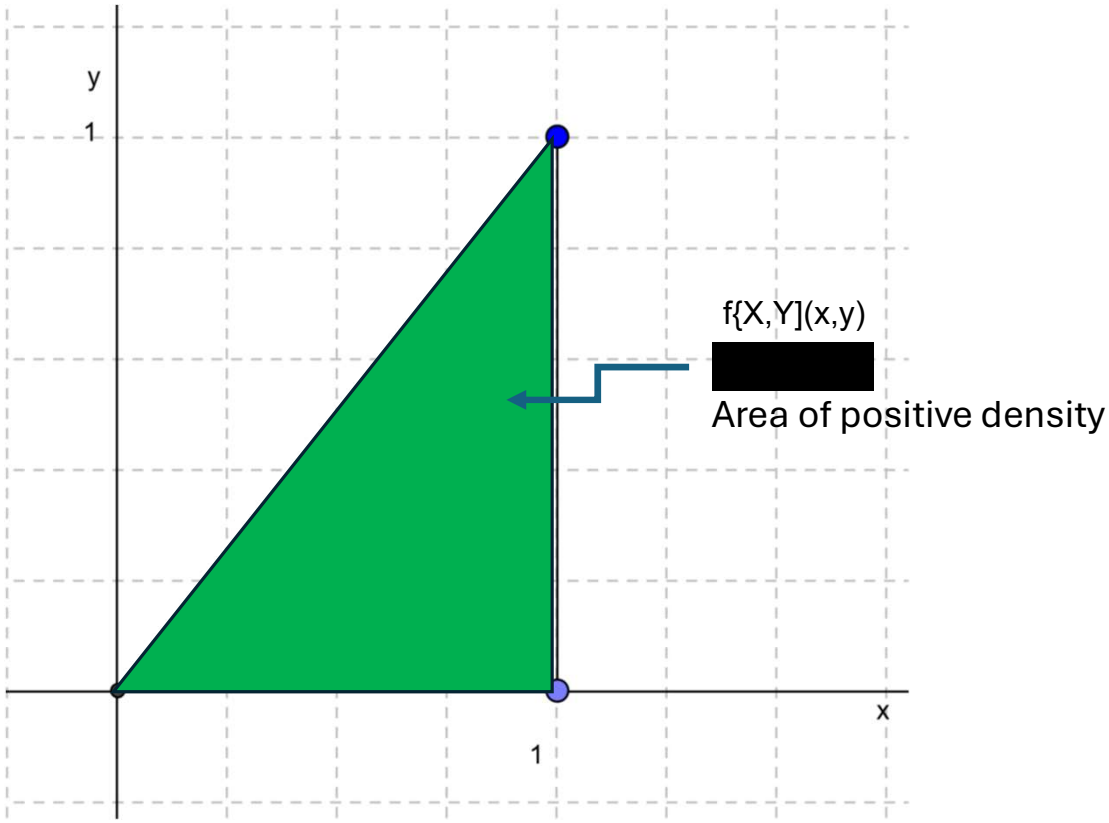
Limits of Integration

Suppose that random variables X and Y are jointly continuous, with joint density function given by,

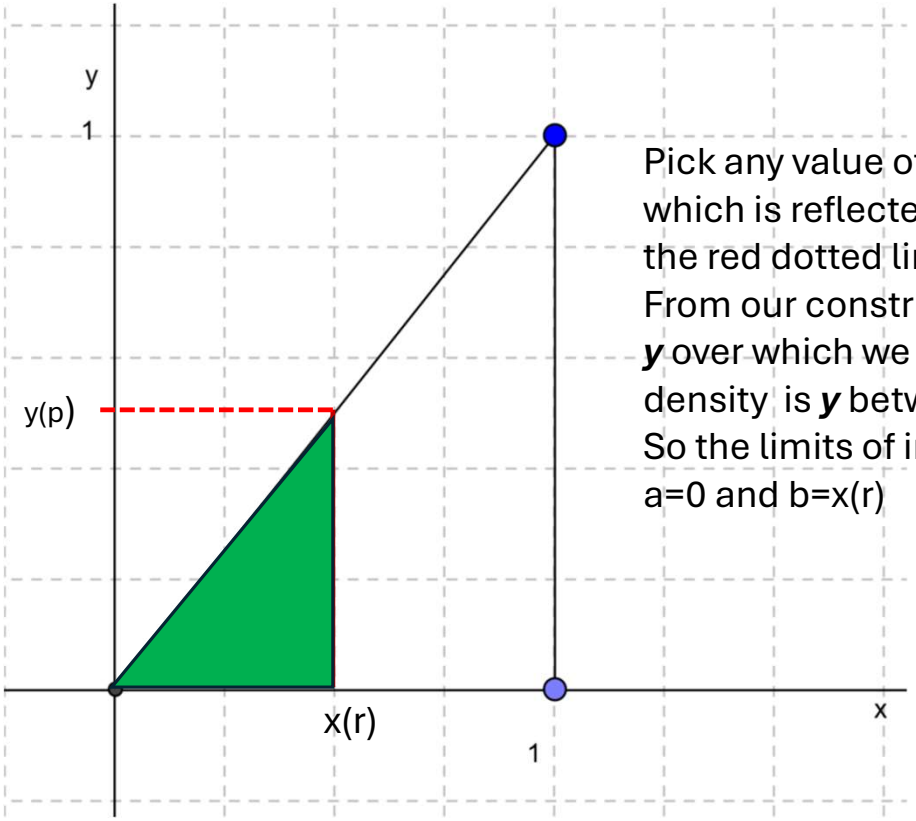
$$f(x, y) = \begin{cases} 2, & 0 \le x \le 1,\ 0 \le y \le x \ \ == 0 \le y \le x \le 1 \\ 0, & \textit{otherwise} \end{cases}$$



