- We care about energy dependence in nuclear reactors because the neutrons with different energies (speeds) see different cross-sections.

- Most nuclear reactors are thermal reactors where the neutrons causing fission have an energy of ~0.025 eV (2200 m/s).

- However, the neutrons born from fission reactions are fast with an energy around 2 MeV.

- Therefore, to keep the chain reaction going, neutrons need to slow from 2 MeV to an energy 8 orders of magnitude lower.

- To slow down neutrons collide with ~~other~~ nuclei some of these nuclei will be light moderator nuclei, some of the others will be fuel (e.g. Uranium).

- During this slowing down process the neutrons will have to cross a resonance energy region where absorption is much more likely.

- Why do we have thermal ~~neutrons~~ reactors?
  1. The fission cross-section is higher for thermal neutrons
  2. When neutrons are moderated, they scatter a lot and are therefore possible to be described by diffusion

Fast Reactors Are possible for several reasons, despite the lower cross-section of fission

1. ν is higher for fission caused by fast neutrons
2. neutrons don't have to cross resonance region.

Most of the analysis we will do applies to thermal reactors.

First, lets recall the definition of $\chi(E)$:

$\chi(E)\,dE$ = probability that a fission neutron is born with energy in dE about E.

It stands to reason that $\int_0^\infty dE\, \chi(E) = 1$ (the neutron has some energy)

Using $\chi(E)$ we can write an energy-dependent diffusion eq.

$$\frac{1}{V(E)}\frac{\partial \phi}{\partial t} - \nabla \cdot D(E) \nabla \phi + \sigma(E)\phi = \int_0^\infty dE'\, \sigma_s(E') f(E' \to E)\, \phi(\vec{r}, E', t) + \chi(E) \int_0^\infty dE'\, \nu\sigma_f(E')\, \phi(\vec{r}, E', t) + Q(\vec{r}, E, t)$$

where σ's can also be function of Space

the equivalent eigenvalue problem is

$$-\nabla \cdot D(E) \nabla \phi + \sigma(E)\phi = \int_0^\infty dE'\, \sigma_s(E') f(E' \to E)\, \phi(\vec{r}, E') + \frac{\chi(E)}{k} \int_0^\infty dE'\, \nu\sigma_f(E')\, \phi(\vec{r}, E, t)$$

$$\phi(\vec{r}_s, E) = 0 \qquad \vec{r}_s \in \partial V^+$$

Now let's make our problem an infinite medium problem. This will do two things: make ∇'s go away and change k to k_∞. k_∞ is just defined as the multiplication factor for an infinite medium of a given material.

This makes our problem

$$\phi(E) = \frac{1}{\sigma(E)} \int_0^\infty dE' \, \sigma_s(E') f(E' \to E) \phi(E') + \frac{\chi(E)}{k_\infty \sigma(E)} \int_0^\infty dE' \, \nu\sigma_f(E') \phi(E')$$

This is not an easy problem to solve (you saw $\sigma_f(E)$).

We can make some simplifications, if we look at particular energy ranges.

<u>Fission Energy Range</u> $> \sim 100$ keV

- In this energy "all" fission neutrons are born.
- Two things can happen when neutrons scatter
  1. They hit light nuclei and lose a lot of energy and leave the energy range (go below 100 keV)
  2. They hit heavy (fuel) nuclei and either don't change energy or inelastically scatter and lose a lot of energy

Therefore, as a first approximation we can ignore the scattering term:

$$\phi_0(E) = \frac{\chi(E)}{k_\infty \sigma(E)} \int_E^\infty dE' \, \nu\sigma_f(E') \phi_0(E')$$

