Problem 1

$$\frac{1}{2}\int_{-1}^{1}\frac{1-h^2}{(1+h^2-2h\mu_0)^{3/2}}\,d\mu_0$$

$$1+h^2-2h\mu_0 = P$$
$$-2h\,d\mu = dP$$
$$d\mu = \frac{dP}{-2h}$$

$$= \frac{1}{4}\frac{1-h^2}{-h}\int \frac{1}{P^{3/2}}\,dP$$

$$= \frac{1}{4}\frac{(1-h^2)}{-h}\frac{-2}{\sqrt{P}}\Bigg| = \frac{1-h^2}{2h}\frac{1}{\sqrt{1+h^2-2h\mu_0}}\Bigg|_{-1}^{1}$$

$$= \frac{1-h^2}{2h}\left(\frac{1}{\sqrt{1+h^2-2h}} - \frac{1}{\sqrt{1+h^2+2h}}\right)$$

$$= \frac{1-h^2}{2h}\left(\frac{1}{\pm(h-1)} - \frac{1}{\pm(h+1)}\right)$$

$$= \frac{1-h^2}{2h}\left(\overset{\text{minus}}{\frac{1}{(1-h)}} - \overset{\text{plus}}{\frac{1}{h+1}}\right)$$

$$= \frac{1-h^2}{2h}\cdot\frac{h+1-1+h}{1-h^2} = \boxed{1}$$

First Moment

$$\frac{1}{2}\int_{-1}^{1}\frac{(1-h^2)\mu_0}{(1+h^2-2h\mu_0)^{3/2}}\,d\mu_0 = \frac{1}{2}\int_{(1+h)^2}^{(h-1)^2}\frac{(1-h^2)}{P^{3/2}}\frac{dP}{-2h}\mu_0$$

$$\frac{P-1-h^2}{-2h} = \mu_0$$

$$= \frac{(1-h^2)}{8h^2}\int_{(1+h^2)}^{(1-h^2)}\frac{P}{P^{3/2}} - \frac{1}{P^{3/2}} - \frac{h^2}{P^{3/2}} = \frac{(1-h^2)}{8h^2}\left(2\sqrt{P}\Big| + \frac{2}{\sqrt{P}}\Big| + \frac{2h^2}{\sqrt{P}}\Big|\right)$$

$$= \frac{(1-h^2)}{4h^2}\left(\overset{\text{minus}}{\pm(h-1)} - \overset{\text{Plus}}{\pm(1+h)} + \overset{\text{minus}}{\frac{1}{\pm(h-1)}} - \overset{\text{Plus}}{\frac{1}{\pm(1+h)}} + \overset{\text{minus}}{\frac{h^2}{\pm(h-1)}} - \overset{\text{Plus}}{\frac{h^2}{\pm(1+h)}}\right)$$

$$= \frac{(1-h^2)}{4h^2}\left(-2h + \frac{2h}{1-h^2} + \frac{2h^3}{1-h^2}\right) = h$$

$$\mu_0^2 = \frac{1}{4h^2}\left[P^2 - 2P(1+h^2) + h^4 + 2h^2 + 1\right]$$

Can I call it a second moment without a tensor?

Second Moment: $\frac{1}{2}\int_{-1}^{1} \frac{(1-h^2)\mu_0^2}{(1+h^2-2h\mu_0)^{3/2}}\, d\mu_0$

case ① $\pm$
case ② $^{+}_{+}$
case ③ $^{-}_{+}$
case ④ $^{-}_{-}$

$$= \frac{(h^2-1)}{16h^3}\left[\frac{2}{3}P^{3/2}\Big|_{(h+1)^2}^{(h-1)^2} + 4(1+h^2)\sqrt{P}\Big|_{(h+1)^2}^{(h-1)^2} - \frac{2(h^2+1)^2}{\sqrt{P}}\Big|_{(h+1)^2}^{(h-1)^2}\right]$$

term ① term ② term ③

$$= \frac{(h^2-1)}{24h^3}\left[P^{3/2}\Big| + 6(1+h^2)\sqrt{P}\Big| - \frac{3(h^2+1)^2}{\sqrt{P}}\Big|\right]$$

