Neutrino oscillations in supernovae and mergers

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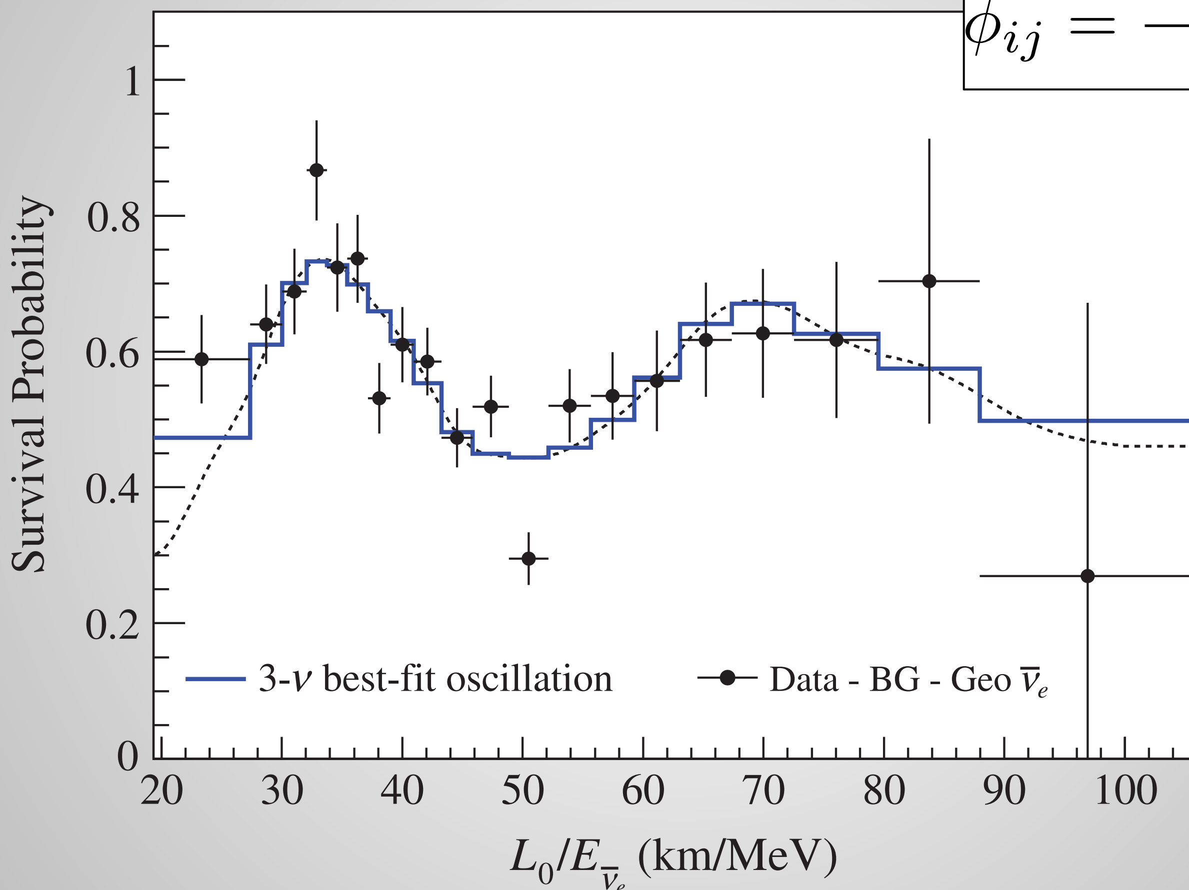
Outline

I. Oscillations in dense media
II. What we know so far
III. Computing oscillations
IV. Summary & prospects

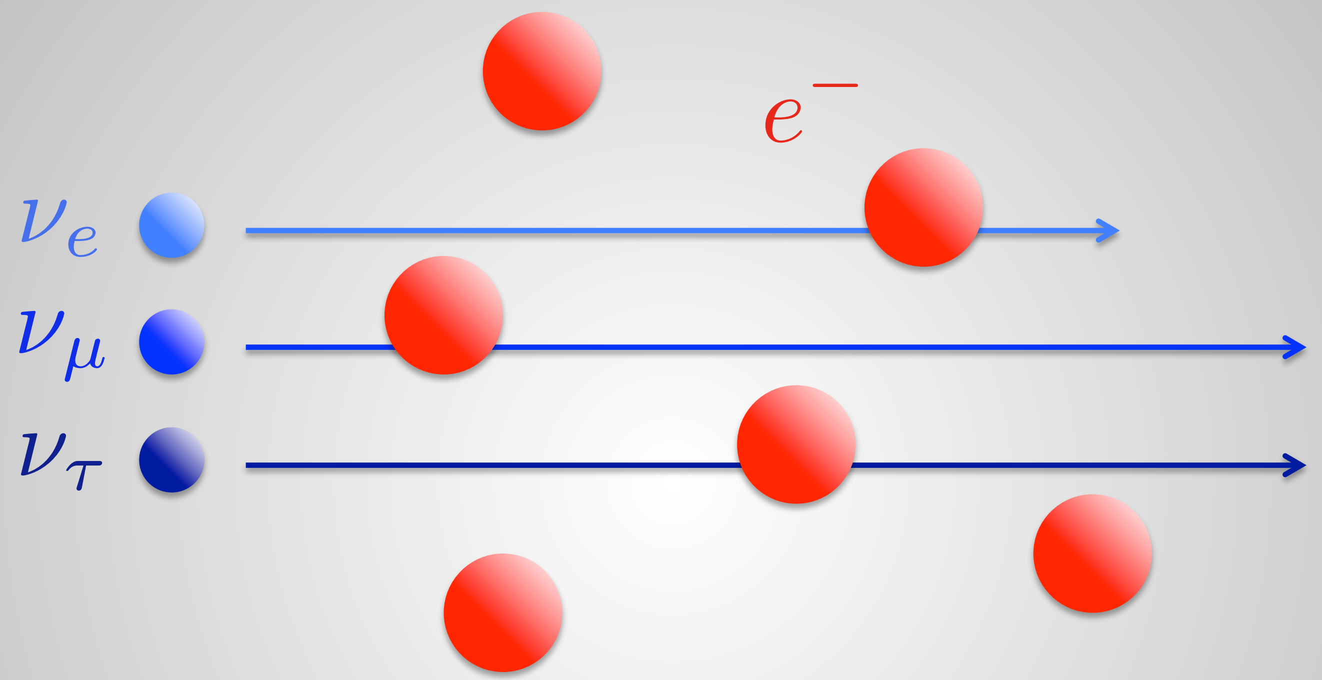
I. Oscillations in dense media

Reactor antineutrino disappearance at KamLAND

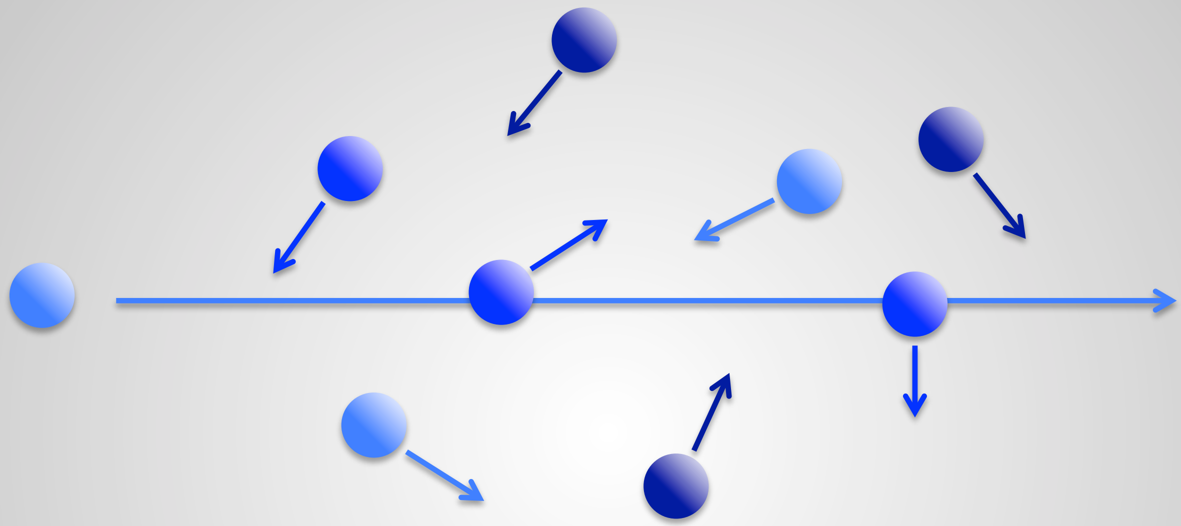


$$\boxed{\phi_{ij} = \frac{\delta m_{ij}^2 L}{2E}}$$

Gando et al. (KamLAND), Phys. Rev. D 88, 033001 (2013)



When a neutrino **forward scatters** on background particles, the flavors are "slowed down" by in-medium effective masses.



Neutrinos contribute to **their own background**. As a result, forward scattering changes oscillations in a nonlinear way.

