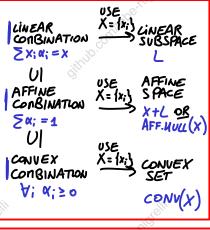


GEONETRY



A YALF-SPACE IS CONVEX, NOT AFFINE {X|QTX≤&}

INTERSECTION OF HIPERPLANES CONTAINING X

IF X, Y: CT... & C... & C... ALSO FOR & C. CONV((x,1))

IF (X;) FINITE,

CANV((x;)) COMPACT

• EMPTY SET DIM
$$(\emptyset) = -1$$
• SINGLETON DIM $(\{x\}) = 0$
• AFFINE SPACE \rightarrow DIM $(x+L) = 0$ IM $(x+L) = 0$
• OIM $(\{x \in \mathbb{R}^d \mid Ax = \ell\}) = d$
• MYPERPLANE DIM $(\{x \in \mathbb{R}^0 \mid a^Tx = \ell\}) = d-1$

DEPENDENCE OF
$$(NOT ALL)$$

A SET $\{X_i\}$ $(NOT ALL)$

AFFINE; $\{\alpha_i\}$ ST. $\{\alpha_i, x_i = 0\}$
 $\{X_i\}$ INDEP EVEN IF $X_1 = 0$
 $\{X_i\}$ INDEP EVEN IF $X_2 = 0$
 $\{X_i\}$ INDEP EVEN IF $X_3 = 0$
 $\{X_i\}$ INDEP EVEN IF $X_4 = 0$
 $\{X_i\}$ INDEP EVEN IF $X_4 = 0$

VECTORS $\subseteq \mathbb{R}^d$

• LINEAR: $\{X_1 - X_n\}$... $\{X_{n-1} - X_n\}$ INDEPENDENT

STANDARD \iff EQUATIONAL

max c^Tx max c^Tx SUB $Ax \leq C$ SUB $A \stackrel{1}{:i} (x \stackrel{1}{y}) = C$ $X \geq 0$ $X \geq 0$ LP (SCAC)

"BASIS" OF AU CARATHEODORY (AFFINE d.oin. SET) X = IRd, oin(X)=d, X $CONV(X) = \left\{ \sum_{i=1}^{d+1} \alpha_i x_i \mid X_i \in X, \ \alpha_i \ge 0, \xi \alpha_i = 1 \right\}$ THE CONVEX HULL CAN BE WRITTEN AS A COMBINATION OF AT MOST d+1 EXTREME POINTS

SEPARATION THEOREM →C, O ⊆ IRO (NONENPTY, CONVEX) OC BOUNDED THERE EXISTS A HYPERPLANE STRONGLY SEPARATING C.D: [C = {x | aTx < & } WALF-SPACES D = {x | aTx > & } (OPEN) SETS) · CONES

NONEMPTY CEIR GENERALISATION OF CONVEX: YX, YEC, YZ, MZO INCLUDES Q AND ALL POINTS ON THE RAY FROM O 1x+mx EC TO ANY POINT OU THE SET AND BEYOND X FINITE ⇒ CONE(X) IS CLOSED CONE IS => CONE IS A X LIN. INDEP => CALLED "PRINITIVE" A FINITELY GENERATED CONE IS THE UNION OF FINITELY MANY PRIMITIVE CONES

TARUAS' LEMMA EQUIVALENTLY, · BxelR st. Ax = C, x=0 EXACTLI ONE OF THE FOLLOWING IS TRUE: · JYEIR M ST. JYTA > OT · & E CONE ({a1 ... an }) ·] MYPERPLANE {x| 4Tx =0} TFAE FOR S = SUP{CTX | AXEB} WITH 1. CONE ((a;)) ON ONE SIDE · VxelPh. Axel -> cTx=8 2. G STRICTLY ON THE 1. by ∈ Rm, y ≥0 → ATy=c, BTy € S

OTHER SIDE

DUALITY

 $max c^Tx$ PRINAL -> ST. AXEB DUAL -> Mlh ST. $\begin{cases} A^{T}y = c \\ y \ge 0 \end{cases}$

NOTE THAT Plinal AND DUAL EXIST IN TWO DIFFERENT SPACES, SO THEY ARE GEONETRICALLY UNRELATED

WEAK LP -> CTX & BTY THE RELATION BETWEEN X, Y FEASIBLE SETS IN THEIR OBJECTIVES

PORD UNBOUNDED DORP IN FEASIBLE

4Tec0

STRONG _ YOU WAVE LP DUALTY 4 CASES 1 4 CASES FOR

BOTH FEASIBLE · UN BOUNDED, INFEASIBLE · BOTH INFEASIBLE

POLYLEORA

TUE DUAL OF THE DUAL IS PRIML AGAIN TUE

CONPLENENTARY SLACHNESS CONDITIONS

FOR XY FEASIBLE SOLUTIONS TO THE DUAL OPTIMAL (=> yT(&-Ax)=0

IN ALL FACES

CONTAINING F1, F2

MINNOWSKI-WEYL

· POLYTOPE (=>) FINITE USIR" ST P= 000(V)

· POLYHEORAN (=)] FINITE V, Y = IR" ST. P = CONV (V) + CONE(4)

FACE = P A SUPPORTING -> NOT ANY HYPERPLANE: PC SPACE = { xep| c7x = 8} MINIMU LARGER FACE ·F1, F2 → 3 F ST F1 SF, F2 SF GET C FROM CTXES YXEP FACE F IS CONTAINED GET C FRON A (SUBSISTER)

YOU CAN DEFINE A N TWO SUBSISTERS A' WITH RANK (A') = RANK(A)-1, AND THE NEQUALITIES { a_1 x = l_1 a_2 x = l_2 FACE = $\{x \mid A'x = \ell', \alpha_1^T x \in \ell_1, \}$

 $\alpha_{2}^{T} \times \leq \ell_{2}$

USE THE EXTREMAL VERTICES: {veP|v&conu(P({x}))}