An LC Filter Design Method for Single-Phase PWM Inverters

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Abstract

In this paper, an analysis and design method of the output LC filter of single-phase PWM inverters is presented. At first, the analytical expressions for the total harmonics of the inductor current and capacitor voltage of the LC filter are derived. As the unique values of the parameters of the LC filter can not be specified based on the total harmonic of the capacitor voltage alone an additional criterion based on the minimum reactive power of the LC filter is used to specify these parameters. Experimental results are included to verify the derived expressions.

1. Introduction

Single-phase voltage-source PWM inverters are commonly used in low and medium power uninterruptible power supplies (UPSs). In such applications, the output of the inverter is usually provided with an output LC filter to reduce the inverter output harmonics. At present, the LC filter is usually designed based on the Fourier series of the inverter output voltage waveform[1]. Accurate results, therefore, cannot be obtained without taking into account a large number of harmonics with the associated complex additions and multiplications. An attempt to derive the analytical expression of the ac side current harmonic of single-phase PWM converters having an inductive filter was reported in [2]. However, extension of the result to the cases of PWM converters having an LC filter has not been reported in the literature.

In this paper, an analysis and design method of the output LC filter of single-phase voltage-source PWM inverter is presented. At first, the analytical expressions of the total harmonics of the output current and voltage of the LC filter are derived. As the unique values of the parameters of the LC filter can not be determined based on the output voltage harmonic specification alone an additional criterion based on the minimum reactive power of the LC filter is used to specify these parameters. The detailed design procedure of the LC filter is outlined. Experimental results are included to verify the derived expressions.

2. Analysis of the Output Current and Voltage Harmonics

The scheme of single-phase voltage-source full-bridge PWM inverter which is used in this investigation is shown in Fig. 1. In this analysis, the followings are assumed :

- The dc source voltage E_d is ripple free and constant.
- The inverter switching devices are assumed as ideal switches.
- The equivalent series resistance of the filter capacitor is neglected.
- The load is a linear load.

2.1 Inductor current harmonic

Based on the circuit of Fig. 1, the output voltage equation of the inverter can be written as

$$v_s = v_o + R_f i_s + L_f \frac{di_s}{dt} \qquad (1)$$

The capacitor voltage v_o and inductor current i_s can be separated into the average (average over one

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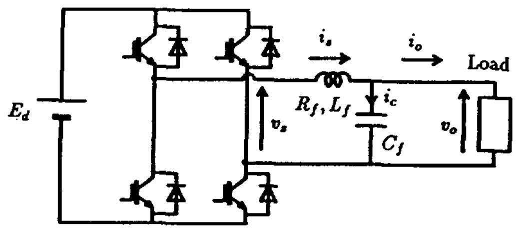

Fig. 1. Single-phase PWM inverter.

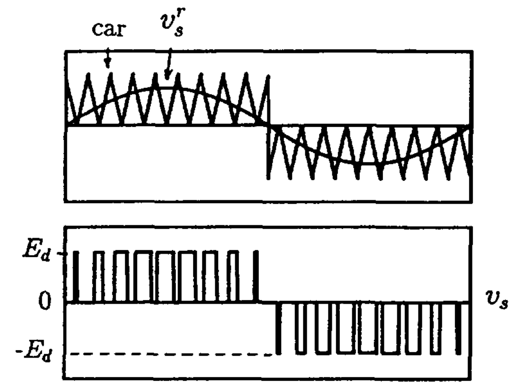

Fig. 2. PWM technique for single-phase inverters.

switching cycle) and the harmonic (ripple) components, that is,

$$\begin{aligned} v_o &= \bar{v}_o + \tilde{v}_o && (2) \\ i_s &= \bar{i}_s + \tilde{i}_s && (3) \end{aligned}$$

Upon substituting eqns. (2)-(3) into eqn. (1), the following equation is obtained

$$v_s = \bar{v}_o + \tilde{v}_o + R_f\left(\bar{i}_s + \tilde{i}_s\right) + L_f\frac{d}{dt}\left(\bar{i}_s + \tilde{i}_s\right) \quad (4)$$

In a well designed filter, the values of $\tilde{v}_o$ and $R_f\tilde{i}_s$ are small compared to $L_f\frac{d\tilde{i}_s}{dt}$ and, therefore, the ripple component of the filter inductor current can be calculated as

$$\tilde{i}_s = \frac{1}{L_f}\int (v_s - \bar{v}_s)dt \quad (5)$$

where

$$\bar{v}_s = \bar{v}_o + R_f\bar{i}_s + L_f\frac{d\bar{i}_s}{dt} \quad (6)$$

is the average value of the output voltage of the inverter.

