DATA ANALYSIS

Week 13: Additional predictors

logistics

- PS6 revisions due Friday
- PS7 opt-out deadline Apr 23
- PS7 due Apr 30
- class participation:
  - Canvas discussion board posts due Apr 30
  - “practice” questions (10 multiple-choice/true-false) due Apr 24
- LAST DAY to submit any late work: May 13

Week 13: Additional Predictors

- Opt-out of Problem Sets (Deadline 3: After Midterm 2) — Apr 23 | 1 pts
- Problem Set 7: First Attempt — Apr 30 | 2.5 pts
- Problem Set 7: Second Attempt — May 8 | 2.5 pts

Class Participation

- Data Around Us! — Apr 30 | 5 pts
- Meme Submission — 1 pts
- Student Practice Questions — Apr 24 | 2.5 pts

12	F: April 12, 2024	**Exam (Midterm) 2**
13	W: April 17, 2024	W13: Additional Predictors
13	F: April 19, 2024	W13 continued…
14	T: April 23, 2024	**Problem Set Opt-out Deadline 3**
14	W: April 24, 2024	W14: Non-Independent/Miscellaneous Data
14	F: April 26, 2024	W14 continued…
15	T: April 30, 2024	**Problem Set 7 due**
15	W: May 1, 2024	W15: Odds and Ends
15	F: May 3, 2024	**Final Exam**
16	W: May 8, 2024	**Wrapping Up!**

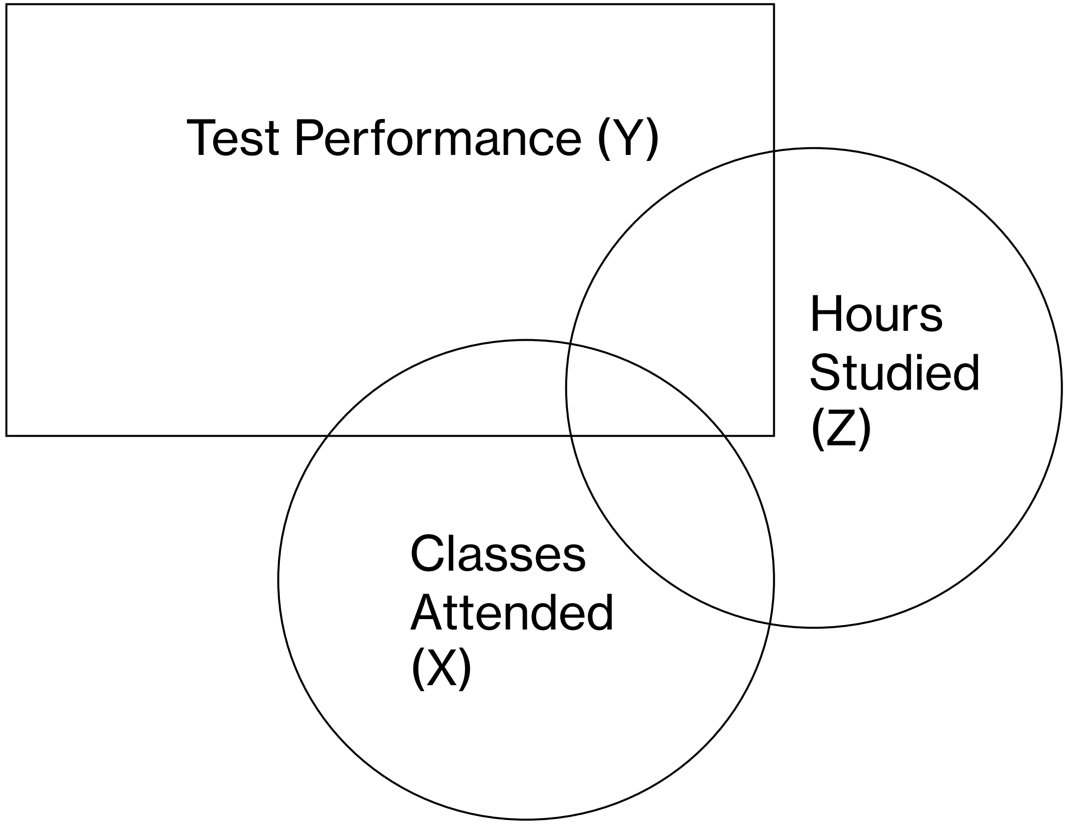
logistics

Letter grade	Points
A	95 - 100+
A-	90 - 94.99
B+	87 - 89.99
B	83 - 86.99
B-	80 - 82.99
C+	77 - 79.99
C	73 - 77.99
C-	70 - 72.99
D	60 - 69.99
F	fewer than 60%

Component	Points
Weekly quizzes	10
Problem sets	35
Exam: Midterm 1	15
Exam: Midterm 2	15
Exam: Final	20
Class participation	5
Extra credit	5
Total	105

additional predictors = complex models

- often, outcomes/dependent variables depend on not just one IV, but several IVs
- in such situations, modeling the variation in our dependent variable using only one variable leads to an impoverished model: we could do better by examining multiple variables
- data = model + error
    - *one IV*: $Y = a + bX + \text{error}$
    - *multiple IVs*: $Y = a + b_1X_1 + b_2X_2 + \ldots + \text{error}$

