NUEN 629
Homework 2
Due: 10/06/15
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1.) Compute three group cross-sections for a homogeneous mixture of graphite and natural uranium where the ratio of graphite to uranium is 150:1. You can assume the Watt-fission spectrum, and that the groups bounds are { 0, 1ev, 100 keV, 20 MeV }.

Group cross sections are defined by:

$$\Sigma_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} dE\ \Sigma(\vec{r},E)\phi(\vec{r},E)}{\int_{E_g}^{E_{g-1}} dE\ \phi(\vec{r},E)}$$

Assuming a homogeneous medium with the uranium to graphite ratio described above the group cross section can be defined as:

$$\Sigma_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} dE(at\%_{12}N^{mix}\sigma_{12}(E) + at\%_{235}N^{mix}\sigma_{235}(E) + at\%_{238}N^{mix}\sigma_{238}(E))\phi(\vec{r},E)}{\int_{E_g}^{E_{g-1}} dE\ \phi(\vec{r},E)}$$

Where $at\%_{12} = 0.993377;\ at\%_{238} = 0.00657;\ at\%_{235} = 4.768e-5$, and $N^{mix} = 0.113e24\ at/cc$. Rearranging,

$$\Sigma_g(\vec{r}) = N^{mix}\left[\frac{at\%_{12}\int_{E_g}^{E_{g-1}} dE\sigma_{12}(E)\phi(\vec{r},E)}{\int_{E_g}^{E_{g-1}} dE\ \phi(\vec{r},E)} + \frac{at\%_{238}\int_{E_g}^{E_{g-1}} dE\sigma_{238}(E)\phi(\vec{r},E)}{\int_{E_g}^{E_{g-1}} dE\ \phi(\vec{r},E)}\right.$$
$$\left. + \frac{at\%_{235}\int_{E_g}^{E_{g-1}} dE\sigma_{235}(E)\phi(\vec{r},E)}{\int_{E_g}^{E_{g-1}} dE\ \phi(\vec{r},E)}\right]$$

Using Short hand:

$$\Sigma_g(\vec{r}) = N^{mix}\left[at\%_{12}\sigma_{g,12} + at\%_{235}\sigma_{g,235} + at\%_{238}\sigma_{g,238}\right]$$

Janus was used to produce these group averaged cross sections assuming a fission spectrum and are shown in Table *1*. Group 1 is the lowest energy group.

Table 1 Three group cross sections for isotopes in the system

	σt (b)	σs (b)	σf (b)	σγ (b)
235G1	131.5515	13.77506	101.2921	16.4844
235G2	13.1442	10.52882	1.89372	0.591047
235G3	7.551755	4.243951	1.215399	0.087956
238G1	9.890571	9.196642		0.693925
238G2	13.09678	12.55621		0.327781
238G3	7.706308	4.732475		0.066865
12G1	4.75749	4.756678		
12G2	5.539658	4.539656		
12G3	2.353363	2.339561		

Three groups cross sections were calculated with the scheme described above and are shown in Table *2*.

Table 2 Three group cross sections for the system

	Σt (1/cm)	Σs (1/cm)	Σf (1/cm)	Σγ (1/cm)
Group 1	0.542088	0.540847	0.000546	0.000604
Group 2	0.631629	0.518962	1.02E-05	0.000247
Group 3	0.269931	0.266156	6.55E-06	5.01E-05

The required text file input for Janis is as follows (energies are in eV):

```
neutron group structure......3 group
1 1e-5 1
2 1 100000
3 100000 2e7
```

A quick MCNP calculation shows flux distributions for the system described and the system without graphite (the fission spectrum is plotted on the same graph for comparison):

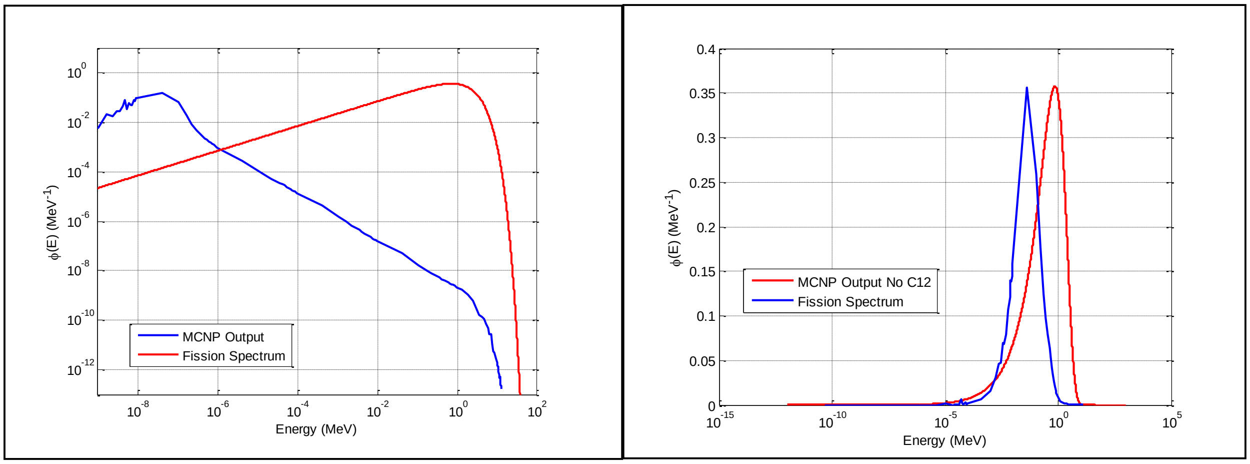


Figure 1. Comparison between MCNP flux distributions and the fission spectrum.

