Development of high-order MHD solvers for astrophysical applications

Sean M. Couch
Michigan State University

TEAMS Collaboration Meeting
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SNAPhU
Stellar and Nuclear AstroPhysics Unit

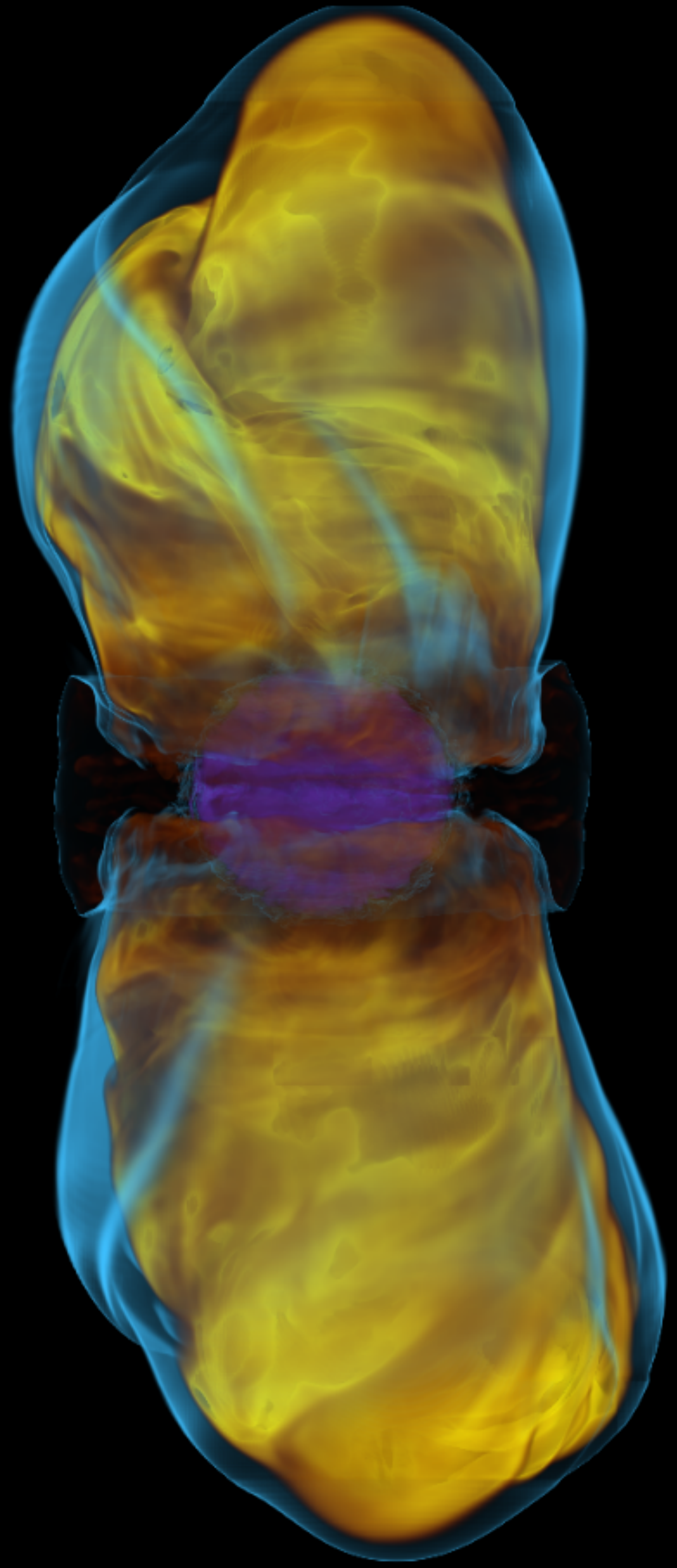
ECP
EXASCALE COMPUTING PROJECT

SciDAC
Scientific Discovery through Advanced Computing

U.S. DEPARTMENT OF ENERGY | Office of Science
Early Career Research Program

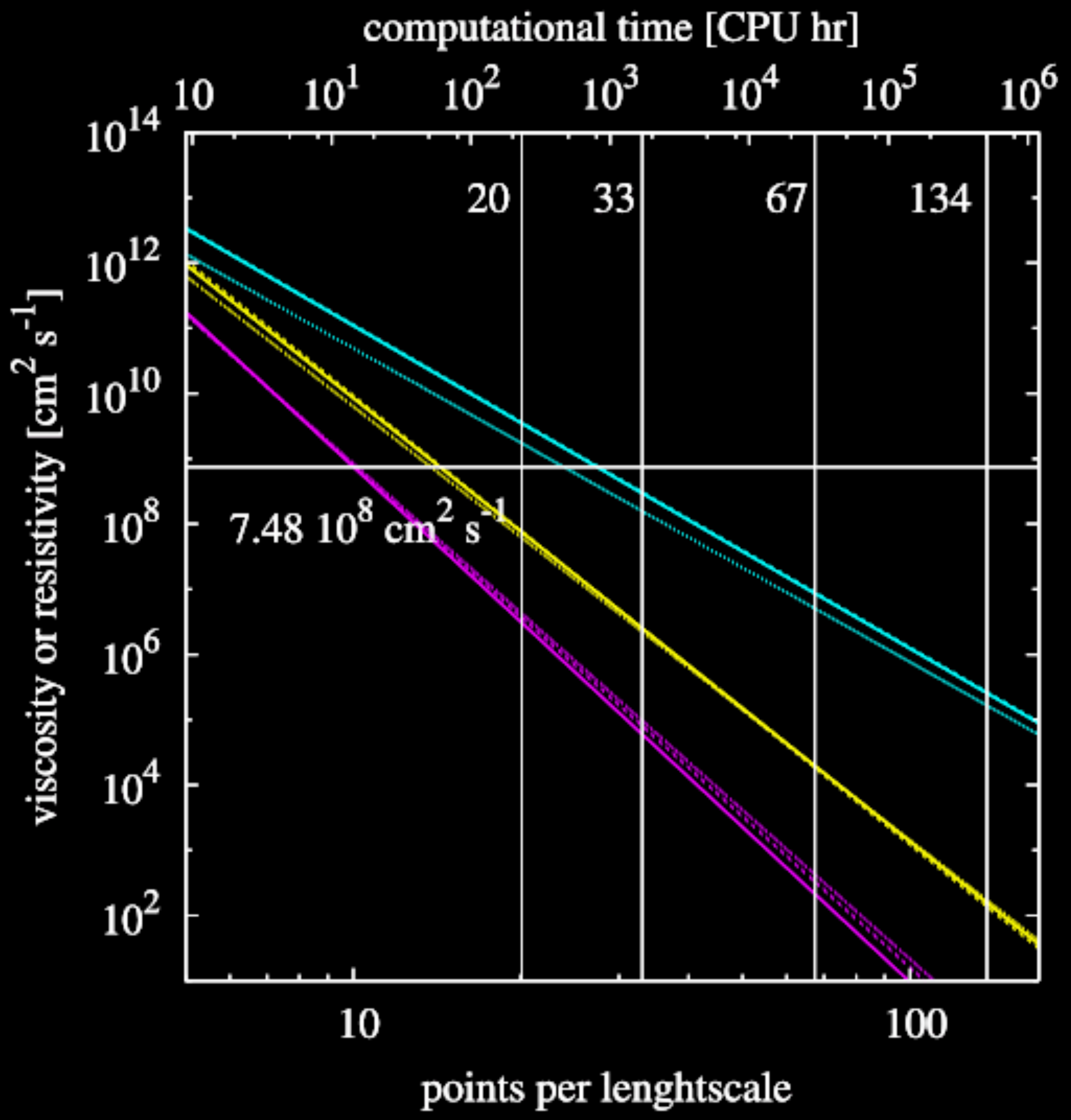
MSU TEAMS Goals

- Develop high-order MHD solver for astro
- Apply it to simulations of CCSNe
- Study pulsar and magnetar birth
- Simulation massive star collapse in 3D (with SBU folk)

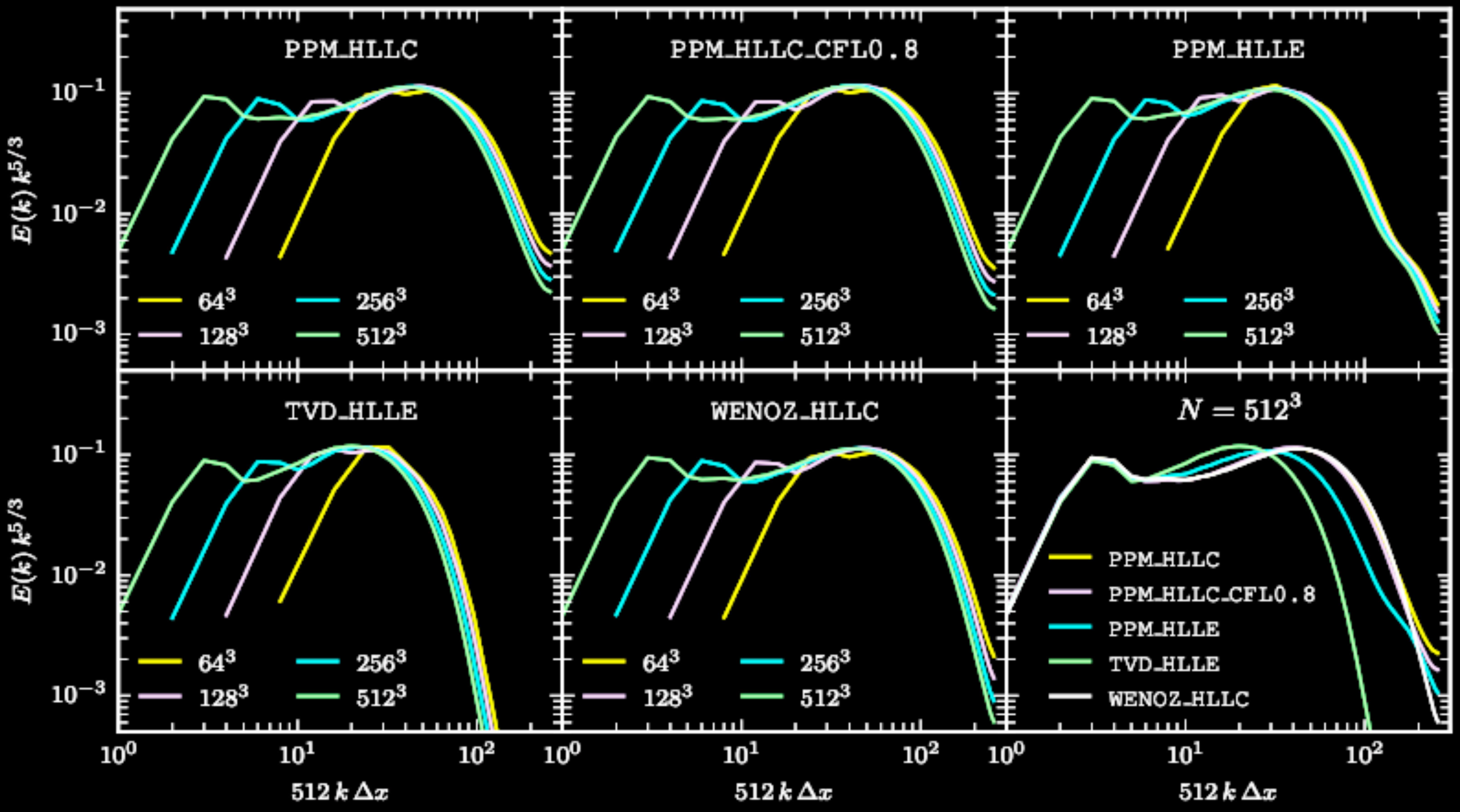


Why High-order?

