Harmony and nonharmonic partials

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We have explored some musical effects of tones with nonharmonic partials. The spacing of partials can be stretched so that each partial frequency f_{ij} present in tones sounded singly or together is given by $f_{ij} = A^{i/12+\log_2 j}$. Here i is the scale step in semitones, j is the partial number, and A is the frequency ratio of a pseudo-octave ($A = 2$ for a true octave). We find that subjects can match the keys of stretched ($A = 2.4$) as well as unstretched passages. Stretched cadences ($A = 2.4$) do not seem final. But, a stretched "cadence" with equally spaced partials that goes from closely spaced (tonally dissonant) to widely spaced (tonally consonant) partials does seem final. Our experiments do not decide finally among three views of harmony: that harmony depends on a fundamental bass or periodicity pitch (Rameau), that harmony depends on the spacing of partials (Helmholtz and Plomp) or that harmony is a matter of brainwashing.

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INTRODUCTION

In this paper we present the results of some experiments involving tones with nonharmonic partials.

All stringed and blown instruments playing steady tones have partials whose frequencies are harmonic or nearly harmonic; that is, the frequencies of the components of the tone are close to 1, 2, 3, 4, 5, 6, etc. times the frequency of the lowest partial, sometimes called the fundamental. It is commonly held that consonance, dissonance, and harmony are associated with this integral relationship among the frequency components which make up musical tones. Indeed, nonharmonic tones, such as those of bells and gongs, do not lend themselves well to harmonic combinations, even though they add a pleasant clangor to the sound of an orchestra.

By using a computer to generate sounds, one can produce tones which include partials of any chosen frequencies.[1] This raises an important question: can tones with nonharmonic partials be used in attaining useful musical effects similar to but different from those of conventional harmony? Our experiments were intended to give a better understanding of the subjective basis of harmony and to explore the musical use of sounds with nonharmonic partials, both sounds with "stretched" partials[2] and sounds with "uniformly spaced" partials.[3]

I. THEORIES OF HARMONY

In a book first published in 1863, Helmholtz[4] proposed that dissonance arises from unpleasant beats between partials whose frequencies are too close together; for example, partials separated by 10–50 Hz. The octave is the most consonant of intervals because all of the harmonic partials of the upper tone coincide in frequency with partials of the lower tone. The fifth is consonant because the frequencies of the fundamentals (first partials) of the two tones have a simple ratio, $\frac{3}{2}$, and because of this the lower partials of the two tones either coincide, or they are considerably separated in frequency and do not beat together objectionably.

Helmholtz's work has been added to by that of others. Particularly, Plomp and Levelt[5] found in 1965 that two (or more) partials that lie within what is called a critical bandwidth produce an unpleasant sensation (unless they differ very, very little in frequency). For frequencies above, say 500 Hz, a critical bandwidth is about $\frac{1}{4}$ octave (a minor third). However, Plomp and Levelt were careful to call the consonance of tones whose partials are separated by more than a critical band a "tonal consonance," and not to imply that this consonance is all there is to musical harmony.

Rameau[6] had another view of harmony. He observed that in a major triad all frequencies present are integer multiples of a *basse fundamentale* or *fundamental bass* which, in the root position of the chord (C, E, G) lies two octaves below the root of the chord (C). Thus, if the frequency of the fundamental bass is f_0, the frequency of the root (C) is $4f_0$, the frequency of the third (E) is $5f_0$, and the frequency of the fifth (G) is $6f_0$. Because Rameau regarded the octave as essentially an equality, he could identify the fundamental bass of the chord with its root.

Neither Rameau nor Helmholtz knew of the phenomenon of residue pitch or periodicity pitch, which Schouten[7] described in 1938. When we are presented with harmonic partials in the absence of the fundamental, we perceive the pitch as the least common denominator of the frequencies present, that is, as the frequency of the missing fundamental. Thus, when we listen to a major triad we might well hear Rameau's fundamental bass. Terhardt has discussed this[8] and has demonstrated[9] the effect.

