Data Communication

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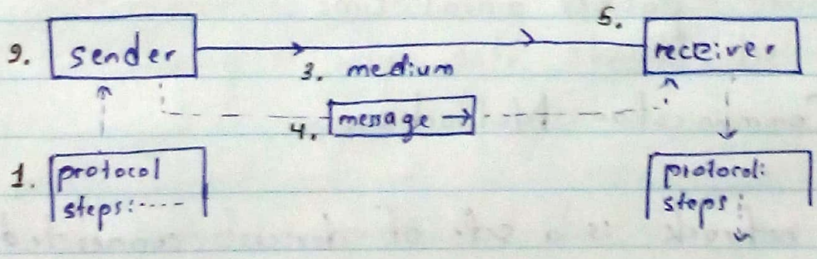
1. Introduction to Datacommunication

Communication means sharing information. Below stated some terminology commonly used in data Communication.

- Telecommunication - communication at a distance
- Data - unarranged raw facts
- Information - arranged data which conveys a message that is understood by human.

Normally data is generated by a device. Human understandable information can be represented as text, voice or video formats. Networks are used to establish to connection between the source and the destination in communication methods.

Communication system component - 5 components

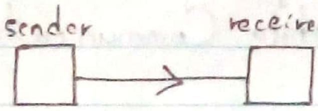


Protocols and standards are the methods and rules respectively that are followed by communitions when transferring data.

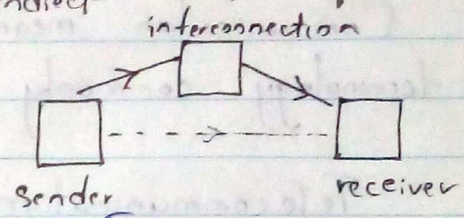
- Protocol:- is a set of rules that govern data communications, it represent the agreement between the communicating devices.
- Medium:- Transmission medium is the physical path, a message travels from sender to receiver.

Medium can be divided generally,

direct communication



intermediate / composite
indirect



Eg:- Telephone calls taken among 2 service providers.

The effectiveness of a data communication depends on,

- Delivery - system must deliver data to the correct destination.
- Accuracy - must deliver data accurately
- Timeliness - data should be delivered without any delays. Late delivered data are useless.
- Jitter - jitter refers to the variation in the packet arrival time.

* Data Communication Networks :-->

A network is a set of devices connected by communication links. These devices are often referred to as nodes.

* Network criteria

To establish a efficient network communication following criteria are important.

1. Performance - depends on number of users, transmission medium, capabilities of connected hardware. Evaluated by 'throughput' and 'delay'.

2. Reliability :- Means the accuracy / correctness of the connection measured by frequency of faliure, time taken to recover from faliure.

3. Security :- protecting data from unauthorized access, protecting data from damage and development.

Protocols and Standards.

Protocols

Even if two devices are connected together using a link they won't be able to understand each other if they communicate / interpret messages in different methods. Therefore a protocol is used to create a understanding between two devices. Then only they will be able to communicate and understand each other.
Key elements of a protocol includes,

- syntax - structure or format of data
- Semantics - meaning of each section of bits
- Timing - when data should be sent and how fast they can be sent.

Standards

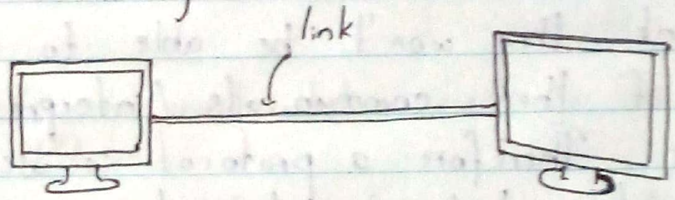
Standard provides guidelines to the users of a particular communication system. Standard can be devided in to two catagories :

- De facto (by fact) - standard that have not been approved by an organization but followed because they are generally accepted in the society
- De jure (by law) - clear written standards set by a set of group/organizations in order to keep control of the communication flow.

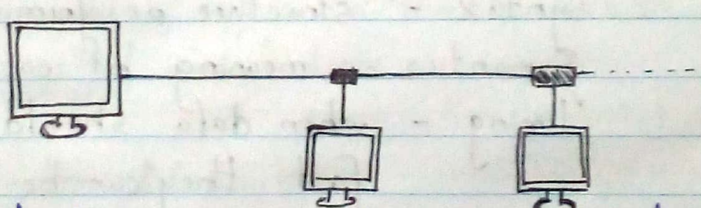
* Line configuration →

When connecting devices for communication there are two possible types of connections;

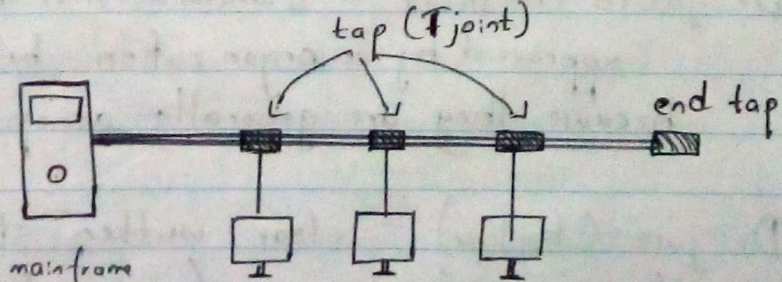
point-to-point :- provides a dedicated link for the communication between sender and receiver. The entire capacity of the link is reserved for transmission between those two devices only



multi-point :- this multi-point connection includes more than two devices to share a single link.



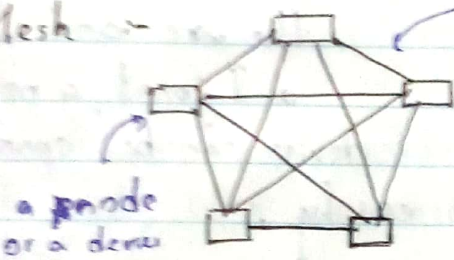
(if several devices use the link simultaneously, the ~~link~~ capacity of the link divided among the devices which is called 'spatially shared' and if the users use the entire capacity but they have to take turns to use the link, it is called a timeshared connection.



Topologies →

Topology is a physical structure that defines the way which network is laid out (how devices are connected) physically. Two or more links form a topology. There are many types of topologies. Below mentioned are, commonly used topologies and their ~~d~~ advantages and disadvantages.

- Mesh :-



dedicated point-to-point links

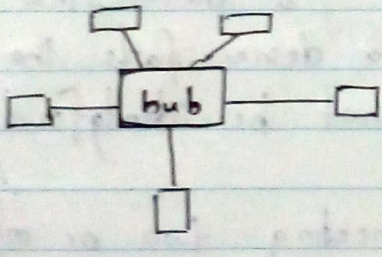
if there's n number of nodes in a half duplex (data can be communicated both directions)

$n(n-1)/2$ number of links are needed

adv : If a connection broken between 2 devices. It can be identified easily and even if that direct connection is lost the communication can be established using indirect method.

dis adv : when using direct connection the no. of links required to connect each other increase rapidly.

- Star :-

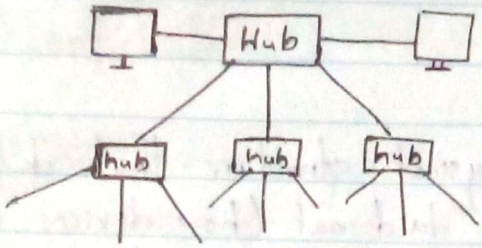


adv : to add a new device only one connection is needed. When a connection failed it can be easily identified.

disadv : When a connection is lost between device and the hub, the device cannot communicate until the connection is reestablished. If the hub fails all the connections terminates.

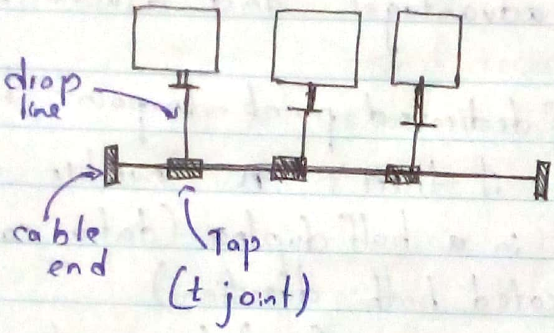
↳ This faliure is called a single point faliure, the faliure in a single point affects entire communication.

- Tree :-



Same as star topo.
(Can connect many devices)

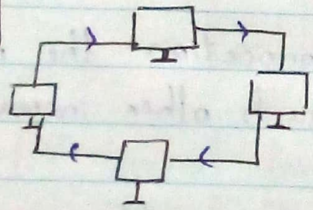
- Bus :-



adv:- low cost - by cutting the wire and using a T-joint, a new device can be connected easily.

disadv :- If the ends are not properly closed (terminated) the ~~circ~~ connection acts as a open circuit causing entire system to fail.

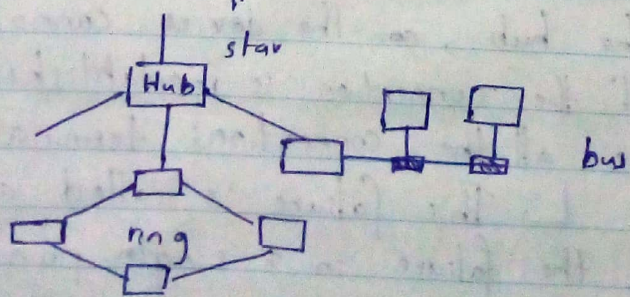
- Ring -



disadv :- the network work as a cycle, also it has a data flow direction. if a device fails the system fails and cannot be identified easily.

- Hybrid - develops connecting two or more topologies of above types.

Eg:-



* Transmission modes →

A Communication system can be catagorized in to 3 type according to their data flow direction

① Data flow

- Simplex
- half - duplex
- full - duplex

- Simplex mode is unidirectional. Only one device can transmit data while other device receives.

~~o Half~~

- half duplex mode both devices can transmit and receive data but only one can tranmit data while other receives at a time.

- full duplex ~~can~~ mode allows both devices to receive and transmit data simultaneously. Here transmition can be done using the shared link → two types of shared link
  - 2 transmission path - one to receive and one to transmit
  - or
  - divide the capacity of the channel between two devices.

Network models

LAN - local area network - privately owned links.

MAN - metropolitant area network - area covering a town or city.

WAN - wide - area network - for long distance transmission over a large geographic areas.

2. Data and Signals

The fundamental requirement for a communication to take place is a signal. Normally human understandable data are not in a form ~~of~~ that can be transmitted over a communication networks.

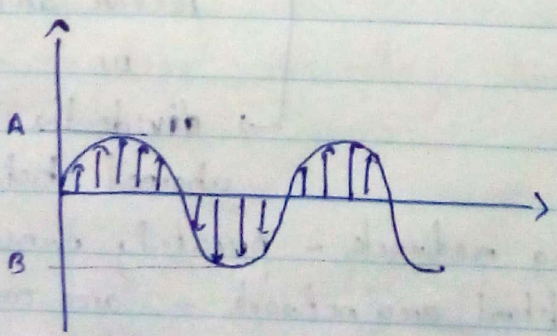
Eg:- a sound wave should be converted into a electro-magnetic signal to transmit over a wire.

Transmission media work by conducting energy along a physical path. Therefore, to transmit data, they must be transformed to electro-magnetic signals. Signals can be categorized into few types,

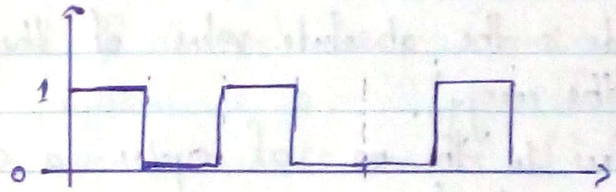
i) Mechanical waves and Electro-magnetic waves
ii) Analog / digital signals
iii) Periodic / Aperiodic signals.

Analog and Digital signals →

Analog signals carries continuous values. This means a wave move from value A to value B passes through an infinite number of values, along it's path.

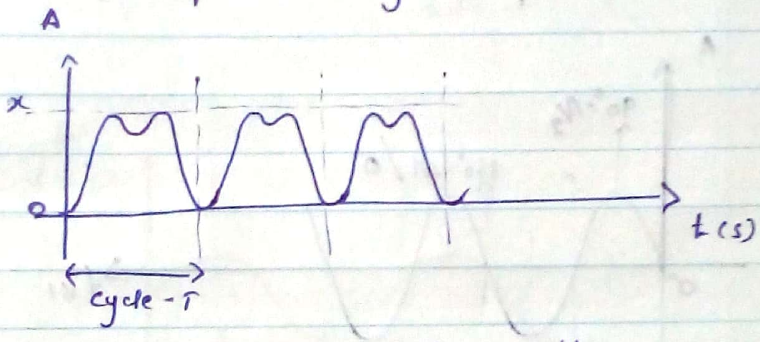


Digital data has few defined discrete values. Normally takes 1's and 0's

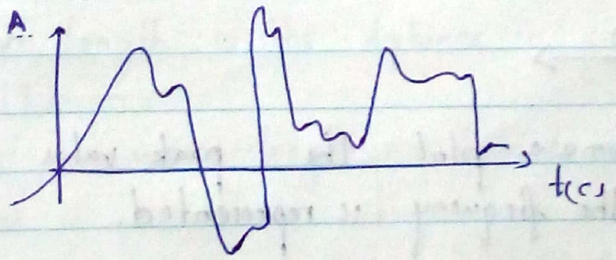


- Periodic and Nonperiodic signals

If a signal shows a repeating pattern over a identical periods it's a periodic signal.



Signals, shows a non repeating pattern are nonperiodic / aperiodic signals.

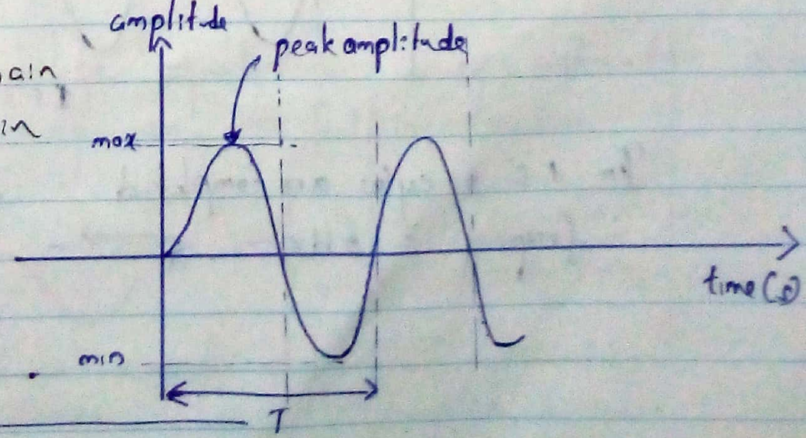


- Representation of a signal

We can represent a signal on a x - y axis graph depending on the scale of x axis we can represent two types →

frequency domain
Time domain

- a fundamental wave/signal



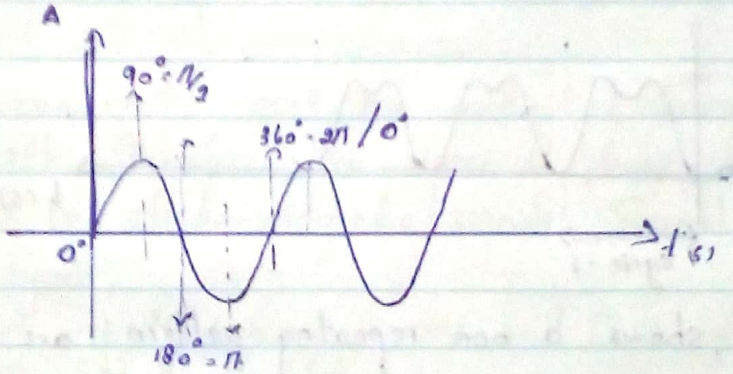
phase - 2π

Peak amplitude is the absolute value of the highest intensity of a signal (the energy).

Frequency refers to the no. of cycles a signal completes in a 1 time unit / 1 s.

$f = \frac{1}{T}$ T - period.

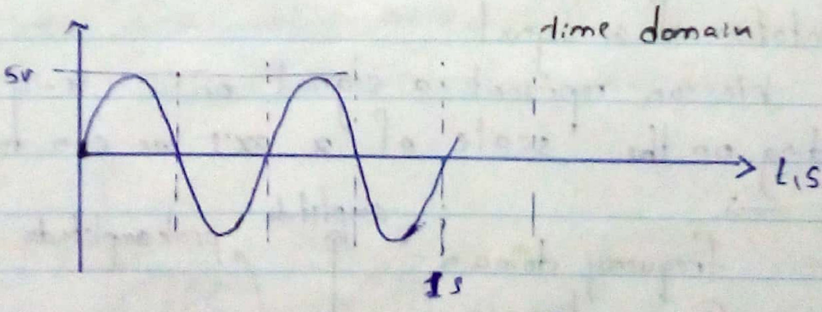
Phase describes the position of the wave form relative to time 0.



- frequency domain →

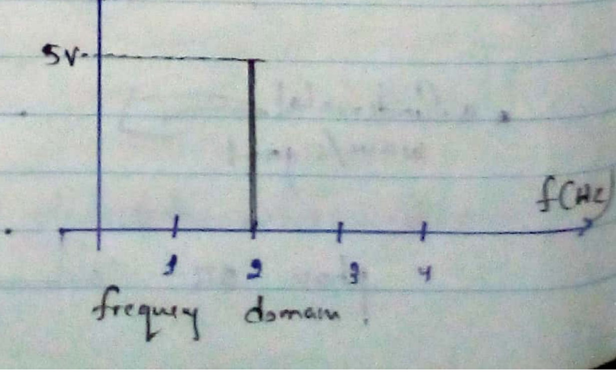
In frequency domain plot the peak value of the signal against the frequency is represented.

Eg:-

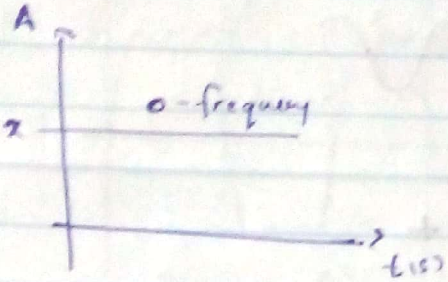


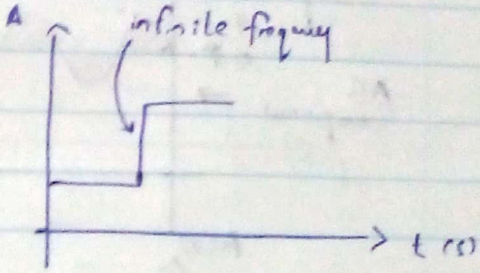
In 1 s 2 cycles are completed

∴ frequency = 2 Hz →

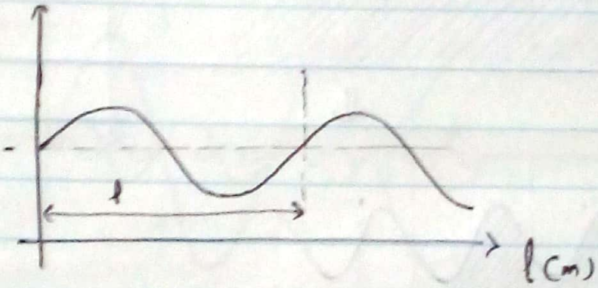


If a signal ~~f~~ does not change (remains at a constant amplitude) ~~it's~~ its frequency is 0. If the ~~frequency~~ signal changes in no time from one level to another it's frequency is said to be infinite.





- wave length



wave length is the distance a signal can travel in one time period (T).

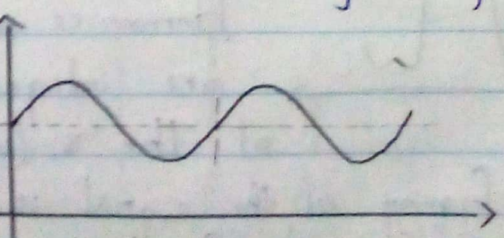
$$\therefore \underset{\lambda}{\text{wave length}} = \text{distance} = \text{propagation speed} \times \text{period}$$

$$= \text{propagation speed} \times \frac{1}{\text{frequency}}$$

(propagation speed = speed of the signal)

- Composite / Complex signals →

We require composite signals to carry information since sine wave cannot convey any useful information

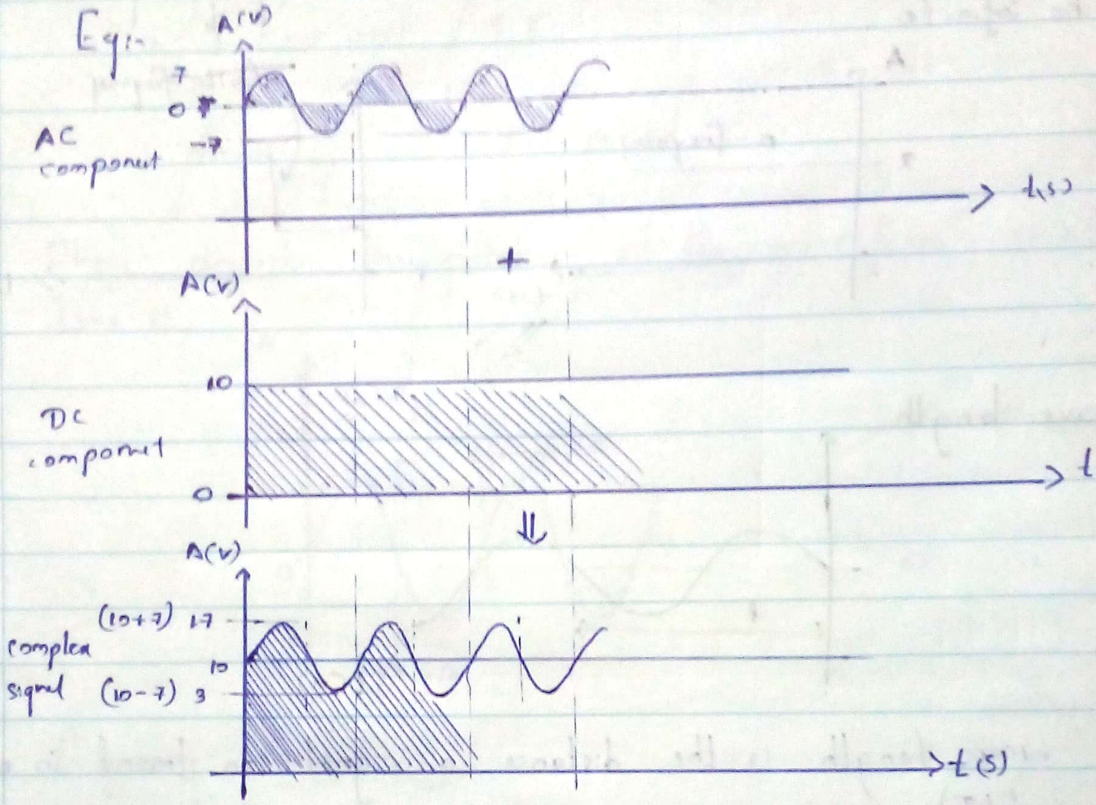


simple sine wave
- no specific information -

Complex signal
- carries information

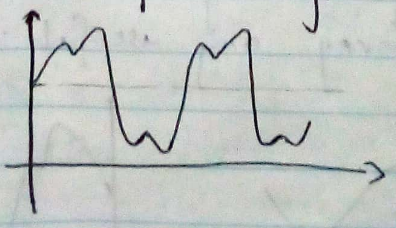
Complex signals occur when 2 or more ~~X~~ different signals added or subtracted from/with each other.

Eg:-



Jean-Baptiste Fourier showed that a composite signal can be decomposed into set of simple sine waves. These simple sine waves are called 'the harmonics' of the composite signals. Harmonics have different ~~of~~ amplitude and frequency values.