"Flops are free."

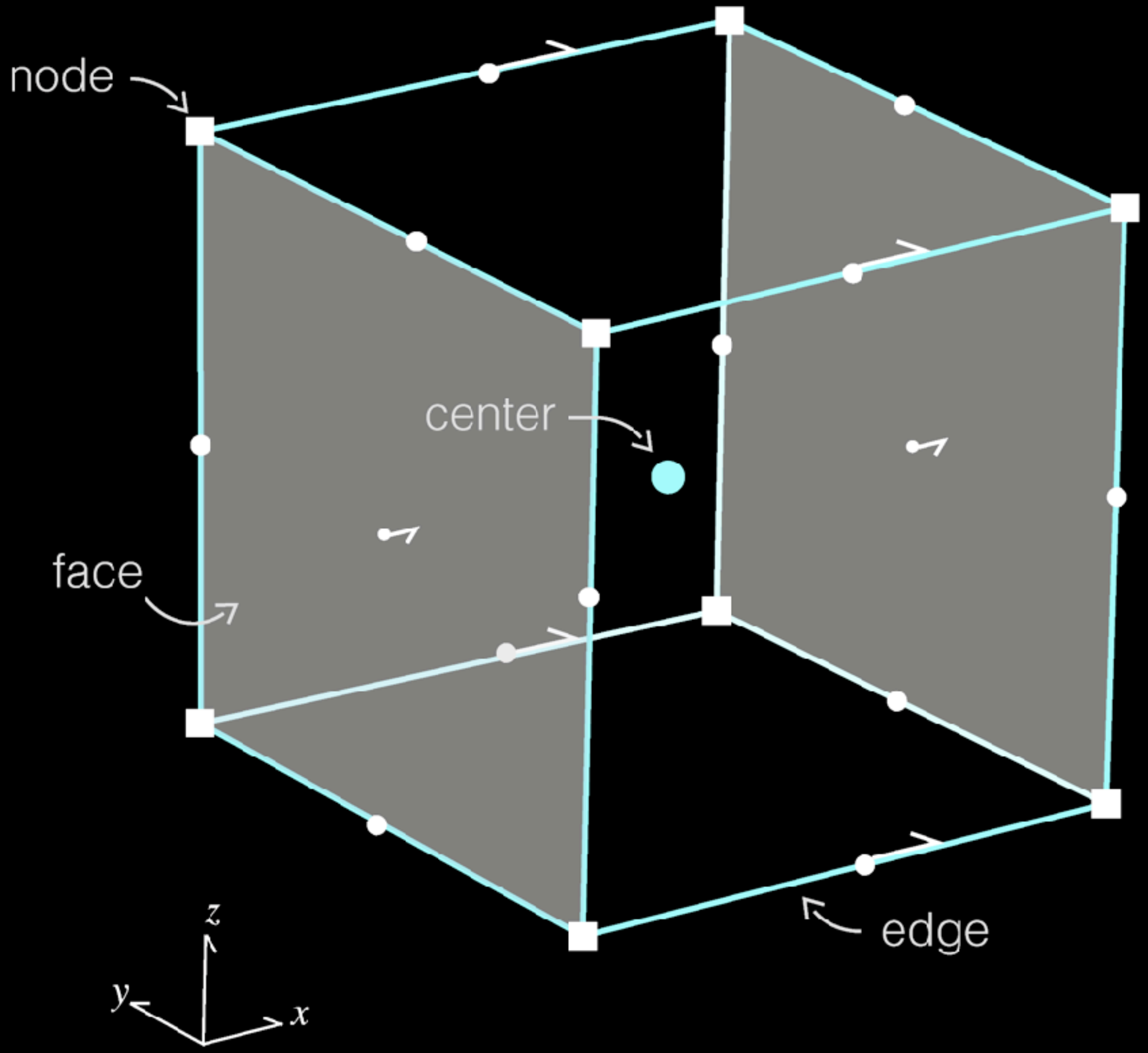


Rembiasz+17



Radice, SMC, Ott '15

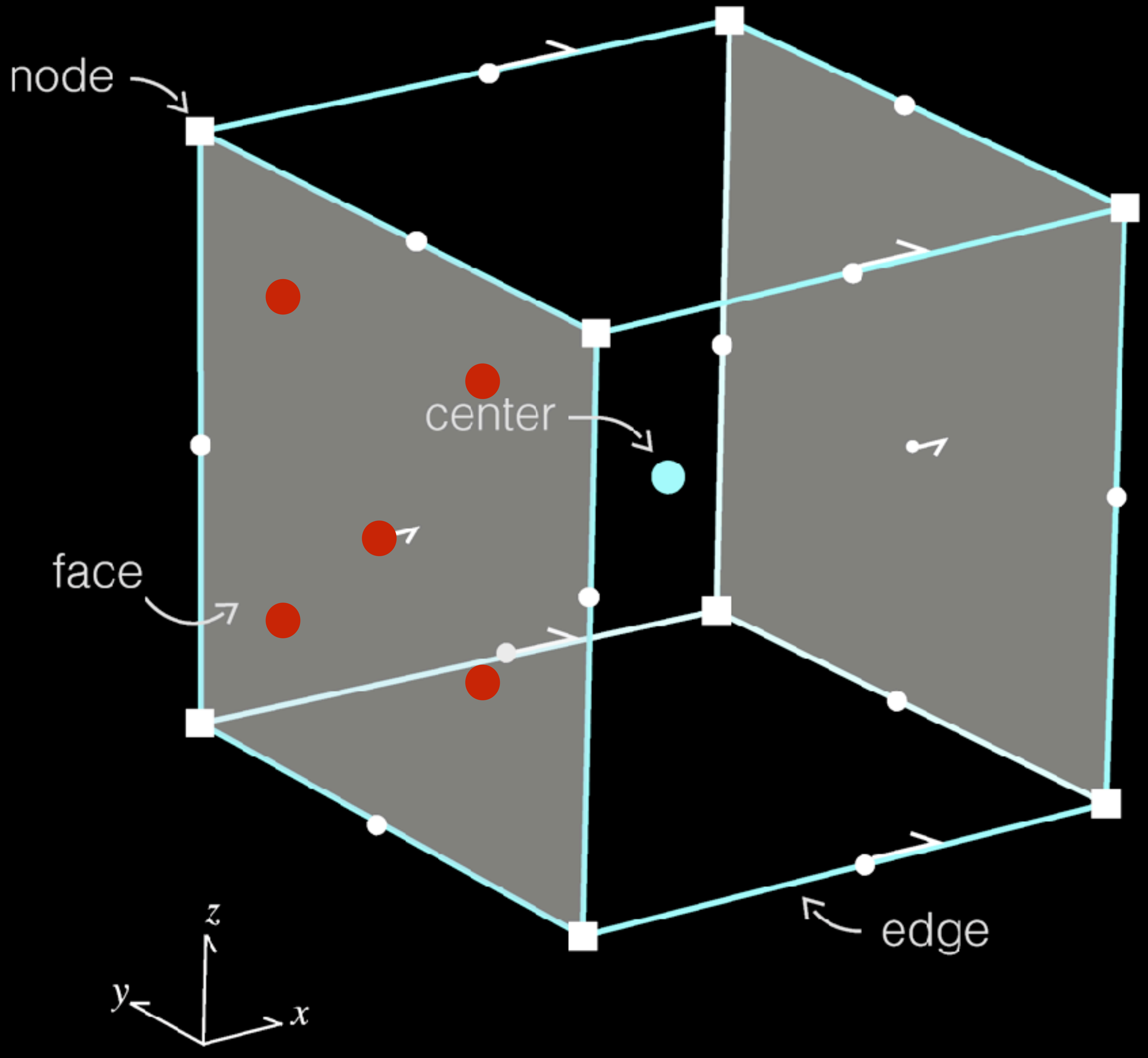
Finite Volume and Order Reduction



Cockett+15

- Evolve cell-averages with face-averaged fluxes. Exactly conservative!
- Spatial, temporal, flux order
- EOS and Riemann solver only exact for point-wise inputs!
- Some benefit from high spatial order (e.g., PPM)