This shows that the Watt-fission spectrum is not the spectrum that this system would have. It should be noted that the MCNP spectrum could be used for this analysis, but JANIS was used instead for speed. Dr. McClarren's code could also be used, and is described below:

Following the example provided in laboratory 1; starting with the steady-state infinite medium problem given by:

$$\left[\sum_{i=1}^{3} at\%_i * N^{mix}\sigma_t^i(E)\right]\psi(\mu,\mathrm{E})$$
$$= \frac{1}{2}\int_0^\infty dE'\left[\sum_{i=1}^{3} at\%_i * N^{mix}\sigma_s^i\left(E_i \rightarrow E_f\right)\right]\phi(E')$$
$$+ \frac{\chi(E)}{2k}\int_0^\infty dE'\left[\sum_{i=1}^{3} at\%_i * N^{mix}\nu_f\sigma_f^i(E')\right]\phi(E')$$

Where $at\%_{12} = 0.993377$; $at\%_{238} = 0.00657$; $at\%_{235} = 4.768e-5$. The average downscattering energy exchange for a neutron is assumed to be:

$$\left(\frac{E_f}{E_i}\right)_{av} = \frac{A^2+1}{(A+1)^2}$$

where for the different isotopes:

$$\left(\frac{E_f}{E_i}\right)_{av,12} \approx 0.857988;\ \left(\frac{E_f}{E_i}\right)_{av,238} \approx 0.991667;\ \left(\frac{E_f}{E_i}\right)_{av,235} \approx 0.99156133.$$

If it is assumed that downscattering is the only type of scattering then the transport equation, integrated over energy (with a normalized fission term) can be written as:

$$\phi(E) = \frac{\sum_{i=1}^{3} at\%_i * \sigma_s^i\left(\left(\frac{E_i}{E_f}\right)_{av,i} E\right)\phi\left(\left(\frac{E_i}{E_f}\right)_{av,i} E\right)}{\left[\sum_{i=1}^{3} at\%_i * \sigma_t^i(E)\right]} + \frac{\chi(E)}{\left[\sum_{i=1}^{3} at\%_i * \sigma_t^i(E)\right]}$$

The cross section data was processed and reproduced below (apologizes for the readability of the graphs):