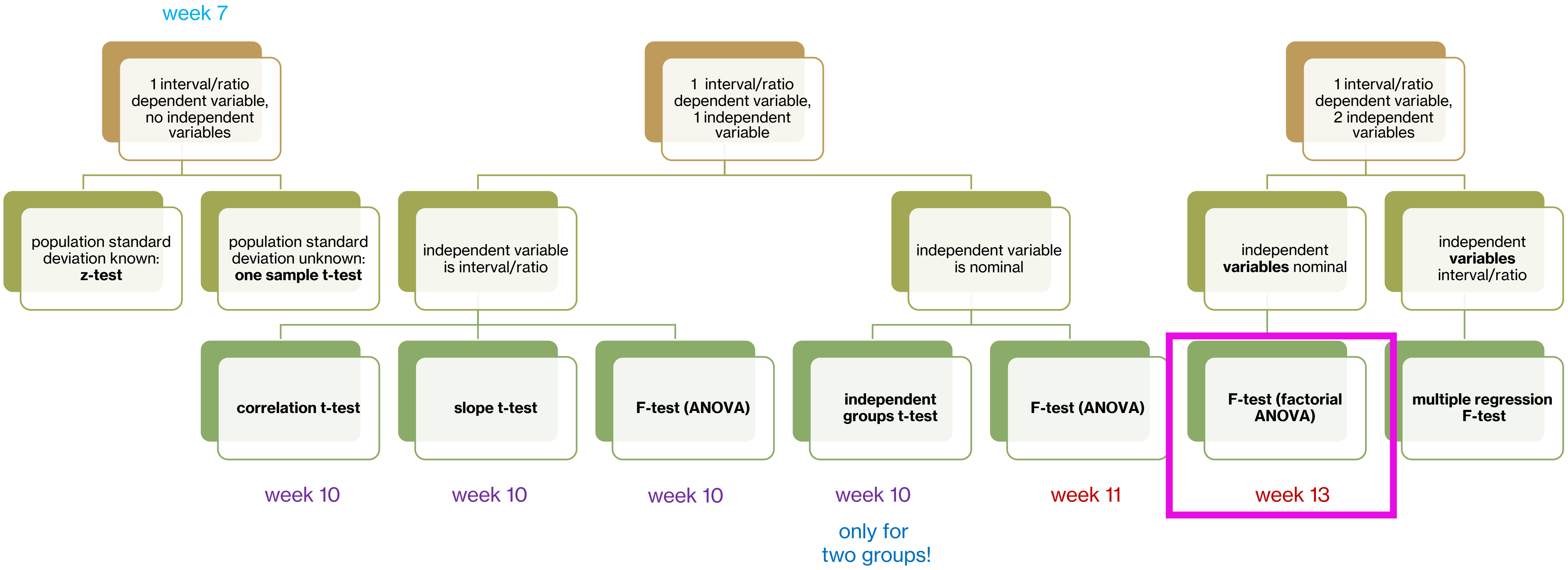


complex models: data types

- for a one DV and one IV situation, we saw how the data could come in different forms
- when more than one IV is involved, several permutations and combinations are possible
  - one DV ~ interval/ratio IV_1 + interval/ratio IV_2
  - one DV ~ interval/ratio IV_1 + nominal IV_2
  - one DV ~ nominal IV_1 + interval/ratio IV_2
  - one DV ~ nominal IV_1 + nominal IV_2
- no fear...general linear models are here!

	one independent variable		
dependent variable	nominal	ordinal	interval/ ratio
nominal			
ordinal			
interval/ratio	F / anova		t / F

hypothesis chart



the tooth growth dataset

- this in-built R dataset contains the “length of odontoblasts (cells responsible for tooth growth) in 60 guinea pigs. each animal received one of three dose levels of vitamin C (0.5, 1, and 2 mg/day) by one of two delivery methods, orange juice or ascorbic acid”
- think about the design of this experiment
  - dependent variable?
  - independent variable(s) and their levels?
  - broad research question?

factorial designs

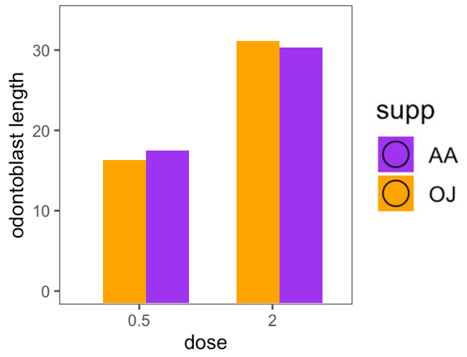
- factorial designs refer to situations where more than one independent variable or "factor" is manipulated in the same experiment (nominal IVs)
- common terminology
  - 2 x 2 factorial design, i.e., two independent variables (number of x's + 1), and each of them had 2 levels
  - 3 x 2 factorial design, i.e., 2 independent variables, one of them had 3 levels, and another had 2 levels
  - 3 x 5 x 4 x 6 factorial design, i.e., you are crazy
- what about our tooth decay design?
  - technically a 3 (dose: 0.5/1/2) x 2 (delivery: OJ, AA) design
  - we will examine a subset of this data that is 2 x 2
  - PS 7 has a problem with a 3 x 2 design! (arousal x task difficulty)

tooth growth dataset: visualization

- let's try to visualize the pattern of **tooth growth** as a function of **dose** and **supplements**
  - **dose**: 0.5 mg and 2 mg
  - **supplements**: OJ and AA
- sketch a possible bar graph of tooth growth based on the research question: is tooth growth impacted by dosage and delivery method of vitamin C?
  - **dose** on x axis
  - **tooth growth** on y axis
  - **supplement** by color

tooth growth dataset: visualization

dose matters
supplement does not matter



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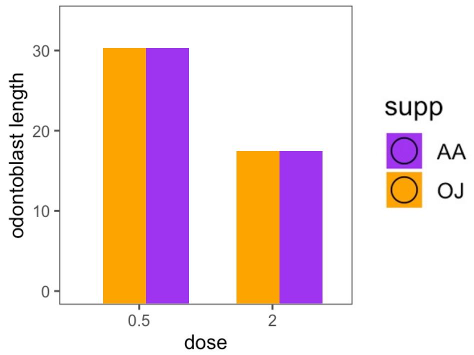
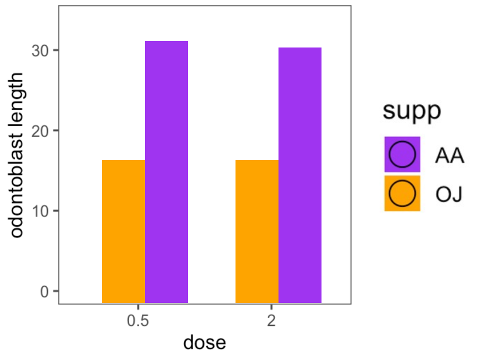