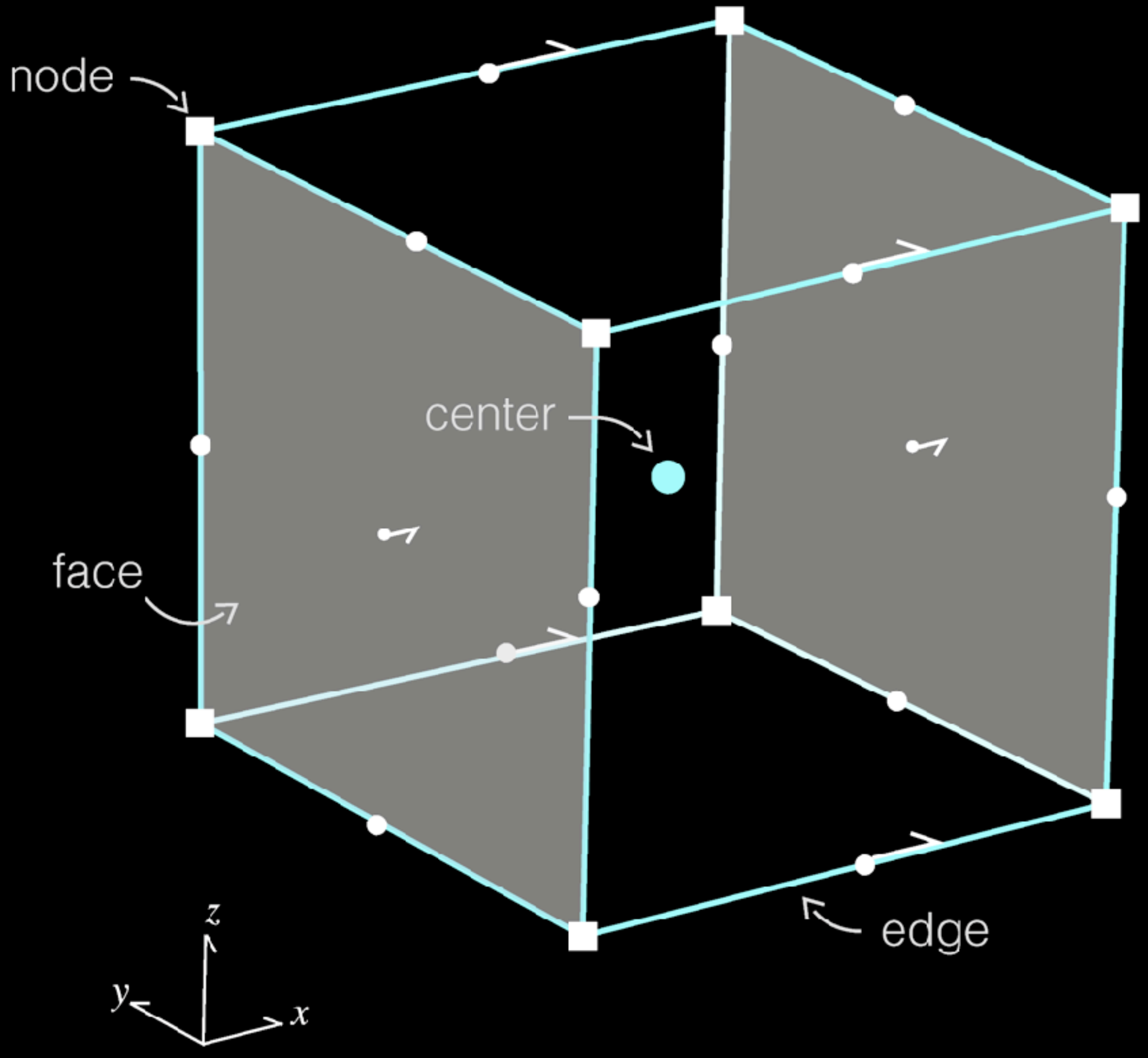
High-order FVM



Cockett+15

- Solve Riemann problem at multiple quadrature points on faces
  - Requires complex multi-D reconstruction!
- Conversions between point-wise and averaged values (larger stencil)

HO FVM W/O Quadrature



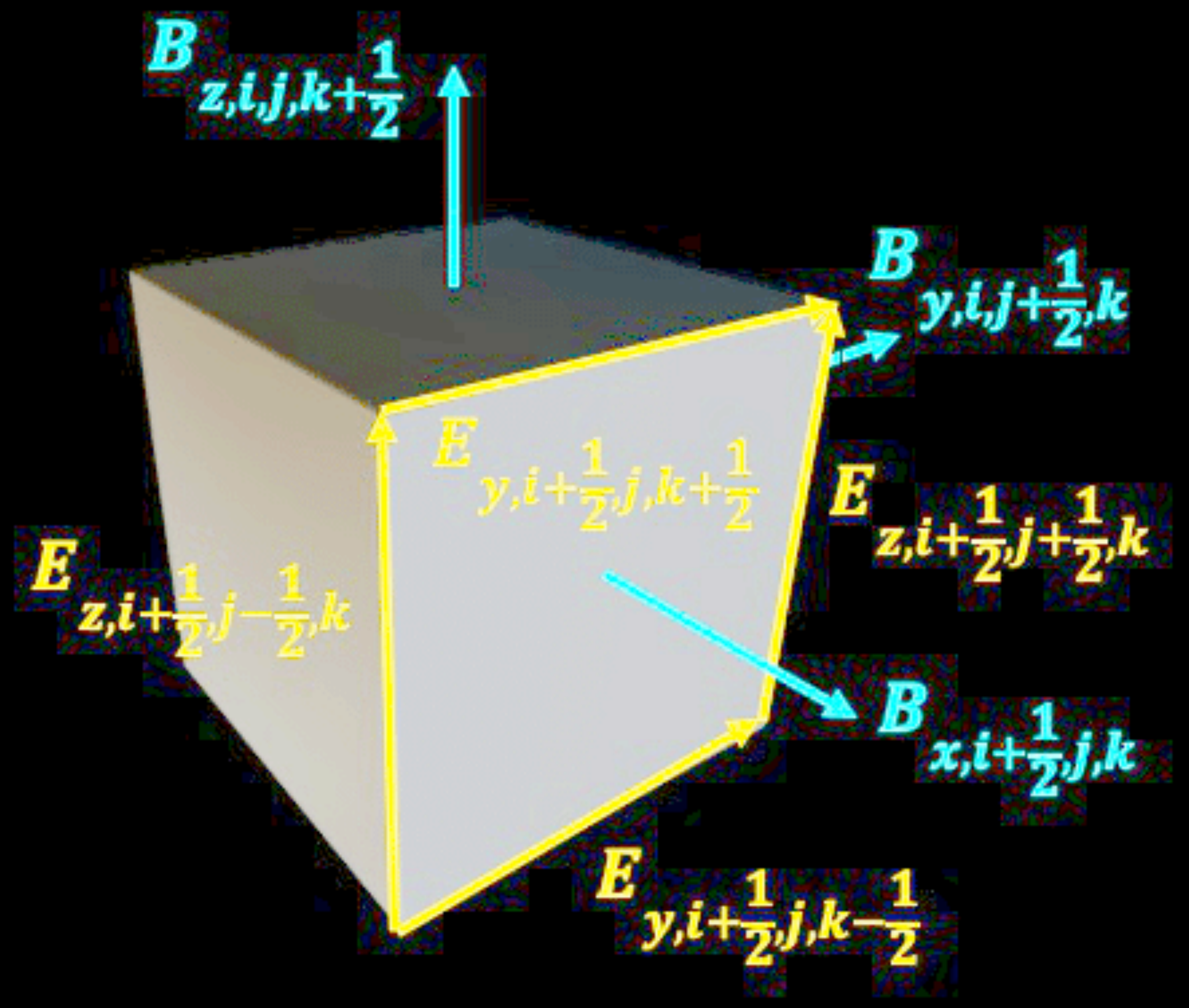
Cockett+15

- Use neighbor information to construct HO fluxes with a single Riemann solve per face (Buchmuller & Helzel '16)
- Or two Riemann solves per face (e.g., McCorquodale & Colella '11, Felker & Stone '18)

Thou Shalt Not Diverge

…magnetic fields

- Constrained transport (some form of staggering the fields)
- Cell centered via vector potential evolution (Helzel+13)
- Divergence cleaning (Dedner ’02)



Base solver: Spark

SMC 2020, in prep.

- Compressible ideal MHD w/ divB cleaning
- Finite volume discretization
- Different spatial reconstruction (5th-order WENO-Z default)
- Runge-Kutta time integration (RK2/RK3 now)
- Time-dependent gravity coupled to update
- General EOS for real gasses

Constrained GLM MHD Equations

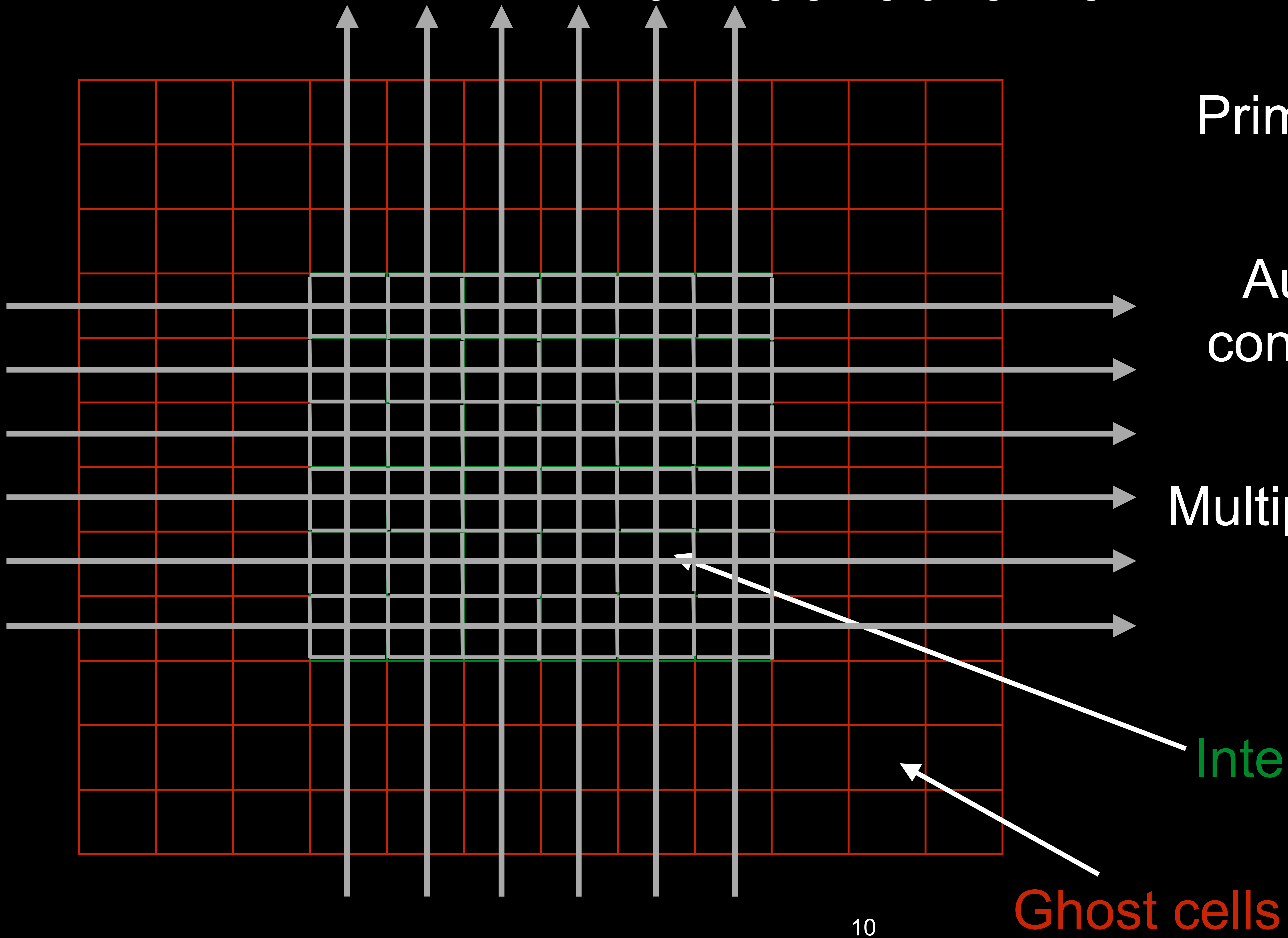
Dedner+02; Mignone, Tzeferacos, Bodo (2010)

$$\frac{\partial \mathbf{U}}{\partial t} = -\frac{\partial \mathbf{F}_i}{\partial x_i} + \mathbf{S}\,,$$

