**SMM641 Problem Set 2**
**Question 1**

a) If the programme's objective was maximise revenue with a single congestion charge, which price would maximise the total revenue? What is the total level of emissions?

**Price:** £8 **Total Revenue:** £1, 349, 009 **Total Emissions**: 45, 845, 714 g/km

b) If the programme's objective was maximise revenue and a peak period pricing strategy is to be implemented, with the price for the non-peak period set at £7, what price would you recommend for the peak period? .

**Price:** NP=£7 & PP = £9 **Total Revenue:** £1, 371, 826 **Total Emissions**: 37,110,105 g/km

Implementing a segmented pricing strategy increases revenue by 1.7%, despite a decrease in overall demand as prices exceed the willingness to pay for some of the 192,000 individuals. This results in a 19% reduction in emissions, with 84% of drivers opting to travel through the congestion zone. The highest revenue is achieved with a Non-Peak price of £7.

c) Suppose the programme's objective is to minimise emissions. The City requires that the revenue should not fall below £1.1 million per day. Assuming a non-peak period price of £7, what price would you recommend for the peak period?

**Price:** NP=£7 & PP = 13 **Total Revenue:** £1, 178, 713 **Total Emissions**: 25, 233, 083 g/km

By prioritising emission reduction over revenue, the Peak price was set higher than in 1(b), discouraging travel during Peak times while leaving the Non-Peak price unchanged. Although this reduces congestion, revenue declines as only 66% of drivers willing to travel through the congestion zone do so at this price. Introducing a dual pricing strategy redistributes demand more evenly between Peak and Non-Peak times, improving average speed uniformity. The difference in average speed between the periods is 4.77 km/h, nearly half the 8.87 km/h gap observed in 1(a). This closer alignment in speeds indicates greater potential for $CO_2$ emission reduction.

**Question 2**

To optimise pricing for Boris Bikes, we propose a multinomial logit (MNL) model with customer choice data that uses historical data to evaluate user preferences and predict how price changes affect demand during peak and non-peak hours whilst incorporating available bike capacity at individual stations. The MNL model accounts for how individuals choose between multiple alternatives, (price and bike availability), enabling predictions of user behavior and forecasting revenue.

Goal

The objective is to optimise system efficiency and revenue by implementing higher prices during peak commuting hours (e.g., 7:30–9:30 am) while balancing demand with bike availability.

Methodology

To optimise the pricing strategy for Boris Bikes, the analysis begins by computing the average gross utilities for both peak and non-peak times, along with their  average variances to model the shape parameter, using a Gumbel distribution. This approach allows us to model demand variability ensuring a more accurate representation of user preferences. Next, calculate attraction values for renting bikes during peak and non-peak times serving as inputs to estimate purchase probabilities for three potential outcomes: renting during peak hours, renting during non-peak hours, and not renting. These probabilities are calculated to ensure their total sum equals one and are interpreted using tools nplotr library in R Studio. It is important to note that interpreting these value from R studio require further calculation.

With these values, we construct an objective function which maximises revenue while adhering to critical constraints, such as ensuring demand does not exceed bike capacity and maintaining that peak pricing remains higher than non-peak pricing. To enhance optimisation, users are segmented into categories, such as commuters and tourists, allowing for tailored strategies. For instance, reserving bikes for commuters at high-demand stations during peak times and implementing higher prices during these hours can effectively manage congestion while increasing revenue. This integrated approach balances user demand, system efficiency, and profitability.

The gross utility function for an individual choosing station *j* can be written as:

$$U_{i,j} = \beta_0 + \beta_1 \text{ x Price}_j + \beta_2 \text{ x Availability}_j + \varepsilon_{i,j}$$

$\varepsilon_{i,j}$ accounts for unobserved factors affecting choice (error term)

Using the MNL model, we can identify peak and non-peak periods by analysing historical demand patterns and price variations. This allows the construction of dynamic pricing strategies to balance demand and maximise revenue. For example, prices can be adjusted in real time through the Santander app based on demand elasticity, improving both operational efficiency and user satisfaction. To test the model by comparing predicted and observed outcomes at selected stations to refine accuracy.

Appendix

Calculation for Question 1(a)

a) when non peak = £8 non peak demand = 13357

peak = £8 peak demand = 155270

non peak speed $= 30 - \left(0.0625 \times \frac{13357}{1000}\right) = 29.17$ km/h

peak speed $= 30 - \left(0.0625 \times \frac{155270}{1000}\right) = 20.3$ km/h

per car:

non peak emission $= 235 - 1.4(29.17) = 194.17$ g/km

peak emissions $= 617.5 - 16.7(20.3) = 278.56$ g/km

total non peak emissions $= 194.17 \times 13357 = 2593418$ g/km

total peak emissions $= 278.56 \times 155270 = 43252295$ g/km

Overall daily emissions $= 45845714$ g/km

Figure 1

Calculation for Question 1(b)