Neutrino quantum kinetics	=	Transport of energy, momentum, spin, *and flavor* by neutrinos

We can think of **neutrino quantum kinetics** as the theory of this equation:

Quantum collision integrals

$$i\left(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}}\right)\rho_{\mathbf{p}} = [H_{\mathbf{p}}, \rho_{\mathbf{p}}] + iC_{\mathbf{p}}$$

3x3 flavor density matrix:

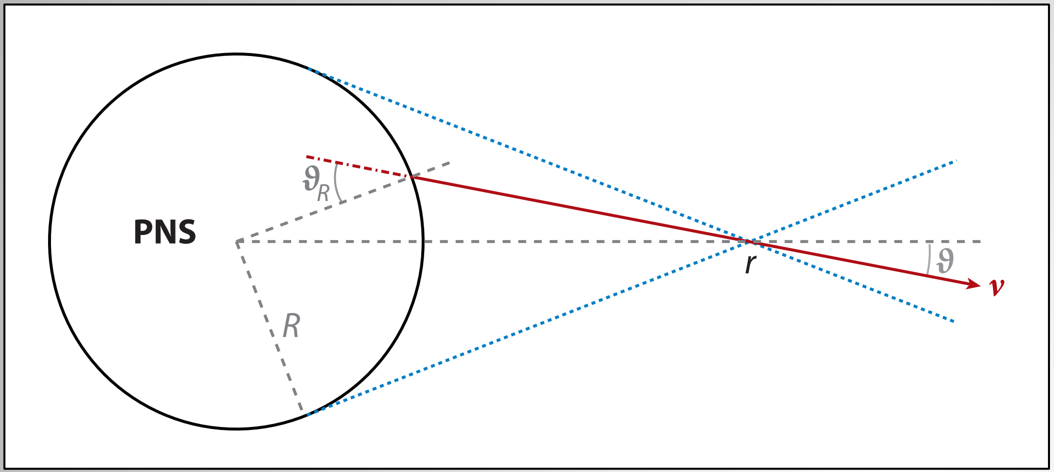
- diagonals = f_{ν_a}
- off-diagonals = coherence densities

$$\boxed{H_{\mathbf{p}} = \underbrace{\frac{M^2}{2E} + \sqrt{2}G_F\Big[N_l}_{\text{MSW}} + \underbrace{\int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')\rho_{\mathbf{p}'}\Big]}_{}}$$

Neutrino-neutrino forward scattering renders the problem **nonlinear** and **geometry-dependent** in a very complicated way.

II. What we know so far

The bulb model of CCSNe



Duan, Fuller, & Qian 2010

How can the kinetic equations be solved in a CCSN environment?

The bulb model simplifies things tremendously:

$$i \cos\theta \frac{d}{dr}\rho(r, E, \theta) = \mathrm{RHS}$$

Assumptions:

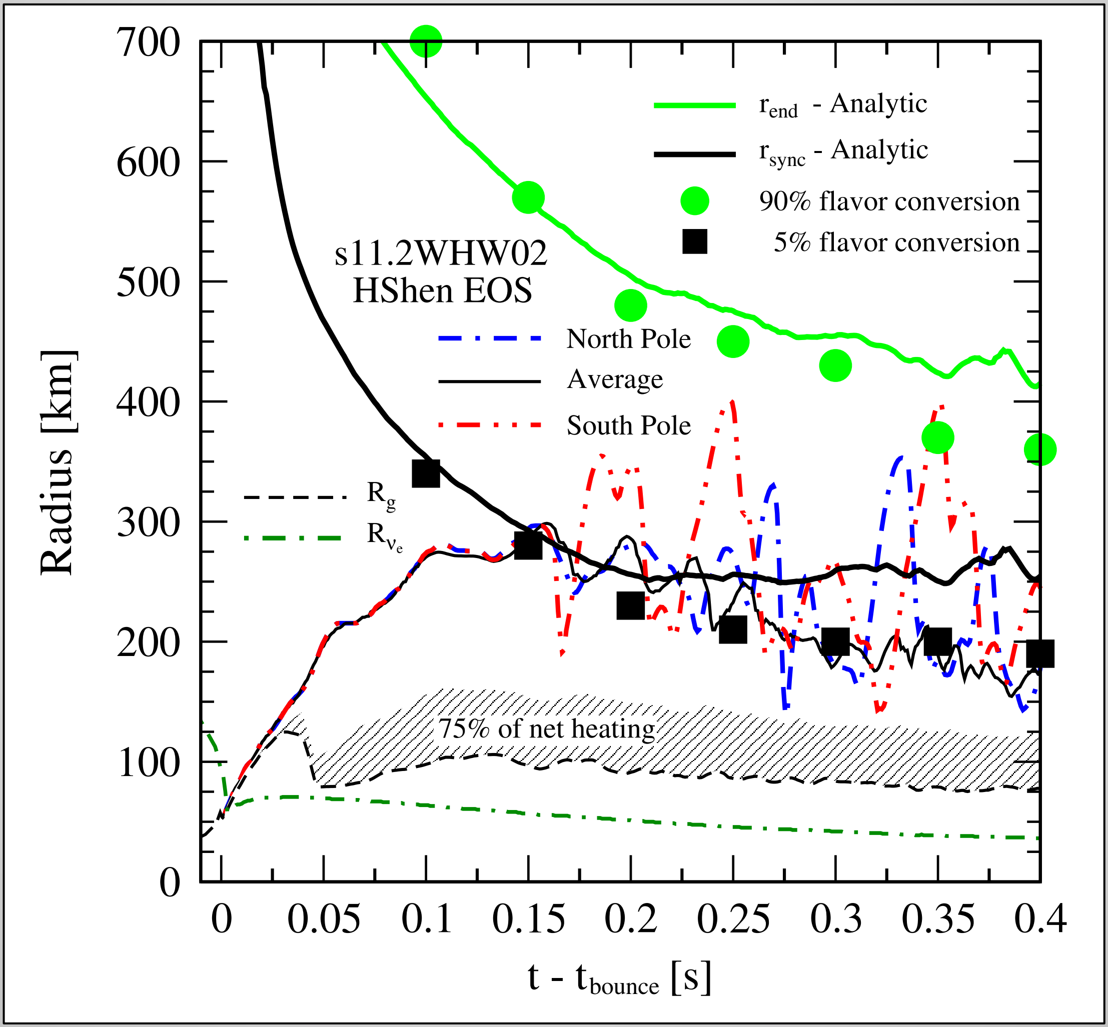
- Spherical symmetry
- Stationarity
- Sharp decoupling with semi-isotropic emission

The bulb model is significant in that it's a **self-consistent model of the SN as a whole**.

e.g., Duan & Friedland 2011

The bulb model: explosion

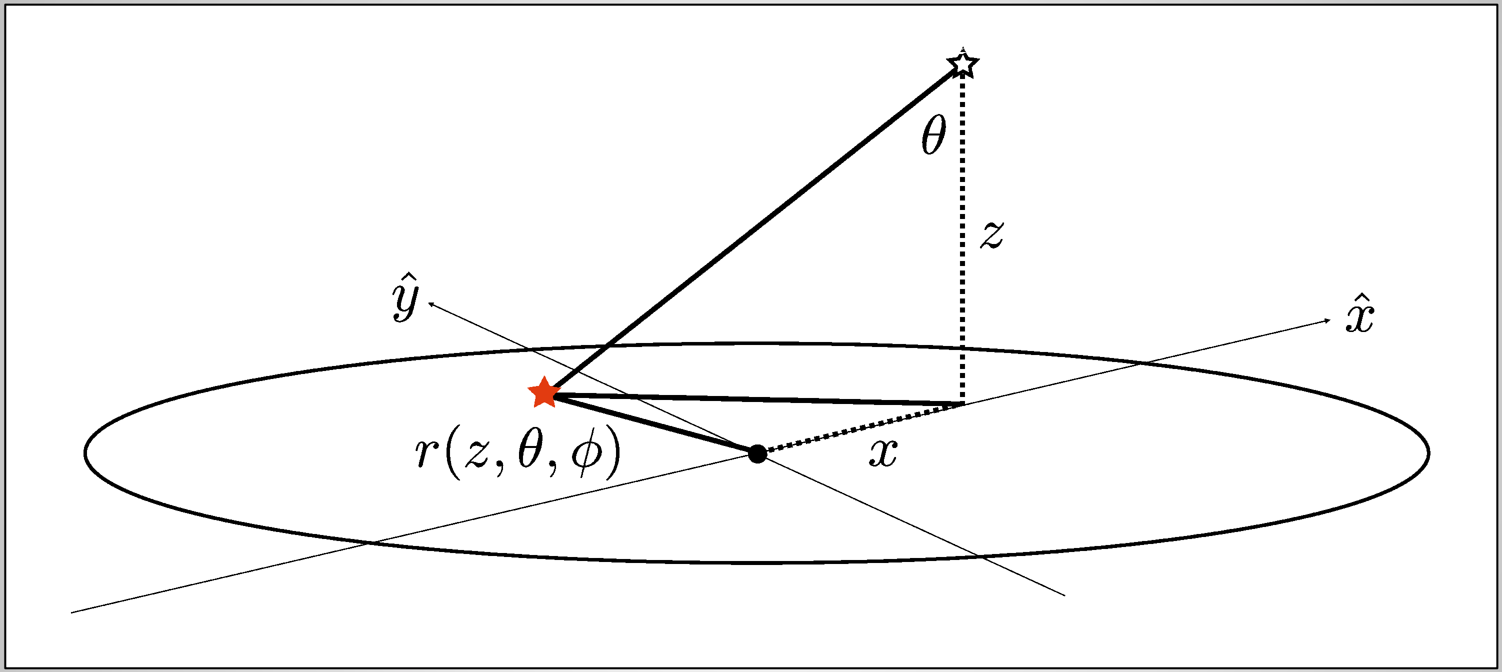
The bulb model predicts **little impact on explosion**:



Dasgupta et al., PRD (2012)

The disk model of NS mergers

Malkus et al., PRD (2012)



In a merger geometry, even the equivalent of the bulb model isn't computationally practical.