The experiments we shall describe are relevant to the two views of harmony described above. But, one might hold that musical harmony is merely a matter of brainwashing; that we accept combinations of tones that we have been taught are correct, and reject those that we have been taught are incorrect. We have some experimental evidence that bears on this.

II. TONES USED AND INTENT OF EXPERIMENTS

We have used in our experiments two sorts of tones with nonharmonic partials. Chiefly, we have used tones with stretched partials, as Slaymaker[2] did. The frequencies f_{ij} the partials of such a tone are given by

$$f_{ij} = A^{(i/12+\log_2 j)} . \quad (1)$$

Here i is the scale step ($i = 12$ for an octave), j is the number of the partial and A is the frequency ratio of the *pseudo octave*. For a true octave, $A = 2$ and

$$f_{ij} = j2^{i/12} \quad (2)$$

we see that f_{ij} gives the frequencies of harmonics of the notes of an equally tempered scale.

Within the accuracy of the equally tempered scale, major triads made up of notes of the true octave scale satisfy the criterion of Rameau; the frequencies of partials are all integer multiples of a fundamental bass. They also satisfy the criterion of Helmholtz. All lower partials either coincide or are well separated. If all the frequencies present in the triad are stretched according to Eq. (1), all the lower partials still coincide or are well separated. So, Helmholtz's criterion will be satisfied in the stretched triad. But, in the stretched triad the partial frequencies are no longer multiples of a fundamental bass frequency. There will no longer be a basis for a periodicity pitch. Rameau's criterion will not be satisfied.

To summarize our conjectures about stretched tones:

(1) Harmonic effects which exist in stretched materials are produced by the interactions of stretched partials.

(2) Harmonic effects which disappear in stretched materials are produced either by periodicity pitch or by brainwashing.

(3) It is hard to separate brainwashing effects from other effects.

Besides the tones mentioned above, one experiment was made with tones in which partials are separated by a fixed fraction of an octave, as described by Pierce[3] and used by him in an eight-tone canon.[10]

III. THE EXPERIMENTS

Our experiments with stretched partials involved:

(1) The generations of stretched and unstretched materials.

(2) Test subjects' ability to identify the key of stretched and unstretched material.

(3) Test subjects' perception of "finality" of cadences for stretched and unstretched materials.

(4) Test subjects' perception of "finality" for unstretched materials with dissonant overtones removed.

(5) Test subjects' judgment of the finality of going from a totally dissonant chord with equally spaced partials to a tonally consonant chord with equally spaced partials.

(6) Observed difference and agreements among musicians and nonmusicians as subjects.

A. Acoustical description of the materials

All the sounds used in the experiments were synthesized on the PDP-10 computer at the Institute for Research and Coordination of Acoustics and Music (IRCAM), Paris, France, by use of the Music V program. Each sound had seven partials whose frequencies were specified by Eq. (1). For $A = 2$, these were in the approximate frequency ratios 1, 2, 3, 4, 5, 6, 8 (note the 7th partial is omitted). The exact partial frequencies as specified by Eq. (1) are equal tempered. That is, the partial frequencies coincide exactly with the fundamental of some note in the equal-tempered scale.

The amplitudes of the partials relative to the fundamental diminished at 9 dB per factor of 2 in frequency ratio between the partial and the fundamental. The value 9 dB per factor of 2 was selected to approximate normal musical instruments which tend to produce a spectrum which decreases faster than 6 dB per octave, but not as fast as 12 dB per octave.

The envelope of the notes was chosen to give a sustained sound with a moderately fast but not percussive attack, a slight diminuation (6 dB) over the duration of the note, and a smooth decay. The attack and decay times were each 10% of the duration of the note. Typical note durations ranged from 0.5–2 s.

The overall timbre can be described as a pleasant, bland, musical sound.

B. Key-sensing experiments

Judging the tonality of a short passage was chosen as an experimental test. Previous experiments[11] show that keys can be successfully identified but with difficulty.

