# MATHS

$df = \vec{\nabla} f \cdot d\vec{l}$

$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$ [DIRECTION OF MAX INCREASE]

$\vec{\nabla} \cdot \vec{f} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$ [STRETCHING EFFECT OF THE FIELD]

$\vec{\nabla} \times \vec{f} = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{x} - \left(\frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z}\right)\hat{y}$ [VECTOR $\perp$ TO ROTATION]

$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$ [IN 2D FIELDS ONLY HAS $\hat{z}$ COMPONENT] [USE THE RIGHT HAND RULE]

- GRADIENT, DIVERGENCE AND CURL ARE DISTRIBUTIVE UNDER ADDITION | $\nabla(fg) = g\nabla f + f\nabla g$

$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\,\delta^3(x)$

THE JACOBIAN EMERGES FROM THE CHANGE OF VARIABLE $\rightarrow V = \frac{4}{3}\pi r^3 \rightarrow d\tau = 4\pi r^2 dr$

| CARTESIAN | SPHERICAL | CYLINDRICAL |
|---|---|---|
| $d\tau = dx\,dy\,dz$ | $= r^2 \sin\vartheta\, d\vartheta\, d\varphi\, dr$ | $= \rho\, d\rho\, d\varphi\, dz$ |
| USE IT TO CONVERT DENSITIES INTO DIFFERENT COORDINATES | $\cdot\ \vartheta$ VERTICAL $[0,\pi]$ <br> $\cdot\ \varphi$ HORIZONTAL $[0,2\pi]$ <br> $\cdot\ r$ RADIAL $[0,\infty)$ | $\cdot\ z$ HEIGHT $(-\infty,\infty)$ <br> $\cdot\ \varphi$ HORIZONTAL $[0,2\pi]$ <br> $\cdot\ \rho$ RADIAL $[0,\infty)$ |

EG. TO INTEGRATE A SURFACE DENSITY IN POLAR COORDINATES YOU ADD THE JACOBIAN TERM $r^2$

**CONSERVATIVE** $\Rightarrow$ **IRROTATIONAL**

- $\vec{E} = \nabla V$
- PATH INTEGRALS ARE ONLY ENDPOINT-DEPENDENT $\downarrow$ WORK IS PATH INDEPENDENT

IRROTATIONAL:
- $\vec{\nabla} \times \vec{E} = 0$
- (STOKES) FLUX OVER A CLOSED LINE IS ZERO

SOLENOIDAL:
- $\vec{\nabla} \cdot \vec{E} = 0$
- (DIVERGENCE) FLUX OVER CLOSED SURFACE IS ZERO

DIVERGENCE: $\int_V (\vec{\nabla} \cdot \vec{f})\, d\tau = \oint_S \vec{f} \cdot d\vec{A}$ (GIVEN A CLOSED OBJECT BOUNDING A SURFACE)

STOKES: $\int_S (\vec{\nabla} \times \vec{f})\, dA = \oint_C \vec{f} \cdot d\vec{l}$

# ELECTROSTATICS

$dq = \lambda\, dl = \sigma\, dA = \rho\, d\tau$

(POTENTIAL: ENERGY / UNIT CHARGE) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\Delta r}$

(FIELD: FORCE / UNIT CHARGE) $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\Delta r}}{\Delta r^2}\, dq = -\nabla V$

(ENERGY) $U(\vec{r}) = qV$

(FORCE) $\vec{F}(\vec{r}) = q\vec{E}$

- $\hat{\Delta r} = \frac{r_1 - r_2}{\Delta r}$
- $\Delta r = r_1 - r_2$

- POINT CHARGE: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\Delta r^2}\hat{\Delta r}$
- DIPOLE $\left(\text{FROM AFAR } \vec{E} \approx \frac{qd}{4\pi\varepsilon_0 r^3}\hat{r}\right)$: $\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\hat{\Delta r}_1}{\Delta r_1^2} - \frac{\hat{\Delta r}_2}{\Delta r_2^2}\right)$ | $V(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \cdot \left(\frac{1}{r_+} - \frac{1}{r_-}\right)$
- INFINITE LINE: $\vec{E}(\vec{r}) = \vec{E}(\rho)\hat{\rho} = \frac{2\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{\rho}\hat{\rho}$ (USE TRIG, $2\int_0^{\frac{\pi}{2}} \dots, \Delta r = \frac{\rho}{\cos\vartheta}$)
- RING: (Z AXIS) $E(z) = \frac{2\pi R z \lambda}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$
- CIRCLE: (Z AXIS) $E(z) = \frac{\sigma\pi}{4\pi\varepsilon_0}\left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2+R^2}}\right)$ [LIMIT BECOMES A PLANE AS $R \to \infty$]

UNIFORM:
- PLANE: $E(\vec{r}) = E(z)\hat{z} = \frac{\sigma}{2\varepsilon_0}\hat{z}$, $V(z) = \frac{\sigma}{2\varepsilon_0}z$
- SHELL: IN $\rightarrow \vec{E} = 0$, $V = \frac{Q}{4\pi\varepsilon_0 R}$ | OUT: $\vec{E} = \frac{Q\,\hat{r}}{4\pi\varepsilon_0 R^2}$, $V = \frac{Q}{4\pi\varepsilon_0 R}$
- SPHERE: IN $\rightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 R^3}$ | OUT: $\vec{E} = \frac{Q\,\hat{r}}{4\pi\varepsilon_0 R^2}$

GAUSS $\Phi_S(\vec{E}) = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{ENC}}{\varepsilon_0}$

- USE SPHERE, CYLINDER OR BOX (SEE KIND OF SYMMETRY)
- PARALLEL FLUX HAS 0 INFLUENCE $\left(\frac{\pi}{2} \rightarrow E \cdot A\right)$

DIFFERENTIAL: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$, $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