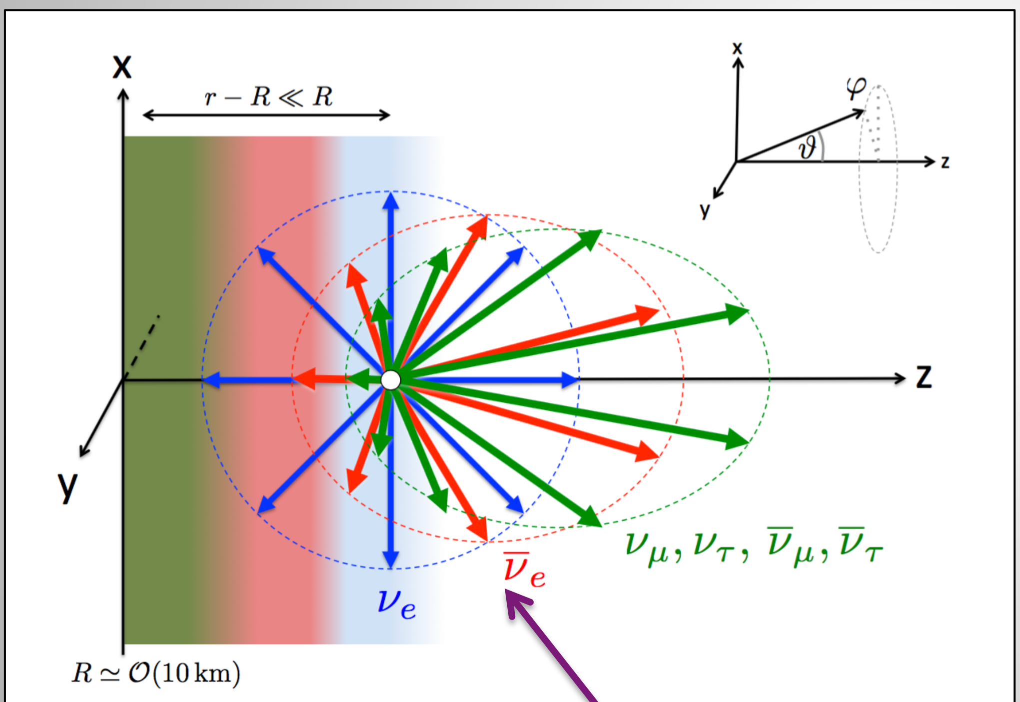
A different perspective on the oscillation problem:

Which waves are solutions to the *linearized* field equations?

And which of these are *unstable*?

Fast flavor conversion

Broken symmetries → **New instabilities** → **Old results must be re-evaluated**



Dasgupta, Mirizzi, & Sen 2017

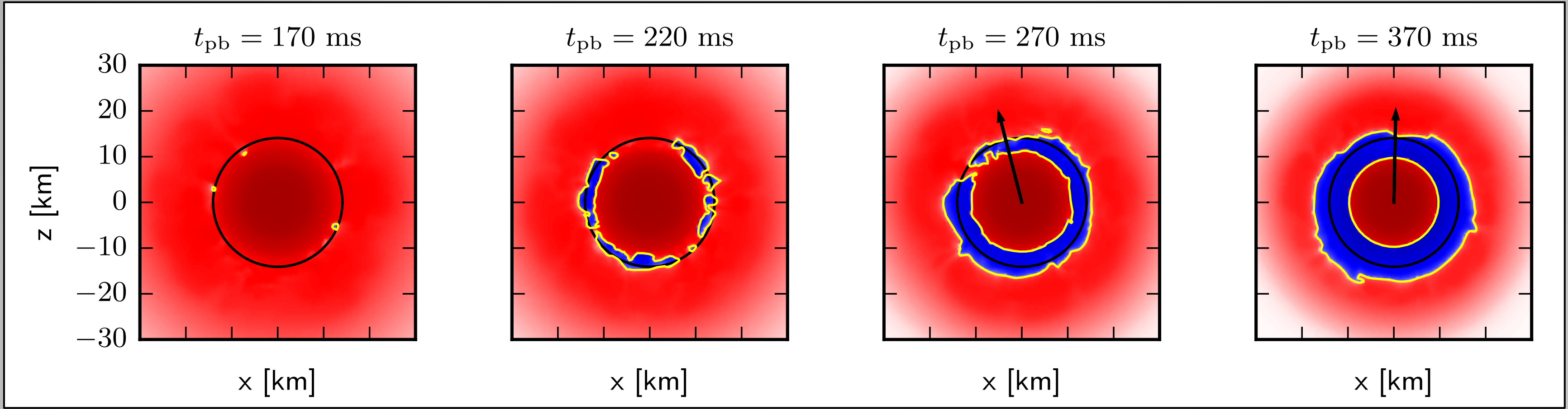
Fast flavor conversion (FFC) is associated with angular crossings in the electron lepton number carried by neutrinos (ELN).

$$\mu^{-1} \sim 1\ \mathrm{mm} - 1\ \mathrm{m}$$
$$\sim 1\ \mathrm{ps} - 1\ \mathrm{ns}$$

ELN crossing due to flavor-dependent opacities

Instabilities in CCSN simulation data

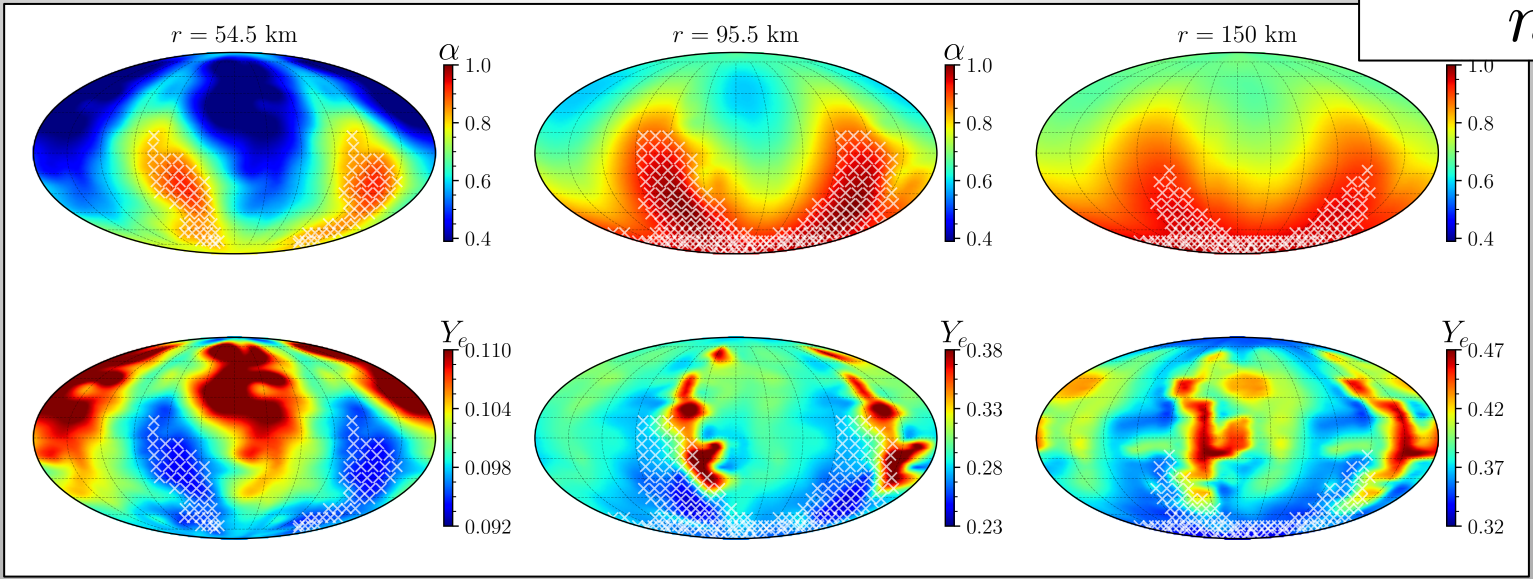
Instabilities in the convective layer of the proto-neutron star…



Glas et al., 1912.00274

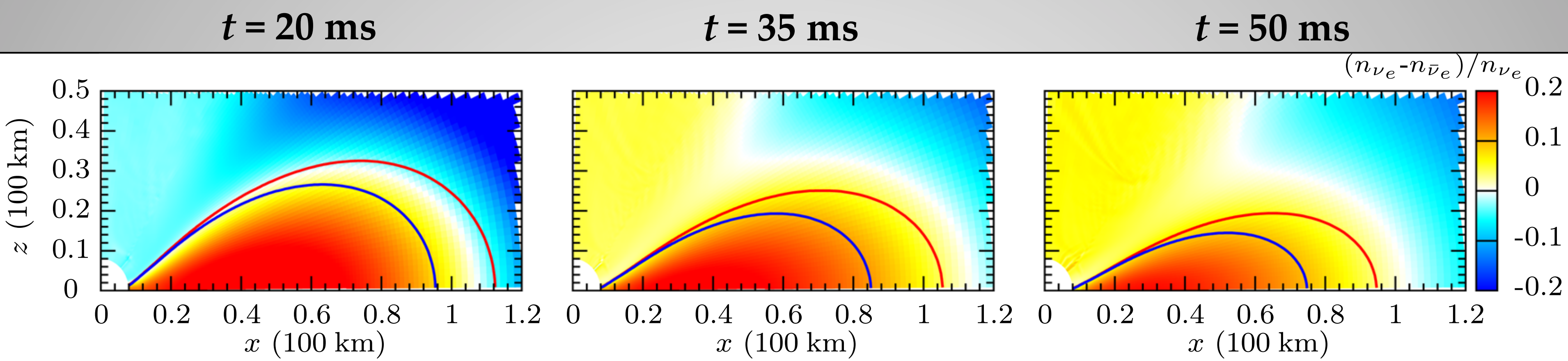
And in (and beyond) the decoupling region…