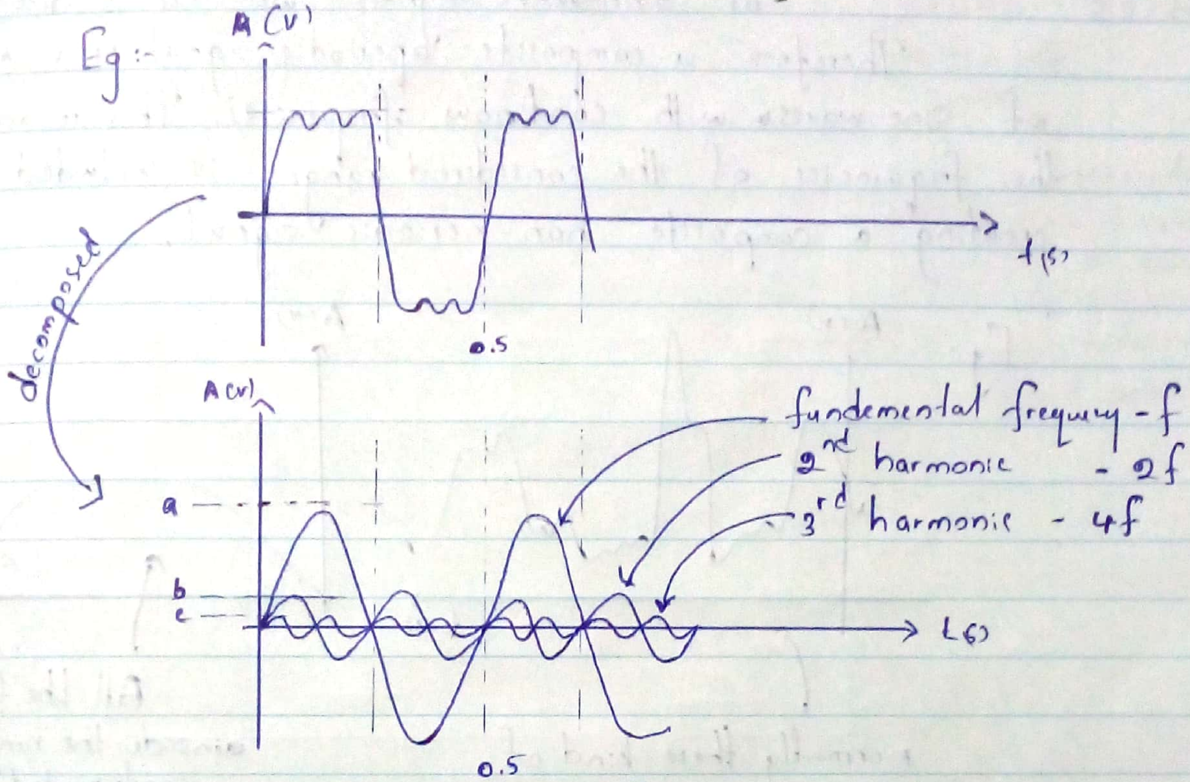
↳ periodic composite signals - periodic signal has a discrete number of frequency. Harmonics of the signal are integer multiplication of the ~~s~~ fundemental signal.



If the fundemental frequency of the signal is 'f' the harmonics have (1f, 2f, 3f, ...) frequencies.

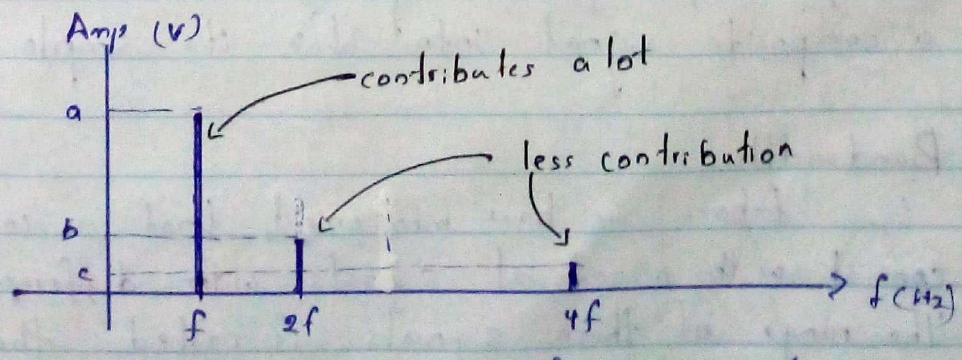
fundemental frequency is the 1st harmonic of a composite signal which has the same frequency of composite signal.

Eg:-



if the fundemental frequency of above = f, the second harmonic has a frequency of 2f since it completes 2 cycles when 1st harmonic (fundemental freq signal) completes 1 cycle. Same way 3rd harmonic = 4f, since it completes 4 cycles.

Normally fundemental frequency signal contributes a larger portion to the composite signal ~~the~~ than other harmoics. Therefore ~~the~~ f.f. signal amplitude is higher than harmoic values.



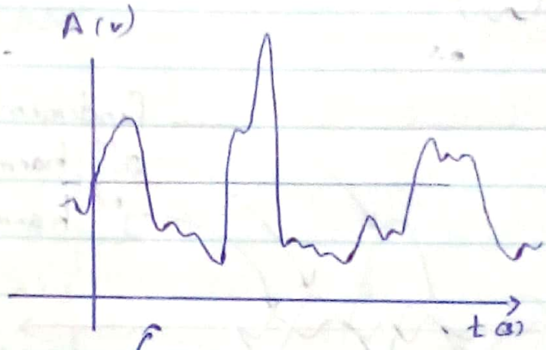
frequency - domain of above signal.

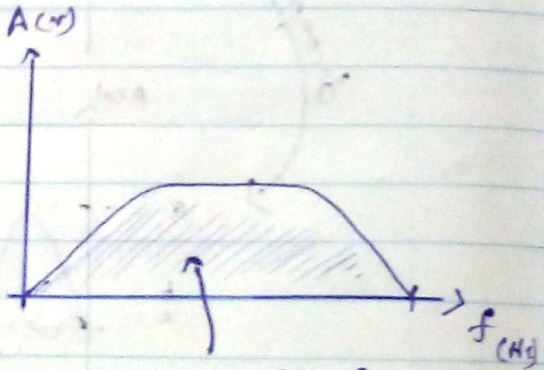
↳ non-periodic / aperiodic composite signals.

- literally, aperiodic signal has an infinite number of sine wave signals when it is decomposed.

Therefore a composite aperiodic signal is a combination of sine waves with continous frequencies. This mean, all the frequencies of the considered range is included when creating a composite non-periodic signal.

Eg:-





normally these kind of signals are the common signals we found in real life.

All the frequencies inside the range are contributed to compose the wave

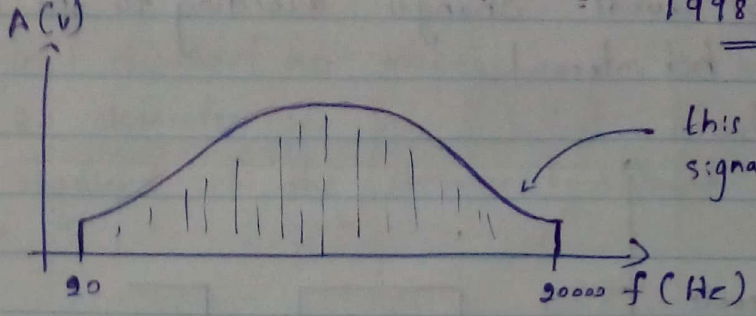
(To decompose a signal into its fundamental waves there are some devices such as spectruscope. decomposing is a difficult task, ~~therefor~~ therefore we can only decompose the signals ~~up~~ to given number of simple waves. Using Fourier series (for composite periodic signals) and Fourier transform (for aperiodic signals) are used to decompose a composite signal into its simple form.)

- **Bandwidth**

Above, we have mentioned that a composite signal consist with a set of signals with different frequencies. The range of these signals is called "the bandwidth".

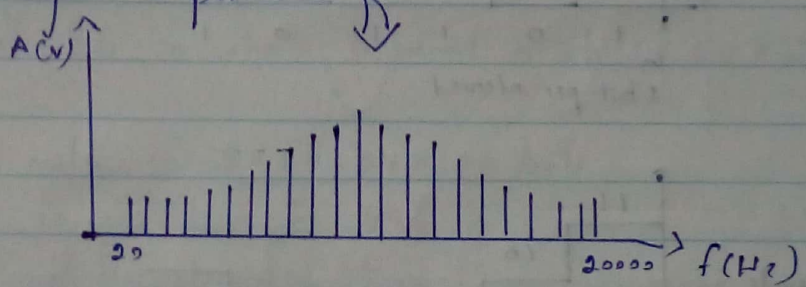
Eg:- If a composite signal contains frequencies between 20 Hz - 20000 Hz

the bandwidth of the signal = (20000 - 20) Hz
= 19980 Hz



this is a aperiodic signal since it contains all the values of frequencies in between 20 Hz and 20000 Hz

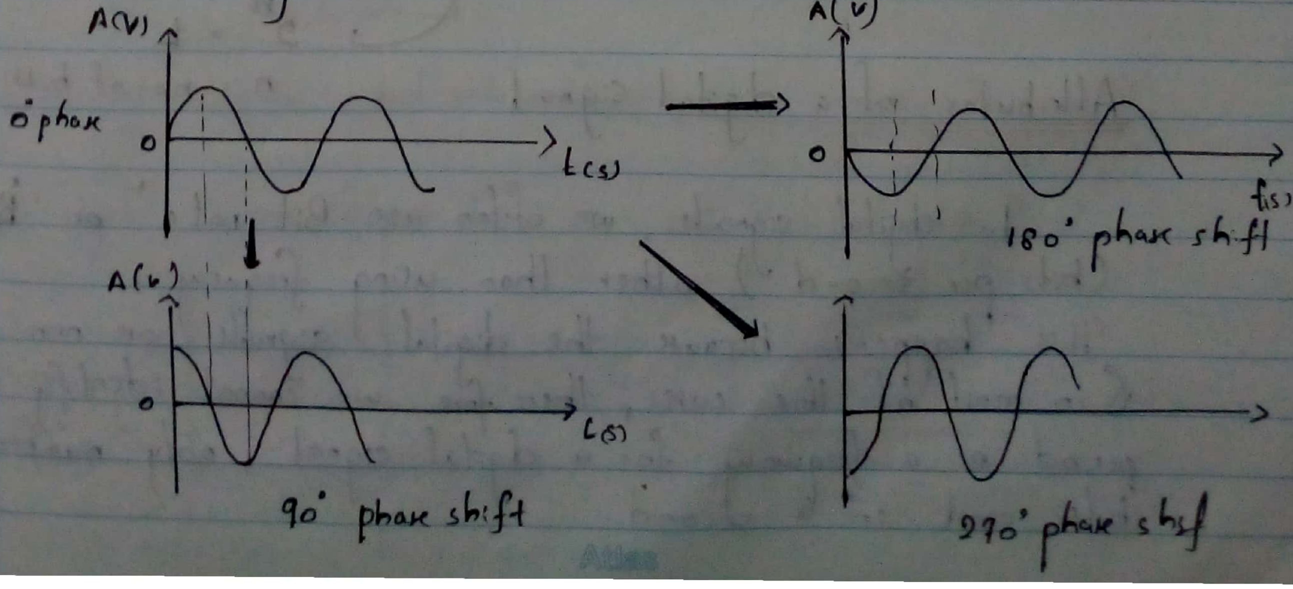
if the signal is periodic



(The bandwidth is the difference between the highest and the lowest frequency of a composite signal.)

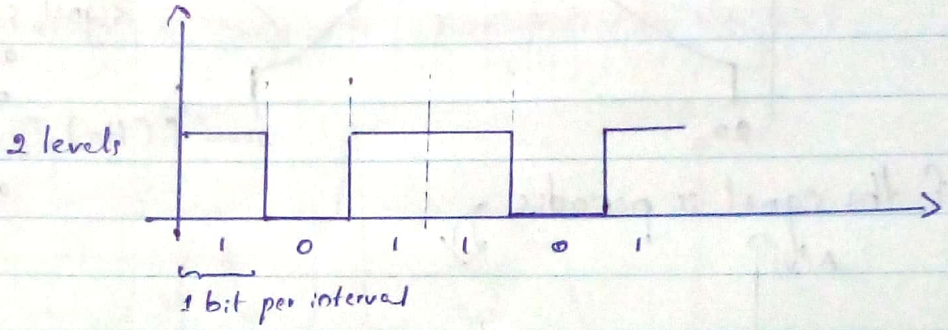
In practical cases ~~smaller~~ harmonics with smaller values (Amplitude) are ignored, because they ~~only~~ contribute less than others. But this ignoring causes distrotions.

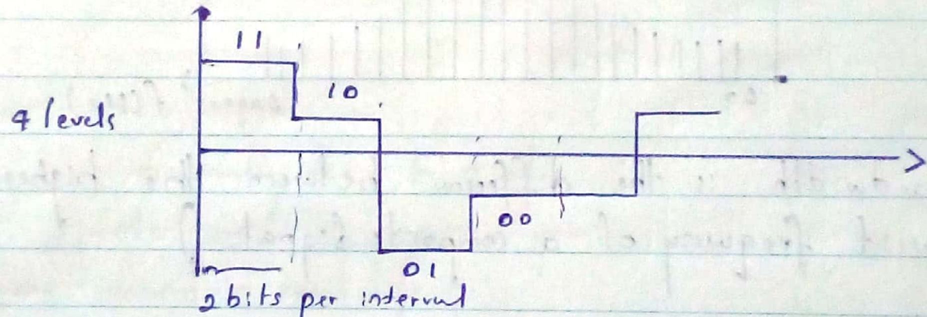
Phase shifting



Digital Signals →

Digital signals have two or more discrete levels. No of levels changes according to the bits used to represent one bit interval.





relationship between levels and no. of bits can be identified using,

$$\text{No. bits per level} = \log_2 L \qquad L - \text{no of levels.}$$

$$2^n = L$$

n = no of bits p

Attributes of a digital signal

In digital signals, we often use 'Bit rate' or 'bps' (bits per second) other than using frequency. This happens because the digital signals are non-periodic in most of the cases, therefore we cannot identify a period or a frequency for a digital signal, only number of bits sent in a second.

Bit rate is the number of bits sent in 1s. The unit is bps.

Mbps - mega bits per second

Eg: A text document has 100 pages with 24 lines each page and 80 characters per line. What should be the bit rate required if the document should be downloaded in 1 min.

bits per character = 8

$\therefore$ no of bits in whole document = 8 bits $\times$ 80 $\times$ 24 $\times$ 100

$$\therefore \text{bit rate} = \frac{8 \times 80 \times 24 \times 100 \text{ bits}}{60 s}$$

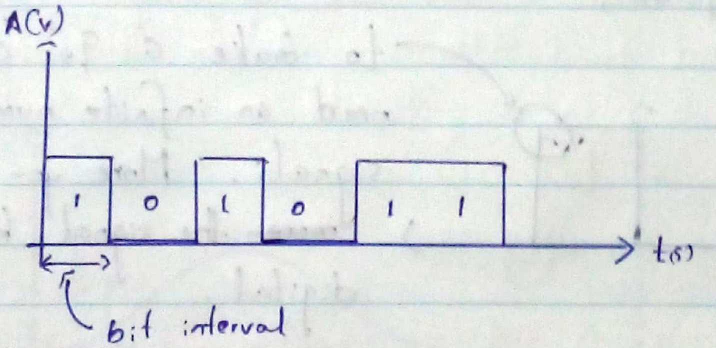
$= \cancel{1.636 \text{ Mbps}}$

$= 25,600$ bps

$= 25.6$ kbps

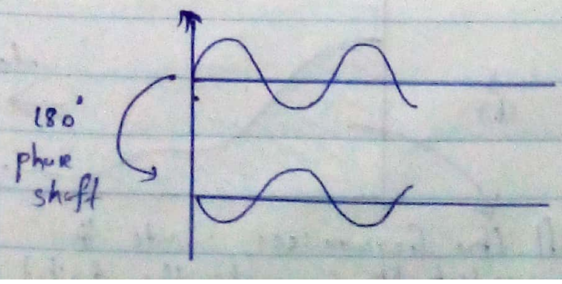
mega - 10^6

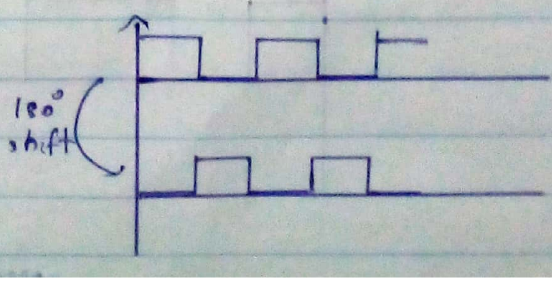
kilo - 10^3



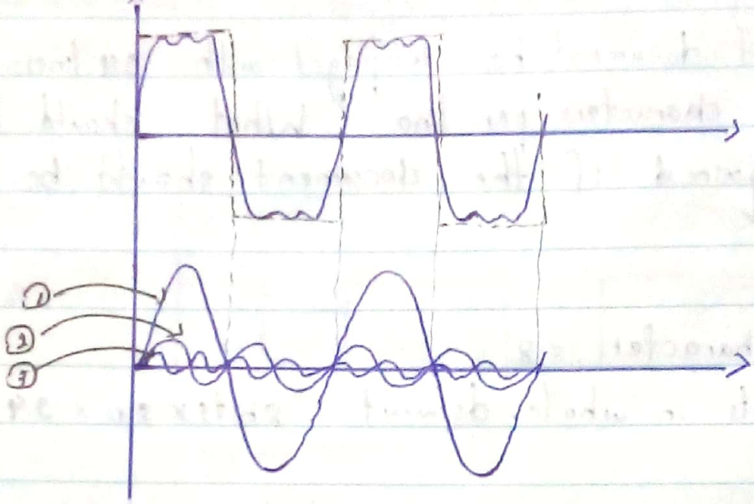
Bit length = bit interval (duration) $\times$ propagation speed.

In digital signals phase shift is called a 'delay'.

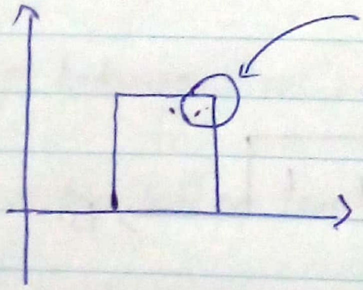




As discussed earlier we can compose a composite signal using set of fundamental frequency signals.

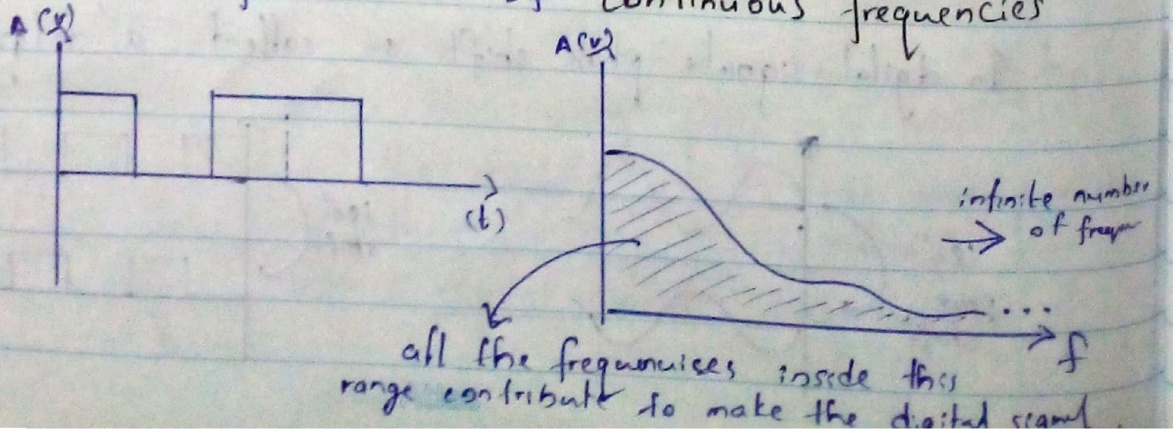


In fourier analysis, a digital signal is also considered as a composite analog signal. But this anolog signal has a infinite number of simple frequency signals (fundamental and harmonic signals). Therefore bandwidth of a digital signal is infinite.

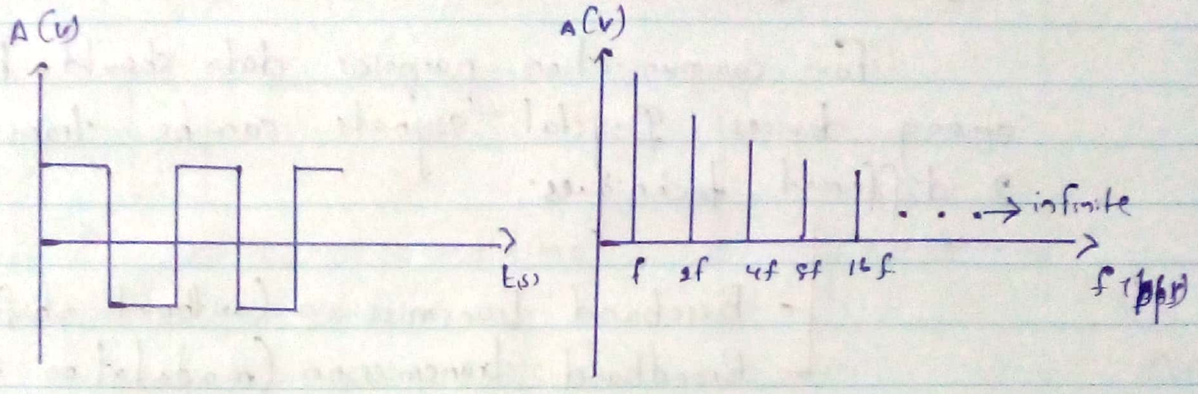


to make a 90° corner you need an infinite number of harmonic signals. More you add harmonics, ~~more~~ the signal becomes more digital.

for a non-periodic digital signal the bandwidth consist with an infinite number of continuous frequencies

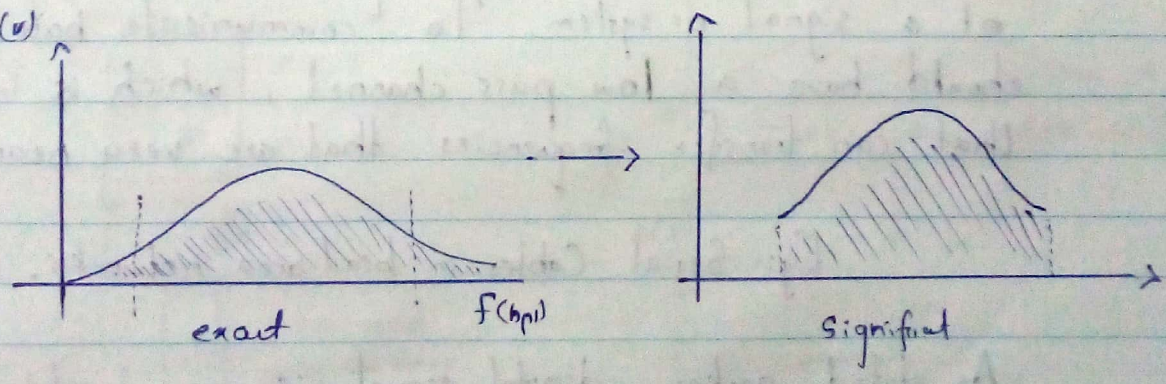


for a period digital signal the bandwidth consist of an infinite number of discrete ~~frequencies~~ frequencies.



- Exact and significant spectrum.

As stated earlier frequency signals with lower amplitudes contribute less to the composite signal. Therefore in practical sessions we only use ~~a set~~ the signals (harmonics) that actually affects ~~in~~ larger propotions to the composite signal. ~~Spct~~ Spectrum representing all the frequency is called ~~the~~ the exact spectrum and the spectrum where the ~~unnessary~~ unnecessary frequency are cut out is called the significant spectrum.



Digital Signal Transmission

For communication purposes data should be transmitted among devices. Digital signals can be transmitted using 2 different techniques.

- Baseband transmission (without shifting the frequency)
- Broadband transmission (modulation transmission)

* Baseband transmission

Baseband transmission is sending a digital signal over a channel (medium) without changing the digital signal to an analog signal.

