

HONG KONG EXAMINATIONS AUTHORITY  
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1986

數學(課程甲) 試卷一  
**MATHEMATICS (SYLLABUS A) PAPER I**

8.30 am–10.30 am (2 hours)

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.  
Full marks will not be given unless the method of solution is shown.

### FORMULAS FOR REFERENCE

<b>SPHERE</b>	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
<b>CYLINDER</b>	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
<b>CONE</b>	Area of curved surface	=	$\pi r l$
	Volume	=	$\frac{1}{3}\pi r^2 h$
<b>PRISM</b>	Volume	=	base area $\times$ height
<b>PYRAMID</b>	Volume	=	$\frac{1}{3} \times$ base area $\times$ height

**SECTION A** Answer ALL questions in this section.

There is no need to start each question on a fresh page.  
Geometry theorems need not be quoted when used.

1. Factorize

(a)  $x^2 - 2x - 3$ ,

(b)  $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$ .

(5 marks)

- 2.

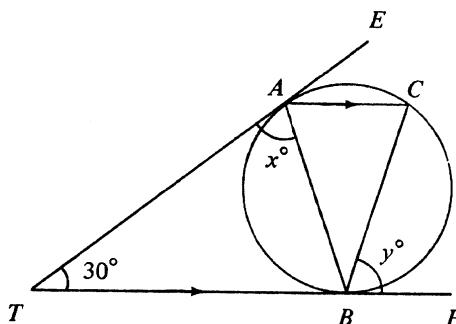


Figure 1

In Figure 1,  $TAE$  and  $TBF$  are tangents to the circle  $ABC$ . If  $\angle ATB = 30^\circ$  and  $AC \parallel TF$ , find  $x$  and  $y$ .

(5 marks)

3. The table below shows the number of students in three classes of a school and their average marks in a test :

Class	No. of Students	Average Mark
F.5A	40	61
F.5B	$x$	70
F.5C	35	50

If the overall average mark of the three classes is 60, find  $x$ .

(5 marks)

4. Solve  $\sin^2 \theta + 7 \sin \theta = 5 \cos^2 \theta$  for  $0^\circ \leq \theta < 360^\circ$ .

(6 marks)

5. (a) Evaluate  $\log_2 8 + \log_2 \frac{1}{16}$ .

(b) If  $2 \log_{10} x - \log_{10} y = 0$ , show that  $y = x^2$ .  
(6 marks)

6.

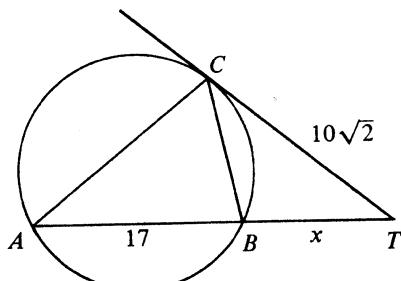


Figure 2

In Figure 2,  $A$ ,  $B$  and  $C$  are three points on the circle.  $CT$  is a tangent and  $ABT$  is a straight line.

(a) Name a triangle which is similar to  $\triangle BCT$ .

(b) Let  $BT = x$ ,  $AB = 17$  and  $CT = 10\sqrt{2}$ . Find  $x$ .  
(6 marks)

7. If  $\frac{1}{m} + \frac{1}{n} = \frac{1}{a}$  and  $m + n = b$ , express the following in terms of  $a$  and  $b$ :

(a)  $mn$ ,

(b)  $m^2 + n^2$ .  
(6 marks)

**SECTION B** Answer any **FIVE** questions from this section.  
Each question carries 12 marks.

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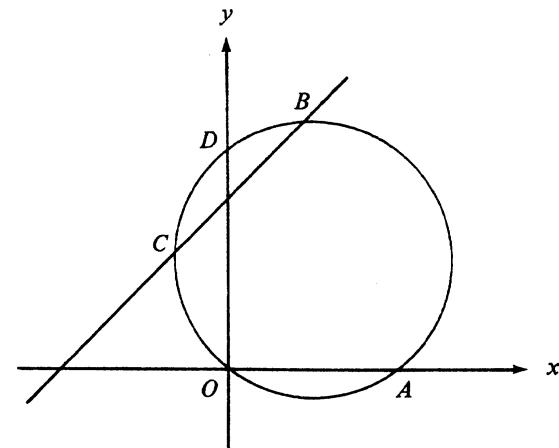


