# NUEN 647
# Uncertainty Quantification for Nuclear Engineering
# Homework 2

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# Contents

# Problem 1

Consider a covariance function between points in 2-D space:

$$k(x_1, y_1, x_2, y_2) = exp[-|x_1 - x_2| - |y_1 - y_2|]$$

Generate 4 realizations of a Gaussian random process with zero mean, $\mu(x, y) = 0$, and this covariance function defined on the unit square, $x, y \in [0, 1]$. For the realizations, evaluate the process at 50 equally space points in each direction. Plot the realizations.

# Problem 2

Assume you have 100 samples of a pair of random variables $(X_1, X_2)$ that have a positive correlation, call this set of pairs, $\mathbf{A_1}$. You then draw another 100 samples and call this set $\mathbf{A_2}$. The Pearson correlation between $(X_1, X_2)$ in $\mathbf{A_1}$ is positive and hte Pearson correlation between $(X_1, X_2)$ in $\mathbf{A_2}$ is negative. What can you say about the Pearson correlation for all 200 samples?

A normalized measure of the relation between two random variables, is the Pearson correlation coefficient, $\rho$. Oftentimes, this is simply called the correlation coefficient or correlation.

$$\rho(X_1, X_2) = \frac{E[X_1 X_2] - E[X_1]E[X_2]}{\sigma_{X1}\sigma_{X2}}$$

The expectation value for a series of realizations is defined:

$$E[g(x)] \approx \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

For the first 100 values:

$$\rho_1 = \frac{\frac{1}{100}\sum_{i=1}^{100} X_{1,i}X_{2,i} - \frac{1}{10000}\sum_{i=1}^{100} X_{1,i}\sum_{i=1}^{100} X_{2,i}}{\sigma_{X1,A1}\sigma_{X2,A1}}$$

$$100\sigma_{X1,A1}\sigma_{X2,A1}\rho_1 = \sum_{i=1}^{100} X_{1,i}X_{2,i} - \frac{1}{100}\sum_{i=1}^{100} X_{1,i}\sum_{i=1}^{100} X_{2,i}$$

$$100\sigma_{X1,A1}\sigma_{X2,A1}\rho_1 + \frac{1}{100}\sum_{i=1}^{100} X_{1,i}\sum_{i=1}^{100} X_{2,i} = \sum_{i=1}^{100} X_{1,i}X_{2,i}$$

Similarly for the second 100 values:

$$\sum_{i=101}^{200} X_{1,i}X_{2,i} = 100\sigma_{X1,A2}\sigma_{X2,A2}\rho_2 + \frac{1}{100}\sum_{i=101}^{200} X_{1,i}\sum_{i=101}^{200} X_{2,i}$$

The Pearson coefficient for all 200 values:

$$\rho_3 = \frac{\frac{1}{200}\sum_{i=1}^{200} X_{1,i}X_{2,i} - \frac{1}{40000}\sum_{i=1}^{200} X_{1,i}\sum_{i=1}^{200} X_{2,i}}{\sigma_{X1,A3}\sigma_{X2,A3}}$$

$$200\sigma_{X1,A3}\sigma_{X2,A3}\rho_3 = \sum_{i=1}^{200} X_{1,i}X_{2,i} - \frac{1}{200}\sum_{i=1}^{200} X_{1,i}\sum_{i=1}^{200} X_{2,i}$$

$$200\sigma_{X1,A3}\sigma_{X2,A3}\rho_3 + \frac{1}{200}\sum_{i=1}^{200} X_{1,i}\sum_{i=1}^{200} X_{2,i} = \sum_{i=1}^{200} X_{1,i}X_{2,i}$$

If we plug in the Pearson for the first 100 and second 100 for the right side of the equation,

$$200\sigma_{X1,A3}\sigma_{X2,A3}\rho_3 + \frac{1}{200}\sum_{i=1}^{200} X_{1,i}\sum_{i=1}^{200} X_{2,i}$$
$$=$$
$$100(\sigma_{X1A1}\sigma_{X2A1}\rho_1 + \sigma_{X1A2}\sigma_{X2A2}\rho_2) + \frac{1}{100}\left(\sum_{i=1}^{100} x_{1,i}\sum_{i=1}^{100} x_{2,i} + \sum_{i=101}^{200} x_{1,i}\sum_{i=101}^{200} x_{2,i}\right)$$

