Measurement and Systems Laboratory Work 2 - 2019

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I. INTRODUCTION

The aim of this paper is to present two experiments, one done on proving the relationship between the rise time and the bandwidth of the system while the other was used to prove the Thevenin theorem.

II. BACKGROUND

A. The Relationship Between Rise Time and Bandwidth

The bandwidth is the range of frequencies at which most of the energies of the signals are stored. [1]. The rise time is defined as the required time for the pulse to rise from 10% to 90% of its steady value [1].

Reference [2] argues that in measurement instruments such as oscilloscopes and spectrum analyzers, the maximum rise time (t_r) that can be measured with reasonable amplitude accuracy is directly related to the instruments -3dB bandwidth (B). His analysis of deriving the relationship for the rise time and bandwidth will be applied.

1) Time domain Analysis: Figure 1 below showcases the first order circuit of which the relationship between the bandwidth and the rise time will be derived.

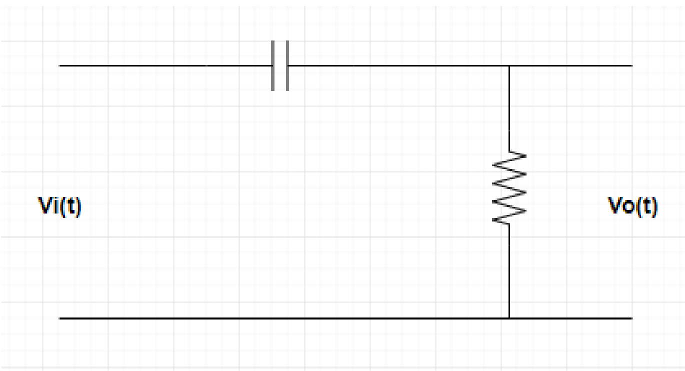

Fig. 1. First order circuit low pass filter

The output voltage of the resistor can be obtained using equation 1 below.

$$V_r(t) = V_i(t) - V_c(t) \tag{1}$$

With the application of first order differential equations, equation 1 can therefore be written as shown in equation 2 below.

$$V_r(t) = V_i(t)\left[1 - e^{\frac{-t}{R\cdot C}}\right] \tag{2}$$

With the assumption of the step input voltage, the speed of the step response can be obtained by solving t, and replacing RC with the circuit time constant (τ) as show below.

$$\frac{V_r(t)}{V_i(t)} = -\tau ln\Big[1 - e^{\frac{-t}{\tau}}\Big] \tag{3}$$

$$t = -\tau ln\left[1 - \frac{V_r(t)}{V_i(t)}\right] \tag{4}$$

With equation 4, the calculation of transitory points can be obtained, with 10% and 90% levels of the swing and obtain the rise time from it as show below,

$$t_r = t(90\%) - t(10\%) = \tau ln(9) \tag{5}$$

with this discovery that the rise time depends on τ (which has the frequency component), frequency domain can therefore be used to solve the remaining piece.

2) Frequency domain Analysis: The impedance of the capacitor is represented by equation 5 below

$$Z_C = \frac{1}{j\omega C} \tag{6}$$

where f is the -3dB cut-off frequency of the bandwidth or B(Hz) and RC is the time constant (τ).

$$\tau = \frac{1}{2\pi{\cdot}f} = \frac{1}{2\pi B} \tag{7}$$

substituting equation 7 into 5 gives equation 8 below,

$$t_r = \left[\frac{1}{2\pi{\cdot}B}\right]\cdot ln(9) = \frac{0.349699}{B} = \frac{0.35}{B} \tag{8}$$

which is the final equation describing the relationship between the bandwidth and the rise time.

B. Determining the Thevenin Equivalence of the Circuit

According to Thevenin's Theorem, any linear circuit containing several voltages and resistances can be replaced by one single voltage in series with a single resistance connected across the load.

Using the Thevenin's theorem, the voltage at across the terminal a-b is the Thevenin's voltage which is simply the open circuit voltage across that terminal. With the clear observation of Figure 2 below, the is seen that the Thevenin's voltage is equivalent to the voltage of the resistor R_2. The Thevenin's voltage can therefore be represented by equation 9 below from using Figure 2 parameters.

$$V_{TH} = \frac{R_2}{R_1 + R_2} \cdot V_{DC} \tag{9}$$

Given both values of resistors to be equal to $10{\times}10^9\Omega$ using the power source of $20V$, the expected value of the V_{TH} is $10V$.

