Math441 TVDOUBLEJ: Project Proposal

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1 Outline:

Project Description: We will study the strategy of choosing players to build a best-performing fantasy sports team.

Models: Here are some **models** we plan to use:

1. A simple model: Given a fixed salary cap, a limited amount of players per team, a limited number of players to choose from, an exact amount of players for each position and region, and k amount of earn-able points (points are rewarded based on players performance according to fantasy sports rules). We want to select players from the pool of available players that allows us to maximize k while staying below the salary cap and does not exceed the total amount of player allowed on a team. This is an IP that is closely related to a scarce resource allocation problem.

2. A more advanced mode1: Much like the previous model, we want to maximize k earn-able points under the same constraints. Except when our position and region constraints are violated we will incur a penalty. These penalties grows exponentially, thus will result in an quadratic function. Our Integer Program now becomes a Quadratic Program.

3. Another advanced model: Line in previous models, we want to maximize k earn-able points under the same constraints. Except when our position and region constraints are violated we will incur a penalty. Unlike the previous model, the size of the penalty will change depending on whether the constraint exceeds within a small amount, resulting in a minor penalty, or exceeds by a large amount, resulting in a major penalty. This is an IP problem.

Software: We plan to use Gurobi software to solve the above IP's.

Some questions we may study: Given an increase in salary cap does it necessarily mean our chances of winning also increases? If so what is the relationship between the amount of money given and the chances of winning i.e. if we increase the salary cap by x amount does the amount of points earned grow linearly or exponentially? At which price point do we experience diminishing returns? At which price point do we see the largest percentage growth in earn-able points? Are there times where we will have an increase in total points by violating our constraints? If we have more players for a certain position will it increase our total points?

Our Simple IP:

Given Information We have (1) n salary cap, (2) m possible players on a team , (3) k earn-able points , and (4) a set of players to choose from,

$$P = \{(p) \mid \ p \text{ is a player available to be selected}\}$$

$$P(i) = \begin{cases} 1 \leq i \leq z & \text{if player } i \text{ is a point guard,} \\ z < i \leq y & \text{if player } i \text{ is a shooting guard,} \\ y < i \leq w & \text{if player } i \text{ is a small forward,} \\ w < i \leq v & \text{if player } i \text{ is a power forward,} \\ v < i \leq u & \text{if player } i \text{ is a center.} \end{cases}$$

We wish maximize the chances of winning. To do this we want to find the maximum amount of earn-able points that does not exceed both the salary cap and the total amount of players allowed for each role. Only players inside the set of allowed players will be available for selection. This is closely related to a scarce resource allocation problem.

The Decision Variables and Their Intuitive Meaning For $i = 1, \ldots, m$, and $j = 1, \ldots, 6$ where m is the total number of in select-able player in set P, s is the number of players selected on the team, j represents a division (1 for Atlantic Division, 2 for Central Division, 3 for SouthEast Division, 4 for NorthWest Division, 5 for Pacific Division, 6 for SouthWest Division) we introduce a decision variable x_{ij} whose intuitive meaning is

$$x_{ij} = \begin{cases} 1 & \text{if player } i \text{ from division } j \text{ is selected to be on the team, and} \\ 0 & \text{otherwise.} \end{cases}$$

Coefficient We also introduce an coefficient s_i, where s_i is a constant which represents the cost to draft player i onto the team.

Formal Description of IP The **objective** is to maximize the number of points obtainable, with consideration of having m players on a team while staying under the salary cap n, we will also change the salary cap to analyze the effects on the model.

The **constraints** are:

$$\sum_{i=1}^{m} x_{i,j} = s, \quad \text{for all } i = 1, \ldots, m$$

[meaning that we have exactly s players on the team]

$$\sum_{i=1}^{m} x_{i,j} s_i \leq n, \quad \text{for all } i = 1, \ldots, m$$

[meaning that the prices of all players together are within the salary cap]

$$\sum_{i=1}^{z} x_{i,j} = 3, \quad \text{for all } i = 1, \ldots, z$$

[meaning that we have exactly 3 players on the team that are point guards]

$$\sum_{i=z+1}^{y} x_{i,j} = 3, \quad \text{for all } i = z+1, \ldots, y$$

[meaning that we have exactly 3 players on the team that are shooting guards]

$$\sum_{i=y+1}^{w} x_{i,} = 2, \quad \text{for all } i = y+1, \ldots, w$$

[meaning that we have exactly 3 players on the team that are small forward]

$$\sum_{i=w+1}^{v} x_{i,j} = 2, \quad \text{for all } i = w+1, \ldots, v$$

[meaning that we have exactly 3 players on the team that are power forward]

$$\sum_{i=v+1}^{u} x_{i,j} = 2, \quad \text{for all } i = v+1, \ldots, u$$

[meaning that we have exactly 3 players on the team that are center]

Our **research group** includes: Joe dela Cruz (JD), Terence Chen (TC), Vincent Lam (VL), and Jeff Zhou (JZ).