FOR WHEN YOU KNOW THE POTENTIAL

$W = \frac{1}{2}\int V\, dq = \frac{\varepsilon_0}{2}\left[\int_V E^2\, d\tau + \oint_S (V\vec{E})\, dA\right] = \frac{\varepsilon_0}{2}\int_{\mathbb{R}^3} E^2\, d\tau$

(WORK IN ASSEMBLING CHARGE $\Rightarrow$ ALWAYS $>0$)

IN A PERFECT CONDUCTOR $W = \frac{V}{2}Q$ (EQUIPOTENTIAL) | FOR POINT CHARGE SYSTEMS: $\sum$ PAIRWISE ENERGIES

$\mu = \frac{\varepsilon_0}{2}E^2$ ("ENERGY DENSITY" OR "ELECTROSTATIC PRESSURE") $\rightarrow$ LOOKS LIKE $\frac{1}{2}mv^2$

WHEN CROSSING SURFACES: (LOCALLY A FLAT SURFACE)

$E^{\perp}_{OUT} - E^{\perp}_{IN} = \frac{\sigma}{\varepsilon_0}$

$E^{\parallel}_{OUT} = E^{\parallel}_{IN}$

$V_{OUT} = V_{IN}$

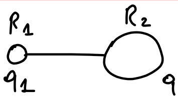
# CONDUCTORS

- A PERFECT CONDUCTOR HAS AN INFINITE NUMBER OF CHARGES INSIDE, SO IT CAN EMULATE ANY FIELD INTERNALLY
- USE GAUSS'S LAW $\Rightarrow$ NO FIELD/CHARGE INSIDE $\left(\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}\right)$
- FARADAY CAGE: $Q_{IN} = -Q_{OUT}$ (IF THE CHARGE IS INSIDE), A SPHERE WITH A CAVITY AND A CHARGE INSIDE HAS UNIFORM CHARGE OUTSIDE

$\downarrow$ THIS BREAKS IF THE CAGE IS CHARGED

$\downarrow$ SAME IF THE $\vec{E}$ IS OUTSIDE

"SPIKEY" CONDUCTORS:




(FAR AWAY)

$q_1 = \left(\frac{R_1}{R_2}\right) \cdot q_2 \Rightarrow q_1 < q_2$ (MORE CHARGE ON THE LARGER SIDE)

$\sigma_1 = \left(\frac{R_2}{R_1}\right) \cdot \sigma_2 \Rightarrow \sigma_1 > \sigma_2$ (MORE FIELD ON THE SMALLER SIDE)

IF YOU KNOW THE POTENTIAL $V_0$, YOU MAY EXPRESS THE POTENTIAL ON DIFFERENT POINTS OF THE SURFACE AND SOLVE FOR THE CHARGE | DON'T ASSUME THE CONDUCTOR IS UNCHARGED

YOU CAN DECOMPOSE THE CONDUCTOR INTO MULTIPLE PRIMITIVE $V(r) = \int E\, d\vec{r} + c$ AND CHANGE THE $+c$ TERM TO ENSURE $V \equiv$ CONST. OVER THE SURFACE

IF TWO CONDUCTORS ARE CONNECTED:


q IF UNCHARGED, OTHERWISE YOU NEED TO KNOW THE POTENTIAL

# CAPACITORS

$C = Q/\Delta V \mid \Delta V = -\int^{+} \vec{E}\cdot d\vec{\ell}$

- PARALLEL $C = C_1 + C_2$
- SERIES $C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$

$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$ (YOU NEED 2: $Q, C, V$)

$= \frac{1}{2}\varepsilon_0 E^2 Ad$ OR $\mu_0 A d$

$\vec{E} = \frac{\Delta V}{d}$

- PLATES : $C = \varepsilon_0 \frac{A}{d}$
- SPHERES : $C = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$
- CYLINDERS: $C = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}$

WITH A DIELECTRIC $\vec{E}_{NEW} = \frac{\vec{E}}{\varepsilon_r}$

# MULTIPOLE EXPANSION

YOU CAN MODEL WITH TAYLOR SERIES ANY LOCALISED CHARGE AS A DIPOLE.

$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{qd}{r^3}\hat{r}$ OR ANY LOCALISED $\vec{B}$

# DIELECTRICS

THE DIELECTRIC BEHAVES AS A WEAKER CONDUCTOR $\Leftrightarrow$ AS $\chi_\varepsilon \to \infty$, THE DIELECTRIC BEHAVES AS A CONDUCTOR

DIPOLE $q^- \xrightarrow{\vec{d}} q^+$

MATTER IS MADE OF DIPOLES THAT CAN ROTATE

BOUND CHARGE $\neq$ FREE CHARGE : NOT IN A DIPOLE OF THE OBJECT

(SINGLE DIPOLE)

- DIPOLE MOMENT $\vec{p} = q\vec{d}$ (IDEAL IF $d \ll 1$)
- TORQUE $N = \tau = \vec{p} \times \vec{E} = I\vec{\alpha}$
- $E^+ - E^- = (\vec{d}\cdot\vec{\nabla})\vec{E}$ } FIELD DIFFERENCE PRODUCED BY A DIPOLE
- $\boxed{U = qV = \vec{p}\cdot\vec{E}}$, $\boxed{\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}}$ → FORCE ON THE DIPOLE FROM AN EXT FIELD
- $\vec{P} = \frac{d\vec{p}}{d\tau}$ (POLARISATION: DIPOLE MOMENT PER UNIT VOLUME)

$= \varepsilon_0 \chi_\varepsilon \vec{E}$ (IN LINEAR DIPOLES)

$\boxed{\sigma_b = -\vec{P}\cdot\hat{n}, \quad \rho_b = -\vec{\nabla}\cdot\vec{P}}$

$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\Delta\hat{r}\cdot d\vec{p}}{\Delta r^2}$

