$X: \Omega \to \mathbb{R}$ (RANDOM VARIABLE)
$X(\omega) = \sum a_i I_{A_i}(\omega)$ (I IS THE BASIS)

→ SIMPLIFIED SETTING BY ASSUMPTION (SUBCASE) →

$B_0(S) := \{X: \Omega \to S\}$
- BOUNDED
- INTEGRABLE

Σ-ALGEBRA
$\forall A \in \Sigma \quad A^c \in \Sigma$
$\forall A, A' \in \Sigma \quad A \cup A' \in \Sigma$

→ $f: \Sigma \to [0,1]$ (PROBABILITY DISTRIBUTION)
- $0 \le f \le 1$
- $\sum f = 1$

$\Delta(C)$ = SPACE OF P. MEASURES

→ $\ell \in \Delta(S)$ AS LOTTERIES

ELEMENTS ARE THE SUPPORT → USUALLY IS C = CONSEQUENCES

INDEPENDENCE
$\forall x,y,z \in X, \quad \forall \alpha \in (0,1)$
$x \succsim y \Rightarrow \alpha x + \bar{\alpha} z \succsim \alpha y + \bar{\alpha} z$

IT'S ALWAYS ABOUT EXISTENCE OF SOME MAPS THAT TURN OBJECTS OF PREFERENCE INTO $\mathbb{R}$

NOTE. SOMETIMES MACCHERONI USES X AND f INTERCHANGEABLY IN THE CONTEXT OF vN-M

vN-M (EU → EXPECTED UTILITY)

$\succsim$ ON SET X
- COMPLETE } WEAK
- TRANSITIVE } ORDER
- ARCHIMEDEAN
- INDEPENDENCE

BOTH ENCODE ENOUGH INFORMATION $\Longleftrightarrow$

$\exists U: X \to \mathbb{R}$ (CARDINALLY UNIQUE, AFFINE) REPRESENTING $\succsim$:

NOTE THAT δ_c = PMF OF $(X = c)$

→ CASE 1 (ABSTRACT vN-M), X = CONVEX SET
$x_1 \succsim x_2 \Longleftrightarrow U(x_1) \ge U(x_2)$

THERE IS A SINGLE PROBABILITY DISTRIBUTION OF X
⇓
THE VIEW ON OPTIMAL CHOICE IS UNIQUE, YOU EITHER HAVE IT OR NOT

→ CASE 2 (vN-M), $X = \Delta_0(C)$ — C CONSEQUENCES
$U(f) = \int_C u(c)\, df(c), \quad f \in \Delta_0(C), \; c \in C$
$= E(u(c))$

$u: C \to \mathbb{R}$ IS A WAY TO RANK CONSEQUENCES
$u(c) := U(\delta_c)$

ARCHIMEDEAN
$\forall x,y,z \in X, \quad x \succ z \succ y$
$\exists \alpha, \beta \in (0,1)$
$\begin{cases} \alpha x + \bar{\alpha} y \succ z \\ z \succ \beta x + \bar{\beta} y \end{cases}$
(THERE IS NO INFINITELY PREFERRED GOOD / IMPLIES DENSITY OF THE SET OF OBJECTS OF PREFERENCE)

UNCERTAINTY AVERSION
$f \sim g \Rightarrow \alpha f + \bar{\alpha} g \succsim f \sim g$

CERTAINTY INDEPENDENCE
$\forall \alpha \in (0,1) \quad x$ = CONSTANT AA ACT
$f \succ g \Rightarrow \alpha f + \bar{\alpha} x \succ \alpha g + \bar{\alpha} x$

AA REVERTS TO vN-M IF $|S| = 1$

AA ACTS CONSTANT IN s WILL SATISFY vN-M BETWEEN THEMSELVES INSIDE AA

IF $|C| = 1$ AND $|S| = 1$ GS BECOMES vN-M

MONOTONICITY
$\forall s \quad f(s) \succsim g(s) \Rightarrow f \succsim g$
(IF A STRATEGY BEATS ANOTHER IN ALL CASES, YOU PREFER IT OVER THE OTHER)

