NUEN 629, Homework 1

Due Date Sept. 17

1 Ayzman

(10 points) Henyey and Greenstein (1941) introduced a function which, by the variation of one parameter, $-1 \leq h \leq 1$, ranges from backscattering through isotropic scattering to forward scattering. In our notation we can write this as

$$K(\mu_0, v' \to v) = \frac{1}{2} \frac{1-h^2}{(1+h^2-2h\mu_0)^{3/2}} \delta(v'-v).$$

Verify that this is a properly normalized $f(\mu_0)$ and compute $K_l(v' \to v)$ for $l = 0, 1, 2$ as a function of g.

2 Bolding

(20 points) In an elastic scatter between a neutron and a nucleus, the scattering angle in the center of mass system is related to the energy change as

$$\frac{E}{E'} = \frac{1}{2}\left((1+\alpha)+(1-\alpha)\cos\theta_{\mathrm{c}}\right),$$

where E is the energy after scattering and E' is the initial energy of the neutron and

$$\alpha = \frac{(A-1)^2}{(A+1)^2}.$$

The scattered angle in the center-of-mass system is related the lab-frame scattered angle as

$$\tan\theta_{\mathrm{L}} = \frac{\sin\theta_{\mathrm{c}}}{A^{-1}+\cos\theta_{\mathrm{c}}}.$$

Also, the distribution of scattered energy is given by

$$P(E' \to E) = \begin{cases} \frac{1}{(1-\alpha)E'}, & \alpha E' \leq E \leq E' \\ 0 & \text{otherwise} \end{cases}.$$

Derive an expression for $K(\mu_0, E' \to E)$, where μ_0 is $\cos\theta_{\mathrm{L}}$. What is the distribution in angle of neutrons of energy in the range $[0.05\,\mathrm{MeV}, 10\,\mathrm{MeV}]$ to energies below 1 eV if the scatter is hydrogen?

3

(70 points) Consider an infinite square lattice of infinitely tall cylindrical UO2 fuel pins in water. A quarter of a pin cell looks for a square lattice is shown in Fig. 1 and an infinite hex lattice in Fig. 2. The cross-section data for each is given in Table 1. The neutron transport equation for this problem is given simply by

$$\hat{\Omega}\cdot\nabla\psi(x,y,\hat{\Omega}) + \Sigma_{\mathrm{t}}\psi(x,y,\hat{\Omega}) = \frac{1}{4\pi}\Sigma_{\mathrm{s}}\phi(x,y) + \frac{Q}{4\pi}.$$

You may choose whichever lattice you wish – square or hex. For one or the other, perform the following:

Table 1: Data for Test Problem

	Fuel	Moderator
Σ_{t} (cm^{-1})	0.1414	0.08
Σ_{s} (cm^{-1})	0	0
Q ($\mathrm{n/cm^3{\cdot}s}$)	1	0

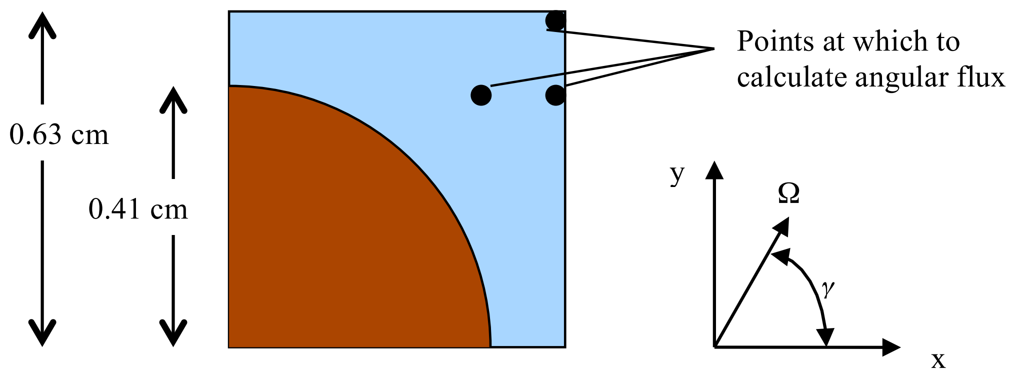

Figure 1: Quarter of a pin cell of infinite square lattice problem.

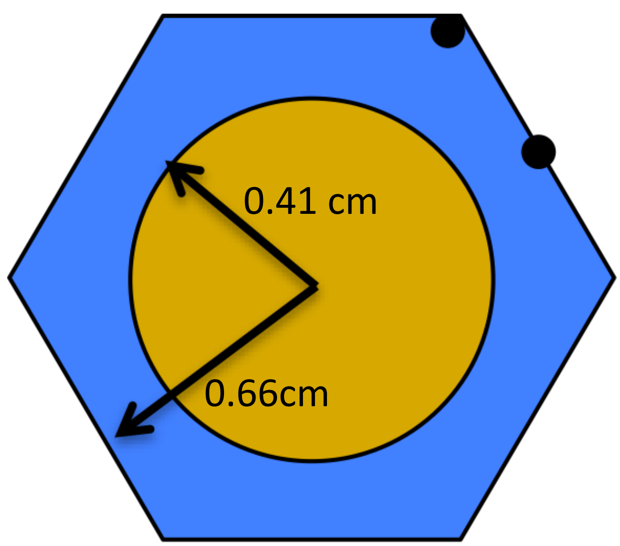

Figure 2: Pin cell of infinite hex lattice problem.

1. Calculate the angular flux as a function of the azimuthal angle, φ, at the spatial points indicated in the figure (two points for the hex lattice; three for the square). Use two different polar angles: $\pi/2$, which means the neutrons are traveling in the x-y plane, and $\pi/8$. Use the dimensions and cross sections from Table 1. Note that to simplify the problem we have abolished scattering. In the highest energy group of a fine-group set, there is very little within-group scattering, so it does not change the problem very much to ignore scattering. We have also assumed that the neutrons are born uniformly in the fuel – a flat radial profile. This isn't precisely true, but again, the simplification does not change the character of the solution that we wish to study. You will need to write a simple computer program for this in whatever language you'd like. You will need to trace rays and compute points of intersection. When you reach the boundary of a pin cell you use a periodic boundary condition to translate the ray across the cell, and then you continue. You will need a strategy to know how far a ray must be traced before you say "enough." Your code should accept as input:

   (a) the number of values of the azimuthal angle, φ at which to calculate the angular flux;

   (b) the precision to which to calculate the angular flux at a given spatial point and given φ. (This tells you when you can say "enough." You can say "enough" when you've traced through τ mean free paths, where $\exp(-\tau) =$ the requested precision.)

2. Convince me that your code calculates the angular flux correctly.

3. Plot the angular flux as a function of φ for each of the two polar angles, for each of the three spatial points (two if hex lattice). Use enough φ values to convince yourself that you have resolved all the significant bumps and wiggles in the angular flux. Discuss your plots, and in particular compare them against what was shown in the notes for square pins. Do the circles make things smoother? Be prepared to present your solutions to the class, and (see part above) be prepared to argue that they are correct.