Approximate Relationship Between Log Of Small Proportional Change And The Proportional Change Itself

Very Useful Approximate Result

- For values close to 1(like small proportional changes or percentages) the logarithm of a value slightly different from 1 is approximately equal to the small difference itself.

- This is of interest as an approximation particularly when we are using log transitions

- This result for the natural logarithm of a small value and the value itself can be explained using the Taylor series expansion* for the natural logarithm function.

- For a value x close to 1, we can express x as $x=1+\epsilon$ where ϵ is a small value (i.e., $\epsilon \approx 0$), meaning using a Taylor expansion means we are expanding the natural logarithm function $\ln(1+\epsilon)$ into a series of terms around the point where $\epsilon=0$.

- This approach allows us to approximate $\ln(1+\epsilon)$ when ϵ is small (close to zero), by expressing it as a sum of simpler terms.

- Using the Taylor expansion of $\ln(1+\epsilon)$ around $\epsilon=0$ we get:

$$\ln(1+\epsilon) \approx \epsilon - (\epsilon^2)/2 + (\epsilon^3)/3 - \cdots \ ,$$

  - where clearly powering very small ϵ is going to 0 and higher powered terms become negligible, so:

$$\ln(1+\epsilon) \approx \epsilon$$

* means of expressing a function around a convenient given point as a sum of the derivatives of the function at the point times the power differenced (noting. ln(1+0) = ln(1) = 0)