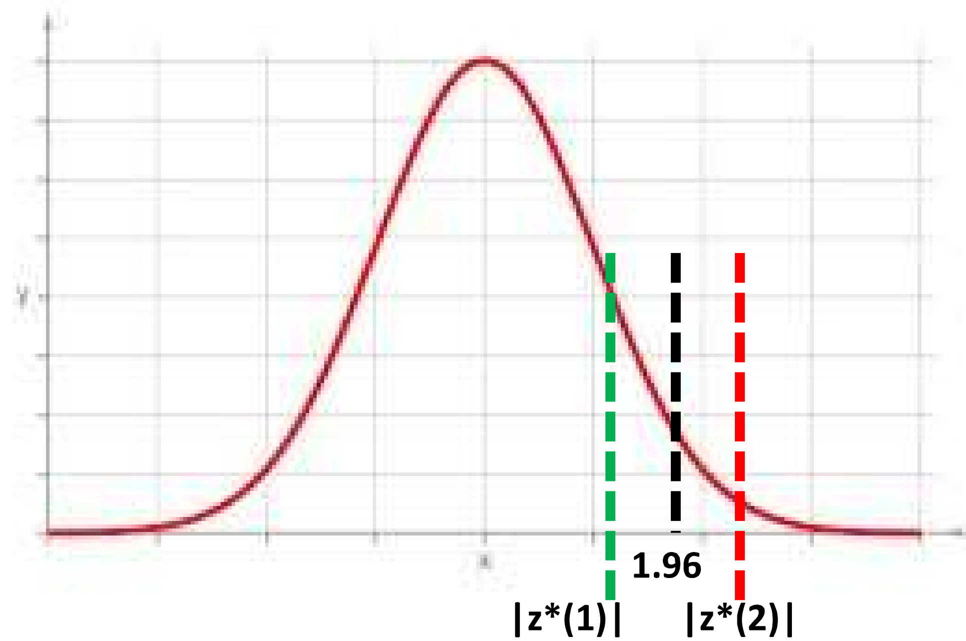
# Towards Hypothesis Testing

# CI & Critical Values (Z Score)

- P(-1.96<=Z*<=1.96) = 0.95
- P(-1.96<= (Xbar-mu)/(sigma/n^0.5) <= 1.96) =0.95
- = P(se*-1.96<= Xbar-mu <= se * 1.96)
- = P(-Xbar – se*1.96<= -mu <= -Xbar + se*1.96)
- = P((Xbar + se*1.96 => mu => Xbar - se*1.96)




If α is set at 0.05 ➔ α/2 =0.025
P(Z>1.96) = 0.025
P(Z>|z*(1)|) > P(Z>1.96) ➔ Fail to reject
P(Z>|z*(2)|) < P(Z>1.96) ➔ Reject|

# Alternative Random Sample Strategies

Let Xbar(1), Xbar(2), …,Xbar(5) be defined in the regular way as the sum of random variables (RVs) divided by the number of RVs in the sample. Each Xbar is formed from an independent random sample (from a very large population) of the RV X which has a finite mean mu and finite variance sigma^2. Without loss of generality lets assume that each Xbar was computed with a random sample with same number of samples in each or n=n(1),n(2),…,n(5).