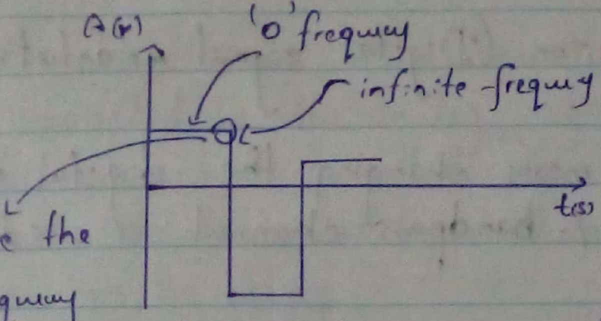
Baseband transmission signals have low frequencies that are close to 0Hz. Also known as low pass / non-modulated transmission. Since the lowest frequency is equal to 0Hz the baseband bandwidth is equal to the highest frequency of a signal or system. To communicate base band transmission should have a low pass channel, which is a channel (medium) that can transfer frequencies that are very near to zero.

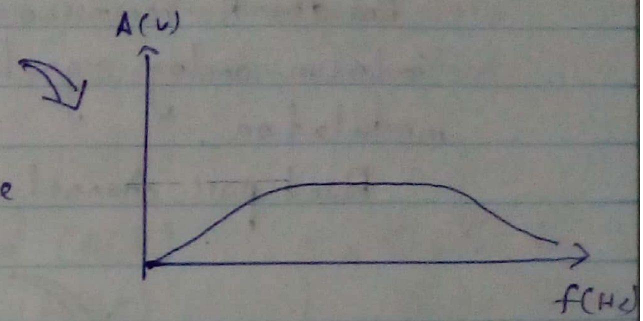
Eg:- Serial Cables in local area networks.

As stated earlier, digital signal is composite analog signal with an infinite bandwidth.

(Simply, this can understand as the amplitude changes from a level to another level with a infinite frequency).

next pg →





here the frequency changes from '0' to ∞ in no time

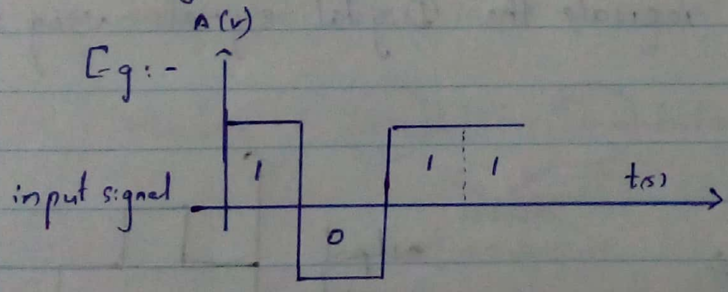
∴ digital signal contains,

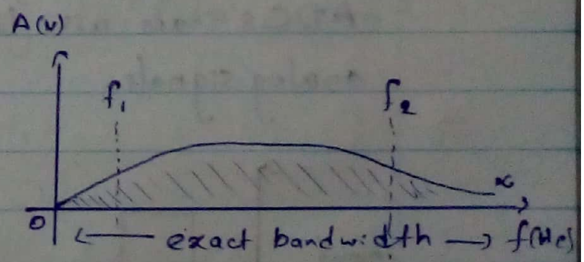
$0 Hz \longrightarrow \infty Hz$

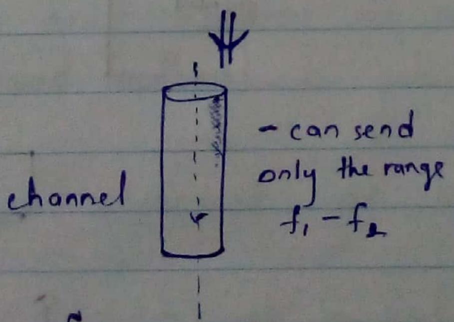
∴ Bandwith $\longrightarrow \infty Hz$

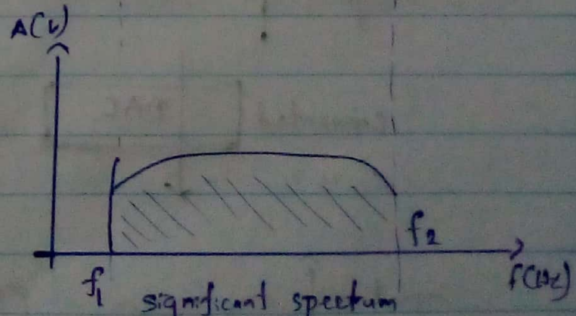
Now, for transmission we should have a medium that can send the signals with all the frequencies ($0 - \infty Hz$). But this ~~is~~ is not possible practically. Therefore we only send the range of signals that actually affects (included in significant spectrum) the composite signal. Even if ~~the~~ some 5 frequencies were cut out still the output signal is identical to the input signal.

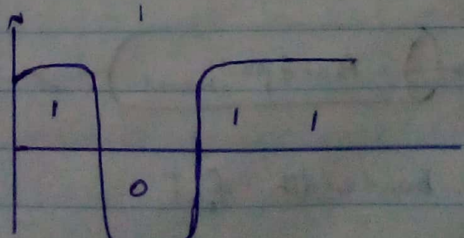
Eg:-









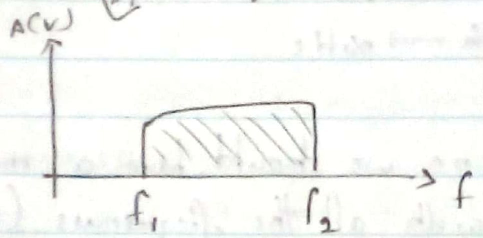


← output - still the data can be derived

* ~~Bro~~ Broadband transmission (Digital signal modulation)

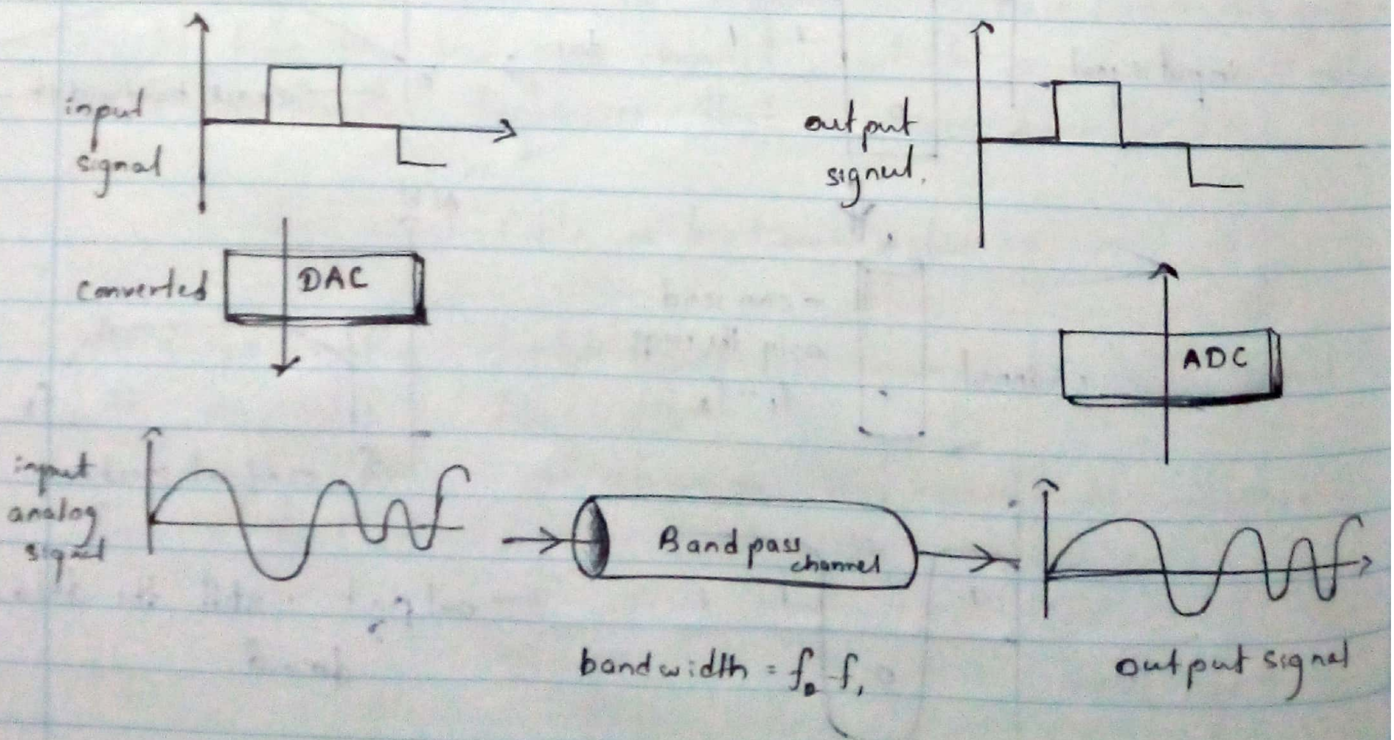
Broadband transmission means changing the digital signal into an analog signal. A bandpass channel is used in this modulation.

Band pass channel - is a channel or a medium that does not transmit signals with frequ that are near to zero



The bandwidth = highest freq - lowest freq
$|f_2 - f_1|$

To convert the digital signals into analog signal DAC's (Digital to Analog Converters) are used. And at the receiver ADCs are use to recreate the Digital signals using received analog signals.



Whenever there's a bandpass channel the digital data should be converted to the analog signal to send over the channel. This convertion is called "the modulation of digital data."
For modulation of digital data, there are number of methods that can be used,

- Amplitude shift keying (ASK)
- Frequency shift keying (FSK)
- Phase shift keying (PSK)
- Quadrature Amplitude modulation (QAM)

→ Digital to Analog conversion.

As stated earlier converters or the modulators are used to D → A and A → D conversions. Modem is an example for a modulator. It has the ability to modulate computer digital data into analog signals and transmit over the telephone lines. ~~with~~ And then demodulate the incoming analog signals over telephone lines in to ~~a~~ digital signals/data.

Modem - Modulator Demodulator

Modem

In the next chapters we will discuss about the modulation types and methods.

3. En Coding

En Coding
↳ converting or putting in to a code

For communication purposes we said earlier that we require conversion methods. This is known as encoding. There are ~~set~~ 4 types of conversion in signal encoding.

- Digital to digital
- Digital to Analog
- Analog to Digital
- Analog to Analog

All of these 4 types uses different types of conversion methods.

* Digital to digital

Why digital to digital encoding?

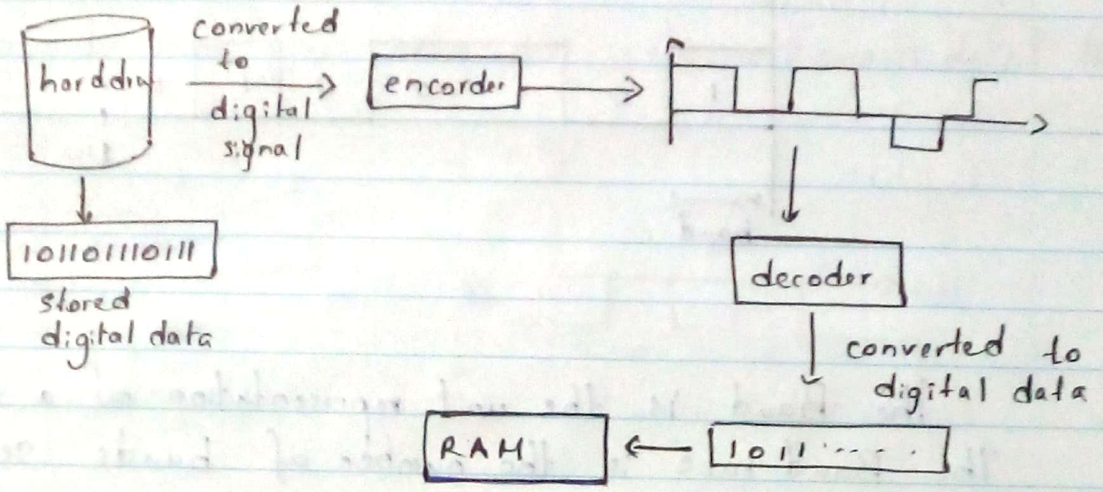
There's a confusion why should we convert digital into digital. But here D → D means converting digital data in to digital signals for transmission.

Digital data – is the form of the digital information stored

Digital signal – is the form that digital data is transferred from one device to another.

digital signals carries digital data.

Eg:- retrieving stored information on a magnetic drive (harddisk) to the random access memory



This process of encoding digital data into digital signals is known as 'Line coding'. In line coding there are several types we can use to encode:

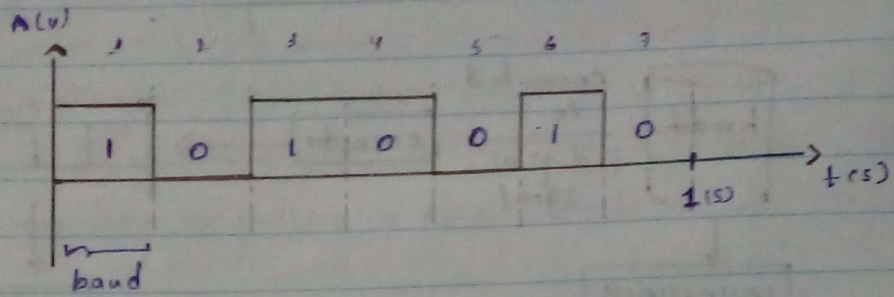
Line coding (D → D conversion)

- → Uni-polar - NRZ
- → polar - NRZ, RZ, Biphase
- → Bi-ploar - AMI, B8ZS, HDB3

Before talking about these conversion methods, let's learn about some attributes / ratios that will require to understand the behavior of above conversion methods.

next pg →

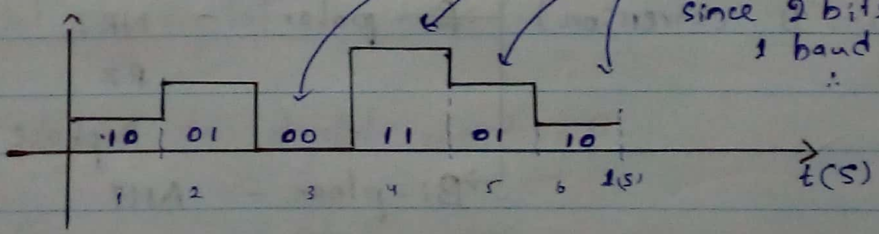
Baud, Baud rate, data rate



The Baud is the unit representation of a signal element. The Baud rate is the number of bauds sent in 1s. On above signal ~~there are~~ 7 bauds are sent in 1s. Therefore the baud rate is 7 baud. The baud rate is also known as the signal rate, pulse rate.

The Data rate is the number of bits sent in 1s. On above signal 7 bits (1 and 0's) are sent in 1s.
∴ The data rate is 7 bps (bits per second).

Eg:-



(Here 4 levels are used since 2 bits are sent in 1 baud ∴ $2^n = 2^2 = 4$ levels are needed)

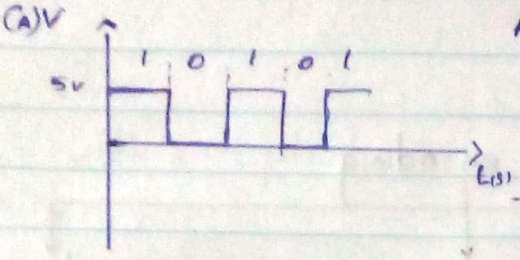
Here 6 bauds in 1s are sent. ∴ The baud rate is 6 baud. 2 bits in baud, that is $2 \times 6 = 12$ bits sent in 1s. Therefore bit rate is 12 bps.

$$\text{Baud rate (S)} = \frac{\text{Bit rate (N)}}{\text{no. bits in one signal element (r)}}$$

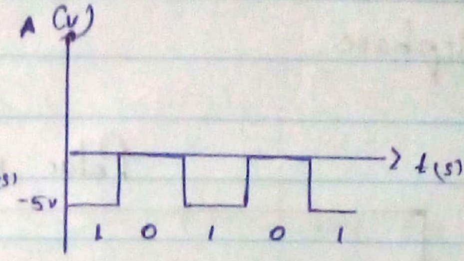
(Here, maximum baud rate required is used) $S = C \times N \times \frac{1}{r}$ (c=1)

Unipolar encoding

Unipolar means, use only one polar or one side of the time axis.



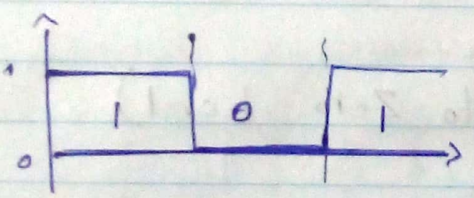
using above



using below

Unipolar system use NRZ (non-return to zero) type to represent data bits. This means signal does not return to zero at the middle of the bit.

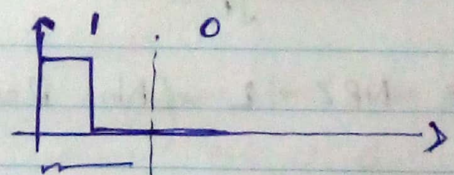
NRZ



vs.

~~RZ~~

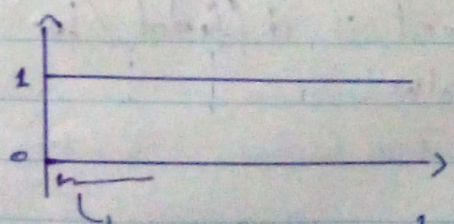
RZ (Return to zero)



↳ in return to zero (RZ) type the signal return to zeros at the middle of the bit.

dis advantage ⇝

When transmitting the same data the starting point and the end point of a signal element cannot be identified that causes loss of clock

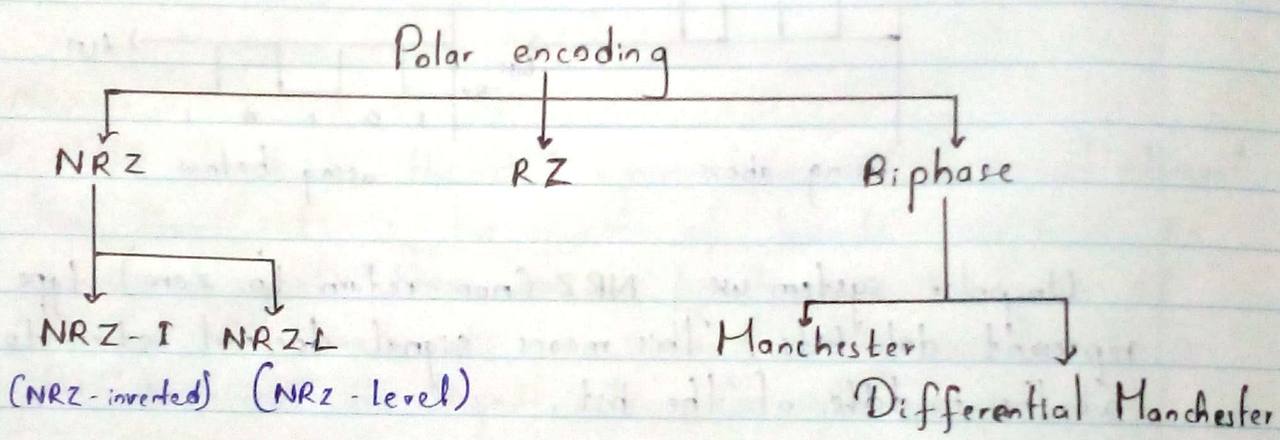


↳ signal element cannot be identified.

And ~~th~~ if 1 is given continuously this requires lot of energy. ~~to~~ Therefore this unipolar system is not used in today communication

* Polar encoding

Polar encoding means using both sides (above and below) of the time axis. This encoding method has 3 types, NRZ (non return to zero), RZ (return to zero) and Biphase.

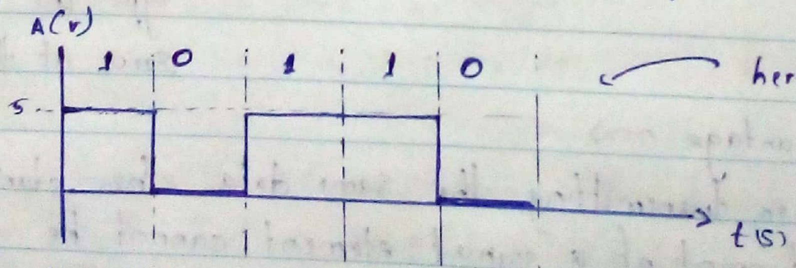


NRZ type

* NRZ – L (Non Return to Zero - Level)

In NRZ–L, the voltage level represent the bit. ~~If the t~~

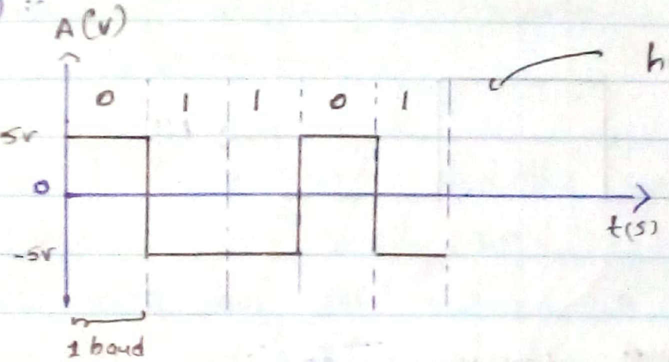
Eg:-



here binary 1 = 5v
binary 0 = 0v

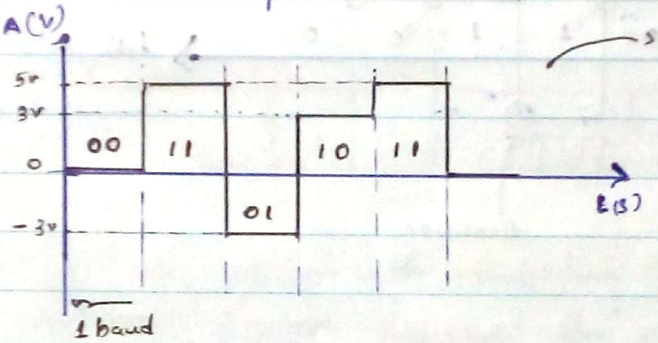
If the level is high it will display one bit value and if the ~~bit~~ voltage level is different / low it will display another ~~disp~~ bit value.

Eg ② :



here binary 1 is -5V
binary 0 is 5V

Eg ③ : with multiple levels

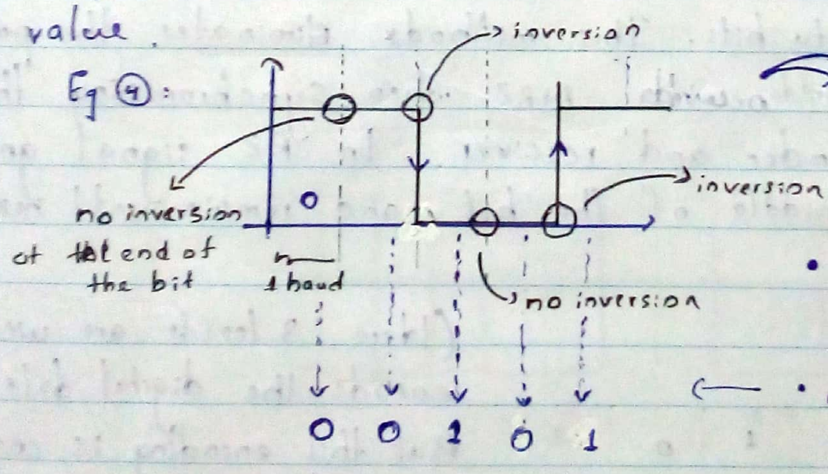


here binary
00 = 0V
01 = -3V
10 = 3V
11 = 5V

* NRZ - I (Non Return to Zero - Invert)

In NRZ-I the transitions will determine the next bit value.