dose does not matter
supplement matters



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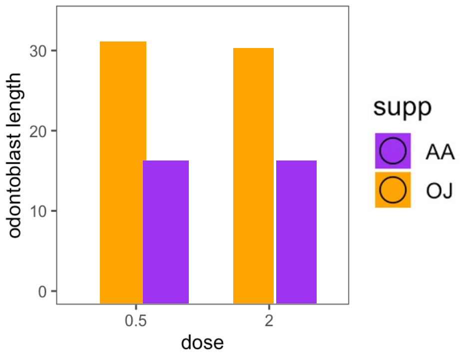
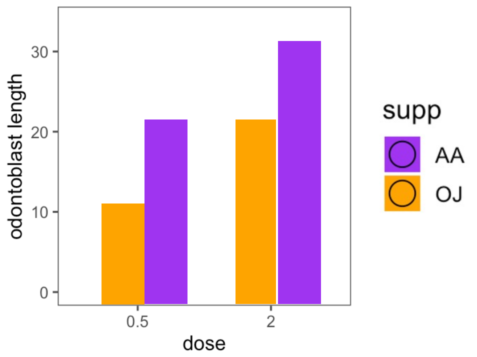


dose matters
supplement matters
dose and supplement
do not influence each other



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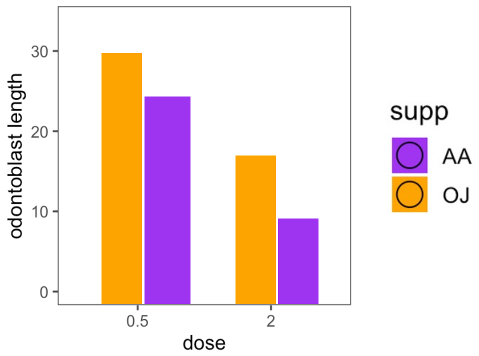

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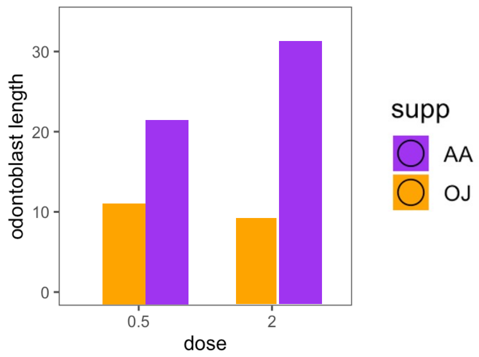

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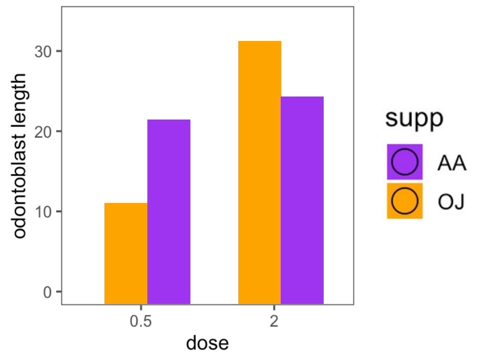
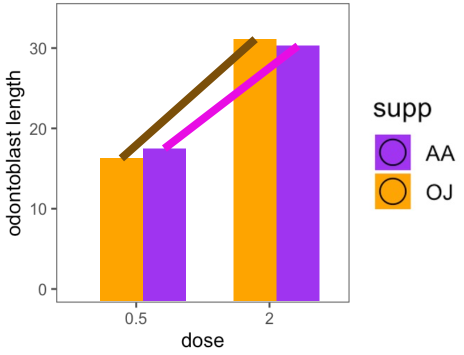


tooth growth dataset: visualization

dose matters
supplement does not matter



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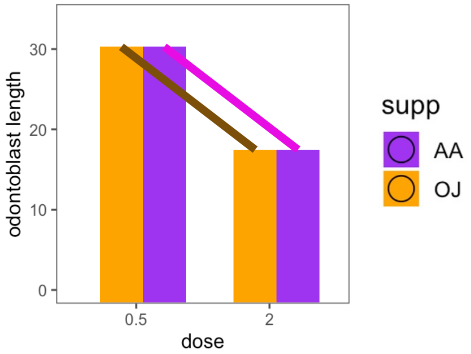
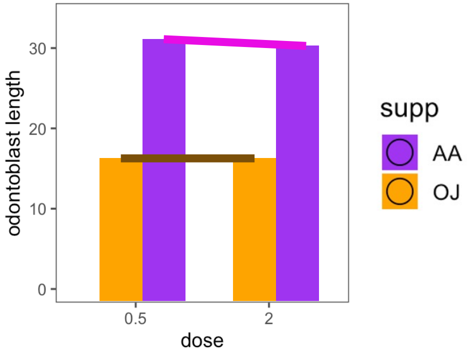


dose does not matter
supplement matters



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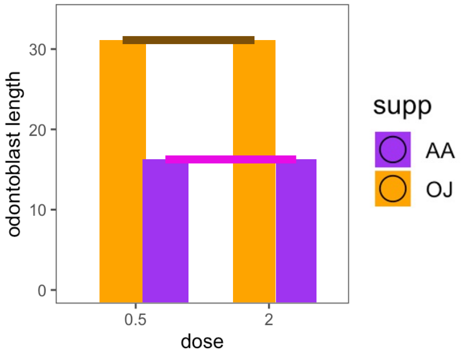
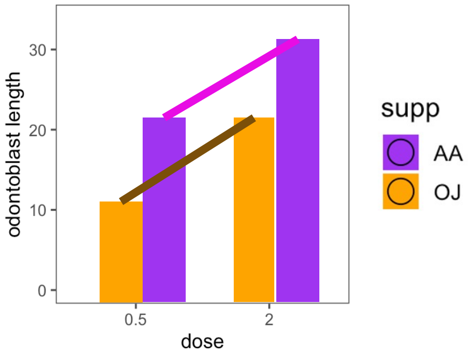


dose matters
supplement matters
dose and supplement
do not influence each other



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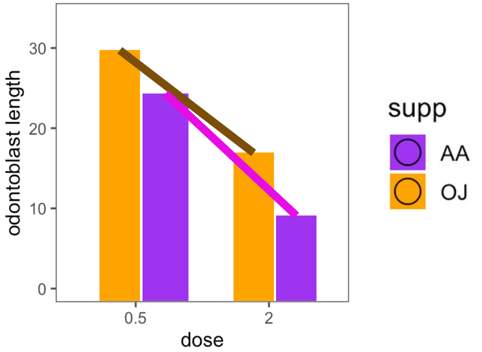

dose matters
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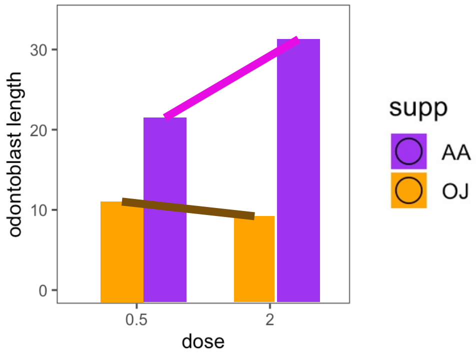

