Advanced Operating Systems GOSSIP Project

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1 Introduction

We designed and implemented a distributed system that uses rumor spreading (gossip) for communication and calculations. The system had to perform the following calculations: minimum, maximum, average, and median. The system had to have the ability to distribute and update a file among nodes.

A key property that we strove toward during the design and implementation was scalability of this system. The system had to have reasonable run times while supporting thousands of nodes. Also, the system was designed to work across multiple machines. After the system was implemented, we tested its functionality through well defined experiments.

2 Methodology

2.1 Node Design and Functionality

Our implementation consists of two basic units, the node and the monitor. A node is responsible for computation and communication with other nodes. The communication uses a message passing paradigm. The monitor is responsible for controlling the simulation, user interaction and debugging.

Each node follows the basic rules of gossip. That is, on every round a node will gossip to itself with probability $1/2$ or with any one of its neighbors with probability $1/(\text{Number of neighbors} * 2)$. Our implementation conducts both push and pull once per gossip round. In addition to the push and pull cycle, each node will simultaneously update its local values. Unlike most example implementations, our nodes conduct all calculations simultaneously. Nodes are first initialized with a fragment of any size. Upon initialization, the node runs local calculations for minimum, maximum, average, and median values. Once this is complete, the node will also replicate its fragment to its

neighbors. All nodes are responsible for a single fragment, but may store any number of fragments as replicas or while passing fragments over the network. When a node receives a new fragment to own, it will reinitialize itself and replicate the new fragment. It is important to note that since each node is responsible for a single fragment, the fragment's number is synonymous with the process ID of it's owner node. We do tend to map actual numbers on top of the node process IDs, but this is as a matter of convenience.

Once a node is done with its initialization, it is ready to gossip. Our implementation requires all nodes to wait for a step signal to be sent before executing each round. This allows us to precisely measure the number of rounds executed and gives us excellent control over the network as a whole.

On each gossip round, all nodes push and pull their local values with their neighbors. Once a node receives new data for any calculation (remember, we do all calculations simultaneously), it will update its local data according to the data received.

For calculating the minimum and maximum values, a simple min/max comparison of the node's current value with the received value is sufficient. Calculating the average is similarly trivial as it is a well documented process of dividing the sum of the current and new average values by two.

$$Max(x_i) = \begin{cases} x_i & \text{if } x_i \geq x_j \\ x_j & \text{else} \end{cases}$$

$$Min(x_i) = \begin{cases} x_i & \text{if } x_i \leq x_j \\ x_j & \text{else} \end{cases}$$

$$Avg(x_i) = \frac{x_i + x_j}{2}$$

Calculating the median can be approached from many angles. Attempting to find the median of medians by recording each median value received was considered. However, this approach essentially requires a node to have knowledge of global state. Recording the most frequent numbers and finding their median was also considered, but again it would be vastly skewed over the whole of the network. Finally, a method similar to averaging was used, where the current median is added to the new median and the sum is divided by two. This was motivated by the method for finding the median in a set of even size.

$$Med(x_i) = \frac{x_{i\_new} + x_{i\_old}}{2}$$

Fragment handling is best examined in two phases. First, each node is the owner of but one fragment. The node will use its fragment for initialization

as discussed, but after this is done the node's duty becomes serving up and updating the fragment. When a node receives a request to store a fragment, it first checks to see if it is the owner of the fragment by looking at the PID assigned to be the owner of the fragment. Remember, our implementation uses PIDs as the actual fragment numbers. If the node owns the fragment, it will replace its current fragment with the new one and reinitialize itself. If a node does not own the fragment it will gossip the fragment along.

When a request for a fragment is issued the following occurs. First, the node checks the request to see if it has the requested fragment. If it doesn't, the node will change the sender field of the request to its own PID and gossip the request on. If it does have the fragment, the node will send it back to the sender of the request. Note that this may not be the originator of the request. When a node issues a request, it sets a special field that indicates which node issued the request. This field does not affect the transmission directly, rather it allows a node to see if the reply is intended for it. If a node gets a reply that is intended for it then it will store the fragment, otherwise it will gossip the fragment reply on.

The second phase of fragment handling is replication. When a node receives a fragment to own, the node will replicate the fragment to its neighbors. Each node then maintains a list of fragments in addition to the fragment it owns. For example, if all nodes have three neighbors, all the nodes will actually hold four fragments its own plus the replicas of the three neighbors.