AA

(SEU) $\succsim$ ON $f: S \to \Delta_0(C)$ WITH vN-M AXIOMS + MONOTONICITY + NONTRIVIALITY
↑ SUBJECTIVE

$\Longleftrightarrow \exists u: \Delta_0(C) \to \mathbb{R}$ (NON CONST) $\exists! P \in \Delta(S)$ ST.
$U(f) = \int_S u(f(s))\, dP(s)$ ← FROM vN-M
(NON-ABSTRACT) $= \int_S \left(\int_C u(x)\, df(x) \right) dP(s)$

A (SUBJECTIVE) PROBABILITY MEASURE IS ASSIGNED TO THE STATES OF NATURE S

AA ACT $f: S \to \Delta_0(C)$
PARAMETRISATION OF f BY $s \in S$ (STATES OF NATURE)
$f(s) = \sum_i^n \alpha_i I_{A_i}(s) = \sum_i^n u(f_{A_i}) I_{A_i}(s)$
FOR ALL $s \in A_i$, α_i IS THE CERTAINTY EQUIVALENT OF f_{A_i}

HORSE RACE BET
YOU DON'T KNOW HOW SICK/TIRED (s) THE HORSE IS TODAY, BUT YOU SUBJECTIVELY ATTRIBUTE A $P(s)$ AND ASSUME $f(s_1) \ne f(s_2)$ IE. A SICK HORSE PERFORMS DIFFERENTLY FROM A HEALTHY ONE

AA EXISTS ALSO IN ABSTRACT VERSION, USING A $\sim (X)$ INNER TERM →

NONTRIVIALITY
$\exists f, g \in$ AA ACTS ST. $f \succ g$

IF $|C| = 1$ GS REVERTS TO AA

GS (MEU) → MINIMUM

$\succsim$ ON $\Delta_0(C)$
- ALL AA AXIOMS EXCEPT INDEPENDENCE
- UNCERTAINTY AVERSION } WEAKENING OF INDEPENDENCE
- CERTAINTY INDEPENDENCE }

$\Longleftrightarrow \exists u: \Delta_0(C) \to \mathbb{R}$ (NON. CONST, AFFINE)
$\exists C \subseteq \Delta(S)$ } AKA "CORE"
UNIQUE UP TO CONVEX CLOSURE
$U(f) = \min_{P \in C} \int_S u(f_s)\, dP(s)$ ← FROM vN-M

ENCODES AMBIGUITY AVERSION

PESSIMISM SOLVES THE ELLSBERG PARADOX BY MINIMISING THE WORST OUTCOME OVER ALL POSSIBLE DISTRIBUTIONS OF THE WORLD

GS. ACCOMODATES FOR SELF-DOUBT IN THE BELIEFS

THE INDIVIDUAL HAS THEORIES EXPLAINING HOW THE WORLD WORKS, EACH WITH AN ASSOCIATED SET OF STATES AND A SUBJECTIVE PROBABILITY MEASURE FOR HOW LIKELY EACH STATE IS

SURE-THING PRINCIPLE
$\forall f, g, h, h' \in$ AA ACTS, $\forall E \in \Sigma$
$fEh \succsim gEh \Rightarrow fEh' \succsim gEh'$

← REMINDS OF INDEPENDENCE

STATE INDEPENDENCE
$\forall c, d \in C \quad \forall E \in \Sigma$
$c \succsim d \Longleftrightarrow \forall h \in AA \quad cEh \succsim dEh$
(YOU PREFER C TO D IFF FOR ALL ..E.h MODIFICATIONS THE ONE WITH C IS PREFERRED)

STAKE INDEPENDENCE
$\forall E, F \in \Sigma \quad cEd \succsim cFd$
$c \succ d, c' \succ d' \qquad \Downarrow \qquad c'Ed' \succsim c'Fd'$

↓ TRANSLATION INVARIANCE OF PREFERENCE

SAVAGE (BROKEN BY ELLSBERG)

$\succsim$ ON f
- COMPLETE, TRANSITIVE
- SURE THING PRINCIPLE
- STATE, STAKE INDEPENDENCE
- NONTRIVIALITY OVER C
- DIVISIBILITY