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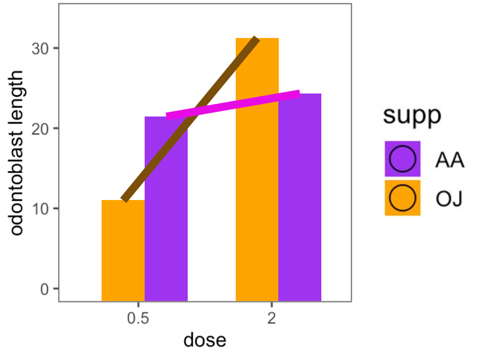
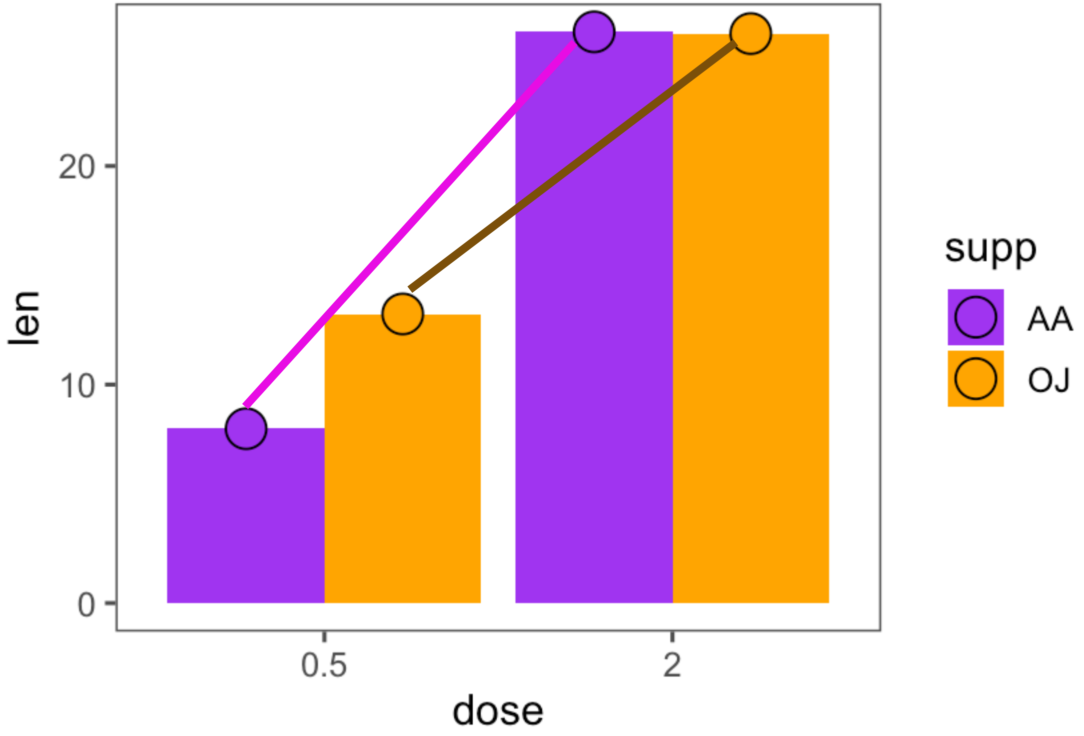


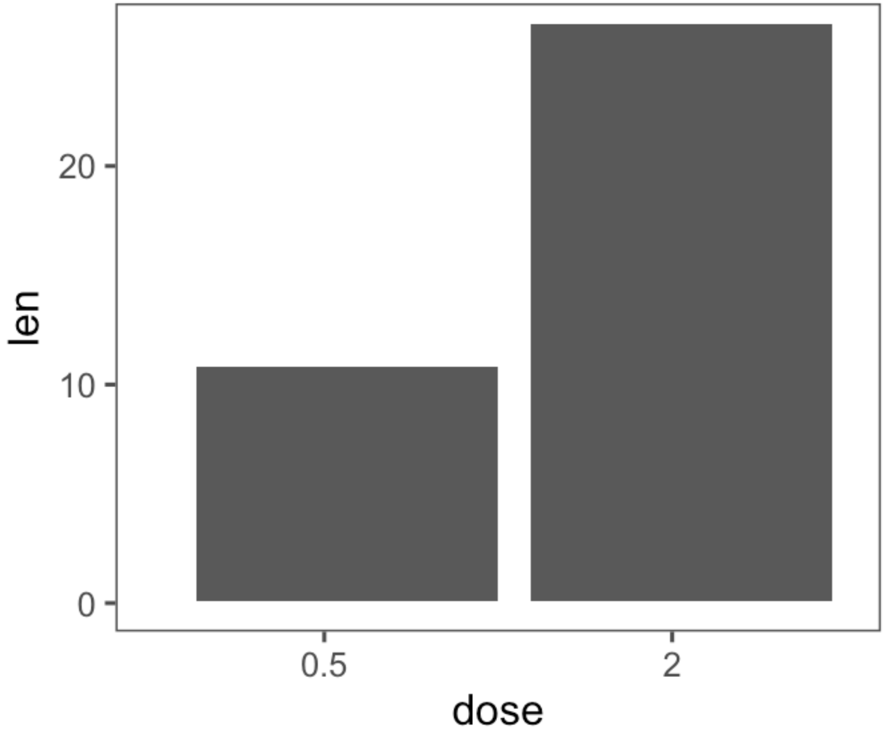
tooth growth dataset: actual pattern

- **dose** matters (0.5 mg << 2 mg)
- **supplement** matters (OJ > AA slightly)
- **dose** and **supplement** <u>influence each other</u>
  - at 0.5 mg, delivery method matters
  - at 2 mg, delivery method stops mattering



