Ms Eibling

AP Physics C

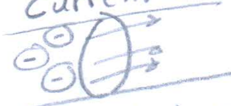
Circuits and Kirchoff's Rules

Wilm Hurst & Light bulb demo

Current

Current is represented by I, and $I = \frac{dQ}{dt}$ or the amount of charge flowing at any given time.

current:



Flow of charge through cross sectional area of wire per unit time. Direction: $+ \to -$

Units: $\frac{C}{s}$ = Amp

We can thus say that

elementary charge

volume that charge moves through

charge increment — $dQ = enAv_d dt$

number of elementary charges/Volume

distance that charge moves through

drift velocity $\sim 10^6$ m/s

$\frac{dq}{dt} = \underbrace{enV \cdot A}_{\text{Electron flux}}$ (#es, speed)

V_d = drift velocity $\sim 10^6$ m/s but due to collisions with ions $\sim 10^{-3}$ m/s

This gives us another definition for current:

$$dQ = enAv_d dt$$

$$\frac{dQ}{dt} = enAv_d$$

$$I = enAv_d$$

← V_d = constant

Only e_s move in conductors, but in plasma both e_s & p_s can move in E-field.

and current density, J

$J \propto E$ for metals at constant temp.

$$J = \frac{I}{A} = env_d$$

*Phet Simulation – V, I and R. Insulators & conductors

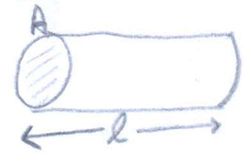
We can restate resistivity of a material, or $\rho = \frac{E}{J}$, which is the electric field through the conductor caused by the voltage and able to move this density of current. The resistivity of a material is affected by temperature, up to the point where other material properties start to change. The equation below works to 373K, or ~100C.

Ex: Thermal couple? oven

$$\rho(T) = \rho_o\left[1 + \alpha(T - T_o)\right]$$

$T < 373K \sim 100°C$

Units: V·m/A



$R = \frac{\rho \cdot l}{A}$

$V = Ed \to$ $\Delta V = El$

$$\rho = \frac{E}{J} = \frac{\Delta V}{Jl}$$

$$\Delta V = \rho Jl$$

$$\Delta V = \frac{\rho l I}{A}$$

From this, we can derive Ohm's Law, which is the big takeaway here.

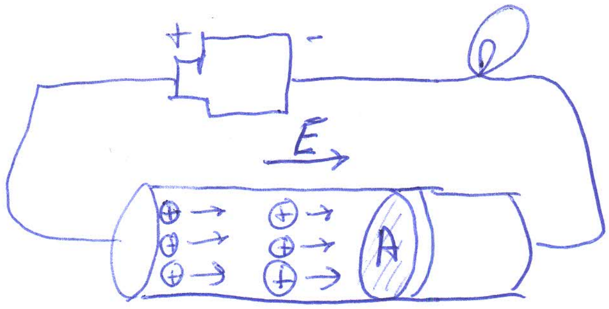
$\Rightarrow \rho = \frac{R \cdot A}{l}$

If we define resistance as resistivity times length divided by cross-sectional area, or $\rho\frac{l}{A}$, we can take that last equation to get the change in voltage over a resistor of some length l and cross-section A being $\Delta V=IR$, which is Ohm's Law. The unit of this resistance is Ohms, or Ω, which is V/Amp.

$R = \rho\frac{l}{A}$

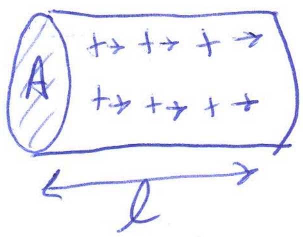
Semi conductors are non-omic.