(b) when NP = £7  NPeak demand = 42852 cars

P = £9  Peak demand = 119096 cars

Non peak speed $= 30 - \left(0.0625 \times \frac{42852}{1000}\right) = 27.32$ km/h

peak speed $= 30 - \left(0.0625 \times \frac{119096}{1000}\right) = 22.56$ km/h

per car

Non peak emissions $= 235 - 1.4(27.32) = 196.75$ g/km

peak emissions $= 617.5 - 16.7(22.56) = 240.81$ g/km

total non peak emissions $= 196.75 \times 42852 = 8,431,131$ g/km

total peak emissions $= 240.81 \times 119096 = 28,678,958$ g/km

overall daily emissions $= \underline{\underline{37,110,085}}$ g/km

Figure 2

Assumptions for Question 1(c)

In Question 1(c), the Multinomial Logit (MNL) model was used to predict the optimal peak-time price to limit $CO_2$ emissions. Alternatively, fitting a linear demand model could be used to estimate the price point, as demonstrated in the accompanying R script. The linear regression suggests an optimal peak price of £11.50, which differs from the MNL solution. This discrepancy arises because linear regression assumes a linear relationship between price and demand. However, as shown in Figure 3, the data exhibits a non-linear pattern, leading to underfitting when using a linear model.

Additionally, the data was scaled from 345 respondents to represent a population of 192,000 individuals, which can amplify discrepancies in prediction accuracy. Given these limitations, the MNL model was chosen for calculating the optimal price as it better captures the nuanced, non-linear relationship between price and demand during both peak and non-peak periods.

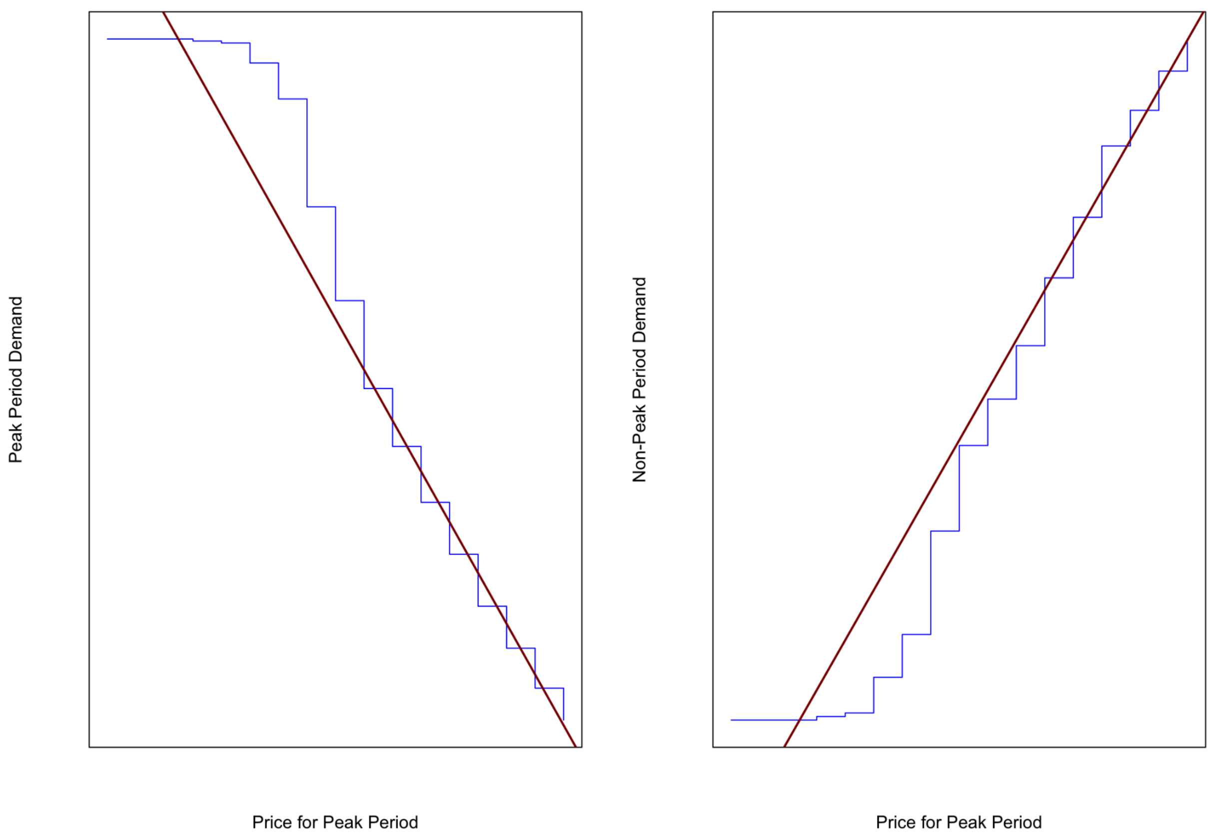


Figure 3