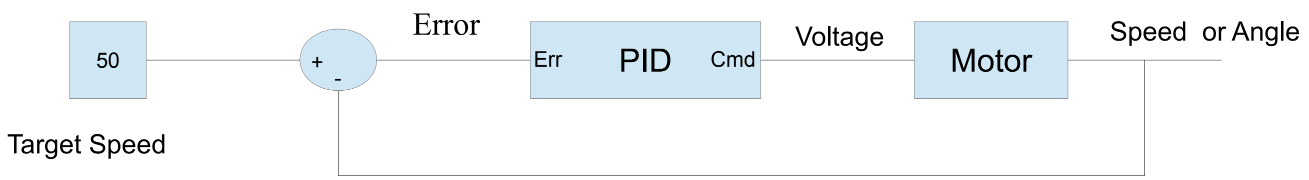
The PID Controller Theory Dan Fabrizio

First, we will discuss the general theory of operation of PID controllers. Then we will show specific examples with the components used in the FRC robot.

The PID controller is commonly used to drive a DC motor. In the diagram below the input to the PID controller is the speed or velocity error and output from the PID controller is the voltage applied to the motor.



feedback from sensor, encoder, or pot

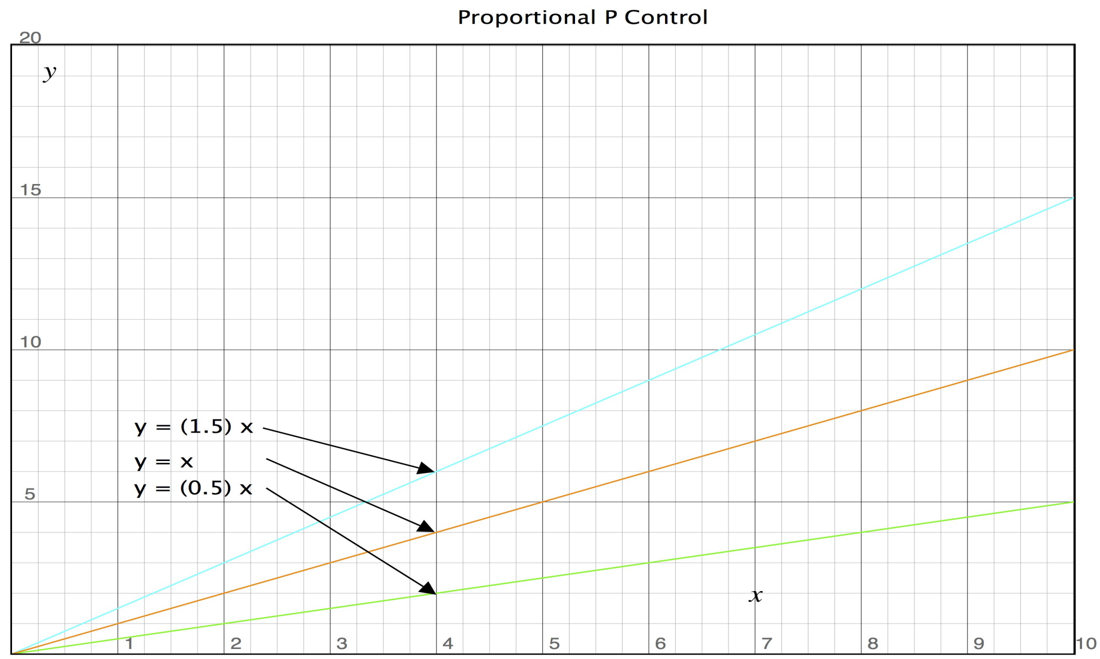
There are 3 circuits in a PID controller. The proportional (P), Integral (I), and Derivative (D) circuits. We will see that the PI circuits are commonly used to control motor speed and the complete PID controller is used for position control.

