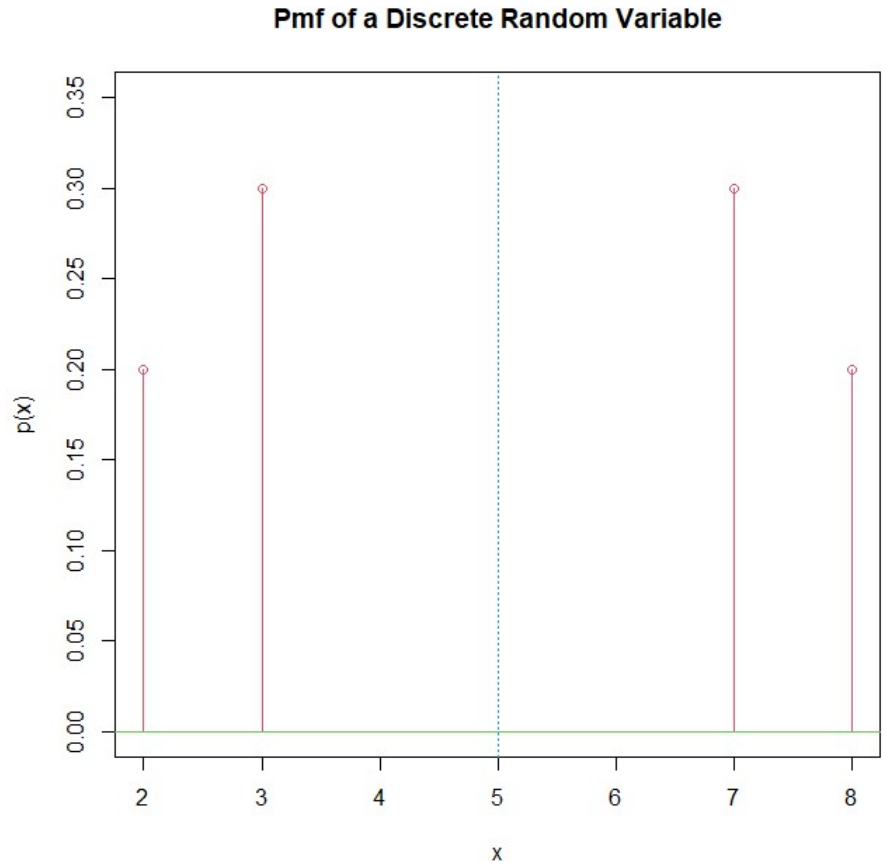
The expectation of a symmetric variable, X and the power of linearity



The expectation?

Note this is a symmetric distribution, i.e. a pdf (or pmf) mirrored through a line/point of symmetric, in this circumstance the values

Proof that for a symmetric distribution the point of symmetric
is the expected value of the random variable

We use transformations and the linearity of the expected value operator

define Y = X - a

Y + a = X

E[Y + a] = E[X]

let a = 5 or the point of symettry

Need to show that E[Y] = 0 and we have E[X] = a = 5

let p() be the pmf for X and g() be the pmf for Y

some the tranformation yields

if x = 2; y = -3 p(x = 2) = g(y = -3) = 0.2

x = 3; y = -2 p(x = 3) = g(y = -2) = 0.3

x = 7; y = 2 p(x = 7) = g(y = 2) = 0.3

x = 8; y = 3 p(x = 8) = g(y = 3) = 0.2

Note symmetry about 0 that is g(y) = g(-y) ==>

Y has the same distribution as -Y ==>

E[Y] = E[-Y] = -E[Y] ==> all are 0

From definition E[X] = E[Y] + 5 = 5 or a