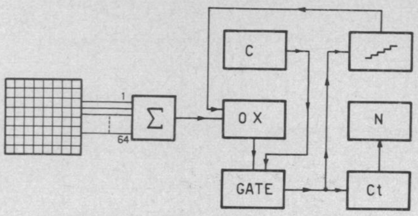

Fig. 10. Area measurement.

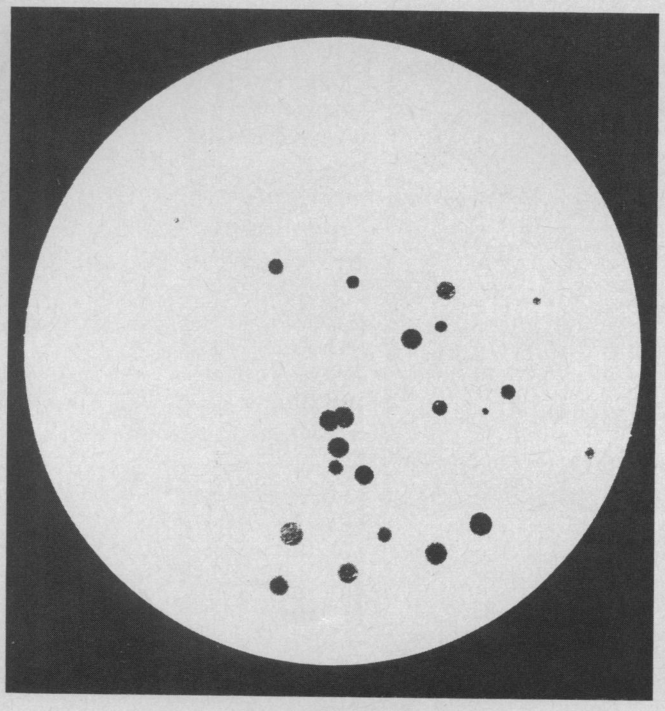

Fig. 11. Bacteria culture.

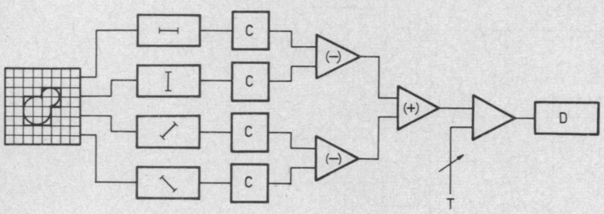

Fig. 12. Form factor extraction.

impossibility to relay all the information to the brain and second, because of the need to give some immediate motor reply whenever the optical signals might call for danger or for any vital decision. With the advent of large-scale integration it is certainly no far-fetched prophecy that a great part of preprocessing in optical input devices will be parallelwise.

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S. Levialdi
Lab. Cibernetica
Consiglio Nazionale delle Ricerche
Naples, Italy

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On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization

***Abstract*—The joint problem of system identification and system optimization is discussed. The bicriterion formulation of the two problems and the ε-constraint formulation viewing the identification problem as a constraint to the optimization problem are introduced. The equivalence theorem relating these two formulations is presented and proved.**

Introduction

In the process of planning, design, operation, or evaluation of large-scale systems, often more than one objective function seems to be both desirable and essential for a meaningful analysis. However, because of the lack of conceptual schemes available for dealing with such multi-objective system models, the systems analyst sacrifices the more realistic modeling for a simplified optimization scheme.

In particular a bicriterion optimization problem arises when both system identification and system optimization problems are considered simultaneously [1], [3]. The integration of system identification and system optimization permits a better understanding of the coupling relations between the two problems and hence enables the systems analyst to reach an optimal policy for a system model which represents the real system more closely and accurately. Clearly, an optimal policy is a meaningful one if and only if the system model closely represents and describes the real system.

Problem Definition [2]

Generally, mathematical programming problems are posed in terms of a single unknown vector, e.g., X. However, in this correspondence the concepts of system state variables X and control variables U will be adopted for the static system models [3]. This notation simplifies the analogy between the static and dynamic systems problems. Consider the static problem

$$\min_{U} f(X,U,\alpha) \tag{1}$$

subject to the constraints

$$\begin{aligned} g_i(X,U,\alpha) &= 0, \qquad i = 1,2,\cdots,n \\ g_i(X,U,\alpha) &\le 0, \qquad i = n+1,\cdots,n+r \end{aligned} \tag{2}$$

where

- X n-dimensional state vector;
- U l-dimensional control vector;
- α k-dimensional vector of unknown parameters; and
- f,g_i functions of class C^2.