This adds a layer of complexity to fragment stores and requests. The protocol described above still is in full effect, but with a couple of extra steps. When a node receives a request to store a fragment, it not only checks that it is the owner of the fragment, but that it has a replica of the fragment. If it has a replica of the fragment, it will update its own copy and then gossip the request along. If a node receives a request for a fragment, the node will again check to see if it has a replica. If there is a replica it will send the replica as a response.

Looking at this as a whole then, we have a system that provides a gossip-based read one, write all functionality. When writing, all nodes with a copy of the fragment are updated either directly or by the replication function of the owner node. When reading, any one of the nodes containing the fragment either owning it or replicating it will respond with the fragment.

2.2   Network Design

The network is constructed as two trees, joined at the leafs. This joint between the trees is the bisection of the network. Starting from the root node in each tree, each node has two children. We build the trees in breadth first

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order to a maximum depth. For simplicity, we restrict ourself to having the same number of nodes in each tree and ensure that the trees are balanced. The trees are stored in Erlang's digraph data structure. We tried have our network have the same properties as discussed in GOSSIP analysis. Our network is undirected and not regular (some nodes have degree 2, most have 3). In real world applications, sensor networks can be sparse. Our network reflects that characteristic. It is irreducible, that is there exist paths from every node to any other nodes. Also, nodes can talk to themselves which makes the network aperiodic and subsequently the transition probability matrix of our network is right stochastic since not all nodes have the same degree.
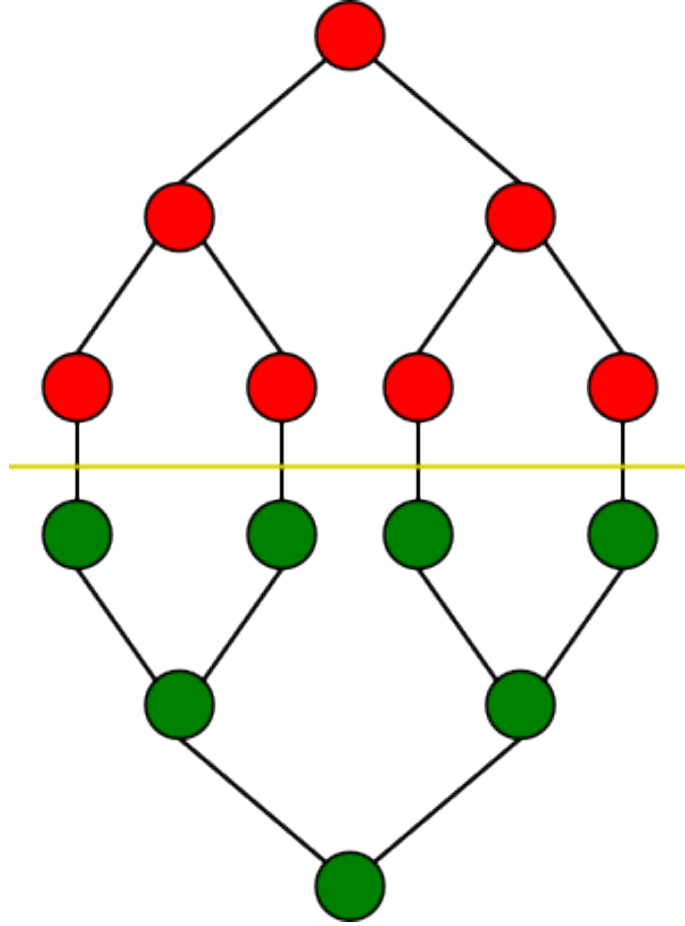

Figure 1: Small scale example of our network. Yellow line is the network bisection.

Our network is not double stochastic, which will affect its conductance. The conductance will not be as high as a regular graphs, but will be good

enough for our experiment. We still expect convergence to happen. We researched matrix normalization techniques such as Richard Sinkhorn's theorem for making a matrix for finding a doubly stochastic matrix from a square matrix and diagonal matrices, but chose not to implement it in our design. Network construction was implemented in a library. First the user supplies a number of desired nodes and gets back the number of nodes in the network. With this information, the user spawns that number of nodes and records their PIDs in a list. Finally, the user supplies this list to a function that creates the digraphs and joins them, returning to the user a final digraph. A few helper methods are also present in this library. The most important is the get_neighbor methods. The user can supply the digraph of the network and a PID and return get a list of the neighbors of this node.

