Ultrasonic Doppler Example:

An ultrasonic Doppler flowmeter is used to measure the flowrate of particles suspended in a liquid. The setup is shown in the figure below. The transmitting transducer is transmitting a continuous sound wave at $f = 1$ MHz and the velocity of the particles is in the range of 5 to 20 m/s. The angle $\theta = 30°$, and the speed of sound in the fluid is $c = 1485$ m/s.

(a) Explain the working principle of this flowmeter and give an approximate expression for the flow velocity, v, in terms of the Doppler shift Δf. You may assume that only the frequency of the Doppler signal is used, not the amplitude.

Hint: You may also assume that v/c is small.

(a) **Solution:** The motion will cause a frequency shift of the received signal, which is proportional to the velocity of the fluid. Using the assumptions given in the problem, we can use equation [16.58] in the text book, giving

$$\Delta f = \frac{2f}{c} (\cos\theta)\, v$$
$$\Longrightarrow v = \frac{c\Delta f}{2f\cos\theta}.$$

(b) What is the minimum sampling frequency required for a analog-to-digital converter (ADC) recording Δf?

(b)

Solution: For a flow of 5 m/s, we have

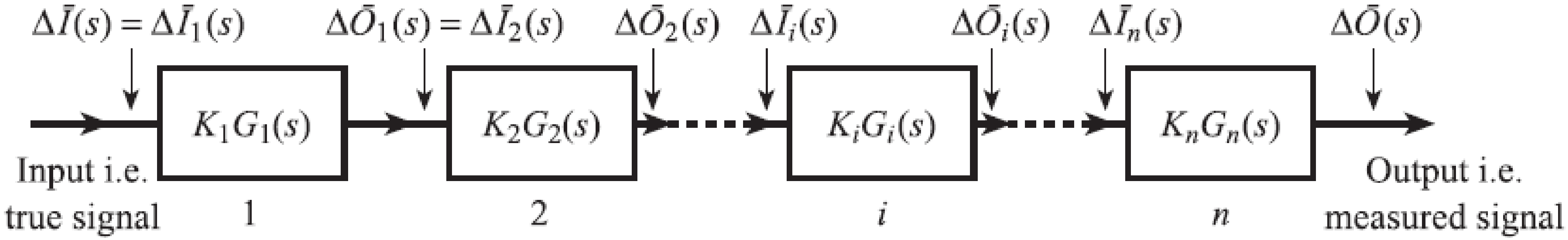
$$\Delta f_5 = \frac{2 \times 10^6}{1485} \cos(30^\circ) \cdot 5 \approx 5831.8 \text{ Hz}$$

For a flow of 20 m/s we have

$$\frac{2 \times 10^6}{1485} \cos(30^\circ) \cdot 20 = 23327 \text{ Hz}$$

The sampling theorem states that we need to sample faster than twice the maxmimum frequency. This means we need to sample faster than $2 \cdot 23327 = 46654$ Hz, i.e. faster than 46.7 kHz.

Dynamic errors in measurement systems



$$\frac{\Delta\bar{O}(s)}{\Delta\bar{I}(s)} = G(s)$$

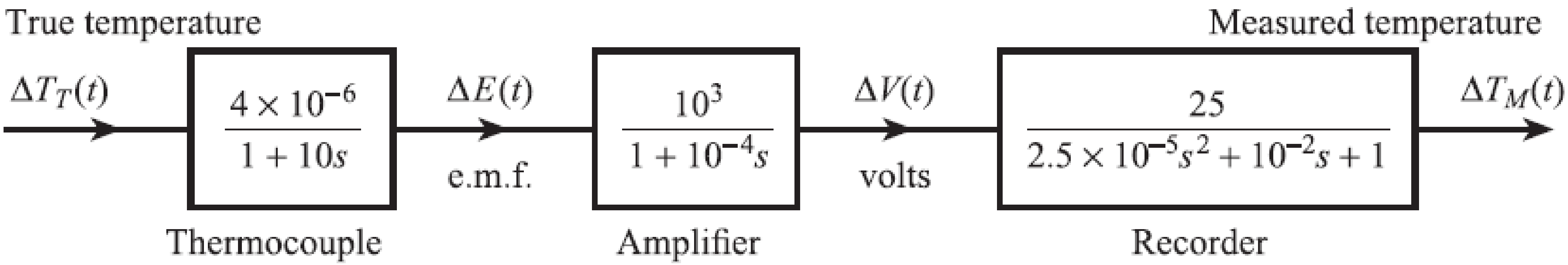
$$E(t) = \mathcal{L}^{-1}[G(s)\Delta\bar{I}(s)] - \Delta I(t)$$

Solution:

$$\begin{aligned} E(t) &= \Delta T_M(t) - \Delta T_T(t) \\ &= -20\{A\, e^{-0.1t} + B\, e^{-10^4 t} - E\, e^{-200t}(1 + 200t)\} \end{aligned}$$

The $Ae^{-0.1t}$ term takes more time to decay

Dynamic errors in measurement systems



We can now calculate the dynamic error of the system for a step input of +20 °C,

$$\Delta \bar{T}_M(s) = 20\frac{1}{s}\frac{1}{(1+10s)}\frac{1}{(1+10^{-4}s)}\frac{1}{(1+1/200s)^2}$$

$$= 20\left\{\frac{1}{s} - \frac{A}{(s+0.1)} - \frac{B}{(s+10^4)} - \frac{Cs+D}{(s+200)^2}\right\}$$

$$\Delta T_M(t) = 20\{u(t) - A\,e^{-0.1t} - B\,e^{-10^4 t} - E\,e^{-200t}(1+200t)\}$$