# EE301 – LINEAR MOTORS

## Learning Objectives

a. Explain the difference between permanent magnets and electromagnets
b. Identify lines of magnetic flux in a permanent magnet, straight line current carrying conductor, and current-carrying coil
c. Define flux density, magnetic field intensity, and magnetic flux
d. Understand the direction of force on a current-carrying conductor in a magnetic field (Lorentz Force Law)
e. Analyze the Lorentz Force Law in a DC linear motor
f. Understand the effect of a changing magnetic field upon a current-carrying closed path conductor (Faraday/Lenz/Electromotive Force)

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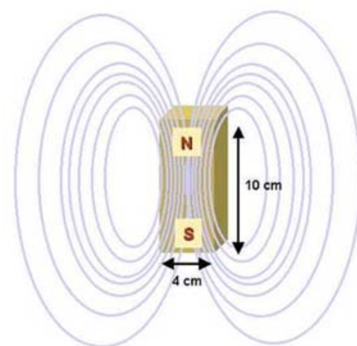
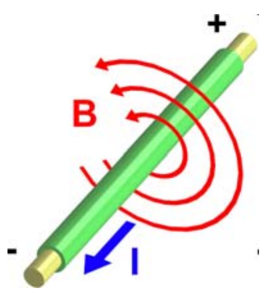
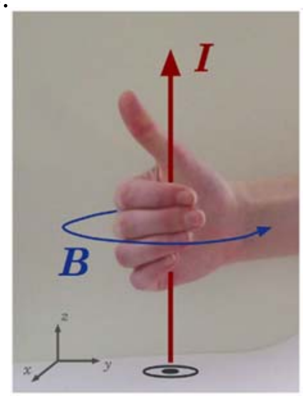
**Magnets and Magnetic Fields** Permanent magnets are devices that are constantly magnetized. Electromagnets, on the other hand, have a magnetic field that exists when electric current is flowing.




Recall from your physics class that all magnets have two poles: north and south, and that opposite poles attract and similar poles repel. Lines of magnetic flux flow from the north pole to the south pole. The magnetic field is strongest close to the magnet, and diminishes with distance from the magnet.

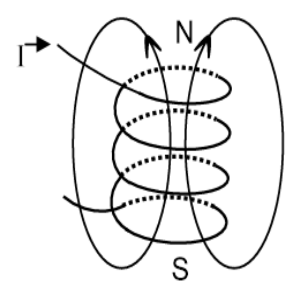
Of particular note for us: current flowing in a wire produces a magnetic field. The direction of the magnetic flux lines associated with the current carrying wire can be found using the Not-Your-Left-Hand Rule:




Right-Hand Rule

## Current Carrying Coil




If current creates a magnetic field, you would surely agree that closely spaced wires create lines of magnetic flux that reinforce each other and create a larger magnetic field. See artist's rendition on the right.

**Magnetic Flux** If you haven't already noticed, we represent the magnetic flux by drawing lines between the north and south poles. Magnetic flux is measured in units of Webers (**Wb**).

The same number of lines leaves the north pole of the magnet as re-enter the south pole. These lines are denser close to the magnet, especially near the poles. The direction of the lines depends on the direction of the current through the coil. Changing the current direction changes the poles of the magnet. The number of lines depends on the strength of the current: higher current produces more lines of flux.

**Magnetic Flux Density (B)** The strength of a magnetic field is defined in terms of the Webers (Wb) per square meter, called the Tesla.

**Lorentz Force Law** Suppose a wire is plopped in a magnetic field, as shown on the right. If we now force a current to flow through the wire (from an external battery), the magnetic field created by the current carrying wire will interact with the existing magnetic field to exert a force on the wire. The magnitude and the direction of this force is given by the Lorentz Force Law:

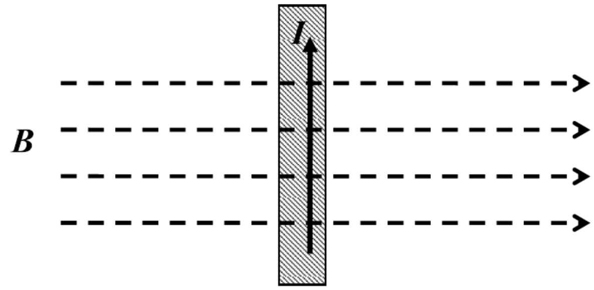


Figure 3. Current Carrying Wire in a Uniform Magnetic Field

$$\boxed{\vec{F}_d = I\vec{L}x\vec{B}}$$

Note that the force is proportional to the current ($I$), the length of the wire ($L$), the magnitude of the magnetic field (as the vector $B$), and the angle between the vectors for $L$ and $B$. This angular dependence is captured as the cross product: $\vec{L}\times\vec{B} = |L||B|\sin\left(\text{angle between } \vec{L} \text{ and } \vec{B}\right)$. Thus, the force on the wire is maximum when the angle between the current and the magnetic field is $90^\circ$, in which case the magnitude of the force is

