Exercise 9.5

1. Find each of the following products by using appropriate identities:

(*i*) $(x-y-1)(x^2+y^2+xy+x+y+1)$

(*ii*) $(2x-3y+1)(4x^2+9y^2+1+6xy+3y-2x)$

(*iii*) $(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$

(*iv*) $(3x-2y-z)(9x^2+4y^2+z^2+6xy-2yz+3zx)$

(*v*) $(x-y+2)(x^2+y^2+4+xy+2y-2x)$

(*vi*) $\left(x+\frac{1}{x}-1\right)\left(x^2+\frac{1}{x^2}+x+\frac{1}{x}\right)$

(*vii*) $\left(x-\frac{1}{x}-3\right)\left(x^2+\frac{1}{x^2}+10-\frac{3}{x}+3x\right)$

2. If $l+m+n=0$, then prove that $\frac{l^2}{mn}+\frac{m^2}{nl}+\frac{n^2}{lm}=3$.

3. Divide $27x^3-8y^3-z^3-18xyz$ by $3x-2y-z$ in any manner you like.

4. (*i*) If $x^2+y^2+z^2=40$ and $xy+yz+zx=12$, then find the value of $x^3+y^3+z^3-3xyz$.

(*ii*) If $x=5, y=7$ and $z=15$, find the value of $\frac{x^3+y^3+z^3-3xyz}{xy+yz+zx-x^2-y^2-z^2}$.

Answers

1. (*i*) $x^3-y^3-3xy-1$

(*ii*) $8x^3-27y^3+1+18xy$

(*iii*) $8a^3+b^3-6ab+1$

(*iv*) $27x^3-8y^3-z^3-18xyz$

(*v*) $x^3-y^3+6xy+8$

(*vi*) $x^3+\frac{1}{x^3}+2$

(*vii*) $x^3-\frac{1}{x^3}-36$

3. $9x^2+4y^2+z^2+6xy+3xz-2yz$

4. (*i*) 224

(*ii*) -27