Robotik

1	2	3	Σ
6	3,5	5,5	15

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Übungsblatt 6

Aufgabe 1

(a) ${}^{W}T_{P} = \text{Trans}(x,y) \cdot \text{Trot}(\theta)$

$${}^{W}T_{P_1} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{P_1}T_{P_2} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(30) & -\sin(30) & 0 \\ \sin(30) & \cos(30) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 & 3 \\ 1/2 & \sqrt{3}/2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

1/1

$${}^{P_2}T_{P_3} = \begin{pmatrix} 1 & 0 & 4/\sqrt{3} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(60) & -\sin(60) & 0 \\ \sin(60) & \cos(60) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 4/\sqrt{3} \\ \sqrt{3}/2 & 1/2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) ${}^{W}T_{R} = {}^{W}T_{P_1} \cdot {}^{P_1}T_{P_2} \cdot {}^{P_2}T_{P_3}$

$${}^{W}T_{R} = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{3}/2 & -1/2 & 3 \\ 1/2 & \sqrt{3}/2 & 3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 4/\sqrt{3} \\ \sqrt{3}/2 & 1/2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

1/2

$${}^{W}T_{R} = \begin{pmatrix} -1 & 0 & -\frac{7\sqrt{3}}{6} \\ 0 & -1 & \cancel{3/2} \\ 0 & 0 & 1 \end{pmatrix} \frac{9}{2}$$

(c) Position $(x_r, y_r) = \left(-\frac{7\sqrt{3}}{6} ; \cancel{\frac{3}{2}}\right) \frac{9}{2}$

Orientierung $(\theta_r) = 180°$

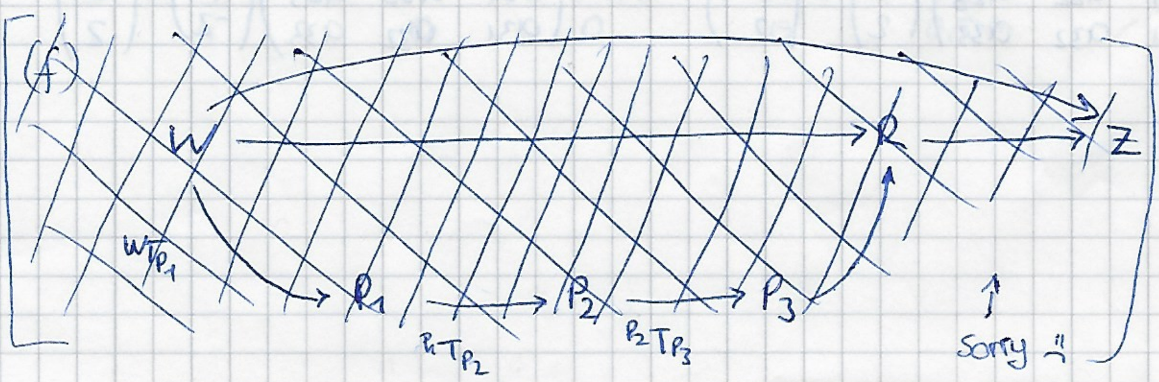
1/1

(d) ${}^{R}T_{Z} = ({}^{W}T_{R})^{-1} \cdot {}^{W}T_{Z}$

$${}^{R}T_{Z} = \begin{pmatrix} -1 & 0 & -\frac{7\sqrt{3}}{6} \\ 0 & -1 & \cancel{\frac{3}{2}}\ \frac{9}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -\frac{7\sqrt{3}}{6} \\ 0 & -1 & \cancel{3/2} \\ 0 & 0 & 1 \end{pmatrix} -\frac{1}{2}$$

2/2.

