Proof

$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

$Var(X-Y) = Var(X) + Var(Y) - 2Cov(X,Y)$

$Var(X \overset{(+)}{-} Y)$

$= E[((X \overset{(+)}{-} Y) - E[X \overset{(+)}{-} Y])^2]$

$= E[(X \overset{(+)}{-} Y)^2 - 2(X \overset{(+)}{-} Y)E[(X \overset{(+)}{-} Y)]$

$+ E^2[(X \overset{(+)}{-} Y)]]$ (1)

$\{$ where $E^2[(X \overset{(+)}{-} Y)] = (E[(X \overset{(+)}{-} Y)])^2 \}$

Linear operation & Combining Terms $= E[(X \overset{(+)}{-} Y)^2] - \underbrace{E^2[(X \overset{(+)}{-} Y)]}_{A}$

$A = -(E[(X \overset{(+)}{-} Y)])^2$

Alternative note $= -(E[X] \overset{(+)}{-} E[Y])^2$

Expanding $= -(E[X])^2 - (E[Y])^2$

$\overset{(-)}{+} 2E[X]E[Y]$

Restating

$(1) = E[X^2] + E[Y^2] \overset{(+)}{-} 2E[XY]$

$- E^2[X] - E^2[Y] \overset{(-)}{+} 2E[X]E[Y]$

Combining Terms

$= Var(X) + Var(Y) \overset{(+)}{-} 2(E[XY]$

$-E[X]E[Y])$

==> From the top

$Var(X \overset{(+)}{-} Y) = Var(X) + Var(Y) \overset{(+)}{-} 2Cov(X,Y)$