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho v_j \\ B_k \\ E \\ \psi \end{pmatrix}\,, \quad \mathbf{F}_i = \begin{pmatrix} \rho v_i \\ \rho v_j v_i - B_j B_i + \delta_{ij}\left(P + \mathbf{B}^2/2\right) \\ B_k v_i - B_i v_k + \delta_{ki}\psi \\ \left(E + P + \mathbf{B}^2/2\right) v_i - v_j B_j B_i \\ c_h^2 B_i \end{pmatrix}\,, \quad \mathbf{S} = \begin{pmatrix} 0 \\ -\rho \partial_j \Phi \\ 0 \\ -\rho v_j \partial_j \Phi \\ -c_h^2/c_p^2 \psi \end{pmatrix}\,,$$

$$E = \rho e + \frac{1}{2}\left(\rho \mathbf{v}^2 + \mathbf{B}^2\right)$$

Flux calculation



Primitive reconstruction with WENO-Z

Auxiliary 1D arrays: contiguous in memory, fit in cache

Multiple Riemann solvers:
HLLC default
HLLE in shocks

OpenMP threading

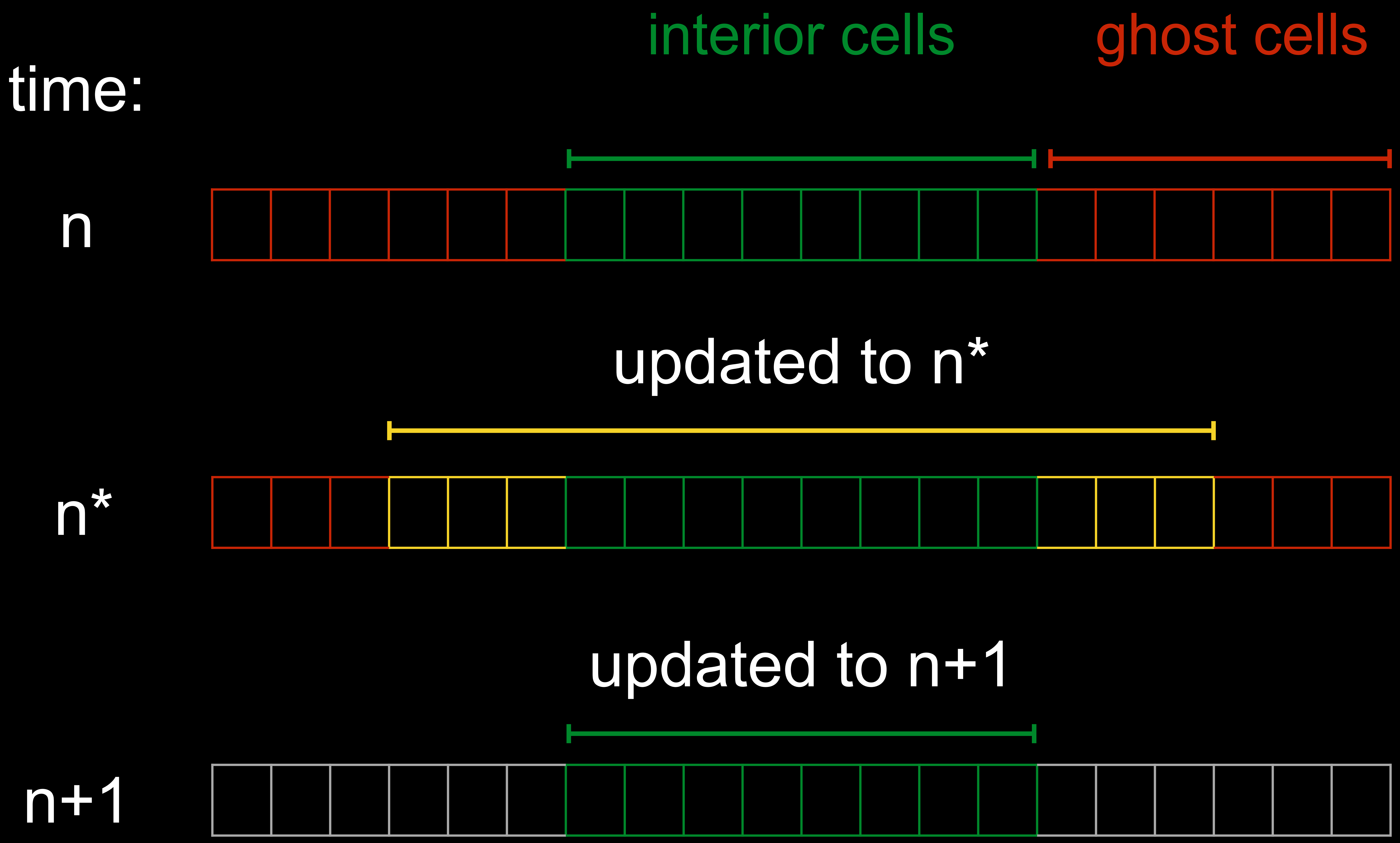
Time integration

SSP RK2:

$$
\begin{aligned}
\mathbf{U}^* &= \mathbf{U}^n + \mathcal{L}(\mathbf{U}^n) \\
\mathbf{U}^{n+1} &= \frac{1}{2}\mathbf{U}^n + \frac{1}{2}\mathbf{U}^* + \frac{1}{2}\mathcal{L}(\mathbf{U}^*)
\end{aligned}
$$

SSP RK3:

$$
\begin{aligned}
\mathbf{U}^* &= \mathbf{U}^n + \mathcal{L}(\mathbf{U}^n), \\
\mathbf{U}^{**} &= \frac{3}{4}\mathbf{U}^n + \frac{1}{4}\mathbf{U}^* + \frac{\Delta t^n}{4}\mathcal{L}(\mathbf{U}^*), \\
\mathbf{U}^{n+1} &= \frac{1}{3}\mathbf{U}^n + \frac{2}{3}\mathbf{U}^{**} + \frac{2}{3}\Delta t^n\mathcal{L}(\mathbf{U}^{**})
\end{aligned}
$$



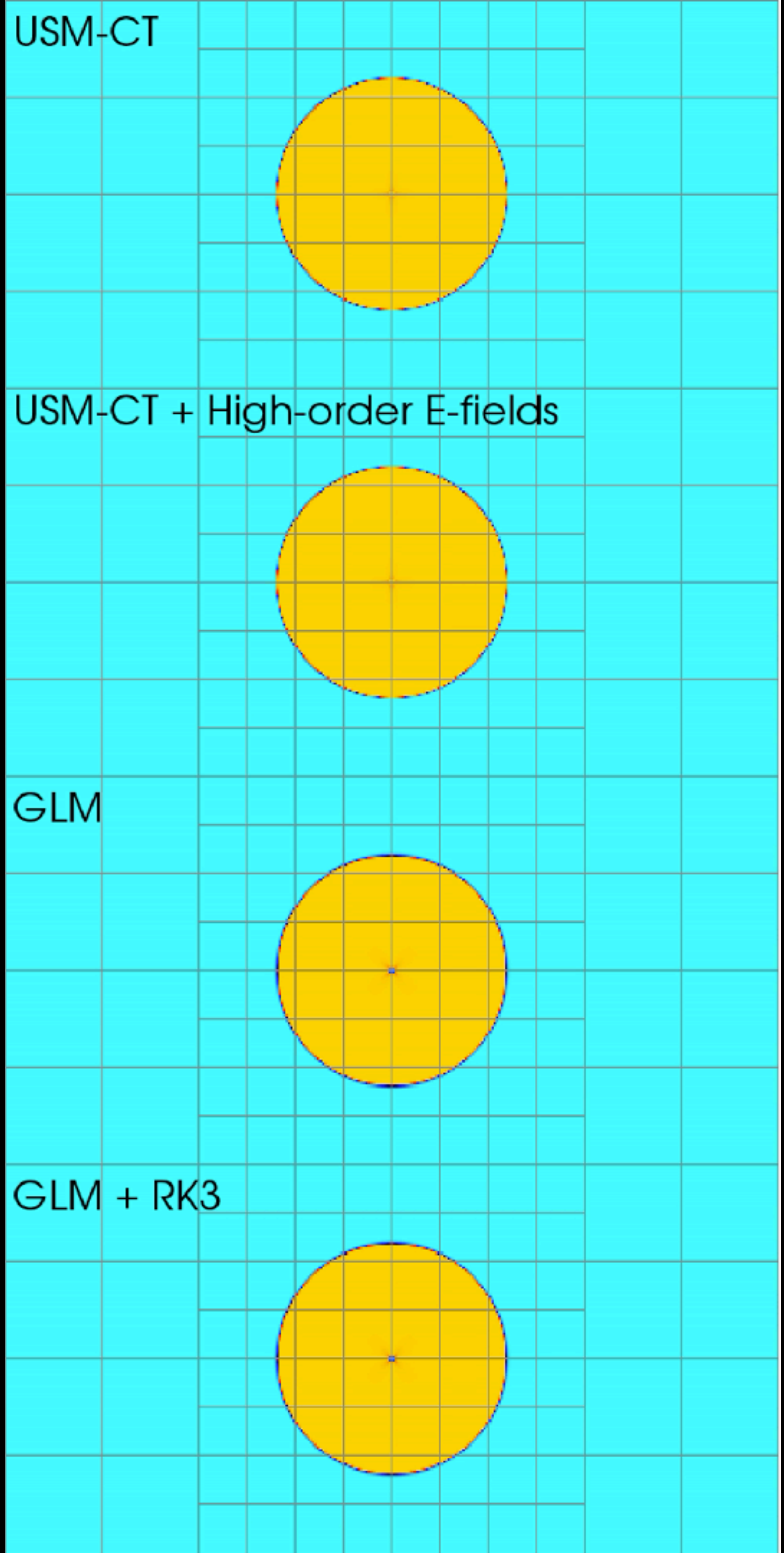
e.g., Gottlieb & Shu (1998)

