2. Microscopic cross-sections for ^{1}H in units of barns for 5 groups were provided from the code NJOY. Given an infinite tank of high-pressure hydrogen, 30 atm, encloses a bare sphere of ^{235}U. Compute the scalar flux φ_g and the current J_g in the hydrogen using the separable, P1 equivalent, and extended Legendre approximations. Compare your solutions graphically.

The neutron transport equation for an infinite medium can be written as:

$$\Sigma_t \Psi = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \int_0^\infty dE' \Sigma_s K_\ell(E' \to E)\phi_\ell + \chi$$

Where:

$$\phi_\ell = \int_{-1}^{1} d\mu \Psi(\mu) \mathrm{P}_\ell(\mu) \qquad\qquad \Psi = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \phi_\ell P_\ell$$

Integrating over an energy bin:

$$0 = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G} \left(\Sigma_{s\ell g' \to g} - \delta_{g'g}\Sigma_{t\ell g'}\right)\phi_{\ell g'} + \chi_g$$

Where:

$$\chi_g = \int_{E_g}^{E_{g-1}} dE\, \chi(E) \qquad\qquad \sum_{g=1}^{G} \chi_g = 1$$

$$\phi_{\ell g} = \int_{E_g}^{E_{g-1}} dE\, \phi_\ell \qquad\qquad \Sigma_{t\ell g} = \frac{1}{\phi_{\ell g}} \int_{E_g}^{E_{g-1}} dE\, \Sigma_t \phi_\ell$$

$$\Sigma_{s\ell g' \to g} = \frac{1}{\phi_{\ell g}} \int_{E_g}^{E_{g-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE'\, \Sigma_s(E') K_\ell(E' \to E)\phi_\ell$$

Adding a term to each side:

$$\Sigma_{t^*g}\Psi_g = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G} \left(\Sigma_{s\ell g' \to g} + \delta_{g'g}\left(\Sigma_{t^*g'} - \Sigma_{t\ell g'}\right)\right)\phi_{\ell g'} + \chi_g$$

Where:

P1 approximation:

$$\Sigma_{t^*g} = \Sigma_{t,0,g}$$

Extended transport approximation:

$$\Sigma_{t^*g} = \Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g \to g'}$$

Noting (from class):

$$\Sigma_{t,0,g} = \frac{1}{\phi_g} \int_{E_g}^{E_{g-1}} dE\, \Sigma_t \phi = \Sigma_{tg}$$

P1 approximation solution:

$$\Sigma_{t,0,g}\Psi_{\mathrm{g}} = \sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell\sum_{g'=1}^{G}\left(\Sigma_{s\ell g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t\ell g'}\right)\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Note:

$$\int_{-1}^{1}P_\ell(\mu)d\mu = \int_{-1}^{1}P_0P_\ell(\mu)d\mu = \frac{2}{2\ell+1}\delta_{0\ell}$$

Flux: integrate over μ $\ell = 0$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t0g'}\right)\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g}\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} - \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g}\right)\phi_{0g'} = \chi_{\mathrm{g}}$$

Expanding into a matrix:

$$\begin{bmatrix} \Sigma_{t1}-\Sigma_{s,1\to1} & \Sigma_{s,2\to1} & \Sigma_{s,3\to1} & \Sigma_{s,4\to1} & \Sigma_{s,5\to1} \\ \Sigma_{s,1\to2} & \Sigma_{t2}-\Sigma_{s,2\to2} & \Sigma_{s,3\to2} & \Sigma_{s,4\to2} & \Sigma_{s,5\to2} \\ \Sigma_{s,1\to3} & \Sigma_{s,2\to3} & \Sigma_{t3}-\Sigma_{s,3\to3} & \Sigma_{s,4\to3} & \Sigma_{s,5\to3} \\ \Sigma_{s,1\to4} & \Sigma_{s,2\to4} & \Sigma_{s,3\to4} & \Sigma_{t4}-\Sigma_{s,4\to4} & \Sigma_{s,5\to4} \\ \Sigma_{s,1\to5} & \Sigma_{s,2\to5} & \Sigma_{s,3\to5} & \Sigma_{s,4\to5} & \Sigma_{t5}-\Sigma_{s,5\to5} \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\end{bmatrix} = \begin{bmatrix}\chi_1\\ \chi_2\\ \chi_3\\ \chi_4\\ \chi_5\end{bmatrix}$$

Solved with a matrix inversion.