The **tasks** we anticipate in this project include:

1. Modeling optimization problems based on synthetic data. [JD is responsible]
2. Harvesting online information regarding the points system for our fantasy league, finding the average amount of points for each player, testing models with different parameters and analyzing these changes/effects. (displaying each data set to graph is needed). [TC is responsible]

3. Generating synthetic data sets based on harvested information and various policies. [VC is responsible]
4. Running optimization algorithms. [JD is responsible]
5. Statisical analysis of the results over various data sets. [JZ is responsible]
6. Taking writeups of the above and organizing them into a coherent report. [VC is responsible]

PROPOSAL:

We will study the best possible outcome of making an NBA fantasy team within the given budget based on a point system the total team will produce at the end of the season. Our **motivation** is that millions of people pay money to join an NBA fantasy league and end up having their money go to waste as they do not know how to allocate their budget to maximize the total points they can obtain throughout the season; we seek to study these trade offs. **The specific policies** we study include the average amount of points each player produces based on their performance from each game, and how we can allocate our budget accordingly to produce the greatest payoff. Additionally, we study how good of a team can be made given different budgets.

Our simplest model of our picking for the best NBA fantasy team is an IP (Integer Program). We are given n as our budget, and m possible players on a team. We want to know the greatest k such that we dont go over our given n and m.

Given n, k, and m, we formulate the following IP: we introduce decision variable x_i for $i = 1, \ldots, n$, whose value is $\{0, 1\}$, and are meant to be (1)

$$x_{i,j} = \begin{cases} 1 & \text{if player } i \text{ is selected to be on the team, and} \\ 0 & \text{otherwise.} \end{cases}$$

To enforce this, we impose the constraints

$$\sum_{i=1}^{m} x_{i,j} = m, \quad \text{for all } i = 1, \ldots, n \tag{1}$$

so that we have exactly m players on the team; the constraints

$$\sum_{i=1}^{p} x_{i,j} s_i \leq n, \quad \text{for all } i = 1, \ldots, n \tag{2}$$

so we establish the salary cap based on all the players prices combined.

$$\sum_{i=1}^{z} x_{i,j} = 3, \quad \text{for all } i = 1, \ldots, z \tag{3}$$

so that we have exactly 3 point guards on the team; the constraints

$$\sum_{i=z+1}^{y} x_{i,j} = 3, \quad \text{for all } i = z+1, \ldots, y \tag{4}$$

so that we have exactly 3 shooting guards on the team; the constraints

$$\sum_{i=y+1}^{w} x_{i,j} = 2, \quad \text{for all } i = y+1, \ldots, w \tag{5}$$

so that we have exactly 2 small forwards on the team; the constraints

$$\sum_{i=w+1}^{v} x_{i,j} = 2, \quad \text{for all } i = w+1, \ldots, v \tag{6}$$

so that we have exactly 2 power forwards on the team; the constraints

$$\sum_{i=v+1}^{u} x_{i,j} = 2, \quad \text{for all } i = v+1, \ldots, u \tag{7}$$

so that we have exactly 2 centers on the team.

In the advanced model, we introduce a quadratic term for the penalty when we exceed the number of players selected for each role.

Our original objective function is maximize 5

$$\sum_{n=1}^{n} c_n x_n \tag{8}$$

And once we incur a penalty, we will add the term of this form to objective function.

$$-\alpha(\sum_{i=1}^{t} x_{i,j} - C)^2, \quad \text{for all } i = 1, \ldots, t \tag{9}$$

(t and C will change depending on the constraints from the simple model (3), (4), (5), (6), (7) we violate.)

In an alternate advanced model, the player role penalty can be minor if the constraint is exceeded by a small amount, but major if the constraint is exceeded by a large amount.

$$(\sum_{i=1}^{t} x_{i,j}) - C = y + z \quad \text{for all } i = 1, \ldots, t, \text{ where } 0 \leq y \leq q, \text{ and } z \in Z \tag{10}$$

t will change depending on the constraints from the simple model (3), (4), (5), (6), (7) we violate. C is the constraint of how many players can have a certain role. q is the parameter of how many players can exceed the constraint but only receive a minor penalty. y is the number of players who are subject to adding a minor penalty to the score. z is the number of players who are subject to adding a major penalty to the score. N is the number of players in that role on the team. The penalty itself will be calculated by

$$(\sum_{n=1}^{n} c_n x_n) - ym - zM \tag{11}$$

Where y and z are defined as followed:
If $C < N \leq q$, then

$$y = C - N$$

$$z = 0$$

If $N > q$, then

$$y = q$$

$$z = N - C - y$$

M and m are just arbitrary constants, where $M >> m$

Our data will be based on online databases that have recorded the performance and stats of players during the 2017-2018 NBA season. We will decide our number of players through the ranking of ESPN and also consist our point system through ESPN's fantasy league.

The simplest model will consist of a single player with no opponent to create the best team possible, that will maximize their season total output of points. The more advanced model will allow the player to go over the constraints with a given penalty and also obtain more points for being under the budget. Our goal is to see how our player will be influenced by the penalty.

Our research group consists of (1) JD, in charge of gathering data, setting up constraints, and generating synthetic data if needed, (2) TC, in charge of creating the IP model and testing the models. (3) VL, in charge of editing the final project and further optimizing the models, (4) JZ, in charge of formatting and running the programs in Gurobi. (Note we plan to create the more advance models together)