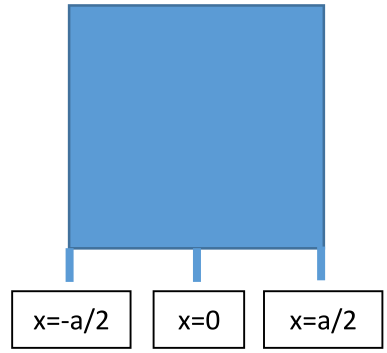
3. Find the solution to the diffusion equation for 1-group, slab geometry, where the material is a pure scatter and the slab width is X:



The one group, slab geometry diffusion equation for slab geometry is:

$$\frac{d^2\phi}{dx^2} = -\frac{q}{D}$$

Has a solution of the form:

$$\phi = -\frac{q}{D}x^2 + C_1 x + C_2$$

General boundary condition:

$$A\phi\left(\frac{a}{2}\right) + BD\left|\frac{\delta\phi}{\delta x}\right|_{\frac{a}{2}} = C; \qquad A\phi\left(-\frac{a}{2}\right) - BD\left|\frac{\delta\phi}{\delta x}\right|_{-\frac{a}{2}} = C$$

Where:

$$\phi\left(\frac{a}{2}\right) = -\frac{q}{D}\frac{a^2}{4} + C_1\frac{a}{2} + C_2; \phi\left(-\frac{a}{2}\right) = -\frac{q}{D}\frac{a^2}{4} - C_1\frac{a}{2} + C_2$$

And:

$$\left|\frac{\delta\phi}{\delta x}\right|_{\frac{a}{2}} = -\frac{aq}{D} + C_1; \left|\frac{\delta\phi}{\delta x}\right|_{-\frac{a}{2}} = \frac{qa}{D} + C_1$$

Applied at right side:

$$A_R\left(-\frac{q}{D}\frac{a^2}{4} + C_1\frac{a}{2} + C_2\right) + B_R D\left(-\frac{qa}{D} + C_1\right) = C_R$$

$$C_2 = \frac{C_R}{A_R} + \left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1\left(\frac{a}{2} + \frac{B_R D}{A_R}\right)$$

Applied at left side:

$$A_L\left(-\frac{q}{D}\frac{a^2}{4} - C_1\frac{a}{2} + C_2\right) - B_L D\left(\frac{qa}{D} + C_1\right) = C_L$$

$$-\frac{A_L q}{D}\frac{a^2}{4} - C_1\frac{A_L a}{2} + A_L C_2 - B_L qa - C_1 B_L D = C_L$$

Plug in C_2

$$-\frac{A_L q}{D}\frac{a^2}{4} - C_1\frac{A_L a}{2} + A_L\frac{C_R}{A_R} + A_L\left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1 A_L\left(\frac{a}{2} + \frac{B_R D}{A_R}\right) - B_L qa - C_1 B_L D = C_L$$

Solve for C_1

$$\frac{A_L C_R}{A_R} + \left(\frac{(A_L qaB_R)}{A_R}\right) - B_L qa - C_L = C_1\left(aA_L + \frac{A_L B_R D}{A_R} + B_L D\right)$$

$$\frac{A_L}{A_R}(C_R + qaB_R) - B_L qa - C_L = C_1\left(aA_L + \frac{A_L B_R D}{A_R} + B_L D\right)$$

$$C_1 = \frac{\left(\frac{A_L}{A_R}(C_R + qaB_R) - B_L qa - C_L\right)}{aA_L + \frac{A_L B_R D}{A_R} + B_L D}$$

$$C_2 = \frac{C_R}{A_R} + \left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1\left(\frac{a}{2} + \frac{B_R D}{A_R}\right)$$

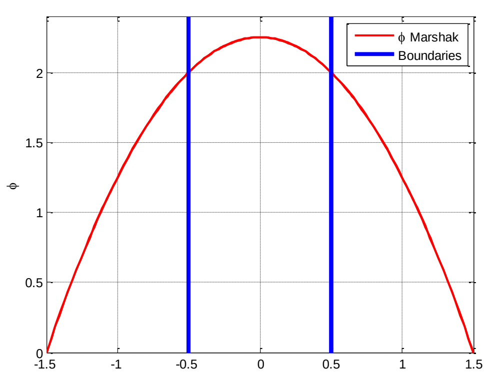
Using the provided constants:

	A	B	C
Vacuum Marshak	0.25	0.5	0
Vacuum Mark	0.5	0.866025	0
Vacuum Dirichlet	1	0	G
Reflecting	0	1	0
Albedo	(1-α)/(2(1+α))	1	0

1. Vacuum Marshak Conditions: $A_R = 0.25 = A_L; B_R = 0.5 = B_L; C_R = 0 = C_L$ (plots assumed $D = 1; a = 1; q = 1$)

$$C_1 = 0$$

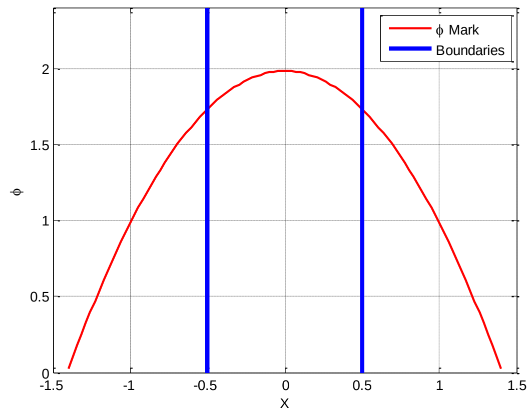
$$C_2 = \left(2qa + \frac{q}{D}\frac{a^2}{4}\right)$$