→ synchronization avoidance

Field Loop Advection

movie of B^2

Spark



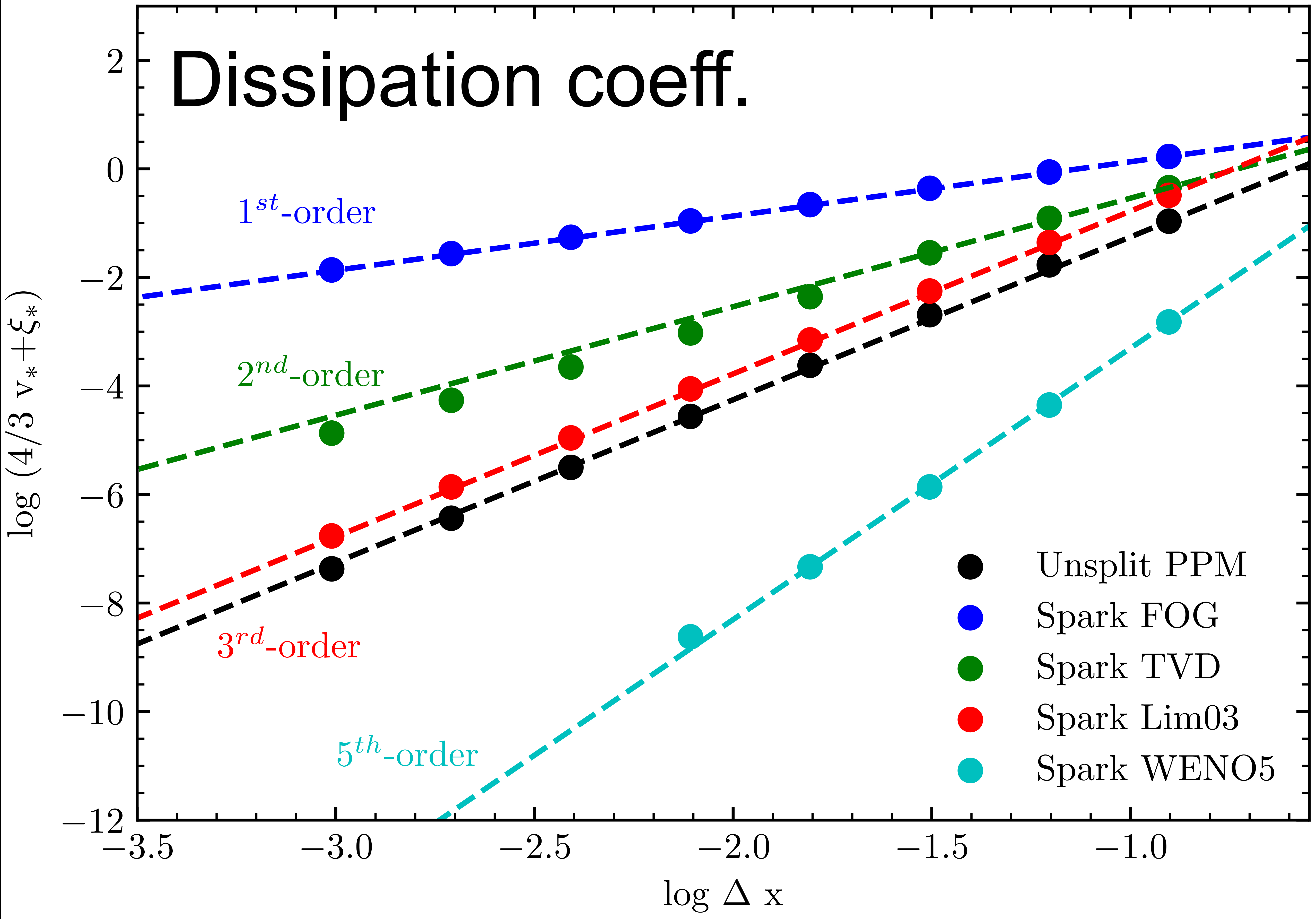
D. Lee & Deane (2009)

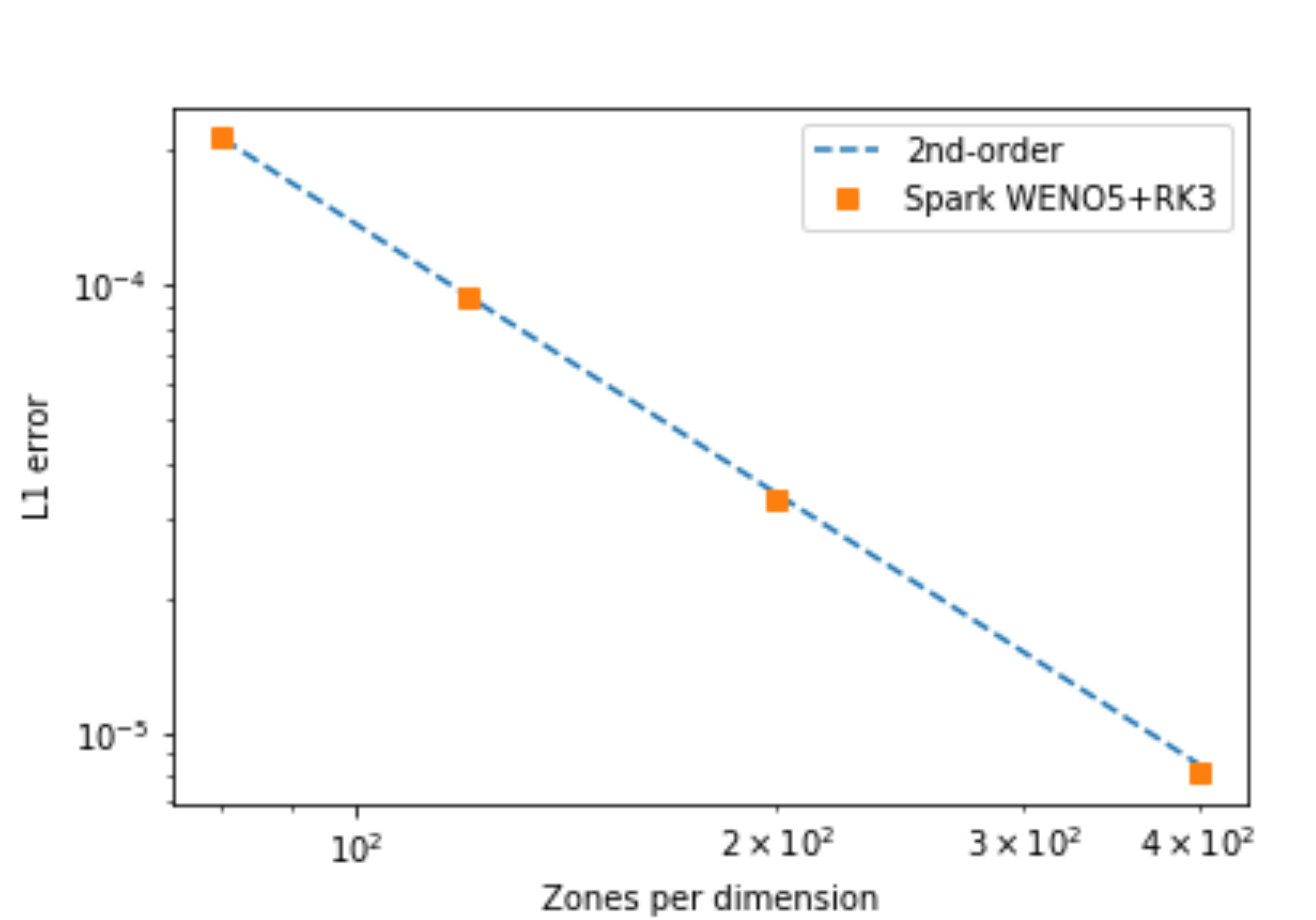
D. Lee (2013)