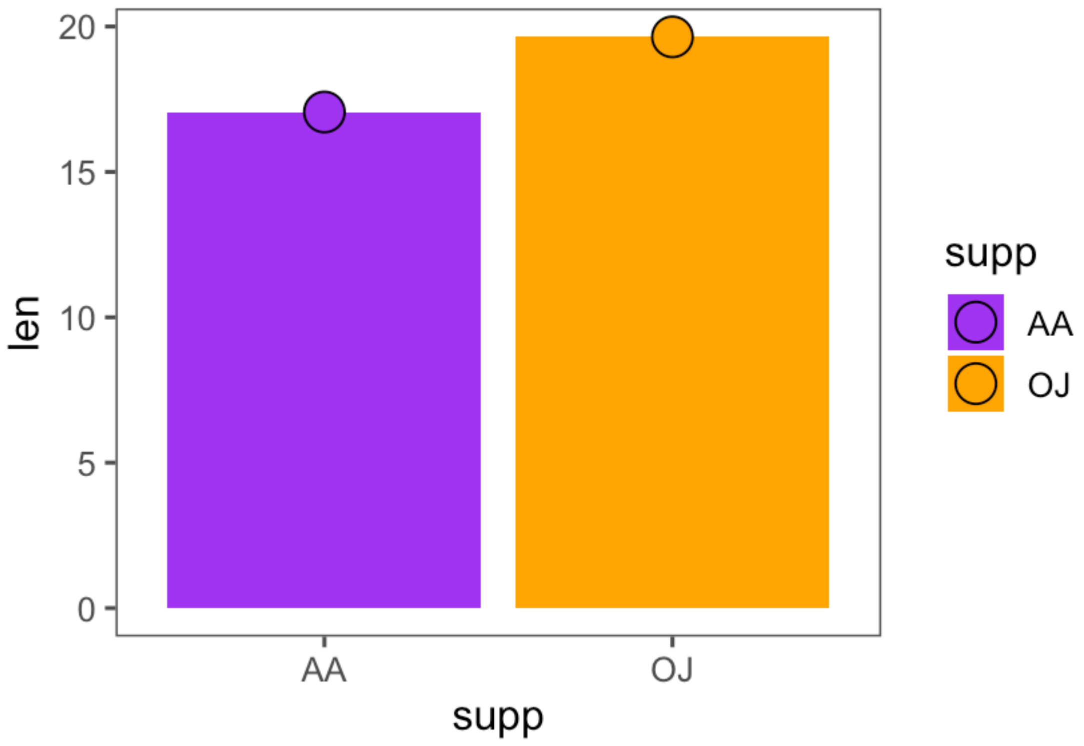
tooth growth dataset: main effects

- **dose** matters (0.5 mg << 2 mg)
  - MAIN effect: the "overall" effect of dose (ignoring delivery method), i.e., difference in tooth growth for 0.5 mg vs. 2 mg
  - $M_{0.5mg} - M_{2mg}$



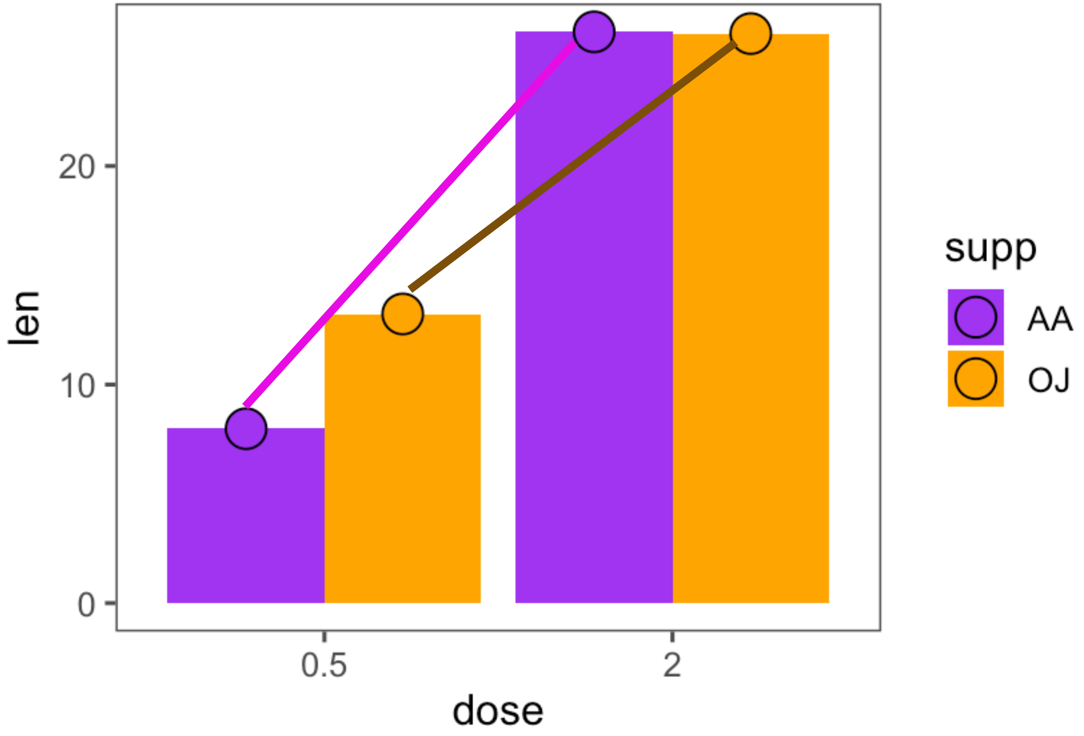
tooth growth dataset: main effects

- **supplement** matters (OJ > AA)
  - MAIN effect: the "overall" effect of supplement (ignoring dose), i.e., difference in tooth growth for OJ vs. AA
  - $M_{OJ} – M_{AA}$



tooth growth dataset: interactions

- **dose** and **supplement** influence each other
  - INTERACTION effect: the difference between differences
    - $OJ_{0.5mg} - OJ_{2mg}$ vs. $AA_{0.5mg} - AA_{2mg}$
- what would the plot look like if there was NO interaction?
  - parallel lines!