Eg ④ :

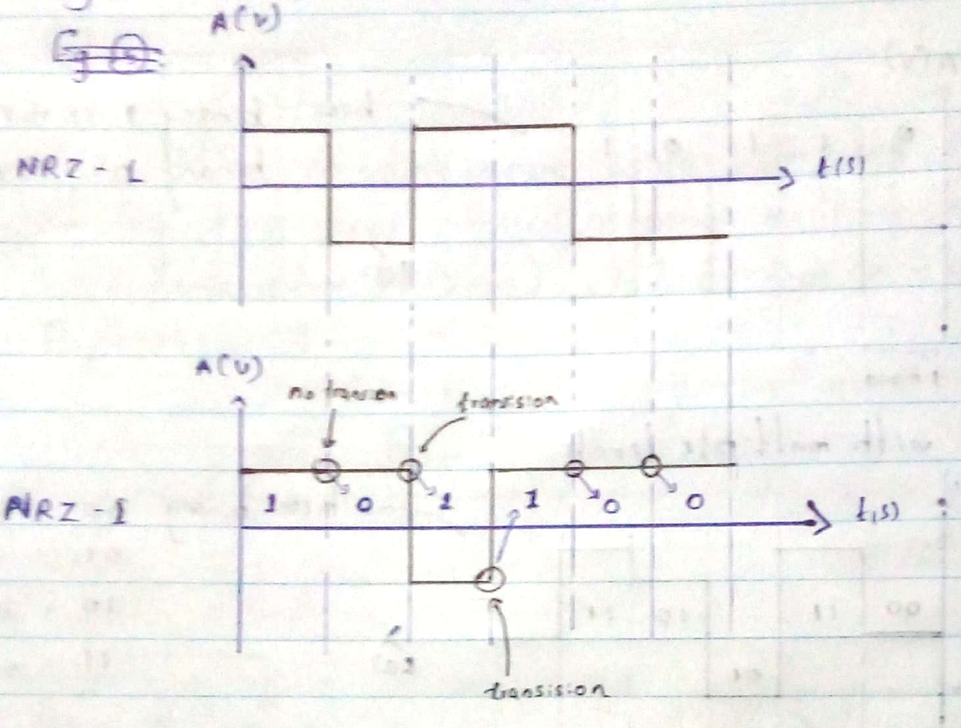


given that the first bit is 0 then,

- if there's no inversion the next bit is 0
- after that there's a inversion, ∴ the next bit is 1

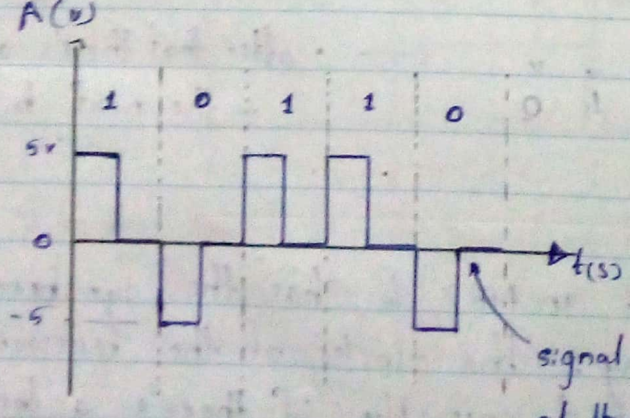
Still both NRZ-I an NRZ-L has the synchronization problems when decoding: The clocks of the receiver and sender cannot synchronized correctly if there's a long sequence of 1's or 0's.

Eg ⑤ :- 1 0 1 1 0 0



* RZ (return to zero)

In return to zero (RZ) method three levels are used to represent data bits. This methods eliminates the ~~prob~~ problems occured with NRZ when synchronizing the clocks of sender and receiver. In RZ signal goes to '0' at the middle of the bit, and remain until next bit.



(Here, 3 levels are used to encode the digital data. But this encoding is complex. Therefore RZ coding is not used in today communications.)

signal becomes '0' at the middle of the bit and remain until next bit.

- Biphase → Differential manchester...
  → manchester...

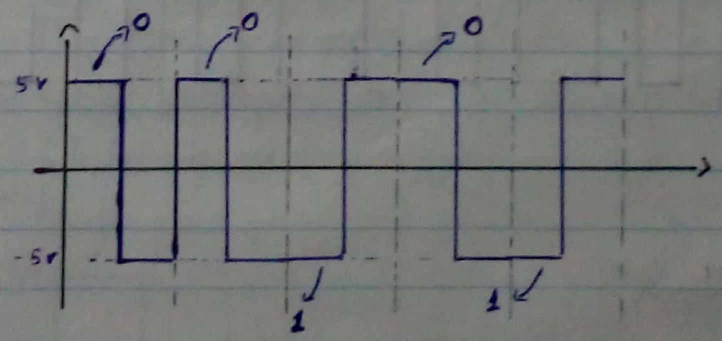
Due to the complexity and the synchronization problems both NRZ and RZ are rarely used in today communication systems. By combining both NRZ and RZ, a two new encoding methods 'Manchester' and 'Differential Manchester' are implemented.

NRZ-L + RZ → Manchester

RZ + NRZ-I → Differential Manchester

- Manchester → made with combining RZ and NRZ-L.
  - Voltage Level ~~decide~~ ~~the~~ of the first half of the bit decides the bit value
  - At the middle of the bit voltage changes to the other level. This transition is used to synchronization

Eg - If ~~the~~ 1's = -5v and 0's = 5v

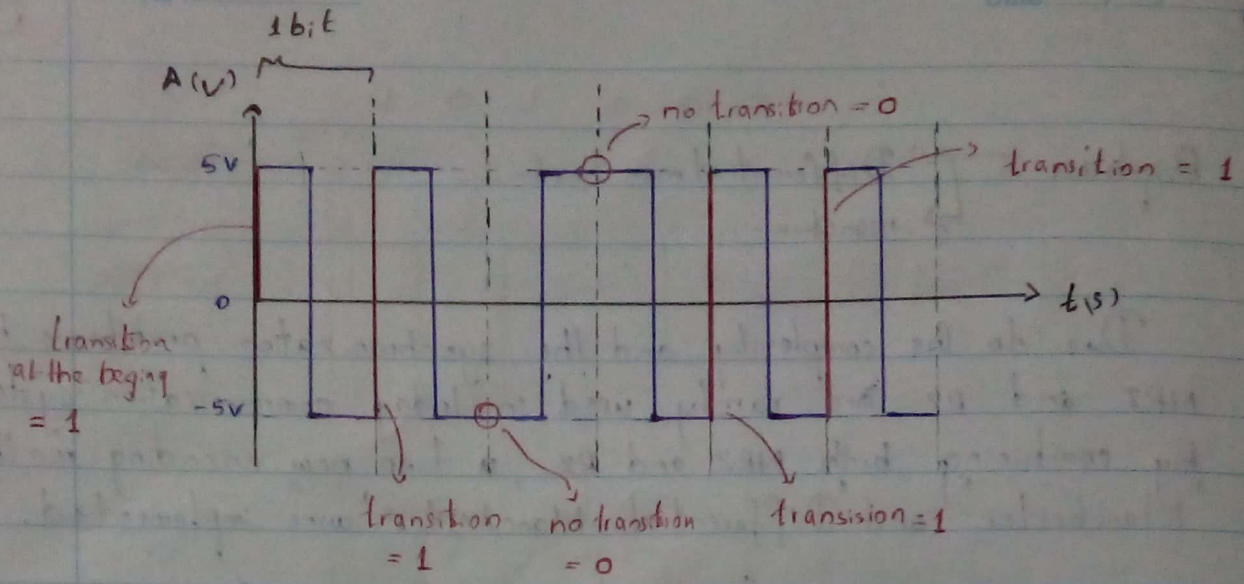


0' zero :

1' one :

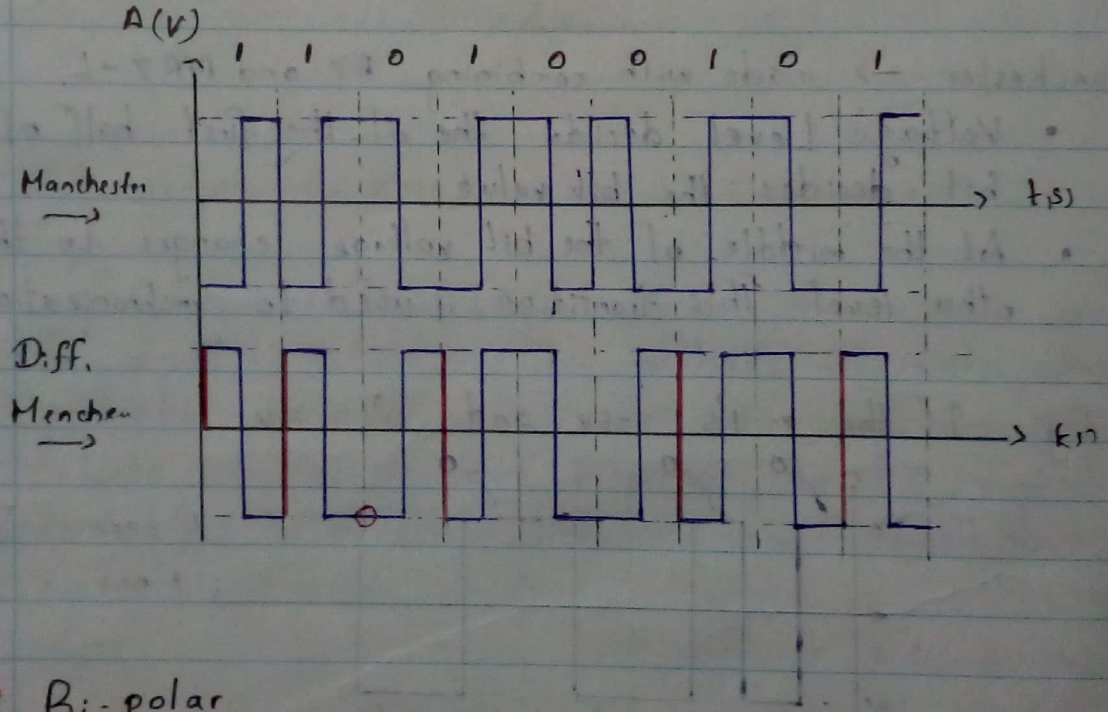
bit pattern = 0 0 1 0 1

- Differential Manchester → combines RZ and NRZ-I
  - transition at the end of the bit decides the bit value of the next bit.
  - If there's a transition next bit is 1, if there's no transition next bit is 0.
  - At the middle of the bit voltage changes from 1 level to another level.



∴ bit pattern = 1 1 0 0 1 1

Eg:- bit pattern = 110100101



* Bi-polar

Bipolar

AMI (Alternate mark inversion)

B8ZS (Bipolar 8-zero substitution)

HDB3 (High density Bipolar-3)

Bipolar encoding is used in long distance communications. It has 3 levels of voltages

- positive
- negative
- zero.

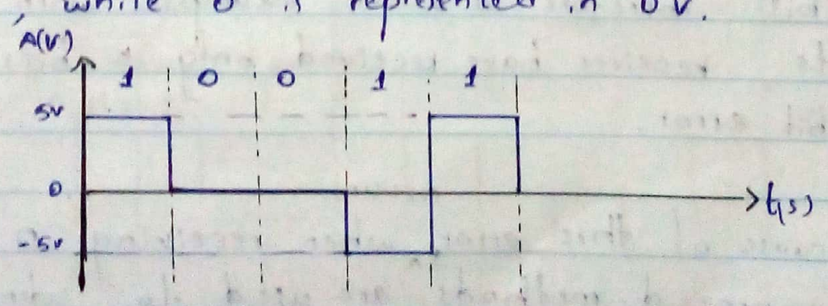
In ~~p~~ bipolar encoding same data bit should be represented in 2 positive negative voltages, while othe data bit type is ~~equal to s.~~ represented by zero voltage.

Eg: If binary '0' is represented by 0 voltage, binary 1 is represented with a positive voltage and a negative voltage alternatively

- Vice versa, if ~~o~~ 1 is represented by 0 voltage binary 0 is represented with positive and negative voltage.

* AMI (Alternate Mark Inversion)

Above 1st type in example is the AMI model encoding. ~~If~~ The binary '1' is represented in ~~at~~ negative and positive voltages, while '0' is represented in 0 V.

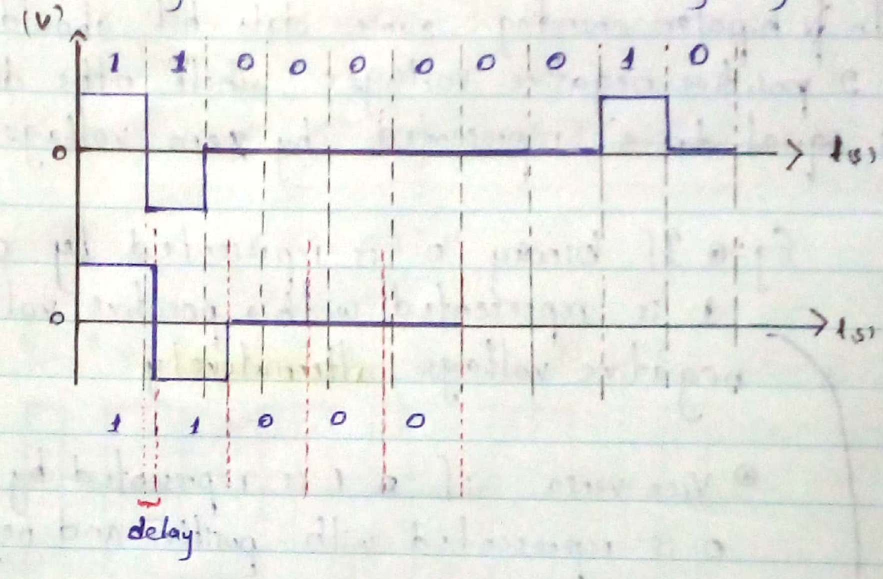


Rule :- If a '1' is represented by a positive voltage the next '1' should be represented in a negative voltage.

~~Sender and re~~

When sender sends the data bits receiver collects them with a short delay, because they work with their own clocks. But in a bipolar system if receiver send continuously binary 0's for a long sequence receiver will loose the bit patter, causing some errors in decoding by loosing data.

Eg :-



In this example after two 1's sender sends a sequence of 0's. Because of the delay receiver lost the bit pattern because when it receives 0's continuesly it cannot understand when the starting and ~~the~~ the end time of a bit. Therefore in above example when sender ~~receives~~ sends 6 bits, receiver have received only 5 bits and there's a 1 bit error.

Because of this error occurs, when receiving 0's continuously two improved methods are used to identify and eliminate the bit pattern error. They are,

B8ZS (Bipolar 8 - zero substitution)

HDB3 (High - density bipolar 3 zeros)

* B8ZS encoding

If in a data stream there's 8 consecutive zeros (8 zeros following each other continuously), those zeros are replaced by a set of voltages by the sender. This replacement of bits is called B8ZS encoding. This replacement is done considering the last voltage level of binary 1.

In AMI coding we know that binary 1 can be represented in positive and negative voltages.

If the last binary 1 bit was represented in positive voltage then the next 8 binary 0's are replaced by below voltage sequence.

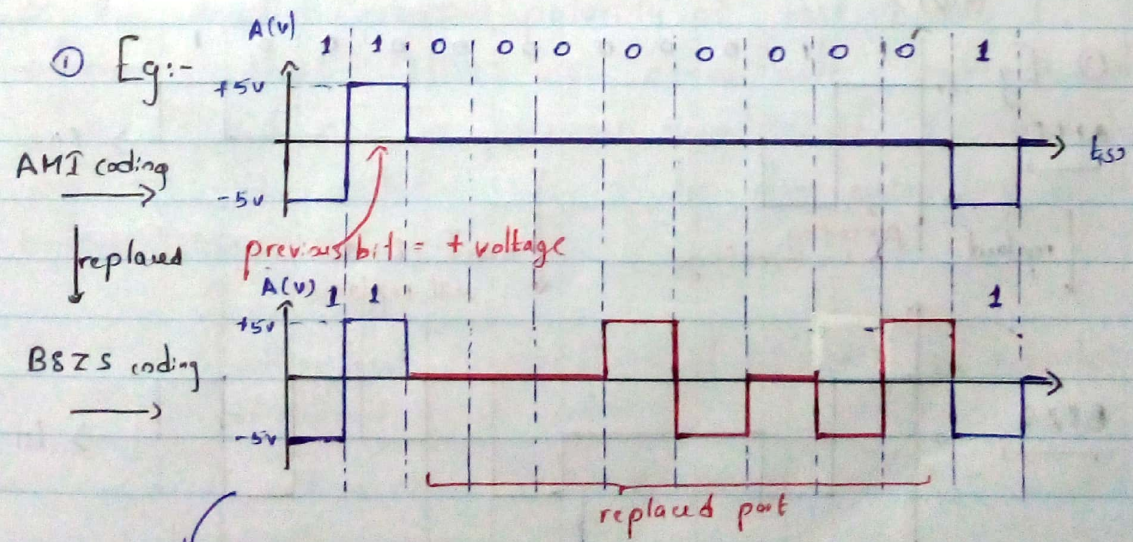
previous bit (positive) → 8 consecutive zeros

+	0	0	0	0	0	0	0	0

↓ will change into

+	0	0	0	+	-	0	-	+

① Eg:-



Now the bit pattern has changed. That means the real information has changed. Receiver does not know the actual bit pattern.
~~But we stated that 1 in AMI there's a rule~~

next pg →

But we stated ealier ~~that~~, in bipolar encoding there's a rule that subsequent 1's & shouldn't have same type of voltage polarity. (If the current 1's is positive the next 1 should be negative) But in above pattern, 2 positive values and 2 negative values comes together. This is a violation of that rule in bipolar encoding. This violation helps to understand the receiver that sender has ~~send~~ sent 8 - 0's continuously. The same theory is applied to the (−) voltage as below.

If the last binary bit represented in (−) voltage, the next 8 0's will be replaced by below pattern.

8 - 0's

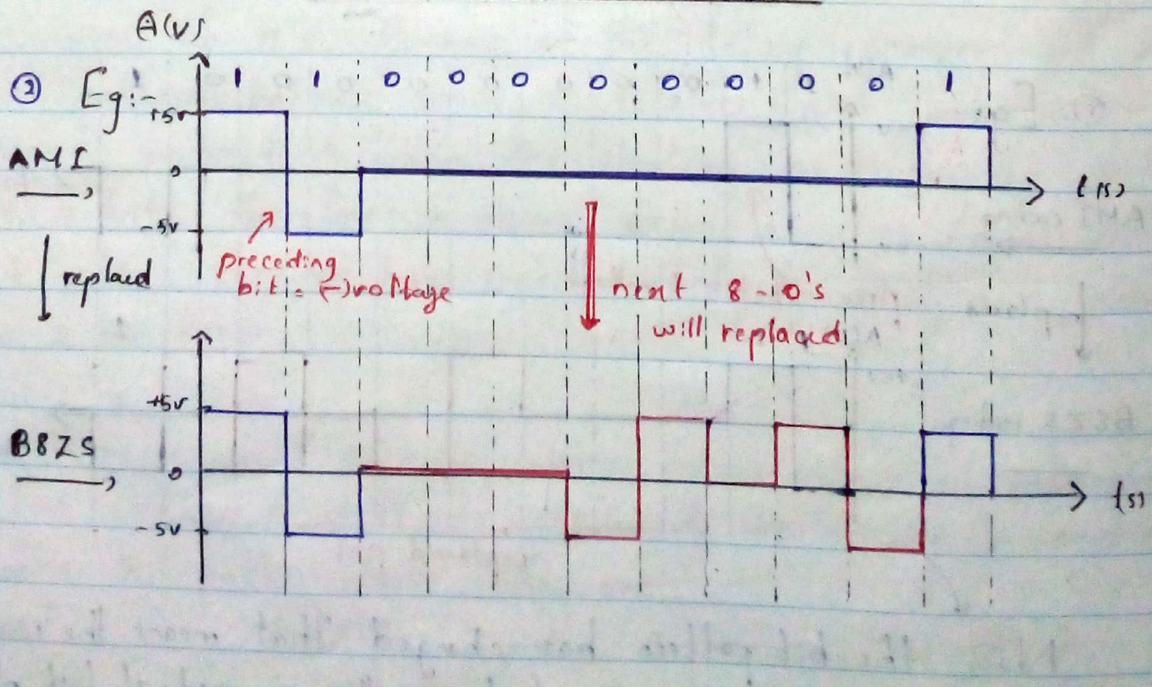
previous binary 1 = negative

−	0	0	0	0	0	0	0	0

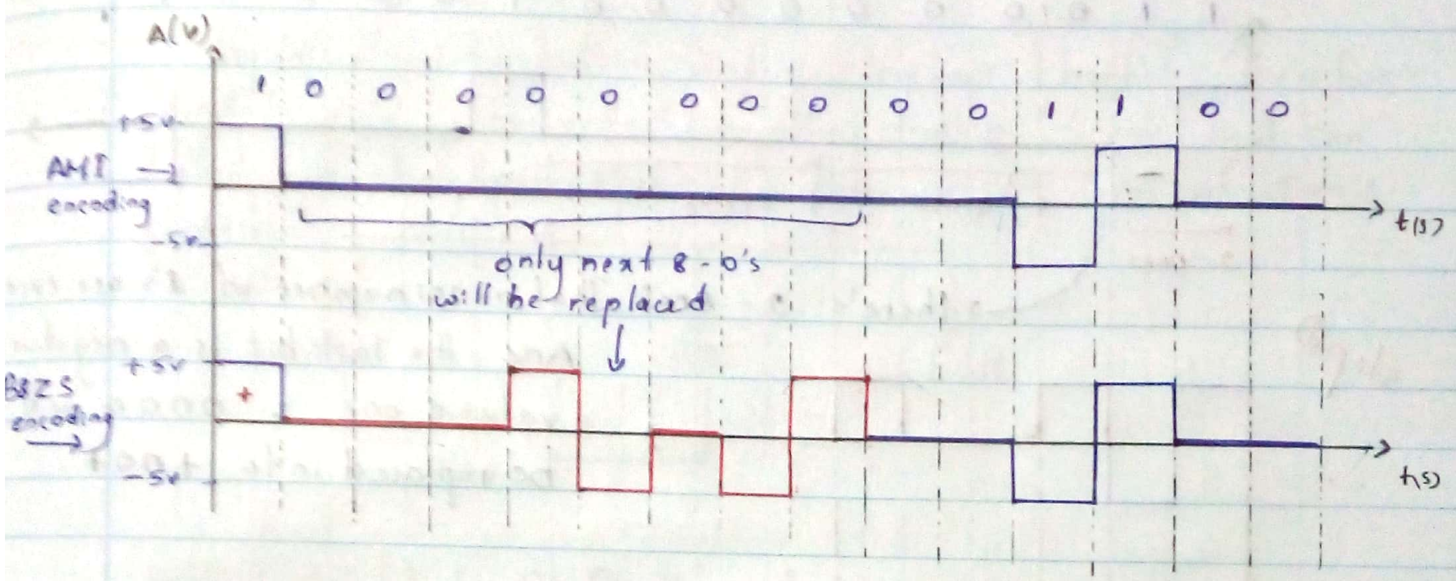
will change to

+	0	0	0	−	+	0	+	−

③ Eg:-



③ Eg:-

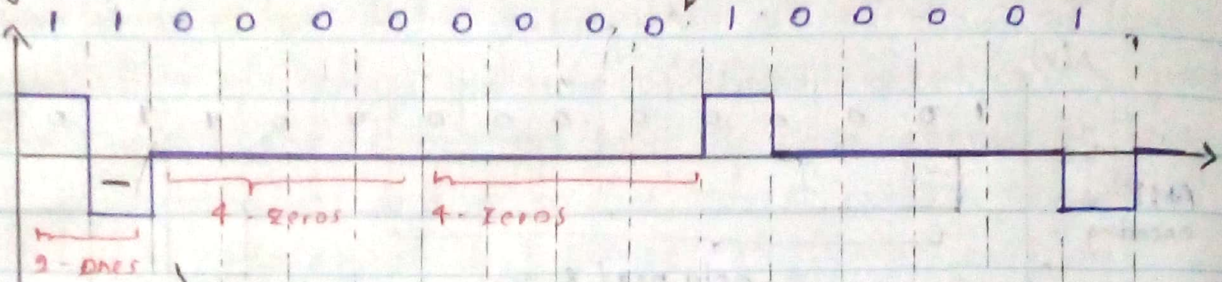


* HDB3 encoding

This encoding type is also derived from bipolar AMI. Here encoding is done for every consecutive 4 - zero's. If there's a set of 4 zeros they ~~are~~ can be encoded in to HDB3. Same as the B8ZS the encoding depends on the positive / negative polarity of last binary 1. But other than that HDB3 uses extra rule, that encoding also depend on the previous number of 1's that have occured. The encoding should be done according to the below table.