	Case ①	case ②	case ③	case ④
Term ①	$2h^3+6h$	$-6h^2-2$	$-2h^3-6h$	$6h^2+2$
$\times h^2-1$	$2h^5+4h^3-6h$	$-6h^4+4h^2+2$	$-2h^5-4h^3+6h$	$6h^4-4h^2-2$
Term ②	$12h^3+12h$	$-12h^2-12$	$-12h^3-12h$	$12h^2+12$
$\times h^2-1$	$12h^5-12h$	$-12h^4+12$	$-12h^5+12h$	$12h^4-12$
Term ③	$\frac{6h^5+12h^3+6h}{(h^2-1)}$	$\frac{6h^4+12h^2+6}{(h^2-1)}$	$\frac{-6h^5-12h^3-6h}{(h^2-1)}$	$\frac{-6h^4-12h^2-6}{(h^2-1)}$

$$\frac{1}{24h^3}\left[12h^5 - \cancel{12h} + 6h^5 + 12h^3 + \cancel{6h} - 2h^5 - 4h^3 + \cancel{6h}\right]$$

$$\frac{1}{3}\left[2h^2+1\right]$$ ~ sorry couldn't get h^2

$$\int_{-1}^{1} K(\mu_0, v'\to v)\,d\mu = 1 \;;\; \int_{-1}^{1} K(\mu_0, v'\to v)\,\mu_0\, d\mu_0 = h$$

$$\int_{-1}^{1} K(\mu_0, v'\to v)\,\mu_0^2\, d\mu = \tfrac{1}{3}\left[2h^2+1\right]$$

$$K_\ell(E' \to E) = \int_{-1}^{1} d\mu_0 \, K(\mu_0, E' \to E) \frac{1}{2^\ell \ell!} \frac{d^\ell}{d\mu_0^\ell}\left[(\mu_0^2 - 1)^\ell\right]$$

$$K_\ell = \int \frac{K}{2^\ell \ell!} \frac{d^\ell}{d\mu_0^\ell}\left[(\mu_0^2 - 1)^\ell\right]$$

$$K_0 = \int K = \boxed{1}$$

$$K_1 = \int \frac{K}{2} 2\mu_0 = \int K \mu_0 = \boxed{h}$$

$$K_2 = \int \frac{K}{8} \frac{d^2}{d\mu_0^2}\left(\mu_0^4 - 2\mu_0^2 + 1\right)$$

$$= \int \frac{K}{8} \frac{d}{d\mu_0}\left(4\mu_0^3 - 4\mu_0\right)$$

$$= \int \frac{K}{2}\left(3\mu_0^2 - 1\right)$$

$$= \frac{3}{2} \cdot \frac{1}{3}\left[2h^2 + 1\right] - \frac{1}{2}$$

$$= h^2 + \tfrac{1}{2} - \tfrac{1}{2}$$

$$= \boxed{h^2}$$ — woo hoo!