Although there are various PWM techniques that can be used to control the inverter, the sampling-based PWM techniques are the most commonly used because the implementation is simpler. The most popular implementation of the sampling-based PWM techniques is based on the comparison between a sinusoidal reference and a high-frequency triangular carrier signals, which is also referred to natural sampling. Fig. 2 shows a PWM technique that is commonly used to control the output voltage of single-phase inverters.

Fig. 3(a) shows one pulse of the output voltage of the inverter over one switching period. The relationships among the time intervals T_{OFF} and T_{ON} in relation to the switching period T_s can be written as

$$\begin{aligned} \frac{T_{OFF}}{T_s} &= 1 - \alpha && (7) \\ \frac{T_{ON}}{T_s} &= \alpha && (8) \end{aligned}$$

where

$$\alpha = \frac{\bar{v}_s}{E_d} = k\sin\omega_r t \quad (9)$$

in which $\omega_r = 2\pi f_r$ and f_r is the fundamental output frequency. The factor k means the modulation index and having a maximum value that is equal to unity. Based on the output voltage in Fig. 3(a) and eqn. (5), expression of the harmonic current that flows through the filter inductor can be written as

$$\tilde{i}_s = \frac{E_d}{L_f}\times\begin{cases} \alpha\frac{T_{OFF}}{2} - \alpha(t - t_o) & \text{for } t_o \le t < t_1 \\ -\alpha\frac{T_{OFF}}{2} + (1-\alpha)(t - t_1) & \text{for } t_1 \le t < t_2 \end{cases} \quad (10)$$

The waveform of this current harmonic is shown in Fig. 3(b). The mean square value of this current harmonic over one switching period can be calculated as

$$\begin{aligned} \tilde{I}_s^2 &= \frac{1}{T_s}\int_{t_o}^{t_o+T_s} \tilde{i}_s^2 dt \\ &= \left(\frac{E_d}{L_f f_s}\right)^2\left[\frac{\alpha^2 - 2\alpha^3 + \alpha^4}{12}\right] && (11) \end{aligned}$$

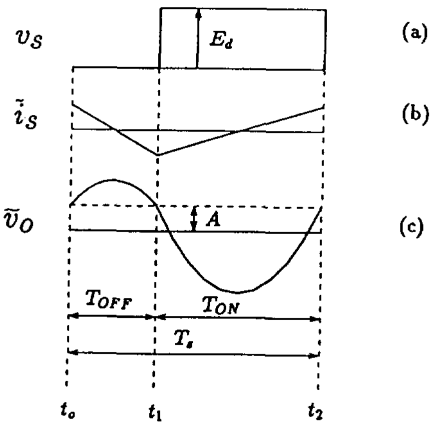

Fig. 3. The detailed output waveforms over one switching period. (a) Inverter output voltage. (b) Inductor current harmonic. (c) Capacitor voltage harmonic.

where f_s is the switching frequency. The rms value of the current harmonic over one period of the fundamental output voltage can be obtained as

$$\begin{aligned} \tilde{I}_{s,av} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \tilde{I}_s^2 d(2\pi f_r t)} \\ &= \frac{E_d}{L_f f_s}\left[\frac{k^2 - \frac{16}{3\pi}k^3 + \frac{3}{4}k^4}{24}\right]^{1/2} \end{aligned} \quad (12)$$

It can be confirmed that the result as given by eqn. (12) is the same as the result in [2]. This current harmonic is important to calculate the additional copper loss in the filter inductor.