SPLIT THE PROBLEM IN TWO CALCULATIONS $= \frac{1}{4\pi\varepsilon_0}\left[\int_V \frac{\rho_b}{\Delta r}d\tau + \int_S \frac{\sigma_b}{\Delta r}dA\right]$

A DIPOLE PRODUCES A FIELD. FROM AFAR: $(\vec{p}\cdot\vec{\nabla})\vec{E} \approx \frac{\vec{p}}{4\pi\varepsilon_0 x^3}$

## POISSON THEOREM:

IF TWO FIELDS ARE EQUAL IN A BOUNDED AREA / VOLUME, THEY ARE THE SAME EVERYWHERE (FOLLOWING $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$)

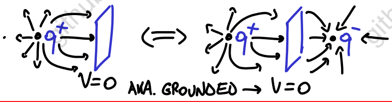
↓

UNIQUENESS OF THE SOLUTION

**IMAGE CHARGES METHOD**

→ CONSTRUCT ANOTHER SYSTEM WHICH IS EQUAL UP TO AN AREA / VOLUME IN ORDER TO FIND THE WHOLE FIELD OF A HARDER SYSTEM, WHICH WOULD HAVE BEEN DIFFICULT

IE.




$\rho = \rho_b + \rho_f \longrightarrow \vec{D} = \varepsilon_0\vec{E} + \vec{P}$ (ELECTRIC DISPLACEMENT)

(BOUND CHARGE) (FREE CHARGE)

$= \varepsilon_0\vec{E} + \varepsilon_0\chi_\varepsilon\vec{E}$ (LINEAR DIELECTRICS) ($\varepsilon_r = \chi_\varepsilon + 1$) HOW EASILY THE MATERIAL POLARISES; RELATIVE PERMITTIVITY

$= \varepsilon_0\varepsilon_r\vec{E}$

$\boxed{\text{OUTSIDE THE DIELECTRIC, } \vec{P} = 0 \Rightarrow \vec{D} = \varepsilon_0\vec{E}}$

$\boxed{\vec{\nabla}\times\vec{D} \neq 0}$

$\varepsilon_0\varepsilon_r = \varepsilon$ PERMITTIVITY

$\Phi_S(\vec{D}) = \oint_S \vec{D}\cdot dA = Q^{FREE}_{ENC}$ ($\varepsilon_0$ IS ON THE LEFT SIDE)

FOR $m,h \ll 1$ $\begin{cases} f(x+m) \approx f(x) + mf'(x) \\ f(x+h) \approx f(x) + hf'(x)\end{cases} \Rightarrow f(x+m) - f(x+h) \approx (m-h)\cdot f'(x)$

LIKEWISE $\vec{E}(\vec{x}_1) - \vec{E}(\vec{x}_2) \approx \vec{\varepsilon}\cdot\nabla\vec{E}(\vec{x}_1)$ FOR $\vec{x}_2 = \vec{x}_1 + \vec{\varepsilon}$ →

THE DIPOLE IS IN A FIELD, THE FORCE IT EXPERIENCES IS GIVEN BY: $\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$ (WHERE $\vec{E}$ IS EXTERNAL)

LIKEWISE, YOU GET ITS ENERGY IN A POTENTIAL FIELD $W = (\vec{p}\cdot\vec{\nabla})V = \vec{p}\cdot E(\vec{r})$

(LINEAR HOMOTROPIC) FIND $\vec{D}$ BY GAUSS' LAW

$\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$

INSIDE DIELECTRIC: $\vec{E} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r}$; OUTSIDE: $\vec{E} = \frac{\vec{D}}{\varepsilon_0}$ } HENCE: $\vec{E}_{IN} = \frac{\vec{E}_{OUT}}{\varepsilon_r}$ (THE FIELD IS REDUCED BY $\varepsilon_r$)

NOTE THAT KIRCHOFF'S LAW WORKS ON THE ASSUMPTION OF INFORMATION TRAVELLING INSTANTANEOUSLY IN THE CIRCUIT

DIPOLE EQUATIONS

$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}}{\Delta r^3}\hat{r}$

$V \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}\cdot\hat{r}}{\Delta r^2}$

# CURRENT

- $I = \frac{dq}{dt}$, $A = \frac{C}{s}$ (STATIONARY CURRENT $I(t) \equiv c$)
- $v_d$ (PARTICLE VELOCITY) → $\langle v_d \rangle \propto c$
- IF PARTICLES ARE $e^-$, $I = -kv_d$ (NEGATIVE)
- $dI = nqv_d \cdot ds$ → CURRENT DENSITY : $\vec{J} = nq\vec{v}_d$
- $I = \Phi_S(\vec{J})$
- $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ (IF $\rho \equiv c$, $\vec{\nabla}\cdot\vec{J} = 0$) → CHARGE IS STILL OR MOVING UNIFORMLY

- $\Delta V = RI$ (OHM'S LAW)
- $P_{(POWER)} = \Delta V\cdot I = RI^2 = \frac{\Delta V^2}{R}$
- $W = Q\Delta V = \Delta V\cdot I\,dt$
- $\binom{CIRCUIT}{\Delta V}$ $\varepsilon = \oint_{CIRCUIT} \vec{E}\cdot d\vec{\ell}$ (EMF)

- CONSERVATION OF CHARGE IN CIRCUITS : $\forall$ VERTICES $\sum I_i = 0$ | OR $\oint_S \vec{J}\cdot d\vec{S} = 0$
- VOLTAGE LAW : $\forall$ CLOSED LOOPS $\sum V_i = 0$
- SOLVE A CIRCUIT BY WRITING ALL EQUATIONS AND SOLVING } SHOULD BE MANY AND ELEMENTARY