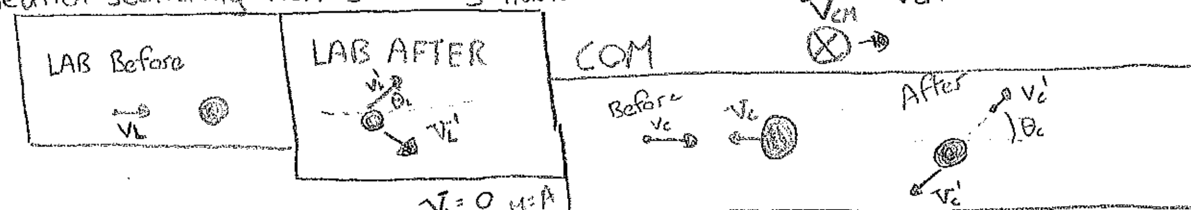
Problem 2

60/670 DUDE I'm not sure what you are asking for

If $Q=0$ little green book LGB pg 90

neutron scattering from stationary nuclei

$V_{CM} = V'_{CM}$

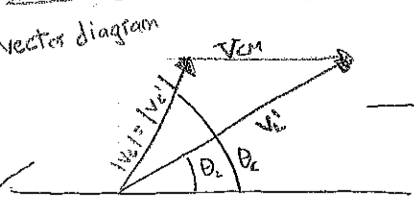


$V'_L = 0$; $m=1$; $M=A$

LGB Pg.90 $\quad V_{CM} = V'_{CM} = \dfrac{mV_L + MV_L}{m+M} = \dfrac{V_L}{1+A}$ $\quad \boxed{V_{CM} = \left(\dfrac{1}{1+A}\right)V_L}$

vector diagram

61/670



$\boxed{V'_c = V_c = -V_{CM} = -\left(\dfrac{1}{A+1}\right)V_L}$

Conservation Dude 61/670

$V'_c = V_c = \dfrac{A}{A+1} V_L \quad V_L = V_{CM}$

$\rightarrow V'_L \sin\theta_L = V'_c \sin\theta_c$

$V'_L \cos\theta_L \, d\theta_L = V'_c \cos\theta_c \, d\theta_c$

$\tan\theta_L \dfrac{d\theta_c}{d\theta_L} = \dfrac{\sin\theta_c}{\cos\theta_c}$

$V'_L \sin\theta_L = V'_c \sin\theta_c$

$V'_L \cos\theta_L = V'_c \cos\theta_c + V_{CM}$

Divide

$\tan\theta_L = \dfrac{V'_c \sin\theta_c}{V_{CM} + V'_c \cos\theta_c} = \dfrac{\sin\theta_c}{\frac{1}{A} + \cos\theta_c}$

$\sigma_L(\theta_L)\sin\theta_L \, d\theta_L = \sigma_{CM}(\theta_c)\sin\theta_c \, d\theta_c$

$\sigma_L(\theta_L) = \sigma_{CM}(\theta_c)\dfrac{\sin\theta_c}{\sin\theta_L}\dfrac{d\theta_c}{d\theta_L} = \sigma_{CM}(\theta_c)\dfrac{V'_L}{V'_c}\dfrac{\sin\theta_c}{\cos\theta_c}\cdot\dfrac{1}{\tan\theta_L}$

$= \sigma_{CM}(\theta_c)\dfrac{V'_L}{V_L}\cdot\dfrac{(A+1)}{A}\cdot\dfrac{1}{\cos\theta_c}\cdot\left(\frac{1}{A} + \cos\theta_c\right)$

$V'^2_L = (V_{CM} + V_c\cos\theta_c)^2 + V_c^2\sin\theta_c^2$

$V'^2_L = V_{CM}^2 + V_c^2 + 2V_{CM}V_c\cos\theta_c$

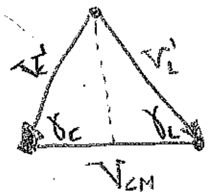
$\dfrac{V'^2_L}{V_L^2} = \left(\dfrac{1}{A+1}\right)^2 + \left(\dfrac{A}{A+1}\right)^2 + 2\cdot\left(\dfrac{1}{A+1}\right)\left(\dfrac{A}{A+1}\right)\cos\theta_c \rightarrow \boxed{\dfrac{V'_L}{V_L} = \dfrac{\sqrt{A^2+1+2A\cos\theta_c}}{A+1}}$

$\sigma_L(\theta_L) = \sigma_{CM}(\theta_c)\sqrt{A^2+1+2A\cos\theta_c}\cdot\dfrac{1}{A}\left(\dfrac{1}{A}\cos\theta_c + 1\right)$

$= \sigma_{CM}(\theta_c)\sqrt{\frac{1}{A^2} + \frac{2\cos\theta_c}{A} + 1}\left(\frac{1}{A}\cos\theta_c + 1\right)$