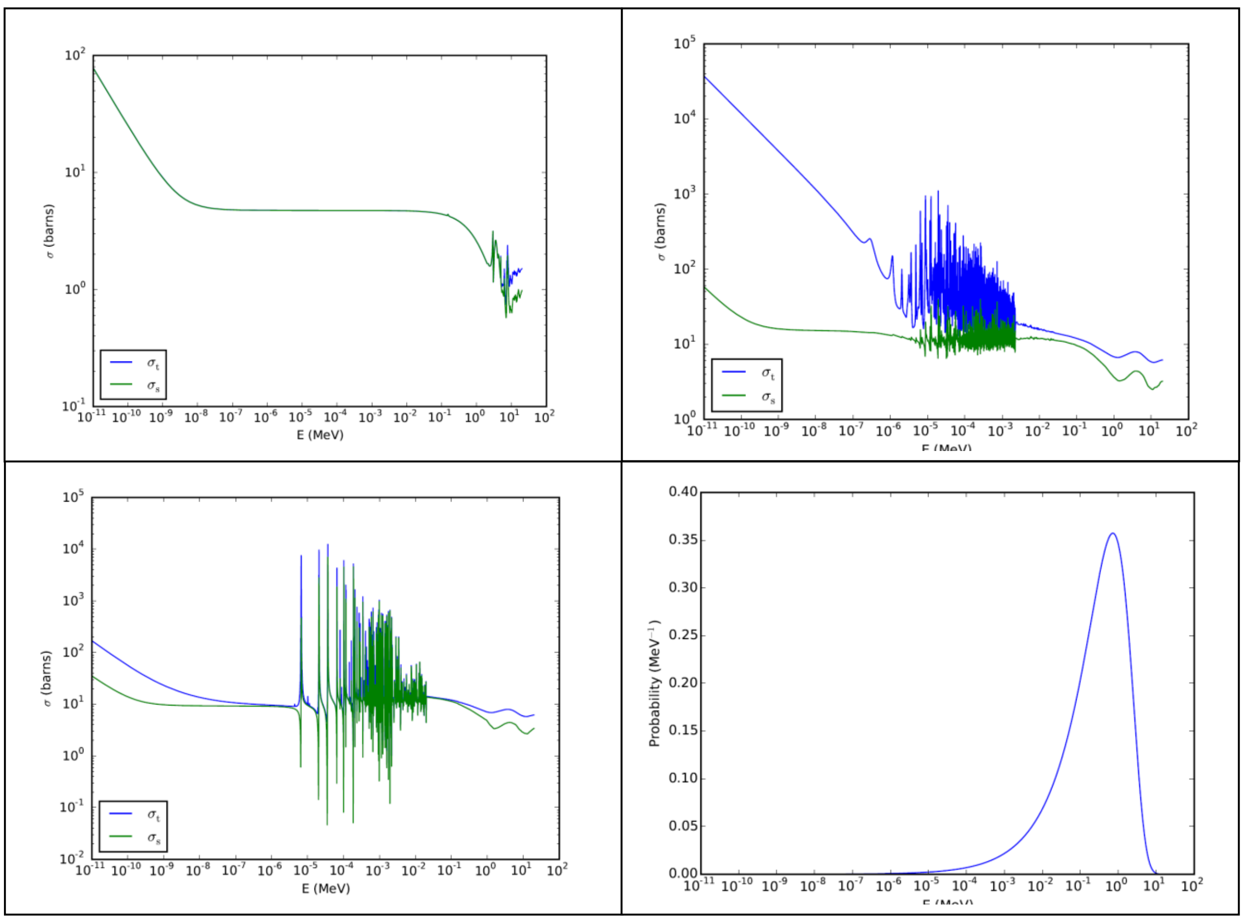


Figure 2: Cross Section data and the fission spectrum for the isotopes in the system.

The spectrum that was used for the averaging:

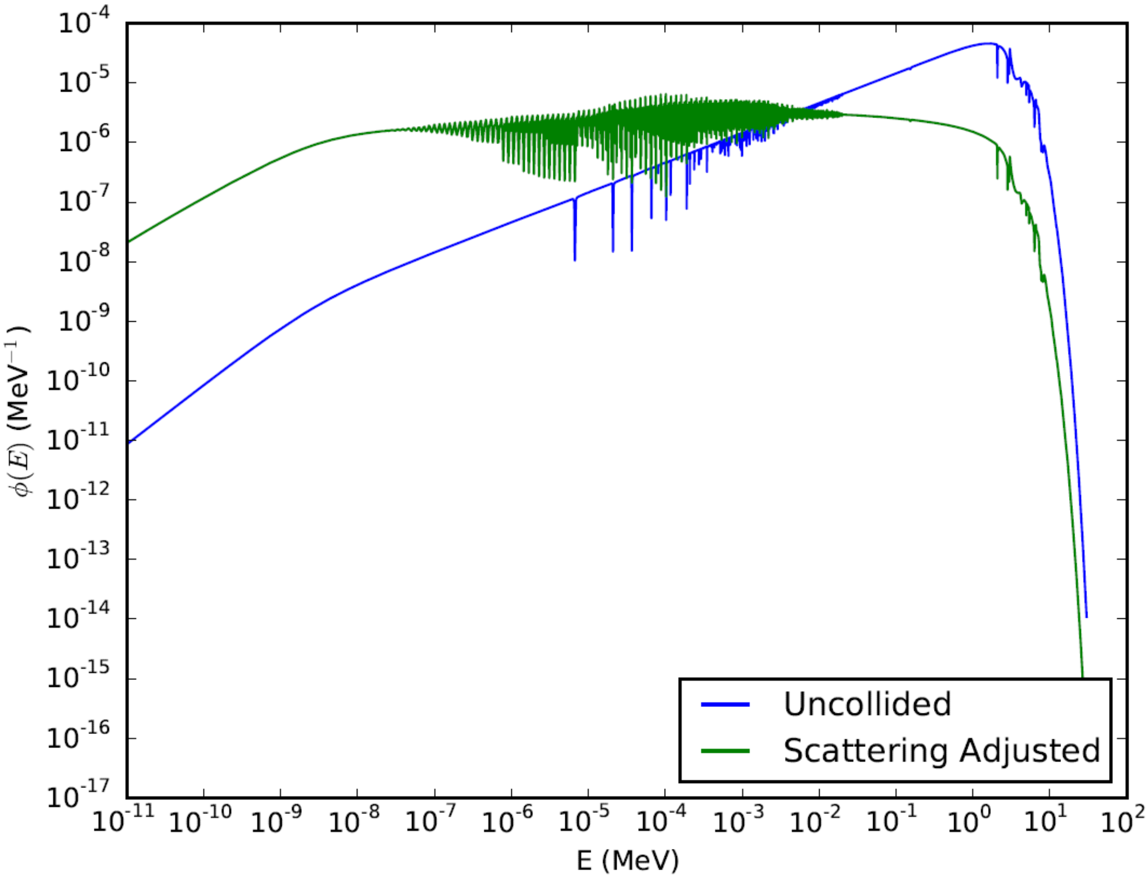


Figure 3. Flux spectrum used for cross section averaging.