## RESISTORS

- CONDUCTIVITY (WIRE) $\sigma = \frac{ne^2\tau}{2m_e}$ ] → $\vec{J} = \sigma\vec{E}$ ($R_{WIRE} = \frac{\ell}{\sigma S}$)
- PARALLEL : $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$
- SERIES : $R_T = R_1 + R_2$

CAPACITOR DISCHARGE : $\Delta V = RI(t) = \frac{Q(t)}{C}$, $I(t) = \frac{d}{dt}Q(t)$

$Q(t) = Q_0 e^{-\frac{1}{RC}t}$

WORK: $dW = \vec{F}\cdot d\vec{\ell} \Rightarrow q(\vec{v}\times\vec{B})\cdot\vec{v}\,dt = 0$

$\vec{F} = \int_V \vec{J}\times\vec{B}\,d\tau = I\int d\vec{\ell}\times\vec{B}$

# MAGNETIC FIELDS

- (LORENTZ FORCE) $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$
- (MAGNETIC FORCE) $\vec{F} = q\vec{v}\times\vec{B}$
- UNIFORM $\uparrow B\cdot\hat{z}$ ⊕ $v_d \neq 0$

CIRCULAR PATH → $(v\cdot\hat{z} = 0)$ $qvB\hat{y} = m\cdot\frac{v^2}{R}$ (CENTR)

HELIX PATH → $(v\cdot\hat{z} \neq 0)$ $r \to \frac{m\cdot v\sin\vartheta}{qB}$

$v_{\parallel}\cdot T = \frac{2\pi m}{qB}\cdot\sin\vartheta$

VELOCITY SELECTOR : $\vec{F}_{EL} = q\vec{E}$ ⊕ $\vec{F}_{MAG} = q\vec{v}\times\vec{B}$ ; $\downarrow\vec{B}, \uparrow\vec{E}$ UNIF. FIELDS → UNDEFLECTED PARTICLES

UNDEFLECTED IF $F_{EL} = F_{MAG} \to v_{\perp} = \frac{E}{B}$

MASS SPECTROMETER : → V SELECTOR → $\uparrow B \odot$ $qvB = m\frac{v^2}{R}$, $R = \frac{mv}{qB}$ ($v \equiv c$)

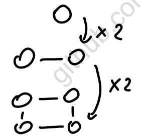
USED IN DIPOLE EXPANSION $(1+x)^n \simeq 1 + nx$ FOR $0 < x \ll 1$

U-SUBSTITUTION

ALMOST ALWAYS, LOOK FOR TRIG. SUBST:

$\frac{z}{R} = \tan\alpha$

$dz = \frac{R}{\cos^2\alpha}d\alpha$ $|\alpha \in [\dots,\dots]$



Multipole expansion is about writing $\vec{B}$ as a Taylor expansion where each term is a multi-pole of increasing number of poles

Magnetic dipoles have $B \propto \frac{1}{z^3}$ because the monopole term in the expansion is zero because B does not have monopoles / $\nabla \cdot B = 0$

## Biot-Savart Law

$\cdot \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \oint_{circuit} \frac{d\vec{\ell}' \times \hat{\Delta r}}{\Delta r^2}$ ⊕ $I = \int_{S'} J \cdot dS$

$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}) \times \hat{\Delta r}}{\Delta r^2} d\tau$ (generally too hard)

· superposition holds for $\vec{B}$

Inverting $\vec{I}$ must invert $\vec{B}$ (use it to show $B=0$)

Given I is coming out, by the right hand rule: you draw a current loop encompassing a current. Use the right hand rule to pick the direction

## Ampere's Law

(use if $\vec{B} \equiv c$ over wire) Ex: $2x \, I_{ENC}$

$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ENC} \rightarrow$

$\int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{S} = \mu_0 \int \vec{J} \cdot d\vec{S} \rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \rightarrow$

Magnetic monopoles don't exist

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \end{cases}$$

$J = J^{free} + J^d$, $J^{displacement} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

## Force on two wires $\left(\frac{dF}{d\ell}\right)$

$I_1 \uparrow \; d \; \uparrow I_2$

$\cdot \vec{F} = I_2 \int d\vec{\ell}_2 \times \vec{B}_1$ (M. field from wire 1 causes force in wire 2)

$= I_2 B_1 \int d\ell \rightarrow \frac{F}{L} = I_2 B_1 = \frac{\mu_0 I_1 I_2}{2\pi d}$

Counts as a change in coordinates — not affecting the result

## $\vec{B}$ from infinite wire with a uniform current

(cylindrical symmetry)

$\cdot d\vec{\ell} \times \hat{\Delta r} \rightarrow d\ell \cos\alpha \; / \; \vartheta$

$\cdot \Delta r = z/\cos\alpha$

$\cdot \ell = z \tan\vartheta \rightarrow d\ell = \left(\frac{z}{\cos^2\vartheta}\right) d\vartheta$

$B = \frac{\mu_0}{4\pi} I \int \frac{z}{\frac{\cos^2\alpha}{\frac{z^2}{\cos^2\alpha}}} \cos\vartheta \, d\vartheta = \frac{\mu_0 I}{4\pi z} \int \cos\vartheta \, d\vartheta \rightarrow \frac{\mu_0 I}{2\pi z} \cdot \hat{\varphi}$

## $\vec{B}$ from a ring with uniform current (on z axis)

(cylindrical symmetry)

$\cdot |d\vec{\ell}' \times \hat{\Delta r}| = \cos\vartheta \, d\ell$

$\cdot \cos\alpha = \frac{R}{\Delta r} \rightarrow \Delta r = \sqrt{R^2 + z}$ (constant)

$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\ell \times \hat{\Delta r}}{\Delta r^2} = \frac{\mu_0}{4\pi} \frac{I}{\Delta r^2} \hat{z} \cos\alpha \int d\ell$

