NUEN 629, Homework 4

Due Date Nov. 19

Solve the following problem and submit a detailed report, including a justification of why a reader should believe your results and a description of your methods and iteration strategies.

1 Vaquer

(150 points + 50 points extra credit) In class we discussed the diamond-difference spatial discretization. Another discretization is the step discretization (this has several other names from other disciplines). It writes the discrete ordinates equations with isotropic scattering as, for $\mu_n > 0$ to

$$\mu_n \frac{\psi_{i,n} - \psi_{i-1,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2}, \tag{1}$$

and for $\mu_n < 0$

$$\mu_n \frac{\psi_{i+1,n} - \psi_{i,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2}. \tag{2}$$

You should be able to modify the codes I have already provided to implement this discretization.

1. (50 points) Your task is to solve a problem with uniform source of $Q = 0.01$, $\Sigma_t = \Sigma_s = 100$ for a slab in vacuum of width 10 using step and diamond difference discretizations. Use 10, 50, and 100 zones ($h_x = 1, 0.02, 0.01$) and your expert choice of angular quadratures. Discuss your results and how the two methods compare at each number of zones.

2. (10 points) Discuss why there is a different form of the discretization for the different signs of μ.

3. (40 points) Plot the error after each iteration using a 0 initial guess for the step discretization with source iteration and GMRES.

4. (50 points) Solve Reed's problem (see finite difference diffusion codes). Present convergence plots for the solution in space and angle to a "refined" solution in space and angle.

5. (50 points extra credit) Solve a time dependent problem for a slab surrounded by vacuum with $\Sigma_t = \Sigma_s = 1$ and initial condition given by $\phi(0) = 1/h_x$. Plot the solution at $t = 1$ s using step and diamond difference. The particles have a speed of 1 cm/s. Which discretization is better with a small time step? What do you see with a small number of ordinates compared to a really large number (100s)?