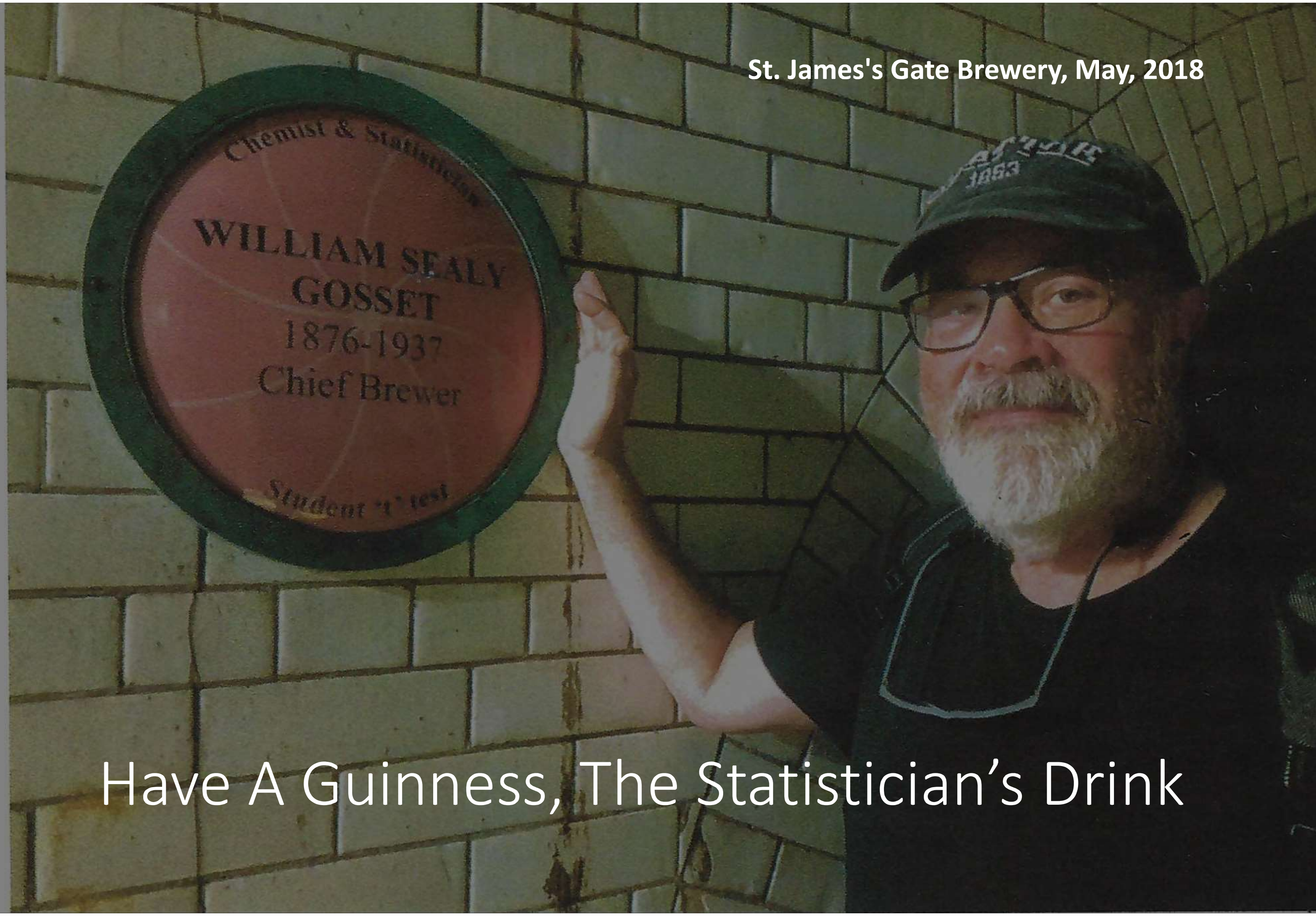
We can prove that each of these Xbar(i), which are RVs, have a mean of mu but a variance of sigma^2/n. Let's suppose we form another RV called Xdoublebar which is the mean of these Xbars.

Let's apply the CLT to Xdoublebar. It asymptotically approach the norm with a mean of mu and a variance of sigma^2/n/5 = sigma^2/(5*n)

Alternatively let's assume that we collect one random sample of the RV X (defined as above) of size 5*n and compute Xbar(all). Note under the CLT, Xbar(all) asymptotically approaches a normal with a mean of mu and a variance of sigma^2/(5*n). Exactly the same result as the alternative sampling strategy