Grouping and setting:

$$\textstyle\sum_{i=1}^{100} x_{1,i} = X_{1,1}$$

$$\textstyle\sum_{i=1}^{100} x_{2,i} = X_{2,1}$$

$$\textstyle\sum_{i=101}^{200} x_{1,i} = X_{1,2}$$

$$\textstyle\sum_{i=101}^{200} x_{2,i} = X_{2,2}$$

$$\textstyle\sum_{i=1}^{200} x_{1,i} = X_{1,3}$$

$$\textstyle\sum_{i=1}^{200} x_{2,i} = X_{2,3}$$

$$200\sigma_{X1,A3}\sigma_{X2,A3}\rho_3 - 100(\sigma_{X1A1}\sigma_{X2A1}\rho_1 + \sigma_{X1A2}\sigma_{X2A2}\rho_2) = \frac{1}{100}\left(X_{1,1}X_{2,1} + X_{1,2}X_{2,2}\right) - \frac{1}{200}X_{1,3}X_{2,3}$$

Setting:

$$\sigma_{X1A1} = \tfrac{1}{100}\textstyle\sum_{i=1}^{100}(x_{1,i}^2 - \mu_{X_{11}}^2) = \tfrac{1}{100}\sigma'_{X1A1}$$

$$\sigma_{X2A1} = \tfrac{1}{100}\textstyle\sum_{i=1}^{100}(x_{2,i}^2 - \mu_{X_{21}}^2) = \tfrac{1}{100}\sigma'_{X2A1}$$

$$\sigma_{X1A2} = \tfrac{1}{100}\textstyle\sum_{i=101}^{200}(x_{1,i}^2 - \mu_{X_{12}}^2) = \tfrac{1}{100}\sigma'_{X1A2}$$

$$\sigma_{X2A2} = \tfrac{1}{100}\textstyle\sum_{i=101}^{200}(x_{2,i}^2 - \mu_{X_{22}}^2) = \tfrac{1}{100}\sigma'_{X2A2}$$

$$\sigma_{X1A3} = \tfrac{1}{200}\textstyle\sum_{i=1}^{200}(x_{1,i}^2 - \mu_{X_{13}}^2) = \tfrac{1}{200}\sigma'_{X1A3}$$

$$\sigma_{X2A3} = \tfrac{1}{200}\textstyle\sum_{i=1}^{200}(x_{2,i}^2 - \mu_{X_{23}}^2) = \tfrac{1}{200}\sigma'_{X2A3}$$

Where $A3$ and $\rho_3$ are for the series added to 200. Plugging these in, and multiplying both sides of the equation by 200.

$$\sigma'_{X1,A3}\sigma'_{X2,A3}\rho_3 - 2(\sigma'_{X1A1}\sigma'_{X2A1}\rho_1 + \sigma'_{X1A2}\sigma'_{X2A2}\rho_2) = 2\left(X_{1,1}X_{2,1} + X_{1,2}X_{2,2}\right) - X_{1,3}X_{2,3}$$

Note: $X_{1,3} = X_{1,1} + X_{2,1}$ and $X_{2,3} = X_{2,1} + X_{2,2}$ and that the right side of the equation simplifies to: $(X_{1,1} - X_{1,2})(X_{2,1} - X_{2,2})$. Then $\rho_3$ is:

$$\rho_3 = \frac{(X_{1,1} - X_{1,2})(X_{2,1} - X_{2,2}) + 2(\sigma'_{X1A1}\sigma'_{X2A1}\rho_1 + \sigma'_{X1A2}\sigma'_{X2A2}\rho_2)}{\sigma'_{X1,A3}\sigma'_{X2,A3}}$$

# Problem 3

Problem3

# Problem 4

Problem 4

# Problem 5

Problem 5