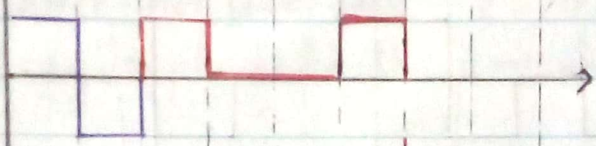
voltage level of previous 1	Number (Count) of 1's since last substitution	
	odd	even
+	000+	-00-
-	000-	+00+

Eg:- Convert below AMI coded sequen into HDB3

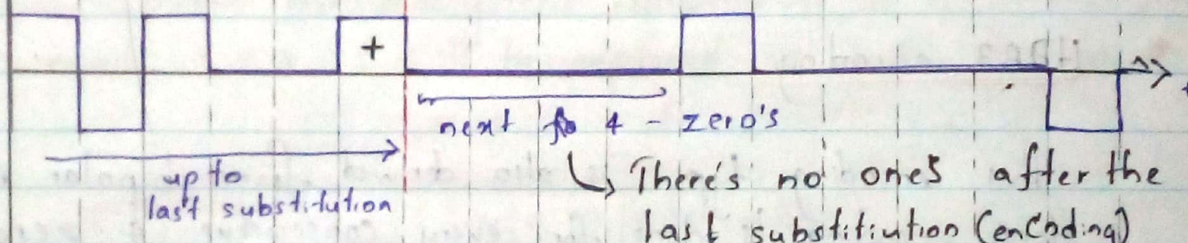


there's 2 - ones. That means number of 1's are even. And the last bit is a negative valued one. ∴ 0000 will be replaced into +00+

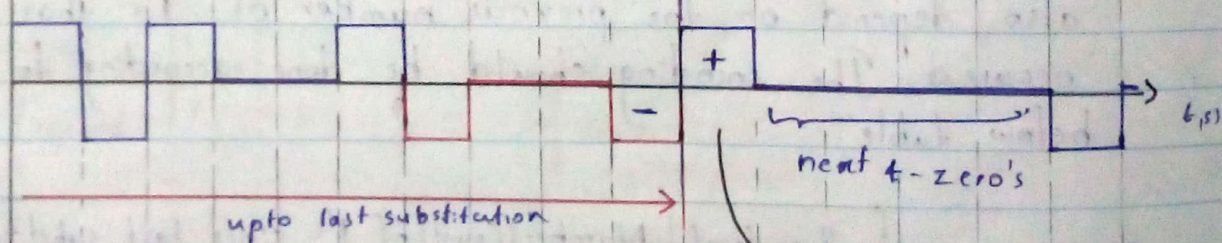
step ①



step ②



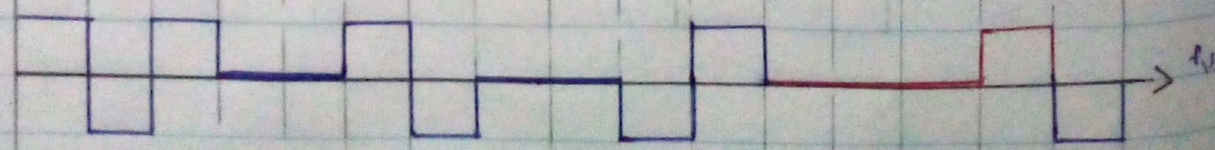
There's no ones after the last substitution (encoding). That means number of value's are even and the last 1 is a positive one. ∴ 0000 will be replaced into -00-



step ③

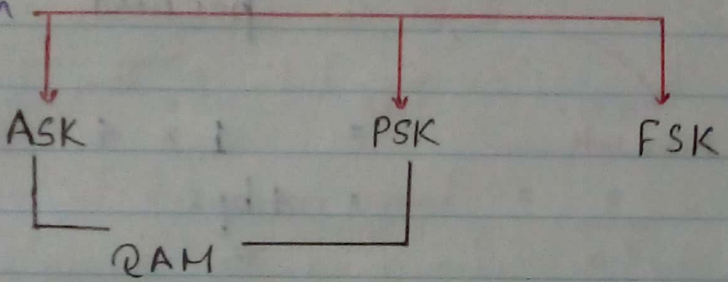
There's a one, binary 1 after the last substitution. That means the number of 1's are odd. And last 1 is positive ∴ 000+ is used.

final result



Digital to Analog encoding →

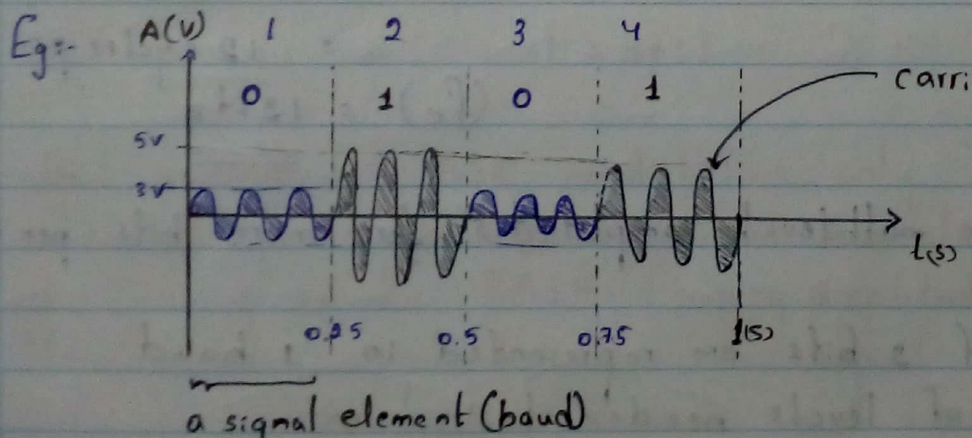
Digital data transmission requires low pass channels. Therefore digital data is converted to the analog signals that can travel over bandpass channels. There are 4 types of D → A conversion



ASK (Amplitude Shift Keying)

Representing digital data (binary bits) using carrier signals that has different amplitude values, while the frequency and the phase remain unchanged, is called ASK.

Eg:-



baud rate = 4 baud
bit rate = 4 bps

In this example 1 signal element (baud) represent 1 bit. The signal that carries the data is called carrier signal. This ASK modulation uses a carrier signal with 2 amplitudes; 5v to binary 1 and 0v to binary '0'. Only amplitude has changed.

Scanned by CamScanner

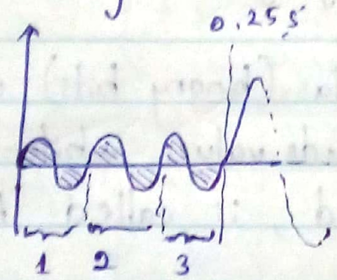
In that example,

- 4 bauds occur in 1 second. That means the baud rate of the modulated ASK signal is 4 baud.
- There's only 1 bit represented by 1 baud
  ∴ 1 bit per baud
  ↳ bit rate = no of bits per baud × no of bauds per second (baud rate)

Here,
Bit rate = Baud rate

= 1 × 4
= 4 bps

- But if you take a closer you can see that 3 cycles of carrier signal occur in 0.25 s.



∴ The frequency of the carrier signal

$= \frac{3 \text{ cycles}}{0.25} \times 1 s$

= 12 cycles per second

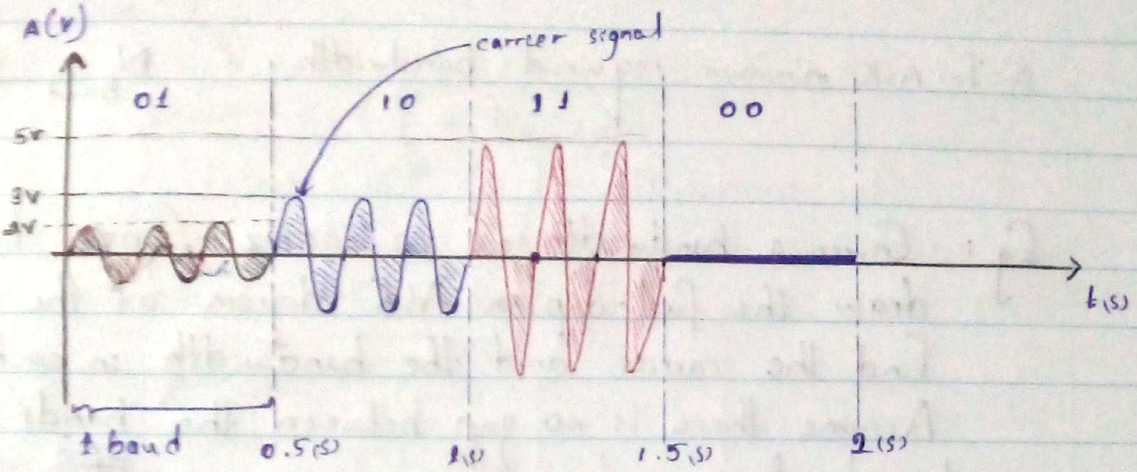
(f_c) = 12 Hz

You can use multilevel amplitudes for more bits per baud.

Eg:- If 2 bits are represented in 1 baud.
No of levels needed to represent different combinations $= 2^n$
$= 2^2 = 4$

Assume,
11 = 5V
10 = 3V
01 = 2V
00 = 0V

next pg →



frequency of the carrier signal $= \frac{3}{0.5} \times 1$

$f_c = 6\,Hz$

baud rate (signal rate) = 4 bauds per 2 seconds

$\therefore$ baud rate $= \frac{4}{2} = 2$ baud

bit rate / data rate = 2 bits per baud × baud rate

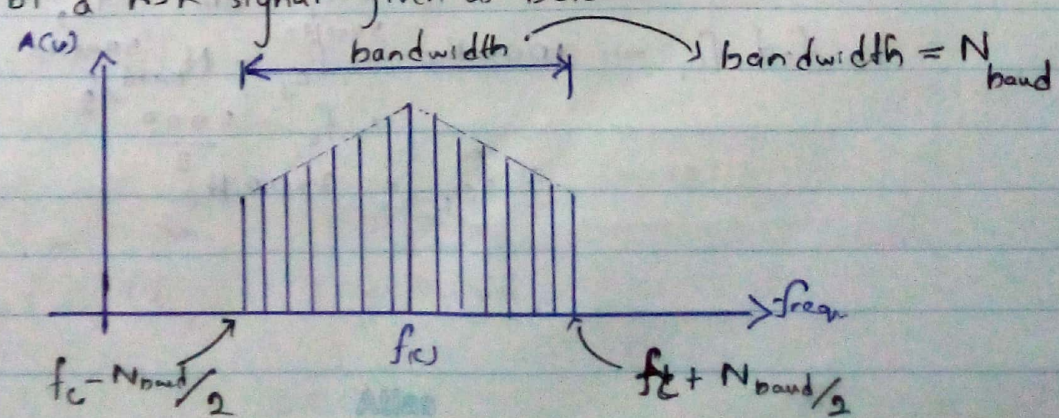
$= 2 \times 2$

$= 4$ bps

* But normally in ASK modulation only 2 levels of amplitudes are used as in the 1st example. This is also known as the BASK (binary amplitute shift keying). Therefore in normal ASK modulated signal

Baud rate = Bit rate (if 1 bit per baud)

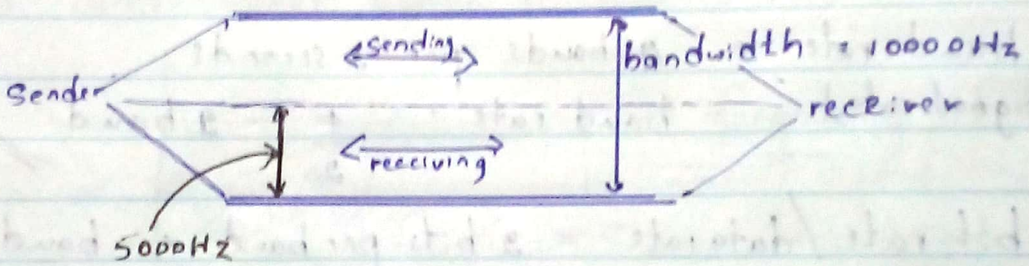
Bandwidth of a ASK signal given as below



∴ In ASK minimum required bandwidth = N_{baud} (baud rate)

Eg :- Given a bandwidth of 10,000Hz (1000 to 11,000Hz) draw the full-duplex ASK digram of the system. Find the carries and the bandwidth in each direction. Assume there is no gap between the bands in the two directions.

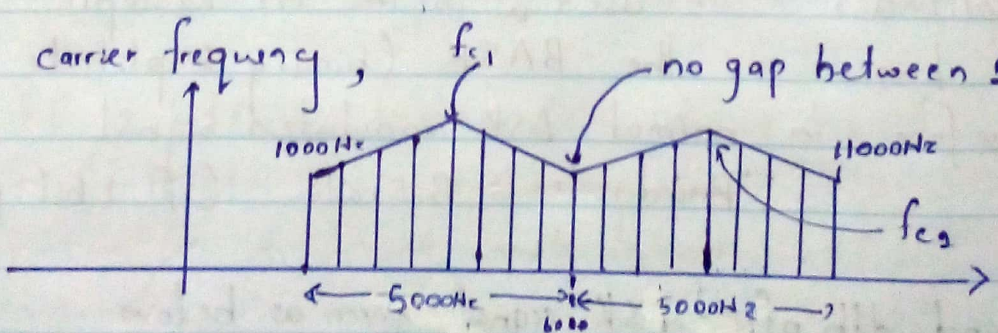
In a full-duplex system bandwidth should be divided into two, for two way communication



∴ Bandwidth for each direction is $= \frac{10000}{2}$

$= 5000Hz$

To find carrier frequency,



to find f_{c_1} → $1000Hz = f_{c_1} - N_{baud}/2$

$= f_{c_1} - \frac{5000}{2}$

∴ $f_{c_1} = 3500 Hz$

to find f_{c_2},

$$11000Hz = f_{c_2} + N_{baud}/2$$

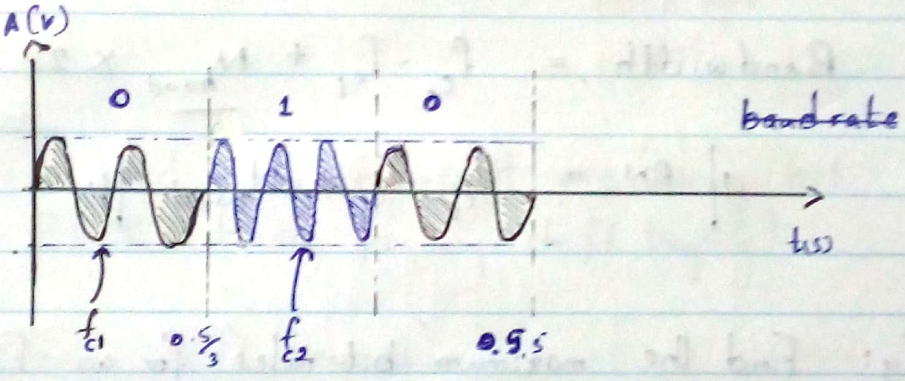
$$= f_{c_2} + 5000/2$$

$$f_{c_2} = \underline{\underline{8500Hz}}$$

FSK (Frequency shift Keying)

In FSK the data is represented with a carrier signal with different frequency values, while amplitude, phase remains unchanged.

Eg:-



$$\text{baud rate} = \frac{3}{0.5} \times 1 = 6 \text{ baud}$$

$$\text{bit rate} = \text{bits per baud} \times \text{baud rate}$$
$$= 1 \times 6$$
$$= \underline{\underline{6 \text{ bps}}}$$

frequeny of c_1 (f_{c_1}) = 2 cyles per $0.5/3$ s

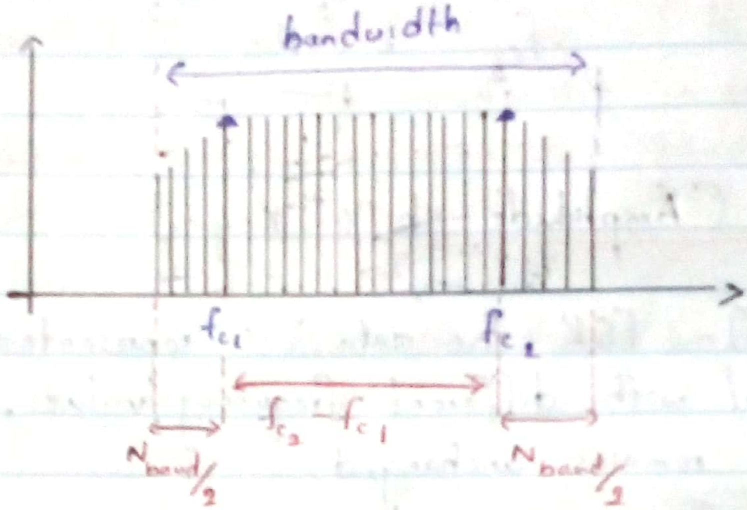
$$\therefore f_{c_1} = \frac{2}{0.5/3} \times 1 = 12Hz$$

frequeny of c_2 (f_{c_2}) = 3 cycles per $0.5/3$ s

$$= \frac{3}{0.5/3} \times 1 = 18Hz$$

Bandwidth of FSK modulation.

The bandwidth of a FSK modulation is given as below.

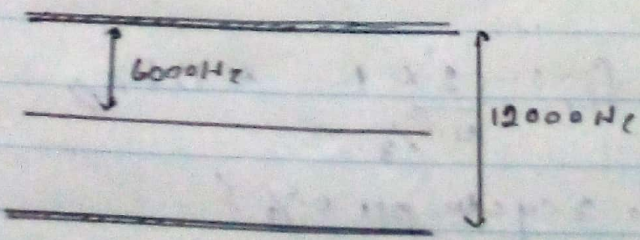


$$\text{Bandwidth} = f_{c_2} - f_{c_1} + \frac{N_{baud}}{2} \times 2$$

$$\boxed{BW = f_{c_2} - f_{c_1} + N_{baud}}$$

Eg:- Find the maximum bit rates for an FSK signal if the bandwidth of the medium is 12,000 Hz and the difference between two carriers is 2000 Hz. Transmission is in full-duplex mode.

If the transmission is in ~~two~~ full duplex mode then the bandwidth should be divided in to two, for receiving and sending.



Therefore the bandwidth of a one direction is

$= 12000/2$

$= 6000\ Hz$

$$BW = \text{baudrate} + (f_{c_2} - f_{c_1})$$

$$6000\,Hz = \text{baudrate} + \underbrace{(f_{c_2} - f_{c_1})}_{2000\,Hz}$$

$\therefore$ baud rate = 4000 baud

Normally in FSK (as ASK) the modulated carrier frequency element (baud) represent only 1 bit. $\therefore$ One bit per baud

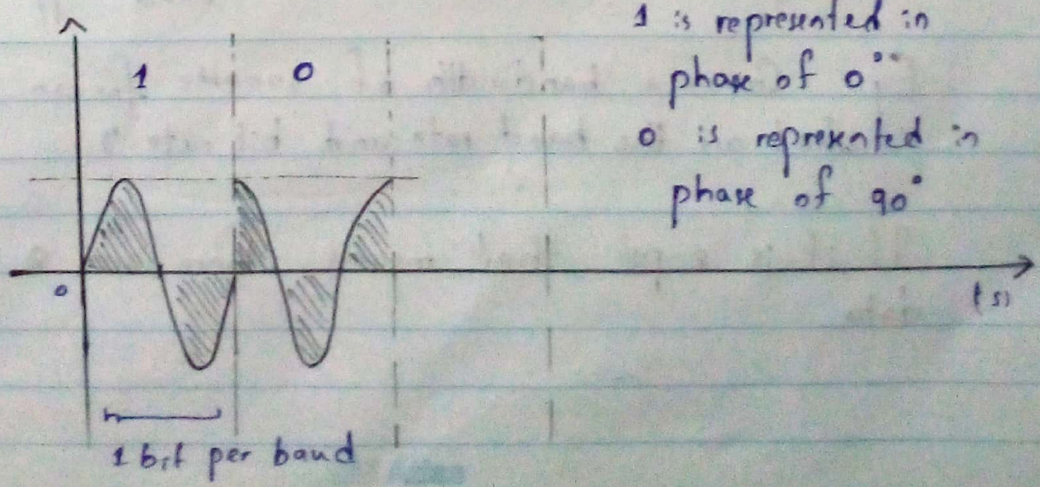
$\hookrightarrow$ bit rate = bits per baud × baud rate
= 1 × 4000
= 4000 bps

When 1 baud only represent one binary bit, that is called BFSK (binary frequency shift keying).

PSK (Phase shift keying)

As ASK, FSK, in phase shift keying (PSK) the data is represented with ~~differen~~ a carrier signal with different phases.

Eg:-

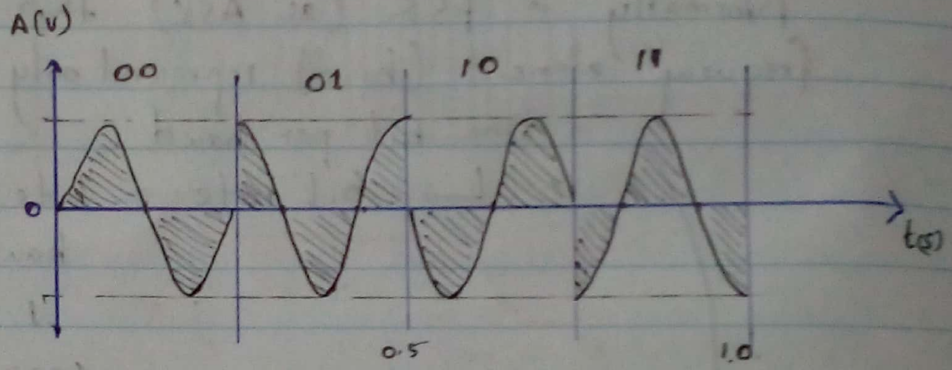


1 is represented in phase of 0°.
0 is represented in phase of 90°

Above example PSK modulation is also called BPSK (same as BASK, BFSK), since 2 different phases are used. But multi level phases also can be used to represent data when there's more than 1 bit per baud.

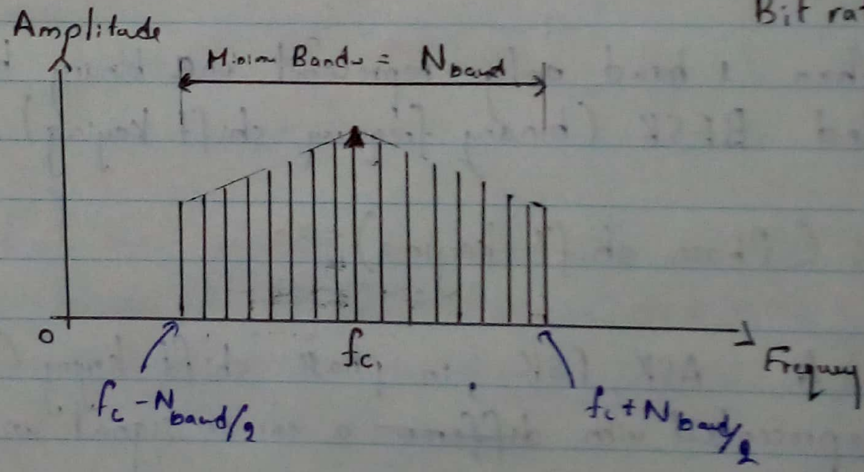
Eg:- 00 - 0°, 01 - 90°, 10 - 180°, 11 - 270°

4 - psk



Baud rate = 4 baud
Bit rate = 4 × 2
= 8 bps

Bandwidth of PSK



In psk the bandwidth = N_{baud}

Eg:- Given a bandwidth of 5000Hz for an 8 - PSK signal, what are the baud rate and bit rate ?

If it is 8-psk, that means there are 8 phases representing data.

2^n = no of phases (n_p) required. (n → no of bits per baud)

$\therefore 2^3 = 8$ phases → therefore 3 bits per baud is transmitted.

