Moment assumptions for $E[\epsilon]$ and $Cov(X, \epsilon)$

In linear regression we have assumptions that the expected value of the error term ϵ is zero, $E[\epsilon]=0$ and that the covariance between the predictors X and the error term is zero, $Cov(X,\epsilon)=0$ These assumptions are closely related to the moments of the error distribution, specifically the first and second moments.

- $E[\epsilon]=0$ -- The expected value represents the first moment of a distribution, is its central tendency mean. $E[\epsilon]=0$ implies that the error term has a mean of zero, ➔ on average, the model's predictions are unbiased.
- $Cov(X,\epsilon)=0$ ➔ there is no linear between the error and the (any) predictor. Without this the model would have some endogenous error variation suggesting that the model has system bias and is omitting some explanatory information (model possesses Omitted Variable Bias [OVB]), leading to biased and inconsistent estimates of the regression coefficients. The existence of the covariance requires a the second moment be finite, i.e. both $E[X^2]$ and $E[\epsilon^2]$ must be finite. Recall $Var(X)=Cov(X,X)=E[X^2]-E^2[X]$.
- The variance is the second central moment $E[(X-E[X])^2]$
- Note that we all have that X and ϵ are independent so it should be clear the $E[\epsilon|X] = E[\epsilon] = 0$

I will leave the discussion of third and fourth moments of ϵ for later.