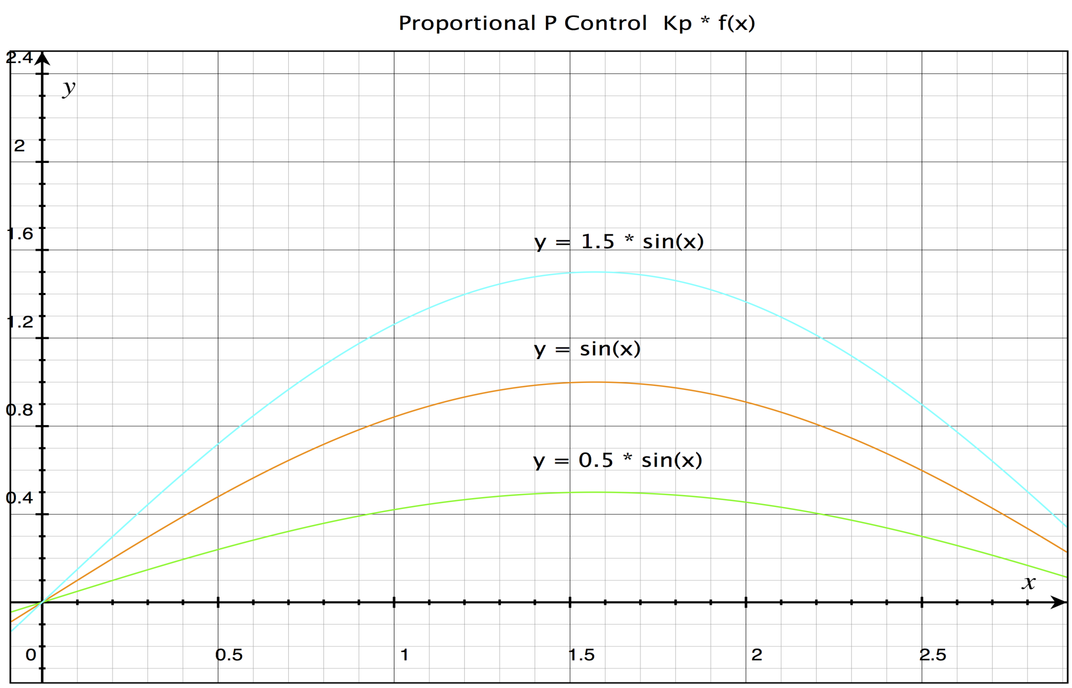
A little calculus theory is needed to understand the relationships with the I and D sections so we will keep this discussion to mathematical concepts and theory of operations. You will not be expected to solve calculus problems to understand these concepts.

The Proportional Section (P) of PID controller.

$y = K_p \cdot x$ Multiply the input x by a constant K_p

$y = K_p \cdot f(x)$ Multiply the input function f(x) by a constant K_p





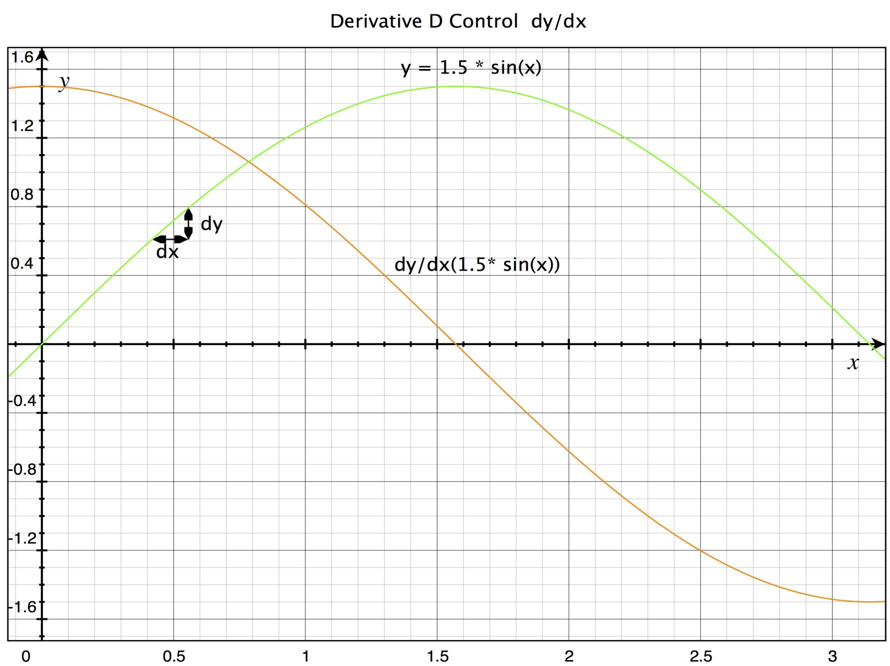
The Derivative Section (D) of the PID controller.

$\dot{y} = dy/dx = \Delta y/\Delta x = m = slope\ of\ the\ line$ The derivative is indicated by the dot over y

It is the rate which the y changes with respect to x. This derivative of y with respect to x is also the slope of a line that is tangent to point x on the curve.