E because there is no upscattering in this range

if we define

$$F = \text{Fission neutron production rate} = \int_0^\infty dE' \, \nu\sigma_f(E') \phi_0(E')$$

$$\phi_0(E) = \frac{\chi(E)}{k_\infty \sigma(E)} F$$

We can take this solution and plug it into original Eq to get

$$\phi_1(E) = \frac{1}{\sigma(E)} \int_E^\infty dE' \, \sigma_s(E') f(E' \to E) \phi_0(E') + \frac{\chi(E)}{k_\infty \sigma(E)} F$$

$$\phi_1(E) = \frac{F}{k_\infty \sigma(E)} \int_E^\infty dE' \sigma_s(E') f(E' \to E) \frac{\chi(E')}{\sigma(E')} + \frac{\chi(E)}{k_\infty \sigma(E)} F$$

$$= \frac{F\chi(E)}{k_\infty \sigma(E)} \left(1 + \frac{1}{\chi(E)} \int_E^\infty dE' \sigma_s(E') f(E' \to E) \frac{\chi(E')}{\sigma(E')} \right)$$

One additional simplification we can make is

$$\sigma_{el}^{fuel}(E') f(E' \to E) = \sigma_{el}^{fuel}(E') \, \delta(E' - E) \quad \text{(no energy change)}$$

elastic scat. fuel

also between E' and ∞ the $\sigma_s(E') f(E' \to E) = \sigma_{el}^{fuel}(E') f(E' \to E)$ because all other scattering goes below E.

This makes

$$\phi_1(E) = \frac{F\chi(E)}{k_\infty \sigma(E)} \left(1 + \frac{1}{\chi(E)} \int_E^\infty dE' \sigma_{el}^{fuel}(E') \delta(E' - E) \frac{\chi(E')}{\sigma(E')} \right)$$

$$= \frac{F\chi(E)}{k_\infty \sigma(E)} \left(1 + \frac{\sigma_{el}^{fuel}(E)}{\sigma(E)} \right)$$

Therefore, in the fission energy range

$\phi(E)$ is proportional to $\dfrac{\chi(E)}{k_\infty \sigma(E)} \left(1 + \dfrac{\sigma_{el}^{fuel}(E)}{\sigma(E)} \right)$

Note: $\dfrac{\sigma_{el}^{fuel}(E)}{\sigma(E)} < 1$

Slowing Down Energy Range ~1eV – ~100 keV

- Almost no fission neutrons born in this region.
- Almost no inelastic scattering in this region

Therefore our diffusion equation becomes

$$\phi(E) = \frac{1}{\sigma(E)} \int_E^\infty dE' \, \sigma_s(E') f(E' \to E) \phi(E')$$

No upscattering because $E_{nuclei} < 1eV$ (11,605 K)

Also, in this region the scattering is elastic so

$$f(E' \to E) = \begin{cases} \frac{1}{E'(1-\alpha)} & E \le E' \le E/\alpha \\ 0 & \text{Otherwise} \end{cases}$$

$$\alpha = \frac{(A-1)^2}{(A+1)^2}.$$

Therefore,

$$\phi(E) = \frac{1}{\sigma(E)} \int_E^{E/\alpha} dE' \, \frac{\sigma_s(E')}{E'(1-\alpha)} \phi(E')$$

There is an analytic solution to this problem when $\alpha = 0$, hydrogen

$$\phi(E) = \frac{1}{\sigma(E)} \int_E^\infty dE' \, \frac{\sigma_s(E')}{E'} \phi(E')$$

$$\phi(E) = \frac{C}{\sigma(E) E} P_{rE} \qquad E_1 = 100eV$$

$$P_{re} = \exp\left[- \int_E^{E_1} \frac{\sigma_a(E') dE'}{\sigma(E') E'} \right]$$ = Prob that a neutron that scatters at energy E_1 does not get absorbed before reaching energy E

$$C = \sigma(E_1)\, E_1\, \phi(E_1).$$

Therefore

$\phi(E)$ is proportional to $\frac{1}{\sigma(E)E}$ for scattering off of hydrogen.