Figure 3

The line  $y - x - 6 = 0$  cuts the circle  $x^2 + y^2 - 6x - 8y = 0$  at the points  $B$  and  $C$  as shown in Figure 3. The circle cuts the  $x$ -axis at the origin  $O$  and the point  $A$ ; it also cuts the  $y$ -axis at  $D$ .

(a) Find the coordinates of  $B$  and  $C$ .  
(4 marks)

(b) Find the coordinates of  $A$  and  $D$ .  
(3 marks)

(c) Find  $\angle ADO$ ,  $\angle ABO$  and  $\angle ACO$ , correct to the nearest degree.  
(3 marks)

(d) Find the area of  $\triangle ACO$ .  
(2 marks)

9.  $2, -1, -4, \dots$  are in A.P.

- (a) Find (i) the  $n$ th term,  
(ii) the sum of the first  $n$  terms,  
(iii) the sum of the progression from the 21st term to  
the 30th term.  
(7 marks)
- (b) If the sum of the first  $n$  terms of the progression is less than  
-1000, find the least value of  $n$ .  
(5 marks)

10.

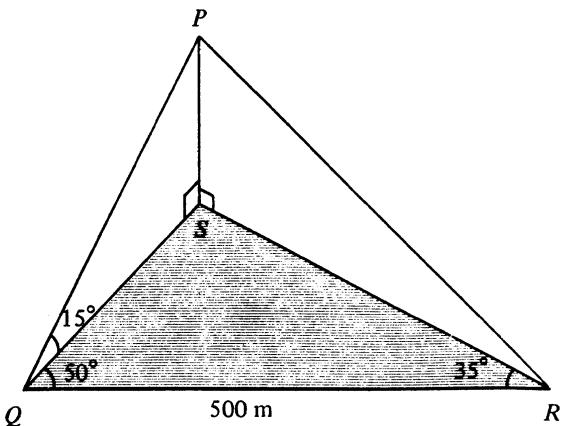


Figure 4

In Figure 4,  $Q$ ,  $R$  and  $S$  are three points on the same horizontal plane.  $QR = 500$  m,  $\angle SQR = 50^\circ$  and  $\angle QRS = 35^\circ$ .  $P$  is a point vertically above  $S$ . The angle of elevation of  $P$  from  $Q$  is  $15^\circ$ .

- (a) Find the distance, in metres, from  $P$  to the plane, correct to  
3 significant figures.  
(6 marks)
- (b) Find the angle of elevation of  $P$  from  $R$ , correct to the  
nearest degree.  
(6 marks)

11. If you attempt this question, you should refer to the separate supplementary leaflet provided.

- (a) (i) On the graph paper provided, draw the following straight lines :

$$x + y = 40,$$

$$x + 3y = 60,$$

$$7x + 2y = 140.$$

- (ii) On the same graph paper, shade the region that satisfies all  
the following constraints :

$$x \geq 0,$$

$$y \geq 0,$$

$$x + y \geq 40,$$

$$x + 3y \geq 60,$$

$$7x + 2y \geq 140.$$

(6 marks)

- (b) A company has two workshops A and B. Workshop A produces 1 cabinet, 1 table and 7 chairs each day; Workshop B produces 1 cabinet, 3 tables and 2 chairs each day. The company gets an order for 40 cabinets, 60 tables and 140 chairs. The expenditures to operate Workshop A and Workshop B are respectively \$1000 and \$2000 each day. Use the result of (a)(ii) to find the number of days each workshop should operate to meet the order if the total expenditure in operating the workshops is to be kept to a minimum. (Denote the number of days that Workshops A and B should operate by  $x$  and  $y$  respectively.)  
(6 marks)

12.