$\approx \dfrac{\sigma_{CM}(\theta_c)\left(\frac{1}{A^2} + \frac{2}{A}\cos\theta_c + 1\right)^{3/2}}{1 + \frac{1}{A}\cos\theta_c}$ $\rightarrow$ if you say $\frac{1}{A^2}\cos\theta_c^2 \approx \frac{1}{A^2}$

The other Angle



$$\rightarrow V_c' \sin\gamma_c = V_L' \sin\gamma_L$$

$$V_c' \cos\gamma_c \, d\gamma_c = V_L' \cos\gamma_L \, d\gamma_L$$

$$\frac{\sin\gamma_c}{\cos\gamma_c} = \frac{d\gamma_c}{d\gamma_L} \tan\gamma_L$$

$$V_L' \sin\gamma_L = V_c' \sin\gamma_c$$
$$V_L' \cos\gamma_L = V_{CM} - V_c' \cos\gamma_c$$

$$\xrightarrow{\text{divide}} \tan\gamma_L = \frac{V_c' \sin\gamma_c}{V_{CM} - V_c' \cos\gamma_c} = \frac{-\sin\gamma_c}{1+\cos\gamma_c}$$

Law of cosines

$$V_L'^2 = V_c'^2 + V_{CM}^2 - 2 V_c' V_{CM} \cos\gamma_c$$

$$\frac{V_L'^2}{V_L^2} = \frac{1}{(A+1)^2} + \frac{1}{(A+1)^2} - \frac{2}{(A+1)^2} \cos\gamma_c$$

$$\frac{V_L'}{V_L} = \frac{\sqrt{2(1-\cos\gamma_c)}}{(A+1)}$$

$$\sigma_L(\gamma_L) \sin\gamma_L \, d\gamma_L = \sigma_{CM}(\gamma_c) \sin\gamma_c \, d\gamma_c$$

$$\sigma_L(\gamma_L) = \sigma_{CM}(\gamma_c) \frac{\sin\gamma_c}{\sin\gamma_L} \frac{d\gamma_c}{d\gamma_L} = \sigma_{CM}(\gamma_c) \frac{V_L'}{V_c'} \frac{\sin\gamma_c}{\cos\gamma_c} \frac{1}{\tan\gamma_L}$$

$$= \sigma_{CM}(\gamma_c) \frac{V_L'}{V_L} (A+1) \frac{(1+\cos\gamma_c)}{\cos\gamma_c}$$

$$= \sigma_{CM}(\gamma_c) \sqrt{2(1-\cos\gamma_c)} \left(1/\cos\gamma_c + 1 \right)$$

$$\cos\theta_L = \frac{V_c' \cos\theta_c + V_{CM}}{V_L'} = \frac{V_c'}{V_L'}\cos\theta_c + \frac{V_{CM}}{V_L'}$$

$$= \left(\frac{A\cos\theta_c + 1}{A+1}\right)\frac{V_L}{V_L'}$$

$\beta = A \quad \mu = \cos\theta_c$

$$= \frac{A\cos\theta_c + 1}{A+1} \frac{A+1}{\sqrt{A^2+1+2A\cos\theta_c}}$$

$$\cos\theta_L = \frac{1+\beta\mu}{\sqrt{\beta^2+1+2\beta\mu}}$$

$$= \frac{A\cos\theta_c + 1}{\sqrt{A^2+1+2A\cos\theta_c}} =$$

$$\cos\gamma_L = \frac{V_{CM} - V_c'\cos\gamma_c}{V_L'} = \frac{V_L}{V_L'}\frac{(1-\cos\gamma_c)}{(A+1)} = \frac{(1-\cos\gamma_c)}{\sqrt{2(1-\cos\gamma_c)}}$$

$$\cos\gamma_L = \frac{1-\gamma\mu}{\sqrt{\gamma^2+1-2\gamma\mu}} \; ; \; \gamma = 1, \; \mu = \cos\gamma_c$$