If the scattering is not hydrogen then we can make an approximation

$$\phi(E) = \frac{C}{\xi\, \sigma(E) E} P_{rE}$$

$$P_{re} = \exp\left[- \int_E^{E_1} \frac{\sigma_a(E')\, dE'}{\xi\, \sigma(E') E'} \right]$$ = Prob. a neutron ~~born~~ having energy E_1 slows to energy E w/o being absorbed

where ξ = average logarithmic energy loss per scatter
= average $\log(E_{init}) - \log(E_{Final})$

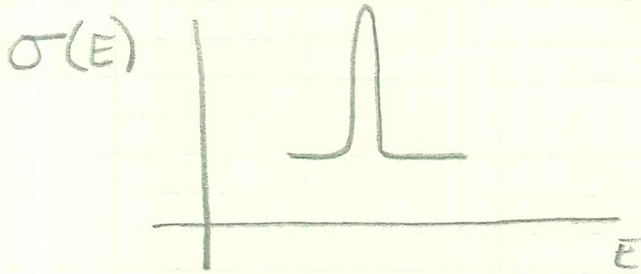
Mod	ξ	$\xi\, \sigma_s / \sigma_a$
H_2O	0.920	71
D_2O	0.590	5670
He	0.425	83
C	0.158	192
^{238}U	0.008	0.009

Energy Self-Sheilding

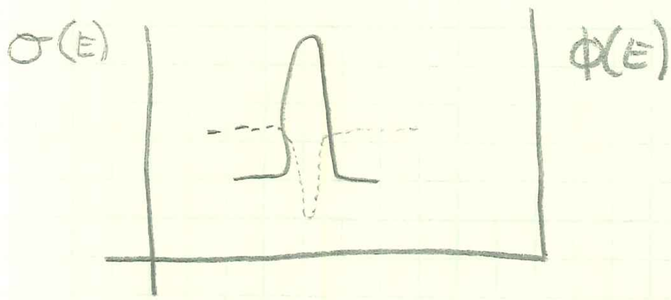
Now in the slowing down energy range

$$\phi(E) \propto \frac{1}{\xi\, \sigma(E) E}$$

Now in a resonance σ goes up



Therefore, $\phi(E)$ dips in the resonance



The scalar flux is depressed in the resonance. This is called energy self-sheilding there are fewer neutrons with energy inside the resonance.

Thermal Energy Range

In the thermal range

- almost no neutrons are born
- neutron energy is comparable to nuclei energy
  this implies upscattering is possible

We need to average over nuclei ~~cross section~~ motion

our balance equation will be

$$\sigma(E)\,\phi(E) = \int_0^{E_{th}} dE'\, \sigma_s(E' \to E)\,\phi(E') + S_{th}(E)$$

rate at which neutrons slow from energies above thermal to E

So we can't solve this for a generic source

If we consider the case where $\sigma_a(E) = 0 = \sigma_f(E)$

then we can re-write our equation as

$$\sigma_s(E)\,\phi(E) = \int_0^\infty dE'\,\sigma_s(E' \to E)\,\phi(E')$$

where we have extended our thermal range to ∞

The solution to this equation is a Maxwellian

$$\phi_M(E) = n_{tot} \frac{2\pi}{(\pi kT)^{3/2}} \left(\frac{2}{m}\right)^{1/2} E\, e^{-E/kT}$$

n_{tot} = total thermal neutron density m = neutron mas

k = Boltzmann Const $\approx \frac{1\,eV}{11605\,K}$ T = material temp in K

It turns out that $\phi_M(E)$ is independent of $\sigma_s(E' \to E)$

The maximum of the maxwellian is at $E = kT$

$$kT \approx \frac{1\,eV}{11605\,K} \cdot (293.15\,K) = 0.0253\,eV \approx 2200\ m/s$$

room temp

<u>Some Reactor physics</u>

We always have some absorption and sources (e.g. from fission).