Current: multiply by μ and integrate ($\ell = 1$ for the same reason $\ell = 0$ above):

$$\Sigma_{t,0,g}\mathrm{J}_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s1g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t1g'}\right)\right)J_{g'} + 0$$

With a non zero matrix:

$$\mathrm{J}_1 = J_2 = J_3 = J_4 = J_5 = 0$$

Extended transport approximation:

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\Psi_{\mathrm{g}}$$
$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G} \left(\Sigma_{s\ell g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'} - \Sigma_{t\ell g'}\right)\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Flux integrate over μ:$\ell = 0$

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\phi_g = \sum_{g'=1}^{G} \left(\Sigma_{s0g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'} - \Sigma_{t0g'}\right)\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

When g=g' it cancels?

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\phi_g - \Sigma_{t,L+1,g}\phi_{0g} + \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\,\phi_{0g} + \Sigma_{t0g'}\phi_{0g'} = \sum_{g'=1}^{G} \left(\Sigma_{s0g'\to g}\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t0g'}\phi_{0g'} = \sum_{g'=1}^{G} \left(\Sigma_{s0g'\to g}\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Which is the same as above.

Current multiply by μ and integrate over μ: $\ell = 1$

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\mathrm{J}_{\mathrm{g}} = \sum_{g'=1}^{G} \left(\Sigma_{s1g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'} - \Sigma_{t1g'}\right)\right)J_{1g} + 0$$

This does not exactly cancel as before, but because the author is tired he will assume the solution is the same as for P1 and:

$$\mathrm{J}_1 = J_2 = J_3 = J_4 = J_5 = 0$$

Filling out the matrix:

$$N\begin{bmatrix} 4.6-2.91518 & 0 & 0 & 0 & 0 \\ 1.67039 & 18.3-14.0197 & 0 & 0 & 0 \\ 0.0167099 & 4.23255 & 20.4-16.0183 & 0 & 0 \\ 0.000166937 & 0.0423257 & 4.35086 & 22.7-4.53876 & 1.53831 \\ 1.04698e-6 & 0.000424355 & 0.0439445 & 17.9734 & 29.8-27.93 \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\end{bmatrix} = \begin{bmatrix}0.99136\\ 0.01379\\ 0\\ 0\\ 0\end{bmatrix}$$

Group boundaries in eV:

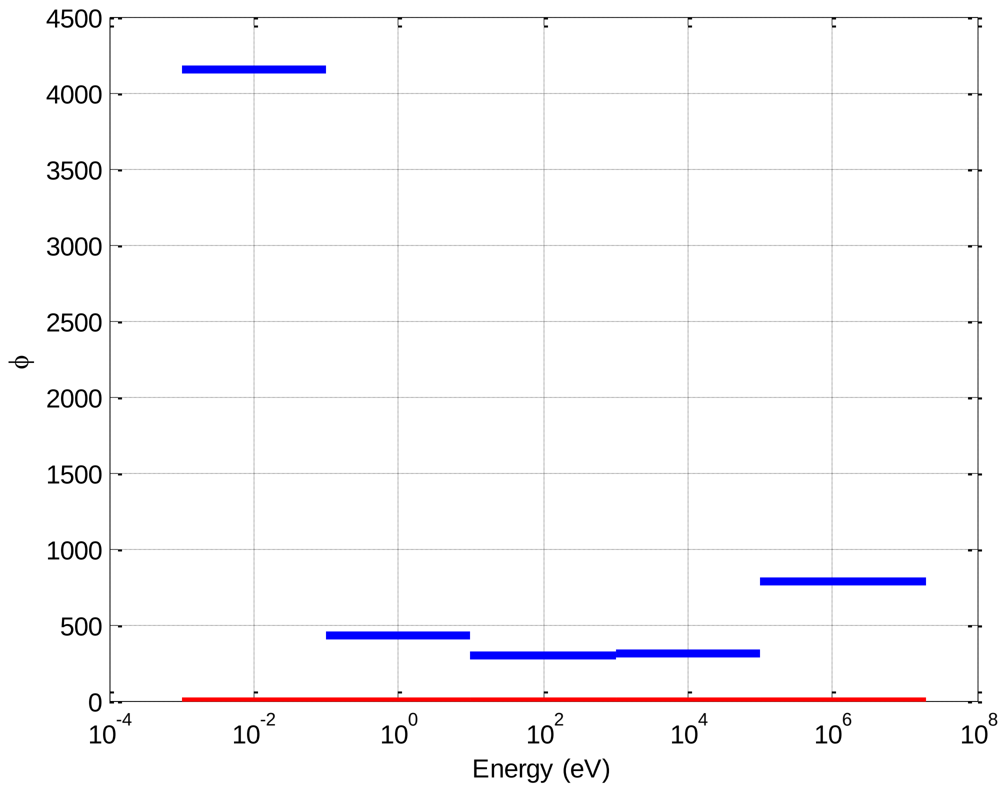
2e+07 100000 1000 10 0.1 0.001

Assuming ideal gas law

$$\frac{n}{V} = \frac{P}{RT} = 3039750Pa * \frac{k*mol}{8.3144598m^3Pa} * \frac{1}{293K} = \frac{1247.77mol}{m^3}$$

$$\frac{1247.77mol}{m^3} * \frac{1m^3}{100^3cm^3} * \frac{0.6022E24atoms}{mol} = 0.000751E24\frac{atoms}{cm^3}$$

Inverse solve:



flux = 782.35 309.91 301 425.19 4148.2

Script for plotting:

```
A =[4.60248-2.91518   0             0             0             0
    -1.67039      18.2957-14.0197   0             0             0
    -0.0167099    -4.23255    20.4196-16.0183     0             0
    -0.00166937   -0.0423257     -4.35086     22.6605-4.53876 -1.53831
    -1.04698e-6   -0.000424355   -0.0439445     -17.9734     29.77991-
27.9344];



A=A*0.000751;

b=[0.99136;0.01379;0;0;0];

flux=A^-1*b

x1=logspace(5,7.3011,30);
x2=logspace(3,5,30);
x3=logspace(1,3,30);
x4=logspace(-1,1,30);
x5=logspace(-3,-1,30);

semilogx(x1,flux(1,1)*ones(1,30),'b','LineWidth',3);
hold on; grid on;xlabel 'Energy (eV)';ylabel '\phi';
semilogx(x2,flux(2,1)*ones(1,30),'b','LineWidth',3);
semilogx(x3,flux(3,1)*ones(1,30),'b','LineWidth',3);
semilogx(x4,flux(4,1)*ones(1,30),'b','LineWidth',3);
semilogx(x5,flux(5,1)*ones(1,30),'b','LineWidth',3);

%Below Zero?
semilogx(x1,zeros(1,30),'r','LineWidth',3);
hold on; grid on;xlabel 'Energy (eV)';ylabel '\phi';
semilogx(x2,zeros(1,30),'r','LineWidth',3);
semilogx(x3,zeros(1,30),'r','LineWidth',3);
semilogx(x4,zeros(1,30),'r','LineWidth',3);
semilogx(x5,zeros(1,30),'r','LineWidth',3);
```