# NUEN 647 Homework 2

## Due: October 19, 2016

1. Consider a covariance function between points in 2-D space:

$$k(x_1, y_1, x_2, y_2) = \exp\left[-|x_1 - x_2| - |y_1 - y_2|\right].$$

   Generate 4 realizations of a Gaussian random process with zero mean, $\mu(x, y) = 0$, and this covariance function defined on the unit square, $x, y \in [0, 1]$. For the realizations, evaluate the process at 50 equally spaced points in each direction. Plot the realizations.

2. Assume you have a 100 samples of a pair of random variables $(X_1, X_2)$ that have a positive correlation, call this set of pairs, $\mathbf{A}_1$. You then draw another 100 samples and call this set $\mathbf{A}_2$. The Pearson correlation between $(X_1, X_2)$ in $\mathbf{A}_1$ is positive and the Pearson correlation between $(X_1, X_2)$ in $\mathbf{A}_2$ is negative. What can you say about the Pearson correlation for all 200 samples?

3. For the following data, compute by hand or via code you write the Pearson and Spearman correlations and Kendall's tau.

| $X_1$ | $X_2$ |
|---|---|
| 55.01 | 82.94 |
| 54.87 | 55.02 |
| 57.17 | 85.18 |
| 36.01 | -84.27 |
| 35.88 | -106.30 |
| 36.33 | -119.65 |
| 43.49 | -112.03 |
| 41.44 | -71.69 |
| 54.43 | -3.50 |
| 36.47 | 140.57 |

4. Demonstrate the tail dependence of a bivariate normal random variable is 0.

5. Another Archimedean copula is the Joe copula with generator

$$\varphi_J(t) = -\log\left(1-(1-t)^\theta\right),$$

and

$$\varphi_J^{-1}(t) = 1-(1-\exp(-t))^{1/\theta}.$$

   (a) Compute the bivariate copula for this generator.

   (b) Derive the upper and lower tail dependence for this copula.

   (c) Compute the value of Kendall's tau for this copula

   (d) Generate 1000 samples from the copula with standard normal marginals and a value of Kendall's tau of 0.6.