# NUEN 629
# Numerical Methods in Reactor Analysis
# Homework 4 & 5 & Project

Due on:
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&
Thursday, December 3, 2015

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# Contents

# Homework 4 Problem Statement

Solve the following problem and submit a detailed report, including a justification of why a reader should believe your results and a description of your methods and iteration strategies.

1. (150 points + 50 points extra credit) In class we discussed the diamond-difference spatial discretization. Another discretization is the step discretization (this has several other names from other disciplines). It writes the discrete ordinates equations with isotropic scattering as, for $\mu_n > 0$ to

$$\mu_n \frac{\psi_{i,n} - \psi_{i-1,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2} \tag{1}$$

and for $\mu_n < 0$

$$\mu_n \frac{\psi_{i+1,n} - \psi_{i,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2} \tag{2}$$

The codes provided in class should be modified to implement this discretization.

(a) (50 Points) Your task (should you choose to accept it) is to solve a problem with uniform source of $Q = 0.01$, $\Sigma_t = \Sigma_s = 100$ for a slab in vacuum of width 10 using step and diamond difference discretizations. Use, 10, 50, and 100 zones ($h_x = 1, 0.02, 0.01$) and your expert choice of angular quadratures. Discuss your results and how the two methods compare at each number of zones.

(b) (10 points) Discuss why there is a different form of the discretization for the different signs of $\mu$.

(c) (40 points) Plot the error after each iteration using a 0 initial guess for the step discretization with source iteration and GMRES.

(d) (50 points) Solve Reed's problem (see finite difference diffusion codes). Present convergence plots for the solution in space and angle to a "refined" solution in space and angle.

(e) (50 points extra credit) Solve a time dependant problem for a slab surrounded by vacuum with $\Sigma_t = \Sigma_s = 1$ and initial condition given by $\phi(0) = 1/h_x$. Plot the solution at $t = 1$ $s$, using step and diamond difference. The particles have a speed of 1 cm/s. Which discretization is better with a small time step? What do you see with a small number of ordinates compared to a really large number (100s)?

# Homework 4 Problem Background

Due to the complicated nature of this course, I provided this background for the lay person (me), so that they might have some grounding for the solution and hopefully believe the results. It should be noted that most of this background information is copied from various points in Dr. McClarren's notes, and is in no way original. Anything intelligent in the following is due to this fact and for any errors, I blame myself.

Beginning with the weighty neutron transport equation.

$$\left(\frac{1}{v}\frac{\delta}{\delta t} + \hat{\Omega}\cdot\nabla + \Sigma_t\right)\psi = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}'\ K(\hat{\Omega}'\cdot\hat{\Omega}, v' \to v)\Sigma_s\psi + \frac{1}{4\pi}\chi\int_0^\infty dE'\ \bar{\nu}\Sigma_f\phi + q$$

Where $K(\hat{\Omega}'\cdot\hat{\Omega}, v' \to v)$ represents the probability of scattering from one angle and energy to another given a scattering event occured and $\Sigma_s$ is the macroscopic scattering cross section. The dependencies for the variables are shown below.

$$\begin{aligned} \Sigma_t(\vec{x}, v, t) \\ \psi(\vec{x}, \hat{\Omega}, v, t) \\ \Sigma_s(\vec{x}, v, t) \\ \chi(\vec{x}, v) \\ \Sigma_f(\vec{x}, v, t) \\ \phi(\vec{x}, v, t) \\ q(\vec{x}, \hat{\Omega}, v, t) \end{aligned}$$

There are 7 free variables (three spatial $[\vec{x}]$, two angular $[\hat{\Omega}]$, one energy $[v]$ and one time $[t]$) in this equation. In the steady state $\left(\frac{\delta\psi}{\delta t} = 0,\ \text{i.e. no time dependence}\right)$, non fissioning $(\Sigma_f = 0)$ case the transport equation reduces to,

$$\left(\hat{\Omega}\cdot\nabla + \Sigma_t\right)\psi = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}'\ K(\hat{\Omega}'\cdot\hat{\Omega}, v' \to v)\Sigma_s\psi + q.$$

In order to reduce this to a single energy the following definitions are helpful (remembering all time dependence is gone).

$$\psi(\vec{x}, \hat{\Omega}) = \int_0^\infty dE\ \psi(\vec{x}, \hat{\Omega}, v(E))$$

$$\Sigma_t(\vec{x}) = \frac{\int_0^\infty dE\ \Sigma_t(\vec{x}, v(E))\psi(\vec{x}, \hat{\Omega}, v(E))}{\psi(\vec{x}, \hat{\Omega})}$$

$$K(\hat{\Omega}'\cdot\hat{\Omega}, v' \to v) = K(\hat{\Omega}'\cdot\hat{\Omega})K(v' \to v)$$

$$\Sigma_s(\vec{x}) = \frac{\int_0^\infty dE \int_0^\infty dE'\ \Sigma_s(\vec{x}, v(E))K(v' \to v)\psi(\vec{x}, \hat{\Omega}, v(E))}{\psi(\vec{x}, \hat{\Omega})}$$

$$q(\vec{x}, \hat{\Omega}) = \int_0^\infty dE\ q(\vec{x}, \hat{\Omega}, v(E))$$

Using these definitions, integrating the transport equation over all energy, and assuming cross sections and sources do not vary in space or angle, our transport equation reduces again to,

$$\left(\hat{\Omega}\cdot\nabla + \Sigma_t\right)\psi(\vec{x}, \hat{\Omega}) = \int_{4\pi} d\hat{\Omega}'\ K(\hat{\Omega}'\cdot\hat{\Omega})\Sigma_s\psi(\vec{x}, \hat{\Omega}') + \frac{q}{4\pi}.$$

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Where the source was divided by $4\pi$ because it was assumed to be isotropic. The final simplification for our problem will be in space. If we assume that our geometry is infinite in $y$ $\left(\frac{\delta}{\delta y}=0\right)$ and $x$ $\left(\frac{\delta}{\delta x}=0\right)$. This also means that $\psi$ depends only on $z$ and $mu$, and if we recall that

$$\hat{\Omega}=(\sqrt{1-\mu^2}cos(\rho),\sqrt{1-\mu^2}sin(\rho),\mu),$$

and

$$\nabla=\left(\frac{\delta}{\delta x},\frac{\delta}{\delta y},\frac{\delta}{\delta x}\right)$$

then our transport equation, and the equation I think we are trying to solve for this homework is.

$$\left(\mu\frac{\delta}{\delta z}+\Sigma_t\right)\psi(z,\mu)=\Sigma_s 2\pi\int_{-1}^{1}d\mu'\ K(\mu_0)\psi(z,\mu')+\frac{q}{4\pi}.$$

Where $\Sigma_s$ was moved outside the integral because it has no angular dependence, $\mu_0$ is the cosine of the scattering angle (representing the probability of scattering into $\mu$), and integration over the azimuthal angle occured because $K(\mu_0)$ is assumed to be uniform and not depend on that angle. This means:

$$K(\mu_0)=\frac{\text{Total events leading to } \mu}{\text{Total scattering events possible}}=\frac{1}{2}$$

Now we employ some Legendre (pronounced Legendary...just kidding) polynomial expansion for the scattering term.

# Homework 5

# Project