \exists m$ s.t. $n > m$
   $\exists n, \forall m$ s.t. $n \leq m$

2. $\exists n, m$ s.t. $m \neq n$
   $\forall n, m$ st. $n = m$

3. $\forall m, m^2 > 0 \implies m > 0$
   $\exists m, m^2 \leq 0 \implies m > 0$

4. $\forall a, b, c \in S, a \; R \; b$ and $b \; R \; c \implies a \; R \; c$

4 question

Given. We prove by induction that any n things are the same. If $n = 0$ or $n = 1$ this is clear. or the induction step, assume we can say that any k things are the same. Let a finite set of $k+1$ things, $x_1, x_2, x_3, \ldots x_k, x_{k+1}$ be given. By induction hypothesis, we get that if we take a subset of k of these things, we get them to be the same. It follows that:

- $x1 = x2 = ... = xk$
- $x2 = ... = x_k = x_{k+1}$

Thus it follows that

$$x_1 = x_2 = ... = x_k = x_{k+1}$$

Thus for any n things, they are the same.

To Find. The error in the proof.

5 question

To Check and Prove. Which of the following results hold.

1. If f is bijective and g is bijective, then $g \circ f$ is bijective.
2. If $f \circ f$ is injective, then f is injective.
3. if $f \circ f$ is surjective, then f is surjective.
4. If $g \circ f$ is surjective, then f is surjective.

Solution.

1. No. Consider $f(x) = x^2$ and $g(x) = x^3$. Then f and g are bijective but $g \circ f = x^6$ is not bijective.
2. Yes. If $f \circ f$ is injective, then f is injective. Let $f(x) = f(y)$. Then $f(f(x)) = f(f(y)) \implies x = y$.
3. No. Consider $f(x) = x^2$. Then $f(f(x)) = x^4$ is not surjective.
4. Yes. If $g \circ f$ is surjective, then f is surjective. Let $y \in Y$. Then $\exists x \in X$ such that $g(f(x)) = y$. Since g is surjective, $\exists z \in X$ such that $g(z) = y$. Thus $f(z) = x$.

6 question

To Check. If $f \circ g$ are bijective then are both f and g bijective?

Solution. No. Consider $f(x) = x^2$ and $g(x) = x^3$. Then $f \circ g = x^6$ is bijective but f and g are not bijective.

7 question

To Show. The set operation $A \backslash B$ is not associative. That is, $(A \backslash B) \backslash C \neq A \backslash (B \backslash C)$

Solution.

To Show. Show that for every

8 question

To Prove. $A \subset B$ and $B \subset C \implies A \subset C$.

Solution.

To Prove. $((P \longrightarrow R) \wedge (Q \longrightarrow R)) \longrightarrow (P \vee Q) \longrightarrow R)$

Solution.
The truth table for the following is as follows

P	Q	R	$P \longrightarrow R$	$Q \longrightarrow R$	$P \vee Q$	$(P \vee Q) \longrightarrow R$	$((P \longrightarrow R) \wedge (Q \longrightarrow R)) \longrightarrow (P \vee Q) \longrightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

9 question

Given. $n \neq 0$ divides k if there is a $q \in \mathbb{N}$ such that $k = n \times q$.

To Check. Whether *divides* as a relation on natural number is reflexive, symmetric, transitive or not.

Solution. Let a relation $\sim$ defined on $\mathbb{N}$ as $\mathbb{N} \times \mathbb{N}$, such that $(k, n) \sim (n, k) \iff \exists q \in \mathbb{N}$ such that $k = n \times q$.

10 question

Given. Panedmic affects 1 person on the first day. After that each person spreads it to a people, where a is average rate of transmission. Number of people infected on n^{th} day is $(n-1)^{th} \times a$. Two programmers wrote the code below.

Naive Coder.

```
fun pandemic(num-days :: Number, rate :: Number) -> Number:
    doc: "counting number of pandemic infected"
    if num-days == 0:
```

```
        1
    else:
        pandemic(num-days - 1, rate) * rate
    end
end
```

Mathematics Coder.

```
fun altpandemic(num-days :: Number, rate :: Number) -> Number:
    doc: "counting using power"
    num-expt(rate, num-days)
end
```

To Prove. Given an input, both the functions give the same output.

Solution.

11 question

To Write. A bijective map from $\mathbb{N} \to \mathbb{Z}$.

Solution. A bijection from $\mathbb{N} \to \mathbb{Z}$ can be given by the function $f(n) = (-1)^n \times \lceil \frac{n}{2} \rceil$.

$$\begin{aligned}
f(0) &= 0 \\
f(1) &= -1 \\
f(2) &= 1 \\
f(3) &= -2 \\
f(4) &= 2 \\
f(5) &= -3 \\
&\vdots \\
f(n) &= (-1)^n \times \left\lceil \frac{n}{2} \right\rceil
\end{aligned}$$

12 question

Given. A relation $\leq$ on a set S is called a pre-order if it is reflexive and transitive. A relation $\equiv$ on S such that $x \equiv y \iff x \leq y$ or $y \leq x$.

To Prove. $\equiv$ is an equivalence relation.

Solution.

13 question

Given. A pre-order is called a partial order if it is antisymmetric, i.e., $x \leq y \vee y \leq x \leftrightarrow x = y$