$$\frac{\cos\theta_L \cdot V_L' - V_{CM}}{V_c'} = \cos\theta_c = \frac{\cos\theta_L V_L' - \frac{1}{(1+A)}V_L}{{}^{A}/_{A+1}V_L}$$

$$\cos\theta_c = \cos\theta_L \frac{V_L'}{V_L}\frac{(A+1)}{A} - \frac{1}{A}$$

Conservation of momentum. In y direction LAB

$$\frac{V_L'}{V_L} = \frac{\sin\theta_c}{\sin\theta_L}\frac{A}{A+1} \qquad V_L' \sin\theta_L = V_L \sin\gamma_L A$$

$= \cos\theta_c$

Assuming: $K(\mu, E' \to E) = P(\theta_c) \cdot P(E' \to E)$

where $P(\theta_c) = \frac{\sigma_{CM}(\theta_c)}{\sigma_s} = \frac{\sigma_s}{4\pi} \cdot \frac{1}{\sigma_s} = \frac{1}{4\pi}$ s-wave

& $P(E' \to E) = \frac{1}{(1-\alpha)E'}$ $\alpha E' \le E \le E'$

I am honestly not sure this is correct. or this

Integrate over energy & Angle $\sim$ $2\pi \int_{\alpha E_i}^{E_i} dE_f \int_{-1}^{1} d\mu \, P(\theta_c) P(E_i \to E_f)$

$$= \frac{4\pi}{4\pi} \cdot \frac{1}{(1-\alpha)E_i} \cdot E_f \Big|_{\alpha E_i}^{E_i} = 1$$

$\sigma_L(\theta_L) \sin\theta_L \, d\theta_L = \sigma_{CM} \sin\theta_c \, d\theta_c$

$\sigma_L \quad \mu_L \, d\mu_L = \sigma_c \, \mu_c \, d\mu_c$

$$\sigma_c \sqrt{\tfrac{1}{A^2} + \tfrac{2\mu_c}{A} + 1} \left(\tfrac{1}{A}\mu_c + 1\right) \cdot \frac{(A\cos\theta_c + 1)}{\sqrt{A^2 + 1 + 2\mu_c}} \, d\mu_L = \sigma_c \, \mu_c \, d\mu_c$$

$$\left(\tfrac{1}{A}\mu_c + 1\right)\left(A\mu_c + 1\right) d\mu_L = \mu_c \, d\mu_c$$

$$\mu_c^2 + A\mu_c + \tfrac{1}{A}\mu_c + 1 \; . \; d\mu_L = \mu_c \, d\mu_c$$

$$\mu_c^2 + \frac{A^2 + 1}{A}\mu_c + 1$$

Sometimes a problem in Right_Direction Subroutine when starting in the middle of an assembly

3.) $$\hat{\Omega}\cdot\nabla\psi + \Sigma_t\psi = \frac{Q}{4\pi}$$

$$\left(\Omega_x\frac{\partial}{\partial x} + \Omega_y\frac{\partial}{\partial y}\right) + \Sigma_t\psi = \frac{Q}{4\pi}$$

$$\frac{d\psi}{ds} + \Sigma_t\psi = Q/4\pi$$

$$\frac{d}{ds}\left(e^{\Sigma_t s}\psi\right) = \frac{Q}{4\pi}e^{\Sigma_t s}$$

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track 1 

$$\int_{-\infty}^{s_0} = \int_{-\infty}^{s_1} ds + \int_{s_1}^{s_0} ds$$

$$\int_{s_1}^{s_0} ds \rightarrow e^{\Sigma_t s}\psi\Big|_{s_1}^{s_0} = 0 \rightarrow e^{\Sigma_t s_0}\psi(s_0) - e^{\Sigma_t s_1}\psi(s_1) = 0$$

