Further Explanation Of t* and Z*

We were running out time when I inadequately answered a question about the Z* and t* transformation and their sampling distribution of normal(0,1) and t(n-1) under the H_0. There is a lot of logic and results which we covered and that we put together here to understand this.

First we recall that all functions of RVs are RVs and in the context we are speaking of ALL RVs have a pdf or pmf. We can apply this statement multiple times in that functions of functions of RVs are RVs

Second recall that through a combination of Chebyshev, the Law of Large Numbers, the definition of convergence, we arrived at the result that states given an $\{X_i\}$ IID random sample (RS) of size n whose underlying RV, X has finite E[] = μ and a Var() = σ^2 , the mean of that RS defined by

$$Xbar = (\sum_{i=1}^{n} Xi)/n$$

which is a RV, converges in distribution to a normal distribution with a mean μ and Var σ^2/n. So now let's form a another RV called Z* =

$$(Xbar - \mu_1)/\sqrt{(\sigma\text{^}2/n)}$$

If we assert that $E[Xbar] = \mu_1$, and if that is true then Z* as a RV has a pdf which converges in distribution to a normal distribution with mean = 0 and σ^2 = 1, to see this assume the CLT and compute E[Z*] and Var(Z*). Which is our sampling distribution for our hypothesis test under H_0. If our assertion is incorrect than Z* will asymptotically follow some other normal with a mean whose absolute value > 0, which will make our realization likely to fall outside of the Upper and Lower critical quantities with a higher probability than the 1 - Confidence Level we set.

However these results require us to have knowledge of what is the parameter σ^2 which we never do. So using an estimator of σ^2 which we notate $s^2 = (\sum_{i=1}^{n}(Xi - Xbar)\text{^}2)/(n-1)$ we can show that

$$t^* = (Xbar - \mu_1)/\sqrt{(s\text{^}2/n)}$$

where t* is a RV which, under the same assertion $E[Xbar] = \mu_1$ being true, has a pdf which converges in distribution to t with n-1 degrees of freedom. If $E[Xbar]! = \mu_1$ or the assertion is not true, then the distribution is other than t(n-1). In this case $Xbar$ is a biased estimator of μ_1, the distribution of the statistic changes. In such cases, the statistic no longer follows a t-distribution centered at zero. Instead, it follows a noncentral t-distribution, which accounts for the bias in the estimation. Once again this will make our realization likely to fall outside of the Upper and Lower critical t(n-1) quantities with a higher probability than the 1 - Confidence Level one we set.