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Encryption Basics

- Encryption is the process of converting information or data into a code, especially to prevent unauthorized access.
- Encryption helps protect data you send, receive, and store using any device.
- Encryption is a way to enhance the security of a message or file by scrambling the contents so that only someone with the right encryption key can unscramble it.

Basically the remainder of a division operation.

 $a \mod b = r$, where r is the remainder when a is divided by b.

Prime Numbers. A $1 < n \in \mathbb{N}$ that is not a product of two smaller natural numbers.

Semiprime. A $n \in \mathbb{N}$ that is the product of two prime numbers.

Coprime. Two numbers are coprime if their greatest common divisor is 1.

Euler's Totient Function

The totient function $\phi(n)$ is defined as the number of positive integers less than n that are coprime to n.

$$\phi(8)$$

$$GCD(1, 8) = 1$$

$$GCD(2,8) = 2$$

$$GCD(3,8) = 1$$

$$GCD(4, 8) = 4$$

$$GCD(5,8) = 1$$

$$GCD(6,8) = 2$$

$$GCD(7,8) = 1$$

 $\phi(8) = 4$, since there are 4 numbers that are coprime to 8.

$$GCD(1, 8) = 1$$

$$GCD(2,8) = 2$$

$$GCD(3,8) = 1$$

$$GCD(4,8) = 4$$

$$GCD(5,8) = 1$$

$$GCD(6,8) = 2$$

$$GCD(7,8) = 1$$

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Similarly,
$$\phi(9) = 6$$
, $\phi(10) = 4$, $\phi(11) = 10$, $\phi(12) = 4$.

- Developed by Ron Rivest, Adi Shamir, and Leonard Adleman in 1977.
- RSA is an algorithm used by modern computers to encrypt and decrypt messages.
- It is an asymmetric cryptographic algorithm.
- It is based on the fact that finding the factors of a large composite number is difficult.

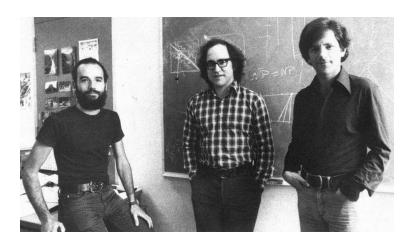


Figure: Rivest, Shamir, Adleman at MIT

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