The code used for the cross section processing was mostly copied from Dr. McClarren the integration portion I wrote is provided below:

```
#Perform Integration
from scipy import integrate
from scipy.integrate import trapz
#Make Functions of things I want to integrate.
X_t_235_phi=interpolate.interp1d(energies,phi_iteration(energies)*sig_t_235_interp(energies),fill_value=0,bounds_error=False)
#Perform the integration For Group 1
EE=1e-6
for xx in range(0, len(energies)):
   if (energies[xx]<=EE):
      index=xx
phi_int_g1=integrate.trapz(phi_iteration(energies[0:index]),energies[0:index])
X_t_235_g1=integrate.trapz(X_t_235_phi(energies[0:index]),energies[0:index])
```

Cross sections are shown on the next page:

Comparison of cross section data calculated from Janis and the data calculated from the python script is shown in Table *3*. The values are similar but different because different spectrum were used.

Table 3 Cross Section Comparison and results from python script

	Python		Janis	
	σt (b)	σs (b)	σt (b)	σs (b)
235G1	179.76	13.98	131.55	13.78
235G2	13.98	10.81	13.14	10.53
235G3	7.78	4.49	7.55	4.24
238G1	10.06	9.21	9.89	9.20
238G2	13.67	13.08	13.10	12.56
238G3	7.95	5.24	7.71	4.73
12G1	4.77	4.77	4.76	4.76
12G2	4.58	4.58	5.54	4.54
12G3	2.59	2.58	2.35	2.34
	Σt (1/cm)	Σs (1/cm)	Σt (1/cm)	Σs (1/cm)
Group 1	0.544	0.543	0.542	0.541
Group 2	0.524	0.524	0.632	0.519
Group 3	0.296	0.294	0.270	0.266

2. Microscopic cross-sections for ^{1}H in units of barns for 5 groups were provided from the code NJOY. Given an infinite tank of high-pressure hydrogen, 30 atm, encloses a bare sphere of ^{235}U. Compute the scalar flux φ_g and the current J_g in the hydrogen using the separable, P1 equivalent, and extended Legendre approximations. Compare your solutions graphically.

The neutron transport equation for an infinite medium can be written as:

$$\Sigma_t \Psi = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \int_0^\infty dE' \Sigma_s K_\ell (E' \to E) \phi_\ell + \chi$$

Where:

$$\phi_\ell = \int_{-1}^{1} d\mu \Psi(\mu) \mathrm{P}_\ell(\mu) \qquad \Psi = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \phi_\ell P_\ell$$

Integrating over an energy bin:

$$0 = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G} \left(\Sigma_{s\ell g' \to g} - \delta_{g'g} \Sigma_{t\ell g'} \right) \phi_{\ell g'} + \chi_g$$

Where:

$$\chi_g = \int_{E_g}^{E_{g-1}} dE \, \chi(E) \qquad \sum_{g=1}^{G} \chi_g = 1$$

$$\phi_{\ell g} = \int_{E_g}^{E_{g-1}} dE \, \phi_\ell \qquad \Sigma_{t\ell g} = \frac{1}{\phi_{\ell g}} \int_{E_g}^{E_{g-1}} dE \, \Sigma_t \phi_\ell$$

$$\Sigma_{s\ell g' \to g} = \frac{1}{\phi_{\ell g}} \int_{E_g}^{E_{g-1}} dE \int_{E_{g'}}^{E_{g'-1}} dE' \, \Sigma_s(E') K_\ell (E' \to E) \phi_\ell$$

Adding a term to each side:

$$\Sigma_{t^*g} \Psi_g = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G} \left(\Sigma_{s\ell g' \to g} + \delta_{g'g} \left(\Sigma_{t^*g'} - \Sigma_{t\ell g'} \right) \right) \phi_{\ell g'} + \chi_g$$

Where:

P1 approximation:

$$\Sigma_{t^*g} = \Sigma_{t,0,g}$$

Extended transport approximation:

$$\Sigma_{t^*g} = \Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g \to g'}$$

Noting (from class):

$$\Sigma_{t,0,g} = \frac{1}{\phi_g} \int_{E_g}^{E_{g-1}} dE \, \Sigma_t \phi = \Sigma_{tg}$$

P1 approximation solution:

$$\Sigma_{t,0,g}\Psi_{\mathrm{g}} = \sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}P_\ell\sum_{g'=1}^{G}\left(\Sigma_{s\ell g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t\ell g'}\right)\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Note:

$$\int_{-1}^{1}P_\ell(\mu)d\mu = \int_{-1}^{1}P_0P_\ell(\mu)d\mu = \frac{2}{2\ell+1}\delta_{0\ell}$$

Flux: integrate over μ $\ell = 0$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t0g'}\right)\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g}\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t,0,g}\phi_{\mathrm{g}} - \sum_{g'=1}^{\infty}\left(\Sigma_{s0g'\to g}\right)\phi_{0g'} = \chi_{\mathrm{g}}$$

