

Gibbs Sampling for estimate of parameter posterior distributions

Modelling y data as a normal distribution with mean = μ , and variance = $1/\tau$. We read in the data, and we start with an estimate of τ to start the gibbs sampling off. Using the posterior distributions derived by hand, we have come up with the μ and τ sampling distributions in the for loop.

```
gibbs = function(tau1) {  
  
  y = scan("y data.txt", what=double())  
  
  y_bar = mean(y); s2 = var(y);  
  n = 100; k = 500;  
  
  mu = rep(0, k); tau = rep(0, k);  
  
  tau[1] <- tau1  
  
  # posterior sampling distributions  
  for (i in 2:(k)) {  
    mu[i] <- rnorm(1, mean = y_bar, sd = sqrt(1/(n*tau[i-1])))  
    k = sum(y^2) - 2*n*y_bar*(mu[i]) + n*((mu[i])^2)  
    tau[i] <- rgamma(1, shape = n/2, scale = (2 / k))  
  }  
  
  result <- list(x1 = mu, x2 = tau)  
  result  
  
}
```

```
means <- function(chain) {  
  return(c(mean(chain$x1), mean(chain$x2)))  
}
```

```
cred_int <- function(chain) {  
  mu_95 <- quantile(chain$x1, 0.95); mu_05 <- quantile(chain$x1, 0.05)  
  tau_95 <- quantile(chain$x2, 0.95); tau_05 <- quantile(chain$x2, 0.05)  
  
  intervals <- matrix(0,2,2)  
  intervals[1,1] <- mu_05; intervals[1,2] <- mu_95;  
  intervals[2,1] <- tau_05; intervals[2,2] <- tau_95;  
  
  return(intervals)  
}
```