So for computing $f_x(x)$, we must get rid of the nuisance variable ***Y***

$$f_x(x) = \int_a^b f_{Yx}(x,y)dy = \int_a^b 2\,dy$$

Okay so what are ***a*** and ***b*** ?

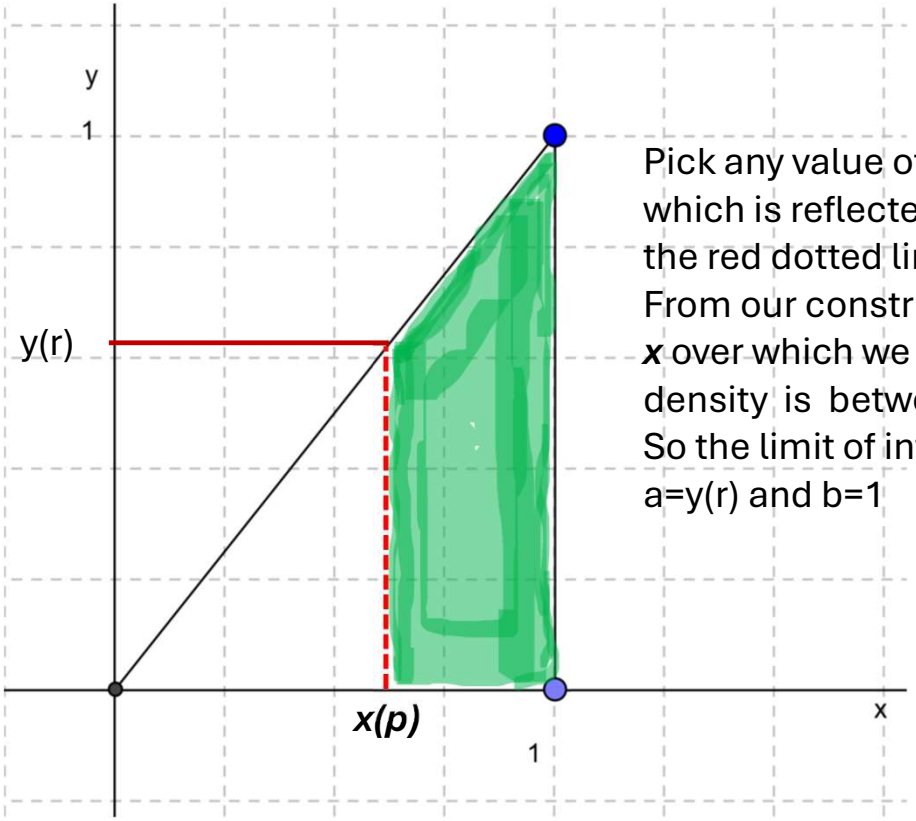


Pick any value of ***x(r)*** between [0,1] (red solid line), which is reflected through the 45 degree line and the red dotted line to a specific ***y(p).***
From our construct we see that the only values of ***y*** over which we can integrate and include positive density is ***y*** between [0,x(r)]
So the limits of integration here are y={0,x(r)} or a=0 and b=x(r)

In a like vein for computing $f_Y(y)$, we must get rid of the nuisance variable ***X***

$$f_Y(y) = \int_a^b f_{YX}(x,y)dx \quad = \int_a^b 2\,dx$$

Okay so what are ***a*** and ***b*** ?



Pick any value of ***y(r)*** between [0,1] (red solid line), which is reflected through the 45 degree line and the red dotted line to a specific ***x(p).***
From our construct we see that the only values of ***x*** over which we can integrate and include positive density is between [y(r),1]
So the limit of integration here are x={y(r),1} or a=y(r) and b=1