Three short passages designated X, M, and T were synthesized. The scores of X and M are shown in Fig. 1. Both X and M are long enough to clearly establish a key. Both end in a clear cadence. T consists of M transposed to a different key. X was synthesized either in the key of M or T.

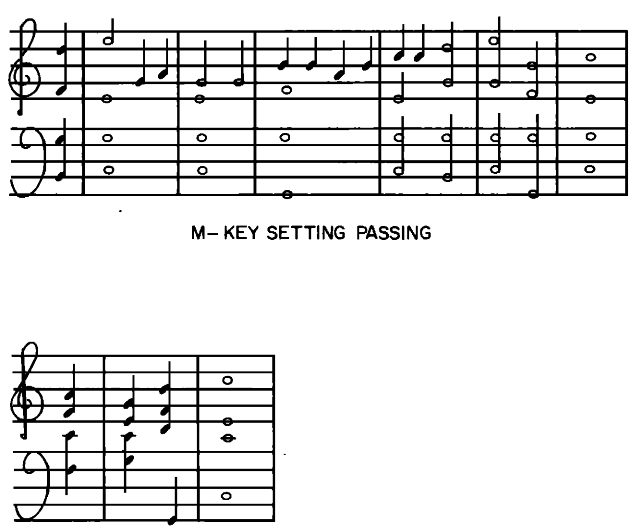

FIG. 1. Scores of passages used for key-sensing tests.

TABLE I. Percentage of correct sensing of key in XMXT tests, by use of musicians and nonmusicians as subjects. 50% correct is chance performance.

	Musicians		Nonmusicians		Everyone	
	Normal	Stretched	Normal	Stretched	Normal	Stretched
Major—minor	100	80	80	60	90	70
Minor 2nd transposition	100	100	85	75	92	87
Minor 3rd transposition	100	95	95	85	97	90

A test was prepared containing 24 test sequences. Each test sequence consists of the sequence XMXT. This sequence was selected over the more tranditional MXT test in order to make the test slightly harder. In the MXT order, the cadence at the end of M is very close to the cadence at the end of X because X is a very short passage.

The 24 tests consisted of six groups:

(1) Four tests where the key of M was 3 semitones below the key of T, unstretched ($A=2$);

(2) four tests where the key of M was 3 semitones below the key of T, ($A=2.4$);

(3) four tests where the key of M was 1 semitone below the key of T, unstretched ($A=2$);

(4) four tests where the key of M was 1 semitone below the key of T, stretched ($A=2.4$);

(5) four tests where M and T were major and minor of the same key, unstretched ($A=2$);

(6) four tests where M and T were major and minor of the same key, stretched ($A=2.4$).

After listening to a test, the subject was asked whether X was in the key of M or T. The correct response was randomized within each of the six groups, but not between groups.

The test was taken by ten subjects. Five subjects (the nonmusicians) had almost no experience as performers and described themselves as having little interest in music. The other five (the musicians) had extensive experience as amateur or semi-professional musicians.

The results of the experiment are shown in Table I. We draw the following conclusions:

(1) Everyone, musicians and nonmusicians alike, performed at better than the chance (50%) for both stretched and unstretched materials. Thus stretching did not destroy the ability to sense key in this test.

(2) In general the larger the key change, the better was the performance.

(3) Normal materials were easier to judge than stretched materials.

(4) Musicians performed better than nonmusicians.

In general, this experiment supports Helmholtz more than Rameau, because the stretching removed periodicity pitch and the fundamental bass, but left a substantial key sensing ability.

C. Cadence studies

The results discussed in the next three sections were all obtained from a single experiment. A test tape for this experiment was prepared containing 25 tests, each of which consisted of two chords of equal duration. The specific chords used will be described below. After listening to a test, the subject was asked to rate the chord pair on a scale of 1–9 according to finality, in other words according to how appropriate the chords would be as the end of a section of music. If the chord pair seemed very final, it was given a high rating, if it seemed very unfinished, it was given a low rating.