$\rho = \frac{E}{J}$ ← Greater resistence ⇒ greter E-field is needed



Current: $I = \frac{dq}{dt}$ Units: $C/s = A$ (Amp)

defined as direction a (+) charge would move.



$$n = \frac{\#\text{charges}}{\text{Volume}} = \frac{\#\text{charges}}{A\,l}$$

$$Q = q_0 \cdot \underbrace{\left(\frac{\#\text{charges}}{\text{Volume}}\right)}_{n} \cdot \overbrace{V_d \cdot A \cdot \Delta t}^{=\text{Volume}}$$

q_0: charge of a proton or electron (e)

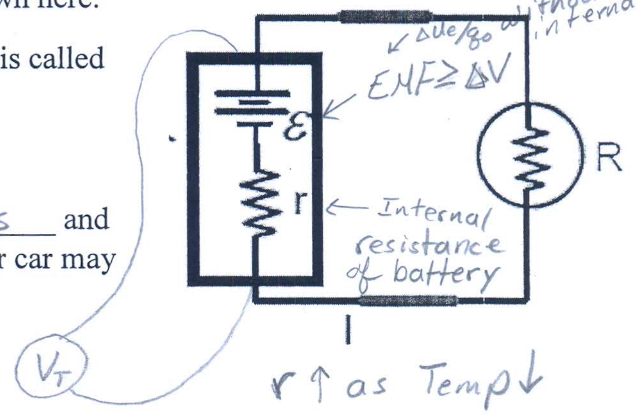
V_d: $\frac{\Delta x}{\Delta t}$

$$\Rightarrow dq = q_0 n V_d A\, dt \quad \text{or} \quad e n V_d A\, dt$$

EMF and Voltage

Electric Potential Diffrence.

What exactly moves electrons? We used to call this EMF, or electromotive force. We now know that it is NOT a force, but Voltage, or the amount of electric potential energy per unit charge. We write EMF as ε, and define it as the Total Voltage of the battery. This is NOT, except in ideal batteries, the same as the ACTUAL voltage, because there is often an inernal resistance, as in the circuit shown here.



Voltage across the battery when there is no circuit is called the open circuit voltage, ε or EMF

Internal resistance increases as the battery ages and as the temperature decreases, which is why your car may not start in winter.

r ↑ as Temp ↓

The EMF of a circuit is calculated thus:

$$V_T = \varepsilon - \underbrace{Ir}_{\Delta V \text{ decrease}}$$

ΔV decrease due to internal resistance of battery. Recall $V = IR$

$$\Rightarrow \boxed{\varepsilon = V_T + Ir} = IR + Ir \quad \Rightarrow \boxed{I = \frac{\varepsilon}{R+r}}$$

Electrical Energy and Power

Electrical potential energy is delivered by the battery to each charge as ΔU=QΔV. This potential energy is generally released by the resistor as heat (or heat and light given a lightbulb resistor, or heat and work, for a motor. To find the rate of work done, or the Power, take the derivative with respect to time. Hold voltage constant.

$$U_e = q_0 V \Rightarrow \Delta U_e = q_0 \Delta V$$

$$P = \frac{W}{\Delta t} \Rightarrow P = \frac{dU_e}{dt} = \frac{d}{dt}\left[q_0 \Delta V\right] \Rightarrow P = \frac{dq}{dt} \cdot \Delta V \Rightarrow \boxed{P = I \cdot \Delta V}$$

Now use Ohm's law substituted in to find 2 more expressions for power.

$$\boxed{P = I^2 R} \qquad \boxed{P = \frac{\Delta V^2}{R}} \qquad \text{Units: } \frac{J}{s} = \text{Watt}(W)$$

Given this, why do long-distance power transmission lines used such a low current and high voltage?

As I↑, the temperature↑ ⇒ R of circuit↑ ⇒ greater loss in energy to enviornment.

High voltage lines allow for the same power with lower current.

$$P = \underset{\downarrow}{I}\,\underset{\uparrow}{V}$$

Resistors in Series and Parallel:

Given the following resistor groupings, find the highest possible and lowest possible resistances and rank them in order of these. For any circuit combination

Number	*Resistances Ω*				*Highest*	*Lowest*
1	1	3	3	7	14 Ω	1
2	3	7	4	15	29 Ω	
3	1	5	9	12	27 Ω	
4	2	2	2	2	8 Ω	0.5 Ω

Parallel

Series

1) $\frac{1}{r_{eq}} = \frac{1}{1} + \frac{1}{3} + \frac{1}{3} + \frac{1}{7} = \frac{38}{21}$