Expanding into a matrix:

$$\begin{bmatrix} \Sigma_{t1} - \Sigma_{s,1\to 1} & \Sigma_{s,2\to 1} & \Sigma_{s,3\to 1} & \Sigma_{s,4\to 1} & \Sigma_{s,5\to 1} \\ \Sigma_{s,1\to 2} & \Sigma_{t2} - \Sigma_{s,2\to 2} & \Sigma_{s,3\to 2} & \Sigma_{s,4\to 2} & \Sigma_{s,5\to 2} \\ \Sigma_{s,1\to 3} & \Sigma_{s,2\to 3} & \Sigma_{t3} - \Sigma_{s,3\to 3} & \Sigma_{s,4\to 3} & \Sigma_{s,5\to 3} \\ \Sigma_{s,1\to 4} & \Sigma_{s,2\to 4} & \Sigma_{s,3\to 4} & \Sigma_{t4} - \Sigma_{s,4\to 4} & \Sigma_{s,5\to 4} \\ \Sigma_{s,1\to 5} & \Sigma_{s,2\to 5} & \Sigma_{s,3\to 5} & \Sigma_{s,4\to 5} & \Sigma_{t5} - \Sigma_{s,5\to 5} \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\end{bmatrix} = \begin{bmatrix}\chi_1\\ \chi_2\\ \chi_3\\ \chi_4\\ \chi_5\end{bmatrix}$$

Solved with a matrix inversion.

Current: multiply by μ and integrate ($\ell = 1$ for the same reason $\ell = 0$ above):

$$\Sigma_{t,0,g}\mathrm{J}_{\mathrm{g}} = \sum_{g'=1}^{\infty}\left(\Sigma_{s1g'\to g} + \delta_{g'g}\left(\Sigma_{t,0,g'} - \Sigma_{t1g'}\right)\right)J_{g'} + 0$$

With a non zero matrix:

$$\mathrm{J}_1 = J_2 = J_3 = J_4 = J_5 = 0$$

Extended transport approximation:

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\Psi_{\mathrm{g}}$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_\ell \sum_{g'=1}^{G}\left(\Sigma_{s\ell g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G}\Sigma_{s,L+1,g\to g'} - \Sigma_{t\ell g'}\right)\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Flux integrate over μ:$\ell = 0$

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\phi_g = \sum_{g'=1}^{G}\left(\Sigma_{s0g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G}\Sigma_{s,L+1,g\to g'} - \Sigma_{t0g'}\right)\right)\phi_{0g'} + \chi_{\mathrm{g}}$$

When g=g' it cancels?

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\phi_g - \Sigma_{t,L+1,g}\phi_{0g} + \sum_{g'=1}^{G}\Sigma_{s,L+1,g\to g'}\,\phi_{0g} + \Sigma_{t0g'}\phi_{0g'} = \sum_{g'=1}^{G}\left(\Sigma_{s0g'\to g}\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

$$\Sigma_{t0g'}\phi_{0g'} = \sum_{g'=1}^{G}\left(\Sigma_{s0g'\to g}\right)\phi_{\ell g'} + \chi_{\mathrm{g}}$$

Which is the same as above.

Current multiply by μ and integrate over μ: $\ell = 1$

$$\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G} \Sigma_{s,L+1,g\to g'}\right)\mathrm{J_g} = \sum_{g'=1}^{G}\left(\Sigma_{s1g'\to g} + \delta_{g'g}\left(\Sigma_{t,L+1,g} - \sum_{g'=1}^{G}\Sigma_{s,L+1,g\to g'} - \Sigma_{t1g'}\right)\right)J_{1g} + 0$$

This does not exactly cancel as before, but because the author is tired he will assume the solution is the same as for P1 and:

$$\mathrm{J}_1 = J_2 = J_3 = J_4 = J_5 = 0$$

Filling out the matrix:

$$N\begin{bmatrix} 4.6-2.91518 & 0 & 0 & 0 & 0 \\ 1.67039 & 18.3-14.0197 & 0 & 0 & 0 \\ 0.0167099 & 4.23255 & 20.4-16.0183 & 0 & 0 \\ 0.000166937 & 0.0423257 & 4.35086 & 22.7-4.53876 & 1.53831 \\ 1.04698e-6 & 0.000424355 & 0.0439445 & 17.9734 & 29.8-27.93 \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\end{bmatrix}=\begin{bmatrix}0.99136\\ 0.01379\\ 0\\ 0\\ 0\end{bmatrix}$$

Group boundaries in eV:

