## Exercise 4.1

**1.** Express each one of the following with rational denominator:

(i) $\frac{2}{\sqrt{7}}$ (ii) $\frac{6}{\sqrt{3}}$ (iii) $\frac{3}{\sqrt{8}}$ (iv) $\frac{1}{\sqrt{12}}$

(v) $\frac{12}{\sqrt{72}}$ (vi) $\frac{14}{\sqrt{98}}$ (vii) $\frac{7}{\sqrt{125}}$ (viii) $\frac{3}{2\sqrt{5}}$

**2.** Rationalise the denominator of each of the following:

(i) $\frac{6}{3+\sqrt{5}}$ (ii) $\frac{2}{\sqrt{3}-1}$ (iii) $\frac{2}{\sqrt{3}+\sqrt{2}}$

**3.** Rationalise the denominator of each of the following:

(i) $\frac{4}{3+\sqrt{3}}$ (ii) $\frac{5}{\sqrt{2}+\sqrt{3}}$ (iii) $\frac{6\sqrt{5}-5\sqrt{3}}{2\sqrt{3}+4\sqrt{5}}$

**4.** Rationalise the denominator of each of the following:

(i) $\frac{\sqrt{14}+\sqrt{21}}{\sqrt{3}}$ (ii) $\frac{\sqrt{2}}{5+\sqrt{2}}$ (iii) $\frac{2+\sqrt{5}}{\sqrt{3}-\sqrt{2}}$ (iv) $\frac{2\sqrt{7}-\sqrt{5}}{3\sqrt{3}-2\sqrt{2}}$

**5.** If $x = \sqrt{7}+\sqrt{3}$, show that $x - \frac{4}{x} = 2\sqrt{3}$.



**6.** If $x = \frac{3+\sqrt{5}}{2}$, show that $x + \frac{1}{x} = 3$. Also, find $x^3 + \frac{1}{x^3}$.

**7.** If $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$, show that (i) $x+\frac{1}{x} = 18$ (ii) $x^2 + \frac{1}{x^2} = 322$.

**8.** If $x = \sqrt{2}+1$, find the value of (i) $x^2+\frac{1}{x^2}$ (ii) $x^3+\frac{1}{x^3}$.

**9.** If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$, $y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, show that $x^2 + xy + y^2 = 35$.

**10.** If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then find $\frac{a^2+ab+b^2}{a^2-ab+b^2}$.

**11.** Simplify: $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

**12.** Determine the rational numbers $a$ and $b$ if $\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1} = a+\sqrt{3}\,b$.

**13.** If $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = a+b\sqrt{6}$, find the values of $a$ and $b$.

**14.** If $a$ and $b$ are both rational numbers, find the values of $a$ and $b$ in each of the following:

(i) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

(ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a+b\sqrt{15}$ (iii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5}-b$

**15.** If $a$ and $b$ are both rational numbers and $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-b\sqrt{3}$, find the values of $a$ and $b$.

**16.** If $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a+7\sqrt{5}\,b$, find the rational numbers $a$ and $b$.

**17.** (i) Find the sum of the squares of the following: $\frac{\sqrt{3}}{\sqrt{2}+1}, \frac{\sqrt{3}}{\sqrt{2}-1}, \frac{\sqrt{2}}{\sqrt{3}}$

(ii) If $A = 5+2\sqrt{6}$, find the value of $\sqrt{A}+\frac{1}{\sqrt{A}}$.

**18.** (i) Given $\sqrt{2} = 1.4142, \sqrt{3} = 1.7321$.



Find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$ correct to three decimal places.

(ii) Evaluate after rationalising the denominator $\left(\frac{25}{\sqrt{40}-\sqrt{80}}\right)$. It is being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$. [*CBSE* 2011]

**19.** Evaluate $\frac{-10}{\sqrt{5}+\sqrt{10}-\sqrt{80}-\sqrt{40}+\sqrt{90}}$. It is being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

# Answers

**1.** (*i*) $\frac{2}{7}\sqrt{7}$ (*ii*) $2\sqrt{3}$ (*iii*) $\frac{3\sqrt{2}}{4}$ (*iv*) $\frac{\sqrt{3}}{6}$

(*v*) $\sqrt{2}$ (*vi*) $\sqrt{2}$ (*vii*) $\frac{7\sqrt{5}}{25}$ (*viii*) $\frac{3}{10}\sqrt{5}$

**2.** (*i*) $\frac{9-3\sqrt{5}}{2}$ (*ii*) $\sqrt{3}+1$ (*iii*) $2\sqrt{3}-2\sqrt{2}$

**3.** (*i*) $\frac{6-2\sqrt{3}}{3}$ (*ii*) $-5\sqrt{2}+5\sqrt{3}$ (*iii*) $\frac{16\sqrt{15}-75}{-34}$

**4.** (*i*) $\frac{1}{3}\sqrt{42}+\sqrt{7}$ (*ii*) $\frac{5\sqrt{2}-2}{23}$ (*iii*) $2\sqrt{3}+2\sqrt{2}+\sqrt{10}+\sqrt{15}$

(*iv*) $\frac{6\sqrt{21}+4\sqrt{14}-3\sqrt{15}-2\sqrt{10}}{19}$



**6.** 18 **8.** 6, $10\sqrt{2}$ **10.** $\frac{4}{3}$ **11.** 10

**12.** $a=4, b=0$ **13.** $a=\frac{3}{5}$, $b=\frac{4}{15}$

**14.** (*i*) $a=\frac{11}{7}$, $b=\frac{6}{7}$ (*ii*) $a=4, b=1$ (*iii*) $a=\frac{9}{11}$, $b=\frac{19}{11}$

**15.** $a = 11, b = 6$ **16.** $a = 0, b = \frac{1}{11}$



**17.** (*i*) $18\frac{2}{3}$ (*ii*) $2\sqrt{3}$ **18.** (*i*) 2.063 (*ii*) –9.5425 **19.** 26.041