Modeling of Passive Solar House in Boston, MA

Introduction

What is a passive solar home?

Passive solar houses are incredible. By definition passive solar houses they use energy from the sun and their surroundings to maintain a livable internal temperature. Everyone should be interested in building one as passive homes are much cheaper to heat and cool since they are designed in such a way to naturally maintain reasonable temperatures.

Passive solar homes are built on the principle of storing energy from the sun. At a base level they require:

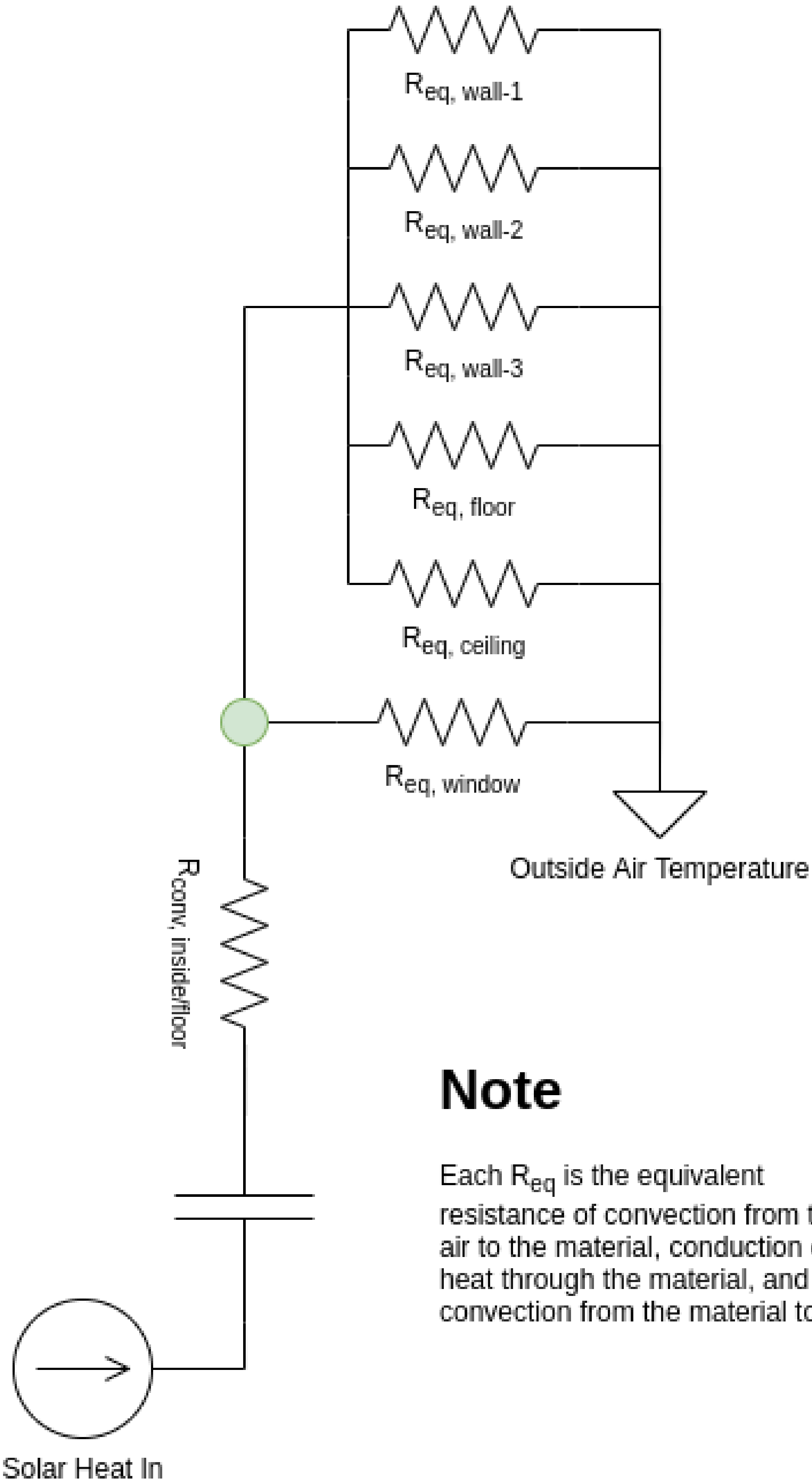
- A large thermal mass (like stone floors) to store energy from the sun
- Large south facing windows to collect sun, specifically in the winter months
- Some form of insulation to reduce heat loss
- Other controls/design choices to block sun in certain situations (eaves, trees, etc)

What's our strategy?