45 degree

Convergence

Yee, Vinokur & Djomehri 2000, J. Comp. Phys 162





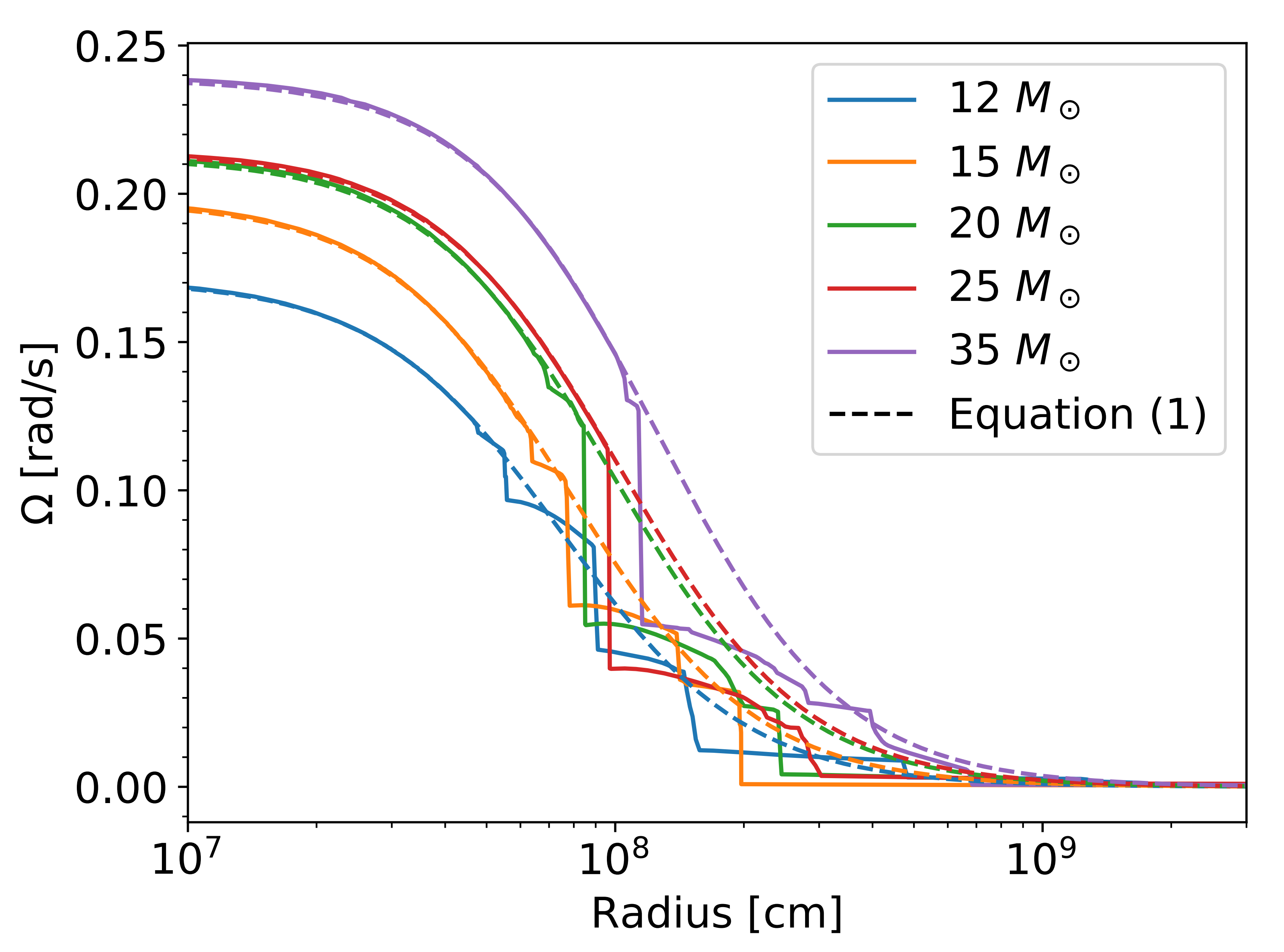
2D sound wave of Rambiasz+17
(see M. Obergaulinger talk)

2D isentropic vortex

Application of Spark

- "Realistic" progenitors from Heger+05
  - not rapid rotators, not very strong B-fields
- High-fidelity neutrino transport
- Full 3D (Cartesian AMR), parameter study:

no rot, no mag	no rot, mag
rot, no mag	rot, mag



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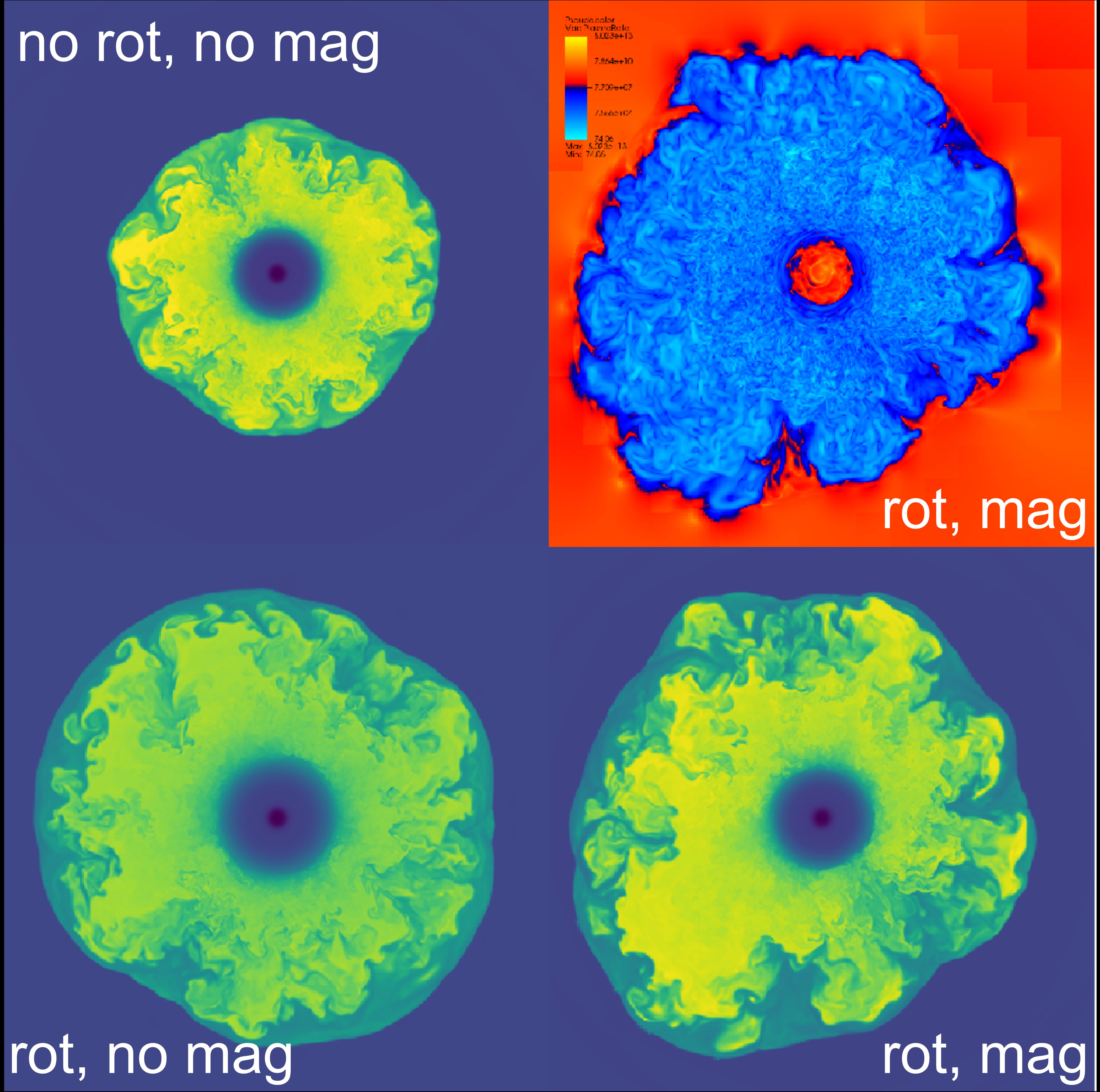


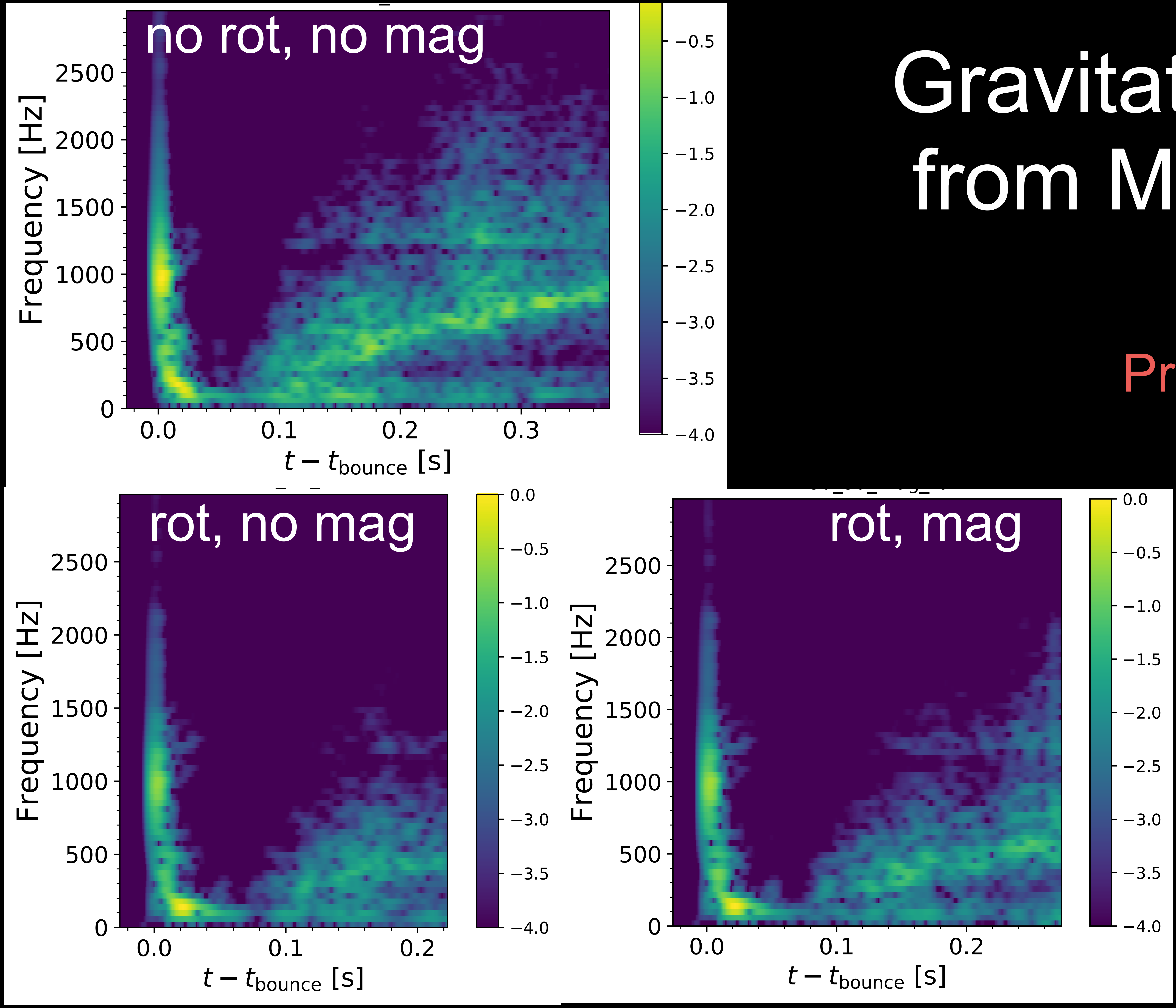
$$\beta = P_{\mathrm{gas}}/P_{\mathrm{mag}}$$

Field amplification beyond flux compression => dynamo action and instabilities

Time = -1.0 (ms)
5.1
5.0
4.0
3.0
2.0
1.0
0.5
Entropy (k_B/baryon)

Gravitational waves from MHD CCSNe

Preliminary!