2.3 Convergence

Our network does not exhibit properties that make an analysis easy. We have to remember the fact that in using gossip we trade accuracy for latency. Therefore, we chose not to do an extensive analysis and instead focus on experimental measurements. We were interested in experimentally determining two qualities. First, we wanted to study how the size of the network and the nature of the computation affects the time to converge on an answer. Second, we wanted to figure out how the size of the network and the computation affect the error over time. Since we could not compute the error during the design of the system, we chose an acceptable point of convergence of when the error was less than or equal to 3% at each node.

3 Results

3.1 Min, Max, Average, and Median

We had to determine whether the basic functionality of the system was working. We did this by continually testing the system during implementation. Here we present the proof that our system functions correctly and satisfies the first three requirements in the specification. We conducted ten runs of the four mathematical calculations specified in the assignment description. This experiment was conducted with a 16,382 node network. Finally, we limited the maximum number of gossip rounds to 200.

The average of all the runs fall within the target delta of less than or equal to 3%. However, it is not enough to simple look at the average error. We also look at the standard deviation of error, the maximum and the minimum

error. This is because of criteria for convergence is that every node's error is less than or equal to 3%. Looking at the standard deviation of the error tells us how quickly we are converging. Looking and the min and max of the error gives us an indication when we have converged. Up to this point, we have talked about the error generically. Each of the mathematical calculations has its own error.
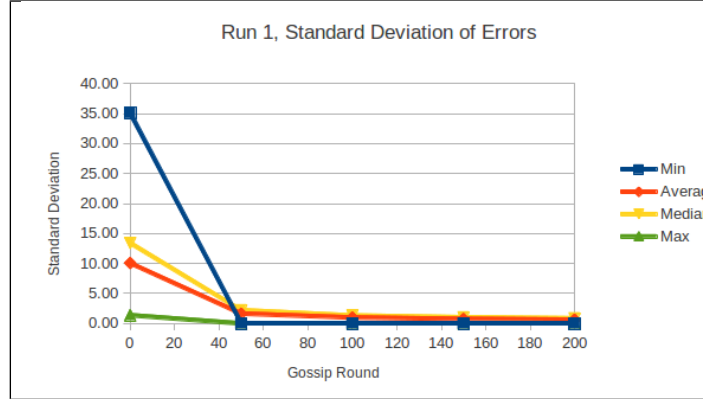

Figure 2: Plot of standard deviation of errors for Run 1.

For the sake of brevity, the results of the first four runs are summarized in Appendix 5.1. One can see that as more and more rounds of gossip happen, the average error drops. The standard deviation of the error decreases as well. The maximum and minimum of errors approach zero as the number of rounds increases. Note minimum of errors of the average and median calculation during Run 1 at round 200. The average is -3.16 and the median is -4.1. In this particular case, those calculation have not converged to useful values based on our criteria. Allowing gossip to continue for those calculation would eventually yield correct answers, since one can see that the average, standard deviation (as shown in Figure 2), min and max of the error during Run 1 are decreasing.

The minimum and maximum calculation converge faster than the other two due to the distribution of their values. This phenomenon will be examined later.

The dependence of the convergence speed on the distribution of the data was alarming at first. However, with enough experimentation we can determine the bounds of the convergence speed.

3.2 Min and Max revisited

After seeing the results of the first experiment, we thought it would be prudent to analyze the min and max calculation at a finer scale. The experimental setup was 16,382 nodes gossiping for 10 rounds. Each round was recorded and this experiment repeated ten times. Appendix 5.2 summarizes our findings and for the sake of brevity on the first run is shown. Figure 11 shows the characteristics of the min and max operations in relation to their errors at a finer scale. Comparing the rate of decrease of the standard deviation between the various calculations, the minimum's standard deviation clearly decreases faster than any other (Figure 3). Looking at the max and min of all errors tells us something interesting. Although the minimum is quickly converging, there remains at least one node whose answer is way off (Figure 4 and 5).
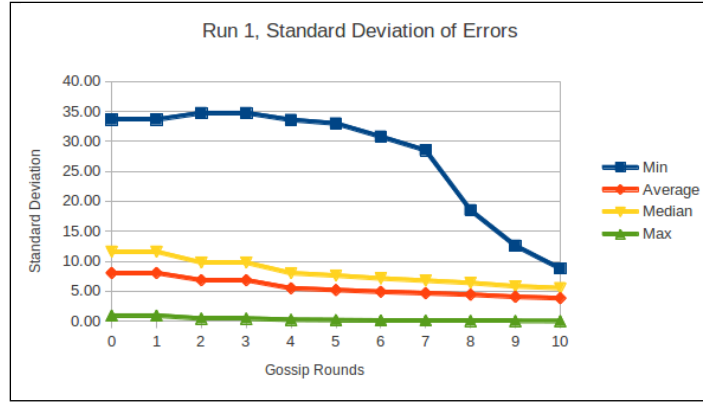