$$\alpha = \frac{n_{\bar{\nu}_e}}{n_{\nu_e}}$$



Abbar et al., 1911.01983

Instabilities in BNSM simulation data

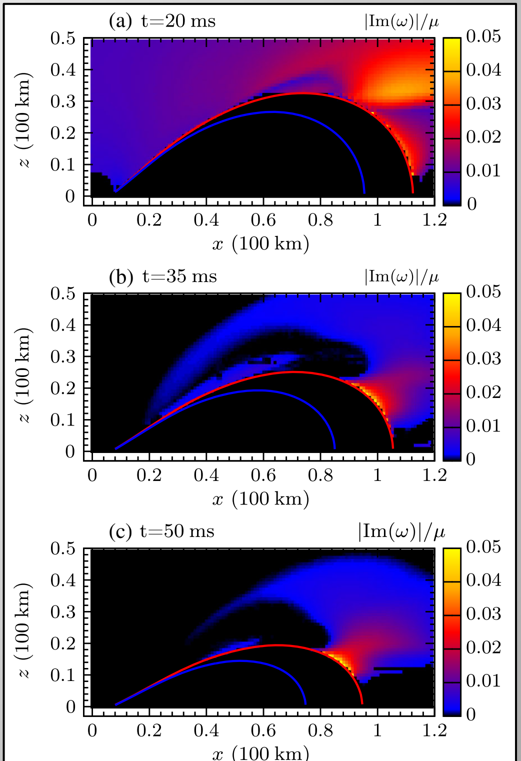


There's also evidence of fast instabilities in the neutrino emission from a **binary NS merger remnant**.

Wu et al., PRD (2017)

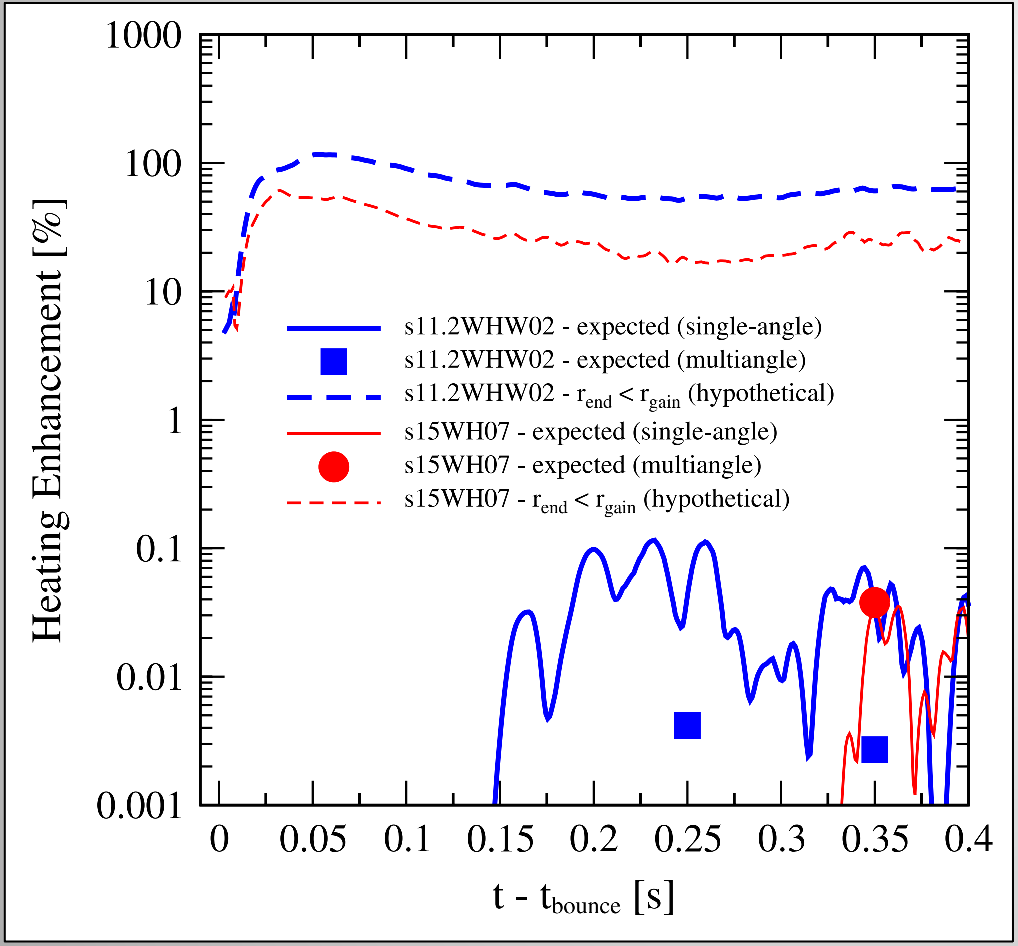
Instability growth rates (*right*) reflect ELN ratios (*above*).

$$\frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\nu_e}}$$



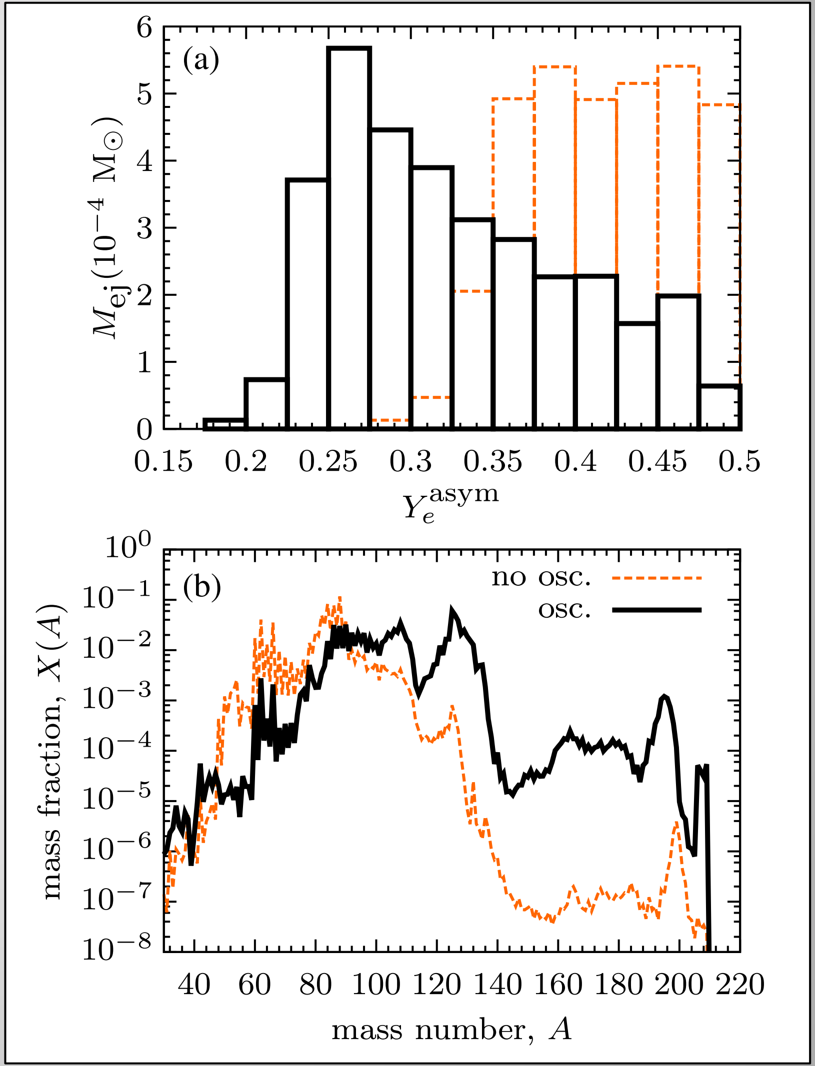
Possible effects of fast instabilities

Heating enhancement in the bulb model when the instability is artificially moved inward:



Dasgupta et al., PRD (2012)

Nucleosynthesis in the neutrino-driven ejecta of a binary NS merger:



Wu et al., PRD (2017)

III. Computing oscillations

Compare the degrees of freedom in a bulb calculation…

$$\rho(r, E, \theta)$$

Radial coordinate — Neutrino energy — Propagation angle

…to those in a fully asymmetric calculation.

$$\rho(t, \mathbf{r}, E, \theta, \phi)$$

- **Temporal resolution**
- **Spatial resolution**
- **Momentum resolution**

It's clear why **spatiotemporal resolution** might need to be high: Oscillations occur on small scales.

$$\mu^{-1} \sim 1 \text{ mm} - 1 \text{ m}$$
$$\sim 1 \text{ ps} - 1 \text{ ns}$$

But why is **momentum** (especially **angular**) **resolution** an issue?