$r_{eq} = 1+3+3+7 = 14\,\Omega$

$r_{eq} = \frac{21}{38}\,\Omega$

2) $\frac{1}{r_{eq}} = \frac{\overset{20}{1}}{3} + \frac{1}{7} + \frac{\overset{15}{1}}{4} + \frac{\overset{4}{1}}{15} = \frac{39}{60} + \frac{1}{7}$

$r_{eq} = 3+7+4+15 = 29\,\Omega$

$= \frac{60+273}{420} = \frac{333}{420}$

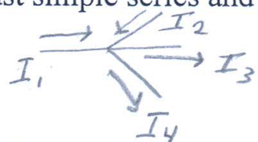
$r_{eq} = \frac{420}{330}\,\Omega$

3) $\frac{1}{r_{eq}} = \frac{\overset{180}{1}}{1} + \frac{\overset{36}{1}}{5} + \frac{\overset{20}{1}}{9} + \frac{\overset{15}{1}}{12} = \frac{251}{180}$

$r_{eq} = \frac{180}{251}\,\Omega$

Kirchoff's Rules:

Kirchoff's rules are required for more complex circuits than just simple series and parallel. The are two, the Junction rule and the Loop rule:

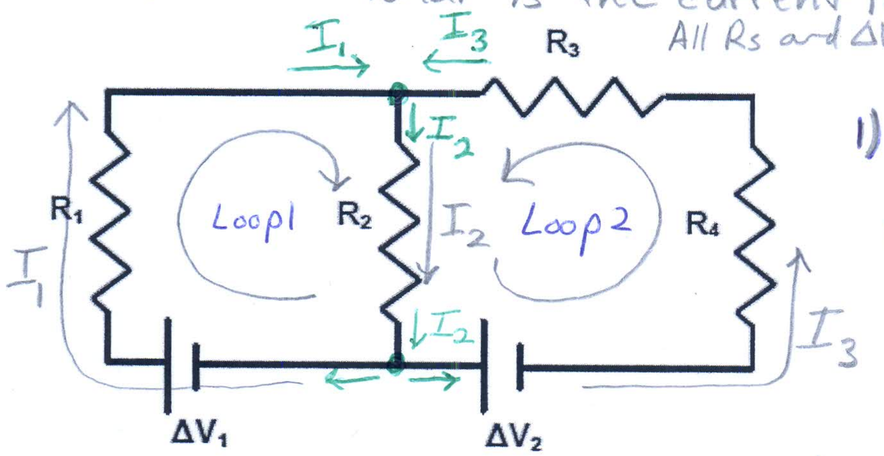
Ex: 

$I_1 + I_2 = I_3 + I_4$

Kirchoff's Junction Rule: The Sum of the currents into any junction is 0. Stated another way, any current that flows in must flow out. This is because charge is conserved.

Kirchoff's Loop Rule: The sum of the potential differences in each loop must equal 0, this includes potentials from EMFs, resisters, capacitors and inductors. This is because of conservation of energy

Let us analyze this circuit: What is the current in each loop? All Rs and ΔVs are known.



1) Pick a direction for loops, chosen direction is not critical. In this example there are 3 loops, but we only need 2 of them.

2) Use Kirchoff's Loop rule to create $\Sigma \Delta V = 0$ equation.

Loop 1: $\Delta V_1 - I_1 R_1 - I_2 R_2 = 0$

Loop 2: $-\Delta V_2 - I_3 R_4 - I_3 R_3 - I_2 R_2 = 0$

Note: ΔV_2 is (−) because it faces the opposite direction of the assumed current in the loop.

3) Use Kirchoff's Junction rule to eliminate one of the currents

$I_2 = I_1 + I_3$

$\Delta V_1 - I_1 R_1 - I_2 R_2 = 0$

$-\Delta V_2 - I_3 \underbrace{(R_4 + R_3)}_{\text{In series}} - (I_1 + I_3) R_2 = 0$

4) If the potential differences ΔV and resistances R are known, then the above system of equations can be used to find I_1, I_2 and I_3. Note: if the answer is negative the current is in the opposite direction of that assumed by the loop.

Current-Measuring Devices.