Figure 3: Plot of standard deviation of errors for Run 1, min/max revisited.
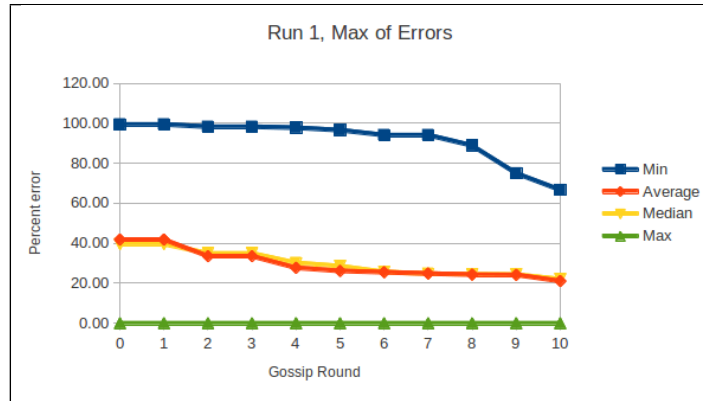
Figure 4: Plot of max of errors for Run 1, min/max revisited.
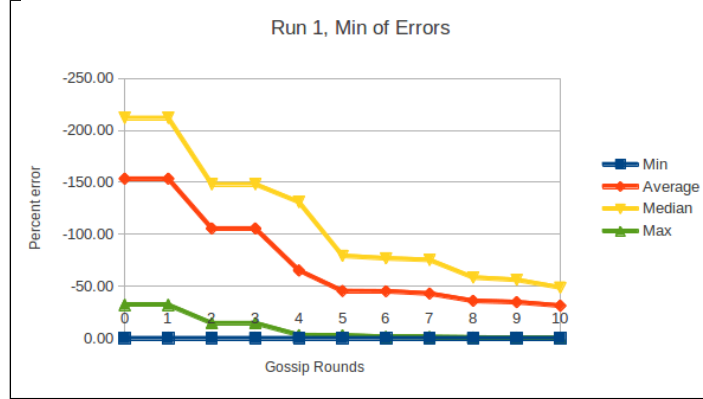
Figure 5: Plot of min of errors for Run 1, min/max revisited.

3.3 Scalability

To test how well our system scales, we conducted experiments with different number of nodes. The results of the experiments are summarized in Figure 6. As the number of nodes in the system increases, the time to converge per node decreases.

Number of nodes	Steps to converge
1022	70
8190	90
16382	100

Figure 6

3.4 Fragments

We also show proof that 4th and 5th requirements are fulfilled by running examples of updating and retrieving fragment data. A network of 16,382 nodes was created and fragments were initialized at each node with pseudo-random data. An arbitrary fragment number was chosen to be updated. The system was passed a new set of data for the fragment to be stored at the destination nodes containing the fragment. The update started at node 1 and the number of gossip rounds needed to store the fragment was recorded. Out of 100 trials it took an average of 896 rounds of gossip to store the fragment.

Retrieving fragment data was tested on the same network. Data from an arbitrary fragment was requested from node 1. The number of gossip rounds it took to retrieve the fragment was recorded. Out of 100 trials it took an average of 529 rounds of gossip to retrieve the fragment.

3.5   Spawning Nodes on Remote Machines

Erlang allows for easy spawning of processes on a remote machine. We were able to take advantage of this to spawn nodes on another Erlang interpreter running on a seperate VM with only slight modification of the code. Both VMs need to have the code compiled and the Erlang interpreter running.