Many absorbers are $1/V_r$ absorbers meaning

$$\sigma_a(V_r) = \frac{Const}{V_r} = \sigma_a(V_0)\frac{V_0}{V_r}$$

where V_r = relative speed b/w neut + nucleus

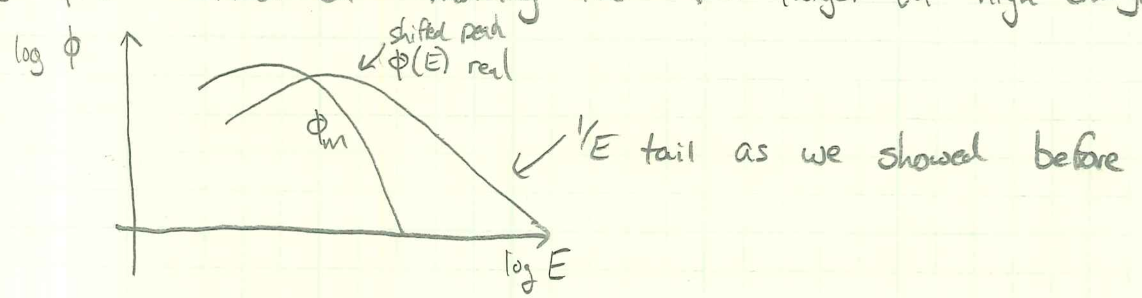
$V_0 = 2200$ m/s (this is just a convenient choice)

Because $\sigma_a(v_r)$ is higher for lower energies, it preferentially absorbs low energy neutrons.
This has the effect of increasing the peak of the Maxwellian
We quantify this with a neutron temperature usually written as

$$T_n = T\left(1 + \frac{C\,\sigma_a(v_0)}{\xi_\phi\,\sigma_s}\right)$$

The effect of $T_n > T$ when absorption is present is called absorption hardening.

The fission source produces high energy neutrons, which has the effect of making the tail larger at high energies



Finally, a real reactor will have leakage
this leakage will be stronger at higher energies, so this has the opposite effect of absorption hardening.
This leakage affect is called diffusion cooling or diffusion softening.

Absorption + Fission Rates

Consider all the neutrons of thermal energies and let's compute their reaction rate density for absorption

$$R_a = \int_0^{E_{th}} dE'\, \sigma_a(v(E'))\,\phi(E',T_n)$$ where $v(E')$ is relative speed b/w neutron and nucleus at temp T

Now $\sigma_a(v(E')) = \sigma_a(v_0)\dfrac{v_0}{v(E')}$

and $\phi(E',T_n) = v(E')\,N(E',T_n)$ by definition

so that

$$R_a = \int_0^{E_{th}} dE'\, \sigma_a(v(E'))\,\phi(E',T_n) = \sigma_a(v_0)\,v_0 \int_0^{E_{th}} dE'\, N(E',T_n)$$

$$= \sigma_a(v_0)\,v_0\,n_{tot}$$

if we define ϕ_0 = "2200 m/s flux" $= v_0 n_{tot}$ then

$R_a = \sigma_a(v_0)\phi_0 \Rightarrow$ thermal absorption rate is independent of the velocity distribution of nuclei and neutrons for $1/v$ absorber

The same is true for fission rate density

$R_f = \sigma_f(v_0)\phi_0$

Note $\sigma_a(v_0)$ and $\sigma_f(v_0)$ are everywhere (Chart of Nuclides, etc.)

<u>The non-$1/v$ absorbers</u>

heavier elements that have resonances so their cross-sections can allow them to absorb neutrons at higher speeds than $1/v$ would suggest.

To get a correction factor that we call a non-$1/v$ factor written as g_x where x is the reaction

To do this we write

$$g_a(T_n) = \frac{\int_0^{E_{th}} dE\, \phi_M(E,T_M)\, \sigma_a(E)}{\sigma_a(v_0)\, \phi_0} = \frac{\text{actual abs rate}}{1/v \text{ abs rate}}$$

So that $R_a = g_a(T_n)\, \sigma_a(v_0)\, \phi_0 = \text{abs. rate corrected}$

There are also $g_f(T_n)$ factors for fission.

Thermal - Averaged Cross-sections

It will be useful to compute averages of cross-sections over the thermal range

for example

$$\sigma_{a,Th} = \text{thermal averaged cross-section} = \frac{\int_0^{E_{th}} dE\, \sigma_a(E)\, \phi(E)}{\int_0^{E_{th}} dE\, \phi(E)}$$

Now if we assume $\phi(E)$ is Maxwellian

The numerator is $g_a(T_n)\, \sigma_a(v_0)\, \phi_0$

The denominator is just an integral over the Maxwellian

$$\int_0^{E_{th}} dE\, \phi_m(E,T_n) = \left(\frac{T_n}{T}\right)^{1/2} \frac{2}{\sqrt{\pi}}\, \phi_0$$