Let's compare to radiative transfer (*i.e.*, transport w/o oscillations):

$$(\partial_t + \hat{\mathbf{p}} \cdot \nabla_{\mathbf{r}})\, I_{\mathbf{p}} = -\,(\kappa_{\mathbf{p},s} + \kappa_{\mathbf{p},a})\, I_{\mathbf{p}} + \epsilon_{\mathbf{p}}$$

Specific intensity

Loss due to **scattering** and **absorption**

Gain due to **scattering** and **emission**

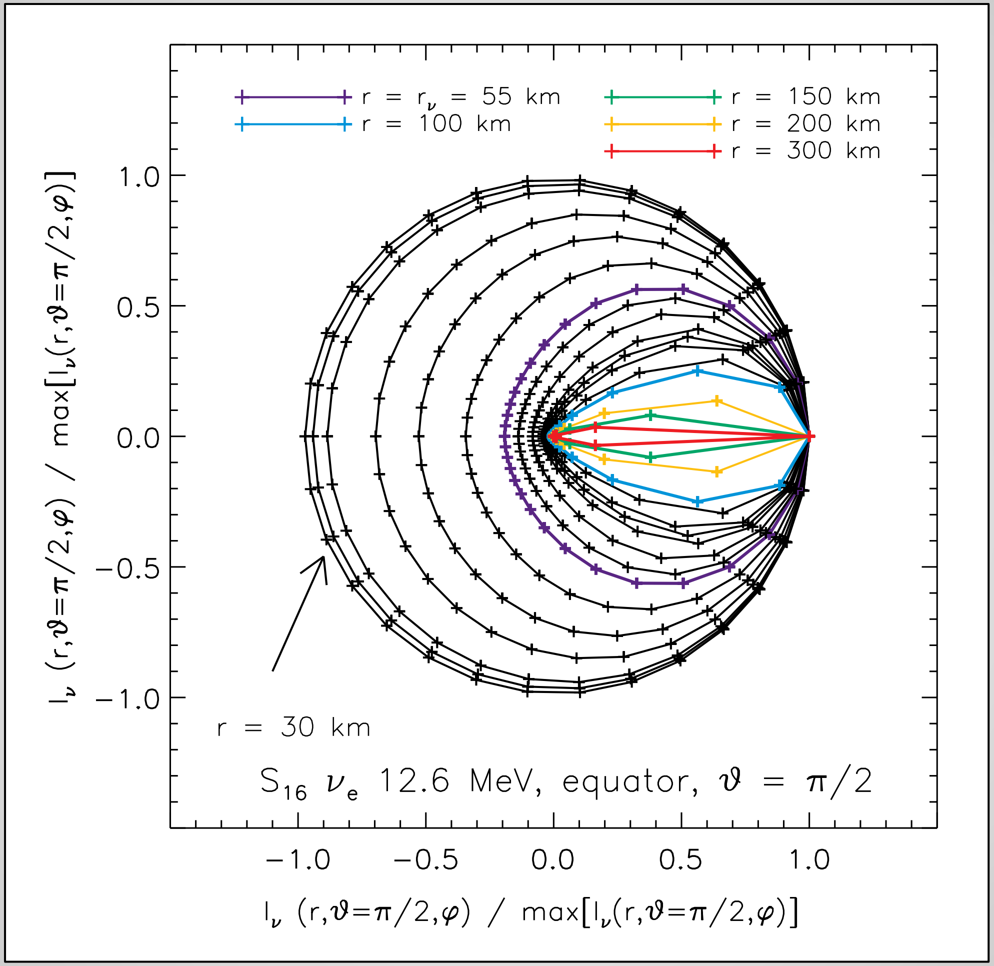
Geometry leads to more **forward-peaked** angular distributions at larger *r*.

Scattering, absorption, and emission **isotropize** the distributions.

As we'll see, oscillations can ***de*-isotropize** flavor states.

Isotropization is why simulations can evolve **just a small number of moments**.

Ott et al., ApJ (2008)



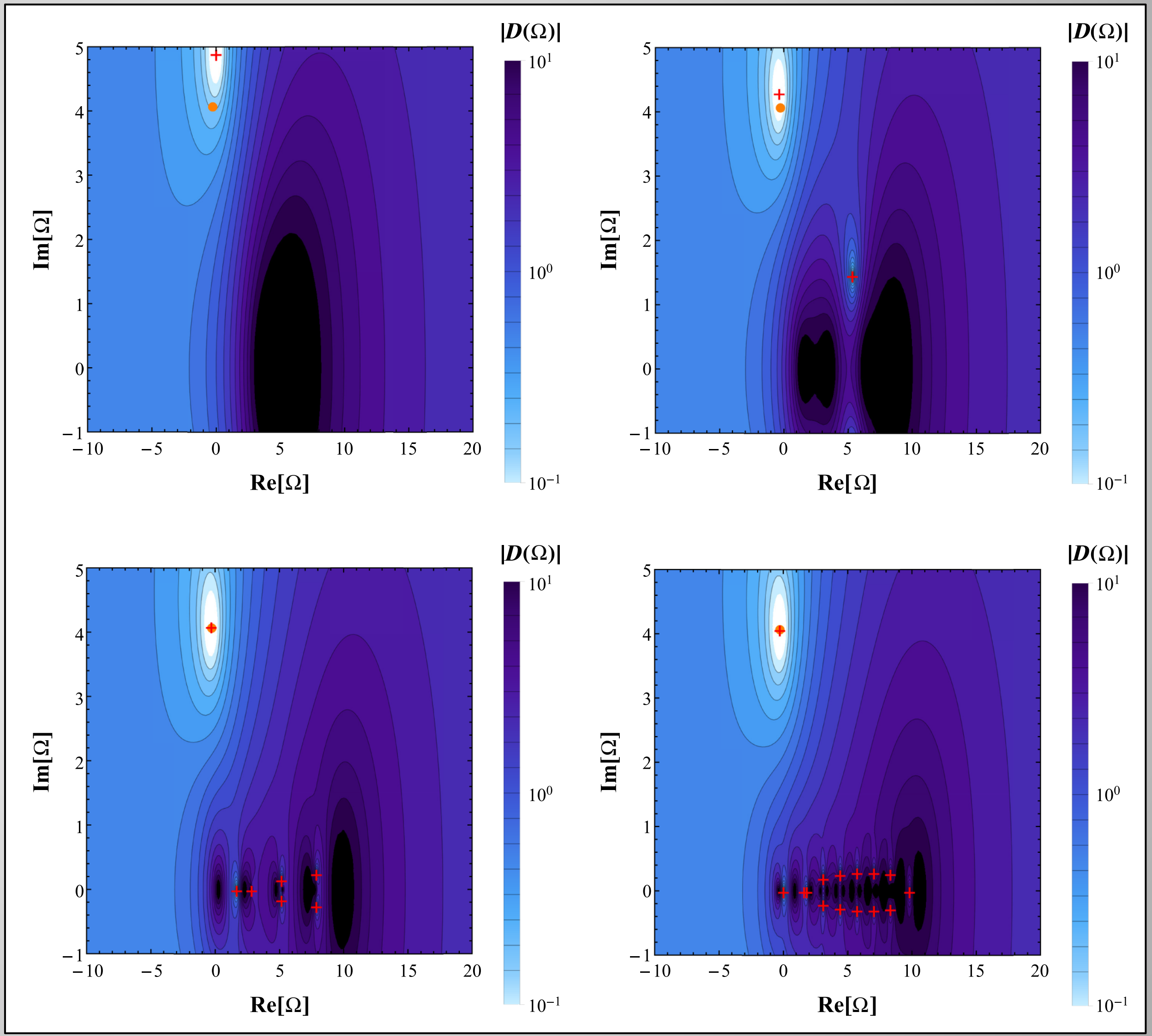
We've been exploring whether this nice feature of the problem can be salvaged when neutrinos oscillate.

The bulb model requires 1000s of angle bins to avoid **spurious instabilities**.

If angular moments are evolved directly, then the touchiness of discretization can be bypassed.

$$I_n = \int_{-1}^{1} dv \frac{v^n g_v}{a + bv}$$

$$(v = \cos\theta)$$



Morinaga & Yamada, PRD 97, 023024 (2018)

On μ^{-1} scales, FFC is principally associated with **low-*l* dynamics**.

Pendulum

Spin

$$\frac{\boldsymbol{\delta} \times \ddot{\boldsymbol{\delta}}}{\mu} + \sigma \dot{\boldsymbol{\delta}} + \mathbf{g} \times \boldsymbol{\delta} = 0$$

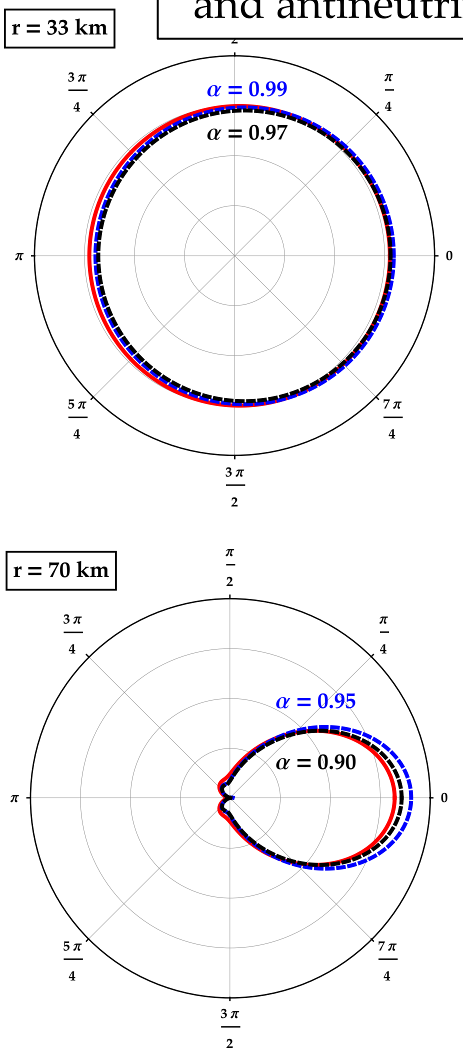
Gravity (coupled to δ in FFC)

This equation applies (with different meanings) to…

- ✧ Slow collective oscillations in SNe
  Hannestad et al. 2006
- ✧ Fast collective oscillations in SNe
  Johns, Nagakura, Fuller, & Burrows 2020
- ✧ Collective oscillations in the early universe with a lepton asymmetry
  Johns & Fuller 2018