$$F = ILB$$

**Linear Motor** A linear motor consists of current source, a moveable wire, and a magnetic field. In the picture shown below, the magnetic field is going into page.

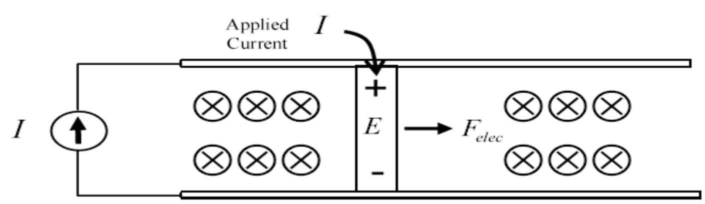


Figure 4: Linear Motor

**Faraday's Law** So, in the picture above, if we apply a current to the wire sitting in the magnetic field, the wire's own magnetic field will interact with the existing magnetic field, inducing a force (the Lorentz Force) that will cause the wire to move. But, would you believe that is not the end of the story? The movement of the conductor (wire) in the magnetic field induces a voltage across the wire, given by Faraday's Law.

$$\boxed{E_{induced} = \left(\vec{u} \text{ x } \vec{\mathrm{B}}\right)L}$$

The polarity of this induced voltage opposes the current from the current source. Note that in the formula above, $u$ us the velocity, since the letter $v$ is already used for something else in EE301. Note also that the geometry above (where the velocity is perpendicular to the magnetic field) means that the magnitude of the induced voltage is given by

$$E_{induced} = uBL$$

So, since you are dying to see how this all fits together, let's talk about the operation of the linear motor.

**Linear Motor Startup** Instead of applying a current source (as in the picture on the prior page), we will apply a voltage source ($V_B$) to our moveable wire sitting in a magnetic field. Initially, the applied voltage $V_B$ is equal to zero (perhaps because a switch is open), the force on the wire is zero, the wire is at rest and the induced voltage across the wire is zero.

**Linear Motor Acceleration** Now, the voltage source is turned on (perhaps a switch is shut). A large current begins to flow in the wire. The value of this current is

$$I = \frac{V_B}{R_{rail}}$$

where the current is limited by the resistance of the moveable wire, $R_{rail}$.

Now, this initial current results in a Lorentz force being applied to the bar: $F = ILB$.

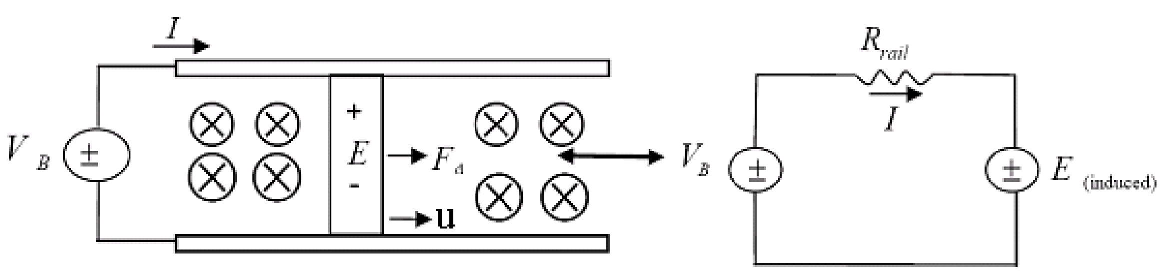
So… the bar begins to move and accelerate.

But… as the bar picks up speed, a voltage is induced across the bar

$$E_{induced} = \left(\vec{u} \text{ x } \vec{\mathrm{B}}\right)L$$

The KVL equation for the circuit becomes

$$V_B - IR_{rail} - E_{induced} = 0 \implies \boxed{I = \frac{V_B - E_{induced}}{R_{rail}}}$$




But…what happens as a result? Answer: As speed ($u$) *increases*, $E_{induced}$ *increases* and current *decreases*.

