Question 2

$O_{MIN} = 0V$

$O_{MAX} = 2.5V$

RSE 1500-090 Temperature Range $-10°C$ to $50°C$

Amplifiers @ constant $22°C$

Resistors $\pm 1\%$ and $50\ ppm/°C$

$V_{CC} = 15V$ & $V_{EE} = -15V$

Arm input $0°$ to $90°$

Table 1: Static Characteristics of the system

Characteristic	Value
RSE Temperature Range	$-40°C$ to $85°C$
RSE Input Range	$0°$ to $90°$
RSE Output Range	0V to 10V
RSE Resolution (%FRO)	0.025%
RSE output impedance	$< 100\Omega$
RSE output Noise	$< 5 mV_{rms}$
RSE Linearity error (%FRO)	$\pm 0.1\%$
RSE Thermal Coefficient	$\pm 0.01\% / °C$
Resistor Tolerances	$\pm 1\%$
Resistor Thermal Coefficient	50 PPM/°C
System Characteristics	
Input Range	$0°$ to $90°$
Input Span	$90°$
Output Range	0V to 2.5V
Output Span	2.5V
Temperature Range	$-10°C$ to $50°C$

(1) Error in RSE 1500-090:

Let's assume the ideal line determined by:

min point $A(0°, 0V)$

max point $B(90°, 10V)$

$$\therefore m = \frac{10}{90} = \frac{1}{9} \, V/°C$$

ie $V_{RSE} = \frac{1}{9}\theta$

where θ is angle of arm.

$$V_{RSE\_Error}(\theta, T) = \left(\frac{1}{9}\ V/^{\circ}C\,(1 + 0.0001T)\right)\theta + \ldots$$

* Note my assumption here is that at 0°C there is no offset so any change in temperature from 0°C causes a shift

This isn't all the error, there is also resolution, noise, linearity, etc.

$$V_{RSE\_Error}(\theta, T) = \left[\left(\frac{1}{9}\ V/^{\circ}C\,(1 + 0.0001T)\right)\left\{\theta\left(1 + \frac{\overset{\text{resolution}}{0.00025}}{2}\right)\right\}\right]\left(1 + \overset{\text{linearity}}{0.001} + \frac{\overset{\text{noise (peak)}}{5\times10^{-3}\times\sqrt{2}}}{10}\right)$$

$$V_{RSE\_Error}(\theta, T) = \frac{1}{9}\ V/^{\circ}C\,(1 + 0.0001T)\,\theta\left(1 + \frac{0.00025}{2}\right)\left(1 + 0.001 + 7.071\times10^{-4}\right)$$

Calculate error FRO:

$$V_{RSE\_Error}(90^{\circ}, 50^{\circ}C) = 10.0684\,V$$

$$Error(90^{\circ}, 50^{\circ}C) = 0.684\,\%$$

$$V_{RSE\_Error}(90^{\circ}, -10^{\circ}C) = 10.0083\,V$$

$$Error(90^{\circ}, -10^{\circ}C) = 0.0830\,\%$$

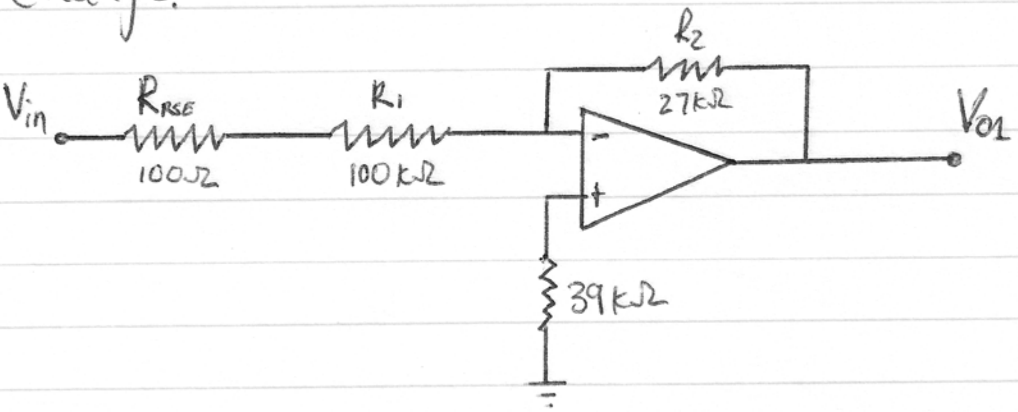
$\therefore$ RSE Max error FRO is 0.684 %

