2.2.2 Desirable properties of point estimators

- We discussed several methods of obtaining point estimators.
- It is possible that different methods of finding estimators will lead to same estimator or different estimators.
- In this section we discuss certain properties, which an estimator may or may not posses, that will guide us in deciding whether one estimator is better than another.

2.2.2.1 Unbiasedness

Definition: Unbiased estimator

An estimator $\hat{\theta}$ $(= t(X_1, X_2, \dots, X_n))$ is defined to be an **unbiased estimator** of θ if and only if

$$E(\hat{\theta}) = \theta$$

- The difference $E(\hat{\theta}) - \theta$ is called as the bias of $\hat{\theta}$ and denoted by

$$Bias(\hat{\theta}) = E(\hat{\theta}) = \theta$$

- An estimator whose bias is equal to 0 is called **unbiased**.

2.2.3 Consistency

Mean-Squared Error

- The *mean-squared error* is a measure of goodness or closeness of an estimator to the target.

Definition: Mean-squared Error (MSE)

The **mean-squared error** of an estimator $\hat{\theta}$ of θ is defined as

$$MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right]$$

- The MSE measures the average squared difference between $\hat{\theta}$ and θ.
- The MSE is a function of θ and has the interpretation

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + \left[Bias(\hat{\theta})\right]^2$$

- Therefore the MSE incorporates two components, one measuring the variability of the estimator (*precision*) and the other measuring its bias (*accuracy*).
- Small value of MSE implies small combined variance and bias.
- If $\hat{\theta}$ is unbiased, then
$$MSE(\hat{\theta}) = Var(\hat{\theta})$$
- The positive square root of MSE is known as the *root mean squared error*
$$RMSE(\hat{\theta}) = \sqrt{MSE(\hat{\theta})}$$

Consistency

- Estimator $\hat{\theta}$ is said to be consistent for θ if $MSE(\hat{\theta})$ approaches zero as the sample size n approaches ∞.

$$lim_{n\to\infty} E\left[(\hat{\theta} - \theta)^2\right] = 0$$

- Mean-squared error consistency implies that the bias and the variance both approach to zero as n approaches ∞.