All 25 chord pairs discussed in the next three sections were randomized on the test tape. Presumably the subjects used the same criteria in judging all 25.

The test was taken by 13 "musicians" and by 17 "nonmusicians" defined according to the above discussed criteria. There were no apparent patterns of differences between the responses of the musicians and the nonmusicians, so the results of these two groups were averaged together.

D. Finality of cadences

In this experiment four chord pairs were judged. These consisted of the stretched ($A=2.4$) and unstretched ($A=2$) forms of the cadence and "anticadence" shown in Fig. 2. As shown, the anticadence ends on the penultimate chord of a normal cadence. Thus it should produce a strong impression of nontermination. The use of chord pairs as cadences implies that the subjects already know the tonic chord. The same key was used

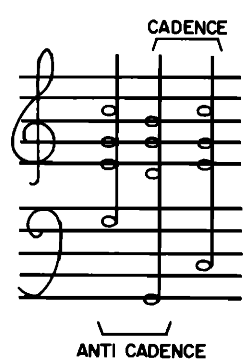

FIG. 2. Cadence (dominant to tonic) and anticadence (tonic to dominant).

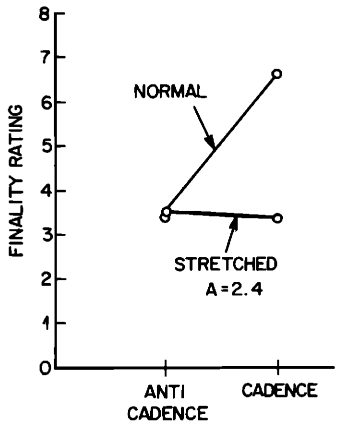

FIG. 3. Finality ratings of stretched and normal cadences and anticadences. With normal (unstretched) partials the cadence is judged as having a great deal of finality and the anticadence as having little finality. With stretched partials, the finalities are essentially equal.

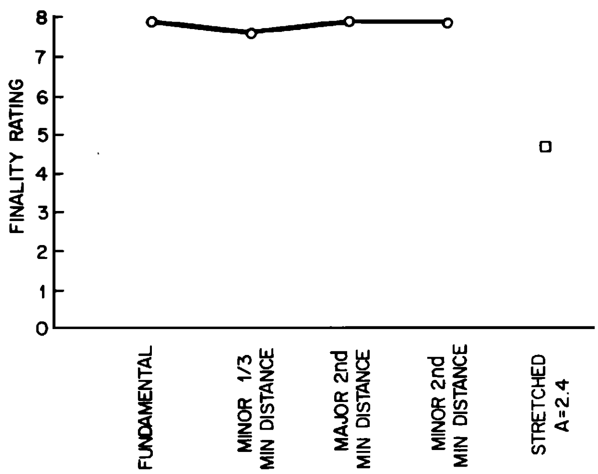

FIG. 5. Judgments of finality of cadence with partials deleted as in Fig. 3 to remove tonal dissonance. For comparison, the square point is for a cadence stretched by $A = 2.4$.

throughout the 25 tests and the results indicate that they clearly knew the tonic.

Results are shown in Fig. 3. It is clear that the unstretched cadence gave a strong sense of finality and the stretched cadence did not. The normal anticadence and the stretched anticadence were equally nonfinal. Thus, stretching seems to destroy the impression of finality in cadences. This supports Rameau rather than Helmholtz.

E. Is finality associated with tonal dissonance?

Following Helmholtz, we might assume that a major cadence consisting of a dominant seventh chord followed by a tonic chord would seem final because we go from a dissonant chord, some of whose partials lie within a critical bandwidth, to a consonant chord whose partials either coincide or are more widely separated.

In this experiment we started with a dominant seventh cadence as shown to the right in Fig. 4. The horizontal lines at the left represent the tones of the two succeeding chords, the dominant seventh above and the tonic below. The dots on the lines show the position in half-steps of the seven partials that are present in each tone (1, 2, 3, 4, 5, 6, 8).