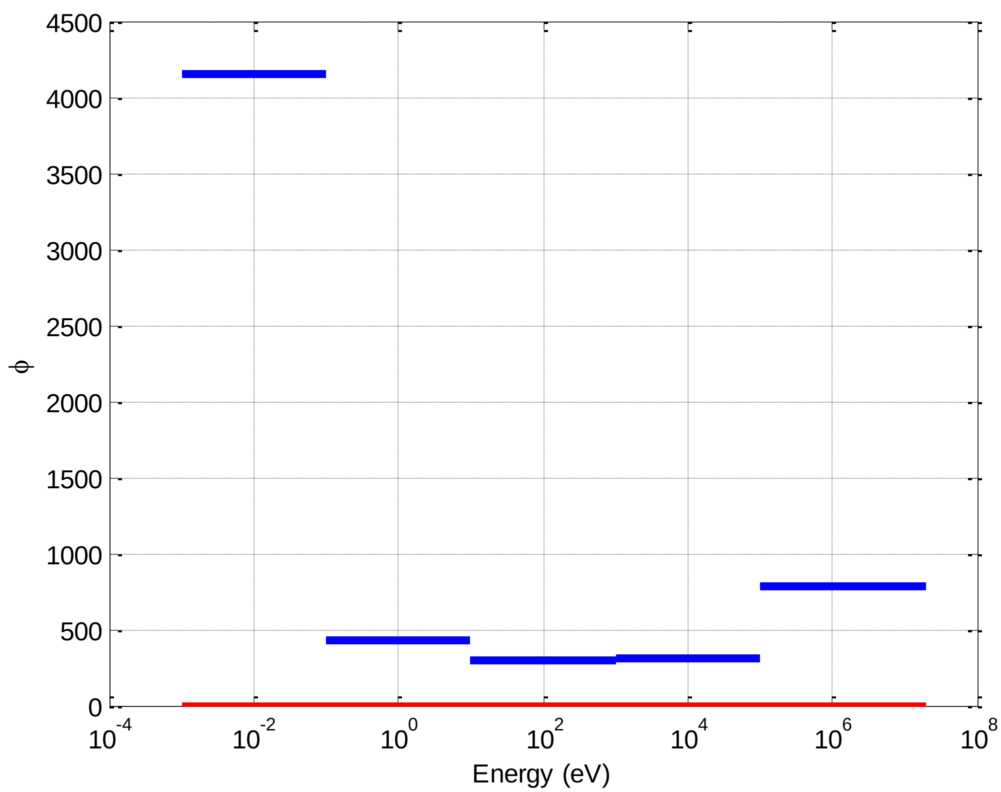
2e+07 100000 1000 10 0.1 0.001

Assuming ideal gas law

$$\frac{n}{V}=\frac{P}{RT}=3039750Pa*\frac{k*mol}{8.3144598m^3Pa}*\frac{1}{293K}=\frac{1247.77mol}{m^3}$$

$$\frac{1247.77mol}{m^3}*\frac{1m^3}{100^3cm^3}*\frac{0.6022E24atoms}{mol}=0.000751E24\frac{atoms}{cm^3}$$

Inverse solve:

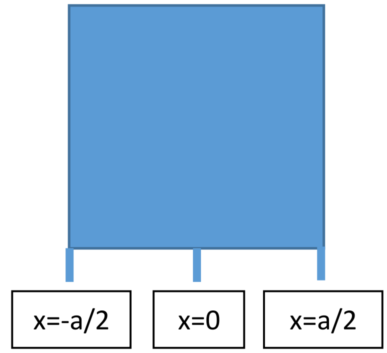


flux = 782.35 309.91 301 425.19 4148.2

Script for plotting:

```
A =[4.60248-2.91518   0             0             0             0
    -1.67039      18.2957-14.0197   0             0             0
    -0.0167099    -4.23255    20.4196-16.0183     0             0
    -0.00166937   -0.0423257     -4.35086     22.6605-4.53876 -1.53831
    -1.04698e-6   -0.000424355   -0.0439445     -17.9734     29.77991-
27.9344];



A=A*0.000751;

b=[0.99136;0.01379;0;0;0];

flux=A^-1*b

x1=logspace(5,7.3011,30);
x2=logspace(3,5,30);
x3=logspace(1,3,30);
x4=logspace(-1,1,30);
x5=logspace(-3,-1,30);

semilogx(x1,flux(1,1)*ones(1,30),'b','LineWidth',3);
hold on; grid on;xlabel 'Energy (eV)';ylabel '\phi';
semilogx(x2,flux(2,1)*ones(1,30),'b','LineWidth',3);
semilogx(x3,flux(3,1)*ones(1,30),'b','LineWidth',3);
semilogx(x4,flux(4,1)*ones(1,30),'b','LineWidth',3);
semilogx(x5,flux(5,1)*ones(1,30),'b','LineWidth',3);

%Below Zero?
semilogx(x1,zeros(1,30),'r','LineWidth',3);
hold on; grid on;xlabel 'Energy (eV)';ylabel '\phi';
semilogx(x2,zeros(1,30),'r','LineWidth',3);
semilogx(x3,zeros(1,30),'r','LineWidth',3);
semilogx(x4,zeros(1,30),'r','LineWidth',3);
semilogx(x5,zeros(1,30),'r','LineWidth',3);
```

3. Find the solution to the diffusion equation for 1-group, slab geometry, where the material is a pure scatter and the slab width is X:



The one group, slab geometry diffusion equation for slab geometry is:

$$\frac{d^2\phi}{dx^2} = -\frac{q}{D}$$

Has a solution of the form:

$$\phi = -\frac{q}{D}x^2 + C_1 x + C_2$$

General boundary condition:

$$A\phi\left(\frac{a}{2}\right) + BD\left|\frac{\delta\phi}{\delta x}\right|_{\frac{a}{2}} = C; \qquad A\phi\left(-\frac{a}{2}\right) - BD\left|\frac{\delta\phi}{\delta x}\right|_{-\frac{a}{2}} = C$$