main effects and interactions

- main effects represent the “overall” effect of one independent variable when ignoring the influence of other variables
- interactions represent the full relationship between multiple independent variables
- when interactions are present in the model, **main effects need to be qualified**, i.e., you cannot truly understand the influence of that variable in isolation

practice question #1

- For a two-factor experiment with 2 levels of factor A and 3 levels of factor B and n = 10 subjects in each treatment condition, how many participants are in <u>each level of factor B</u>?
  - 10
  - 20
  - 30
  - 60

practice question #2

- A two-factor research study is used to evaluate the effectiveness of a new blood-pressure medication. In this two-factor study, Factor A is medication versus no medication and factor B is male versus female. The medicine is expected to reduce blood pressure for both males and females, but it is expected to have a much greater effect for males. What pattern of results should be obtained if the medication works as predicted?
  - significant main effect for factor A (medication).
  - a significant interaction.
  - a significant main effect for factor A and a significant interaction.
  - none of the above.

practice question #3

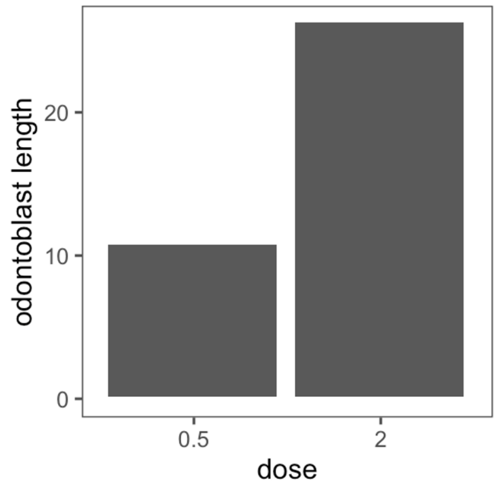
- In a line graph showing the results from a two-factor experiment, the levels of factor A (A1 and A2) are presented on the X-axis and separate lines are used to display the means for B1 and B2. If the points on the line for B1 are consistently 10 points lower than the corresponding point on the line for B2, what pattern of results is indicated?
  - an indication of an overall A-effect
  - an indication of an overall B-effect
  - an indication of a significant interaction
  - no claims can be made

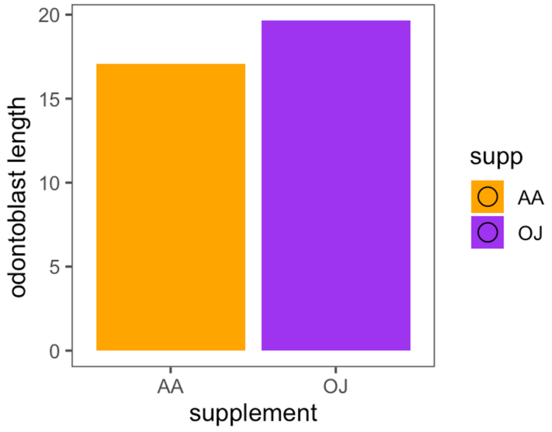
practice question #4

- In a line graph showing the results from a two-factor experiment, the levels of factor A (A1 and A2) are presented on the X-axis and separate lines are used to display the means for B1 and B2. If the points on the line for B1 are consistently **at least** 10 points lower than the corresponding point on the line for B2, what pattern of results is indicated?
  - an indication of an overall A-effect
  - an indication of an overall B-effect
  - an indication of a significant interaction
  - no claims can be made

building a factorial model

- we can start with three simple models
- grand mean model : toothGrowth ~ **grand mean**
- main effect 1: toothGrowth ~ **dose**
  - model = dose means
  - obtain $SS_{dose\_model} = SS_{total} - SS_{Y-\hat{Y}_{dose\_model}}$
- main effect 2: toothGrowth ~ **supp**
  - model = supplement means
  - obtain $SS_{supp\_model} = SS_{total} - SS_{Y-\hat{Y}_{supp\_model}}$





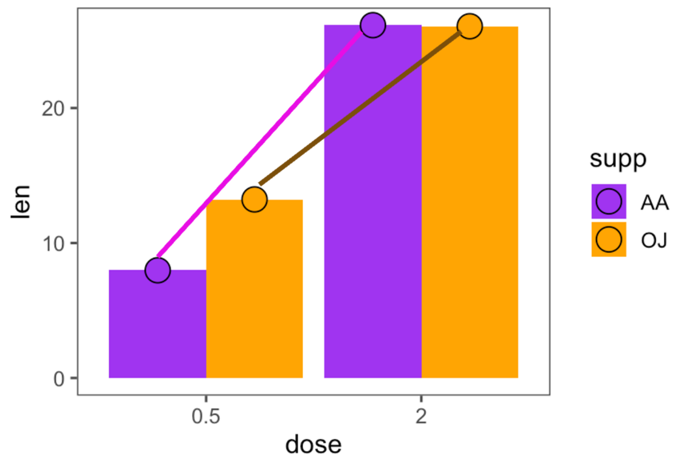
activity: compute the means

- use the tooth growth data
- compute all the means and come back

supplement	dose=0.5	dose=2
AA	7.98	26.14
OJ	13.23	26.06

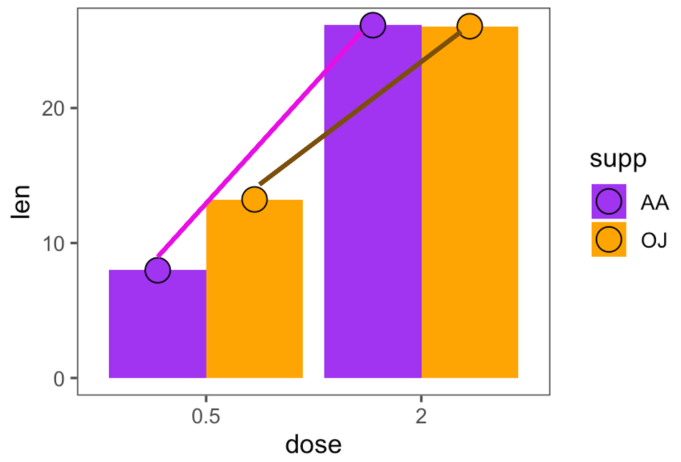
AA_overall	17.06
OJ_overall	19.645
dose_0.5	10.605
dose_2	26.1

activity: build the models



- build the **grand mean** model
  - obtain SS_{total}
- build the **dose** model using dose means
  - obtain $SS_{dose_{model}}$
- build the **supplement** model using supplement means
  - obtain $SS_{supp_{model}}$

