Math 441

NBA Fantasy Sports: Team Building

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Our Goal

Formulate a model that optimizes the team building process for an NBA fantasy team

Why?

Fantasy sports is a multimillion dollar industry, with about 60 million people playing in North America.

The Basic Model

Description

- Integer program
- m total players allowed on a team
- u budget
- n total selectable players (Top ranked 100 players)
- ϵ_i estimated points player i will earn in the upcoming season
- s_i salary of player i

The Basic Model

Decision variable

We Introduce the decision variable x_i, where

$$x_i = \begin{cases} 1 & \text{if player } i \text{ is selected, and} \\ 0 & \text{otherwise.} \end{cases}$$

The Basic Model

Organization of Positions

- Divide the set of all players, into five subsets representing the five possible positions.
- The fives subset are defined as:

$$\begin{cases} P_1 & \text{is the set of all point guards} \\ P_2 & \text{is the set of all shooting guards} \\ P_3 & \text{is the set of all small forwards} \\ P_4 & \text{is the set of all power forwards} \\ P_5 & \text{is the set of all centers} \end{cases}$$

where $\{P_1, P_2, P_3, P_4, P_5\} \subseteq \{1, \ldots, n\}$

The Basic Model

Organization of Divisions

- Divide the set of all players, into six subsets representing the six possible divisions.
- The six divisions are defined as:

$$\begin{cases} D_1 & \text{is the set of all Atlantic Division players} \\ D_2 & \text{is the set of all Central Division players} \\ D_3 & \text{is the set of all SouthEast Division players} \\ D_4 & \text{is the set of all NorthWest Division players} \\ D_5 & \text{is the set of all Pacific Division players} \\ D_6 & \text{is the set of all SouthWest Division players} \end{cases}$$

where $\{D_1, D_2, D_3, D_4, D_5, D_6\} \subseteq \{1, \ldots, n\}$

The Basic Model

Estimating future points

Calculation for ϵ_i

Let γ_1 = total points earned for player i in 2016-2017

Let γ_2 = total points earned for player i in 2017-2018

$$\epsilon_i = \frac{((\gamma_1 / \text{games played by player i in 2016-2017}) + (\gamma_2 / \text{games played by player i in 2017-2018}))82}{2}$$

The Basic Model

Estimating Salary

Calculation for S_i

Let ϕ_1 = salary for player i in 2017-2018

Let ϕ_2 = salary for player i in 2018-2019

$$S_i = \frac{\phi_1 + \phi_2}{2}$$

The Basic Model

Constraints

- Player count constraint

$$\sum_{i=1}^{n} x_i = m, \quad \text{for all } i = 1, \ldots, n$$

- Budget constraints

$$\sum_{i=1}^{n} x_i s_i \leq u, \quad \text{for all } i = 1, \ldots, n$$

The Basic Model

Constraints

- Position constraint

$$\sum_{j=0}^{5} \sum_{i \in P_j} x_i = p_j, \quad p_j = \begin{cases} 3 & \text{if } j = 1, 2 \\ 2 & \text{if } j = 3, 4, 5 \end{cases}$$

In other words, for our basic model a team must have:

- 3 point guards and 3 shooting guards
- 2 small forwards, 2 power forward and 2 centers

The Basic Model

Constraints

- Division Constraints

$$\sum_{k=0}^{6} \sum_{i \in D_k} x_i = 2, \text{for all } k = 1, 2, \ldots, 6$$

In other words, a team must have 2 players for each conference.

The Basic Model

Objective function

- Our goal is to maximize the total amount of points a team will earn thus our objective function is as as follows:

$$\text{Maximize} \sum_{i=1}^{u} \epsilon_i x_i$$

Refined Model

Refined Model

Changes to Basic Model

- Introduces a penalty system.
- Allow teams to go over the amount, while incurring a penalty
- Quadratic Program

Goal of Change

- Analyzing the most efficient team build up, in terms of player positions.
- Are teams are better off having more players from a single role?

The Refined Model

Quadratic Penalty

- We introduce a quadratic penalty

$$\alpha(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2)$$

$$\text{such that } y_j = \left(\sum_{i \in P_j} x_i\right) - p_j, \text{ for j} = 1,\ldots,5$$

- No penalty added, if

$$\left(\left(\sum_{i \in P_j} x_i\right) - p_j \leq 0\right) \text{ then } \; y_i = 0$$

The Refined Model

Objective function

- The new objective function for our refined model is

$$\text{Maximize } \sum_{i=1}^{n} \epsilon_i x_i - \alpha(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2)$$

Advanced Model

Advanced Model

Further Changes

- Introduces two different types of penalties.
- Minor Penalty
- Major Penalty

Goal of Change

- To further analyze the effects of the penalties under a more refined system.
- More realistic and fair approach towards determining the penalty.

The Advanced Model

Adding new variables

- C = number of players allowed on position before penalty
- q = threshold for major penalty
- N = current number of players in that position
- y = number of players who will incur a minor penalty
- z = number of players who will incur a major penalty
- m and M are constants where $M >> m$

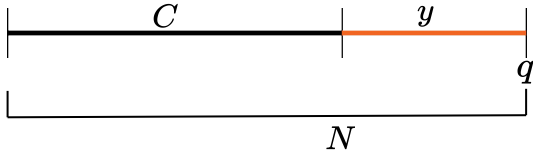
The Advanced Model

Changes to Penalty

- Calculation new penalty is
  $-ym - zM$

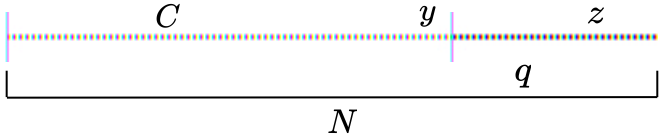
Minor Penalty: $(C < N \leq q)$

Then $y = N - C$ and $z = 0$



Major Penalty: $(N > q)$

Then $y = q - C$ and $z = N - q$



The Advanced Model

Objective function

- The new objective function for our advanced model is

$$\text{Maximize}\left(\sum_{i=1}^{n} \epsilon_i x_i\right) - ym - zM$$

Future Analysis

1. Diminishing returns
2. Value of positions
3. Effects of penalties