The functions for spawning nodes were modified to spawn every other node on the remote system. The network generation was done in the same way as when using a single VM, so the nodes have the same degree as the single system implementation (3 for most nodes). Because of this, all functions required approximately the same number of gossip rounds to converge. However, due to additional latency introduced by the network communication, operations took roughly twice as long to complete.

4   Conclusion

Our system has been tested and functions per specification. It scales up well and can achieve an above than expected level of accuracy. As with any project, there are things one wishes to have done. After verifying the correctness of the system, we wanted to conduct a few experiments to figure out some quantitative characteristics of the system. One can see that there is a definite relationship between the number of nodes and time to convergence. There is also a difference in time to convergence between the different operations and that the distribution of values has an influence. We wanted to estimate the rate of convergence for every mathematical operation. The first experiment provided us with some data that can be used to make a general analysis, but a deeper statistical analysis is need to come up with an actual model. Also, the network topology we chose, though simple, is very inflexible in how many nodes we could spawn. If we had more time, we would have probably chosen a better topology. One that had characteristics that lend it better to easier analysis.

5 Appendix

5.1 Min, Max, Avg, Med

Run 1				
Average of errors	Min	Average	Median	Max
Round				
0	55.08	-1.01	-1.87	-0.44
50	0.00	-0.83	-1.30	0.00
100	0.00	-0.90	-1.39	0.00
150	0.00	-0.94	-1.46	0.00
200	0.00	-1.00	-1.53	0.00
Standard deviation of errors	Min	Average	Median	Max
Round				
0	35.09	10.08	13.42	1.38
50	0.00	1.64	2.21	0.00
100	0.00	0.99	1.33	0.00
150	0.00	0.76	1.03	0.00
200	0.00	0.63	0.87	0.00
Max of errors	Min	Average	Median	Max
Round				
0	99.64	42.64	43.68	0.00
50	0.00	3.63	7.02	0.00
100	0.00	2.18	3.94	0.00
150	0.00	1.32	2.16	0.00
200	0.00	0.51	0.90	0.00
Min of Errors	Min	Average	Median	Max
Round				
0	0.00	-261.92	-335.65	-95.69
50	0.00	-12.03	-12.59	0.00
100	0.00	-6.67	-7.80	0.00
150	0.00	-4.44	-5.45	0.00
200	0.00	-3.16	-4.10	0.00

Figure 7

Run 2				
Average of errors	Min	Average	Median	Max
Round				
0	49.66	-0.24	-0.61	-0.33
50	0.00	0.43	0.97	0.00
100	0.00	0.37	1.01	0.00
150	0.00	0.35	1.03	0.00
200	0.00	0.32	1.02	0.00
Standard deviation of errors	Min	Average	Median	Max
Round				
0	35.23	7.38	10.46	0.74
50	0.00	1.13	1.54	0.00
100	0.00	0.53	0.69	0.00
150	0.00	0.37	0.48	0.00
200	0.00	0.29	0.38	0.00
Max of errors	Min	Average	Median	Max
Round				
0	99.78	37.62	43.17	0.00
50	0.00	5.61	7.90	0.00
100	0.00	2.82	3.86	0.00
150	0.00	1.95	2.88	0.00
200	0.00	1.30	2.27	0.00
Min of Errors	Min	Average	Median	Max
Round				
0	0.00	-91.55	-97.32	-39.08
50	0.00	-3.84	-5.28	0.00
100	0.00	-1.70	-2.17	0.00
150	0.00	-0.98	-0.94	0.00
200	0.00	-0.58	-0.30	0.00

Figure 8

Run 3				
Average of errors	Min	Average	Median	Max
Round				
0	52.07	-0.57	-1.20	-0.36
50	0.00	-0.02	-0.19	0.00
100	0.00	0.01	-0.15	0.00
150	0.00	0.00	-0.17	0.00
200	0.00	-0.01	-0.21	0.00
Standard deviation of errors	Min	Average	Median	Max
Round				
0	34.54	8.18	11.31	0.77
50	0.00	1.55	2.11	0.00
100	0.00	0.68	0.88	0.00
150	0.00	0.52	0.67	0.00
200	0.00	0.36	0.46	0.00
Max of errors	Min	Average	Median	Max
Round				
0	99.35	36.38	36.16	0.00
50	0.00	6.44	7.44	0.00
100	0.00	2.25	2.96	0.00
150	0.00	1.49	1.87	0.00
200	0.00	0.81	0.91	0.00
Min of Errors	Min	Average	Median	Max
Round				
0	0.00	-273.60	-252.20	-39.28
50	0.00	-6.73	-10.12	0.00
100	0.00	-2.98	-4.44	0.00
150	0.00	-2.13	-3.08	0.00
200	0.00	-1.05	-1.76	0.00