starting guesses here for the chain

```

test_result1 = gibbs(tau1 = 1)
test_result2 = gibbs(tau1 = 0.5)

# try a 'bad' starting guess
test_result3 = gibbs(tau1 = 0.000001)

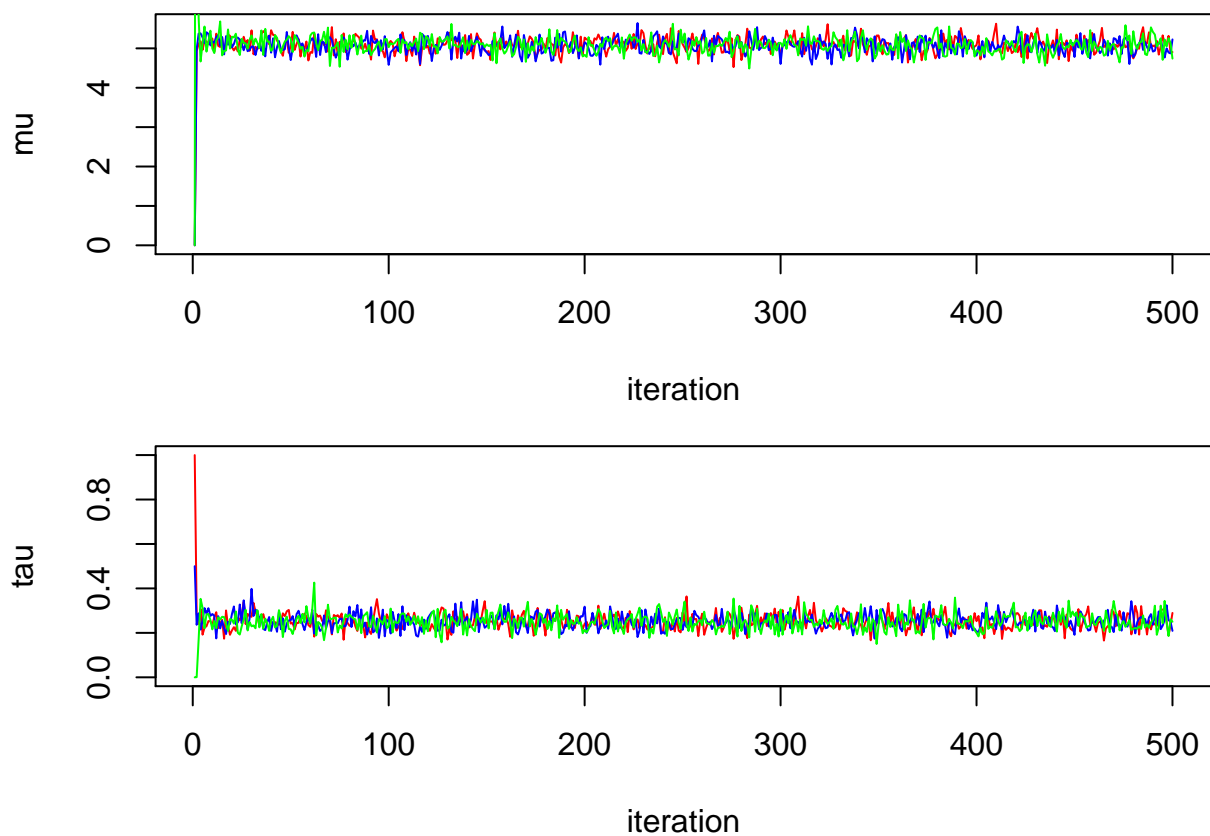
nreps = 500;

par(mfrow=c(2,1), mar=c(4,4,1,1))

plot(1:nreps, test_result1$x1, type="l", col="red", ylim = c(0, max(test_result1$x1, test_result2$x1)),
points(1:nreps, test_result2$x1, type="l", col="blue")
points(1:nreps, test_result3$x1, type="l", col="green")

plot(1:nreps, test_result1$x2, type="l", col="red", ylim = c(0, max(test_result1$x2, test_result2$x2)),
points(1:nreps, test_result2$x2, type="l", col="blue")
points(1:nreps, test_result3$x2, type="l", col="green")

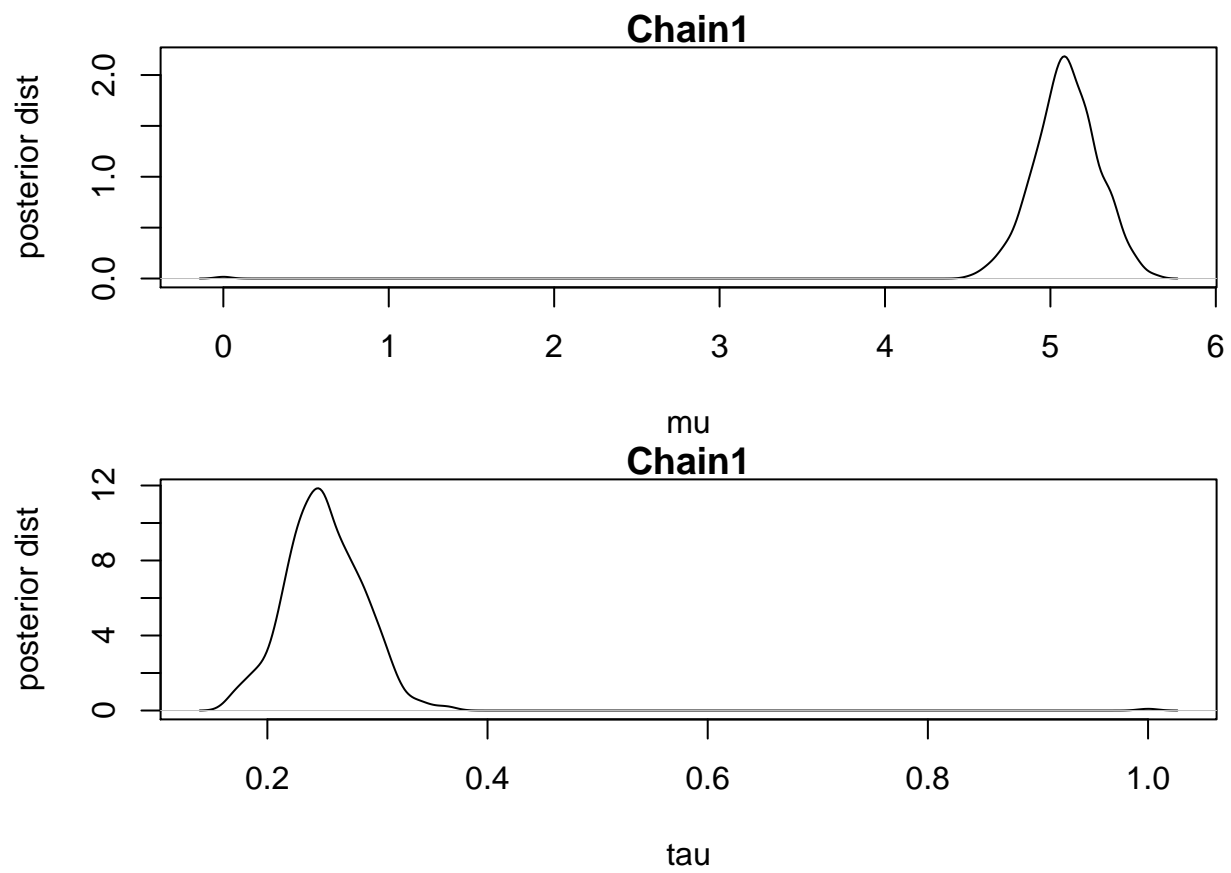
```