2) Error due to Amp 1:

* The RSE 1500-900 has an (output) impedance $< 100\Omega$ so I'm going to take worst case, when it 99.999.... $\approx 100\Omega$.

** The amplifier section operates in a controlled environment of 22°C ie the temperature doesn't change! I'm going to assume that the amps and resistors dissipate the heat into the air instantaneously so their temperature is always 22°C. I'm also going to assume that the resistors have no drift due to temperature change at 0°C so there is a affect on the resistance due to the ambient temperature being 22°C. X Tests are done on resistors between 22°C and 25°C

* The op amps are "ideal" so the 39kΩ attached to the positive pin in Amp 1 and the 4K7 attached to the positive pin in Amp 2 do not serve a purpose and do not affect the system if their values change.



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Ideally we'd have:

$$R_{RSE} = 0\,\Omega$$
$$R_1 = 100\,k\Omega$$
$$R_2 = 27\,k\Omega$$

$$V_{o2} = -0.27\,V_{in}$$

To maximize error I'm going to maximize the input resistance error in the positive direction and maximize the feedback resistance in the negative direction. I don't need to try the other way round because of the $R_{RSE} = 100\,\Omega$ adding to R_1, I know R_1 should be larger for maximum error.

$T = 22^\circ C$

$$R_1 = 100\,k\Omega\,(1 + \underbrace{0.01}_{\text{accuracy}} + \underbrace{0.000050(22)}_{\text{temperature drift}})$$
$$= 101.11\,k\Omega$$

$$\therefore R_{in} = 101.11\,k\Omega + 100\,\Omega = 101.21\,k\Omega$$

$$R_2 = 27\,k\Omega\,(1 - 0.01 + 0.000050(22))$$
$$= 26.7597\,k\Omega$$
$$= R_F$$

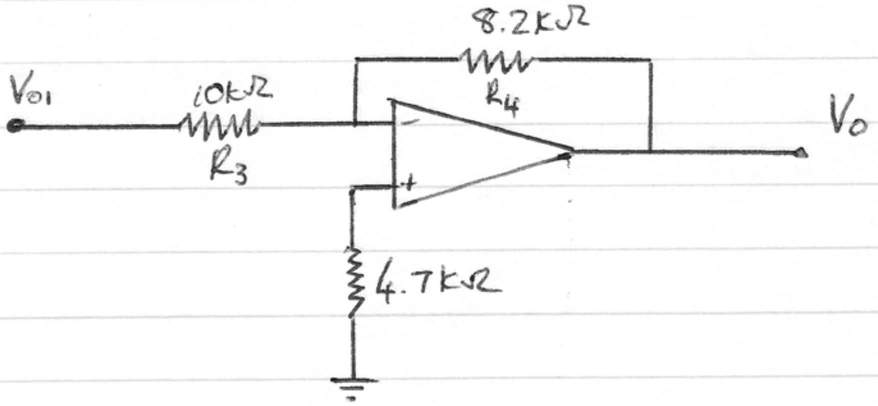
$$\therefore V_{o2} = -\frac{26.7597}{101.21} V_{in}$$

$$= -0.2643977868\ldots V_{in}$$

$$\text{error} = \left| \frac{-0.2643\ldots - (-0.27)}{-0.27} \right| \times 100\% = 2.075\%$$

3 Error due to Amp 2:

* Since these op amps are ideal Their output impedance is zero.