Figure 9

Run 4				
Average of errors	Min	Average	Median	Max
Round				
0	50.42	-0.78	-1.45	-0.38
50	0.00	-0.06	-0.23	0.00
100	0.00	0.23	0.07	0.00
150	0.00	0.41	0.24	0.00
200	0.00	0.52	0.34	0.00
Standard deviation of errors	Min	Average	Median	Max
Round				
0	35.18	8.76	11.96	0.89
50	0.00	1.68	2.35	0.00
100	0.00	0.83	1.20	0.00
150	0.00	0.56	0.82	0.00
200	0.00	0.49	0.73	0.00
Max of errors	Min	Average	Median	Max
Round				
0	99.50	39.99	40.20	0.00
50	0.00	6.29	8.18	0.00
100	0.00	2.77	4.40	0.00
150	0.00	2.00	2.84	0.00
200	0.00	1.57	2.09	0.00
Min of Errors	Min	Average	Median	Max
Round				
0	0.00	-284.06	-312.06	-77.62
50	0.00	-10.53	-15.79	0.00
100	0.00	-3.40	-5.86	0.00
150	0.00	-1.49	-2.73	0.00
200	0.00	-0.79	-1.62	0.00

Figure 10

5.2   A closer look at Min and Max

Run 1				
Average of errors	Min	Average	Median	Max
Round				
0.00	58.56	-0.54	-1.12	-0.48
1.00	58.56	-0.54	-1.12	-0.48
2.00	48.25	-0.44	-0.89	-0.27
3.00	48.25	-0.44	-0.89	-0.27
4.00	38.00	-0.24	-0.59	-0.16
5.00	33.83	-0.16	-0.44	-0.13
6.00	26.56	-0.18	-0.45	-0.08
7.00	20.70	-0.20	-0.47	-0.06
8.00	7.14	-0.24	-0.53	-0.02
9.00	3.21	-0.23	-0.49	-0.01
10.00	1.54	-0.19	-0.42	0.00
Standard deviation of errors	Min	Average	Median	Max
Round				
0.00	33.61	8.03	11.58	0.93
1.00	33.61	8.03	11.58	0.93
2.00	34.70	6.83	9.77	0.44
3.00	34.70	6.83	9.77	0.44
4.00	33.55	5.49	8.00	0.23
5.00	32.96	5.19	7.60	0.19
6.00	30.77	4.90	7.13	0.13
7.00	28.48	4.66	6.76	0.10
8.00	18.50	4.41	6.38	0.05
9.00	12.57	4.06	5.82	0.03
10.00	8.78	3.84	5.51	0.02
Max of errors	Min	Average	Median	Max
Round				
0.00	99.45	41.80	39.56	0.00
1.00	99.45	41.80	39.56	0.00
2.00	98.28	33.52	34.98	0.00
3.00	98.28	33.52	34.98	0.00
4.00	97.78	27.70	30.15	0.00
5.00	96.55	26.13	28.57	0.00
6.00	94.12	25.49	25.63	0.00
7.00	94.12	24.84	24.81	0.00
8.00	88.89	24.29	24.40	0.00
9.00	75.00	24.15	24.29	0.00
10.00	66.67	21.06	22.05	0.00
Min of Errors	Min	Average	Median	Max
Round				
0.00	0.00	-153.45	-211.91	-32.10
1.00	0.00	-153.45	-211.91	-32.10
2.00	0.00	-105.57	-148.10	-14.55
3.00	0.00	-105.57	-148.10	-14.55
4.00	0.00	-65.15	-130.91	-3.09
5.00	0.00	-45.43	-79.25	-2.77
6.00	0.00	-45.18	-76.84	-1.42
7.00	0.00	-42.94	-75.33	-1.21
8.00	0.00	-35.97	-58.40	-0.81
9.00	0.00	-34.75	-56.17	-0.40
10.00	0.00	-31.32	-48.78	-0.40

Figure 11