Spark-Joy: HO FVM

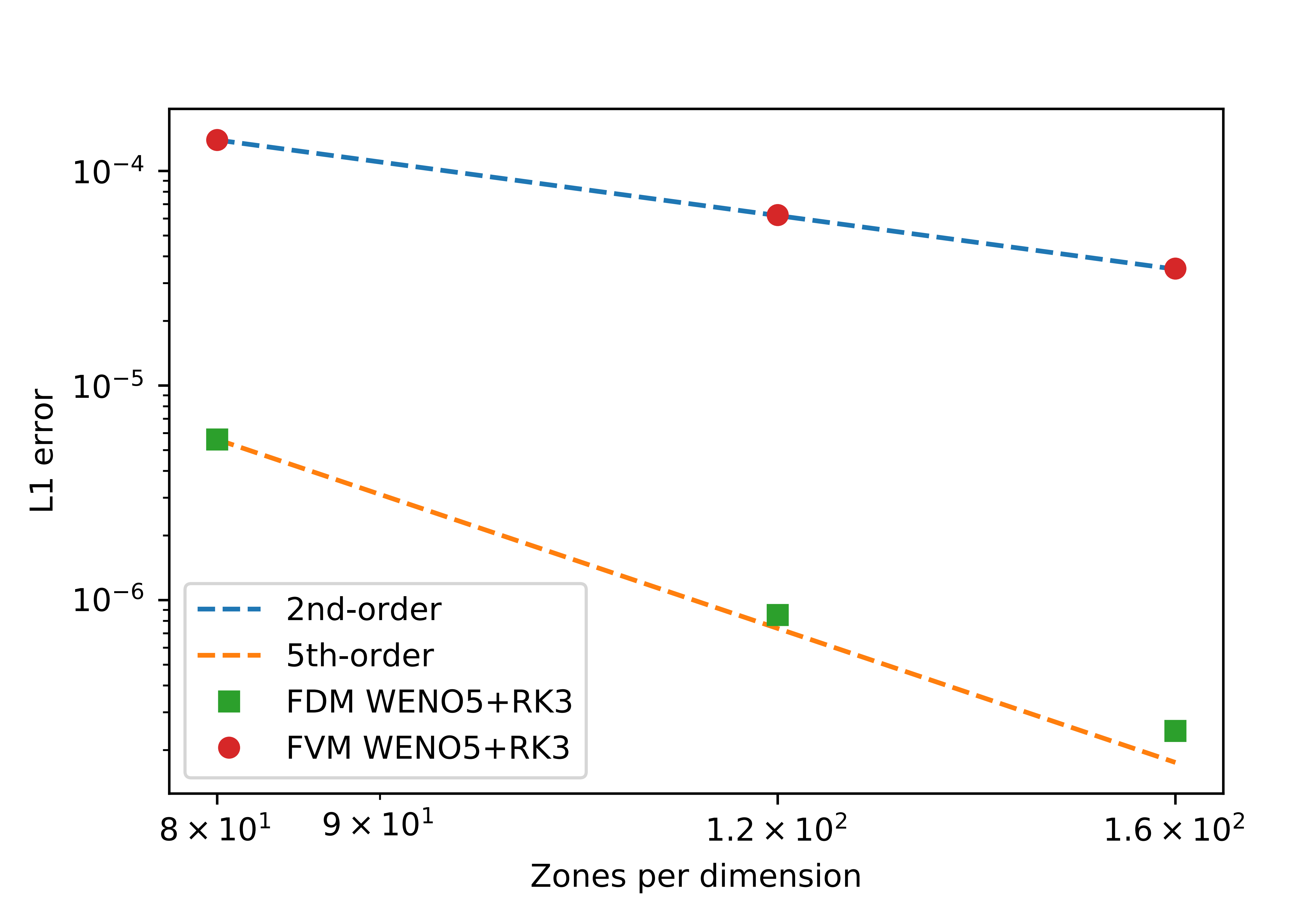
- Original Buchmuller & Helzel approach reconstructed conserved variables. We reformulated for primitive reconstruction (bigger stencil…)
- Many conversions between average and point-wise values
- Implemented and giving correct solutions, but only 2nd-order convergence (debugging…)

Is HO FVM Worth It?

…or, Who cares about conservation?

- Compare to HO finite difference
- “Flux conservative” FD: primitive interpolation and use of Riemann solver. Identical to FV at 2nd order
- Simple to implement
- 1D HO flux correction of Chen+16
- Used in WhiskeyTHC (Radice+)

Isentropic vortex test

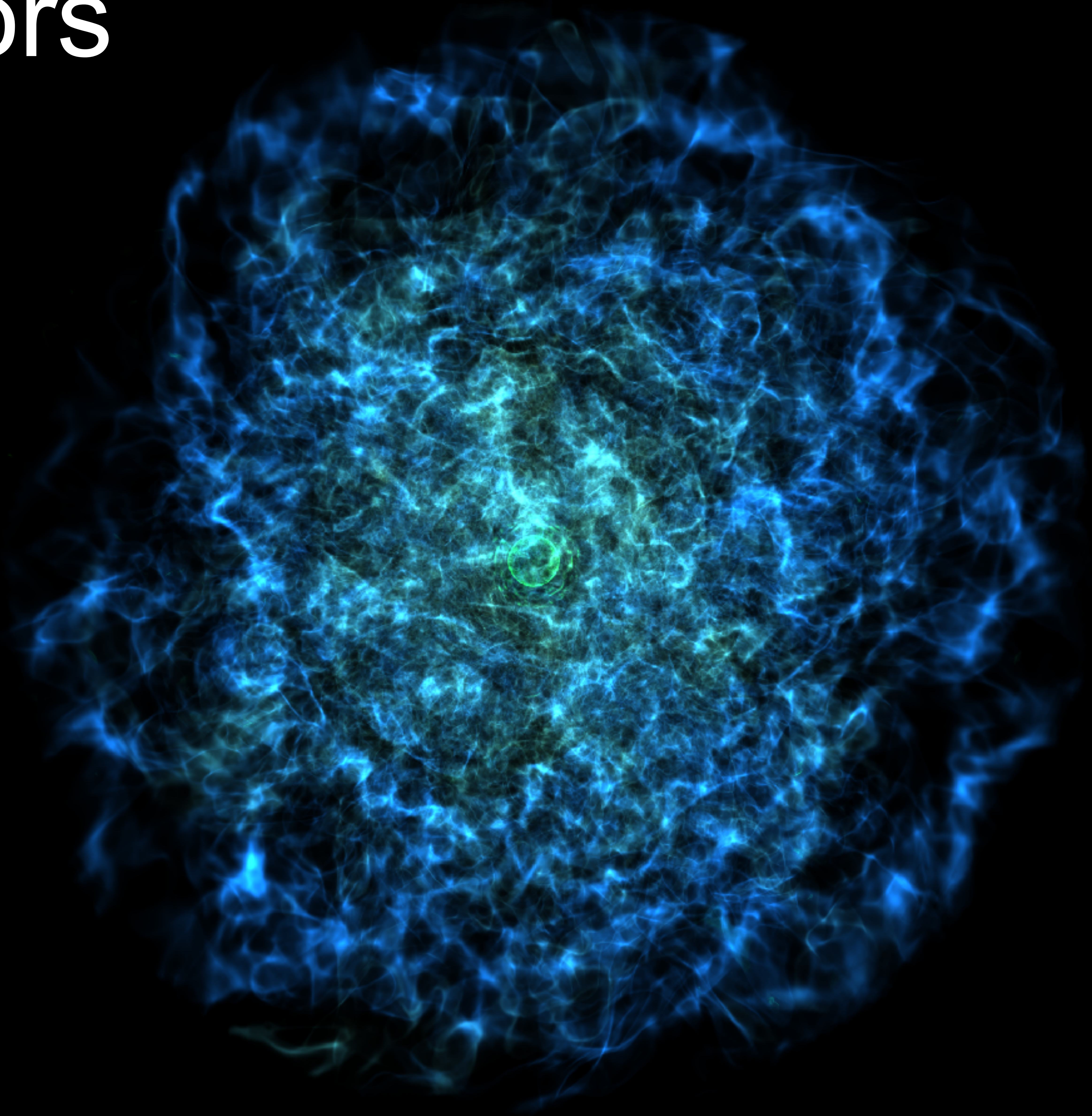


Sedov Blastwave Velocity

So why not just use FDM?

Next-gen 3D Progenitors

- Full 4-pi 3D
- High resolution
- Physically self-consistent core neutronization
- Inner core (r < 1000 km) follows 1D MESA model
- Data will be public



