Exercise 9.6

1. Factorise each of the following algebraic expressions by taking out the common factor:

(i) $46x^2 + 2x + 10y^2$ (ii) $6x^3y^2 - 9x^2y^3 + 12x^4y^4$

(iii) $3x(3a-1) - 6xy(3a-1)$ (iv) $a^2b(x-y)^2 - ab^2(x-y)$

(v) $p(a+b+c) - q(a+b+c) + r(a+b+c)$

(vi) $(4x-7y)^2 + 3(4x-7y)z$

(vii) $(a+b+c)^2 + x(a+b+c) - y(a+b+c)$

(viii) $a(3x-4y)^2 + 2b(4y-3x)$

2. Factorise each of the following expressions by grouping the terms:

(i) $x^3 + x - 3x^2 - 3$ (ii) $ab(x^2+1) + x(a^2+b^2)$

(iii) $1 - c^2 + b^2 - c^2b^2$ (iv) $x^2 - \left(a + \frac{1}{a}\right)x + 1$

(v) $ab(x^2+y^2) + xy(a^2+b^2)$ (vi) $ab(x^2+4) - x(a^2+4b^2)$

(vii) $x^3 - x^2 - ax + x + a - 1$ (viii) $x^3 + xy(1-3x) - 3y^2$

3. Factorise each of the following expressions by first making the perfect square:

(i) $1 - 6x + 9x^2$ (ii) $x^2 - x + \frac{1}{4}$

(iii) $49x^2 - 126xy + 81y^2$ (iv) $4x^2 - 4\sqrt{3}x + 3$

(v) $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$

4. Factorise each of the following (difference of two squares):

(i) $64x^2y^2 - 1$ (ii) $1 - (b-c)^2$

(iii) $81(x+y)^2 - 64(x-y)^2$ (iv) $3xy - 243xy^5$

(v) $25x^2 - 10x + 1 - 36z^2$ (vi) $16x^4 - 1$

(vii) $x^2y^2 + 2xy + 1 - 16p^2$ (viii) $x^4 + 14x^2 + 48$

5. Find the factors of each of the following algebraic expressions by splitting the middle term:

(i) $4x^2 - 17x - 21$ (ii) $10x^2 + 3x - 4$

(iii) $2x^2 + 3\sqrt{3}x + 3$ (iv) $15x^2 - 77xy + 10y^2$

(v) $6x^2 + 11x - 35$ (vi) $16\sqrt{5}x^2 - 50x + 5\sqrt{5}$

6. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) $5x^2 + 13x - 6$ (ii) $7x^2 - 25x - 12$

7. Factorise each of the following algebraic expressions expressible as the sum or difference of two cubes :

(i) $8a^5b^2 + 27a^2b^5$ (ii) $x^5 + 27x^2$

(iii) $27a^3 + 8b^3 - 9a - 6b$ (iv) $8x^6 - 1$

(v) $x^3y^3 - 512$ (vi) $64x^3 - 27y^3$

(vii) $3x^{13} - 192x^7y^6$ (viii) $27x^3 - \frac{125}{x^3}$

(ix) $x^3 + 8y^3 + 6x^2y + 12xy^2$ (x) $8x^3 + y^3 + 12x^2y + 6xy^2$

(xi) $8p^3 - 27q^3 - 36p^2q + 54pq^2$

8. Factorise each of the following algebraic expressions:

(i) $8x^3 - 64y^3 + 27z^3 + 72xyz$ (ii) $216\,m^3 - 8n^3 - 27\,l^3 - 108\,mnl$

(iii) $27x^3 - 8y^3 - z^3 - 18xyz$ (iv) $x^3 - y^3 + 6xy + 8$

(v) $x^3 - y^3 - 3xy - 1$

(vi) $2\sqrt{2}x^3 + 8y^3 - 27z^3 + 18\sqrt{2}xyz$ *[NCERT Exemplar]*

(vii) $x^3 + \frac{1}{x^3} - 2$ (viii) $8a^3 + b^3 - 6ab + 1$

(ix) $x^3 - y^3 - 9xy - 27$

9. If $x + y - 1 = 0$, prove that $x^3 + y^3 + 3xy = 1$.

10. Factorise : $(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$

11. Prove that if $p = 2 - a$, then $a^3 + 6ap + p^3 - 8 = 0$.

12. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$ and $a^3 + b^3 + c^3 = 36$, find the value of abc.

13. Resolve into factors $x^6 + 4x^3 - 1$

[Hint: Given expression $= (x^2)^3 + (x)^3 + (-1)^3 - 3 \cdot x^2 \cdot x \cdot (-1)]$

14. If $x = y = 333$ and $z = 334$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

[Hint: $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\,[(x - y)^2 + (y - z)^2 + (z - x)^2]$

15. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 5$ and $a^3 + b^3 + c^3 = 9$, find the value of abc.

