Proof 1 LS 3

Let $X \sim U(0,1)$ $f_Y(y)$

$Y = X^2$

$\Rightarrow f_X(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{ow} \end{cases}$

$F_X(x) = \dfrac{x-a}{b-a} = x$ if $a=0, b=1$

Note X^2 is a invertible fcn over support

$f_Y(y)$ has a support $(0 \leq y \leq 1)$

So we have

$F_Y(y) \triangleq P(Y \leq y) = P(X^2 \leq y) = P(X \leq \sqrt{y})$

which from above $= \dfrac{\sqrt{y}-0}{1-0} = \sqrt{y}$

find $f_Y(y)$:

We have from above

$F_Y(y) = F_X(X \leq \sqrt{y})$

$\dfrac{\partial F_Y(y)}{\partial y} = \dfrac{\partial F_X(\sqrt{y})}{\partial y} =$

$f_Y(y) = f_X(\sqrt{y}) \dfrac{\partial x}{\partial y}$ by chain rule

$= f_X(\sqrt{y}) \dfrac{\partial \sqrt{y}}{\partial y}$

$= 1 \cdot \dfrac{1}{2\sqrt{y}} = \begin{cases} \dfrac{1}{2\sqrt{y}} & 0 < y \leq 1 \\ 0 & \text{ow} \end{cases}$

$E[Y]$

$= \int_0^1 y \frac{1}{2} y^{-1/2} dy = \int_0^1 \frac{y^{1/2}}{2} dy =$

$\frac{1}{3} y^{3/2} \Big|_0^1 = \frac{1}{3}$

check $\int_0^1 \frac{1}{2\sqrt{y}} dy = \frac{2}{1}\frac{1}{2} y^{\frac{1}{2}} = y \Big|_0^1 = 1$

Proof 2 LS 3

Start w/ the following result proved in HW 2

if X is a continuous RV with pdf $f_X(x)$ let h be invertible function on X where h^{-1} is define and differentiable. Define $Y = h(X)$ which is also a continuous RV

Let $g_Y(y)$ be the pdf of Y

Then $g_Y(y) = f_x(h^{-1}(y)) \cdot \left| \frac{d}{dy} h^{-1}(y) \right|$

Proof 2

Let $Y = X^2$ $X \sim U(0,1)$ so $X = \begin{cases} 1 & 0<x<1 \\ 0 & \text{(w)} \end{cases}$

note $h(x) = X^2$, X^2 is continuous and over the support $h^{-1}(y)$ is defined as $\sqrt{y}$ or positive sqrt root of y because of the definition of the support and $\frac{\partial h^{-1}(y)}{dy} = \frac{\partial \sqrt{y}}{dy} = \frac{1}{2} y^{-1/2}$ and $f_Y(y) = 1 \cdot \frac{1}{2} y^{-1/2}$ value

Using above result and substituting

$E[Y] = \int_0^1 y\, f_Y(y)\, dy = \int_0^1 y \cdot 1 \cdot \frac{1}{2} y^{-1/2}\, dy =$

$\int_0^1 \frac{1}{2} y^{1/2} dy = \frac{1}{3} y^{3/2} \Big|_{y=0}^{1} = \frac{1}{3} - 0 = 1/3$

Proof 3

Law of Unthinking (Uninformed, unconcerned, unconscious) Statistician

pdf of X is known $== f_X(x)$

$g(x) \triangleq Y$ but $f_Y(y)$ is unknown

$$E[Y] = \sum_x g(x) f_X(x)$$

$$E[Y] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

- Note that the $\int$ or Σ can be computed only over range or points for which $f_X(x) > 0$
- The function $g(X)$ must be well behaved and integrable or summable over the supp of X