2.2 Capacitor voltage harmonic

From the circuit of Fig. 1, the filter capacitor current can be calculated as

$$i_c = i_s - i_o \quad (13)$$

The capacitor and inverter output currents can be separated into the average and harmonic components, that is,

$$i_c = \bar{i}_c + \tilde{i}_c \quad (14)$$
$$i_s = \bar{i}_s + \tilde{i}_s \quad (15)$$

Substituting eqn. (3) and eqns. (14)-(15) into eqn. (13), the following equation is obtained

$$\bar{i}_c + \tilde{i}_c = \bar{i}_s + \tilde{i}_s - \bar{i}_o - \tilde{i}_o \quad (16)$$

As the average and harmonic components in the left and right hand sides of the above equation should be the same, the average and harmonic components of the filter capacitor current can be obtained as

$$\bar{i}_c = \bar{i}_s - \bar{i}_o \quad (17)$$
$$\tilde{i}_c = \tilde{i}_s - \tilde{i}_o \quad (18)$$

If the capacitance of the filter capacitor is quite large then almost all of the filter inductor current harmonic is flowing into the filter capacitor. Thus, the harmonic component of the filter capacitor current can be approximated as

$$\tilde{i}_c \simeq \tilde{i}_s \quad (19)$$

Thus, the result of eqn. (12) is also useful to specify the ripple current rating of the filter capacitor. Due to the harmonic current that flows through the filter capacitor, the voltage across the capacitor will fluctuate arround the average value. The ripple component of the filter capacitor voltage can be calculated as

$$\tilde{v}_o = \frac{1}{C_f}\int \tilde{i}_c dt = \frac{1}{C_f}\int \tilde{i}_s dt \quad (20)$$

Substituting eqn. (10) into eqn. (20) and performing the integration, the following result is obtained

$$\tilde{v}_o = \frac{E_d}{L_f C_f} \times \begin{cases} A + \alpha T_{OFF}\frac{(t-t_o)}{2} - \frac{\alpha}{2}(t-t_o)^2 & \text{for } t_o \le t < t_1 \\ A - \alpha T_{OFF}\frac{(t-t_2)}{2} + (1-\alpha)\frac{(t-t_1)^2}{2} & \text{for } t_1 \le t < t_2 \end{cases} \quad (21)$$

The constant A can be determined from the fact that the average value of eqn. (21) over one switching period should be zero, and the result is

$$A = \frac{\alpha^3(1-\alpha) - \alpha(1-\alpha)^3}{12 f_s^2} \quad (22)$$

Fig. 3(c) shows the waveform of the filter capacitor voltage harmonic. The mean square value of the filter capacitor voltage harmonic over one switching period can be obtained as

$$\tilde{V}_o^2 = \frac{1}{T_s}\int_{t_o}^{t_o+T_s} \tilde{v}_o^2 dt \quad (23)$$

Substituting eqn. (21) into eqn. (23) and performing the integration, the following result is obtained

$$\tilde{V}_o^2 = \left(\frac{E_d}{L_f C_f f_s^2}\right)^2 \left[\frac{\alpha^2 - 5\alpha^4 + 6\alpha^5 - 2\alpha^6}{720}\right] \quad (24)$$

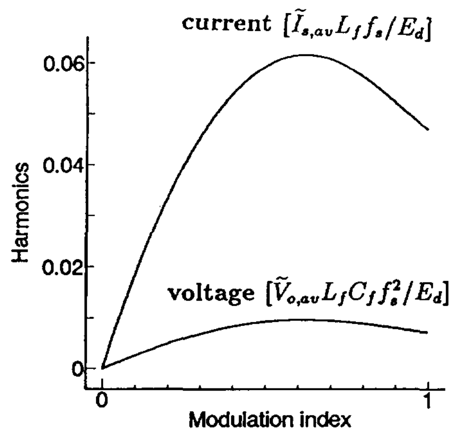

Fig. 4. The inductor current and capacitor voltage harmonics as a function of the modulation index.

The rms value of the output voltage harmonic over one period of the fundamental output voltage can be obtained as

$$\tilde{V}_{o,av} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \tilde{V}_o^2 d(2\pi f_r t)}$$
$$= \frac{E_d}{L_f C_f f_s^2}\left[\frac{k^2 - \frac{15}{4}k^4 + \frac{64}{5\pi}k^5 - \frac{5}{4}k^6}{1440}\right]^{1/2} \quad (25)$$

Fig. 4 shows the rms value of the filter inductor current and capacitor voltage harmonics as a function of the modulation index. The harmonics will be maximum when the modulation index is equal to 0.617.