$$\boxed{\psi(s_0,\hat{\Omega}) = \psi(s_1,\hat{\Omega})e^{-\Sigma_t^{M}d_1}}$$

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track 2 

$$\int_{s_2}^{s_1} ds \rightarrow e^{\Sigma_t s_1}\psi(s_1) - e^{\Sigma_t s_2}\psi(s_2) = \int_{s_2}^{s_1}\frac{Q}{4\pi}e^{\Sigma_t s}$$

$$e^{\Sigma_t s_1}\psi(s_1) = e^{\Sigma_t^{F} s_2}\psi(s_2) + \frac{Q}{\Sigma_t 4\pi}\left[e^{\Sigma_t s_1} - e^{\Sigma_t s_2}\right]$$

$$\boxed{\psi(s_1) = e^{-\Sigma_t^{F}d_2}\psi(s_2,\hat{\Omega}) + \frac{Q}{\Sigma_t 4\pi}\left[1 - e^{-\Sigma_t^{F}d_2}\right]}$$

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track 3 

$$\boxed{\psi(s_2) = \psi(s_3,\hat{\Omega})e^{-\Sigma_t^{M}d_3}}$$

$$\psi(s_i,\hat{\Omega}) = \psi(s_{i+1},\hat{\Omega})e^{-\Sigma_t^{i+1}d_{i+1}} + \frac{Q_{i+1}}{\Sigma_t 4\pi}\left[1 - e^{-\Sigma_t^{i+1}d_{i+1}}\right]$$

Set $I = \frac{Q}{4\pi\Sigma_t^F}$

$$\psi(s_0,\hat{\Omega}) = e^{-\Sigma_t^F d_2 - \Sigma_t^M d_1}\,\psi(s_2,\hat{\Omega}) + I\left[e^{-\Sigma_t^M d_1} - e^{-\Sigma_t^F d_2 - \Sigma_t^M d_1}\right]$$

$$\psi(s_0,\hat{\Omega}) = e^{-\Sigma_t^F d_2 - \Sigma_t^M d_1 - \Sigma_t^M d_3}\,\psi(s_3,\hat{\Omega}) + I\left[e^{-\Sigma_t^M d_1} - e^{-\Sigma_t^F d_2 - \Sigma_t^M d_1}\right]$$

Make sure code alternates Between Moderater & fuel

and set:

$$P_i = -\sum_{j=1}^{i} \Sigma_t^j d_j$$

if Σ_t^j = odd $\Rightarrow \Sigma_t^j = \Sigma_t^M$

if Σ_t^j = even $\Rightarrow \Sigma_t^j = \Sigma_t^F$

This Makes

Always positive

$$\psi(s_0,\hat{\Omega}) = e^{P_3}\,\psi(s_3,\hat{\Omega}) + I\left[e^{P_1} - e^{P_2}\right]$$

lets Make Sure we Know our pattern: Fuel is next

$$\psi(s_0,\hat{\Omega}) = e^{P_4}\,\psi(s_4,\hat{\Omega}) + I\left[e^{P_3} - e^{P_4}\right] + I\left[e^{P_1} - e^{P_2}\right]$$

then Moderator:

$$\psi(s_0,\hat{\Omega}) = e^{P_5}\,\psi(s_5,\hat{\Omega}) + I\left[e^{P_3} - e^{P_4}\right] + I\left[e^{P_1} - e^{P_2}\right]$$

then fuel

$$\psi(s_0,\hat{\Omega}) = e^{P_6}\,\psi(s_6,\hat{\Omega}) + I\left[e^{P_5} - e^{P_6}\right] + I\left[e^{P_3} - e^{P_4}\right] + I\left[e^{P_1} - e^{P_2}\right]$$

$$\psi(s_0,\hat{\Omega}) = I\sum_{i=1}^{\infty}\left[(-1)^{i+1} e^{P_i}\right]$$

Stop when P_i is over 9,000!!!