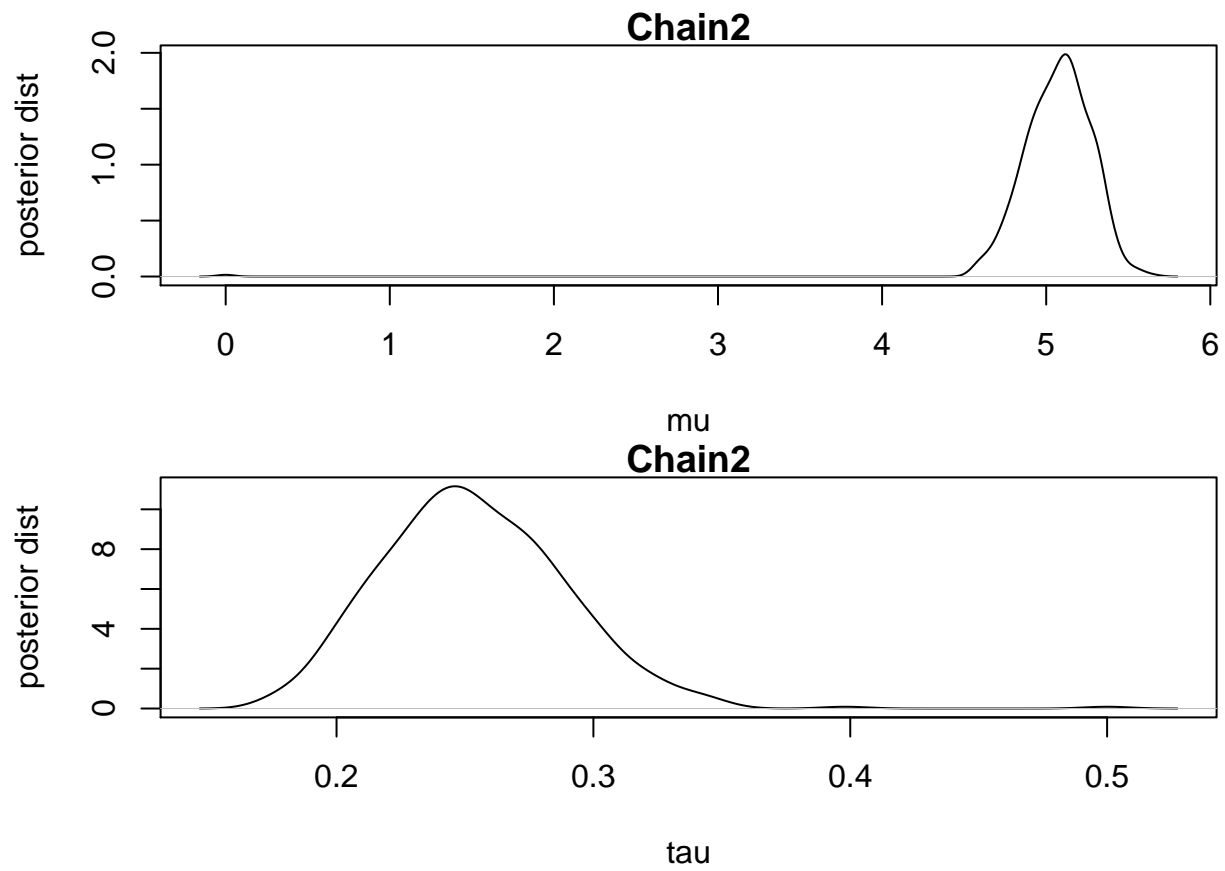
```

# Densities of 1st chain
plot(density(test_result1$x1), ylab="posterior dist", xlab="mu", main="Chain1")
plot(density(test_result1$x2), ylab="posterior dist", xlab="tau", main="Chain1")

```



```
# Densities of 2nd chain
plot(density(test_result2$x1), ylab="posterior dist", xlab="mu", main="Chain2")
plot(density(test_result2$x2), ylab="posterior dist", xlab="tau", main="Chain2")
```



```
(mean_test_result1 <- means(test_result1))
```

```
## [1] 5.086460 0.253068
```

```
(mean_test_result2 <- means(test_result2))
```

```
## [1] 5.0639462 0.2537444
```

```
(cred_int_test_result1 <- cred_int(test_result1))
```

```
##           [,1]      [,2]
## [1,] 4.7718470 5.3975394
## [2,] 0.1925715 0.3084869
```

```
(cred_int_test_result2 <- cred_int(test_result2))
```

```
##           [,1]      [,2]
## [1,] 4.7419524 5.3639195
## [2,] 0.1989986 0.3123465
```

Note: As we can see from the trace plots - all three converge quickly to the correct values of $\mu = 5$, and $\tau = 0.25$. The green line is the worst starting position, but still converges quickly.