

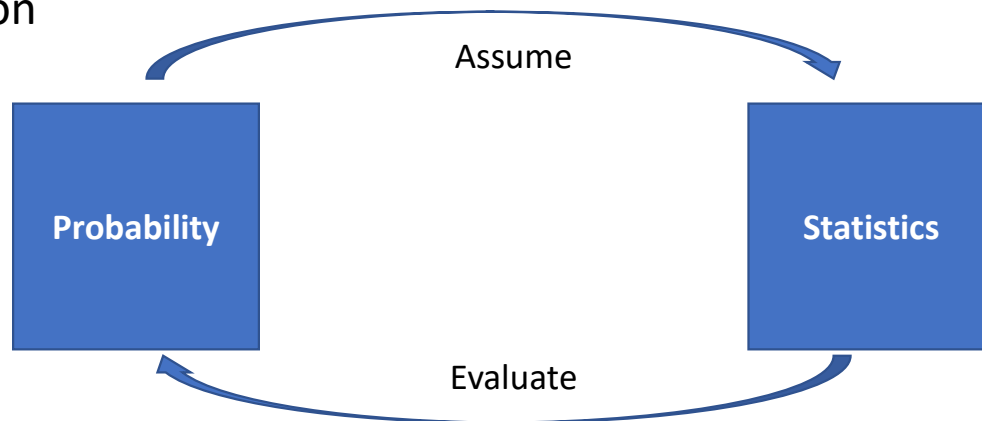
LS 2 Preliminaries To RVs And Distributions

Review Important Laws And Results

- Law of Additive Probability
- Law of Total Probability
- Law of Multiplicative Probability
- Conditional Probability
- Bayes Rule
- Law of Complementary Probability

The Interplay – Probability And Statistics

- Probability and statistics are closely related fields of study
- Use much of the terminology, often no distinction is noted
- But the difference hinges on knowledge
- In probability the tuple (Ω, \mathcal{F}, P) is known
- In statistics we assume a probability model is true make assertions and perform analyses to support or reject the notion



Transitioning To Probability Rules Or Laws And Random Variables

- Up to now we have talked about events in qualitative manner
 - Getting a disease, liking stats, getting a heads
 - Probability has been given to us
- Transitioning in a more general model of the assignment of probability with more general events
- Need a new concept or function introduce the concept of a Random Variables (RV)
 - Definition a RV is a function which maps the outcome of a random experiment (an outcome which is a priori not known with certainty) to the real number line.
- Interestingly a Random Variable is neither random nor a variable.

Notation

- Capital latin letters (X , Y are popular ones) are random variables
- Lower x or y are realization or specific values on the RNL
- So
- $f_X(X=x)$ or $f_X(x) \equiv$ probability density function (pdf) (or probability mass function) which describes the relative likelihood of realization of X being x
- $F_X(X \leq x) \equiv$ Cumulative density function (CDF) or the probability that the realization of X is less than or equal x

RVs and Discrete Distributions

- Concept of Random Variables (RV's)
 - A function that maps the outcome of a random experiment to the real number line
- Functions of RV's are RV's
- Discrete distributions: Outcome space is countable infinite or finite and discrete
- Mapping descriptions RV's versus that of probability
- $X = \begin{cases} 0 & \text{if failure} \\ 1 & \text{if success} \end{cases}, \quad P_X(X=x) = \begin{cases} 0.3, & x=0 \\ 0.7, & x=1 \end{cases}$
- Notation
- $f_X(X=x)$ or $f_X(x) \equiv$ [redacted] probability mass function (PMF)
- $F_X(X \leq x) \equiv$ Cumulative density function (CDF) or the probability that the realization of X is less than or equal x
- Functions of probability laws in, e.g. pbinom, dbinom, rbinom, qbinom
- Simulation versus closed form solutions

Continuous Distributions

- Second major dist'n type (there are hybrids)
- The outcome space for RV is entire real number line.
- Consequences for the way we think about prob under such circumstances
 - Not countable
 - Density exists at every value of random variable, probability of value is 0.
 - Non zero probability only defined over an interval not at a single value
 - Integration rather than summation to compute probability