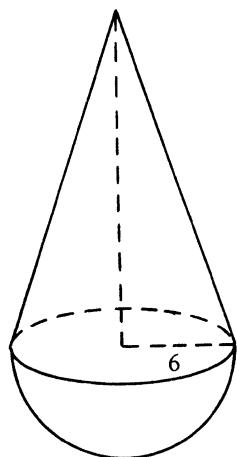


Figure 5a

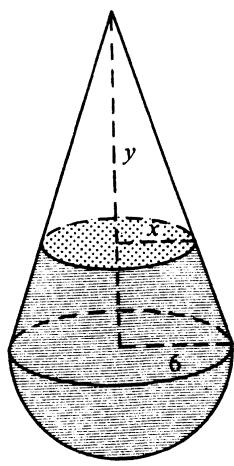


Figure 5b

Figure 5a shows a solid consisting of a right circular cone and a hemisphere with a common base which is a circle of radius 6. The volume of the cone is equal to  $\frac{4}{3}$  of the volume of the hemisphere.

- (a) (i) Find the height of the cone.  
 (ii) Find the volume of the solid. (Leave your answer in terms of  $\pi$ .) (6 marks)
  
- (b) (i) The solid is cut into two parts. The upper part is a right circular cone of height  $y$  and base radius  $x$  as shown in Figure 5b. Find  $\frac{x}{y}$ .  
 (ii) If the two parts in (b)(i) are equal in volume, find  $y$ , correct to 1 decimal place. (6 marks)

13. A box contains wooden blocks of 5 different shapes  $A, B, C, D$  and  $E$ . For each shape, there are 5 different colours red, orange, yellow, green and blue. For each colour of each shape, there is one block of each of the sizes  $L, M$  and  $S$ . (Hint: There are altogether 75 blocks in the box.)

- (a) When a block is picked out randomly from the box, what is the probability that it is of
  - (i) red colour ?
  - (ii) blue colour and shape  $C$  ?
  - (iii) size  $S$ , shape  $A$  or  $E$  but not yellow ?(6 marks)
  
- (b) Two blocks are drawn at random from the box, one after the other. The first block drawn is put back into the box before the second is drawn. Find the probability that
  - (i) the first block drawn is of size  $L$  and the second block is of size  $S$ ,
  - (ii) one of the blocks drawn is of size  $L$  and the other of size  $S$ ,
  - (iii) the two blocks drawn are of different sizes.(6 marks)

14. If you attempt this question, you should refer to the separate supplementary leaflet provided.

Figure 6 shows the graph of  $y = x^4 + x$  for  $0 \leq x \leq 2$ .

- (a) Draw a suitable straight line on the figure and use it to find the approximate value of the root of the equation

$$x^4 - x - 1 = 0$$

in the interval  $0 \leq x \leq 2$ , correct to 1 decimal place.

(4 marks)

- (b) By the method of magnification, find the approximate value of the root in (a), correct to two decimal places.

(4 marks)

- (c) Use the result in (b) to find the approximate value of the root of the equation

$$(x - 1)^4 = x$$

in the interval  $1 \leq x \leq 3$ , correct to two decimal places.

[Hint : Put  $x = y + 1$ ]

(4 marks)