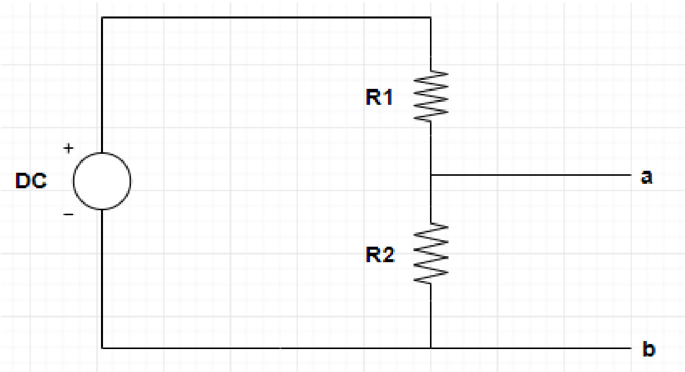

Fig. 2. Voltage divider circuit

The corresponding value of the R_{TH} is obtained short-circuiting the voltage source and measuring the equivalent circuit resistance with reference to terminal a-b. This implies the resistor R_1 and R_2 will now be in parallel. The equivalent parallel resistance is therefore modeled by equation 10 below.

$$R_{TH} = \frac{R_1 \cdot R_2}{R_1 + R_2} \tag{10}$$

With the given parameters aforementioned and substituting them into equation 10, gives the expected value of R_{TH} to be equal to $10{\times}5^9\Omega$. Therefore the equivalent Thevenin circuit for Figure 1 is shown in Figure 2.

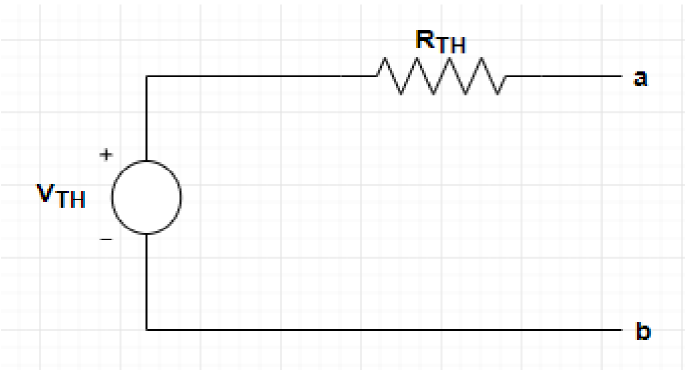

Fig. 3. Thevenin equivalent circuit

C. Square From Sine Wave Using Fourier Series

The square wave can be obtained by the addition of infinite sine waves at different frequencies. The equation below shows the Fourier series representation of the square wave in terms of the sine wave.

$$x(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{sin(2\pi(2 \cdot k - 1)f \cdot t}{2 \cdot k - 1} \tag{11}$$

Which can further be represented by the equation shown underneath?

$$x(t) = \frac{4}{\pi}\left[sin(2\pi f \cdot t) + \frac{1}{3}sin(3{\times}2\pi f \cdot t) + \frac{1}{5}sin(5{\times}2\pi f \cdot t).....\right] \tag{12}$$

This equation shows that the square wave is formed by the combination of the infinite sine waves at different frequencies or harmonics, where f represent the fundamental frequency.

the section ASPECTS OF BANDWIDTH The primary objective of this experiment was to observe the relationship between the bandwidth and rise time, using the function generator and the oscilloscope. The filter of the oscilloscope is set to 25kHz.