~~(f)~~

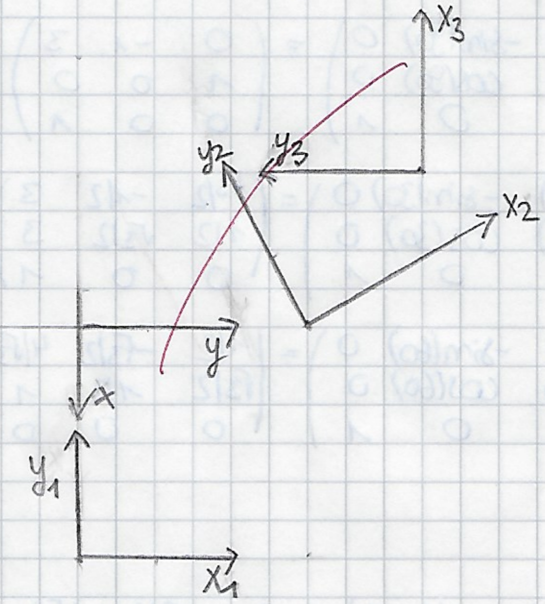


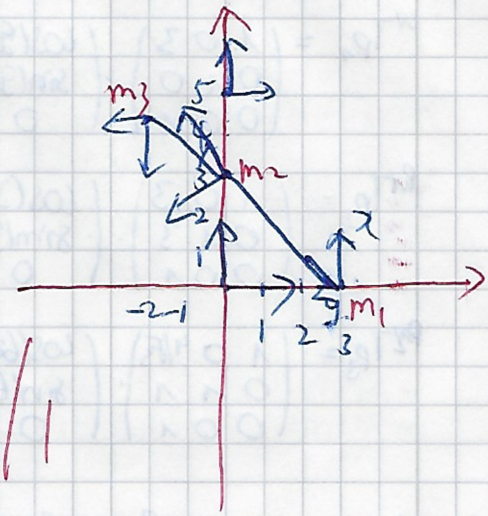
(f)

$$\begin{tikzcd}
W \arrow[rr, "{}^W T_{P_3}"] \arrow[d, "{}^W T_{P_1}"'] \arrow[rrr, bend left, "{}^W T_Z"] & & P_3 = R \arrow[r, "{}^R T_Z"] & Z \\
P_1 \arrow[r, "{}^{P_1} T_{P_2}"'] & P_2 \arrow[ur, "{}^{P_2} T_{P_3}"'] & &
\end{tikzcd}$$

1/1

(e)





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Aufgabe 2

(a) $P_3' = P_3$ somit Rotation um Punkt P_3.

$P_1' = P_2$, $P_2' = P_3$ $\Rightarrow$ $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ \sqrt{3} \\ 0 \end{pmatrix}$ und $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 \\ \sqrt{3} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

$P_3' = P_3$ $\Rightarrow$ $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -1 \\ -\sqrt{3} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -\sqrt{3} \\ 0 \end{pmatrix}$

$P_4' = P_5$, $P_5' = P_4$ $\Rightarrow$ $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$ und $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

$2 \cdot a_{11} = -1 \rightarrow a_{11} = -1/2$
$2 \cdot a_{21} = \sqrt{3} \rightarrow a_{21} = \sqrt{3}/2$
$2 \cdot a_{31} = 0 \rightarrow a_{31} = 0.$

$-a_{11} + \sqrt{3} \cdot a_{12} = 2 \rightarrow a_{12} = \sqrt{3}/2$
$-a_{21} + \sqrt{3} \cdot a_{22} = 0 \rightarrow a_{22} = 1/2$
$-a_{31} + \sqrt{3} \cdot a_{32} = 0 \rightarrow a_{32} = 0$

$-a_{11} - \sqrt{3} \cdot a_{13} = -1 \rightarrow a_{13} = \sqrt{3}/2$
$-a_{21} - \sqrt{3} \cdot a_{22} = -\sqrt{3}$
$-a_{31} - \sqrt{3} \cdot a_{32} = 0$

$2 \cdot a_{13} = 0 \rightarrow a_{13} = 0$
$2 \cdot a_{23} = 0 \rightarrow a_{23} = 0$
$2 \cdot a_{33} = -2 \rightarrow a_{33} = -1$

$$\Rightarrow R = \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

✓

2/2

(c) Vorteile:

- Kompakte Darstellung der Rotationen ✓
- geringer Rechenaufwand – bei was? $-\frac{1}{2}$ P
  nicht bei Rotation eines Vektors

Nachteile:

- Keine Translation ✓
- Komplexe Quaternionmultiplikationen. ✓

1,5/2

Aufgabe 3

(a)

Glied	Gliedtyp	Gelenkart
1	Typ 6	D–S
2	Typ 8	S–D
3	Typ 7	D–S
4	–	–

✓ 2/2

(b)

Glied	l_n	α_n	θ_n	d_n	
1	0	90°	90°	1 m	✓
2	0	−90°	+90°	0,5 m	−0,5
3	0,5 m	90°	90°	0,5 m	✓
4	0	0	0	0,5 m.	✓

3,5/4