END OF PAPER

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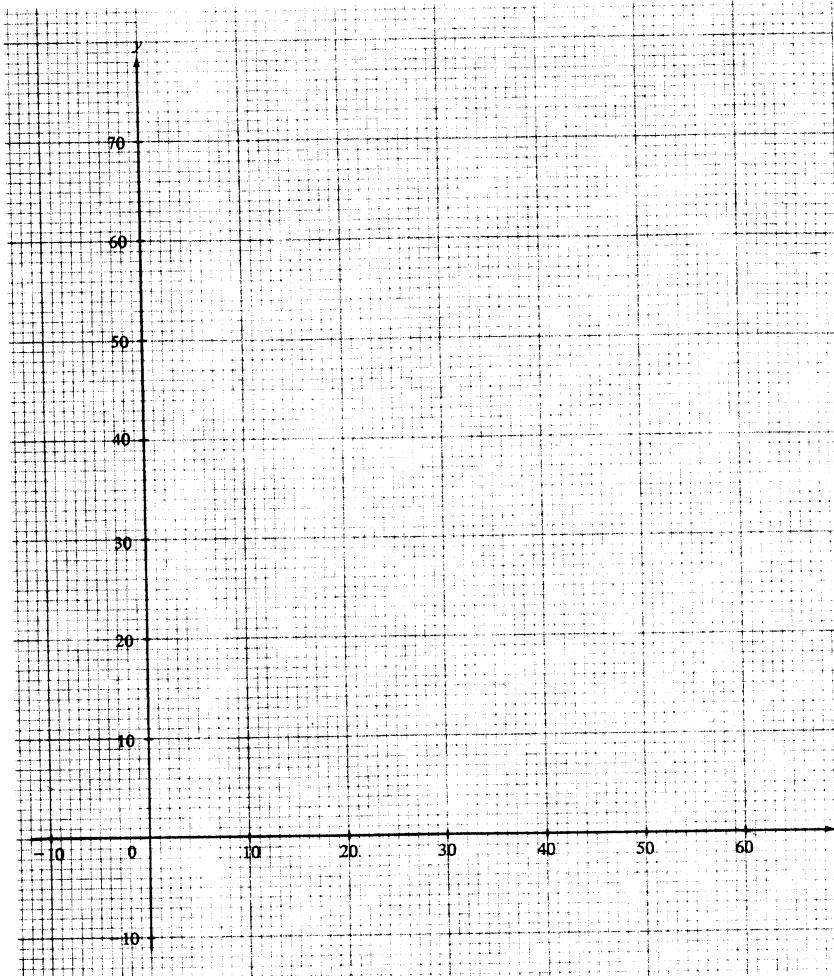
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1986

數學(課程甲) 試卷一(附頁)

**MATHEMATICS (SYLLABUS A) PAPER I  
(SUPPLEMENTARY LEAFLET)**

Total Marks  
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11. If you attempt this question, fill in the details in the first three boxes above and tie this sheet inside your answer book.



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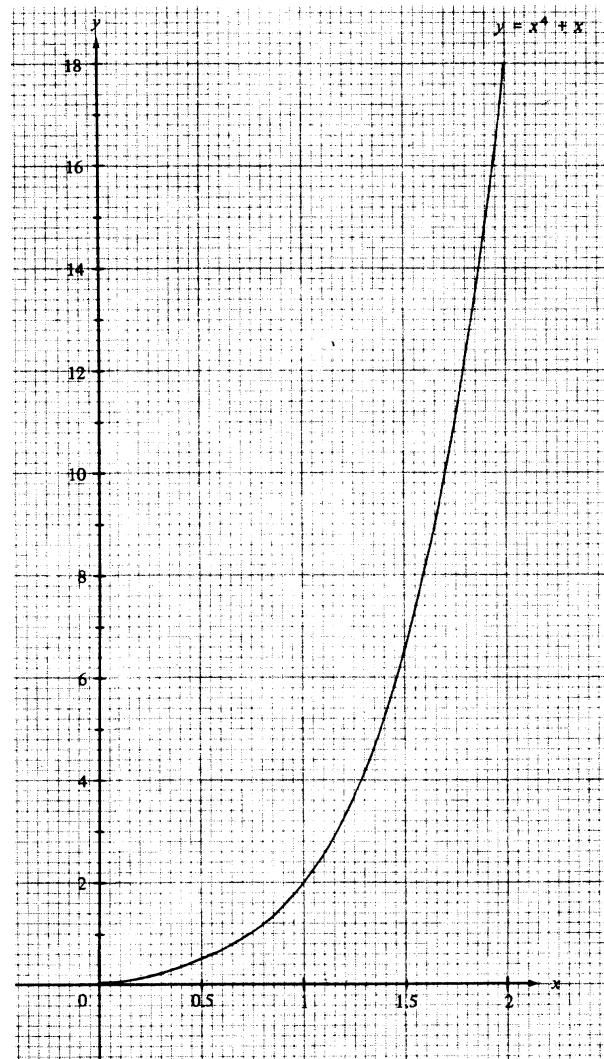