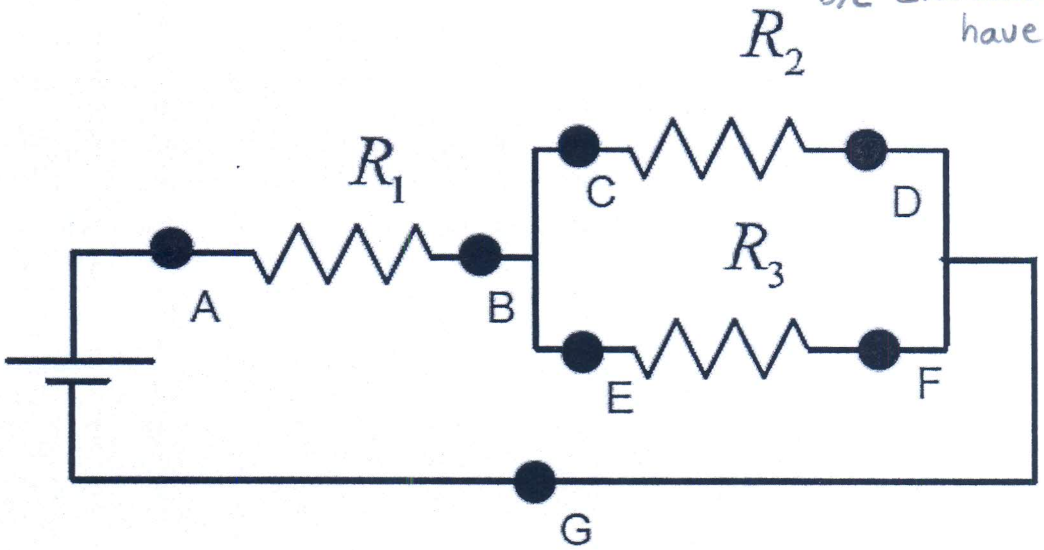
A D'Arsonval Galvnometer can be used to determine the current, electric potential, and the resistance in a circuit based on the interaction of an electric current with a magnetic field. This is an analog device and is rather old fashioned compared to digital ammeters, voltmeters and ohmmeters.

A D'Arsonval Galvnometer is comprised of a coil of fine wire that is placed in a permanent magnetic field. A spring is attached to the coil and when a current is present the magnetic field exerts a torque on the coil.

Ammeter: An ammeter measures current in a circuit, and must be installed in series with the element that it's trying to measure. b/c Elements in series have the same current.

Voltmeter: A voltmeter measured the Potential Difference ie Voltage between two points in a circuit, and is placed in parallel with the element that it's trying to measure.

b/c Elements in parallel have the same ΔV



In the circuit above indicate where you would place your probes to measure:

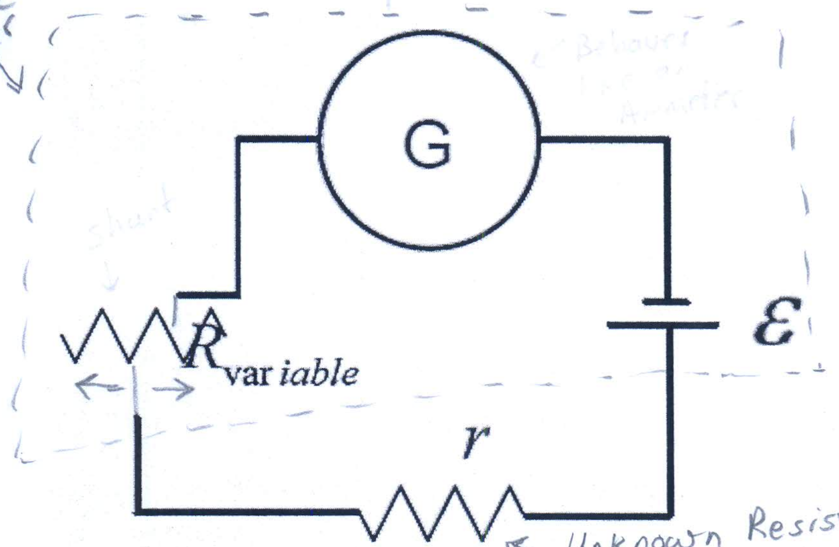
	R1	R2	R3	Battery B
Current	A R_1 A A or B	A R_2 A C or D	E or F R_3 A A	A A A A G
Voltage Electric Potential Difference	A R_1 B V	C R_2 D V	V E R_2 F	A V G
	A & B	C & D	E & F	A and G

G, B or A
Same loop

Ohmmeter: In order to measure the resistance in a circuit an Ohmmeter, which consists of a battery of known $\mathcal{E}$, a variable resistor and a galvanometer, is used. The deflection of the galvanometer is dependent on the equivalent resistance of the variable resistor, ε, and the unknown resistor r. Since all quantities are known, the ohmmeter is calibrated to find r.