$= \frac{\mu_0}{4\pi} \cdot I \cdot \frac{R \cdot \hat{z}}{(R^2+z^2)^{3/2}} \cdot 2\pi R = \frac{\mu_0 R^2 I}{2(R^2+z^2)^{3/2}} \hat{z}$

(From afar $z \gg R$) $\frac{\mu_0}{2\pi} \cdot \frac{\pi R^2}{z^3} \sim \frac{1}{2\pi\varepsilon_0} \cdot \frac{p}{d^3}$ (dipole) → since there are no magnetic monopoles, the expansion behaves as a dipole

## Magnetic vector potential

· Equivalent of $V(\vec{r})$ for $\vec{B}(\vec{r})$

$\cdot \vec{\nabla} \cdot \vec{B} = 0 = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$

$\cdot \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$

$= \nabla(\vec{\nabla} \cdot \vec{A}) - \nabla^2 A$ (cancel with a transformation)

$\begin{cases} A' = A + \nabla f \\ \nabla^2 f = -\vec{\nabla} \cdot \vec{A}' \neq 0 \end{cases}$

$\boxed{\nabla^2 A = -\mu_0 \vec{J}}$

## Gauge transformations

$\begin{cases} V' = V + c \\ A' = A + \nabla f \end{cases}$ (properties are kept)

When you are obtaining the $\vec{B}$ field of an object via integrations of a smaller object with known $d\vec{B}$, integrate wrt $dI$

Rotating disk $= \int B \, dI$ of circles

Magnetic moment of a sphere $= \int B \, dI$ of circles

## Integral law

(I const)

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r})}{\Delta r} d\tau = \frac{\mu_0}{4\pi} I \int \frac{d\ell}{\Delta r}$

## Solenoid

$L \gg 1$, $I$, $R$, $d \ll 1$

$\cdot n = \frac{\text{turns}}{\text{length}}$,

1. Find $\hat{B}$
2. Find symm.
3. Use Ampere more times

· Cylindrical symmetry $\rightarrow \hat{r}$: flipping the current inverts the $\vec{B}$ direction ⊕ the shape is still the same $\Rightarrow B_r = 0$

$\cdot \hat{\varphi}$: $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ENC} \approx 0 \Rightarrow B_\varphi = 0$ (tightly wound)

$\cdot \hat{z}$: (outside) · top/bottom have no contribution (h) *

· sides: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{IN}$

$h[B(a) - B(\ell)] = 0$ •

$B(a) = B(\ell) = B(\infty) = 0$

(inside) $\oint \vec{B} \cdot d\vec{\ell} = (*) + \left(\begin{smallmatrix} B \equiv c \text{ over} \\ h \text{ segment} \end{smallmatrix} \bullet\right) = Bh = \mu_0 I_{IN} = \mu_0 n h I$

$B_z = \mu_0 n I$ → makes sense as it is symmetri

## Boundary conditions

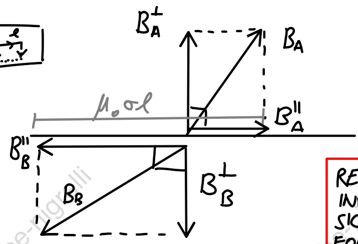
$I \downarrow$

(cross surface → A discontinuous)

$\cdot B^\perp_{ABOVE} = B^\perp_{BELOW}$

$\cdot B^{\parallel}_{ABOVE} - B^{\parallel}_{BELOW} = \mu_0 \sigma \ell$, $\boxed{\sigma = \frac{dK}{dx}}$

Density of I per side




($A_{BELOW} \cong A_{ABOVE}$, like V)

Remember to invert the sign of current for negative streams ($e^-$ flow)

$\boxed{B_{ABOVE} - B_{BELOW} = \mu_0 (I \times \hat{n})}$

Pauli exclusion principle: $\sum m_i = 0$ in diamagnets

As electric dipoles polarise from $\vec{E}$, magnetic dipoles "magnetise" (aka. align) in $\vec{B}$

# $\vec{B}$ in matter

· $e^-$ can orbit
· $e^-$ can spin
} dipoles with moment $\vec{m} = I\vec{S}$ (surface)

· with $\vec{B}_{EXT}$, dipoles align to m. field — diamagnets: opposite (see orbits); ferro/para magnets: parallel (see spin)

## $\vec{B}$ non-uniform

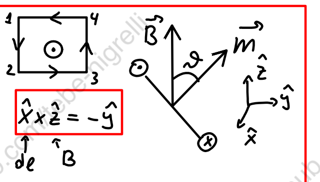
$(\vec{B} \equiv c \Rightarrow \vec{F}_{closed\ loop} = 0)$

$\vec{F} = (\vec{m} \cdot \vec{\nabla})\vec{B} \iff \vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$

$\vec{B}$ induced disappears: · Para: attract one pole → ∥ to $\vec{B}$ → add to the field
· Dia: · repel both · poles → ⊥ to $\vec{B}$ → subtract from the field
$\vec{B}$ induced stays: · Ferro: attract both poles → ∥ to $\vec{B}$, attracts field lines → aquires and keeps a field

Iron becoming magnetic

## Square uniform circuit + field




$\cdot \vec{F}_{1,2} = -F_{3,4} = I \int d\vec{\ell} \times \vec{B}$ $\boxed{\hat{x} \times \hat{z} = -\hat{y}}$

$= -IBa\hat{y}$

Result in net torque: * $\leftarrow$ $N = \vec{m} \times \vec{B}$ } tries to align the circuit ⇒ result is parallel

$= aF\sin\vartheta \hat{x} = \underbrace{Ia^2}_{m} B \sin\vartheta \hat{x}$

$\cdot \vec{F}_{2,3} = -\vec{F}_{4,1} = I \int d\vec{\ell} \times \vec{B}$ $\quad \hat{y} \times \hat{z} = \hat{x}$ (with $\cos\vartheta$) } expect there to be some cancellation