If (X^0,U^0) solves (2) and all $\nabla g_i(X^0,U^0,\alpha)$ are linearly independent then the implicit function theorem guarantees the existence of solution

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$X(U)$, for all U near U^0. With this justification (2) is now rewritten as

$$g_i(X,U,\alpha) \le 0, \qquad i = 1,2,\cdots,m \tag{3}$$

where

$$m = 2n + r.$$

The p-dimensional vector Y represents the system output which can be observed and is given by

$$Y = H(X,U,\alpha) \tag{4}$$

where H is a p-dimensional vector of functions of class C^2. It is assumed that the J observations of the output can be made, and these are denoted by

$$\hat{Y}^j, \qquad j = 1,2,\cdots,J$$

where $\hat{Y}^j$ denotes a known value.

The parameter identification problem is given by

$$\min_{\alpha} \sum_{j=1}^{J} [Y^j - \hat{Y}^j]^T W^j [Y^j - \hat{Y}^j] \tag{5}$$

where W^j is an $n \times n$ positive definite weighting matrix. Of course the value of α determined by (5) must also satisfy the constraints (3). By substitution the parameter identification problem becomes

$$\min_{\alpha} G(\alpha) \tag{6}$$

where

$$G(\alpha) = \sum_{j=1}^{J} [H(\hat{X}^j,\hat{U}^j,\alpha) - \hat{Y}^j]^T W^j [H(\hat{X}^j,\hat{U}^j,\alpha) - \hat{Y}^j]$$

and the integrated problem can be written as

$$\min_{U,\alpha} [f(X,U,\alpha),G(\alpha)]$$

subject to

$$g(X,U,\alpha) \le 0.$$

Problem A

Find an efficient point for the vector minimization problem

$$\min_{U,\alpha} [f(X,U,\alpha),G(\alpha)] \tag{7}$$

subject to

$$g(X,U,\alpha) \le 0.$$

Problem B(ε)

Consider

$$\min_{U,\alpha} f(X,U,\alpha)$$

subject to

$$G(\alpha) \le \varepsilon$$

$$g(X,U,\alpha) \le 0.$$

Let

$$V = (X,U,\alpha).$$

Equivalence Theorem

Let $\varepsilon \ge \min G(\alpha)$, let V^* solve Problem B(ε), and assume that, if V^* is not unique, then V^* is an optimal solution of Problem B(ε) with minimal $G(\alpha)$ value. Then V^* solves Problem A.

Proof: Assume V^* does not solve Problem A. Then there is a $\hat{V}(\hat{X},\hat{U},\hat{\alpha})$ satisfying (7) such that either

$$f(\hat{V}) < f(V^*), \qquad G(\hat{\alpha}) \le G(\alpha^*) \tag{8}$$

or

$$G(\hat{\alpha}) < G(\alpha^*), \qquad f(\hat{V}) \le f(V^*). \tag{9}$$

But (8) contradicts the fact that V^* solves Problem B(ε), while (9) contradicts the hypothesis that V^* is an optimal solution of Problem B(ε) with smallest $G(\alpha)$. Hence the theorem is proved.

Consider now solving Problem B(ε) with the minimal value of ε such that Problem B(ε) is feasible. Let this ε be ε^*. Clearly, if the original solution of the least square model fitting problem

$$\min G(\alpha)$$

(let this solution be α^*) is unique and there exists a vector (X,U) such that $g(X,U,\alpha^*) \le 0$, then

$$\varepsilon^* = G(\alpha^*)$$

and Problem B(ε) may be solved by first minimizing $G(\alpha)$ and then solving the optimization problem

$$\min f(X,U,\alpha^*) \tag{10}$$

subject to

$$g(X,U,\alpha^*) \le 0 \tag{11}$$

that is, the optimization and identification problems may be decomposed. Hence we need consider an integrated formulation (i.e., solving Problem B(ε)) only if α^* is not unique or if it is not feasible, i.e., if there exists no (X,U) such that $g(X,U,\alpha^*) \le 0$, since then the optimization problem (10) and (11) has no feasible solution.

Yacov Y. Haimes
Syst. Eng. Div.
School Eng.
Case Western Reserve Univ.
Cleveland, Ohio

Leon S. Lasdon
Operations Res. Dep.
School Management
Case Western Reserve Univ.
Cleveland, Ohio

David A. Wismer
Eng. Syst. Dep.
School Eng. and Appl. Sci.
Univ. California
Los Angeles, Calif.

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