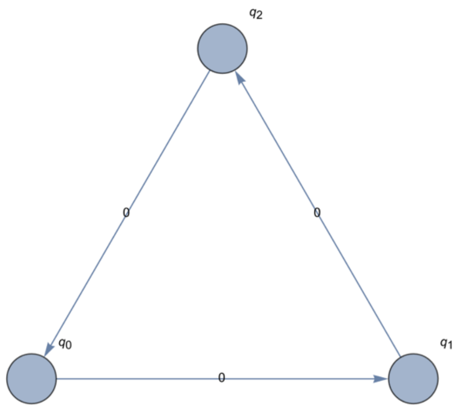
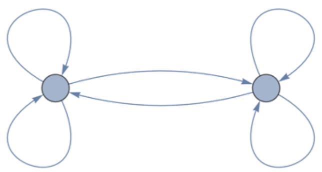
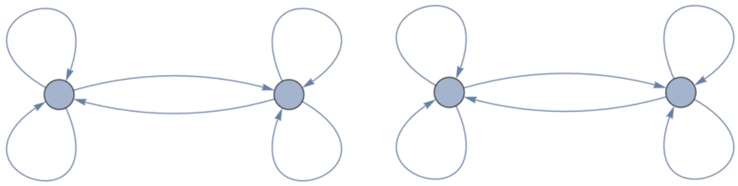
The motivation behind our project comes from the fact that biological organisms can be viewed as computers executing specific algorithms. Genetic regulatory networks, specifically, can be thought of as computational objects called finite state machines, or automata in short. An automata describes a dynamical system with a finite number of states and events. For example,



This is a very simple automaton with 3 states q1, q2, and q3. There is one event, labelled 0 (zero), which tells us how the states transform. Take the following automata (called the **flip-flop** automata).

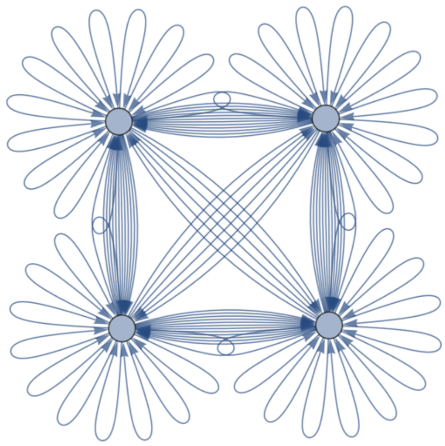


Now, we can define an algebraic operation called the direct product which takes 2 automata and "runs them simultaneously" , separately from each other

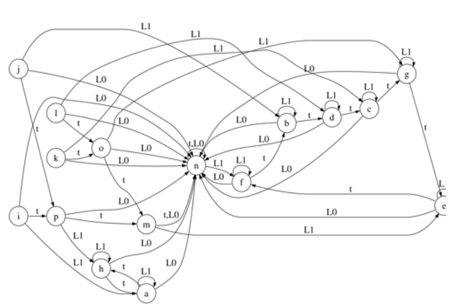


This is another automata. This way, we have built a new automata from 2 small automata. In a sense, that's what an algebraic structure broadly is – it contains a set of elements, and an operation between them. We can define another operation called the wreath product of 2 automata. This is essentially a generalisation of the direct product, but includes a flow of information between them (so their computations can be related, unlike in the direct product case).

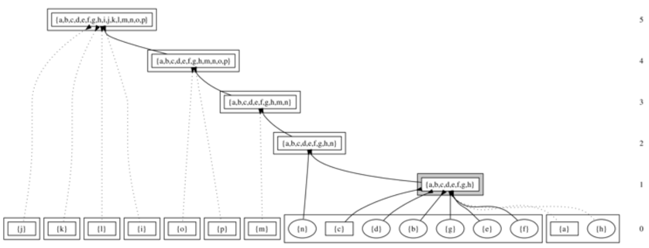
This is the wreath product of the **flip-flop** automata with itself:



Now, here’s where we use our main result. The krohn-rhodes theorem says that any finite state machine, with any number of states and events, can be “built up” from basic automata using the wreath product of very simple automata. A very direct application of the Krohn Rhodes theorem is to understand genetic regulatory networks. For example, this image on the left is the automata representing the genetic regulatory network of lac operon in E coli. Transition L1 denotes the introduction of lactose, L0 the depletion of lactose. And to the right is the decomposition obtained using the krohn-rhodes theorem:



STATE	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p
A	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
Op	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
ZYA	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
lactose	present								absent							



Now, how is all this relevant? Well, in synthetic biology, researchers have to design different genetic networks that perform a specific action. Thus, we want to create a website which curates the automata for different networks and allows users to build their automata by using different algebraic operations, such as the wreath product. Such a tool currently does not exist, and moreover, the powerful krohn - rhodes theory has not been used in biology literature yet.

In conclusion, the intersection of pure mathematics and biology shows that we can build organisms out of simpe finite state machines, and study their properties algebraically.

We are currently in the process of implementing the decomposition of complicated in the backend website *(the code for the decomposition has already been implemented in a computer algebra software called GAP. The package is called SGP DEC).*