An ohmmeter is placed in series with the resistor, and not in a circuit.

inside meter



Unknown Resistor or circuit element with resistance.

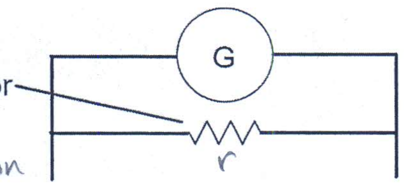
$V = IR$

$$R_{eq} = \frac{V}{I} \Rightarrow R_{var} + r = \frac{\mathcal{E}}{I}$$

$$\Rightarrow r = \frac{\mathcal{E}}{I} - R_{var}$$

Both voltmeter and ammeter require a shunt, which is a resistor to allow the safe measuring of very high values.

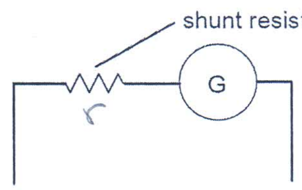
Ammeter:



r is known

$I = I_r + I_G$

Voltmeter



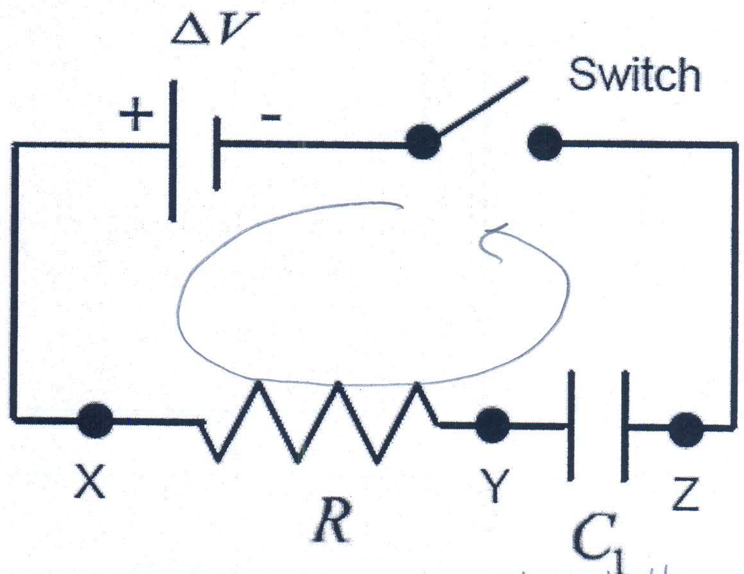
very large R, so $I \simeq 0$

$V = V_r + V_G$

$C = \frac{Q}{\Delta V}$

capacatance

RC Circuits:



When the switch is closed, the charge on the capacitor starts at ___0___, and so the battery can ___charge___ the capacitor. $I = max$

At a later time, the voltage on the capacitor is ___equal___ to the voltage on the battery. $I = 0$

A charged capacitor acts as a ___potential diffrece like a battery___, while an uncharged capacitor acts ~~as a~~ like ___a wire___. When charged, the charge on the capacitor is given by ___$Q = C \cdot \Delta V$___

So let's analyze this using Kirchoff's Loop Rule to find the charge on the capacitor at any given moment:

① Loop Rule

$$\Delta V - \underbrace{iR}_{\Delta V_R} - \underbrace{\frac{q}{C}}_{\Delta V_C} = 0$$