Then the baud rate is = 5000 baud (since in psk the baud rate = bandwidth).

and the bit rate = 5000×3

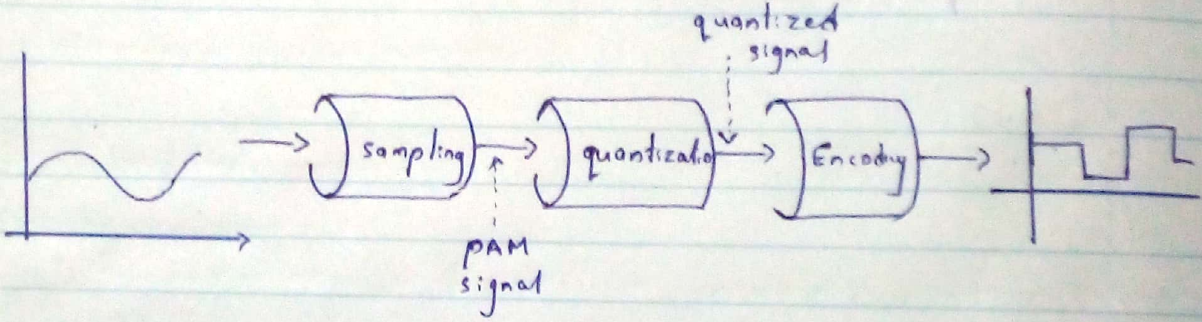
= 15000 bps.

QAM

Analog to Digital Conversion

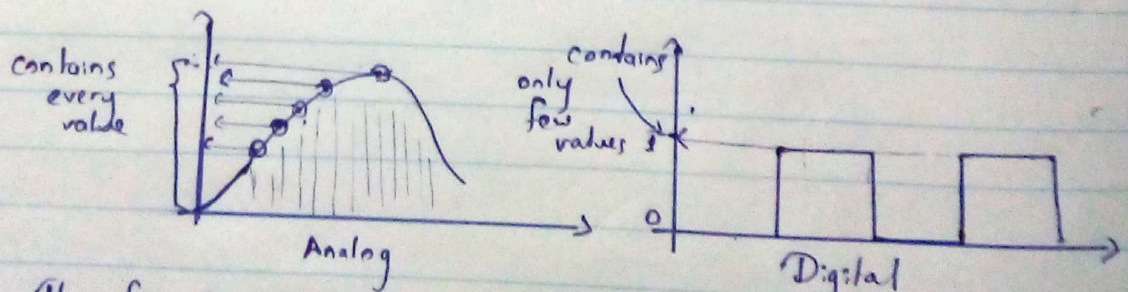
The common technique used to convert Analog signals into Digital signal is PCM (pulse code modulation). This PCM is a process of 3 steps.

- Sampling
- Quantization
- Encoding



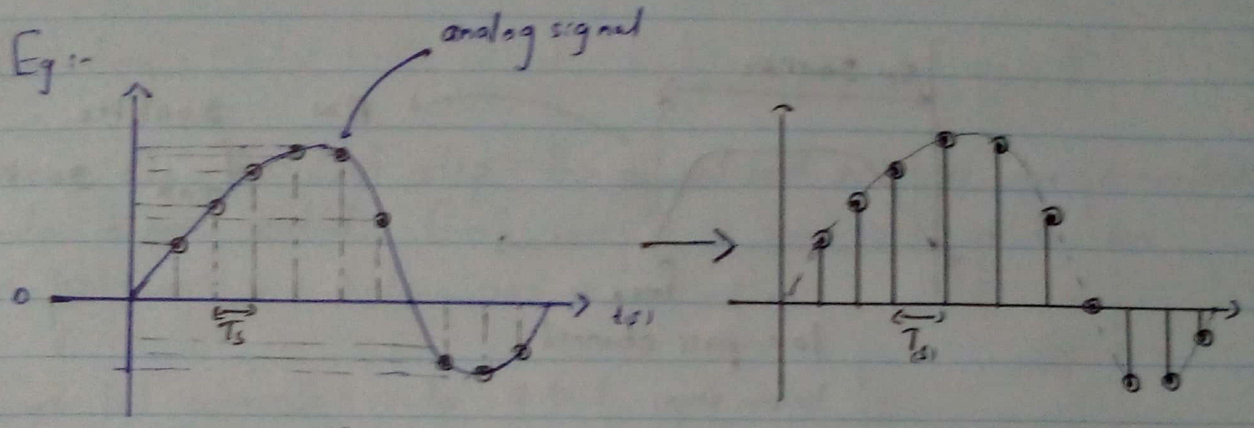
Sampling

An analog signal is a continuous signal, but digital signal have discrete values



Therefore to convert an analog signal to digital we should take only ~~selectete~~ few selected amplitude values between selected periods. This process is called sampling. The period selected to sampling is called sample interval or period (T_s). The inverse of sampling interval is called sampling frequency (f_s) $= 1/T_s$

Eg:-



for every T_s - sampling interval, the amplitude of the analog is taken

$f_s = \frac{1}{T_s}$

sampled signal (PAM signal)

This sampling process also known as PAM (pulse amplitude modulation).

* Nyquist theorem →

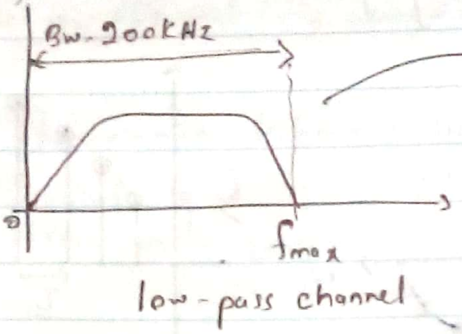
To decide how to choose the sampling interval (T_s) is described by the Nyquist theorem. In Nyquist theorem it says that,

The sampling rate (f_s) should be atleast twice the highest frequency of the original signal.

$$\therefore \text{Sampling rate } f_{(s)} = \frac{1}{T_{(s)}} = 2 \times f_{max}$$

Eg:- A complex low-pass signal has a bandwidth of 200kHz. What is the ~~maximum~~ minimum sampling rate?

Earlier we said that a low-pass channel bandwidth starts from 0 Hz. If the bandwidth is 200kHz, then maximum frequency is 200kHz.



$BW = 200kHz = f_{max} - 0$

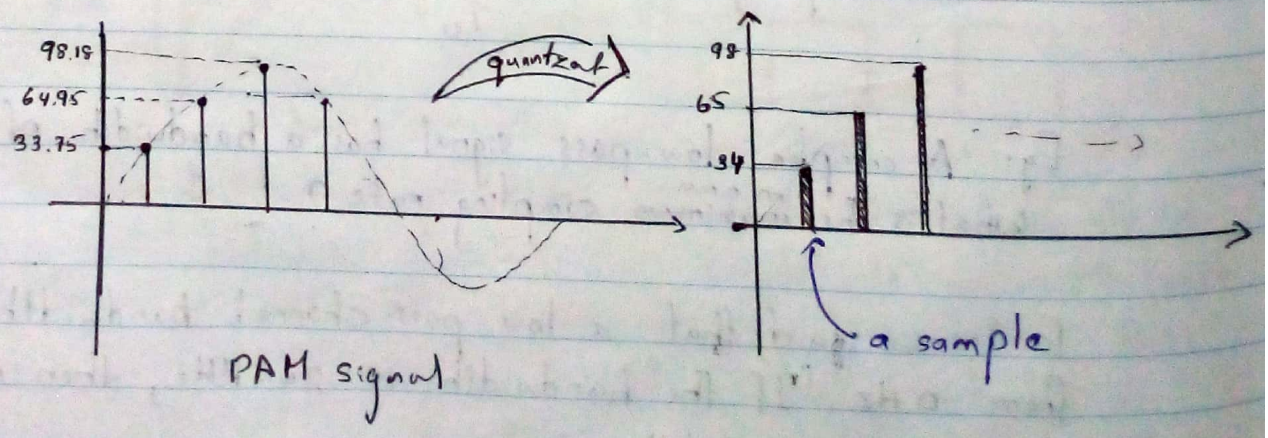
$\therefore f_{max} = 200kHz$

Then according to the Nyquist theorem,
the minimum sampling rate $= \cancel{200}\ 2 \times f_{max}$
$= 2 \times 200$
$(f_s) = 400kHz$

- Quantization

Quantization is kind of a rounding numbers to integer values. When you sample and get the amplitude values, you get non-integral values (values with decimal places). Therefore before encoding, these values are converted to (rounded to) integer values using special calculations. This process is called quantization.

Eg:-



Encoding

In this last step the quantized numbers are converted into binary codes. Each sample is encoded in to an n-bit word.

Eg:- if converted into 8-bit code word

quantized numbers	8-bit code word
+98	1 1 1 0 0 0 1 0
+65	1 1 0 0 0 0 0 1
+34	1 0 1 0 0 0 1 0
-45	0 0 1 0 1 1 0 1

here, + is 1, - is 0

Signbit

Then the encoded binary bits are transferred.

Bandwidth of PCM

$$BW_{PCM} = n_b \times BW_{analog}$$

Bandwidth of PCM = number of bits per sample × Bandwidth of the analog signal.

Eg:- What is the bandwidth a channel that sends a digitized low pass analog signal with 8KHz frequency. ~~The~~ bits per sample is 8.

If it is a lowpass channel the minimum bandwith of the analog signal is 8KHz. ($B_w = f_{max} - 0 = 8KH_z - 0 = 8KH_z$)

$\therefore BW_{PCM} = 8KHz \times 8 bits$

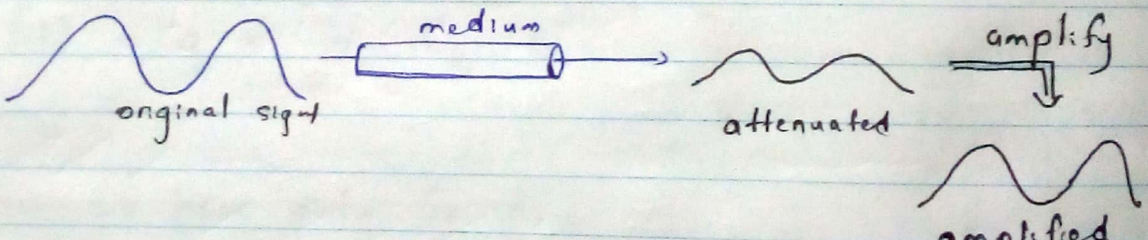
$= 64 KHz$

Transmission Impariments

When transmit through a medium, signal looses it's original qualities, so that the output signal is not identical to the input signal. This effect is called transmission impariments.

There are 3 main reasons for signal impariments
- Attenuation
- Distortion
- Noise

- Attenuation - means loss of energy due to resistance of the transmitting medium. Amplifiers are used to gain that lost energy back by amplifying the signal.



Calculating the power of a signal

$$\beta = 10\log(I/I_0)$$

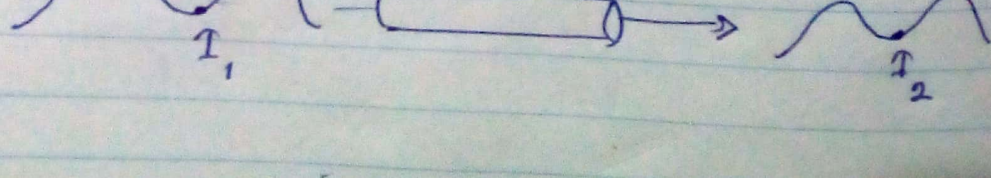
I - signal intensity

I_0 - reference intensity

↳ units - Wm^{-2}

↳ units . dB

Calculating the loss or gain



loss/gain of energy = power of the ~~original~~ Attenuated signal − power of the original signal

$$= 10\log\left(\frac{I_2}{I_0}\right) - 10\log\left(\frac{I_1}{I_0}\right)$$

$$= 10\left[\log \frac{I_2}{I_0} \Big/ \left(\frac{I_1}{I_0}\right)\right]$$

$$= 10\left[\log \frac{I_2 \times I_0}{I_0 \times I_1}\right]$$

$$= 10\log\left[\frac{I_2}{I_1}\right] \longrightarrow$$ if > 1 gain of energy
if < 1 loss of energy

- **Distortion** - means signal changes its form of shape.
  - occurs in composite signals.
  - occurs due to the different propagation speeds of the signals when traveling in a medium.
    - ↳ This makes the components of the composite signal reach to the output in different times.

- **Noise** - can be catagorized into four types
  - ↳ Thermal noise - due to agitation of electrons in a conductor. (විසිර යාම)
  - Intermodulation - noise - mixing up 2 or more seperate signals when sharing a single transmission medium.
  - Cross talk - result of branching several conductors together in a single cable. This causes electromagnetic fields affect on signal transmission
  - Impluse noise -

next pg →

- impulse noise - irregular pulses or noise spikes of short duration generated by incidents such as lightning or spark due to loose contact in electrical circuits. This is the main reason of bit-errors in digital data communicat.

- Shanon Capacity
  - this uses to calculate the data rate for a noisy channel

SNR (S/N) - signal to noise ratio

$$\frac{S}{N} \quad \begin{matrix} \leftarrow \text{average received signal power} \\ \leftarrow \text{average noise or interference power} \end{matrix}$$

$$\text{Capacity} = \text{bandwidth} \times \log_2 (1 + S/N)$$

$$\{\{ \; C = B \log_2 (1 + SNR) \; \}\}$$

~~if S/N = 0~~

$$\cancel{C = B \log_2 (1 + 0)}$$

$$\cancel{C/B = \log_2 (1) = 0}$$

if SNR given in deciBells (dB)

$$\{\{ \; SNR_{dB} = 10 \log_{10} (S/N) \; \}\}$$

Eg:- if SNR = 36 dB S/N = ?

$$36 = 10 \log_{10} (S/N)$$

$$\therefore 10^{3.6} = S/N = 3981$$

Throughput →

is a measurement of how we can actually send data through a network. Bandwidth specifies the limit (potential) of the link or medium that can send data per second. But throughput specifies the actual speed that medium transmit data since actual transmission speed (limit) is less than bandwidth

Eg:- A network with bandwidth of 10Mbps can ~~process~~ pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of the network?

$$\text{Throughput} = \frac{12000 \times 10{,}000}{60\,s} = 2 \times 10^6 \text{ bits}^{-1}$$

$$= \underline{\underline{2\text{ Mbps}}}$$

Propagation Time

- measures the time required for a bit to travel from the source to the destination

$$\text{propagation time} = \frac{\text{Distance}}{\text{propagation speed}}$$ (speed of the network)

Transmission Time

- time required for transmission of a message, depends on the size of the message and the bandwidth of channel

$$\text{Transmission time} = \frac{\text{Message size}}{\text{Bandwidth}}$$

Latency (Delay)

- defines how long it take for an entire message to completely arrive at the destination from the time the first bit is sent from the source.

Latency = propagation time + transmission time + queing time + processing delay

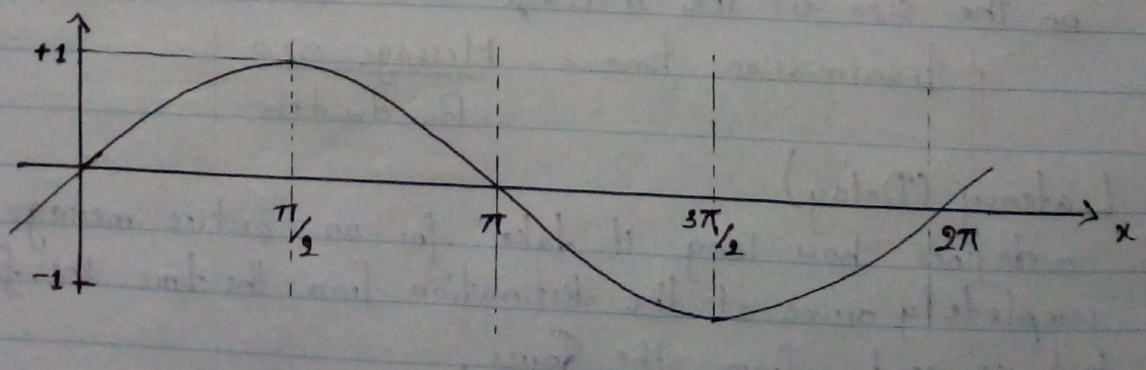
4. Fourier Series and Fourier Transform

Joseph Fourier is the founder of the fourier series and fourier transform. The idea was developed for the periodic signals then expanded it in to aperiodic signals.

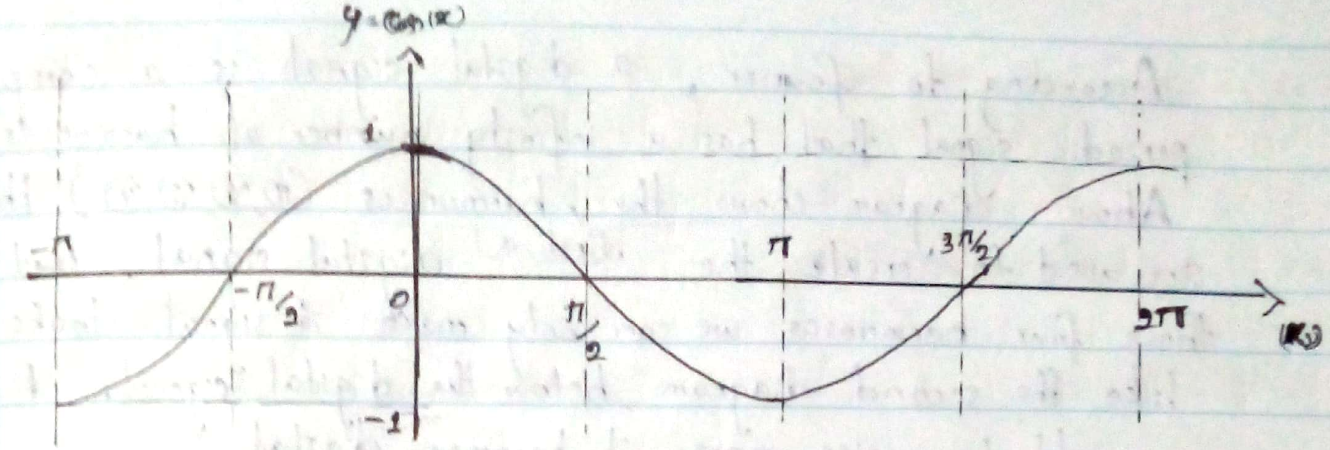
We know that sine waves are continuous periodic signals. These are also known as sinusoidal signals. Joseph Fourier said that the periodic signals can be created using set of sine waves. (This is the simple explanation of fourier series).

simply we can identify that, A periodic signal is a composite signal made up with different sine wave with different frequencies. The fourier series is used to break down these composite periodic signals ~~and~~ in to simple sine and ~~cosin wa~~ cosine waves and analyze them.

(cosine wave is also a periodic signal. Fourier series includes both sine and cosine waves).



simple sine wave



simple cosine wave

$\sin 0 = 0$

$\sin \pi/6 = \frac{1}{2}$

$\sin \pi/4 = \frac{1}{\sqrt{2}}$

$\sin 3\pi/2 = \frac{\sqrt{3}}{2}$

$\sin \pi/2 = 1$

$\sin \pi = 0$

$\sin 2\pi = 0$

$\cos 0 = 1$

$\cos 2\pi = 1$

$\cos \pi/4 = \frac{1}{\sqrt{2}}$

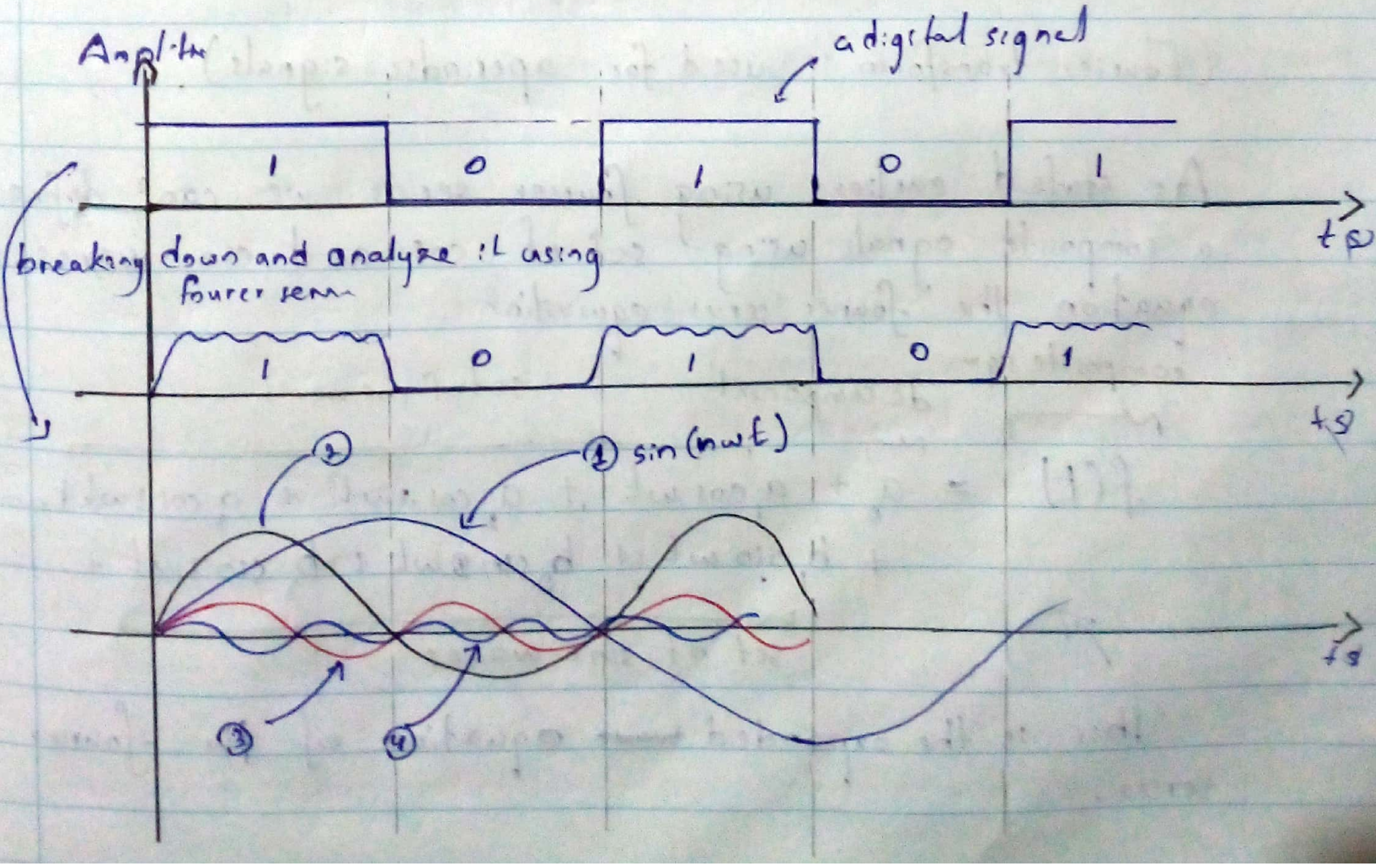
$\cos \pi/6 = \frac{\sqrt{3}}{2}$

$\cos \pi/2 = 0$

$\cos \pi/3 = \frac{1}{2}$

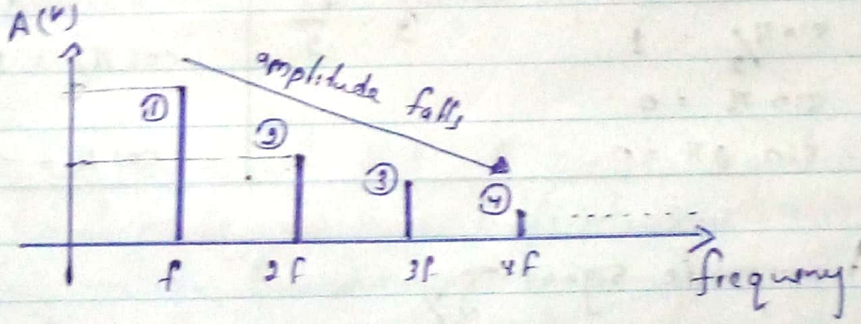
$\cos \pi = -1$

A composite signal



According to fourier, a digital signal is a composite periodic signal that has a infinity number of harmonics. Above diagram shows the first 4 harmonics (1, 2, 3, 4) that are used to create the digital signal. But using those four harmonics we can only make a signal looks like the second diagram, below the digital signal. More you add harmonics more it becomes digital.