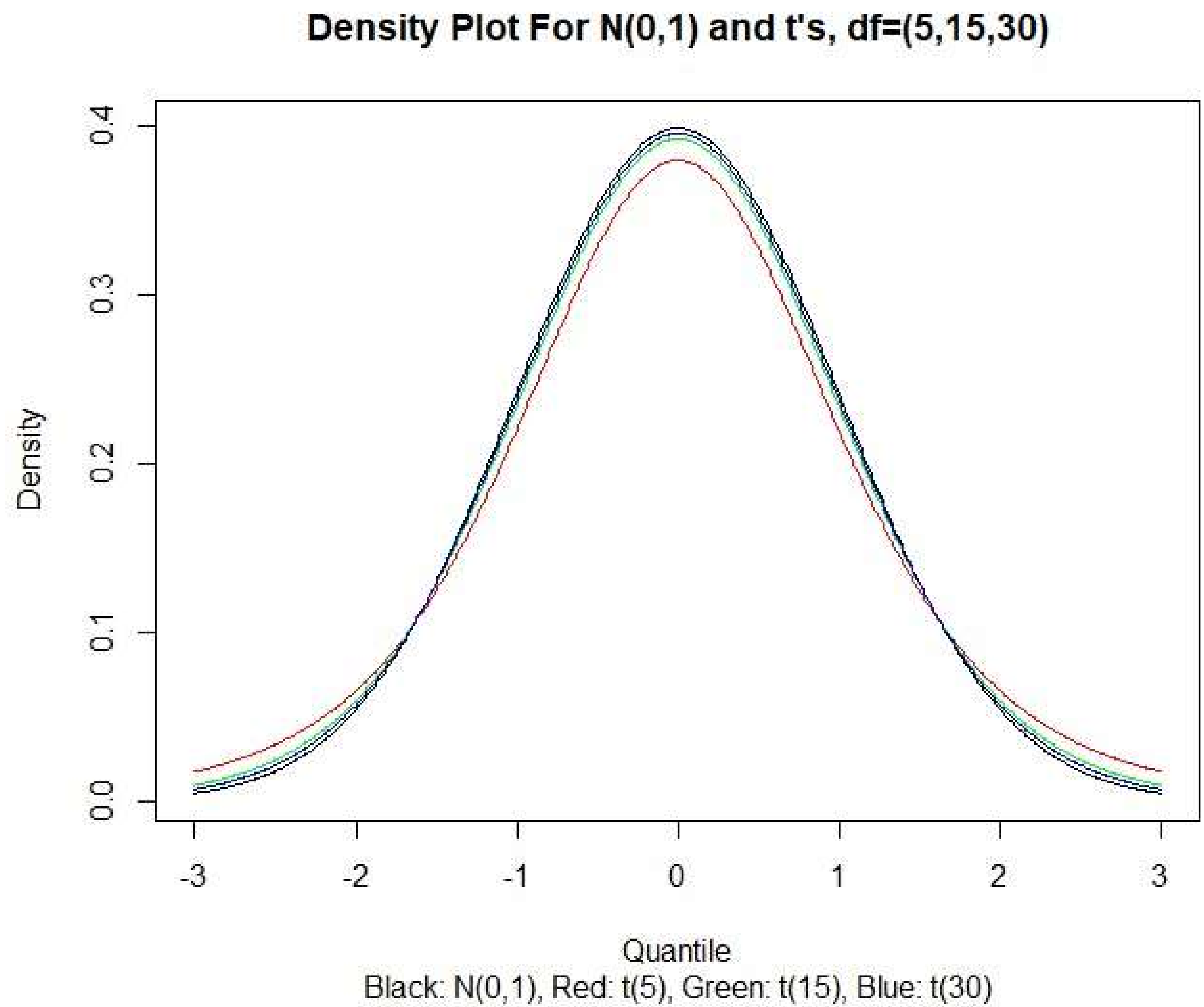
# William Sealy Gosset & Student's t (from https://en.wikipedia.org/wiki/Student's_t-distribution