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if we Know all our d_j ~ this is trivial

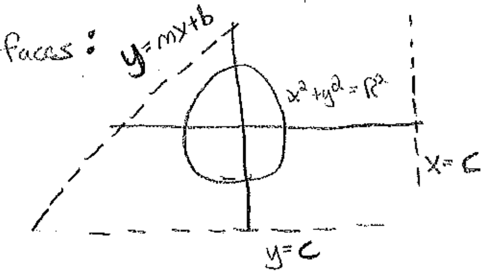
I could Be over simplifying, But I assumed Because the cylinder is infinitely tall that there will be no polar angle dependance ($\pi/2$ & $\pi/8$ are the same)

Need distances $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; Current location: (x_1, y_1)

Direction: $x' = \cos\theta$; $y' = \sin\theta$

$\hat{\Omega} = \hat{e}_x \cos\theta + \hat{e}_y \sin\theta$

Different Surfaces:



Line Eq. for direction & location

$y - y_1 = m(x - x_1)$

$m = \frac{\sin\theta}{\cos\theta}$ $\quad y = m(x - x_1) + y_1$

$x = \frac{1}{m}(y - y_1) + x_1$

if $\cos\theta = 0$ $\quad x = C = x_1$

if $\sin\theta = 0$ $\quad y = C = y_1$

Points of intersection:

Common Line: $y_L - y_{1L} = m_L(x_L - x_{1L})$

$\cos\theta \neq 0$ $\quad m_L(x - x_{1L}) + y_{1L} = m(x - x_1) + y_1$

$\cos\theta_L \neq 0$ $\quad m_L x - m x = m_L x_{1L} - m x_1 + y_1 - y_{1L}$

$m_L \neq m$ $\quad \boxed{x_2 = \frac{m_L x_{1L} - m x_1 + y_1 - y_{1L}}{m_L - m}} \Rightarrow \boxed{y_2 = m(x_2 - x_1) + y_1}$

$\cos\theta = 0$ $\quad \boxed{x_2 = x_1}$

$\cos\theta_L \neq 0$ $\quad \boxed{y_2 = m_L(x_2 - x_{1L}) + y_{1L}}$

$\boxed{\text{if } \sin\theta = 0 \Rightarrow m = 0 \;\; \& \;\; \sin\theta_L = 0 \Rightarrow m_L = 0}$

if $\cos\theta = 0$ $\quad$ Parallel no intercept or on top of each other

& $\cos\theta_L = 0$ $\quad d = 9000$

if $m_L = m$ $\quad$ Same as above

$\cos\theta \neq 0$ $\quad \boxed{x_2 = x_{1L} \Rightarrow y_2 = m(x_2 - x_1) + y_1}$

$\cos\theta_L = 0$

for a circle: $y = \pm\sqrt{R^2 - x^2}$

$x = \pm\sqrt{R^2 - y^2}$ $\quad$ if $\cos\theta = 0$

if $\cos\theta = 0$ $\quad x_2 = x_1 \Rightarrow y_2 = \pm\sqrt{R^2 - x_2^2}$ (if real)

if $\sin\theta = 0$ $\quad y_2 = y_1 \Rightarrow x_2 = \pm\sqrt{R^2 - y_2^2}$ (if real)

otherwise $\quad m(x_2 - x_1) = \pm\sqrt{R^2 - x_2^2}$ $\qquad x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$

$m^2(x_2^2 - 2x_2 x_1 + x_1^2) + x_2^2 - R^2 = 0$

$x_2^2(1 + m^2) + x_2(2x_1 m^2) + x_1^2 m^2 - R^2 = 0$ $\qquad y = \pm\sqrt{R^2 - x^2}$

How do we know we are going in the right direction?

which side?

$$\hat{n} \cdot \langle x_2 - x_1 , y_2 - y_1 \rangle > 0$$

Multiple points on a circle?

take the shorter of the two that are heading in the right direction, and are real (not imaginary)

if Both points are real & in the right direction, you could skip the next irrative step

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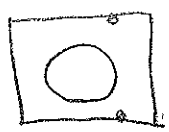
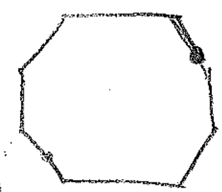
translation at Boundaries?