↳ It does not need to find all the harmonics upto infinity. Because ~~other~~ ~~the~~ only first few harmonics contribute ~~to~~ a larger portion to the amplitude of the composite signal.



(Fourier transform is used for aperiodic signals).

As stated earlier, using fourier series we can define a composite signal using set of cos and sin waves. Below equation the fourier series equation.

$$\underbrace{f(t)}_{\text{composite sig-}} = \underbrace{a_0}_{\text{dc component}} + \overbrace{a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \ldots}^{\text{set of cos waves}} + \underbrace{b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \ldots}_{\text{set of sine waves}}$$

This is the expanded ~~wave~~ equation of the fourier series.

Definitions

- Σ - sum
- $\sum_{i=1}^{n}(x_i)$ - Sum of the x_i from $i = 1$ to n

to n

from $i = 1$

$$\sum_{i=1}^{n}(x_i) = (x_1 + x_2 + \dots + x_n)$$

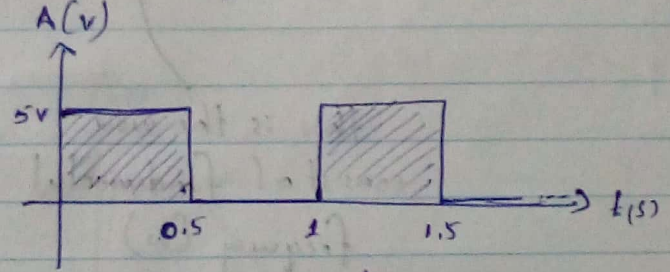
- using Σ we can write,

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(n\omega t) + b_n\sin(n\omega t)\right)$$

This is called Trigonometric fourier series

- a_0 - the dc component
  is the average value of the signal.

Eg:-



average amplitude (a_0)

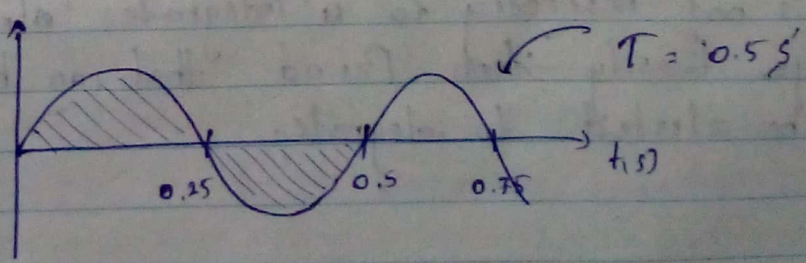
$$= \frac{\text{Area of the signal in 1 time period}}{\text{the time period}}$$

$$= \frac{5 \times 0.5 + 0 \times (1 - 0.5)}{1} \text{ V}$$

dc component (a_0) = 2.5 V

- period of a signal (T) - time taken to complete one cycle

Eg:-



- The frequency of a signal $(f) = \frac{1}{\text{period}} = \frac{1}{T}$

- The fundemental frequency (ω_0) = is the frequency of the fundemental harmonic. All the other harmonics are integer multiplications of the fundemental harmoic.

$$\omega_0 = \frac{2\pi}{T}$$

Eg:-

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots\ldots$$

this is the wave consist of fundemental frequency (ω)

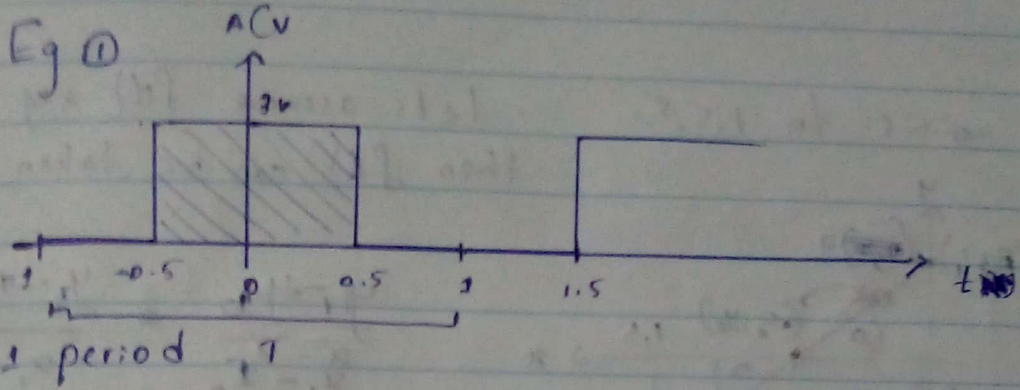
others are integer multiplications of funde. freq.

There are sometime that you cannot get the area of the signal for a given period without intergration in such cases below equation is used to calculate the dc component.

$$a_0 \text{ (dc component)} = \frac{1}{T}\int_0^T f(t)\, dt$$

It is not nesecery to integrate from 0 to T always. ~~Easily ident.~~ Period that can be easily identified can be selected to integrate.

Eg ①

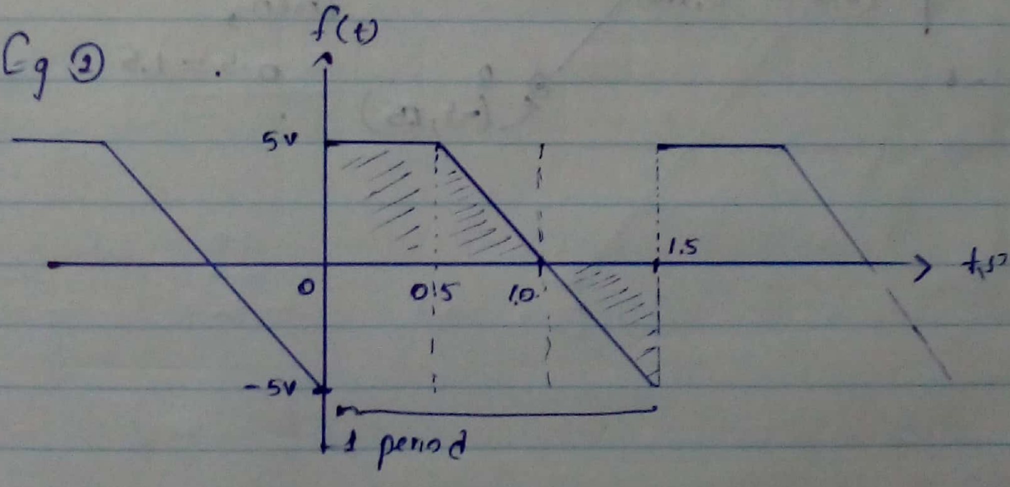


$$T = 1 - (-1) = 2s$$

$$\therefore a_0 = \frac{[0-(-0.5)] \times 3v + (0.5 \times 3v) + 0 \times (1-0.5)}{2}$$

$$= \frac{3}{2} = 1.5V$$

Eg ②



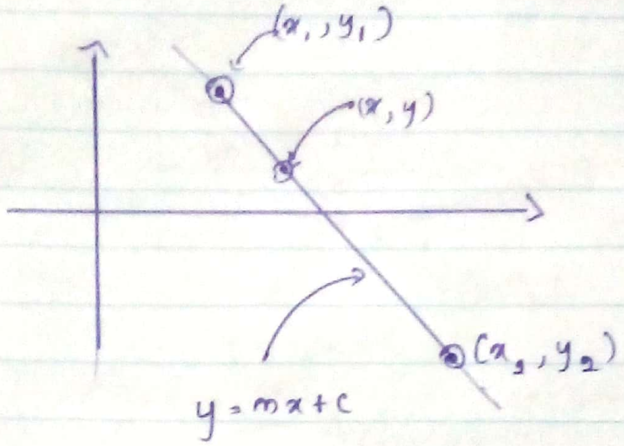
$T = 1.5\,s$

f(t) can be ~~de~~ written as below

$$f(t) = \begin{cases} 0 - 0.5 \; ; \; f(t) = 5 \text{ (a constant)} \\ 0.5 - 1.5 \; ; \; f(t) \text{ is decreasing} \end{cases}$$

↓

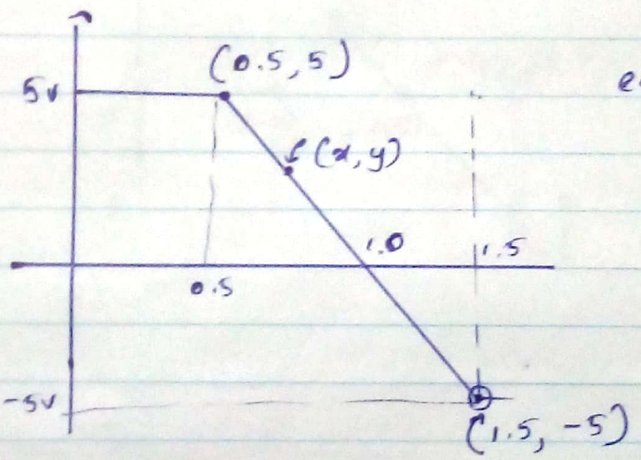
from 0.5 s to 1.5 s we have to derive a equation for f(t) from 0.5 s to 1.5 s



if you know 2 points on the line you can find the equation of the line using,

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_1 - y}{x_1 - x}$$

(x, y) is any point on the line



equation of line,

$$\frac{5 - (-5)}{0.5 - (1.5)} = \frac{5 - y}{0.5 - x}$$

$$\frac{10}{-1.0} = \frac{5 - y}{0.5 - x}$$

$$10(0.5 - x) = -(5 - y)$$

$$5 - 10x = -5 + y$$

$$\therefore \quad y = 10 - 10x$$

$$y = -10x + 10$$

$$\downarrow \quad \downarrow \; \downarrow \quad \downarrow$$

$$y = m\,x + c$$

Then

$$f(t) = \begin{cases} 0 - 0.5 \; ; & f(t) = 5 \\ 0.5 - 1.5 \; ; & f(t) = -10t + 10 \end{cases}$$

- Now let's find the dc component. We have to find the area of the signal from 0 – 0.5 s and 0.5 s – 1.5 s separately since f(t) is different in these time periods

$$\therefore a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{0.5} 5\,dt + \int_{0.5}^{1.5}(-10t+10)\,dt\right]$$

$$= \frac{1}{1.5}\left\{\underbrace{[5(t)]_0^{0.5}} + \underbrace{(-10)\int_{0.5}^{1.5}(t-1)\,dt}\right\}$$

$$= \frac{1}{1.5}\left\{5(0.5) - 5(0) - 10\left[\frac{t^2}{2} - t\right]_{0.5}^{1.5}\right\}$$

$$= \frac{1}{1.5}\left[2.5 - 0 - 10\left(\left[\frac{(1.5)^2}{2} - 1.5\right] - \left[\frac{(0.5)^2}{2} - 0.5\right]\right)\right]$$

$$= \frac{1}{1.5}\left[2.5 - 10\left(\frac{2.25}{2} - 1.5 - \frac{0.25}{2} + 0.5\right)\right]$$

$$= \frac{1}{1.5}\left\{2.5 - 10(0)\right\}$$

$$a_0 = 1.67\ V$$

This can also calculated simply,

$$a_0 = \frac{5 \times 0.5 + \frac{1}{2} \times 5(1-0.5) + \frac{1}{2}(-5)(1.5-1.0)}{1.5}$$

$$= 1.67\,V$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

To find the fourier series for a given signal we should find the a_n and b_n. They can be given as below.

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt$$

or

$$= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega t)\,dt$$

→ both can be used

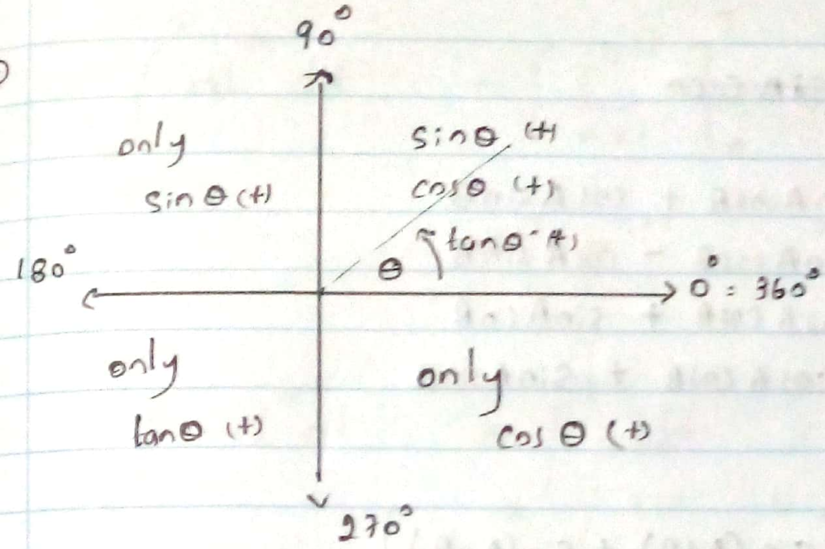
$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

or

$$= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega t)\,dt$$

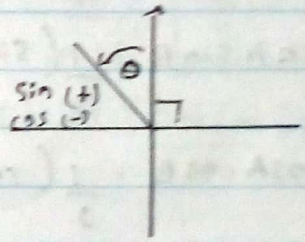
before starting fouriers series lets take a look at some trigonometric equations that are useful for the calculations

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x \neq -\cos x$
- ~~cot~~ $\tan x = \dfrac{\sin x}{\cos x}$ ◦ $\tan(-x) = -\tan x$



$90 + \theta \longrightarrow$

$\sin(90^\circ + \theta) = +\cos\theta$



$\cos(90^\circ + \theta = -\sin\theta$

$180 + \theta$

$\sin(180 + \theta) = -\sin\theta$

$\cos(180 + \theta) = -\cos\theta$

$180 - \theta$

$\sin(180 - \theta) = \sin\theta$

$\cos(180 - \theta) = -\cos\theta$

$270 + \theta$

$\sin(270 + \theta) = -\cos\theta$

$\cos(270 + \theta) = +\sin\theta$

$270 - \theta$

$\sin(270 - \theta) = -\cos\theta$

$\cos(270 - \theta) = -\sin\theta$

$\sin^2\theta + \cos^2\theta = 1 \longrightarrow 1 - \cos^2\theta = \sin^2\theta$

$1 - \sin^2\theta = \cos^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

~~$\sin(C+D) = \sin C \cos$~~

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin A\cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$

$\cos A\sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

$\cos A\cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$

$\sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$\int_0^T \sin(t)\,dt = [\cos(t)]_0^T$

$\int_0^T 5\,dt = [5t]_0^T$

$\int_0^T \cos(t)\,dt = [-\sin t]_0^T$

$\int_0^T 5t\,dt = \left[5\frac{t^2}{2}\right]_0^T$

$\int_0^T 5t^2\,dt = \left[5\frac{t^3}{3}\right]_0^T$

- $$\int_0^T m t^n \, dt = m\left[\frac{t^{n+1}}{n+1}\right]_0^T$$

$$= \frac{m}{n+1}\left[T^{n+1} - 0^{n+1}\right] = \frac{m}{n+1}\left(T^{n+1}\right)$$

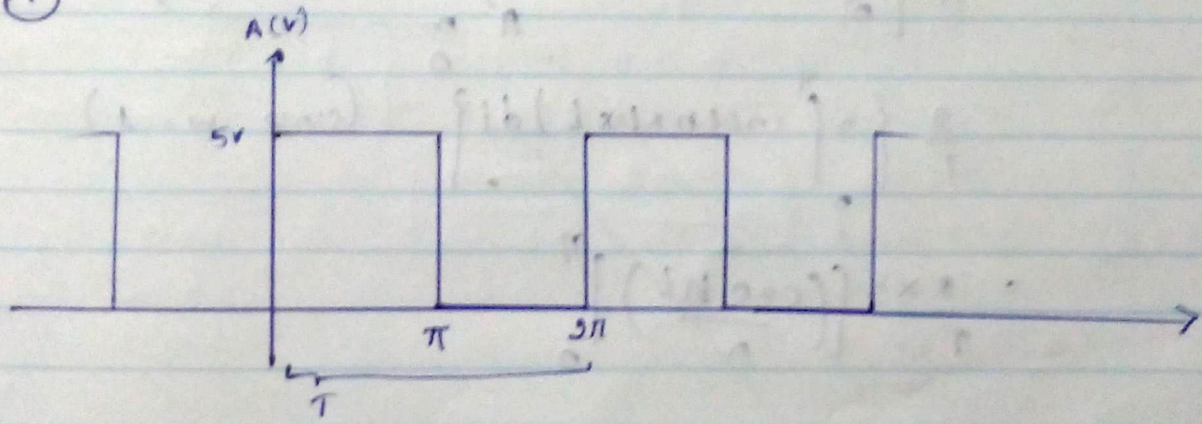
- $$\int_x^y m t^n \, dt = m \int_x^y t^n \, dt$$

$$= m \left[\frac{t^{n+1}}{n+1}\right]_x^y = \frac{m}{(n+1)}\left[y^{n+1} - x^{n+1}\right]$$

- $\int \sin(nt) = \frac{-\cos(nt)}{n}$
- $\int \cos(mt) = \frac{\sin(mt)}{m}$

Fourier series →

Ex: 1



The period is $T = 2\pi$

$\therefore f$ (frequen) $= \frac{1}{T} = \frac{1}{2\pi}$

fundemental frequency $(\omega) = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ Hz}$

dc component $a_0 = \frac{5V \times \pi + 0 \times (2\pi - \pi)}{2\pi}$

$= 2.5V$

$$f(t) = \begin{cases} 0 - \pi & 5V \\ \pi - 2\pi & 0V \end{cases} \qquad T : 0 \to 2\pi$$

let's find a_n first,

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t)\, dt$$

$$= \frac{2}{T} \left[\int_0^{\pi} 5 \times \cos(n\omega t)\, dt + \underbrace{\int_{\pi}^{2\pi} 0 \cos(n\omega t)\, dt}_{0} \right]$$

$$= \frac{2}{T} \left[5 \left[\frac{\sin(n\omega t)}{n\omega} \right]_0^{\pi} \right]$$

$$= \frac{10}{n\omega T} \left\{ \sin(n\omega\pi) - \sin(n\omega \times 0) \right\}$$

n is a integer, w is 1 Hz } $n \times 1 = m$; m is any integer

$$\therefore = \frac{10}{n\omega T} \left\{ \underbrace{\sin m\pi}_{0} - \underbrace{\sin 0}_{0} \right\}$$

$\sin \pi = 0$

$\sin 2\pi = 0$

$\sin 3\pi = 0$

$\vdots$

$\sin m\pi = 0$

$$a_n = \underline{\underline{0}}$$

lets find the b_n next,

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t)\, dt$$

$$= \frac{2}{T} \left[\int_0^{\pi} 5 \sin(n\omega t)\, dt + \underbrace{\int_{\pi}^{2\pi} 0 \times \sin(n\omega t)\, dt}_{0} \right]$$

$$b_n = \frac{2}{T}\left[5\int_0^{\pi} \sin(n\omega t)\,dt\right]$$

$$= \frac{10}{T}\left[\frac{-\cos(n\omega t)}{n\omega}\right]_0^{\pi}$$

$$= \frac{-10}{n\omega T}\left[\cos(n\omega t)\right]_0^{\pi}$$

$$= \frac{-10}{2\pi n}\left\{\cos(n\omega\pi) - \underbrace{\cos(0)}_{1}\right\}$$

$\omega = \frac{2\pi}{T}$

$\therefore \omega T = 2\pi$

$$b_n = -\frac{5}{\pi n}\left\{\cos(n\pi) - 1\right\} \quad (\because \omega = 1)$$

for even n values

$n = 0, 2, 4, 6, \ldots$

$\cos(n\pi) = +1$

Eg:- $\cos 2\pi = 1$

$\cos 0 = 1$

for odd 'n values

$n = 1, 3, 5$

$\cos(n\pi) = -1$

Eg:- $\cos\pi = -1$

$\cos 3\pi = -1$

Therefore

for even n

$$\underbrace{b_n = -\frac{5}{\pi n}\{+1 - 1\}}$$

$$b_n = 0$$

for odd n

$$b_n = \frac{-5}{\pi n}\{-1 - 1\}$$

$$b_n = \frac{10}{\pi n}$$

b_n exists only for $n =$ odd values only.

$$\therefore f(t) = \underbrace{a_0}_{2.5} + \sum_{n=1}^{\infty}\left(\underbrace{a_n \cos(n\omega t)}_{0} + \underbrace{b_n \sin(n\omega t)}_{\text{only for odd n values}}\right)$$

Then for above signal we can write the fourier expansion as below.

$$f(t) = 2.5 + \sum_{n=1}^{\infty}\left(0 + \frac{10}{\pi n}\sin(n\omega t)\right)$$

$$f(t) = 2.5 + \sum_{n=1}^{\infty}\frac{10}{\pi n}\sin(n\omega t) \qquad (n = 1, 3, 5, \ldots) \quad \omega = 1$$

if you are required to find the first ~~five~~ three components of the $f(t)$.

$$f(t) = 2.5 + \frac{10}{\pi \times 1}(\sin(1\omega t)) + \frac{10}{\pi \times 3}(\sin(3\omega t)) + \frac{10}{5\pi}(\sin(5\omega t))$$

coefficients

$$= 2.5 + \underbrace{\frac{10}{\pi}(\sin t)}_{1st\ component} + \underbrace{\frac{10}{3\pi}(\sin(3t))}_{2^{nd}} + \underbrace{\frac{10}{5\pi}(\sin(5t))}_{3^{rd}} \qquad (\omega = 1)$$

Ex 2 :- Data com tutorial 2 - ques 7

$$f(t) = \begin{cases} 1 - t & 0 \leq t < 1 \\ 1 + t & -1 \leq t < 0 \end{cases}$$

a) sketch the curve for $-4 \leq t \leq 4$

$t = 0 \to 1$ $\quad f(t) = 1 - t$ $\qquad (t, f(t))$

$= 1 - (+1) = 0 \qquad (1, 0)$

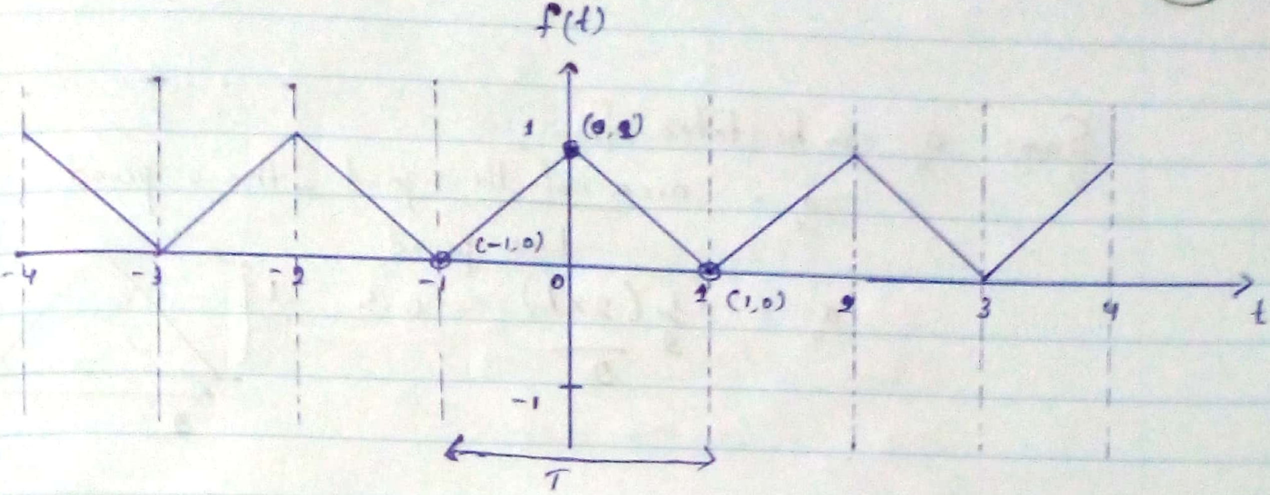
$f(t) = 1 - 0$

$= 1 \qquad (0, 1)$

$t = -1 \to 0$

$f(t) = 1 + t = 1 + (-1) = 0 \qquad (-1, 0)$

$f(t) = 1 + t = 1 + 0 \qquad (0, 1)$

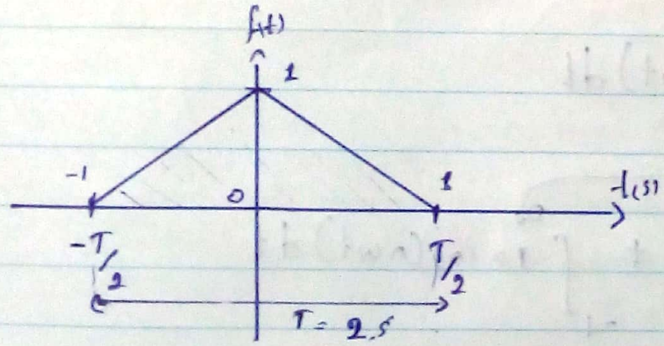


b) Identify the period and the frequency of the signal

period (T) = $(1 - (-1)) = 2s$

frequency (f) = $\frac{1}{T} = \frac{1}{2s} = 0.5\ Hz$

c) find the fourier series of the signal.