② put in diferential form

$$\Delta V - \frac{dq}{dt}R - \frac{q}{C} = 0$$

$V = IR$

$C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C}$

③ Solve for $\frac{dq}{dt}$

$$\frac{dq}{dt} = \frac{\Delta V}{R} - \frac{q}{RC} = \frac{\Delta VR - q}{RC}$$

④ put all qs on one side

$$\frac{1}{\Delta VR - q}\,dq = \frac{1}{RC}\,dt$$

⑤ Multiply both sides by -1

$$\frac{1}{q - \Delta VR}\,dq = -\frac{1}{RC}\,dt$$

⑥ Intigrate

$$\int_0^q \frac{1}{q - \Delta VR}\,dq = \int_0^t -\frac{1}{RC}\,dt$$

$$\Big|_0^q \ln(q - \Delta VR) = \Big|_0^t -\frac{t}{RC}$$

$$\ln(q - \Delta VR) - \ln(-\Delta VR) = -\frac{t}{RC}$$

⑦ simplify

$$\ln\left(\frac{\Delta VR - q}{\Delta VR}\right) = -\frac{t}{RC}$$

$e \qquad e$

⑧ raise to e

$$\frac{\overset{Q}{\Delta VR} - q}{\underbrace{\Delta VR}_{Q}} = e^{-\frac{t}{RC}}$$

⑨ simplify

$$\frac{Q - q}{Q} = e^{-\frac{t}{RC}}$$

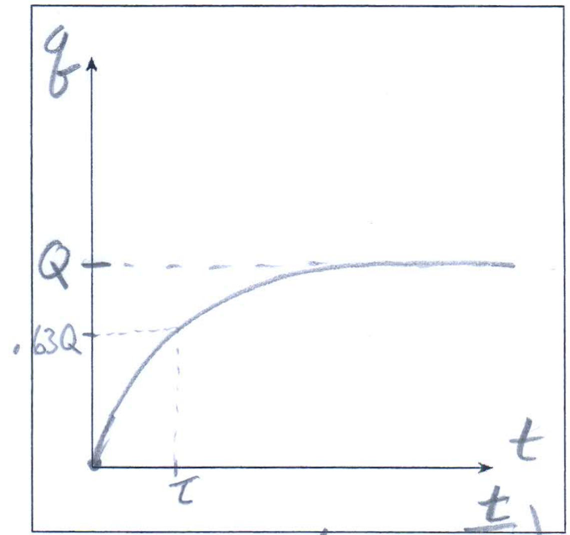
$$\boxed{q = Q\left[1 - e^{-\frac{t}{RC}}\right]}$$

Now let's find the current over time by taking the time derivative of the charge:

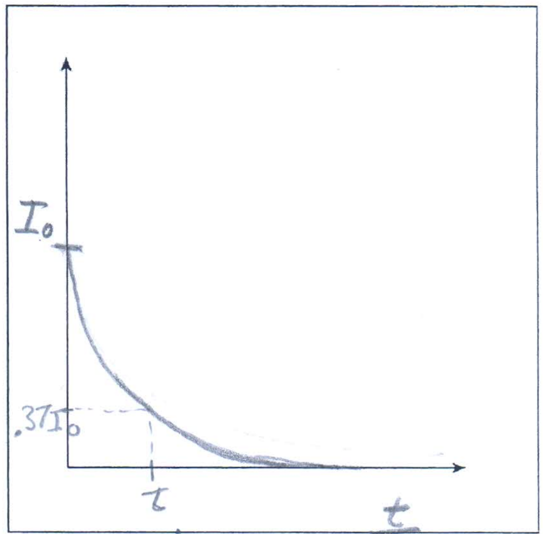
$$i = \frac{dq}{dt} = \underbrace{\frac{Q}{RC}}_{I_0} e^{-t/RC} = I_0 e^{-t/RC}$$

$$\frac{\cancel{Q}}{R\left(\frac{\cancel{Q}}{\Delta V}\right)} = \frac{\Delta V}{R} = I$$

Let's plot both charge q and current i on the graphs below:



$q = Q(1-e^{-\frac{t}{RC}})$



$i = I_0 e^{-\frac{t}{RC}}$

So you notice that both i and q are dependent on ___RC___, this is called the ___time constant___ and is represented by ___τ___. We first saw one of these when we looked at ______________.