In practice, only the output voltage harmonic is usually specified. However, as it is shown by eqn. (25), the unique values of the inductance and capacitance of the LC filter can not be specified based on the output voltage harmonic alone. In the next section, an additional criterion based on the minimum reactive power of the LC filter is used to specify the parameters of the LC filter.

3. Design of the Output LC Filter

3.1 Minimization of the reactive power

As it is shown in the previous section, an additional criterion is required to specify the inductance and capacitance of the LC filter. Although a criterion based on the minimum cost, size, losses, etc. can be used as an additional criterion, a criterion based on the minimum reactive power is used in this paper. Minimization of the reactive power also indirectly minimizes the size, losses, and cost of the filter. Additional criterion based on the minimum reactive power was also used in [1]. However, as the harmonics of the LC filter are given in the Fourier series form the closed form expressions of the inductance and capacitance of the LC filter were not obtained.

The reactive power of the LC filter can be calculated as

$$P_r = \omega_r L_f(\overline{I}_s^2 + \tilde{I}_{s,av}^2) + \omega_r C_f\left(\overline{V}_o^2 + \tilde{V}_{o,av}^2\right) \quad (26)$$

where $\overline{I}_s$ and $\overline{V}_o$ are the rms value of the fundamental component of the inductor current and load voltage, respectively. As the harmonic components are much smaller than the fundamental components, eqn. (26) can be simplified into

$$P_r \simeq \omega_r L_f \overline{I}_s^2 + \omega_r C_f \overline{V}_o^2 \quad (27)$$

The rms value of the inductor current can be calculated as

$$\overline{I}_s = \left[\overline{I}_{or}^2 + \left(\overline{I}_{oi} - \omega_r C_f \overline{V}_o\right)^2\right]^{1/2} \quad (28)$$

where $\overline{I}_{or}$ and $\overline{I}_{oi}$ are the rms values of the real and imaginary components of the load current. Substituting eqn. (28) into eqn. (27), the following equation is obtained

$$P_r = \omega_r L_f\left[\overline{I}_{or}^2 + \left(\overline{I}_{oi} - \omega_r C_f \overline{V}_o\right)^2\right] + \omega_r C_f \overline{V}_o^2 \quad (29)$$

Eqn. (25) can be rewritten as

$$C_f = K\frac{E_d}{L_f f_s^2 \tilde{V}_{o,av}} \quad (30)$$

where

$$K = \left[\frac{k^2 - \frac{15}{4}k^4 + \frac{64}{5\pi}k^5 - \frac{5}{4}k^6}{1440}\right]^{1/2} \quad (31)$$

Substituting eqn. (30) into eqn. (29), the reactive power as a function of the inductance of the filter can be obtained

$$P_r = \omega_r L_f\left(\overline{I}_o^2 + \frac{\omega_r^2 E_d^2 K^2 \overline{V}_o}{L_f^2 f_s^4 \tilde{V}_{o,av}^2} - \frac{2\omega_r E_d K \overline{I}_{oi}\overline{V}_o}{L_f f_s^2 \tilde{V}_{o,av}}\right) + \frac{\omega_r E_d K \overline{V}_o^2}{L_f f_s^2 \tilde{V}_{o,av}} \quad (32)$$

where $\overline{I}_o$ is the rms value of the load current. The minimum reactive power is then calculated as

$$\frac{\partial P_r}{\partial L_f} = 0 \quad (33)$$

Table. 1. Experimental conditions.

E_d (V)	f_s (Hz)	f_r (Hz)	L_f (mH)	C_f (μF)	R_L (Ω)	L_L (mH)
150	4000	50	2.1	10.2	4.25	3.2

The result is

$$L_f = \frac{\overline{V}_o}{\overline{I}_o f_s}\left\{K\frac{E_d}{\tilde{V}_{o,av}}\left[1+4\pi^2\left(\frac{f_r}{f_s}\right)^2 K\frac{E_d}{\tilde{V}_{o,av}}\right]\right\}^{1/2} \tag{34}$$

After the inductance of the filter is calculated then the capacitance of the filter can be calculated by eqn. (30).

