LS 8 Moment Results In Regression

The term moments arose from physics and te tendency of forces creating rotation about a point or axis. The term was introduced by Karl Pearson into statistics

Results

- **Cov(X,e)=0**

Where X is a predictor and $e = y - X\beta$

$cov(X,e)=E[(X-E(X))(e-E(e))]$ (by substitution)

$=E[(X-E(X))((y-X\beta)-E(y-X\beta))]$ (and then by expanding)

0

$= E[Xy]-E[X^2\beta]-E(Xy)+E(X^2\beta)]$

$= 0$

- **E[e|X]=0**

By the law of iterated expectations: $E[e|X]=E[E[Y|X]|X]$

Given that $e=E[Y|X]$ this simplifies to: $E[e|X]=E[e|X]$

This implies that $E[e|X]$ is a constant, and since it's conditioned on X, it must be equal to its expectation, which is 0.

Results (Cont'd)

- **E[e]=0**

This follows directly from the previous result. Since E[e|X]=0 for any value of X, then E[e]=0

- **If g is a function of X, then Cov(g(X),e)=0**

Using the law of iterated expectations and the properties of conditional expectations:
Cov(g(X),e)=E[g(X)e]−E[g(X)]E[e]
Since E[e]=0, this simplifies to: Cov(g(X),e)=E[g(X)e]
Now, notice that e=E[Y|X], so: E[g(X)e]=E[g(X)E[Y|X]]
By the law of iterated expectations: E[g(X)e]=E[E[g(X)Y|X]]
Since g(X) is a function of X, it is constant with respect to X, and therefore independent of Y given X. Hence, we can factor it out of the conditional expectation: E[g(X)e]=E[g(X)]E[Y|X]
Given that E[Y|X]=e this further simplifies to: E[g(X)e]=E[g(X)]e
Finally, since e is a constant (a function of X), its expectation is itself: E[g(X)e]=eE[g(X)]
Therefore, Cov(g(X),e)= 0

Results (Cont'd)

- **V(e|X)=V(Y|X)**

By the definition of conditional variance: $V(e|X)=E[(e-E[e|X])^2|X]$
Since $E[e|X]=0$, this simplifies to: $V(e|X)=E[e^2|X]$
Now, notice that $e=E[Y|X]$ so: $V(e|X)=E[E[Y^2|X]|X]$
Again, by the law of iterated expectations: $V(e|X)=E[Y^2|X]$
This is exactly the definition of conditional variance $V(Y|X)$

- **V(e)=E[V(Y|X)]**

By the law of total variance: $V(e)=E[V(e|X)]+V(E[e|X])$
Since $V(E[e|X])=0$ (it's a constant), $V(e)=E[V(e|X)]$
Using the result from point 4, we have: $V(e)=E[V(Y|X)]$