$\tau = RC$

You find ___τ___ graphically by ___percentage___, which means that you can find it from a graph by looking when a graph has reached ___63%___ of its max or minimum.

ie $\boxed{q(t=\tau)} = Q(1-e^{-\frac{\tau}{\tau}}) = Q(1-0.37) = \boxed{0.63Q}$

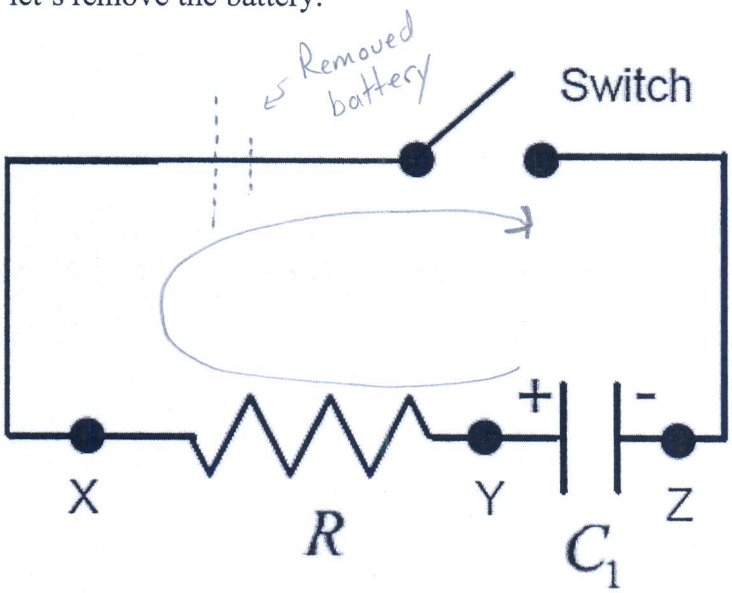
$\boxed{i(t=\tau)} = I_0 e^{-\tau/\tau} = \boxed{0.37 I_0}$

$\tau = RC$

- if $R\uparrow \Rightarrow I\downarrow \Rightarrow$ takes longer to charge
- if $C\uparrow \Rightarrow$ capacitor can hold more charg for a given $\Delta V \Rightarrow$ takes longer to charge

$q = Q(1-e^{-\frac{t}{\tau}})$, $i = I_0 e^{-\frac{t}{\tau}}$

Now let's remove the battery.



What happens to the current when the switch is closed?

The capacitor acts like a battery.

- I moves in the opposite direction of the original current.
- At start ΔV and I_o will be max and then will decrease to 0 over time.

Using Kirchoff's loop rule, find the current and charge expressions for the discharging capacitor.

$C = \frac{Q}{\Delta V}$

$i = \frac{dq}{dt}$

$$\frac{Q}{C} - iR = 0 \Rightarrow i = \frac{Q}{RC} \quad \text{Since } i\downarrow \quad \frac{dq}{dt} = -\frac{q}{RC}$$

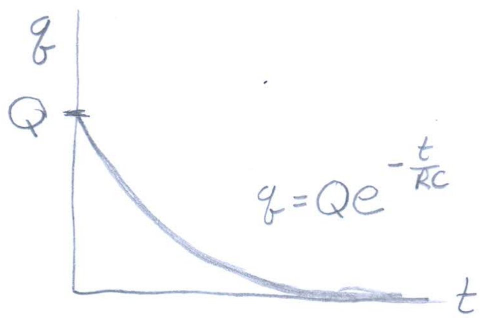
② Intigrate to find $q(t)$

$$\int_Q^q \frac{1}{q}\,dq = \int_0^t -\frac{1}{RC}\,dt \Rightarrow \Big|_Q^q \ln q = \Big|_0^t -\frac{t}{RC}$$

$$\Rightarrow \ln(q) - \ln(Q) = -\frac{t}{RC}$$

$$e^{\ln\left(\frac{q}{Q}\right)} = e^{-\frac{t}{RC}}$$

$$\boxed{q = Qe^{-t/RC}}$$



③ Differentiate to find $i(t)$.

$$\frac{dq}{dt} = -\frac{Q}{RC}e^{-t/RC} = -I_o e^{-t/RC}$$

$$\boxed{I = \frac{\Delta V}{R} \qquad \Delta V = \frac{Q}{C}}$$