

# Envelope Theorem

## One choice variable and one constraint

Let  $F$  and  $G$  be continuously differentiable functions of  $x$  and  $\theta$ . For any given  $\theta$ , let  $x^*(\theta)$  maximize  $F(x, \theta)$  subject to  $c \geq G(x, \theta)$ , and let  $\lambda^*(\theta)$  be the associated value of the Lagrange multiplier. Suppose that  $x^*(\theta)$  and  $\lambda^*(\theta)$  are also continuously differentiable functions, and that the constraint qualification  $G_1(x^*(\theta), \theta) \neq 0$  holds for all values of  $\theta$ . Then, the maximum value function defined by

$$V(\theta) = \max_x F(x, \theta) \text{ subject to } c \geq G(x, \theta)$$

satisfies

$$V'(\theta) = F_2(x^*(\theta), \theta) - \lambda^*(\theta)G_2(x^*(\theta), \theta).$$