**Linear Motor at Steady State (frictionless case)**
If there are no frictional forces (or other loads) on the wire, eventually $E_{induced}$ will match $V_B$ and

$$I = \frac{V_B - E_{induced}}{R} = 0$$

Thus, no current will flow through the wire, which means Lorentz force will be zero. The linear motor will maintain a constant speed. This condition is steady state.

## Linear Motor Operation (friction or load exists)

If frictional forces (or other loads) exist, these can be treated as a force ( $F_{load}$ ) that opposes the Lorentz force. When the Lorentz force equals $F_{load}$ , a steady state condition will be reached, and the bar will maintain a constant speed.

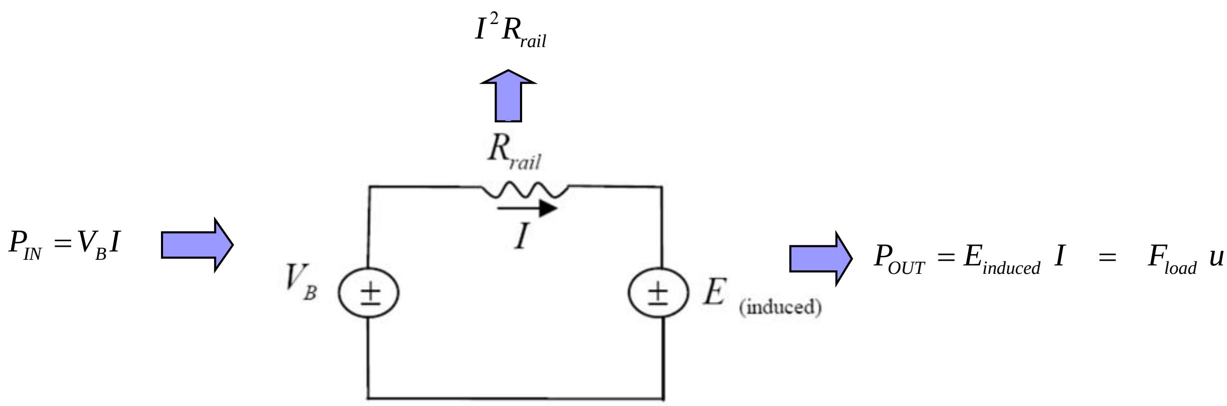
The steady state current can be determined:

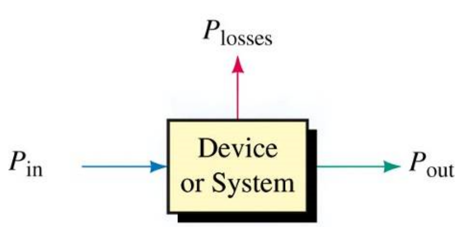
$$\vec{F}_d = I\vec{L}x\vec{B} = F_{load} \quad \Rightarrow \quad \boxed{I = \frac{F_{load}}{BL}}$$

This means the steady state speed can be calculated:

$$\left.\begin{aligned} I &= \frac{V_B - E_{induced}}{R} \\ E_{induced} &= \left(\vec{u} \text{ x } \vec{\mathrm{B}}\right)L \end{aligned}\right\} \quad \boxed{u = \frac{E_{ind}}{BL} = \frac{V_B - IR_{rail}}{BL}}$$

## Power Balance




$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

Example: A 100V linear motor operates with a magnetic field of 0.5 T, and a mechanical loading of 1.0N. The effective length of the bar is 0.1m, and the rail resistance is 0.02 Ω. Find the current flowing through the motor and the velocity of the bar when steady-state conditions are achieved.

Solution:

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Example: A 10 kW (output power) roller coaster that reaches 100 kph. Maximum B-field is 3 T. 450 V DC source. The desired efficiency is 95%. Find the required rail resistance, the source current, and the bar length.

Solution:

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Example: A 240 V linear motor operating in a field with $B = 0.8$ and at a velocity of 26.8 m/sec produces an output power of 20,000 watts at an efficiency of 92%. Determine:

a. The current I
b. The length of the bar
c. The resistance of the rail

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Example 4 A 12 V linear motor is operating in a field with $B = 0.4$. The motors bar is 0.2 meters long, and the rail resistance is 0.025 ohms. The motor is operating against a load of 0.5 N. Determine:

a. The current I
b. The velocity of the linear motor