Angular features of FFC

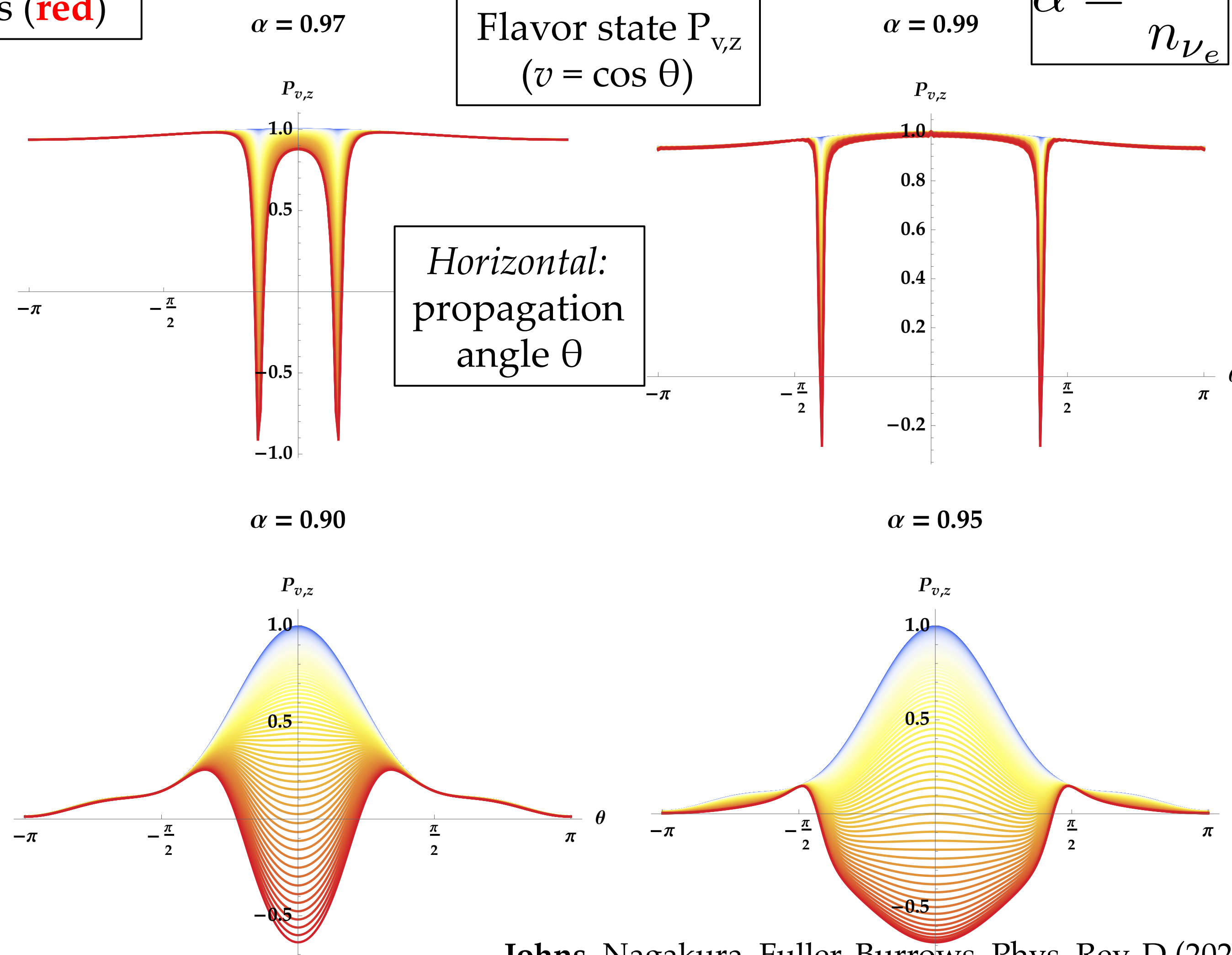
Angular distributions of neutrinos (**black** & **blue**) and antineutrinos (**red**)



Vertical: Flavor state $P_{v,z}$ ($v = \cos\theta$)

$$\alpha = \frac{n_{\bar{\nu}_e}}{n_{\nu_e}}$$

Horizontal: propagation angle θ

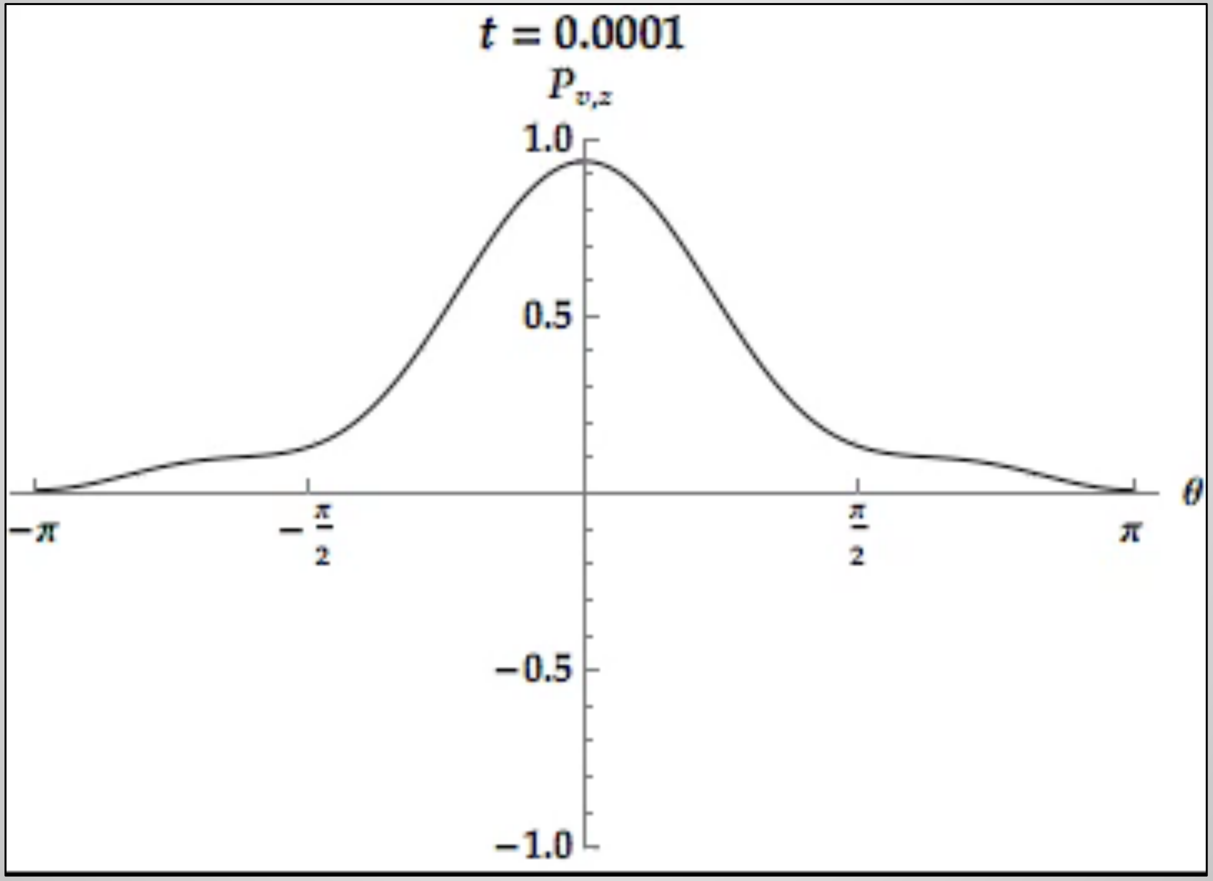


Johns, Nagakura, Fuller, Burrows, Phys. Rev. D (2020)

On the one hand, FFC shows **qualitatively different behavior depending on the angular distributions**. On the other hand, **all major features are captured by *l* ≤ 3**.

But the dynamics isn't closed at low l. On longer time scales, more moments get involved:

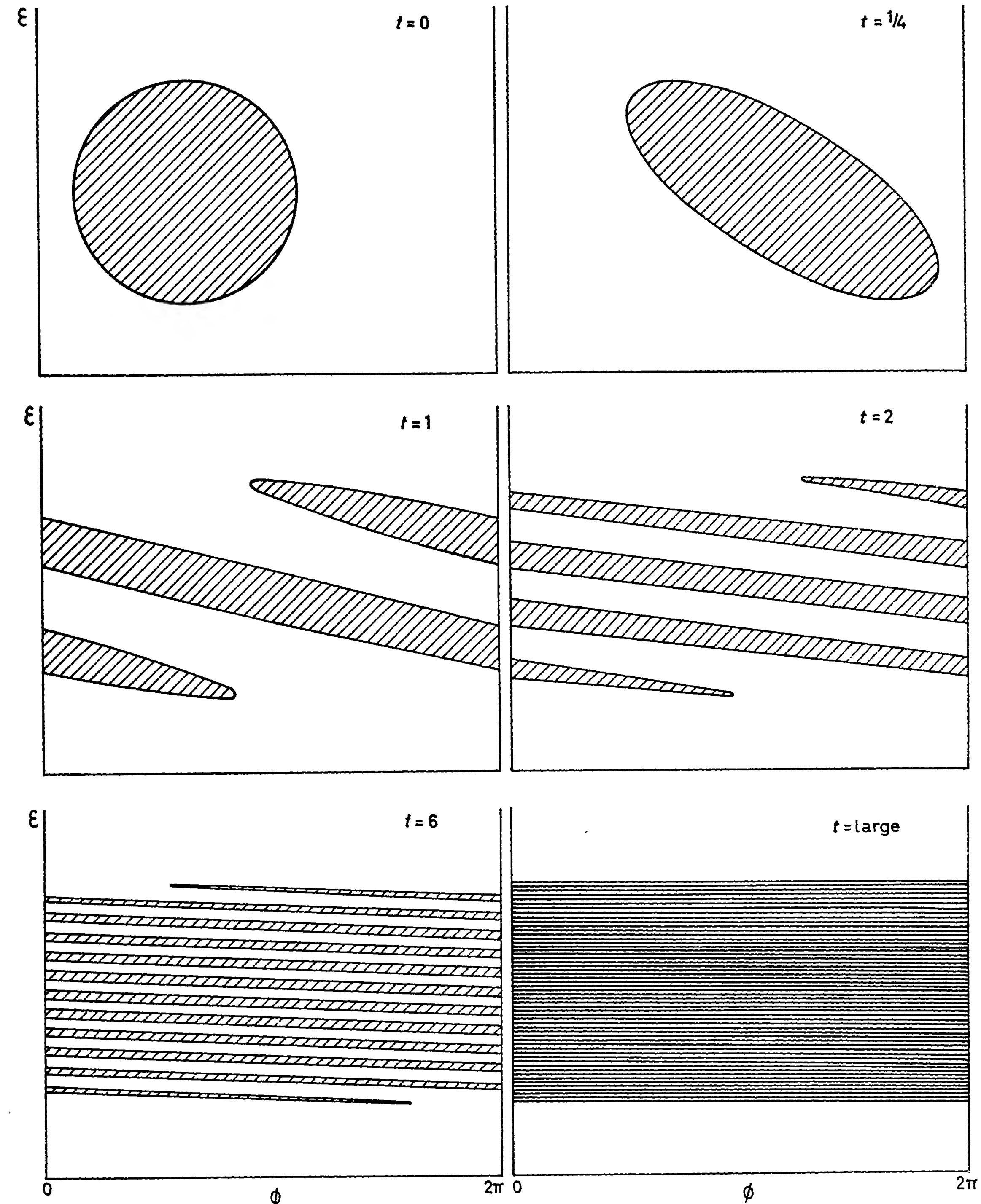
r = 70 km, α = 0.90, time in units of ω^{-1}



Do oscillations of the flavor field eventually settle down?

We're working to understand how **collective oscillations** interact with **collisionless relaxation**.

via the development of small-scale structure in phase space

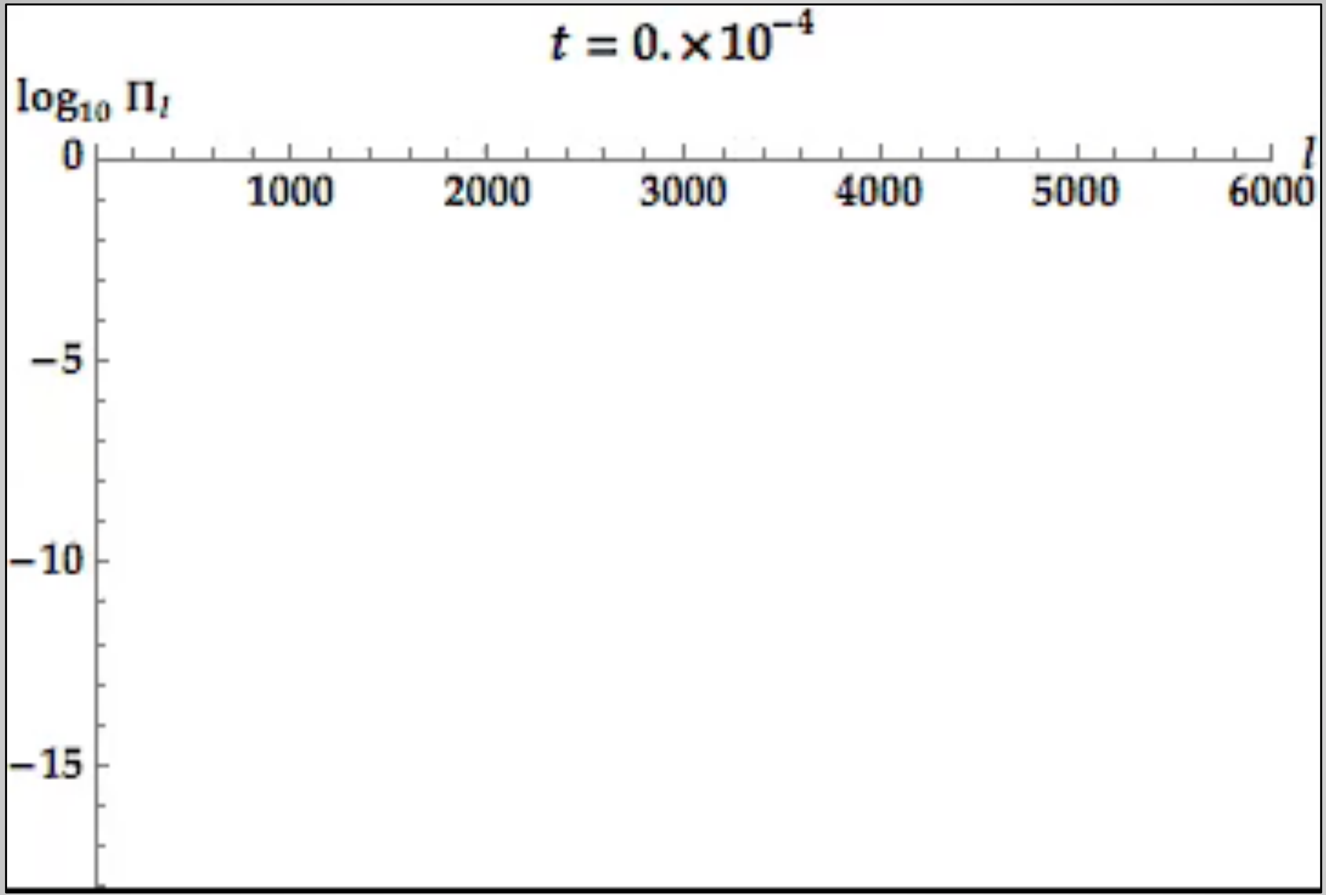


The flavor-field analogue of

- **Violent relaxation** in grav. systems
- **Filamentation** in plasmas
- **Turbulence** in fluids

Lynden-Bell, MNRAS (1967)

Fast oscillations expedite & amplify **momentum-space cascade**:



lth Legendre moment in momentum space

Fast
Slow

Although **oscillations** are captured by low-l evolution, **relaxation** depends on "inertial-range" dynamics.

Johns, Nagakura, Fuller, Burrows, in prep.

$$\rho(t, \mathbf{r}, \underbrace{E, \theta, \phi}_{\text{See previous slides}})$$

$$\rho(\underbrace{t, \mathbf{r}}, \underbrace{E, \theta, \phi})$$

See previous slides

Time-averaging / Reynolds decomposition, coarse-graining, filtering, …

e.g., the filtered Navier-Stokes equation depends on a **subgrid-scale stress tensor**:

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

A common closure posits that subgrid physics just adds to the molecular viscosity & resolved pressure.

Why not do something similar for the flavor field?

Fluid: Convection transfers energy to smaller scales until microphysics (*viscosity*) becomes important.

vs.

Flavor: Convection and microphysics (*nonlinear, nonrandomizing forward scattering*) both transfer energy across scales.

Summary & prospects

Approaches to the oscillation problem:

1. Highly simplified models (*e.g.*, bulb)
2. Linear stability analysis
3. ***Realistic quantum transport***

0^{th} order: No oscillations.

1^{st} order: Instability → flavor equilibrium.

2^{nd} order: Perhaps moments + closure in **p**-space, ??? in **x**-space and time.

At the very least, we'd like to perform *and justify* the 1^{st}-order calculation.

A cunning derivation of the oscillation formula in vacuum…

1. Assume that each mass state (i = 1, 2, 3) propagates as a **plane wave of momentum *p***:

$$\psi_i \sim e^{ipx - iE_i t}$$

2. Assume that the neutrino is **measured at *t* ≈ *L***.

3. **Expanding** $E_i = \sqrt{p^2 + m_i^2}$ **in m_i/p**, one obtains a phase difference

$$(E_i - E_j)\, t \cong \frac{\delta m_{ij}^2 L}{2p}$$

For subtleties, see **Johns**, IJMPA (2019) & references

SN neutrino signal without self-coupling

$$H_{\mathbf{p}} = \frac{M^2}{2E} + \sqrt{2}G_F\left[N_l + \cancel{\int d\Gamma'(1 - \mathbf{v}\cdot\mathbf{v}')\rho_{\mathbf{p}'}}\right]$$

