ANALYSIS 1

MATH 201

ASSIGNMENT 2

Remark. *This assignment is due on **Monday August 12** by **23:59** and will count towards your final grade as described in the course grading policy. Late submissions will be handled in accordance with course policies. When writing your arguments, you can use any result that has been proved in the class, mentioned on a previous assignment, or mentioned on the this assignment in a question preceding the one you are considering as long as you explicitly state the result that you are using. The number of points each question is worth is mentioned in square brackets at the beginning of the question. Your final score for this assignment will be scaled as described in the course policies.*

Question A. [2] Let $a \in \mathbb{R}$. Show that the set $(-\infty, a)$ is open, and that $(-\infty, a]$ is not open.

Question B. [2] Determine all $x \in \mathbb{R}$ which satisfy the following equation:

$$|x+3| + |x-2| = 9.$$

Question C. [2] Prove that $\mathbb{Z}^{\mathrm{c}}$ is open in $\mathbb{R}$.

Question D. [2] Let $S \subseteq \mathbb{R}$. Show that S is open if and only if it satisfies the following property: for all $x \in S$, there exists $\delta > 0$ such that $B[x, \delta] \subseteq S$.

Question E. [2] Let $S \subseteq \mathbb{R}$ be an open set such that S is non-empty and bounded above. Prove that $\sup(S) \notin S$.

Question F. [8] Calculate the closures, interiors, and limit points of the following subsets of $\mathbb{R}$:

(1) $(0, 3]$
(2) $\mathbb{Z}$
(3) $\bigcup_{n \in \mathbb{Z}_{>0}} (\frac{1}{n+1}, \frac{1}{n})$
(4) $\mathbb{Q} \cap (0, 1)$

Question G. [4] Construct examples of:

(1) A infinite subset of $\mathbb{R}$ which is not an interval, and is neither open nor closed.
(2) A subset of $\mathbb{R}$ which is bounded and has exactly two limit points.

Question H. [3] Let $S \subseteq \mathbb{R}$. Let $x \in \mathbb{R}$. Show that $x = \inf(S)$ if and only if it satisfies the following two conditions:

(1) The element x is a lower bound for S.
(2) Given $n \in \mathbb{Z}_{>0}$, there exists some $y \in S$ such that $y < x + \dfrac{1}{n}$.

Question I. [7] Consider the subset $S := \{\frac{1}{n} - \frac{1}{m} \mid m, n \in \mathbb{Z}_{>0}\} \subseteq \mathbb{R}$.

(1) Draw a picture of S.
(2) Calculate supremum and infimum of S. Justify your answer.
(3) Show that S is neither open nor closed.
(4) Calculate the interior of S. Justify your answer.
(5) Calculate the closure of S. Justify your answer.