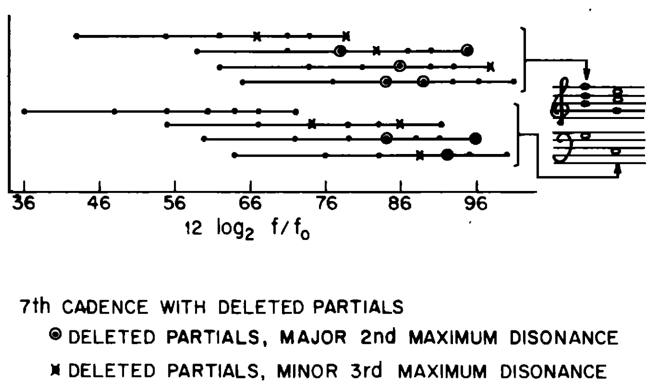

FIG. 4. Frequencies of partials of 7th cadence. $f_0 = 16.37$ Hz. The dots on the horizontal lines show the frequencies of the partials of a dominant seventh chord (above) and a tonic chord (below). The circles show deletions to make the minimum distance between partials a major second, and the crosses show additional deletions to make the minimum distance between partials a minor third.

We see that in the dominant seventh chord there are five pairs of partials a half step apart (very dissonant) and three pairs a whole step apart. When the circled dots are deleted, none of the remaining partials is less than a whole tone apart, and the chord sounds less dissonant. When the crossed dots are deleted as well, none of the remaining partials are less than a minor third apart and the chord sounds tonally consonant.

We note also that in the tonic chord, two partials must be deleted in order to make the least interval between partials a major second, and two more must be deleted in order to make the least interval between partials a minor third. Thus, the tonic chord itself has some tonal dissonance.

Subjects were asked to judge the finality of the dominant seventh cadences without deletion of partials, and with deletions to render the dominant seventh less tonally dissonant. The results are shown in Fig. 5. All the cadences were given a very high finality rating. The removal of partials makes almost no difference in the ratings.

We should note that a musician identified the chords correctly even when partials had been deleted so as to make the least interval between the partials in the dom-

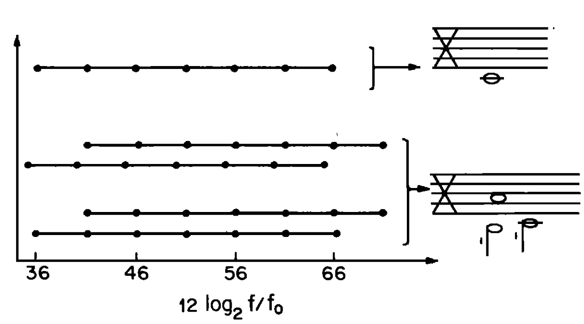

FIG. 6. A cadence with uniformly spaced partials, going from a tonally dissonant to a tonally consonant chord. $f_0 = 16.37$ Hz. The clef sign X was inverted to indicate that the staff is not normal musical notation. The plot of the partials is the basic description of the sounds.

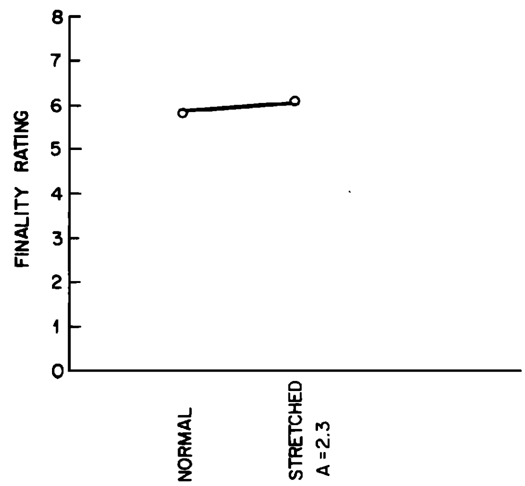

FIG. 7. Judgments of finality of the cadence of Fig. 6. It is judged as final with or without stretching.

inant seventh a minor third. We should also note that both the dominant seventh chord and the tonic chord were noticeably less harsh, less tonally dissonant, after this deletion of partials.