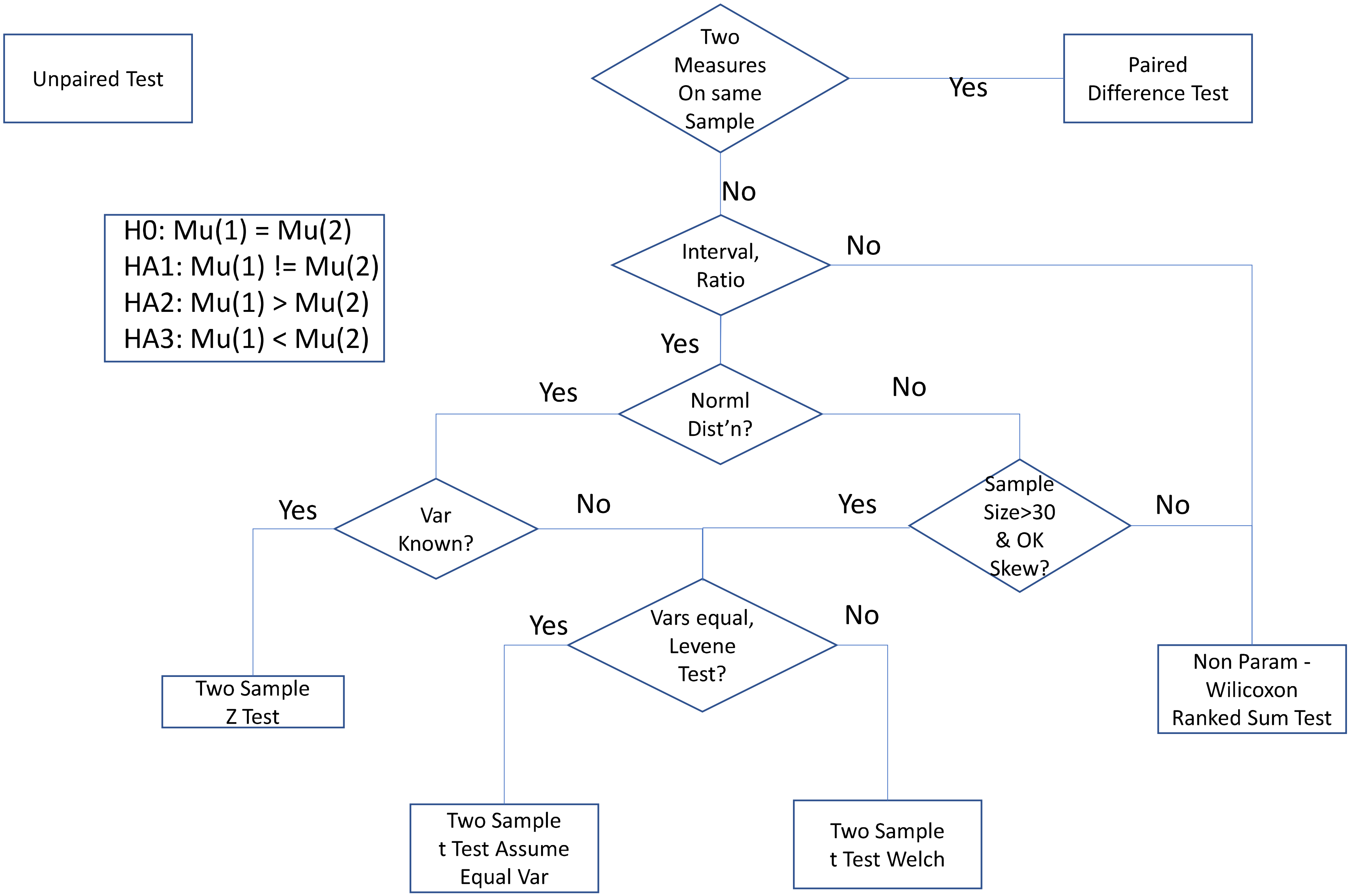
- **Student's *t*-distribution** (or simply the ***t*-distribution**) family of continuous probability distributions that arise when estimating the mean of a normally-distributed population in situations where the sample size is small and the population's standard deviation is unknown. It was developed by English statistician William Sealy Gosset under the pseudonym "Student".
- The *t*-distribution is symmetric and bell-shaped, like the normal distribution, but has heavier tails, meaning that it is more prone to producing values that fall far from its mean.
- Gosset worked at the Guinness Brewery in Dublin, Ireland as Head Brewer and Head Experimental Brewer, and was interested in the problems of small samples – for example, the chemical properties of barley and malts where sample sizes might be as few as 3
- Trained by Karl Pearson, a biostatistician, Gosset's work was popularized, supported and expanded upon by Sir Ronald Fisher.

