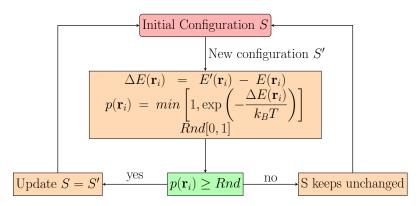
# Canonical Monte Carlo Metropolis algorithm



Temperature=const.,  $E = E_D + E_{dip} + E_{ar} + E_e$ 

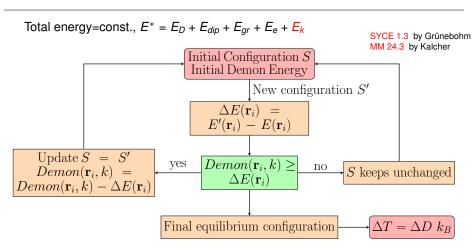


Model for relaxor: Ma, Albe, and Xu, Phys. Rev. B 91, 184108 (2015)



## Micrcanonical Monte Carlo Multi-demon algorithm





Ponomareva & Lisenkov, Phys. Rev. Lett. (2012).



### Lattice-based Monte Carlo Energy terms



$$E = E_D + E_{dip} + E_{ar} + E_e$$

► Landau potential

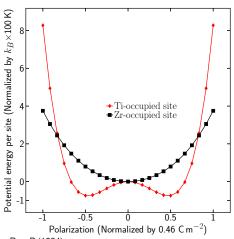
$$E_D = V_0 \sum_{i} \left[ \frac{\mathbf{a}_i}{2} (P_x^2(\mathbf{r}_i) + P_y^2(\mathbf{r}_i)) + \frac{\mathbf{b}_i}{4} (P_x^4(\mathbf{r}_i) + P_y^4(\mathbf{r}_i)) + \frac{\mathbf{c}_i}{6} (P_x^6(\mathbf{r}_i) + P_y^6(\mathbf{r}_i)) \right]$$

$$\begin{cases} \textbf{a}_i = -13.7128 \times 10^8 \, \text{J m C}^{-2}, \, \textbf{b}_i = 28.908 \times 10^9 \, \text{J m}^5 \, \text{C}^{-4} & \text{(Ti-occupied site)} \\ \textbf{a}_i = 6.112 \times 10^8 \, \text{J m C}^{-2}, \, \textbf{b}_i = 1.4454 \times 10^9 \, \text{J m}^5 \, \text{C}^{-4} & \text{(Zr-occupied site)} \end{cases}$$



## Simulation concept Behavior of $Ba(Zr_xTi_{1-x})O_3$







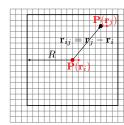
## Lattice-based Monte Carlo Energy terms



$$E = E_D + E_{dip} + E_{ar} + E_e$$

▶ Dipole-dipole interaction

$$E_{dip} = V_0^2 \frac{1}{8\pi\epsilon_0} \sum_{i,j} \frac{1}{\epsilon_{ij}} \left[ \frac{\mathbf{P}(\mathbf{r}_i) \cdot \mathbf{P}(\mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - \frac{3[\mathbf{P}(\mathbf{r}_i) \cdot (\mathbf{r}_i - \mathbf{r}_j)][\mathbf{P}(\mathbf{r}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{|\mathbf{r}_i - \mathbf{r}_j|^5} \right], \tag{1}$$



Servoin et al., Phys. Rev. B (1980). Hlinka & Márton, Phys. Rev. B (2006). Nayak et al., RSC Adv. (2014).



#### **Lattice-based Monte Carlo** Energy terms



$$E = E_D + E_{dip} + E_{ar} + E_e$$

Dipole-dipole interaction

$$E_{dip} = V_0^2 \frac{1}{8\pi\epsilon_0} \sum_{i,j} \frac{1}{\epsilon_{ij}} \left[ \frac{\mathbf{P}(\mathbf{r}_i) \cdot \mathbf{P}(\mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - \frac{3[\mathbf{P}(\mathbf{r}_i) \cdot (\mathbf{r}_i - \mathbf{r}_j)][\mathbf{P}(\mathbf{r}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{|\mathbf{r}_i - \mathbf{r}_j|^5} \right],$$

$$\epsilon_{ij} = \begin{cases} 12.0 \\ 144.0 \\ 12.0(1-x) + 144.0x \end{cases}$$

 $\epsilon_{ij} = \begin{cases} 12.0 & \text{ both sites } i \& j \text{ are occupied by } \angle_{i}, \\ 12.0(1-x) + 144.0x & \text{if site } i \& j \text{ are occupied by Ti and Zr, respectively.} \end{cases}$ 

Servoin et al., Phys. Rev. B (1980). Hlinka & Márton, Phys. Rev. B (2006). Navak et al., RSC Adv. (2014).