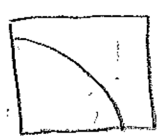
can either do



translation with the same angle



or reflection 90°

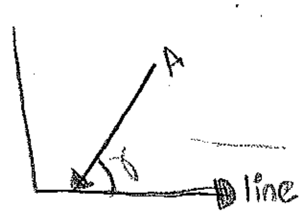


and

keep the same position.

I think reflection would be easier because otherwise you need to hard code some geometric expectations.

With reflection you mirror every time you encounter a line



$A \cdot B = \|A\| \cdot \|B\| \cos\gamma$

Actually reflection is probably wrong. I'm doing translation.

$y - y_v = m_v(x - x_v)$ $\qquad$ $x^2 + y^2 = R^2$

$y = m_v(x - x_v) + y_v \rightarrow (1)$

$x^2 + [m_v(x - x_v) + y_v]^2 = R^2$

$x^2 + [m_v x - m_v x_v + y_v][m_v x - m_v x_v + y_v] = R^2$

$x^2 + m_v^2 x^2 - m_v^2 x_v x + m_v x y_v - m_v^2 x_v x + m_v^2 x_v^2 - m_v x_v y_v$

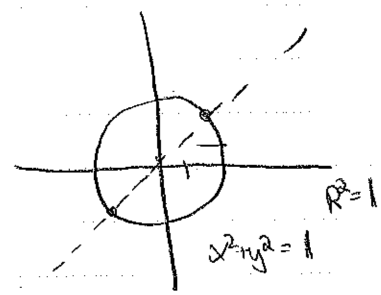
$y_v m_v x - y_v m_v x_v + y_v^2 = R^2$

$x^2[1 + m_v^2] + x[-m_v^2 x_v + m_v y_v - m_v^2 x_v + y_v m_v] + m_v^2 x_v^2 - m_v x_v y_v - m_v x_v y_v + y_v^2 = R^2$

$a = 1 + m_v^2$

$b = -2m_v^2 x_v + 2m_v y_v$

$c = m_v^2 x_v^2 + y_v^2 - 2m_v x_v y_v - R^2$



$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$y = m_v(x_{1,2} - x_v) + y_v$

$y = x$

$y - y_v = m_v(x - x_v)$

$y_v = 0$

$m_v = 1 \quad x_v = 0$

expect points

$(1,1)$ & $(-1,-1)$

$m_v = 1$

$R = 0.41$

$x_v = 0.63$

$y^2[1]$

How do we know we are going in the right direction?

unchecked $\hat{n} \cdot \langle x_2 - x_1, y_2 - y_1 \rangle > 0$

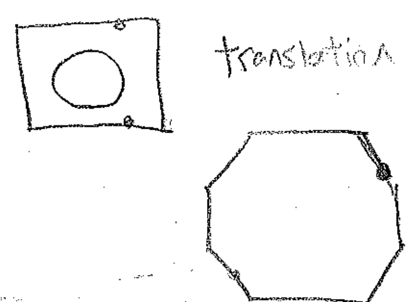
Multiple points on a circle?

take the shorter of the two that are heading in the right direction, and are real (not imaginary)

if Both points are real & in the right direction, you could skip the next iterative step

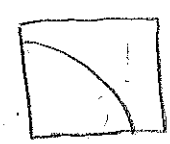
translation at Boundaries?

can either do

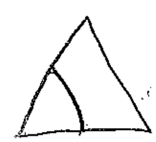


translation with the same angle

or reflection 90°



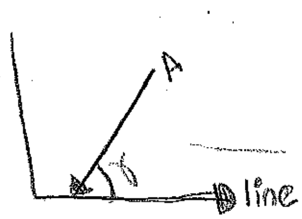
and



keep the same position.

I think reflection would be easier because otherwise you need to hard code some geometric expectations.

With reflection you mirror every time you encounter a line



$A \cdot B = \|A\| \cdot \|B\| \cos\gamma$

Actually reflection is probably wrong. I'm doing translation.