$\Longleftrightarrow \exists v: \Delta(C) \to \mathbb{R}$ (NON CONSTANT, CARD. UNIQUE)
$\exists! P \in \tilde{\Delta}(S)$ ← SPACE OF CONVEX RANGED PROBABILITY MEASURES
$U(f) = \int v(f)\, dP$
P IS THE DM'S SUBJECTIVE BELIEFS OVER THE EVENTS

$\ne$

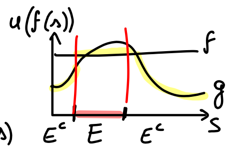
AA
- v DOES NOT NEED AFFINITY
- CONVEXITY OF X NOT NEEDED } MORE APPLICABLE
- P MUST BE CONVEX/NON RANGED/ATOMIC } LESS APPLICABLE

CONVEX-RANGED PROBABILITY MEASURE
$\forall E \in \Sigma, \quad \forall \alpha \in (0,1)$
$\exists F: F \subseteq E,$
$P(F) \le \alpha \cdot P(E)$

DIVISIBILITY
$f \succ g \quad \forall c \in C$
$\exists$ PARTITION $\{E_i\}$ OF S IN Σ ST.
$\forall i \quad cE_i f \succ g$ AND $f \succ cE_i g$

↓ "PREFERENCES CANNOT BE CHANGED BY MINUTE CHANGES" → COMPARE WITH INTEGRABILITY AND CHANGES OF MEASURE 0

$\forall f, g: S \to \Delta(S)$
$(fEg)(s) = \begin{cases} f(s) & s \in E \\ g(s) & s \in E^c \end{cases}$
$(fEg)(s) = f(s) I_E(s) + g(s) I_{E^c}(s)$



ELLSBERG PARADOX

- URN = {30 RED, 60 BLUE OR YELLOW} = 90

EVENTS →	R	B	Y
A.	1	0	0
B.	0	1	0
C.	1	0	1
D.	0	1	1

BETS AKA AA ACTIONS

BY THE SURE THING PRINCIPLE, THE MODIFICATION OF THE COMMON PARTS OF TWO ACTS MUST PRESERVE THEIR PREFERENCE ORDERING:
- IN THE PARADOX, $A \succ B$
- C, D ARE OBTAINED BY CHANGING THE VALUE AT $s = Y$
- WE EXPECT $C \succ D$, BUT $D \succ C$ ✗

} THIS IS A VIOLATION OF THE SURE THING PRINCIPLE

YOU CAN'T USE SAVAGE THEN IN THE ELLSBERG SETTING

CONSTANT AA ACTS CAN BE VISUALISED AS $\Delta_0(C)$, SPACE OF AA ACTS IS F_0 (MARINACCI) AKA. $\Delta_0(C)^S$ FOR S SPACE, δ_c IS THE LOTTERY OF $X = c$

↑ CERTAIN "ACTS" CAN BE SEEN AS LOTTERIES

IN THE GS. SETTING:
$U(f) = \min_{P \in C} \int_{S \in \text{SUPPORT OF } P} u(f(s))\, dP(s)$

$U(A) = \min_{P \in C} \left[\int_{s \in \{R,B,Y\}} u(f(s))\, dP(s) \right]$ WITH $u(f(s)) = E(f(s))$
$= u(R) \cdot P_s(R) + u(R) \cdot P_s(R) + u(R) \cdot P_s(R)$

SET OF ALL THE POSSIBLE PROBABILITY MEASURES IN THE PROBLEM →
$C = \left\{ \begin{array}{l} \{(R; \frac{1}{3}), (B; \frac{2}{3}), (Y; 0)\} \\ \{(R; \frac{1}{3}), (B; \frac{2}{3} - \varepsilon), (Y; \varepsilon)\} \\ \circ\circ\circ \\ \{(R; \frac{1}{3}), (B; 0), (Y; \frac{2}{3})\} \end{array} \right\} \subseteq \Delta(S)$

$\begin{cases} U(A) = \min\{1/3\} = 1/3 \\ U(B) = \min\{0, \frac{2}{3}\} = 0 \\ U(C) = \min[\frac{1}{3}, 1] = 1/3 \\ U(D) = \min\{\frac{2}{3}\} = 2/3 \end{cases}$