activity: build the models

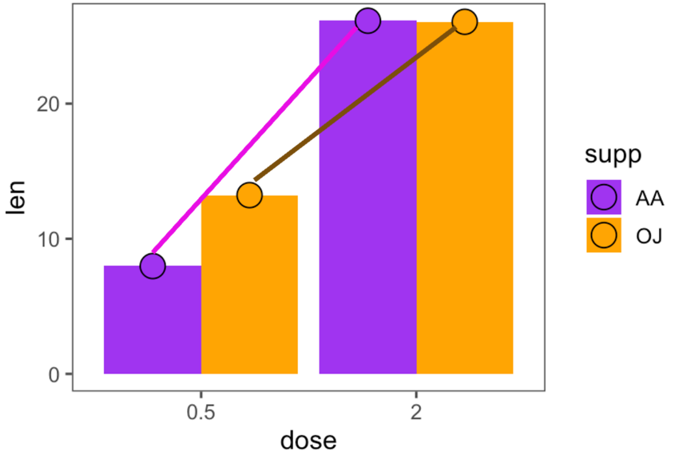


- build the **grand mean** model
  - obtain $SS_{total} = 3056.29975$
- build the **dose** model using dose means
  - obtain $SS_{dose_{model}} = 2400.95025$
- build the **supplement** model using supplement means
  - obtain $SS_{supp_{model}} = 66.82225$

SStotal	3056.29975

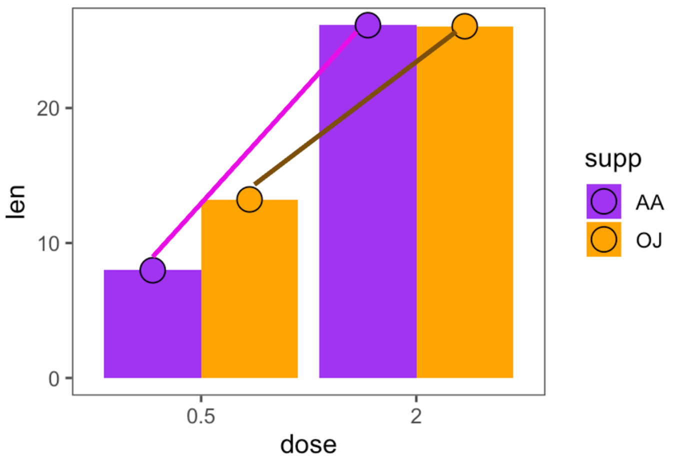
	SS
supplement_model	66.82225
dose_model	2400.95025

building a complex model



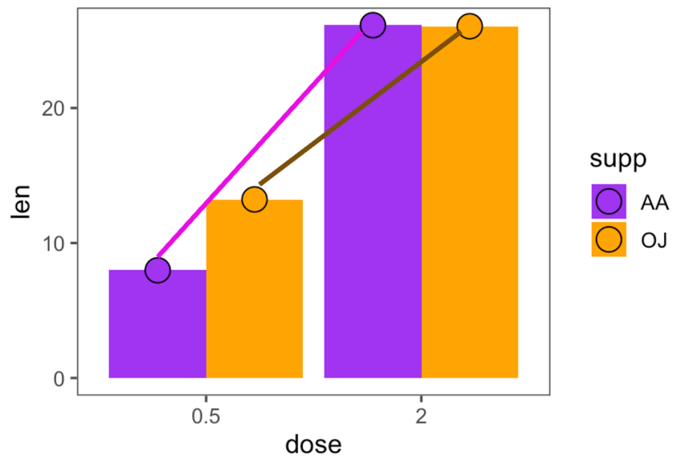
- next, we fit our more complex model
- interaction model: toothGrowth ~ **dose + supp** + (**dose**)(**supp**)
  - substitutes each value with the respective sub-mean of the factorial design
  - obtain $SS_{full\_model} = SS_{total} - SS_{Y-\hat{Y}_{full\_model}} = SS_{total} - SS_{error}$
- how much variance is explained by the interaction ($SS_{interaction}$)?
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
- the interaction represents the part of the “full model” that is not explained by the simple models of only dose and only supplement

activity: build full model



- build full model using <u>all</u> sub-group means
  - $SS_{error} = ??$ (the error left over from the full model)
    - also called $SS_{residuals}$
  - $SS_{full\_model} = SS_{total} - SS_{error} = ??$
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
  - $SS_{interaction} = ??$

activity: build full model



- build full model using <u>all</u> sub-group means
  - $SS_{error} = 517.505$ (the error left over from the full model)
    - also called $SS_{residuals}$
  - $SS_{full\_model} = SS_{total} - SS_{error} = 2538.79475$
  - $SS_{interaction} = SS_{full\_model} - SS_{dose_{model}} - SS_{supp_{model}}$
  - $SS_{interaction} = 71.02225$

	SS
supplement_model	66.82225
dose_model	2400.95025
interaction	71.02225
residuals	517.505
SStotal	3056.29975

NHST for factorial ANOVA

step 1: build grand mean model

step 2: build factor 1 model

step 3: build factor 2 model

step 4: build full model

step 5: build F table & conduct **<u>ALL</u>** F tests!

step 1

(1) "summarizing" data using a single grand mean (ignoring all group labels)

(2) compute SS_{total}

step 2

(1) "summarize" data using the means for levels from the **<u>first</u>** independent variable

(2) compute SS_{model1} and SS_{error1}

step 3

(1) "summarize" data using the means for levels from the **<u>second</u>** independent variable

(2) compute SS_{model2} and SS_{error2}

step 4

(1) "summarize" data using the means for the **<u>full</u>** 2x2 design (i.e., each of the 4 means)

(2) compute $SS_{fullmodel}$ and SS_{error}

(3) compute $SS_{interaction}$

step 5

(1) create F table

(2) find $F_{critical}$

(3) compute $F_{observed} = \frac{MS_{model}}{MS_{error}}$

(4) find p-value for F-score

(4) decide!

testing significance (F-test)

- we conduct individual F-tests for each type of possible effect using the remaining error ($SS_{residual}$) from the full model

$$F(df_1, df_2) = \frac{MS_{model}}{MS_{error}} = \frac{SS_{model}/df_{model}}{SS_{error}/df_{error}}$$

