Exercise 7.1

1. (i) The remainder obtained when the polynomial $p(x)$ is divided by $(b - ax)$ is

   (a) $p\left(-\dfrac{b}{a}\right)$ (b) $p\left(\dfrac{a}{b}\right)$ (c) $p\left(\dfrac{b}{a}\right)$ (d) $p\left(-\dfrac{a}{b}\right)$

   [CBSE 2011]

   (ii) Let $p(x) = x^3 - 2x^2 + x - 2$. Find the remainder by using division process and also by using remainder theorem, when $p(x)$ is divided by:

   (a) $x - 2$ (b) $x + 1$

2. $p(x)$ is the polynomial $4x^3 - 12x^2 + 11x - 2$. Use the remainder theorem to find the remainder, when $p(x)$ is divided by:

   (i) $x - \dfrac{1}{2}$ (ii) $x + \dfrac{1}{2}$ (iii) $x - \dfrac{3}{2}$

3. By using remainder theorem, show that $g(x)$ is a factor of $f(x)$, given $f(x) = x^3 - x^2 + 11x + 69$, $g(x) = x + 3$.

4. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. Find the values of p and q.

5. $6x^2 + ax + 7$ when divided by $x - 2$ gives the remainder 13, find the value of a.

6. (i) If $x - 3$ is a factor of $x^2 + kx + 9$, then find the value of k.

   (ii) Find the value of a, if $x - a$ is a factor of $x^5 - a^2x^3 + 2x + a + 1$.

   (iii) Prove that $x^2 - 5x + 6$ is a factor of $2x^4 - 17x^3 + 49x^2 - 52x + 12$.

   [**Hint:** $x^2 - 5x + 6 = (x - 2)(x - 3)$. Now, show both $x - 2$ and $x - 3$ are factors of the given polynomial.]

7. Find the remainder when $p(y) = y^2 + 4y + 2$ is divided by $y + 2$.

8. If $p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$, find the remainder. Also, verify the result by actual division.

9. If $x^3 + x^2 + x + a$ is divisible by $x - 1$, find the value of a.

10. Find the value of a if the division of $ax^3 + 9x^2 + 4x - 10$ by $(x + 3)$ leaves a remainder 5. -

11. Two polynomials $p(x) = ax^3 + 3x^2 - 13$ and $q(x) = 2x^3 - 5x + a$ when divided by $x + 2$ gives the same remainder. Find the value of a.

12. (i) Let r_1 and r_2 be the remainders when the polynomials
    $$p(x) = x^3 + x^2 - 5kx - 7 \text{ and } q(x) = x^3 + kx^2 - 12x + 6$$
    are divided by $x + 1$ and $x - 2$ respectively.
    Find the value of k if $2r_1 - r_2 = 10$.

    (ii) If the polynomial $p(x) = 2x^3 + kx^2 - 3x + 5$ and $q(x) = x^3 + 2x^2 - x + k$, when divided by $(x - 2)$ leaves the remainder r_1 and r_2 respectively. Find the value of k if $r_1 - r_2 = 0$.

    [CBSE 2010]

13. (i) If $(x^{11} + 1)$ is divided by $(x + 1)$, find the remainder.

    (ii) If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, find the remainder.

14. The polynomials $kx^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + k$ leave the same remainder when divided by $(x - 3)$, find the value of k.

15. When $x^5 - 5x^4 + 9x^3 - 6x^2 - 16x + 13$ is divided by $x^2 - 3x + a$, the quotient and remainder are $x^3 - 2x^2 + x + 1$ and $-15x + 11$, respectively. Find the value of a.

16. If $Ax^3 + 31x^2 - Bx - 10$ is exactly divisible by $2x^2 + 9x - 5$, find the values of A and B.

17. Show that $x - 1$ and $x + 4$ are factors of the polynomial $x^3 + x^2 - 10x + 8$.

18. Determine the value of 'a' such that $(x - 4)$ is a factor of the polynomial
$$p(x) = 2x^3 + ax^2 + 27x - 28.$$ 　　　 [CBSE 2001 C]

19. If $x - 1$ is a factor of $g(x) = x^2 + kx + 1$, then find k and hence prove that $(x - k)$ is a factor of
$$p(x) = x^3 + 3x^2 + 3x + 2.$$

20. (i) Find the value of k so that $x - 2$ is a factor of $2x^3 - 6x^2 + 5x + k$.

　(ii) If $ax^3 + bx^2 + x - 6$ has $x + 2$ as a factor and leaves remainder 4 when divided by $(x - 2)$. Find the values of a and b. 　　　 [CBSE 2011]

21. Let $f(x) = x^3 + kx^2 + hx + 6$. Find the value of h and k so that $(x + 1)$ and $(x - 2)$ are factors of $f(x)$.

22. (i) Show that $(x - 5)$ is a factor of the polynomial
$$p(x) = 3x^3 - 16x^2 - 5x + 50.$$ 　　　 [CBSE 2001 C]

　(ii) Find k so that $3x^4 + 8x^3 - 4kx + k$ may be divisible by $x - 2$.

　(iii) If $(x + a)$ is the HCF of $x^2 + px + q$ and $x^2 + 1x + m$, then find the value of 'a'.

23. (i) Use factor theorem to determine if $(x - 1)$ is a factor of:
$$x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$$

　(ii) Given $p(x) = 2x^5 + 3x^2 - 3x - 2$ and $q(x) = x - 1$. Find by actual division whether $q(x)$ is a factor of $p(x)$. Verify your answer by factor theorem.

　(iii) If remainder is same when polynomial $p(x) = x^3 + 8x^2 + 17x + ax$ is divided by $(x - 2)$ and $(x + 1)$. Find the value of a. 　　　 [CBSE 2011]

　(iv) If $(x + 5)$ is a factor of $x^3 + 2x^2 - 13x + 10$, find the other factors.

Answers

1. (i) (c) $p\left(\dfrac{b}{a}\right)$ (ii) (a) 0 (b) -6 2. (i) 1   (ii) -11 (iii) 1

4. $p = -3, q = -6$ 5. $a = -9$

6. (i) $k = -6$ (ii) $a = -1/3$ 7. -2 8. 1

9. $a = -3$ 10. $a = 2$ 11. $a = 5/9$

12. (i) $k = \dfrac{7}{3}$ (ii) $k = -\dfrac{1}{3}$

13. (i) 0   (ii) -63 14. $k = -1$ 15. $a = 2$

16. $A = 6, B = -3$ 18. -13 20. (i) $k = -2$ (ii) $a = 0, b = 2$

21. $h = 1, k = -4$ 22. (ii) $k = 16$ (iii) $\dfrac{q-m}{p-1}$

23. (i) $x - 1$ is not a factor of the given polynomial.
   (ii) $x - 1$ is a factor of $p(x)$ (iii) $a = -28$ (iv) $(x - 2)(x - 1)$