$= IBa\cos\vartheta$

* Applying $\ddot{x} = -kx$, rotation

$N = I_k \alpha = \vec{F} \times \vec{r}$

$= m \cdot \ddot{\vartheta} = -m \cdot B\sin\vartheta$ (small angles $\sin\vartheta \approx \vartheta$)

$\ddot{\vartheta} = -m\vartheta \rightarrow \vartheta = \sin(\sqrt{B}\vartheta)$, $\omega = \sqrt{B} = \frac{2\pi}{T}$ } $T = \frac{2\pi}{\sqrt{B}}$ ($4 \times B \Downarrow T/2$)

The effects of $\vec{B}$ on an object are split into an equivalent $(J_b + K_b)$ system

## $\vec{B}$ and orbits

$I = \frac{Q}{T} = -\frac{e}{T}$ (single $e^-$ orbiting charge)

⊕ $T = \frac{2\pi}{\omega} = \frac{2\pi R}{v} \rightarrow \boxed{I = -\frac{e v_0}{2\pi R}}$

$m = I\vec{S} = I\pi R^2 = -\frac{1}{2} e v_0 R \hat{z}$

$\cdot q\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{R^2} = m_e \frac{v_0^2}{R} = [\ldots]$ } without $\vec{B}$ influence

(orbits around proton) $v_0 = \frac{q}{\sqrt{4\pi\varepsilon_0 m_e R}}$

· Orbit adding $\vec{B}$: $F_{TOT} = F_{EL} + F_{MAG}$ $\quad R \; e^- \; \uparrow\hat{B}$

(abstract induction only looking at stable situation) $= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{R^2} + evB = m_e \frac{v^2}{R}$ (assuming R constant)

$\boxed{\frac{m_e}{R}(v^2 - v_0^2) = evB}$ with $\Delta v \ll 1$ $\boxed{v - v_0 \approx \frac{eB}{2m_e}}$

## Magnetisation

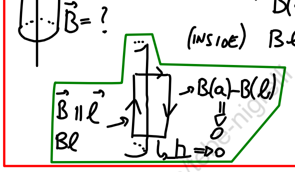
(density of magnetic moment)

$\vec{m} = \vec{M}(\vec{r}) \cdot d\tau$ (magnetic moment) $\iff \vec{P} = \frac{d\vec{p}}{d\tau}$

$\vec{m} = I\vec{S}$ ($I$ current, $S$ surface $\cdot \hat{n}$) } quantised version

$$\begin{cases} \vec{J}_b = \vec{\nabla} \times \vec{M} & \text{(inner current density)} \\ \vec{K}_b = \vec{M} \times \hat{n} & \text{(surface current density)} \end{cases}$$

$\vec{M} \equiv c \Rightarrow \vec{J}_b = 0$



Polarised, no $B_{EXT}$

$\vec{M} = M \cdot \hat{z}$

$\vec{B} = ?$

1. Compute $\vec{J}_b = 0$, $\vec{K}_b = M\hat{\varphi} \Rightarrow$ by RHR, $B \parallel \hat{z}$

2. $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ENC}$

(outside) $B(a) - B(\ell) = 0 \Rightarrow \vec{B} = 0$

(inside) $B\ell = \mu_0 K_b \ell$

$\Downarrow$

$\vec{B} = \mu_0 \vec{K}_b$

$\vec{B} \parallel \vec{\ell}$, $B\ell$, $\rightarrow B(a) - B(\ell) \Downarrow 0$, $h \Rightarrow 0$

**MAGNETISATION**
$\vec{m} = \frac{1}{2}\int d\vec{l} \times I = \vec{S} I$

→ FINDING $\vec{m}$ OF AN OBJECT
$\vec{m} = \int d\vec{m}$, $d\vec{m} = S \cdot dI$
1. FIND S
2. FIND $dI = \frac{dq}{T}$, $\omega = \frac{2\pi}{T}$

REMEMBER THAT IN SPHERICAL COORDINATES,
$\sigma dA = \sigma R^2 \sin\vartheta d\vartheta d\varphi \,\Big|\, \varphi \in [0, 2\pi),\ \vartheta \in [0, \pi)$

THE ONLY DIFFERENCE WITH $\vec{D}$ IS THAT $\vec{P}$ DOESN'T ABSORB THE $\varepsilon_0$ COEFFICIENT

**AMPERE'S LAW IN MATTER**

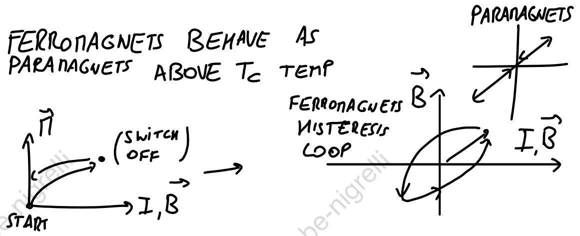
$\boxed{\vec{B} = \mu_0(\vec{H} + \vec{M})} \rightarrow \boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} = \mu_0(\vec{J}_f + \vec{J}_b)$ (DUE TO: $\vec{E}$, $\vec{B}$) $\xrightarrow{\text{BECOMES}} \vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}$

$\boxed{\oint \vec{H} \cdot d\vec{l} = I^{FREE}_{ENCLOSED}}$

$\vec{J}_b$ IS LINKED TO $\vec{B}$
$\vec{J}_f$ CAN BE OBTAINED FROM $\vec{B}, \vec{M}, \varepsilon$

· FERROMAGNETS BEHAVE AS PARAMAGNETS ABOVE $T_c$ TEMP




· LINEAR MEDIA $\boxed{\vec{M} = \chi_m \vec{H}}$ ($\chi_m$ MAGNETIC SUSCEPTIBILITY) → "HOMOGENEOUS" ($\chi_m \equiv c$); "ISOTROPIC" (SAME IN ALL DIRECTIONS) ↓ REMEMBER THIS IS SUPPOSED TO BE A TENSOR