D. Input Sinusoidal Wave

The function generator was used to generate sinusoidal wave used as input to the function generator at different frequencies. Table 1 below shows the obtained results from the experiment for varying sinusoidal frequencies. The

TABLE I
EFFECT OF THE INPUT FREQUENCY ON THE AMPLITUDE, RISE TIME, FALL AND PERIOD

Input kHz	Amplitude	Rise (μs)	Fall (μs)	Period (μs)
5	4.96	59.8	58	201
10	5.0	28	28	100
25	2.6	12	12	40

More detailed results for each sinusoidal under specific frequency is shown by Figure 1-3, Appendix A. From Table 1 and Figures in Appendix A, it is observed that as the input frequency of the input sinusoidal increases, it has no major effect on the amplitude of the output but the increase in frequency increases the rise time. When input frequency reached 25kHz it was witnessed that the input amplitude was reduced from 5Vp-p to 2.6Vp-p. This remarkable discovery shows the oscilloscope has the build in low pass filter, as it is observed that frequencies below 25kHz are allowed to pass without altering the amplitude of that signal but once the signal has frequency over 25kHz the system prevents it by attenuating its amplitude. This results can be observed from Figure 4 below, which shows the amplitude relationship with the frequency of the signal.

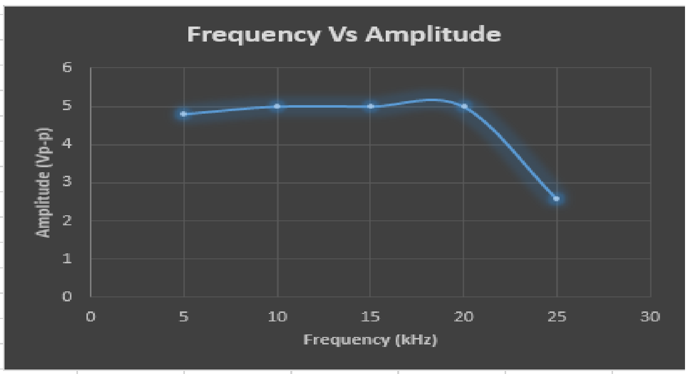

Fig. 4. Behaviour of the amplitude and different frequencies

The relationship between the rise time and the bandwidth can now be obtained using Table 1 with the assistance of equation 13 below.

$$constant = t_r \cdot B = \frac{\sum_{n=5}^{25} t_{rn} \cdot B_n}{N} \tag{13}$$

It is expected that the value of the constant to more similar to '0.35' constant provided by equation 8 to ensure the equation is valid which will validate the relationship between the bandwidth and the rise time. Substituting the values in Table 1 into equation 13 gives the results shown below.

$$constant = \frac{(5 \times 59.8 + 10 \times 28 + 25 \times 12) \cdot 10^{-3}}{3} \tag{14}$$

The definition provided by

$$constant = \frac{0.299 + 0.28 + 0.3}{3} \tag{15}$$

$$constant = 0.293 \tag{16}$$

Equation 8, provided by [2] seems to be valid as the obtained constant tally with the obtained constant equation 16 but with a difference of 16.285%.

E. Input Square Wave

Here the square wave generated from the signal generator was used as an input to the oscilloscope. The Frequency of the square wave was changed while observing the output on the oscilloscope cut-off still set at 25kHz.

I was observed that with low frequencies the square wave from the signal generator becomes less distorted as shown in Figure 8, Appendix B. As the frequency increases, the distortion of the square wave increases as well, these results are shown Figure 9 as compared with Figure 8. It is further observed from the results that when as the value of the square wave frequency reaches 10kHz, the square wave is turned into the sine wave, the reason for this behavior will be explained more on analysis.

III. THE USE OF MEASUREMENT TOOL

A. Proof of Thevenin Using Digital Multimeter

The digital multimeter was used in this application to prove the Thevenin theorem aforementioned using the same parameters for resistors and a voltage. The Thevenin voltage which is the voltage across the terminal a-b was measured to be 6.73V. The short circuit current flow throw the terminal a-b was measured to be 1.6μA.Therefore equation 15 below is used to determine the R_{TH} of the circuit.

$$R_{TH} = \frac{V_{TH}}{I_{SC}} \tag{17}$$

Using the above equation the value of the R_{TH} is calculated to be $4.20625 \times 10^9 \Omega$.Both the values of the Thevenin voltage and resistance and different from the expected values calculated from Prelab.

B. Using Oscilloscope

The oscilloscope was also used to prove the Thevenin's theorem. The Thevenin voltage across the terminal a-b was measured to be 1.7V and the current was measured to be zero.

Even though the proofs for Thevenin theorem using two different measurements were incorrect, the results of the digital multimeter were better compared to those of the oscilloscope.