So that

$$\sigma_{a,th} = \sigma_a(v_0)\, \frac{\sqrt{\pi}}{2} \sqrt{\frac{T}{T_n}}$$ and

$$\sigma_{f,th} = \sigma_f(v_0)\, \frac{\sqrt{\pi}}{2} \sqrt{\frac{T}{T_n}}$$

We can use these results to find out some other important quantities

Recall

$$P_{re} = \exp\left[-\int_{E}^{E_1} \frac{\sigma_a(E')}{\xi\,\sigma(E')E'}\,dE'\right]$$ = prob. that neutron having energy E_1 slows to energy E w/o being absorbed

Therefore

$$\exp\left[-\int_{E_{th}}^{E_1} \frac{\sigma_a(E')}{\xi\,\sigma(E')E'}\,dE'\right]$$ = prob. of a neutron entering the slowing down region (in energy) will slow to thermal

This has a special name: resonance escape probability $= p$

Now let's try to evaluate it

In the slowing down energy range the scattering is nearly all potential scattering. In this case the neutron does not enter nucleus. Potential scattering is independent of energy so $\sigma_s(E) \approx \sigma_p$

$$p = \exp\left[-\frac{1}{\sigma_p \xi}\int_{E_{th}}^{E_1} \frac{\sigma_a(E')\sigma_p}{\sigma(E')E'}\,dE'\right]$$

$\sigma(E) = \sigma^{fuel}(E) + \sigma^{mod}(E)$ and $\sigma^{mod}(E) \gg \sigma^{fuel}$ (lots of scatter few fuel atoms)

INfinitely dilute limit

then $\sigma(E) \approx \sigma_p \Rightarrow$

$$p = \exp\left[\frac{-1}{\sigma_p \xi}\int_{E_{th}}^{E_1} \frac{\sigma_a(E')}{E'}\,dE'\right]$$

Now for a given Nuclei we define

$$I^{\infty}_{ra,i} = \int_{E_{th}}^{E_1} \frac{\sigma^{micro}_{a,i}(E')}{E'} dE'$$

$\Sigma_{a,i}$ = macro abs x-sect for nuclei i $= N_i \cdot \sigma^{micro}_{a,i}$ (N_i: # dens)

= Infinitely dilute abs. resonance integral

Now p becomes

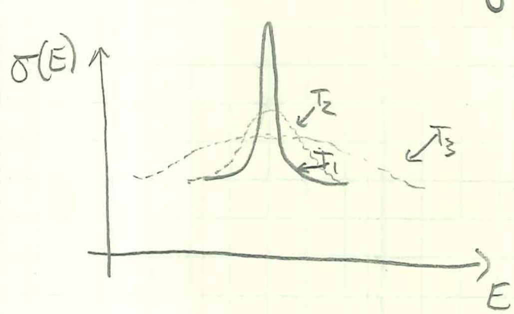
$$p = \exp\left[\frac{-1}{\xi_\varphi \Sigma^{mod}_p} \sum_i^{\#\,nuclides} N_i I^{\infty}_{ra,i}\right]$$

Note there are also fission resonance integrals.

$$I^{\infty}_{rf,i} = \int_{E_{th}}^{E_1} \frac{\sigma^{micro}_{f,i}(E')}{E'} dE' = \text{infinitely dilute fission resonance integral.}$$

Doppler Broadening

- The cross-sections depend on the relative speed of the neutron and the nucleus.
- x-sections are generally averaged over the nuclei's speed dist (Maxwellian)
- When T↑ there is a wider range of nuclei speeds this has the affect of broadening resonances.



$T_3 > T_2 > T_1$

This also has the affect of increasing resonance abs. as T↑ because

$$\phi(E) \propto \frac{C}{\xi_\varphi \Sigma(E) E}$$

$$\text{Abs rate dens.} \propto \frac{C\,\Sigma_a(E)}{\xi_\varphi E(\Sigma_s(E)+\Sigma_a(E))} = \frac{C}{\xi_\varphi E}\left(1 - \Sigma_s(E)/\Sigma_a(E)\right) + O\left(\Sigma_s(E)/\Sigma_a(E)\right)$$