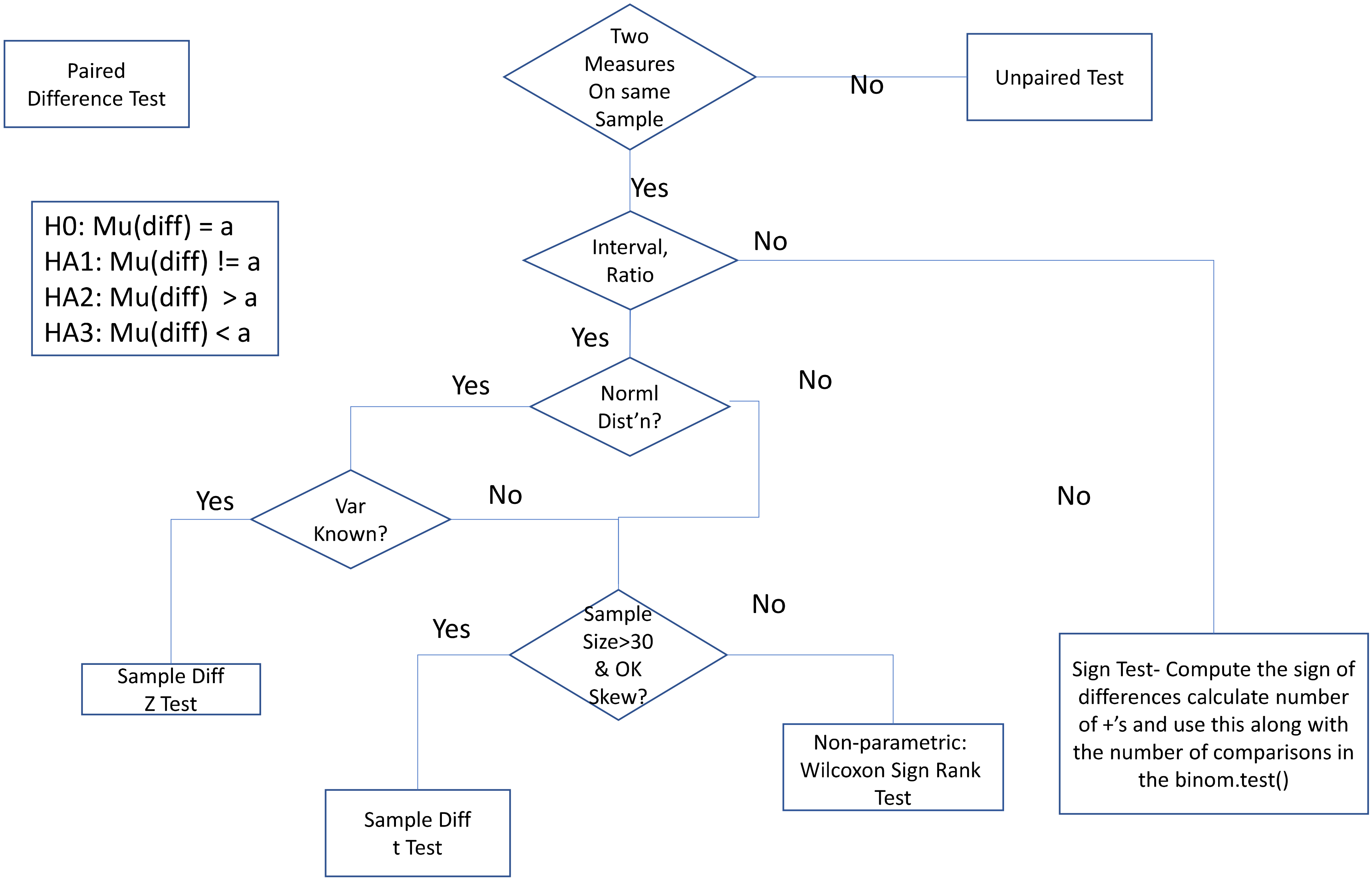



St. James's Gate Brewery, May, 2018

# Have A Guinness, The Statistician’s Drink


Density Plot For N(0,1) and t's, df=(5,15,30)
Density
0.0
0.1
0.2
0.3
0.4
-3
-2
-1
0
1
2
3
Quantile
Black: N(0,1), Red: t(5), Green: t(15), Blue: t(30)


Unpaired Test
H0: Mu(1) = Mu(2)
HA1: Mu(1) != Mu(2)
HA2: Mu(1) > Mu(2)
HA3: Mu(1) < Mu(2)
Two Measures On same Sample
Yes
Paired Difference Test
No
Interval, Ratio
No
Yes
Norml Dist'n?
Yes
No
Var Known?
Yes
No
Sample Size>30 & OK Skew?
Yes
No
Vars equal, Levene Test?
Yes
No
Two Sample Z Test
Non Param - Wilicoxon Ranked Sum Test
Two Sample t Test Assume Equal Var
Two Sample t Test Welch


Paired Difference Test
H0: Mu(diff) = a
HA1: Mu(diff) != a
HA2: Mu(diff) > a
HA3: Mu(diff) < a
Two Measures On same Sample
No
Unpaired Test
Yes
Interval, Ratio
No
Yes
Norml Dist'n?
Yes
No
Var Known?
Yes
No
No
Sample Diff Z Test
Sample Size>30 & OK Skew?
Yes
No
Sample Diff t Test
Non-parametric: Wilcoxon Sign Rank Test
Sign Test- Compute the sign of differences calculate number of +'s and use this along with the number of comparisons in the binom.test()