16. If $x = a(b - c)$, $y = b(c - a)$, $z = c(a - b)$, then prove that

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = \frac{3xyz}{abc}.$$

Answers

1. (i) $2(23x^2 + x + 5y^2)$ (ii) $3x^2y^2(2x - 3y + 4x^2y^2)$

(iii) $3x(3a - 1)(1 - 2y)$ (iv) $ab(x - y)(ax - ay - b)$

(v) $(a + b + c)(p - q + r)$ (vi) $(4x - 7y)(4x - 7y + 3z)$

(vii) $(a + b + c)(a + b + c + x - y)$ (viii) $(3x - 4y)(3ax - 4ay - 2b)$

2. (i) $(x^2 + 1)(x - 3)$ (ii) $(ax + b)(bx + a)$

(iii) $(1 - c^2)(1 + b^2)$ (iv) $(x - a)\left(x - \frac{1}{a}\right)$

(v) $(ay + bx)(ax + by)$ (vi) $(ax - 4b)(bx - a)$

(vii) $(x-1)(x^2-a+1)$

(viii) $(x^2+y)(x-3y)$

3. (i) $(1-3x)(1-3x)$

(ii) $\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$

(iii) $(7x-9y)(7x-9y)$

(iv) $(2x-\sqrt{3})(2x-\sqrt{3})$

(v) $\left(\frac{x}{y}+\frac{y}{x}\right)\left(\frac{x}{y}+\frac{y}{x}\right)$

4. (i) $(8xy+1)(8xy-1)$

(ii) $(1+b-c)(1-b+c)$

(iii) $(17x+y)(x+17y)$

(iv) $3xy(1+9y^2)(1+3y)(1-3y)$

(v) $(5x-1+6z)(5x-1-6z)$

(vi) $(4x^2+1)(2x+1)(2x-1)$

(vii) $(xy+1+4p)(xy+1-4p)$

(viii) $(x^2+8)(x^2+6)$

5. (i) $(x+1)(4x-21)$

(ii) $(5x+4)(2x-1)$

(iii) $(x+\sqrt{3})(2x+\sqrt{3})$

(iv) $(x-5y)(15x-2y)$

(v) $(2x+7)(3x-5)$

(vi) $(2x-\sqrt{5})(8\sqrt{5}x-5)$

6. (i) $(x+3),(5x-2)$

(ii) $(x-4),(7x+3)$

7. (i) $a^2b^2(2a+3b)(4a^2-6ab+9b^2)$

(ii) $x^2(x+3)(x^2+9-3x)$

(iii) $(3a+2b)(9a^2+4b^2-6ab-3)$

(iv) $(2x^2-1)(4x^4+2x^2+1)$

(v) $(xy-8)(x^2y^2+64+8xy)$

(vi) $(4x-3y)(16x^2+9y^2+12xy)$

(vii) $3x^7(x+2y)(x-2y)(x^2+4y^2+2xy)(x^2+4y^2-2xy)$

(viii) $\left(3x-\frac{5}{x}\right)\left(9x^2+15+\frac{25}{x^2}\right)$

(ix) $(x+2y)(x^2+4xy+4y^2)$

(x) $(2x+y)(4x^2+4xy+y^2)$

(xi) $(2p-3q)(4p^2-12pq+9q^2)$

8. (i) $(2x-4y+3z)(4x^2+16y^2+9z^2+8xy+12yz-6xz)$

(ii) $(6m-2n-3l)(36m^2+4n^2+9l^2+12mn+18ml-6nl)$

(iii) $(3x-2y-z)(9x^2+4y^2+z^2+6xy-2yz+3zx)$

(iv) $(x-y+2)(x^2+y^2+4+xy+2y-2x)$

(v) $(x-y-1)(x^2+y^2+1+xy+x-y)$

(vi) $(\sqrt{2}x+2y-3z)(2x^2+4y^2+9z^2-2\sqrt{2}xy+6yz+3\sqrt{2}xz)$

(vii) $\left(x+\frac{1}{x}+1\right)\left(x^2+\frac{1}{x^2}-x-\frac{1}{x}\right)$

(viii) $(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$

(ix) $(x-y-3)(x^2+y^2+9+xy-3y+3x)$

10. $3(a+b-2c)(b+c-2a)(c+a-2b)$ **12.** $abc=6$

13. $(x^2+x-1)(x^4-x^3+2x^2+x+1)$ **14.** 1000

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