Figure 6

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HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1986

數學(課程乙) 試卷一  
**MATHEMATICS (SYLLABUS B) PAPER I**

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FORMULAS FOR REFERENCE

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PRISM	Volume	= base area × height
PYRAMID	Volume	= $\frac{1}{3} \times$ base area × height

**SECTION A** Answer ALL questions in this section.  
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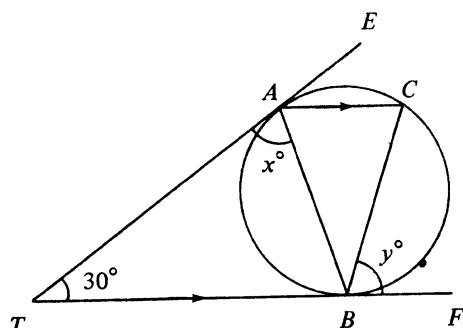


Figure 1

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If the overall average mark of the three classes is 60, find  $x$ .  
(5 marks)

4. Solve  $\sin^2 \theta + 7 \sin \theta = 5 \cos^2 \theta$  for  $0^\circ \leq \theta < 360^\circ$ .  
(6 marks)

5. It is given that  $z$  varies directly as  $x^2$  and inversely as  $y$ . If  $x = 1$  and  $y = 2$ , then  $z = 3$ . Find  $z$  when  $x = 2$  and  $y = 3$ .  
(6 marks)

6.

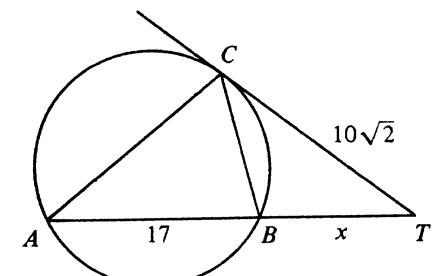


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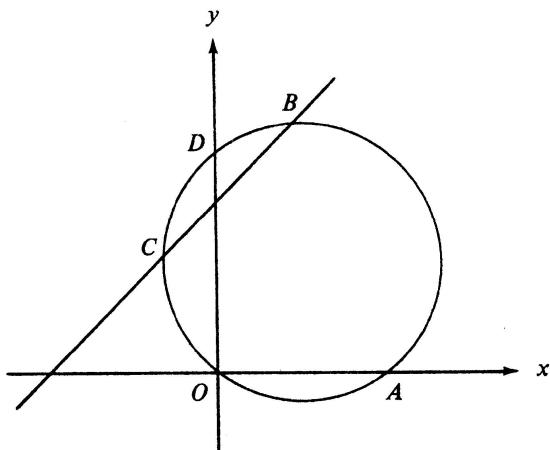


Figure 3

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- Find the coordinates of  $B$  and  $C$ . (4 marks)
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- Find  $\angle ADO$ ,  $\angle ABO$  and  $\angle ACO$ , correct to the nearest degree. (3 marks)
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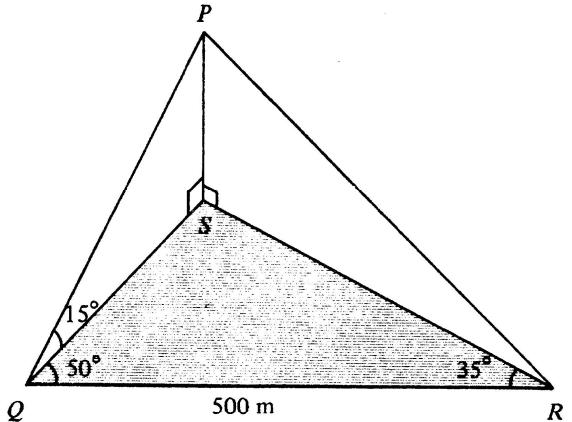


