1. Find the value of each of the following:

(i) $625^{-3/4}$

(ii) $\left(\dfrac{1}{216}\right)^{-2/3}$

(iii) $\left(\dfrac{243}{32}\right)^{2/5}$

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2. Find the value of each of the following:

(i) $\dfrac{8^{-2/3}}{16^{-3/4}}$
(ii) $\left(\dfrac{256}{625}\right)^{-1/2}$
(iii) $(32^{-2/5})^{1/2}$

3. Find the value of $\dfrac{1}{27^{-1/3}} + \dfrac{1}{625^{-1/4}}$.

4. Find the value of $\left(\dfrac{16}{81}\right)^{-3/4} \div \left(\dfrac{243}{32}\right)^{3/5}$ of $\left(\dfrac{9}{4}\right)^{-1/2}$.

5. Simplify: $\dfrac{a^{x\,(y\,-\,z)}}{a^{y\,(x\,-\,z)}} \div \left\{\dfrac{a^y}{a^x}\right\}^z$.

6. Simplify: $\left(\dfrac{5c^4 b^3}{a^2}\right)^3 \times \left(\dfrac{a^2}{4b^2 c^5}\right)^2$.

7. Simplify: (i) $\left(\dfrac{x^{\frac{4}{3}}}{y^2}\right)^{-\frac{1}{2}} \div \left(\dfrac{y^3}{x^{\frac{1}{2}}}\right)^{\frac{4}{3}}$
(ii) $\left\{81^{-3/4} \times \dfrac{16^{1/4}}{6^{-2}} \times \left(\dfrac{1}{27}\right)^{-4/3}\right\}^{1/3}$

8. Prove that $9^{3/2} - 3 \times 5^0 - \left(\dfrac{1}{81}\right)^{-1/2} = 15$.

9. Prove that $\sqrt{\dfrac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \dfrac{3}{2}$.

10. Prove that:

$$\left(\dfrac{64}{125}\right)^{-2/3} + \dfrac{1}{\left(\dfrac{256}{625}\right)^{1/4}} + \left(\dfrac{\sqrt{25}}{\sqrt[3]{64}}\right) = \dfrac{65}{16}$$

11. Prove that:

(i) $\dfrac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \dfrac{3}{2}$
(ii) $\dfrac{6^{n+3} - 32.6^{n+1}}{6^{n+2} - 2.6^{n+1}} = 1$

12. Prove that $\dfrac{3^{n+4} - 9.3^{n+1}}{9.3^{n+1}} = 2$.

13. Simplify:

$$\left(\dfrac{x^a}{x^b}\right)^{a+b-c} \times \left(\dfrac{x^b}{x^c}\right)^{b+c-a} \times \left(\dfrac{x^c}{x^a}\right)^{c+a-b}$$

14. Simplify: $\sqrt[qr]{\dfrac{x^q}{x^r}} \times \sqrt[rp]{\dfrac{x^r}{x^p}} \times \sqrt[pq]{\dfrac{x^p}{x^q}}$.

15. Simplify: $\left(\dfrac{x^a}{x^{-b}}\right)^{a^2+b^2-ab} \times \left(\dfrac{x^b}{x^{-c}}\right)^{b^2+c^2-bc} \times \left(\dfrac{x^c}{x^{-a}}\right)^{c^2+a^2-ac}$.

16. If $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$ and $abc = 1$, then prove that $x + y + z = 0$.

17. Show that $\dfrac{16 \times 2^{n-1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}} = \dfrac{1}{14}$.

18. If $2^x = 4^y = 8^z$ and $\dfrac{1}{2x} + \dfrac{1}{4y} + \dfrac{1}{4z} = 4$, find the value of x.

19. If $2^x = 3^y = 6^{-z}$ find the value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$.

20. Simplify: $\left(\dfrac{x^a}{x^b}\right)^{a+b} \times \left(\dfrac{x^b}{x^c}\right)^{b+c} \times \left(\dfrac{x^c}{x^a}\right)^{c+a} \div \dfrac{1}{x^2 a^2}$.

21. If $x^p = y^q = (xy)^{pq}$, show that $p + q = 1$.

22. If $3^x = 5^y = (75)^z$, show that $z = \dfrac{xy}{(2x + y)}$.

23.   (i) If $\dfrac{9^x \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then find the value of x.

     (ii) If $\left(\dfrac{3}{4}\right)^{2x-3} = \left(\dfrac{4}{3}\right)^{x-1}$, then find the value of x.

     (iii) Find the value of x, if $(\sqrt{6})^{2x+4} = 216$.

24. Simplify: $9^{-3} \times \dfrac{(16)^{1/4}}{(6)^{-2}} \times \left(\dfrac{1}{27}\right)^{-4/3}$.

25. Simplify:

   (i) $\left(\dfrac{15^{\frac{1}{3}}}{9^{\frac{1}{4}}}\right)^{-6}$ [CBSE 2011]   (ii) $\sqrt[3]{(343)^{-2}}$ [CBSE 2010]

26. If $9 \times (81)^x = \dfrac{1}{27^{x-3}}$, find x.

27. If $x + y + z = 0$, show that $a^{x^2 y^{-1} z^{-1}} \times a^{x^{-1} y^2 z^{-1}} \times a^{x^{-1} y^{-1} z^2} = a^3$.

Answers

1. (i) $1/125$ (ii) 36   (iii) $9/4$
2. (i) 2   (ii) $25/16$ (iii) $1/2$

3. 8

4. $\dfrac{2}{3}$

5. 1

6. $\dfrac{125c^2b^5}{16a^2}$

7. (i) $1/y^3$ (ii) 6

13. 1

14. 1

15. $x^{2(a^3+b^3+c^3)}$

18. $7/16$

19. 0

20. x^2a^2

23. (i) 3   (ii) $4/3$ (iii) 1

24. 8

25. (i) $\dfrac{3}{25}$ (ii) $\dfrac{1}{49}$

26. 1