Designing a passive solar house for the Boston area where it gets very cold in the winter will require careful design of the house. In attempt to maintain a stable internal temperature, our first model/design will incorporate the following features:

- Extremely large south facing window
- Eaves that protect the windows/thermal mass during the summer
- A tile floor that act's as a large thermal mass
- No windows in any other walls
- A very tiny floorplan (~200sqft)
- Insulation in the ceiling, walls, and floor

The Model



Note

Each R_{eq} is the equivalent resistance of convection from the air to the material, conduction of heat through the material, and convection from the material to

Governing ODE's

The behavior of the various parts of the house can be modeled using a set of Ordinary Differential Equations. (ADD MORE EXPANATION ETC HERE)

This simple house makes use of just 2 ODEs. The first ODE describes the rate of change of the temperature of the floor (T_{floor}) in realtion to time. This value depends on the amount of energy coming into the floor via solar radiation (Q_{in}). The floor also loses energy to its surroundings through the walls. This is accounted for in the second part of the equation. This part takes into account the resistance from the floor to the outside air, as well as the temperature differential.

The inside air temperature can be modeled as a linear function of the floor temperature. THis is described by the second ode.

ODE describing rate of change of the Temp of the floor

$$\frac{dT_{floor}}{dt} = \frac{Q_{in}}{C_{floor}} - \frac{T_{floor} - T_{out}}{C_{floor} \cdot R_{total}}$$

ODE describing rate of change of the Temp of the air inside the house

$$\frac{dT_{inside\ air}}{dt} = \frac{R_{parallel}}{R_{total}} \cdot \frac{dT_{floor}}{dt}$$

Assumptions

Key assumptions allow the simplification of this complex system to the above equations. These include:

- Air doesn't enter or leave the house
- Insulation is purely a resistance, it doens't store any heat
- The only heat storage is in the large thermal mass
- Outside air is a constant temperature
- Radiation only hits the window and the heat storage unit
- Heat transfer and resistance is the same for all walls, floors, and ceilings
- Heat storage unit is at spatially uniform temperature

Parameters

Time

```
% Number of days to run the model
days = 14;

% Timespan for the model
time_span = [0:60*60*24*days];
```

Dimensions

```
% Internal dimensions of inside air
house.x = 5.0; % meters
house.y = 3.0; % meters
house.z = 5.0; % meters
```

```
house.sq_ft = house.x * house.z * 10.7639; % ft^2
house.eave = 0.9; % m % Length of the eave of the house

% Window
window.h = 0.7;
window.y = 2.6; % m;
window.z = house.z; % m % Runs the full length of the house
window.area = window.y * window.z; % m^2
window.y_offset = 0.2; % m % Height off the ground

% Floor
floor.x = house.x;
floor.z = house.z;
% PARAM %
floor.y = 0.05; % meters
floor.density = 3000.0; % kg/m^3
floor.spec_heat = 800.0; % J/kg*K
floor.mass = floor.x * floor.y * floor.z * floor.density; % kg
floor.T_0 = 0; % C
floor.sa = 2*floor.x*floor.y + 2*floor.x*floor.z + 2*floor.y*floor.z; % m^2 % Surface a

% Walls
house.ceiling_floor_area = house.z * house.x; % m^2
house.side_wall_area = house.x * house.y; % m^2
house.anti_window_wall_area = house.z * house.y; % m^2 % Wall opposite window
house.window_wall_area = house.anti_window_wall_area - (window.area); % m^2

% PARAM %
insulation.thickness = 0.05; % meters
insulation.k = 0.04; % W/m*K % Thermal conductivity constant
insulation.total_area = 2*house.ceiling_floor_area + 2*house.side_wall_area + house.ant
```

Air Temperature

```
outside_air.T = -3; % degrees celcius
outside_air.h = 30.0; % W/m^2*K

inside_air.T_0 = 0; % degrees celcius
inside_air.h = 15.0; % W/m^2*K
```

Heat

```
% W/m^2
solar_flux = -361 .* cos((pi.*time_span)./(12*3600)) + 224 .* cos((pi.*time_span)./(6*3
floor.heat_in = solar_flux .* window.area; % W
```