$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_R \text{ RELATIVE PERMITTIVITY}} \vec{H}$

$\vec{B}$ IN CYLINDER GIVEN CONST. MAGNETISATION

A) GET $\vec{J}_b, \vec{K}_b \Rightarrow$ GET $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ENC}$ ⇒ GET $\vec{B} = B\hat{B}$
$\begin{cases} \vec{J}_b = \vec{\nabla} \times \vec{M} = 0 \\ \vec{K}_b = \vec{M} \times \hat{n} = M\hat{\varphi} \end{cases} \Rightarrow \begin{matrix} B(OUT) = 0 \\ B(IN) = \mu_0 K_b \end{matrix}$ (COMPUTE VIA AMPERE'S LAW)

B) $\oint \vec{H} \cdot d\vec{l} = J^{ENC}_{FREE}$ (FASTER) $\Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \ldots$
$\oint \vec{H} \cdot d\vec{l} = 0 \rightarrow \vec{H} = 0 \leftarrow J^{ENC}_f$ IS ZERO, NO P.D. ANYWHERE
$\frac{\vec{B}}{\mu_0} - \vec{M} = 0 \Rightarrow \vec{B} = \mu_0 \vec{M}$

**SUPERCONDUCTOR:**
$\chi_m = -1 \Rightarrow \mu_R = 0 \Rightarrow \vec{B} = 0$ (REPEL ALL FIELD LINES)

**MAGNETIC MOMENT**
$\vec{m} = I\vec{S}$ (AREA OF THE CIRCUIT)
EVALUATE WITH $d\vec{m} = S d\vec{I}$

$\vec{B}$ FROM DISK WITH $\omega \equiv c$ (EVALUATED AT THE CENTER) $A = \pi r^2$
$\vec{B}(z) = \frac{\mu_0}{2}\frac{R^2 I}{(R^2+z^2)^{3/2}}\hat{z}$ (OF CIRCLE) $\rightarrow dI = \frac{q}{T} = \frac{\sigma dA}{2\pi/\omega} = \frac{\sigma(2\pi r)\omega dr}{2\pi} = \sigma\omega r dr$


(WITHOUT $\vec{H}$ YOU CANNOT GET $J_b, K_b$) $\boxed{\oint \vec{H} \cdot d\vec{l} = I^{ENC}_F} \longrightarrow$ THEN COMPUTE $\vec{B}, \vec{M}$ (LINEAR)
$\vec{I} = \vec{J}S$ (S = DIST FROM $\hat{z}$ AXIS)
$\chi_m$ (DIAMAGNETIC MATERIAL)
$(0 < s < R)$: $H 2\pi s = I^{ENC}_F = J\pi s^2$, $\vec{H} = \frac{J_F s}{2}\hat{\varphi}$
$(R < s)$: $H 2\pi s = J_F R^2$

WHEN WRITING THE SYSTEM REMEMBER TO KEEP SIGNS CONSISTENT: $F_{\vec{E}} > F_{\vec{B}}$ EXPECTED, $F_{\vec{E}} - F_{\vec{B}} =$ (SOMETHING EXPECTED TO BE POS)

**FARADAY'S LAW**
$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t}\left(\int_S \vec{B} \cdot d\vec{A}\right)$
(RELATES EMF IN CIRCUIT) = (CHANGE IN FLUX)
BOUNDARY PATH; AREA ENCLOSED BY PATH

DISK: $\vec{B}(z) = \hat{z}\int \frac{\mu_0}{2}\frac{R^2 dI}{(R^2+z^2)^{3/2}} = \hat{z}\frac{\sigma\omega\mu_0}{2}\int_0^R \frac{r^3 dr}{(r^2+z^2)^{3/2}}$
$B = \sigma\omega\frac{\mu_0}{2}\left[\frac{r^2+2z^2}{\sqrt{r^2+z^2}}\right]_0^R$

REMEMBER THAT CURRENT DENSITY AND CHARGE DENSITY ARE INDEPENDENT } $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

**INDUCTANCE**

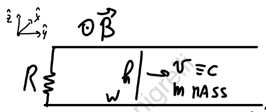
$\Phi_B(\vec{B}_A) = M \cdot I_A$ → M PURELY GEOMETRIC; → $M_{1,2} = M_{2,1}$; → $I_1 = I_2 \Rightarrow \Phi_1 = \Phi_2$


$M = \frac{\mu_0}{4\pi}\oint_1\oint_2 \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\Delta r}$ ("NEUMANN" FORMULA — THE 1800' GUY, NOT THAT ONE)

SELF INDUCTANCE: $\Phi_S(\vec{B}) = LI \rightarrow L := \frac{\oint_S \vec{B} \cdot d\vec{A}}{I}$ (SELF-I.) $> 0$ (CONSERVATION OF ENERGY)
$+I \Rightarrow +\vec{B}$

BACK-EMF: $\varepsilon = \frac{\partial \Phi_A(\vec{B}_A)}{\partial t} = -L\frac{dI}{dt}$ $\boxed{\varepsilon_{NEW} = \varepsilon - \varepsilon_{BACK}}$

$\varepsilon_{INDUCED} = -\frac{d}{dt}\Phi_S(\vec{B})$ ] → THE MINUS, AKA LENZ' LAW, GIVES CONSERVATION OF ENERGY
(USING $\varepsilon = \oint \vec{E} \cdot d\vec{l}$ + STOKES) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
ALSO, $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_S}{\partial t} = -L\frac{\partial I}{\partial t}$
USE IT TO FIND CAPACITANCE