$y = mx + b$ Equation for line. Slope is m, b is y intercept

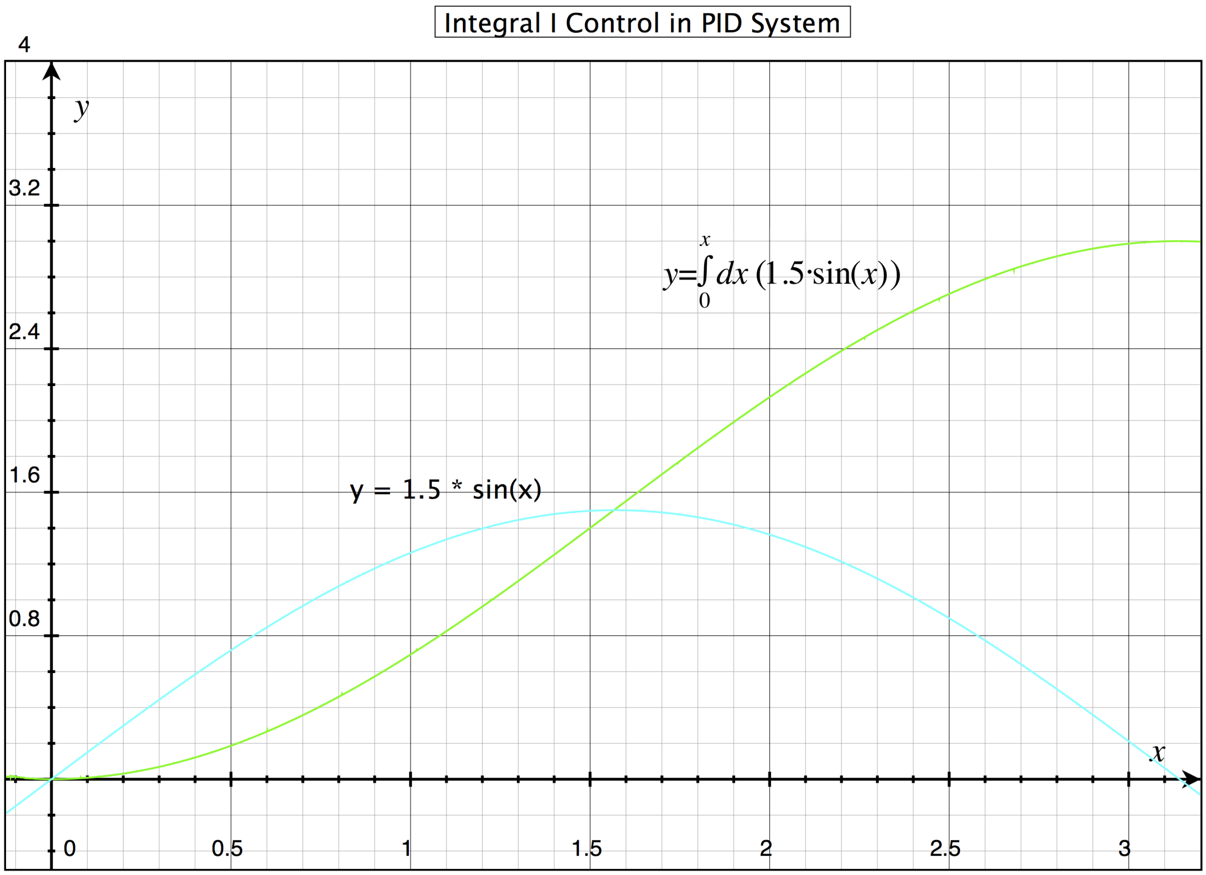
If dx is very small, then the derivative of y with respect to x is dy/dx. The slope is positive when the values in the curve are increasing, negative when decreasing, and zero when constant.



The Integral Section (I)

$$y = \int f(x)\,dx$$

The Integral function in theory solves for the area under a curve. For example: If f(x) is a sine wave that represents the voltage level of a AC signal, then area under the curve of the sine wave signal is considered the average voltage of the signal.



As you can see in the diagram above the integral of the sin(x) or the area under the curve is increasing even as y decreases. As the sine wave approaches zero, the integral or area stays somewhat constant.

Acceleration, Velocity, and Distance

Now that we understand the basics of each section of the PID controller, we can apply these concepts to our motor control system.

Velocity and Distance Relationship Defined:

$$velocity = dDistance / dTime = dx / dt$$

$$\int velocity = \int dx / dt$$

$$x = Distance = \int velocity\ dt$$

Velocity is the change in distance divided by the change in time. Which is the same as the derivative of distance with respect to time. Distance and position are interchangeable. If we integrate both sides and solve, we can find the Distance or position.

Acceleration and Velocity Relationship Defined:

$$Acceleration = dVelocity / dTime = dv / dt$$

$$\int acceleration = \int dv / dt$$

$$v = velocity = \int acceleration\ dt$$

Acceleration is the change in velocity divided by the change in time. Which is the same as the derivative of velocity with respect to time. If we integrate both sides of the equation and solve, we can find the velocity.