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- Find the distance, in metres, from  $P$  to the plane, correct to 3 significant figures. (6 marks)
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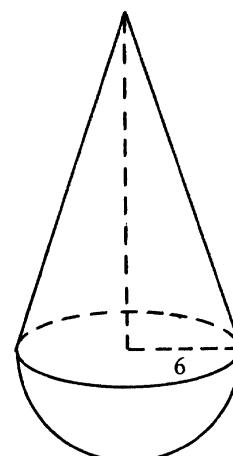


Figure 5a

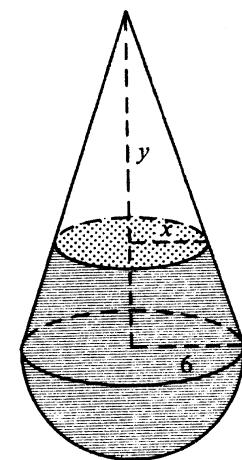


Figure 5b

Figure 5a shows a solid consisting of a right circular cone and a hemisphere with a common base which is a circle of radius 6. The volume of the cone is equal to  $\frac{4}{3}$  of the volume of the hemisphere.

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- (6 marks)
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- (i) the first block drawn is of size  $L$  and the second block is of size  $S$ ,
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  - (iii) the two blocks drawn are of different sizes.
- (6 marks)

14. If you attempt this question, you should refer to the separate supplementary leaflet provided.

Figure 6 shows the graph of  $y = ax^2 + bx + c$ .

- (a) Find the value of  $c$  and hence the values of  $a$  and  $b$ .  
(4 marks)
- (b) Solve the following equations by adding a suitable straight line to Figure 6 for each case. Give your answers correct to 1 decimal place.
- (i)  $(x + 2)(x - 3) = -1$ ,
  - (ii)  $x^2 - 2x - 1 = 0$ .
- (8 marks)

**END OF PAPER**

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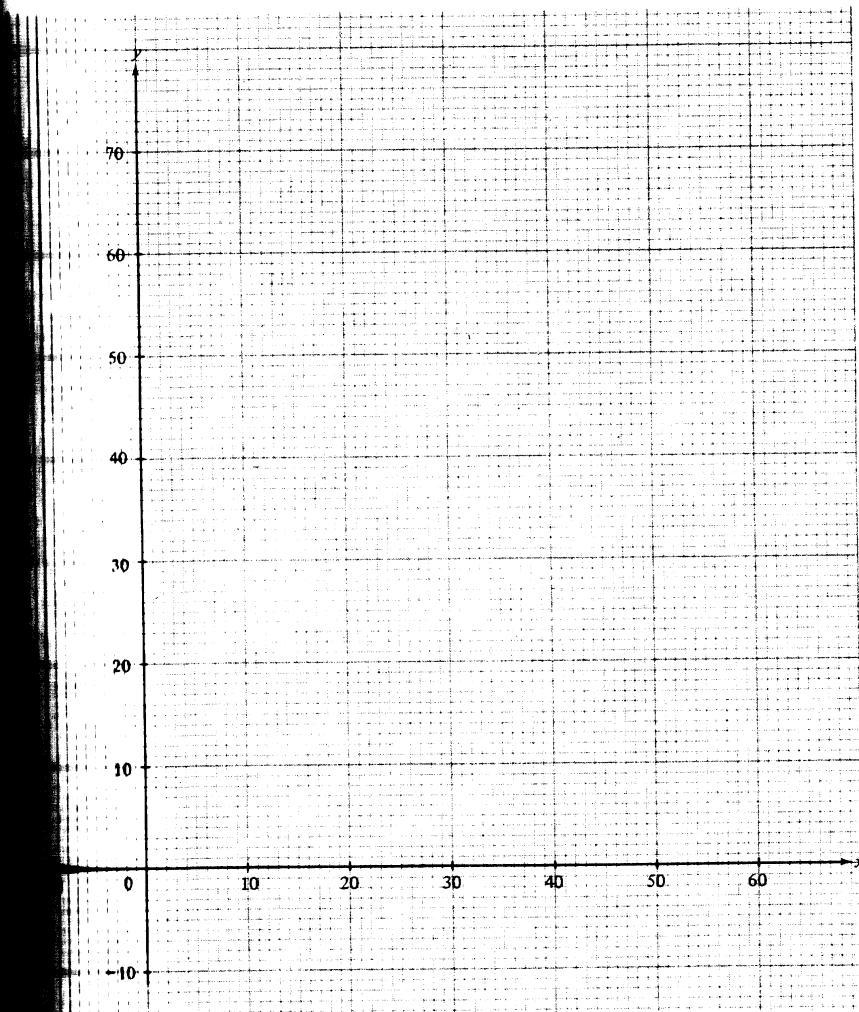
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1986