Where:

$$\phi\left(\frac{a}{2}\right) = -\frac{q}{D}\frac{a^2}{4} + C_1\frac{a}{2} + C_2; \phi\left(-\frac{a}{2}\right) = -\frac{q}{D}\frac{a^2}{4} - C_1\frac{a}{2} + C_2$$

And:

$$\left|\frac{\delta\phi}{\delta x}\right|_{\frac{a}{2}} = -\frac{aq}{D} + C_1; \left|\frac{\delta\phi}{\delta x}\right|_{-\frac{a}{2}} = \frac{qa}{D} + C_1$$

Applied at right side:

$$A_R\left(-\frac{q}{D}\frac{a^2}{4} + C_1\frac{a}{2} + C_2\right) + B_R D\left(-\frac{qa}{D} + C_1\right) = C_R$$

$$C_2 = \frac{C_R}{A_R} + \left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1\left(\frac{a}{2} + \frac{B_R D}{A_R}\right)$$

Applied at left side:

$$A_L\left(-\frac{q}{D}\frac{a^2}{4} - C_1\frac{a}{2} + C_2\right) - B_L D(\frac{qa}{D} + C_1) = C_L$$

$$-\frac{A_L q}{D}\frac{a^2}{4} - C_1\frac{A_L a}{2} + A_L C_2 - B_L qa - C_1 B_L D = C_L$$

Plug in C_2

$$-\frac{A_L q}{D}\frac{a^2}{4} - C_1\frac{A_L a}{2} + A_L\frac{C_R}{A_R} + A_L\left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1 A_L\left(\frac{a}{2} + \frac{B_R D}{A_R}\right) - B_L qa - C_1 B_L D = C_L$$

Solve for C_1

$$\frac{A_L C_R}{A_R} + \left(\frac{(A_L qaB_R)}{A_R}\right) - B_L qa - C_L = C_1\left(aA_L + \frac{A_L B_R D}{A_R} + B_L D\right)$$

$$\frac{A_L}{A_R}(C_R + qaB_R) - B_L qa - C_L = C_1\left(aA_L + \frac{A_L B_R D}{A_R} + B_L D\right)$$

$$C_1 = \frac{\left(\frac{A_L}{A_R}(C_R + qaB_R) - B_L qa - C_L\right)}{aA_L + \frac{A_L B_R D}{A_R} + B_L D}$$

$$C_2 = \frac{C_R}{A_R} + \left(\frac{(qaB_R)}{A_R} + \frac{q}{D}\frac{a^2}{4}\right) - C_1\left(\frac{a}{2} + \frac{B_R D}{A_R}\right)$$

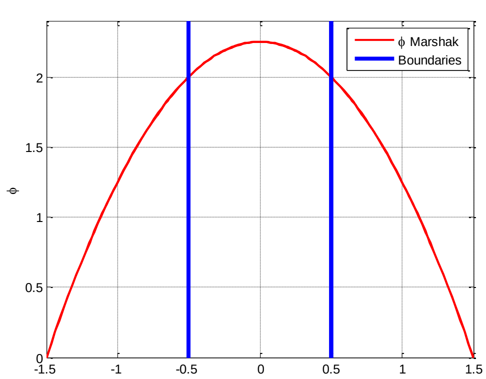
Using the provided constants:

	A	B	C
Vacuum Marshak	0.25	0.5	0
Vacuum Mark	0.5	0.866025	0
Vacuum Dirichlet	1	0	G
Reflecting	0	1	0
Albedo	(1-α)/(2(1+α))	1	0

1. Vacuum Marshak Conditions: $A_R = 0.25 = A_L; B_R = 0.5 = B_L; C_R = 0 = C_L$ (plots assumed $D = 1; a = 1; q = 1$)

$$C_1 = 0$$

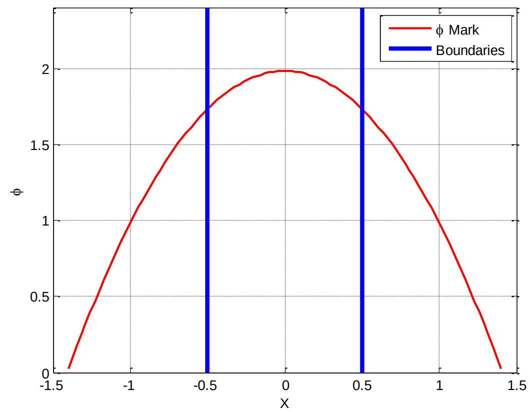
$$C_2 = \left(2qa + \frac{q}{D}\frac{a^2}{4}\right)$$