Ideally we'd have

$$V_o = -0.82\,V_{o1}$$

Let $R_3 = 10k\Omega(1 + \overset{\text{accuracy}}{0.01} + \overset{\text{temperature drift}}{0.000050(22)})$ x

$= 10.111k\Omega$

$R_4 = 8.2k\Omega(1 - 0.01 + 0.000050(22))$ x

$= 8.12702k\Omega$

Now $V_o = -0.80378\ldots V_{o1}$

$$\text{error} = \left| \frac{-0.80378\ldots - (-0.82)}{-0.82} \right| \times 100\%$$

$= 1.978\%$

Let $R_3 = 10k\Omega(1 - 0.01 + 0.000050(22))$
$= 9.911\,k\Omega$

$R_2 = 8.2k\Omega(1 + 0.01 + 0.000050(22))$
$= 8.29102\,k\Omega$

Now $V_o = -0.8365\ldots\ V_{o1}$

$$\text{error} = \left| \frac{-0.8365\ldots - (-0.82)}{-0.82} \right| \times 100\%$$

$= 2.018\%$

$\therefore$ Max error for amp 2 is 2.018%

4) Final errors:

Table 2: Errors in each section of the circuit

Section	Error
RSE 1500-090	0.684 %
Amp 1	2.075%
Amp 2	2.018%

The error for the whole system would be determined using:

$$\text{error} = \left\{ \left[\prod_{i=1}^{n} (1 + e_i) \right] - 1 \right\} \times 100\%$$

In this case the total error is then:

$$\text{error} = \left[(1+0.00684)(1+0.02075)(1+0.02018) - 1\right] \times 100\%$$

$$= 4.847\%$$ ← will be less due to temperature drifts being calculated incorrectly.

b) Ideally at full scale rotation we'd have

$$V_0 = 10V(-0.27)(-0.82) = 2.214V$$

Which is in range of the 0V–2.5V of the system, at full error we'd have:

$$V_0 = 10V(1+0.00684) \times (-0.27 \times (1+0.02075)) \times (-0.82(1+0.02018))$$
$$= 2.321V$$

still in range.

I think the gains are pretty good because they are very close to the maximum output of the system of 2.5V.

Introducing full error (2.321V) it uses 92.84% of the maximum output. Since no op amps are ideal this 7% gap is useful because it should cater for the errors introduced by the op amps.

Question 2

a) * As stated in the data sheet the RSE 1500 series has a frequency response of 50 Hz and a gain of −3dB at that point. I'm going to assume it's a first order Butterworth lowpass filter with a cutoff frequency of 50 Hz.

* I'm going to assume the amps (amp 1 and amp 2) form part of the dynamic part of Bentley's model, since they're separate from the sensor. The amps are ideal so their frequency response is the same for all frequencies, therefore, they just apply a gain and do not attenuate frequencies at different levels.

The gain of the amps is 0.2214 (calculated in Question 1)