數學(課程乙) 試卷一(附頁)

MATHEMATICS (SYLLABUS B) PAPER I  
(SUPPLEMENTARY LEAFLET)

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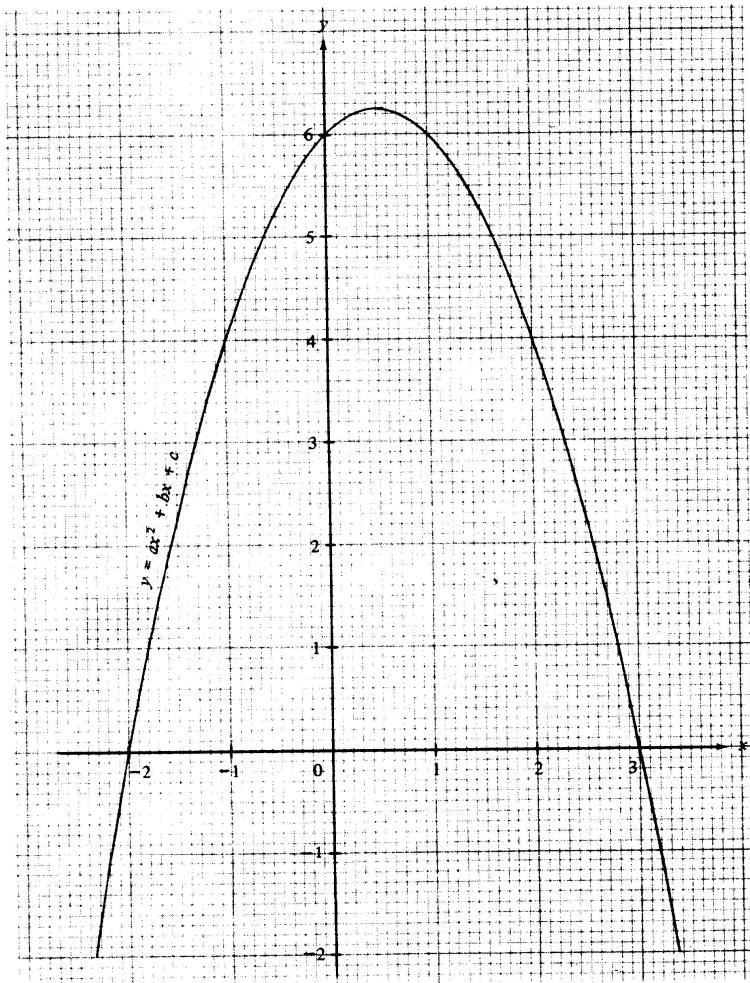


Figure 6