Test 1   ST 15a

1) mit Newton-Iteration oder Bisektionsverfahren

$f'(x) = 3x^2 + e^{1-x\sin x} \cdot (\sin x + x\cos x)$

$x_0 = 1.6$

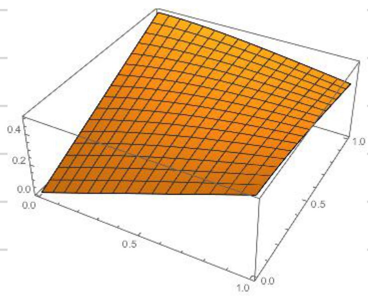
$\Rightarrow \underline{x^* \approx 1.0366}$ vgl. Aufgabe1.m

2) a) $\left| \begin{array}{l} 2y - e^{-xy} = 0 \\ \ln(1+y) - 4x = 0 \end{array} \right|$

Fixpunktgleichung: $x = \frac{1}{4}\ln(1+y)$ $y = \frac{1}{2}e^{-xy}$ $\Rightarrow g(x,y) = \begin{pmatrix} \frac{1}{4}\ln(1+y) \\ \frac{1}{2}e^{-xy} \end{pmatrix}$

Fixpunkt-Iteration $\vec{x}^{(n+1)} = g(\vec{x}^{(n)})$ vgl. Aufgabe2.m

$\Rightarrow \underline{x^* \approx 0.0975, \quad y^* = 0.4773}$



b) $g'(x,y) = \begin{pmatrix} 0 & \frac{1}{4(1+y)} \\ -\frac{1}{2}e^{-xy}\cdot y & -\frac{1}{2}e^{-xy}\cdot x \end{pmatrix}$

$$\|g'(x,y)\|_\infty = \max_{x,y\in[0,1]^2} \left\{ \frac{1}{4(1+y)}, \overbrace{\frac{1}{2}e^{-xy}(x+y)} \right\} =$$

$$= \max_{x,y\in[0,1]^2} \left\{ \frac{1}{4}, \frac{1}{2} \right\} = \frac{1}{2} =: C$$

$$\|x^{(n)} - x^*\|_\infty \leq \frac{C^n}{1-C}\|x^{(1)} - x^{(0)}\|_\infty \leq 10^{-3}$$

$$x^{(1)} = g(\underset{\begin{pmatrix}0\\0\end{pmatrix}}{x^{(0)}}) = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} \Rightarrow \|x^{(1)}\|_\infty = \frac{1}{2}$$

$$\Rightarrow \frac{1\cdot\cancel{2}}{2^n\,\cancel{2}} \leq 10^{-3} \Rightarrow 10^3 \leq 2^n \Rightarrow \underline{n \geq 10}$$

3) $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ vgl. Aufgabe3.m

a) $Q_1 \approx \begin{pmatrix} -0.7071 & -0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\hookrightarrow A_1 \approx \begin{pmatrix} -1.4142 & 0 & -0.7071 \\ 0 & 0 & 0.7071 \\ 0 & 1 & 0 \end{pmatrix}$

b) $Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$

$\hookrightarrow A_2 \approx \begin{pmatrix} -1.4142 & 0 & -0.7071 \\ 0 & -1 & 0 \\ 0 & 0 & -0.7071 \end{pmatrix}$

c) $Q \approx \begin{pmatrix} -0.7071 & 0 & 0.7071 \\ -0.7071 & 0 & -0.7071 \\ 0 & -1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} -1.4142 & 0 & -0.7071 \\ 0 & -1 & 0 \\ 0 & 0 & -0.7071 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2.1213 \\ -1 \\ -0.7071 \end{pmatrix}$

$\Rightarrow \underline{x = 1, \quad y = 1, \quad z = 1}$

4) $u = a \cdot e^{-\frac{x^2}{2\sigma^2}}$ vgl. Aufgabe 4.m

$\log u = \log a - \frac{1}{2\sigma^2} x^2 \Rightarrow A = [1, x_i^2]_{i=1,\dots,5}$

$A = \begin{pmatrix} 1 & 16 \\ | & 4 \\ | & 0 \\ | & 4 \\ 1 & 16 \end{pmatrix}$ QR-Zerlegung

$\rightsquigarrow R \begin{pmatrix} u \\ v \end{pmatrix} = Q^T \log u$

$\begin{pmatrix} -2.2361 & -17.88 \\ 0 & 14.96 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -11.2 \\ -1.75 \end{pmatrix}$

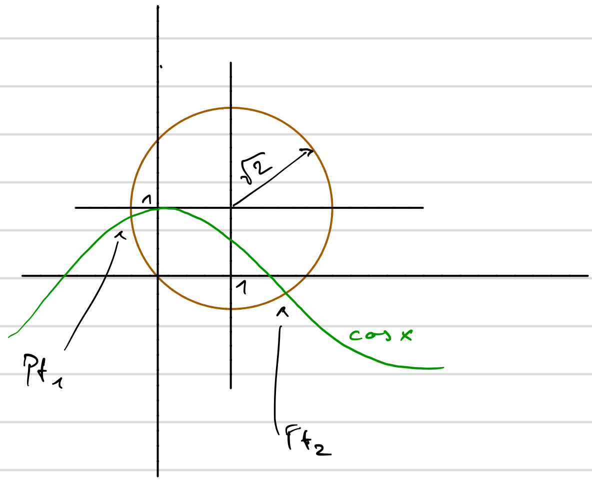
$\Rightarrow u \approx 5.94 \quad v \approx -0.1167$

$-\frac{1}{2\sigma^2} = -0.1167 \Rightarrow \sigma = \frac{1}{\sqrt{2 \cdot 0.1167}} = \underline{2.07}$

5) $\begin{aligned} C_1 + C_3 &= 297 \,\mu F \\ C_1 + C_2 &= 253 \,\mu F \\ C_3 &= 99 \,\mu F \\ C_1 &= 201 \,\mu F \\ C_2 + C_3 &= \underbrace{149}_{b} \,\mu F \end{aligned} \Rightarrow A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$\Rightarrow$ Aufgabe 5.m $\quad \underline{C_1 = 200.5 \,\mu F, \; C_2 = 52 \,\mu F, \; C_3 = 97.5 \,\mu F}$

6) $\left| \begin{array}{r} (x-1)^2 + (y-1)^2 = 2 \\ y = \cos x \end{array} \right|$



$$F(x,y) = \begin{pmatrix} (x-1)^2 + (y-1)^2 - 2 \\ y - \cos x \end{pmatrix}$$ Newton-Verfahren.

$$F'(x,y) = \begin{pmatrix} 2(x-1) & 2(y-1) \\ \sin x & 1 \end{pmatrix}$$

$PT_1 = (-0.4117,\ 0.9164)$
$PT_2 = (1.7628,\ -0.1908)$

vgl. Aufgabe6.m