2. Vacuum Mark Conditions:

$$C_1 = 0$$

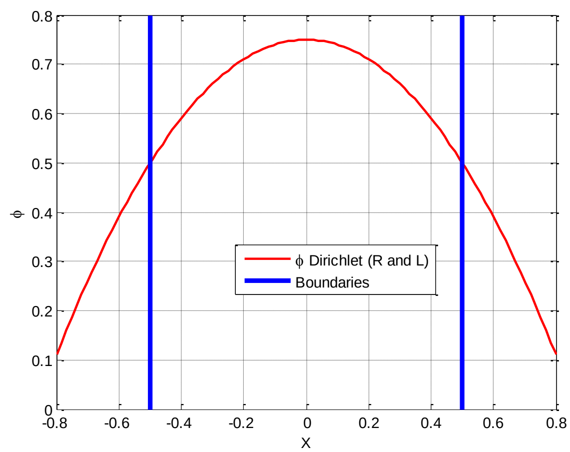
$$C_2 = qa\sqrt{3} + \frac{q}{D}\frac{a^2}{4}$$



3. Vacuum Dirichlet Conditions
   a. C=0.5

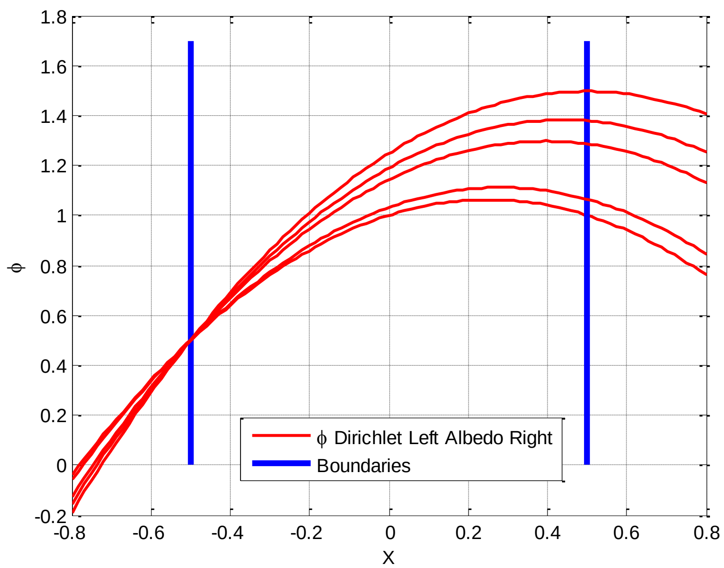
$$C_1 = 0$$

$$C_2 = 0.5 + \left(\frac{q}{D}\frac{a^2}{4}\right)$$

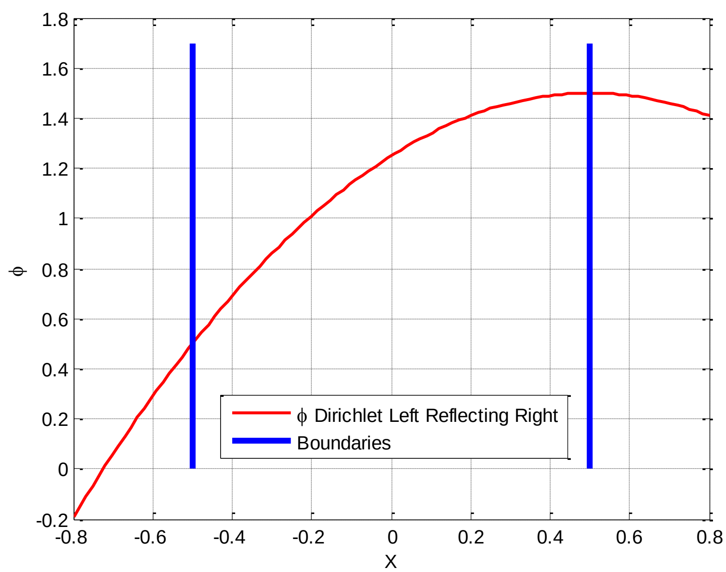


4. Vacuum Dirichlet conditions on the left and albedo on the right

   CL=0.5. Albedo varied from 0 to 1



5. Vacuum Dirichlet condition on the left and reflecting on the right



The albedo condition can represent either a reflecting or vacuum condition. Extrapolation distances were consistent with expectations.

Code for plotting reproduced below:

```
D=1;a=1;q=1;alpha=0.999;
%Boundary Conditions:
CL=0.5;CR=0;BL=0;BR=1;AL=1;AR=(1-alpha)/(2*(1+alpha));

%Plot Solution
C1=((AL/AR)*(CR+q*a*BR)-BL*q*a-CL)/(a*AL+(AL*BR*D)/AR+BL*D);
C2=CR/AR+((q*a*BR)/AR+(q*(a^2))/(D*4))-C1*(a/2+(BR*D)/AR);

x=linspace(-0.8,0.8);
phi=(-q./D).*(x.^2)+C1.*(x)+C2;
plot(x,phi,'r','LineWidth',2);

%Plot Bars
hold on
y=linspace(0,1.7);
xn=ones(1,100).*-0.5;xp=ones(1,100).*0.5;
plot(xn,y,'b','LineWidth',3);
plot(xp,y,'b','LineWidth',3);
grid on;xlabel 'X';ylabel '\phi';
legend ('\phi Dirichlet Left Reflecting Right','Boundaries');
ylim([0,0.8]);
```