· WRITE FLUX
· DIFFERENTIATE
· SOLVE

$\Phi' > 0$, $B_{INDUCED}$ OPPOSES
SO $I = \circlearrowleft$
$\Phi_S(\vec{B}) = B \cdot wh$
$\varepsilon = -\frac{\partial\Phi}{\partial t} = Bhv = IR$
$I_{INDUCED} = \frac{Bhv}{R}$

$\vec{F} = I\int d\vec{l} \times \vec{B}$ | $\hat{F} = \Leftarrow$
$F = IBh$
$\vec{F} = -\frac{B^2h^2v}{R}\hat{q}$
$W = \int_0^\infty \vec{F} \cdot d\vec{l} = \int_0^\infty P(t)dt$
(SOLVE AS AN ODE)

SIMPLIFIES IF $\vec{B}$ IS ZERO ALMOST EVERYWHERE / OTHERWISE DO U-SUBST: $V = f(\ldots) \rightarrow dt = f'(\ldots)d\ldots$

ENERGY STORED IN $\vec{B}$: $W = \frac{1}{2}\frac{1}{\mu_0}\int_{\mathbb{R}^3} B^2 d\tau \leftrightarrow W = \frac{1}{2}\varepsilon_0\int_{\mathbb{R}^3} E^2 d\tau$ (ENERGY DENSITY)

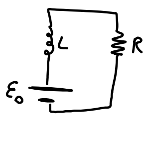
ZERO TERM IF $\vec{E}$ IS FIXED

$\begin{cases} \vec{\nabla} \times \vec{B} = \mu_0\left[J^F + \varepsilon_0\frac{\partial \vec{E}}{\partial t}\right] \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{cases}$ (DISPLACEMENT CURRENT) $\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \cdot \vec{E} = \frac{\varrho}{\varepsilon_0} \end{cases}$ INVARIANT UNDER LORENTZ TRANSF.

$\vec{J}_d = \varepsilon_0\frac{\partial \vec{E}}{\partial t} \longrightarrow \vec{\nabla} \cdot \vec{J}_d = \frac{\partial \varrho}{\partial t}$ → THIS IS THE CURRENT THAT ONE VISUALISES TRANSITING IN A CIRCUIT BETWEEN PLATES

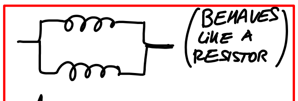
$\Delta V = \int \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi(\vec{B})}{\partial t}$; $\Phi_S(\vec{B}) = LI$; $\frac{\partial}{\partial t}(\Phi_S(\vec{B})) = L\frac{\partial I}{\partial t}$; $\Delta V = -L\frac{\partial I}{\partial t}$

IN QUESTIONS WHERE YOU ARE ASKED L "PER UNIT LENGTH $d\ell$" COMPUTE $\Phi_S(B) \cdot d\ell = LI \leftarrow I = f(\ell)$
HENCE $\frac{\Phi_S(B)}{I} = Lf(\ell)$




$\varepsilon_0 - L\frac{dI}{dt} = RI$
$I' = \frac{\varepsilon_0}{L} - \frac{R}{L}I$, $\frac{dI}{dt} = \left(\frac{\varepsilon_0 - RI}{L}\right)$
$\frac{1}{\varepsilon_0 - RI}dI = \frac{1}{L}dt$, $\frac{\ln(\varepsilon_0 - RI)}{-R} = \frac{t}{L} + c$
$\varepsilon_0 - RI = Ae^{-\frac{R}{L}t}$, $I(t) = \left(\varepsilon_0 - Ae^{-\frac{R}{L}t}\right)/R = \frac{\varepsilon_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

(ENERGY) $\varepsilon_0 = RI + L\frac{dI}{dt} \oplus dq = Idt$ $\boxed{W = \int Vdq}$ THIS MAY BE SIMPLER TO COMPUTE IN CASES WITH $B = 0$ OUT OF THE OBJECT
$\varepsilon_0 Idt = RI^2dt + LIdI$ (FROM BATTERY) (THROUGH RESISTOR) (THROUGH INDUCTOR) $\longrightarrow W = \frac{LI^2}{2} = \frac{1}{2\mu_0}\int_{\mathbb{R}^3} B^2 d\tau$




$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2}$

**MAGNETOSTATICS:**
IF $J \equiv c$, $\begin{cases} \vec{\nabla} \times \vec{B} = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 \end{cases}$ $\vec{B}$ WILL BEHAVE AS $\vec{E}$ IN A VACUUM

| $\vec{E}$ | $\vec{B}$ |
|---|---|
| $\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \varepsilon_0\chi_r\vec{E} = \varepsilon_0\varepsilon_r\vec{E}$ | $\vec{B} = \mu_0\vec{H} + \vec{M} = \mu_0\vec{H} + \mu_0\chi_m\vec{H} = \mu_0\mu_r\vec{H}$ (LINEAR) |
| $\varepsilon_r = (1 + \chi_r)$ | $\mu_r = (1 + \chi_m)$ |
| $\vec{P} = \varepsilon_0\chi_r\vec{E}$ | $\vec{M} = \mu_0\mu_r\vec{H}$ |
| $\vec{E}_p = \frac{\vec{E}_{OUT}}{\varepsilon_r} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r}$ | $\vec{H} = \frac{\vec{B}}{\mu_0\mu_r}$ |

| | |
|---|---|
| $\begin{cases} \sigma_b = \vec{P} \cdot \hat{n} \\ \varrho_b = \vec{\nabla} \cdot \vec{P} \end{cases}$ | $\begin{cases} K_b = \vec{M} \times \hat{n} \\ J_b = \nabla \times \vec{M} \end{cases}$ |
| $\int \sigma_b dA + \int \varrho_b d\tau = Q$ | $\int K_b dl + \int J_b dA = I$ |
| $\oint_S \vec{D} \cdot d\vec{A} = Q^{FREE}_{ENC}$ | $\oint_C \vec{H} \cdot d\vec{l} = I^{FREE}_{ENC}$ |