This portion of the experiment appears to argue against Helmholtz, that is, against acoustic dissonance as a basis for harmonic effects.

F. Cadence with equally spaced partials

In the experiment using a dominant seventh chord we noted that, even in this strongly dissonant chord, relatively few partials lie close together. Perhaps Helmholtz's effects are weak simply because too few partials interact closely. To study this hypothesis we generated a chord pair as shown in Fig. 6 using notes with partials equally spaced along a musical scale. The partials are separated by a major fourth in this specific example. As a result of this spacing the first chord of the pair is very dissonant, almost all being one half step apart and the second chord is very consonant, almost all being coincident.

Figure 7 shows that this chord pair is given a relatively high finality rating. Furthermore, this rating, as we would expect, is not diminished by stretching ($A = 2.3$) the material. Thus, if one makes strong enough interactions between an impression of finality can be conveyed by a transition to an acoustically consonant situation. This would argue for Helmholtz's tonal consonance view of harmony.

IV. SUPPLEMENTARY OBSERVATIONS AND MATERIAL

Before trying to draw further conclusions from the experimental results described above, it may be well to describe some further observations.

It is our observation that melodies are easily recognized despite stretching. A stretched ($A = 2.4$) version of the round, "Are You Sleeping, Brother John," was instantly recognized by an audience. The melody of harmonized stretched ($A = 2.4$) versions of "The Country Carol" and "Old Hundred" were recognized, but these had also been played unstretched.

In the harmonized stretched "Old Hundred" it seemed difficult to distinguish the inner voices. In fact, in single tones stretched $A = 2.4$, one tended to hear the partials as separated sounds rather than as fused into a tone of a single pitch. We believe that such fusion depends in part on the phenomenon of residue or periodicity pitch.[7] This has been noted by Cohen.[12] She has observed further that the degree of fusion of a stretched tone depends on the envelope of the tone, and is greatest[13] for an exponentially decreasing amplitude which gives a "struck" quality.

We observe that stretched tones sounded singly tend to fall apart into a group of partials, but when a sequence of such tones is played as a known melody the tones are heard as the individual notes of the melody.

It appears that whether or not a collection of stretched partials is heard as a single tone can depend on both the time evolution of the partial amplitudes and on the context (melodic or otherwise) in which the tones are heard.

V. SUMMARY AND CONCLUSIONS

The intent of the experiments performed was to try to decide among three explanations of harmonic effects: (1) Rameau's fundamental bass, which can be related to Schouten's residue pitch or periodicity pitch; (2) the tonal consonance of Helmholtz and Plomp, which depends on the spacings of the partials that are present; (3) brainwashing, that is, learned expectations and reactions. The experiments did not distinguish clearly among these views.

We found:

(1) Subjects can identify keys of both stretched and unstretched materials in an XMXT test.

(2) Stretching destroys the perception of finality of cadences.

(3) Removing dissonant partials does not change the perception of the finality of cadences.

Results (1) above may be a melodic rather than a harmonic effect, and, as we have noted, melody seems more robust under stretching than harmony does.

Result (2) argues for either Rameau or brainwashing.

Result (3) argues somewhat for brainwashing. We are taught that resolution is going from dissonance to consonance and in going from the dominant seventh to the tonic we accept the cadence as correct and final even when the tonal dissonance has been removed. But, cadences went from the dominant to the tonic before the dominant seventh was used.

Nonetheless, by using tones with equally spaced partials, we can get a sense of finality by going from a tonally dissonant chord, to a tonally consonant chord. This argues for Helmholtz–Plomp.

We may observe that traditional harmony as we know it is like language, a complex but learnable art that is very difficult to explain.

The fact seems to us salient and important that under stretching melody is more robust than harmony.

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