$$20\log(0.2214) = -13.096 \text{ dB}$$

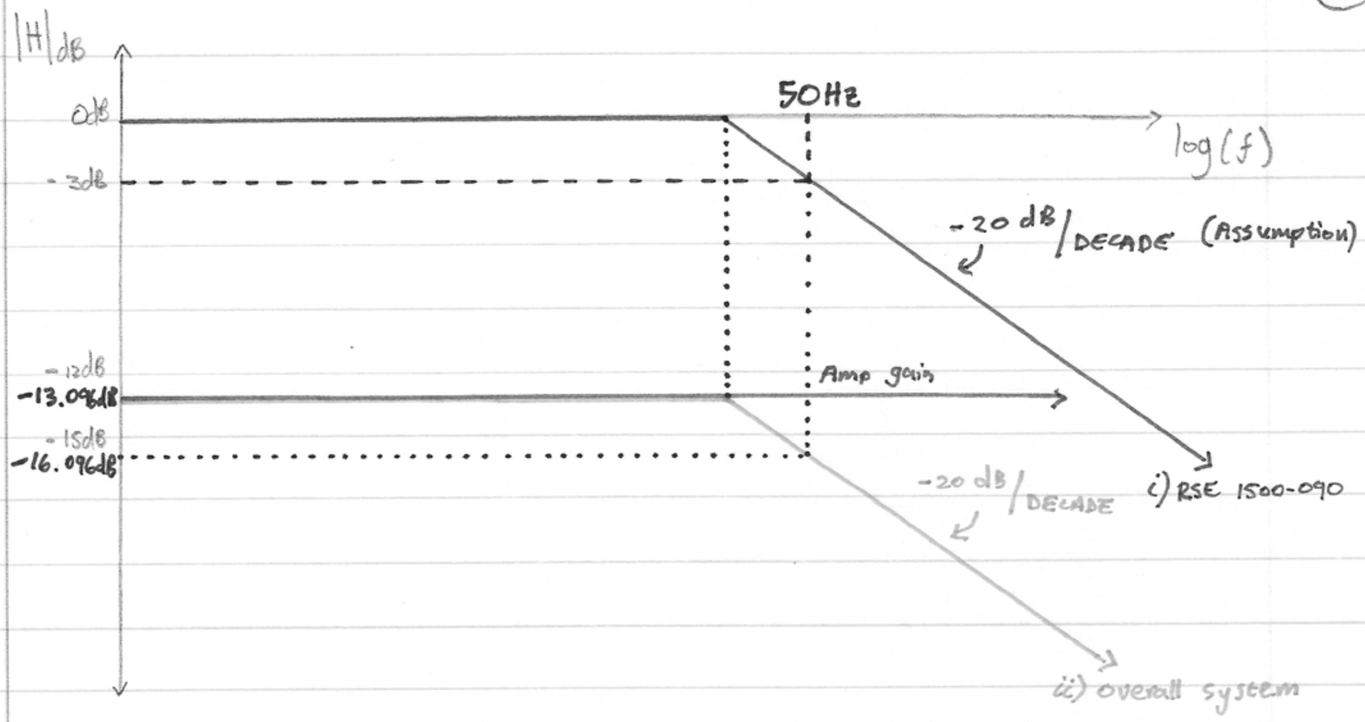


Figure 1: Bode Plots of system Elements and system as a whole.

b) ~~This must mean aliasing error since we can't reduce the static error to 1%.~~

* As mentioned before, I'm assuming it's a Butterworth filter.

$$\boxed{|H| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}}}$$

Butterworth Transfer characteristic

in this case $n=1$ (first order filter)

$$\omega_c = 2\pi(50\,Hz)$$ cutoff frequency

$$= 100\pi$$

I'm looking for the frequency where the signal is 99% to 100% of what it should be.

As mentioned before the op amps are ideal so I can ignore their gain and focus on the RSE low-pass filter.

∴ I need:

$$|H| = 0.99$$

$$0.99 = \frac{1}{\sqrt{1 + \left(\frac{\omega}{100\pi}\right)^2}}$$

$$\therefore \omega = 44.765 \ldots$$

$$f = 7.125 \text{ Hz}$$

∴ At a frequency of about 7 Hz the error due to the dynamic characteristics should be less than 1%

Question 3

* Ideally I'd want to reduce the number of amps in the system since each amp introduces error. However, in order to have only 1 amp in the system it would need to be non-inverting but that can only have gains > 1. Therefore, I need two non-inverting amps.

* In order to reduce error I need the amps to have the "largest" possible gain each, ie, both amps will be the same.

My maximum gain:

$$G_{MAX} \times (10V \times 1.00684) = 2.5V$$

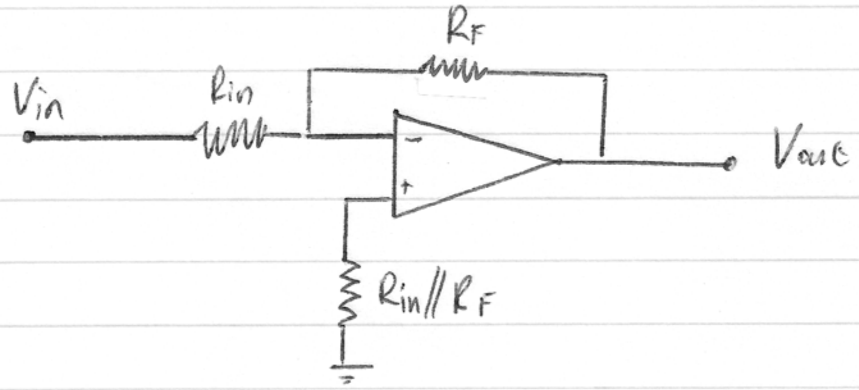
$$\therefore G_{MAX} = 0.2483...$$

$$\sqrt{G_{MAX}} = 0.498298726...$$

So IDEALLY each amp would have a gain of 0.498298... but obviously they have errors so this should be the MAXIMUM gain for each amp.

Designing the amp using standard resistances:

* Designing Amp 2 first since Amp 1 has the 100Ω impedance from the RSE!



$$V_{out} = -\frac{R_F}{R_{in}} V_{in}$$

* test : $\frac{\frac{1.01}{0.99} - 1}{1} \times 100\% = 2.02\%$

$\left| \frac{\frac{0.99}{1.01} - 1}{1} \right| \times 100\% = 1.98\%$ ∴ max R_F for max error

$$\therefore \frac{1.01 R_F}{0.99 R_{in}} = 0.4983$$

$$\therefore R_F = 0.48843 \ldots R_{in}$$

$330k\Omega \rightarrow 161.18k\Omega$

$370k\Omega \rightarrow 180.7196k\Omega$

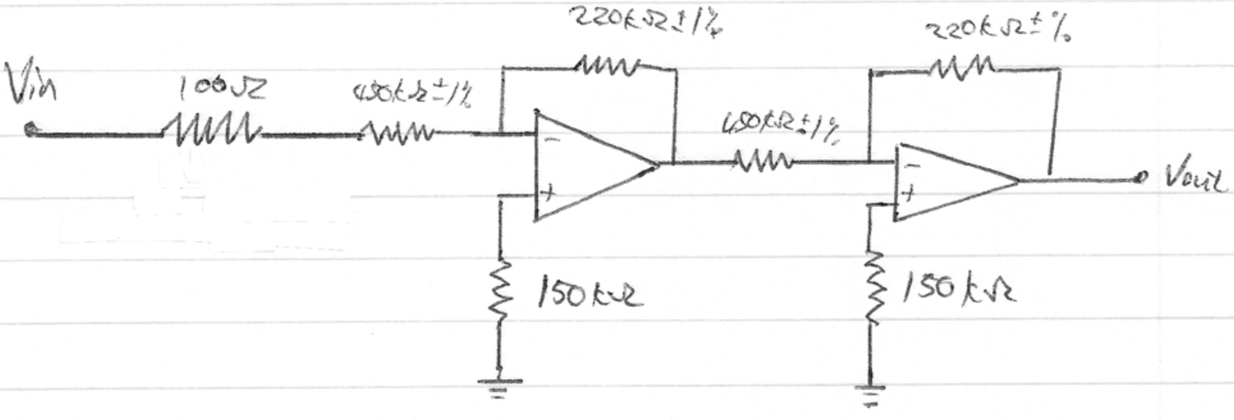
$410k\Omega \rightarrow 200.25688k\Omega$

$450k\Omega \rightarrow 219.7941395k\Omega$

X NEVER ROUND UP!! ALWAYS DOWN FOR GAIN

$\therefore$ Using $R_{in} = 450k\Omega \pm 1\%$
$R_F = 220k\Omega \pm 1\%$

$R_{in} // R_F = 147.76k\Omega \Rightarrow 150k\Omega$



At FRO: $V_{out} = (10V \times 1.00684)\left(-\frac{220 \times 1.01}{(450 \times 0.99) + 100\Omega}\right)\left(-\frac{220 \times 1.01}{450 \times 0.99}\right)$

$= 2.50412313...$

* Assuming the range saturates at 2.5V and doesn't damage the ADC:

X

error = 0.165% FRO