Experimentation:

ID	Thresholds	Cost	Initial H	Generated	Expanded	Expanded/s	Time(s)
1	41, 42, 45, 47, 49, 51, 53, 55, 57	57	41	499,911,606	253,079,569	3,987,492	63.47
2	43, 45, 47, 49, 51, 53, 55	55	43	18,983,862	9,777,817	4,073,012	2.4
3	41, 43, 45, 47, 49, 51, 53, 55, 57, 59	59	41	455,125,298	229,658,364	3,973,364	57.79
4	42, 44, 46, 48, 50, 52, 53, 56	56	42	82,631,583	41,689,061	3,964,194	10.52
14	41, 42, 45, 47, 49, 51, 53, 53, 55, 57, 59	59	41	937,956,626	475,109,940	4,001,567	118.73
88	43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65	65	43	6,195,467,140	3,176,234,880	4,047,826	784.68

The 2 main improvements that I sought to do intuitively (I did not read the paper) were to only use one node, the initial_node, and to not go back to where you were previously. As only one node was used – no dynamic memory allocation had to be used, speeding up the program. Secondly, I realized that there was no point in reverting the state to where you had been previously, so I tracked that and did not allow the algorithm to apply that action.

```
-bash-4.1$ 15puzzle 1.puzzle
14 13 15  7
11 12  9  5
 6  0  2  1
 4  8 10  3

Initial Estimate = 41
Threshold = 41 43 45 47 49 51 53 55 57
Solution = 57
Generated = 499,911,606
Expanded = 253,079,569
Time (seconds) = 63.47
Expanded/Second = 3,987,492
```

```
-bash-4.1$ 15puzzle 2.puzzle
13  5  4 10
 9 12  8 14
 2  3  7  1
 0 15 11  6

Initial Estimate = 43
Threshold = 43 45 47 49 51 53 55
Solution = 55
Generated = 18,983,862
Expanded = 9,777,817
Time (seconds) = 2.40
Expanded/Second = 4,073,012
```

```
-bash-4.1$ 15puzzle 3.puzzle
14  7  8  2
13 11 10  4
 9 12  5  0
 3  6  1 15

Initial Estimate = 41
Threshold = 41 43 45 47 49 51 53 55 57 59
Solution = 59
Generated = 455,125,298
Expanded = 229,658,364
Time (seconds) = 57.79
Expanded/Second = 3,973,725
```

```
-bash-4.1$ 15puzzle 4.puzzle
 5 12 10  7
15 11 14  0
 8  2  1 13
 3  4  9  6

Initial Estimate = 42
Threshold = 42 44 46 48 50 52 54 56
Solution = 56
Generated = 82,631,583
Expanded = 41,689,061
Time (seconds) = 10.52
Expanded/Second = 3,964,194
```

```
-bash-4.1$ 15puzzle 14.puzzle
 7  6  8  1
11  5 14 10
 3  4  9 13
15  2  0 12

Initial Estimate = 41
Threshold = 41 43 45 47 49 51 53 55 57 59
Solution = 59
Generated = 937,956,626
Expanded = 475,109,940
Time (seconds) = 118.73
Expanded/Second = 4,001,567
```

```
-bash-4.1$ 15puzzle 88.puzzle
15  2 12 11
14 13  9  5
 1  3  8  7
 0 10  6  4

Initial Estimate = 43

Threshold = 43 45 47 49 51 53 55 57 59 61 63 65
Solution = 65
Generated = 6,195,467,140
Expanded = 3,176,234,880
Time (seconds) = 784.68
Expanded/Second = 4,047,826
```