f(t) is defined for $-T/2$ to $T/2$

∴ Integration $\int_{-T/2}^{T/2}$

a_0 (dc component) $= \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$

$$= \frac{1}{2}\left[\int_{-1}^{0}(1+t)\,dt + \int_{0}^{1}(1-t)\,dt\right]$$

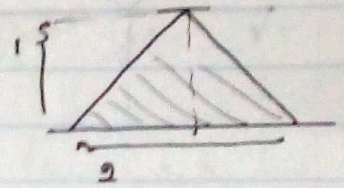
$$= \frac{1}{2}\left\{\left[t + \frac{t^2}{2}\right]_{-1}^{0} + \left[t - \frac{t^2}{2}\right]_{0}^{1}\right\}$$

$$= \frac{1}{2}\left\{0 - \left(-1 + \frac{1}{2}\right) + \left(1 - \frac{1}{2}\right) - 0\right\} = \frac{1}{2} = 0.5$$

Same a_0 can be taken from,

$$a_0 = \frac{\text{area of the signal within a period}}{\text{time period}}$$

$$a_0 = \frac{\frac{1}{2}(2\times 1)}{2} = 0.5$$



$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega t)\,dt$$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2}$

ω(fundemental frequency) $= \pi$

$$= \frac{2}{T}\Big[\underbrace{\int_{-1}^{0}(t+1)\cos(n\omega t)\,dt}_{I_1} + \underbrace{\int_{0}^{1}(1-t)\cos(n\omega t)\,dt}_{I_2}\Big]$$

$$I_1 = \int_{-1}^{0}(t+1).\cos(n\omega t)\,dt$$

$$= \int_{-1}^{0} t.\cos(n\omega t)\,dt + \int_{-1}^{0} 1\times\cos(n\omega t)\,dt$$

$\int u\,d(v) = uv - \int v\,d(u)$ ගුණිතයෙන අනුකලනය

$$= \int_{-1}^{0} t.\cos(n\omega t)\,dt$$

අනුකලනය කරන්න.

$$= \int_{-1}^{0} t\, d\left(\frac{\sin(n\omega t)}{n\omega}\right)$$

ගුණිතය

$$= \left[t\left(\frac{\sin(n\omega t)}{n\omega}\right)\right]_{-1}^{0} - \int_{-1}^{0}\frac{\sin(n\omega t)}{n\omega}\,dt$$

$$= \left[\frac{t\sin(nwt)}{nw}\right]_{-1}^{0} - \frac{1}{nw}\int_{-1}^{0}\sin(nwt)\,dt$$

$$= \left[\frac{t\sin(nwt)}{nw}\right]_{-1}^{0} - \frac{1}{nw}\left[-\frac{\cos(nwt)}{nw}\right]_{-1}^{0}$$

$$= \left[\frac{t\sin(nwt)}{nw}\right]_{-1}^{0} + \frac{1}{(nw)^2}\left[\frac{\cos(nwt)}{nw}\right]_{-1}^{0}$$

$$\therefore I_1 = \left[\frac{t\sin(nwt)}{nw}\right]_{-1}^{0} + \frac{1}{(nw)^2}\left[\cos(nwt)\right]_{-1}^{0} + \int_{-1}^{0}\cos(nwt)\,dt$$

$$= \sin 0 - \frac{(-1)\sin(nw(-1))}{nw} + \frac{1}{(nw)^2}(\cos 0 - \cos(-nw)) + \left[\frac{\sin(nwt)}{nw}\right]_{-1}^{0}$$

$$= -\frac{\sin(nw)}{nw} + \frac{1}{(nw)^2}(1 - \cos nw) + \left(\sin 0 - \frac{\sin(-nw)}{nw}\right)$$

$\sin(-\theta) = \sin\theta$ $\quad$ $\cos 0 = 1$ $\quad$ $\cos(-\theta) = +\cos\theta$ $\quad$ 0 $\quad$ $-\sin(-\theta) = +\sin\theta$

$$I_1 = \cancel{\frac{\sin(nw)}{nw}} + \frac{1}{(nw)^2}(1 - \cos nw)$$

$$I_2 = \int_0^1 (1-t)\cos(nwt)\,dt$$

$$= \int_0^1 \cos(nwt) - \int_0^1 t.\cos(nwt)\,dt$$

$$= \left[\frac{\sin(nwt)}{nw}\right]_0^1 - \int_0^1 t\,d\,\frac{\sin(nwt)}{nw}$$

$$= \left[\frac{\sin(nwt)}{nw}\right]_0^1 - \left\{\left[\frac{t\sin(nwt)}{nw}\right]_0^1 - \int_0^1 \frac{\sin(nwt)}{nw}\,dt\right\}$$

$$= \left[\frac{\sin(nwt)}{nw}\right]_0^1 - \left[\frac{t\sin nwt}{nw}\right]_0^1 + \left[-\frac{\cos nwt}{(nw)^2}\right]_0^1$$

$$= \underbrace{\frac{\sin(nw)}{nw} - \underbrace{\sin 0}_{0} - \left[\frac{\sin(+nw)}{nw} - 0\right]}_{0} - \left[\frac{\cos(nwt)}{(nw)^2}\right]_0^1$$

$$I_2 = -\frac{1}{(n\omega)^2}\left(\cos(n\omega t)\right]_0^1$$

$$= \frac{-1}{(n\omega)^2}\left(\cos n\omega - \underbrace{\cos 0}_{1}\right)$$

$$= \frac{-1}{(n\omega)^2}(\cos n\omega - 1)$$

$$I_2 = \frac{+1}{(n\omega)^2}(1 - \cos n\omega)$$

$$\therefore a_n = \frac{2}{T}\left(I_1 + I_2\right)$$

$$= \frac{2}{T}\left(\frac{1}{(n\omega)^2}(1-\cos n\omega) + \frac{1}{(n\omega)^2}(1-\cos n\omega)\right)$$

$\frac{2}{T} = \frac{2}{2} = 1$

$T = 2$

$$a_n = \frac{2(1-\cos n\omega)}{(n\omega)^2}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega t)\,dt$$

$$= \frac{2}{T}\left[\underbrace{\int_{-1}^{0}(1+t)\sin(n\omega t)\,dt}_{I_1} + \underbrace{\int_0^1 (1-t)\sin(n\omega t)\,dt}_{I_2}\right]$$

$$I_1 = \int_{-1}^{0} 1\times\sin(n\omega t)\,dt + \int_{-1}^{0} t\cdot\sin(n\omega t)\,dt$$

$$= \left[-\frac{\cos(n\omega t)}{n\omega}\right]_{-1}^{0} + \int_{-1}^{0} t\; d\left(-\frac{\cos n\omega t}{n\omega}\right)$$

$$= \left(-\frac{\cos(n\omega t)}{n\omega}\right)_{-1}^{0} + \left(-t\frac{\cos n\omega t}{n\omega}\right)_{-1}^{0} - \int_{-1}^{0}\left(-\frac{\cos n\omega t}{n\omega}\right)dt$$

$$= \left[-\frac{\cos(n\omega t)}{n\omega}\right]_{-1}^{0} + \left[-\frac{t\cos n\omega t}{n\omega}\right]_{-1}^{0} + \int_{-1}^{0} \frac{\cos n\omega t}{n\omega}\, dt$$

$$= \left[-\frac{\cos(n\omega t)}{n\omega}\right]_{-1}^{0} + \left[-\frac{t\cos n\omega t}{n\omega}\right]_{-1}^{0} + \frac{1}{(n\omega)^2}\left[\sin(n\omega t)\right]_{-1}^{0}$$

$$= -\frac{\cos 0}{n\omega} - \left(-\frac{\overbrace{\cos(-n\omega)}^{\cos n\omega}}{n\omega}\right) + 0 - \left(-\frac{(-1)\cos(-n\omega)}{n\omega}\right) + \frac{1}{(n\omega)^2}\left[\sin(n\omega t)\right]_{-1}^{0}$$

$$= \frac{-1}{n\omega} + \cancel{\frac{\cos n\omega}{n\omega}} - \cancel{\frac{\cos(n\omega)}{n\omega}} + \frac{1}{(n\omega)^2}\left\{\sin 0 - \sin(-n\omega)\right\}$$

$$I_1 = \frac{-1}{n\omega} + \frac{1}{(n\omega)^2}(\sin n\omega) \quad //$$

$$I_2 = \int_0^1 (1-t)\sin(n\omega t)\, dt$$

$$= \int_0^1 \sin(n\omega t)\,dt - \int_0^1 t\sin(n\omega t)\,dt$$

$$= \left[-\frac{\cos n\omega t}{n\omega}\right]_0^1 - \left[\int_0^1 t\, d\left(-\frac{\cos(n\omega t)}{n\omega}\right)\right]$$

$$= \left[-\frac{\cos n\omega t}{n\omega}\right]_0^1 - \left\{\left[-\frac{t\cos(n\omega t)}{n\omega}\right]_0^1 - \int_0^1 -\frac{\cos n\omega t}{n\omega}\, dt\right\}$$

$$= \left[-\frac{\cos n\omega t}{n\omega}\right]_0^1 + \left[\frac{t\cos n\omega t}{n\omega}\right]_0^1 - \left[\frac{\sin n\omega t}{(n\omega)^2}\right]_0^1$$

$$= \cancel{-\frac{\cos n\omega}{n\omega}} - \left(-\frac{\overbrace{\cos 0}^{1}}{n\omega}\right) + \cancel{\frac{\cos n\omega}{n\omega}} - 0 - \left(\frac{\sin n\omega}{(n\omega)^2} - \sin 0\right)$$

$$I_2 = \frac{1}{n\omega} - \frac{\sin n\omega}{(n\omega)^2} \quad //$$

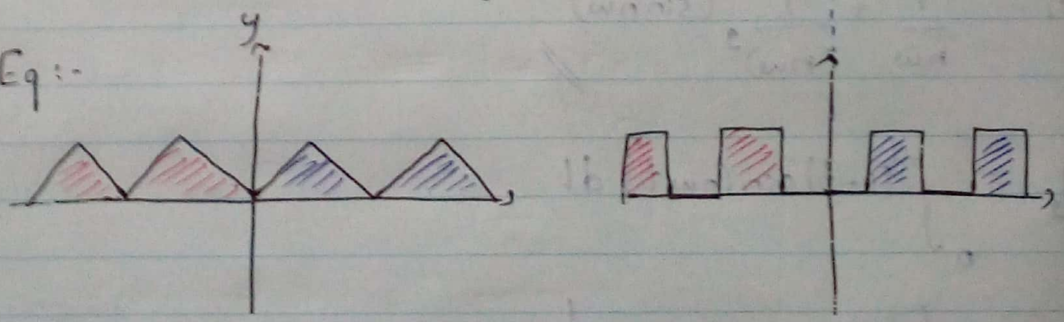
$$\therefore b_n = \frac{2}{T}\left[I_1 + I_2 \right]$$

$$= \frac{2}{T}\left[\underbrace{-\frac{1}{nw} + \frac{\sin(nw)}{(nw)^2} + \frac{1}{nw} - \frac{\sin(nw)}{(nw)^2}}_{0} \right]$$

$$b_n = 0$$

* Normally b_n becomes 0 in even symmetric signals. Which means the right hand side to the y axis of the signal is the mirror image of the left hand side.

Eg:-



If a given signal is a even symmetric signal you can say

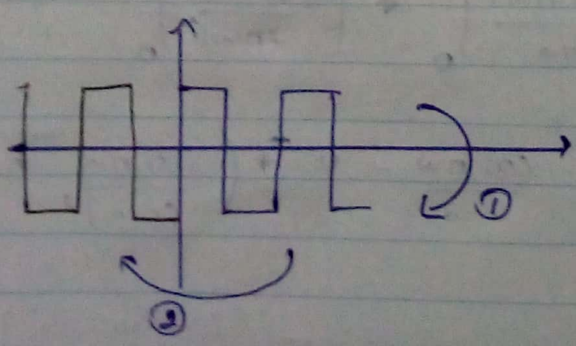
$$\therefore b_n = 0.$$

If the signal is odd symmetric $a_n = 0$

that means

$$f(t) = -f(t)$$

Eg:-



① right side upside down

② and that upside down is equal to the mirror image of left side now,

$$a_0 = 0.5\,V$$

$$a_n = \frac{2(1-\cos n\omega)}{(n\omega)^2} \qquad \omega = \frac{2\pi}{2} = \pi$$

$$b_n = 0$$

$n\omega$ = frequency of the signal

∴ fourier series;

$$f(t) = 0.5 + \sum_{n=1}^{\infty} \overbrace{\frac{2(1-\cos(n\pi))}{(n\pi)^2}}^{a_n} \cos n\omega t$$

$1 - \cos(n\pi)$

- when n is even $\cos n\pi = 1$
  $\therefore 1-\cos n\pi = 1-1 = 0$
- when n is odd $\cos n\pi = -1$
  $\therefore 1-(-1) = 2$

∴ a_n exist only for $n = 1, 3, 5, \ldots$

$$\therefore f(t) = 0.5 + \sum_{n=1}^{\infty} \frac{2(1-\cos n\pi)}{(n\pi)^2}(\cos n\pi t)$$

$(\because n = 1, 3, 5, \ldots\ldots)$

1st component, 2nd component, 3rd component

d) Plot the spectrum of the signal using first five components.

There are two types of spectrums,

1. Amplitude spectrum
2. Phase spectrum

- Calculating amplitude

Amplitude of a given component $= \sqrt{a_n^2 + b_n^2}$

Eg:- amplitude of the 1st component
$= \sqrt{a_1^2 + b_1^2}$

- Calculating phase

Phase of a given component $= \theta$

$\tan\theta = \cancel{\tan} \left(\frac{b_n}{a_n}\right)$

$\therefore \theta = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

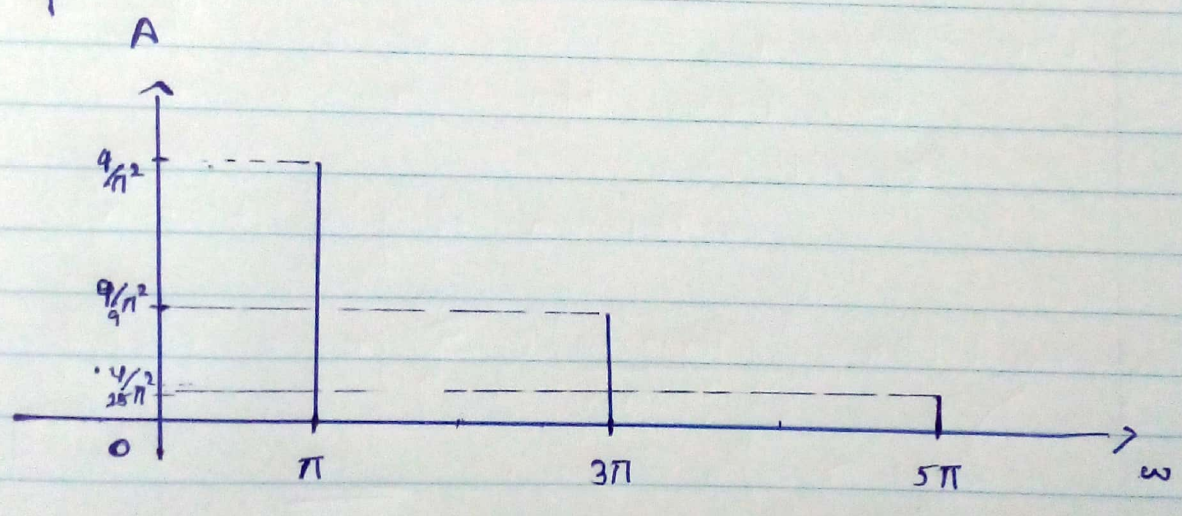
Eg:- phase of the 3rd component
$\theta = \tan^{-1}\left(\frac{b_3}{a_3}\right)$

Component	Amplitude	Phase
1st component $n = 1$	$A_1 = \sqrt{a_1^2 + \underbrace{b_1^2}_{0}}$ $= a_1$ $\therefore A_1 = \frac{2(1-\cos\pi)}{(\pi)^2}$ $= 4/\pi^2 = 0.406$	$\tan^{-1}\left(\frac{0}{a_1}\right) = \tan^{-1} 0$ $= 90^\circ$ frequency of the component $= n\omega$ $n\omega = 1 \times \pi$ $= \pi$

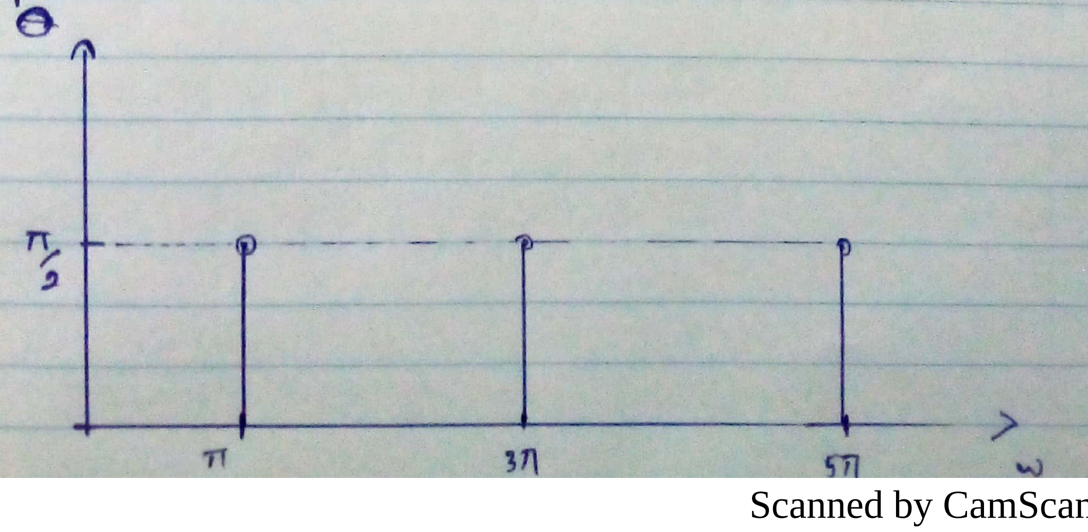
2^{nd} component $n = 3$	$A_2 = a_3$ $= \dfrac{2(\overset{2}{\overbrace{1 - \cos 3\pi}})}{(3\pi)^2}$ $= \dfrac{4}{9\pi^2}$	$\theta = 90^\circ$ freq. of the component $= 3 \times \pi$ $= 3\pi$
3^{rd} component $n = 5$	$A_3 = \dfrac{2(2)}{(5\pi)^2} = \dfrac{4}{25\pi^2}$	$\theta = 90^\circ$ freq. comp.. $= 5\pi$

→ Amplitude - frequency spectrum

Amplitude is drawn against the ~~fundamental~~ frequency of the component.



Phase spectrum



Complex exponential fourier series.

previously we stated that

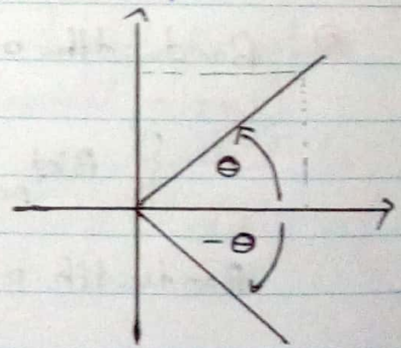
$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

This is trigonometric fourier series. Same fourier expansion can be derived from exponential functions and complex number. This is called complex exponential fourier series. It's easier to do calculations with exponential functions, therefore this complex exponential fourier series is commonly used in electrical engineering signal analysis.

This complex exponential fourier series derived using eular's formula.

$j = \sqrt{-1}$

Eular's formula →



$$e^{j\theta} = \cos\theta + j\sin\theta \rightarrow ①$$

$$e^{j(-\theta)} = \cos(-\theta) + j\sin(-\theta)$$

$$e^{-j\theta} = \cos\theta - j\sin\theta \rightarrow ②$$

① + ②

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\therefore \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

① - ②

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$$

$$\therefore \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

replacing θ with (ωt)

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$\theta \rightarrow$ replaced with (ωt), because

$$\boxed{\omega = \theta / t}$$ ω - frequency

Now we can replace $\sin(\omega t)$ and $\cos \omega t$ with trigonometric fourier series.

in complex numbers

$$j = \sqrt{-1} \quad \therefore j^2 = \sqrt{-1} \times \sqrt{-1} = (\sqrt{-1})^2 = -1$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j \times j} \times j = \frac{j(e^{j\omega t} - e^{-j\omega t})}{2j^2}$$ $\quad j^2 \rightarrow -1$

$$= -j\frac{(e^{j\omega t} - e^{-j\omega t})}{2}$$

$$\therefore f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \times \left(\frac{e^{nj\omega t} + e^{-nj\omega t}}{2}\right) + b_n\left(-j\left(\frac{e^{nj\omega t} - e^{-nj\omega t}}{2}\right)\right)\right]$$

$$\cancel{a_0 + \sum_{n=1}^{\infty} a_n}$$

$$\frac{1}{2}\left(a_n e^{nj\omega t} + a_n e^{-nj\omega t} - jb_n e^{nj\omega t} + jb_n e^{-nj\omega t}\right)$$

$$\frac{1}{2}\left(e^{nj\omega t}(a_n - jb_n) + e^{-nj\omega t}(a_n + jb_n)\right)$$

$$\therefore f(t) = a_0 + \sum_{n=1}^{\infty}\left(e^{jn\omega t}\frac{(a_n - jb_n)}{2} + e^{-jn\omega t}\frac{(a_n + jb_n)}{2}\right]$$

But this is bit complex to memorize

Therefore,

let, $\frac{a_n - jb_n}{2} = c_k$ and $\frac{a_n + jb_n}{2} = c_{-k}$

Then we can write

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(e^{jwnt} \cdot c_k + e^{-jnwt} \cdot c_{-k} \right)$$

when $n \rightarrow +\infty$ $c_k \rightarrow +\infty$ | when $n \rightarrow +\infty$ $c_k \rightarrow -\infty$

∴ That means you are adding, $(e^{jnwt} c_k)$ from $-\infty$ to $+\infty$

$$\therefore f(t) = a_0 + \sum_{n=-\infty}^{+\infty} c_k \cdot e^{jnwt}$$ ← complex fourier series

now we have only to find c_k. c_k can be found using below equation.

$$c_k = \frac{1}{T} \int_0^T f(t) e^{-jnwt} \, dt$$

Eg:- c_k of the first component

$$= \frac{1}{T} \int_0^T f(t) e^{-j \times 1 \times wt} \, dt$$

$$= \frac{1}{T} \int_0^T f(t) e^{-jwt} \, dt$$

It's easier to work with 'e' functions, since,

$$\int e^{x}\,dx = e^{x}$$

$$\int e^{nx}\,dx = \frac{\cancel{n}e^{nx}}{n}$$

$$\int e^{njwt}\,dt = \frac{e^{njwt}}{(njw)}$$

Fourier Transform $\longrightarrow$