2. Vacuum Mark Conditions:

$$C_1 = 0$$

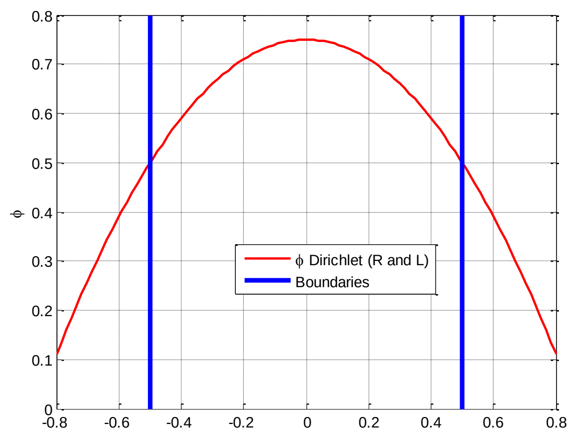
$$C_2 = qa\sqrt{3} + \frac{q}{D}\frac{a^2}{4}$$



3. Vacuum Dirichlet Conditions
    a. C=0.5

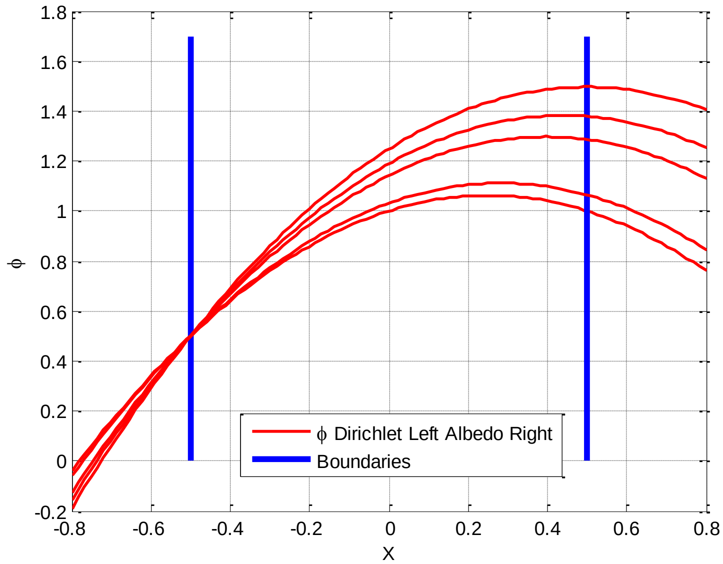
$$C_1 = 0$$

$$C_2 = 0.5 + \left(\frac{q}{D}\frac{a^2}{4}\right)$$

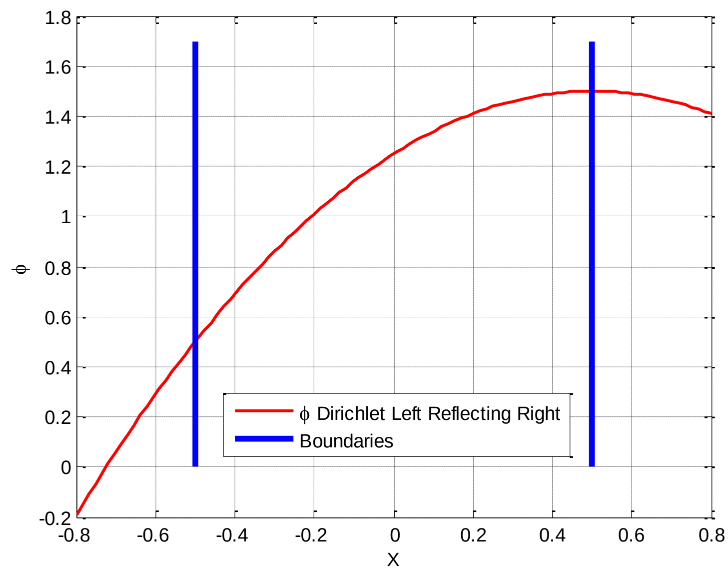


4. Vacuum Dirichlet conditions on the left and albedo on the right

CL=0.5. Albedo varied from 0 to 1



5. Vacuum Dirichlet condition on the left and reflecting on the right



The albedo condition can represent either a reflecting or vacuum condition. Extrapolation distances were consistent with expectations.

Code for plotting reproduced below:

```
D=1;a=1;q=1;alpha=0.999;
%Boundary Conditions:
CL=0.5;CR=0;BL=0;BR=1;AL=1;AR=(1-alpha)/(2*(1+alpha));

%Plot Solution
C1=((AL/AR)*(CR+q*a*BR)-BL*q*a-CL)/(a*AL+(AL*BR*D)/AR+BL*D);
C2=CR/AR+((q*a*BR)/AR+(q*(a^2))/(D*4))-C1*(a/2+(BR*D)/AR);

x=linspace(-0.8,0.8);
phi=(-q./D).*(x.^2)+C1.*(x)+C2;
plot(x,phi,'r','LineWidth',2);

%Plot Bars
hold on
y=linspace(0,1.7);
xn=ones(1,100).*-0.5;xp=ones(1,100).*0.5;
plot(xn,y,'b','LineWidth',3);
plot(xp,y,'b','LineWidth',3);
grid on;xlabel 'X';ylabel '\phi';
legend ('\phi Dirichlet Left Reflecting Right','Boundaries');
ylim([0,0.8]);
```