Proof of Variance

$E[(X-E[X])^2]$

$= E[X^2 - 2XE[X] + E^2[X]]$

$= E[X^2] - 2E^2[X] + E^2[X]$

$= E[X^2] - E^2[X]$

if Proof of Covariance

$E[(X-E[X])(Y-E[Y])]$

$= E[XY - XE[Y] - YE[X] + E[X]E[Y]]$

$= E[XY] - 2E[Y]E[X] + E[X]E[Y]$

$= E[XY] - E[Y]E[X]$

Interesting Additional Results

$$E[X] \triangleq c, \quad E[c] = c$$

$$E[aX+b] = E[aX] + E[b] =$$
$$a\,E[X] + b$$

$$Var(aX+b) = a^2\, Var(X) + 0$$

$$Var(\text{constant}) = 0$$

$$Var(-X) = Var(-1 * X)$$
$$= (-1)^2\, Var(X) = Var(X)$$

$$Var(X+Y) = E[((X+Y) - E[X+Y])^2]$$
= by a number steps
$$= Var(X) + Var(Y) + 2\,Cov(X,Y)$$
and results from above
$$Var(X-Y) = Var(X) + Var(Y)$$
$$-2\,Cov(X,Y)$$

$$Cov(aX, bY) = ab\,E[X \cdot Y]$$
$$- a\,E[X] \cdot b\,E[Y]$$
$$\Rightarrow Cov(X,X) = Var(X)$$

$$Cov(a,Y) = 0$$

$$Corr(X,Y) = \rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X), Var(Y)}}$$

Proof

$$Var(X+Y) =$$

$$E\left[\left((X+Y) - E[(X+Y)]\right)^2\right]$$

$$= E\left[(X+Y)^2 - 2(X+Y)E[(X+Y)] + E^2[(X+Y)]\right]$$

where $\left(E[(X+Y)]\right)^2 \triangleq E^2[(X+Y)]$

$$= E[X^2 + Y^2 + 2XY] - \underbrace{E^2[(X+Y)]}_{(E[X])^2 + (E[Y])^2 + 2E[X]E[Y]}$$

The rearranging term & linearity of Expectation

We

$$= Var(X) + Var(Y) + 2\,Cov(X,Y)$$