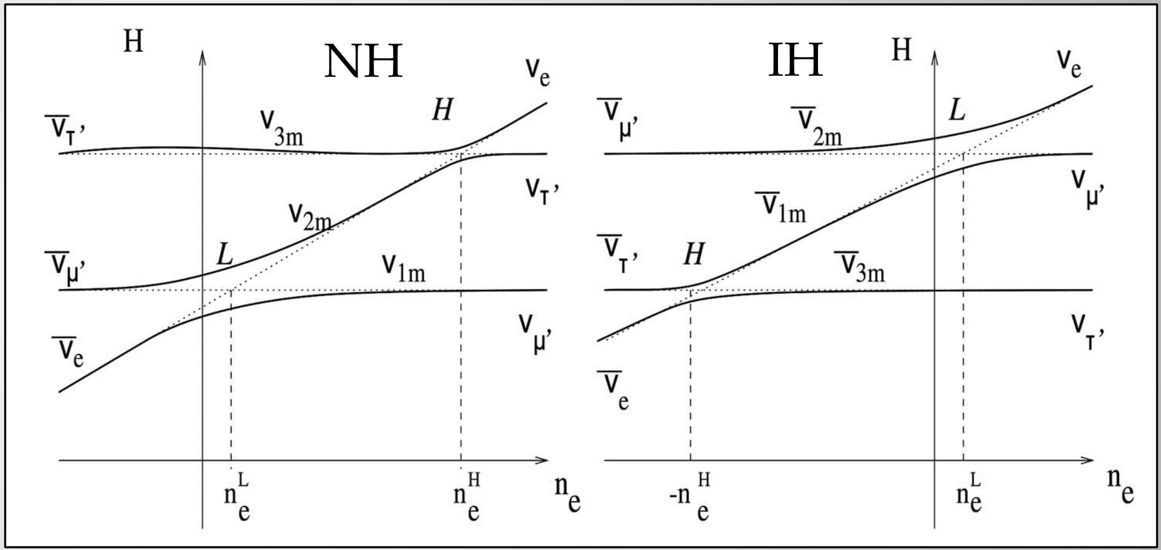
Adiabatic MSW

NH:

$$F_{\nu_e} \approx F^0_{\nu_x},\ F_{\bar{\nu}_e} \approx 0.7F^0_{\bar{\nu}_e} + 0.3F^0_{\nu_x}$$

IH:

$$F_{\nu_e} \approx 0.3F^0_{\nu_e} + 0.7F^0_{\nu_x},\ F_{\bar{\nu}_e} \approx F^0_{\nu_x}$$



G. Raffelt

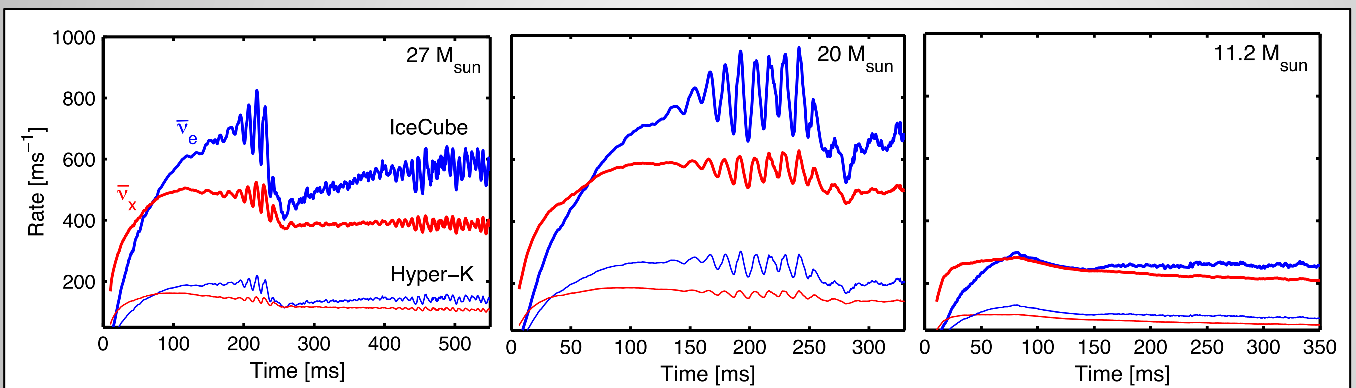
Adiabaticity is potentially compromised by
- the steep **density gradient** at the shock and
- **turbulence** in the wake of the shock.

Loreti et al., PRD (1995)
Schirato & Fuller (2002)
Kneller & Volpe, PRD (2010)

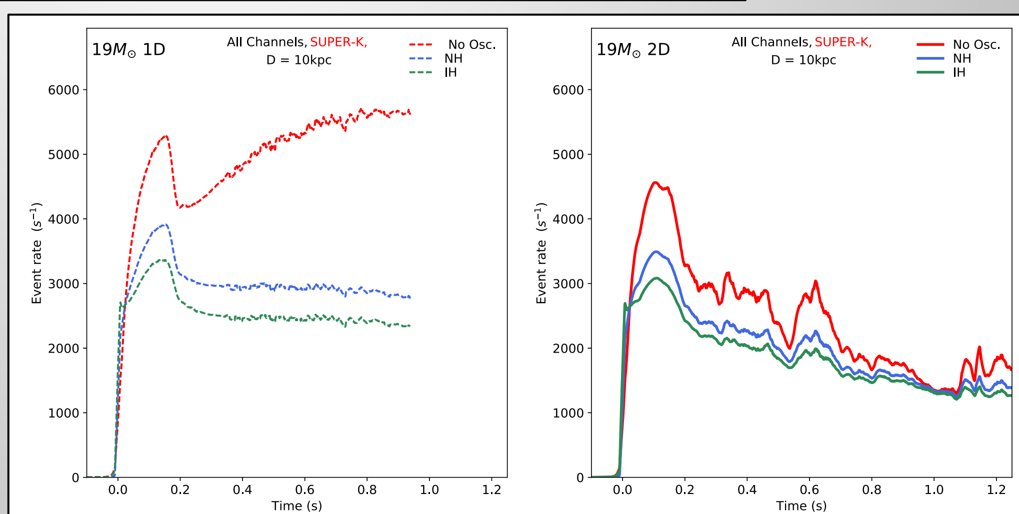
SN neutrino signal without self-coupling

Some anticipated features of the signal:

- The **neutronization peak** (if IH).
- A **faster post-bounce rise** of $\bar{\nu}_e$ luminosity in IH than in NH.
- Signal modulation due to **SASI** (depending on direction).
- Rapid variation due to **turbulent convection**.
- **Successful vs. failed explosion** and **BH formation**.

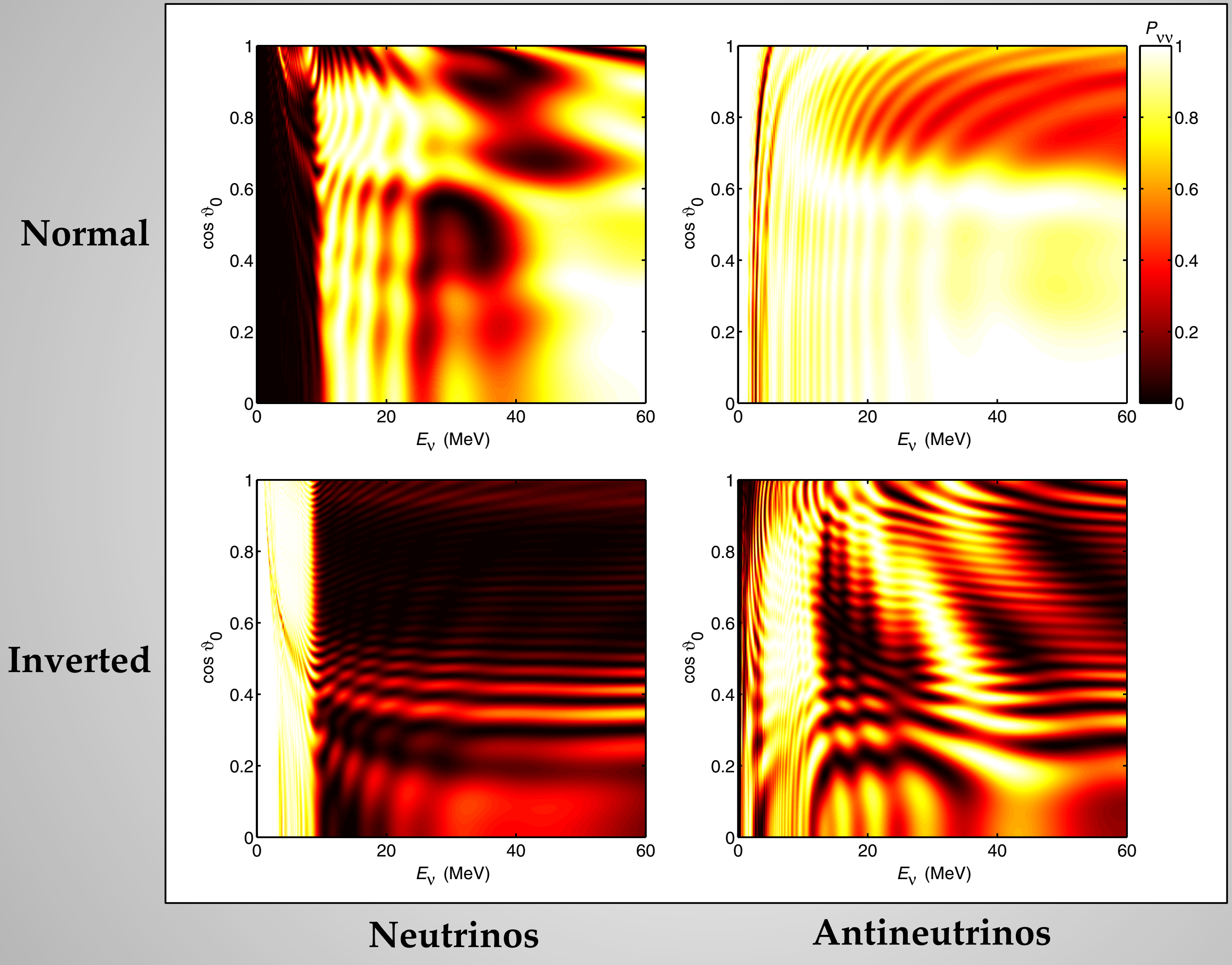


Tamborra et al.,
PRD (2014)



Seadrow et al.,
MNRAS (2018)

The bulb model: spectral swaps



Duan et al.,
PRL 97,
241101 (2006)