Carl Fields
MSU grad

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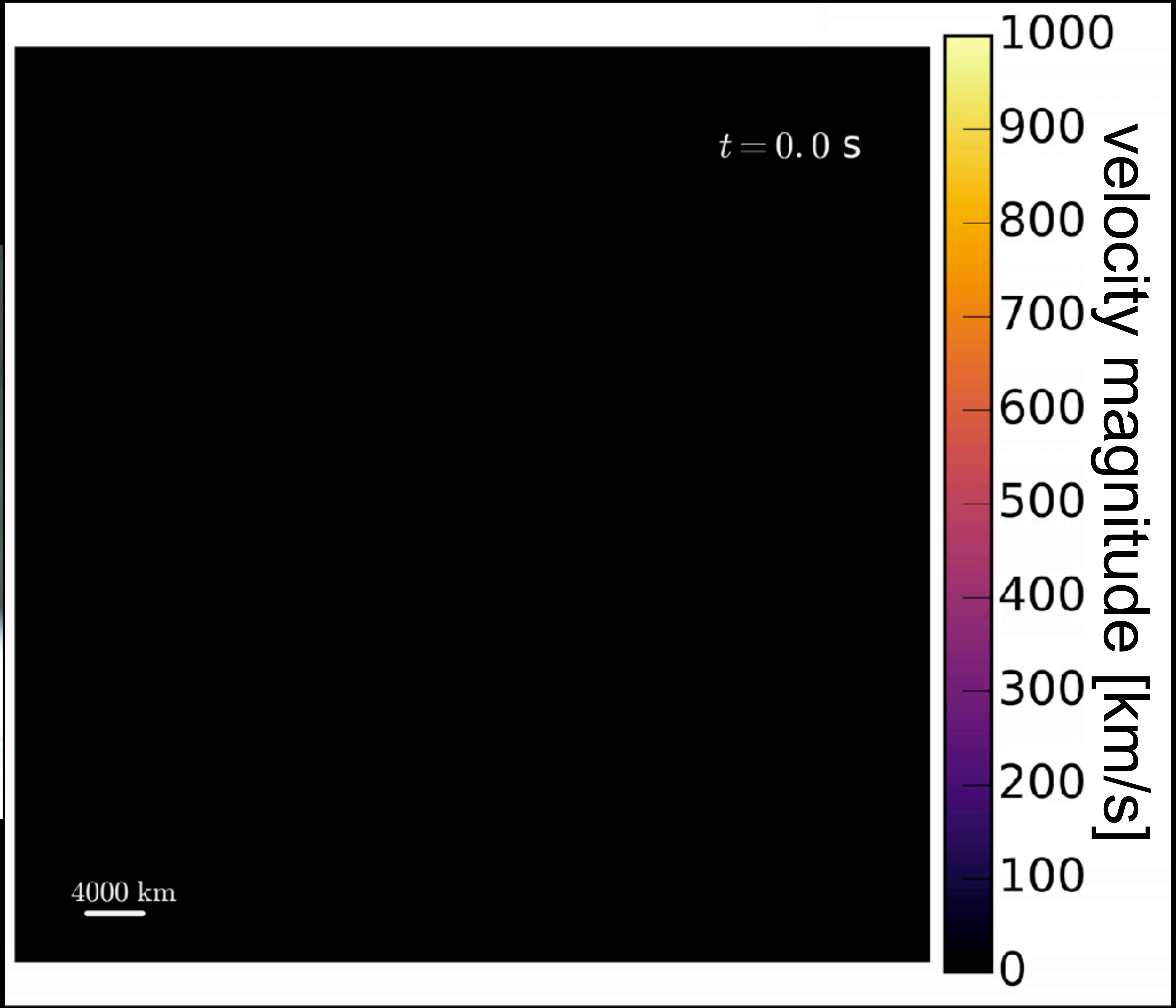
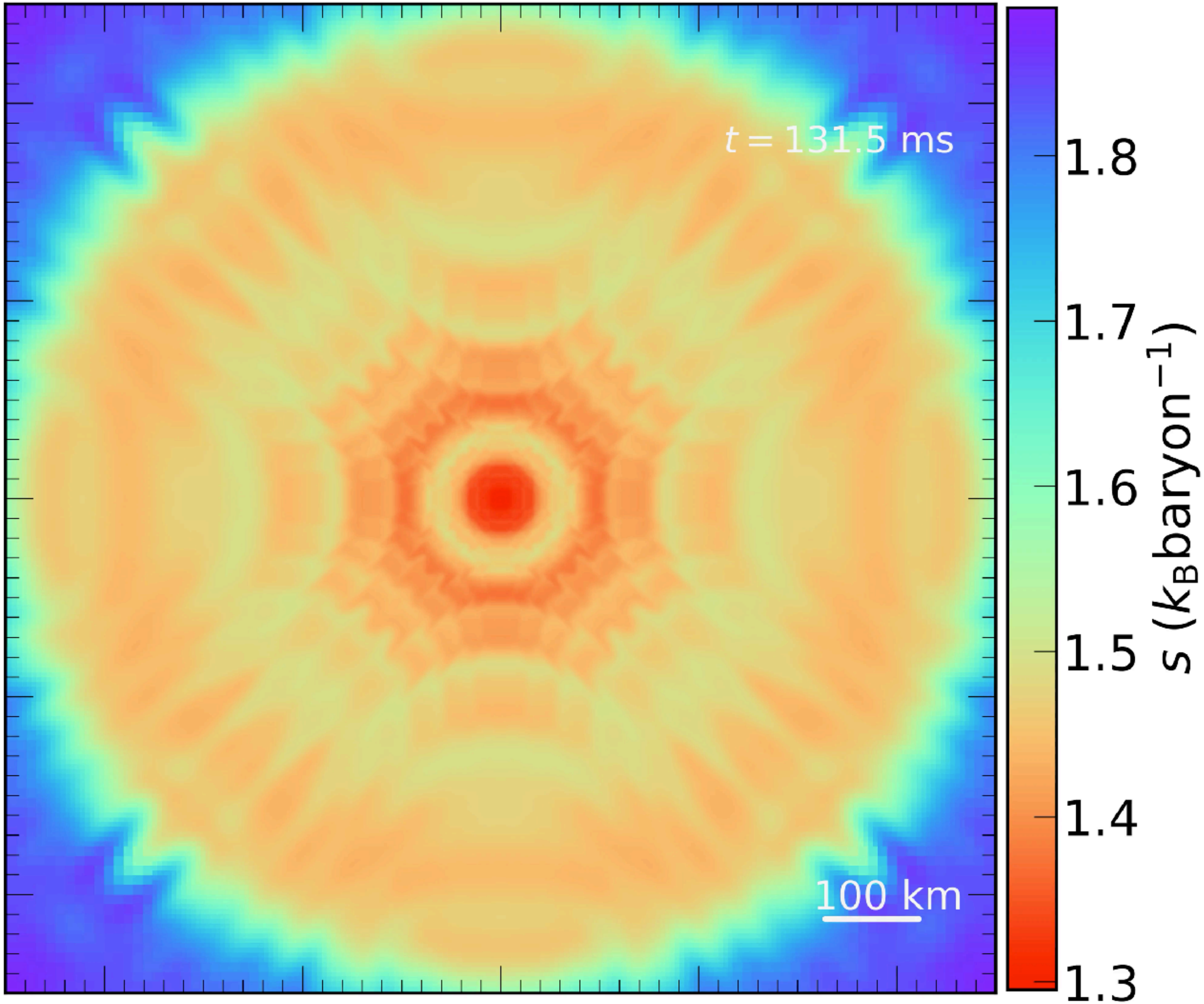
Polar Bear
Cartesian Bear

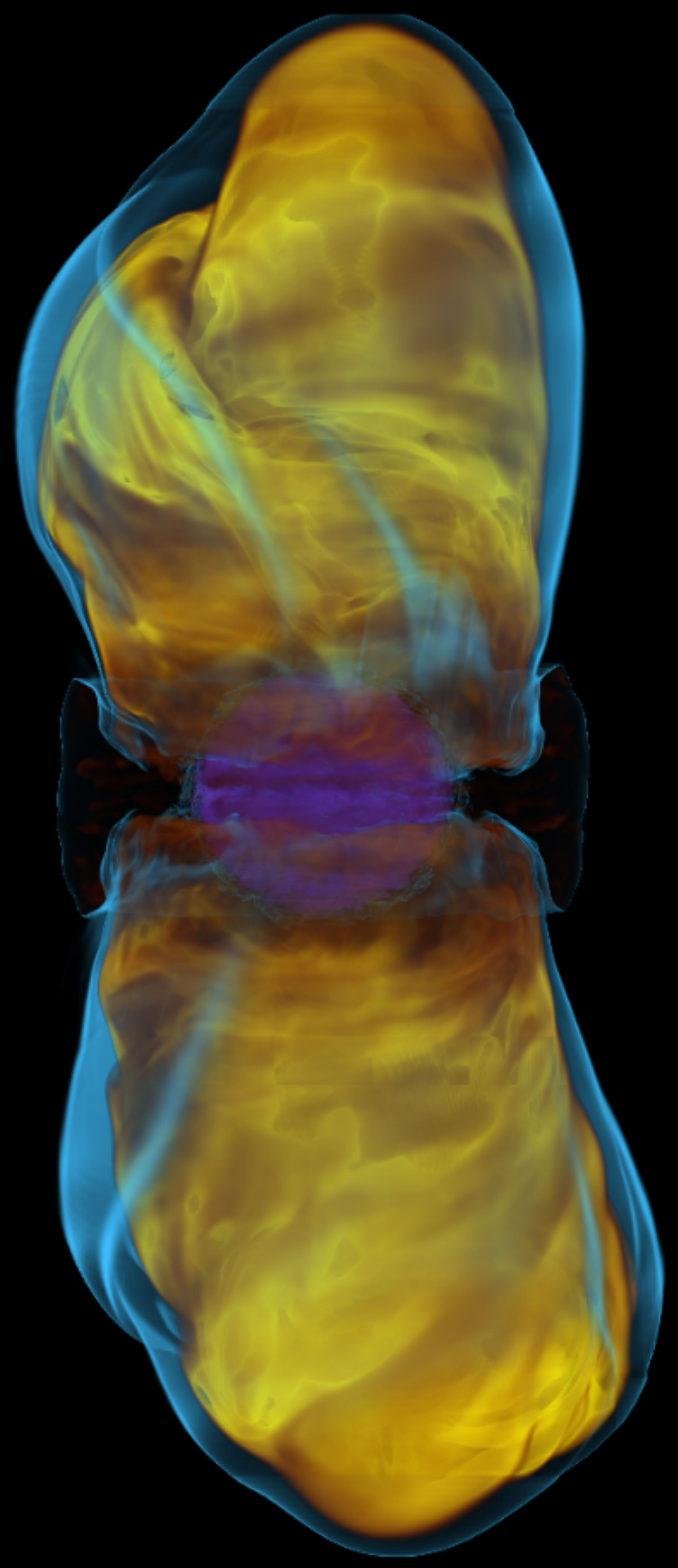

$t = 0.0$ s
4000 km
1000
900
800
700
600
500
400
300
200
100
0
velocity magnitude [km/s]

With initial perturbations



$t = 131.5$ ms
100 km
1.8
1.7
1.6
1.5
1.4
1.3
s ($k_{\rm B}$baryon^{-1})

Summary



- High-order accurate MHD is important for our problems on modern computers
- Spark: a new, performant, accurate MHD solver in the FLASH5 framework
- Being used in high-fidelity 3D CCSN sims to study impact of rotation & B-fields
- Continue work on HO FVM variant. Will compare to FDM
- Work continues on 3D progenitors. Next step: more initial masses and MAESTROeX