3.2 Design procedure

Based on the previous analysis, the design procedure of the LC filter can be divided into the following steps:

1) Based on the nominal dc source voltage E_d and nominal load voltage $\overline{V}_o$, we can calculate the nominal modulation index. Because the voltage drop across the filter inductor can not be determined before the parameters of the filters are specified, this voltage drop can be assumed to be negligible. This assumption is justified because the voltage drop across the inductor is compensated in part by the filter capacitor. In order to calculate the nominal modulation index, therefore, the rms value of the output voltage of the inverter can be assumed equal to the rms value of the load voltage, that is,

$$k = \sqrt{2}\frac{\overline{V}_o}{E_d} \tag{35}$$

The result is then used to calculate the factor K by using eqn. (31).

2) Based on the nominal load current, $\overline{I}_o$, fundamental output frequency, f_r, switching frequency, f_s, and the specified value of the total harmonic of the load voltage, $\tilde{V}_{o,av}$, the optimum value of the inductance of the filter can be calculated by using eqn. (34).

3) The capacitance of the filter is then calculated by using eqn. (30).

If the dc source voltage varies widely during the operation, the worst value of the dc voltage that results in the higher value of the output voltage harmonic should be used in this design.

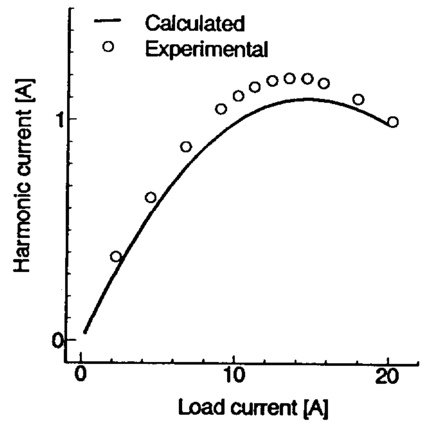

Fig. 5. The experimental and calculated results of the inductor current harmonic.

4. Experimental Results

In order to verify the derived expressions, a small single-phase full-bridge IGBT inverter with the circuit as shown in Fig. 1 was constructed. A small dead time of 4 μs is used to prevent a short circuit through the upper and lower arms switching devices of the inverter. The load consists of a resistance and an inductance in series connection. Due to the limitation of the available components in the laboratory, the inductance and capacitance of the LC filter which are used in this experiment do not indicate the optimum values. The experimental conditions are shown in Table 1.

In order to measure the total harmonic components, the waveforms of the filter inductor current and capacitor voltage over one fundamental output frequency are recorded by using a digital storage oscilloscope. The data stored in the oscilloscope is then processed by a personal computer to determine the rms values of the fundamental and harmonic components.

Fig. 5 shows the experimental and calculated results of the filter inductor current harmonic as a function of the load current. A good agreement between the calculated and experimental results can be appreciated from this figure.

Fig. 6 shows the experimental and calculated results of the filter capacitor voltage harmonic as a

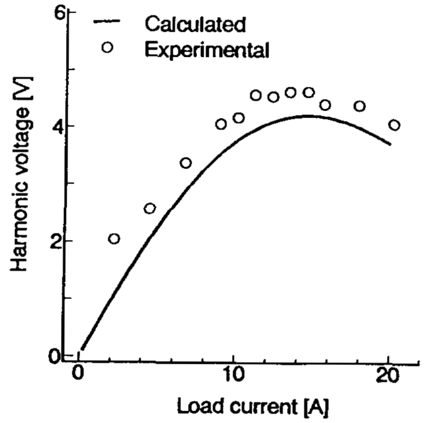

Fig. 6. The experimental and calculated results of the capacitor voltage harmonic.

function of the load current. Once again, a good agreement between the calculated and experimental results is obtained.

5. Conclusion

In this paper, analysis and design methods for the LC filter of single-phase PWM inverter are presented and verified by experimental results. The obtained closed form expressions of the inductance and capacitance of the LC filter eliminate the time consuming conventional Fourier series method. Extension of the results to the cases of three-phase PWM inverters is left for future investigation.

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