

Day At the Farm

RV: $T \triangleq$ wta of a tomato

RV: $P \triangleq$ " " " " pumpkin

RV: $W \triangleq$ amt of rainfall

Given

$$W \sim U(0,6) \quad \text{Var}(W) = \frac{1}{12} (6)^2 = 3$$

$$E[W] = 6/2 = 3$$

$$T = .1 + W + U_T$$

$$U_T \sim N(0, 7/3)$$

$$U_T \perp W \Rightarrow \text{Cov}(U_T, W) = 0$$

$$P = 10 + 2 \cdot W + U_P$$

$$U_P \sim N(0, 12)$$

$$U_P \perp W \Rightarrow \text{Cov}(U_P, W) = 0$$

VarCov(W, T)

$$\begin{bmatrix} \frac{1}{12} (6^2) = 3 \\ \\ 3 \end{bmatrix}$$

$$\text{Cov}(W, .1 + W + U_T)$$

$$= \text{Cov}(W, .1) + \text{Cov}(W, W)$$

$$+ \text{Cov}(W, U_T) =$$

$$0 + 3 + 0 = 3$$

$$= \text{Var}(.1 + W + U_T)$$

$$= 0 + 3 + 7/3$$

$$= 16/3$$

$$\begin{bmatrix} 3 & 3 \\ 3 & 16/3 \end{bmatrix}$$

VarCov(W, P)

$$\begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\begin{aligned} \text{Cov}(W, P) &= \text{Cov}(W, 10 + 2W + U_P) \\ &= 0 + 2 \cdot 3 + 0 = 6 \end{aligned}$$

$$\begin{aligned} \text{Var}(P) &= V(10 + 2W + U_P) \\ &= 0 + 4 \cdot 3 + 12 \\ &= 24 \end{aligned}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 24 \end{bmatrix}$$

3 Which fruit has larger covariance
 $\text{Cov}(T, W) = 3$, $\text{Cov}(P, W) = 6$

4. Rescale to Correlation

$$\text{Tom: } \rho_{T, W} = 3 / \sqrt{3 \cdot 16/3} = 0.75$$

$$\begin{aligned} \text{Pump: } \rho_{P, W} &= \frac{6}{\sqrt{3 \cdot 24}} = \frac{6}{\sqrt{36 \cdot 2}} = \frac{1}{\sqrt{2}} \\ &= 0.707 \end{aligned}$$