Resistances/Capacities

```
floor.heat_cap = floor.spec_heat * floor.mass; % J/K

% Resistance through all the insulation
R_air_wall = (inside_air.h * insulation.total_area)^-1; % Convection, K/W
R_wall = insulation.thickness / (insulation.k * insulation.total_area); % Conduction, K
R_wall_outside = (outside_air.h * insulation.total_area)^-1; % Convection, K/W
```

```
R_wall_total = R_air_wall + R_wall + R_wall_outside; % K/W

% Resistance through the window
R_air_window = (inside_air.h * window.area)^-1; % Convection, K/W
R_window = (window.h * window.area)^-1; % Conduction, K/W
R_window_outside = (outside_air.h * window.area)^-1; % Convection, K/W
R_window_total = R_air_window + R_window + R_window_outside; % K/W

R_loss = (sum([R_wall_total R_window_total].^-1))^-1;

R_floor_air = (inside_air.h*floor.sa)^-1; % Resistance from floor to inside air

R_total = R_floor_air + R_loss; % Total resistance from floor to outside air
```

ODEs

ODE describing rate of change of the Temp of the floor

$$\frac{dT_{floor}}{dt} = \frac{Q_{in}}{C_{floor}} - \frac{T_{floor} - T_{out}}{C_{floor} \cdot R_{total}}$$

ODE describing rate of change of the Temp of the air inside the house

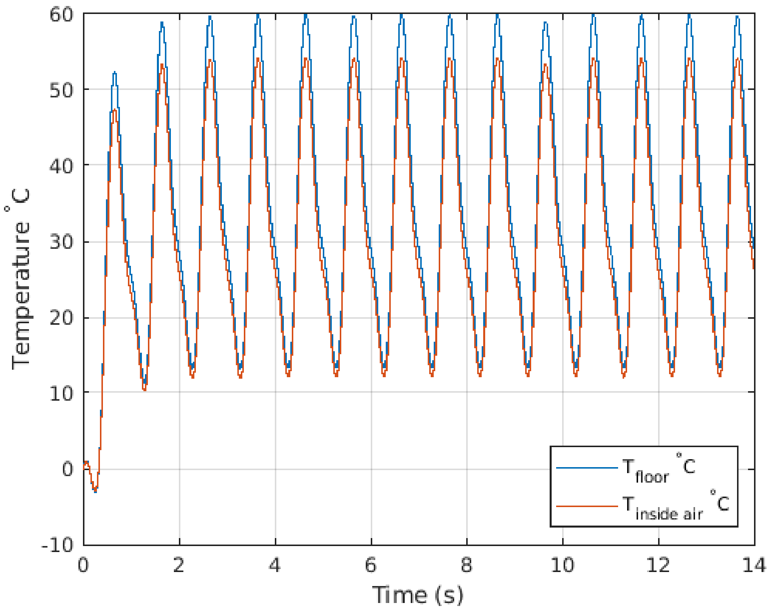
$$\frac{dT_{inside\ \ air}}{dt} = \frac{R_{parallel}}{R_{total}} \cdot \frac{dT_{floor}}{dt}$$

```
Qin = @(t) (-361 * cos((pi*t)/(12*3600)) + 224 * cos((pi*t)/(6*3600)) + 210) * window.a

f = @(t,T) [...
    Qin(t)/floor.heat_cap - (T(1)-outside_air.T)/(R_total*floor.heat_cap);...
    (R_loss/R_total) * (Qin(t)/floor.heat_cap - (T(1)-outside_air.T)/(R_total*floor.hea
];
[t,T_sol] = ode45(f,time_span,[0 0]);
```

Graphed,

```
plot(t/(60*60*24), T_sol(:,1), "DisplayName", "T_{floor} ^{\circ}C");
hold on;
grid on; legend("Location", "Southeast");
xlabel("Time (s)"); ylabel("Temperature ^{\circ}C");
plot(t/(60*60*24), T_sol(:,2), "DisplayName", "T_{inside air} ^{\circ}C");
hold off;
```


60
50
40
30
20
10
0
-10
Temperature °C
0
2
4
6
8
10
12
14
Time (s)
T_{floor} °C
$T_{inside\ air}$ °C