- degrees of freedom
  - $df_{1i} = k_i - 1$
  - $df_{interaction} = product\ of\ all\ df_{1i}$
  - $df_2 = \mathrm{n}\ - \text{product of } k_i$

df for toothGrowth dataset

n	k	term	df	
40	2 (AA vs. OJ)	supplement	2-1 = 1	
	2 (0.5 mg vs 2 mg)	dose	2-1 = 1	
		interaction	1 x 1 = 1	
		residual	40 – (2*2) = 36	error or within

practice question

- For an experiment involving 2 levels of factor A and 3 levels of factor B with a sample of n = 5 in each treatment condition, what is the value for df_{within}?
  - 20
  - 24
  - 29
  - 30

practice question

- The results of a two-factor analysis of variance produce df = 2, 36 for the F-ratio for factor A and df = 2, 36 for the F-ratio for factor B. What are the df values for the AxB interaction?
  - 1, 36
  - 2, 36
  - 3, 36
  - 4, 36

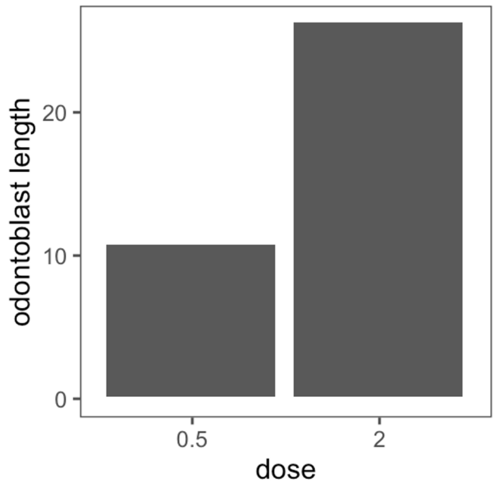
testing significance (F-test)

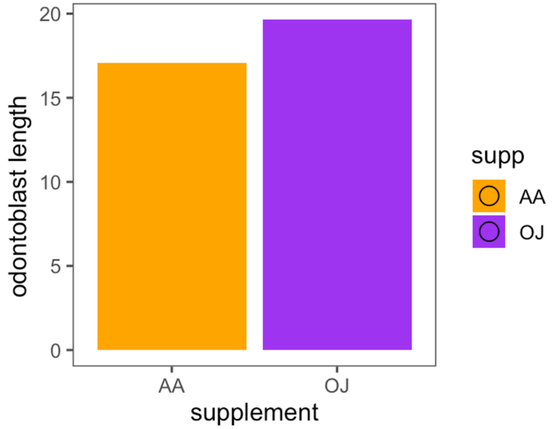
k		SS	df	MS	F_observed	F_critical	check	p_value
2	**supplement_model**	66.82225	1	66.82225	4.648459435	4.1132	TRUE	0.0378
2	**dose_model**	2400.95025	1	2400.95025	167.0210124	4.1132	TRUE	less than 0.0001
	interaction	71.02225	1	71.02225	4.940630525	4.1132	TRUE	0.0326
	residuals	517.505	36	14.37513889				
	SStotal	3056.29975						

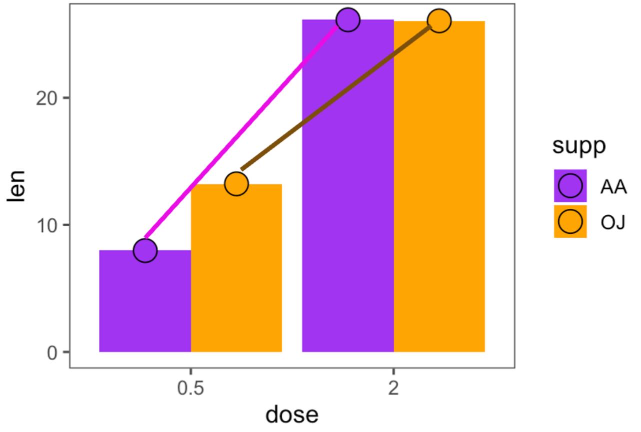
[Sheets solution](#)
[video tutorial](#)

post-hoc tests

- once the "overall" F-tests show that substantial variation is explained by some combination of independent variables, we can dive in and explore specific effects
- sometimes, researchers have specific hypotheses about main effects and/or the interaction(s)
- these hypotheses can be tested using pairwise t-tests/one-way ANOVAs, but must be corrected for multiple comparisons







next time

- **before** class
  - *watch:* Hypothesis Testing (Factorial ANOVA) [33 min]
  - *explore:* Problem Set 7!
  - *post*: Data Around Us OR practice questions (class participation)
- **during** class
  - review for midterm 2!

optional: building a complex model

- what is our model's equation?
  - toothGrowth ~ a + b (dose) + c (supp) + d (dose) (supplement)
  - simple coefficients signify main effects (b and c)
  - product coefficients signify interactions
  - "intercept" (a) signifies the mean of toothGrowth when all other coefficients = 0
  - NOTE: this is no longer a line!
- what are the values of a, b, c, and d?
  - nominal independent variables are converted to 0s and 1s ("dummy codes")
  - intercept (a): dose and supp are both 0, i.e., predicted mean toothGrowth in the $AA_{0.5mg}$ group
  - b: dose = 1, supp = 0, i.e., change in toothGrowth from $AA_{0.5mg}$ to AA_{2mg}
  - c: supp = 1, dose = 0, i.e., change in toothGrowth from $AA_{0.5mg}$ to $OJ_{0.5mg}$
  - d: supp = 1, dose = 1, i.e., difference of differences, i.e., $(OJ_{0.5mg} - OJ_{2mg}) - (AA_{0.5mg} - AA_{2mg})$
- this is called **dummy coding** or setting up **contrasts** in your model

	0	1
dose	0.5mg	2mg
supp	AA	OJ

optional: continuous IVs

- the same framework in general holds for interval/ratio-level independent variables
  - *multiple regression*: $Y = b_1X_1 + b_2X_2 + ... + a + error$
- here, the coefficients represent the change in Y as a function of the specific independent variable (X_i) when "controlling for" the effect of other variables
- just as the linear correlation is structurally equivalent to the slope of a line, *partial* correlations are structurally equivalent to the coefficients from a multiple regression

optional: multiple regression in Excel

- fitting a (multiple) regression model in Sheets / Excel
- LINEST(Y, range of X columns/predictors, TRUE, FALSE)