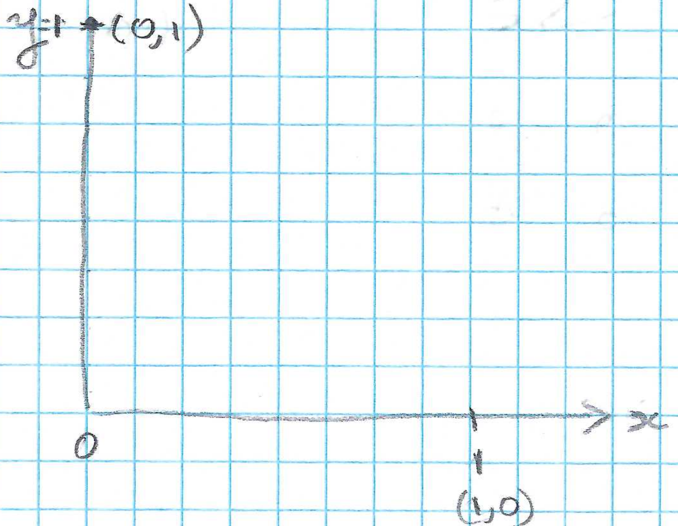
LSZ

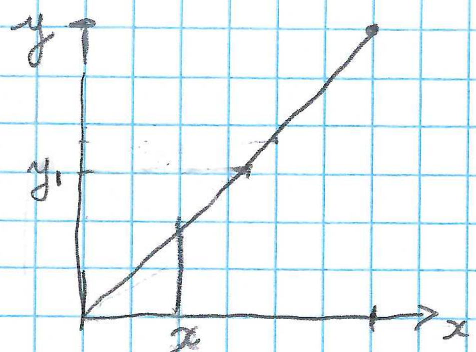
$$f_{X,Y}(x,y) = \begin{cases} c & 0 \leq x \leq 1,\ 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$



Given an x what is the limits of integration for y inttersecting with the supp

find C

How ?



given an x area of integration goes from 0 to $x = y$

$$1 = \int_0^1 \int_0^x c\, dy\, dx = c \int_0^1 \int_0^x 1\, dy\, dx$$

$$= c \int_0^1 y \Big|_{y=0}^{x} dx = c \int_0^1 x\, dx$$

$$= c \frac{x^2}{2} \Big|_{x=0}^{1} = \frac{c}{2} = 1$$

$$\Rightarrow c = 2$$

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 \le y \le x \le 1 \\ w & \end{cases}$$

$$f_X(x) = ?$$

Need to integrate out "nuisance" var

$$\Rightarrow \quad f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$= \int_{-\infty}^{0} f_{X,Y}(x,y) + \int_{0}^{x} f_{X,Y}(x,y)\,dy$$

$$+ \int_{x}^{\infty} f_{X,Y}(x,y)\,dy$$

$$= 0 + \int_{0}^{x} 2\,dy + 0 = 2y\Big|_{y=0}^{x} = 2x$$

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ w & \end{cases}$$

$$f_{Y|X}(y|x) \quad \text{for } x \in [0,1]$$

$$= f_{X,Y}(x,y) \,/\, f_X(x)$$

if $0 \le y \le x$

$$= 2/2x = \frac{1}{x}$$

$$= f_{Y|X}(y|x) = \begin{cases} 1/x & 0 < y \le x \\ 0 & \text{ow} \end{cases}$$

Given a y what is the x limits
of integration intersecting with the supp

Bonus $\quad f_Y(y) = ?$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$$

$$= \int_y^1 2\,dx$$

$$= 2x \Big|_{x=y}^{1} = 2 - 2y$$