IV. DISCUSSION AND ANALYSIS

The relationship of bandwidth provided by [2] which shows that the product of rise time and bandwidth is equal to 0.35 was proven to be correct by obtaining the product of 0.293. The difference was obtained to be 16.285% which could have occurred due to human error especially in the positioning of the cursors in the oscilloscope to record the measurements. The systematic error could also be a flaw since it is impossible to produce the perfect system, therefore it can happen that the system is not 100% calibrated.

It is observed that increasing input frequency increases have no effect on the amplitude of the output but it reduces the rise time of the system as expected and knowledge that frequency is inversely proportional to time. Once the frequency of input reaches to 25kHz which equal to the filter frequency of the oscilloscope, it was observed that the amplitude of the input was attenuated, this due to the filter present in the oscilloscope, therefore the attenuation behavior shows that the low pass filter is present there.

Applying the square wave as the input to an oscilloscope at the frequency of 500Hz it is observed that the system output has the noise. With increasing frequency, a remarkable observation is obtained. It is observed that increasing the frequency has no major impact on the amplitude of the input but does improve the rise time and reduces the noise of the output signal as observed from Figure 8-10.

At 10kHz it is observed that the generated square is turned into the sine wave. The sine wave produced from the square is the exact replica of the pure sine from the signal generator at 10kHz. This discovery proves that the square wave is produced of the infinite sine harmonics. When the frequency of the square wave is at 10kHz it implies the fundamental frequency is that 10kHz. Therefore using equation 12, it shows that the second and the third harmonics will be 30kHz and 50kHz respectively, as the harmonics increase their frequencies also increases. The filter of the oscilloscope was set to 25kHz since it is a low pass filter, it does not allow high frequencies beyond 25kHz to pass. The only harmonic that will pass from the filter is the first harmonic only at 10kHz, as observed that the second harmonic and upwards have frequencies greater than 25kHz.

In the case when the bandwidth of the measurement system is unknown, especially the oscilloscope. We can use the function generator and feed its sinusoidal signal output into the oscilloscope. The frequency of the signal from the signal generator will slowly be increased from zero while observing the results of the oscilloscope. The point where the amplitude of the system is attenuated, the corresponding value of the frequency in the function generator is the bandwidth of the oscilloscope.

Using the oscilloscope, the Thevenin equivalent circuit was measured to 1.7V compared to the 6.73V obtained using the digital multimeter. This shows the multimeter to better in terms of voltage DC voltage and current measurement. Due to the nature of the oscilloscope to act as a low pass filter, it shows that it has the capacitor within it which limit the dc current flow. Therefore the current measured by the system becomes zero. The results show that the digital multimeter has the resistor component which is in parallel with the terminal a-b, as it was witnessed that the Thevenin resistance and Thevenin voltages were reduced.

V. FUTURE RECOMMENDATIONS

Due to time constraint, it was not possible to repeat the investigation again and again to validate the quality of results, but in future, it is recommended that experiments are repeated.

It is also important to use different instruments of the same type during such experiments since there are possibilities that the currently used instruments might have some errors which can falsify the results.

VI. CONCLUSION

The experiment was conducted to determine the relationship between rise time and bandwidth. The obtained results show that the product of the two variables gives the constant which almost 0.3 whereby the theory shows the constant 0.35, therefore the obtained results validate the theory of the relationship. It was further observed that the square wave is composed of infinite sine harmonics whereby setting the cut-off frequency of the oscilloscope to 25kHz restricted all the harmonics of sine waves within the square wave greater than 25kHz, therefore with the fundamental frequency of 10kHz for the square wave the only harmonic of the sine wave remaining was that with the frequency of 10kHz which was equivalent to the pure sine wave of the generator at 10kHz. If the bandwidth of the oscilloscope is unknown, the sine wave can be used whereby if it is attenuated by almost 30% of its initial value then that corresponding frequency is the bandwidth.

REFERENCES

[1] C. Ni, C. Gan, X. Ma, and C. Wu. "Dynamic bandwidth allocation with optimized excess bandwidth distribution and wavelength assignment in multi-wavelength access network." In *2013 IEEE International Conference of IEEE Region 10 (TENCON 2013)*, pp. 1–4. Oct 2013.

[2] M. Buzuayene. "Rise Time vs. Bandwidth and Applications.", dec 2008. URL https://interferencetechnology.com.

APPENDIX A

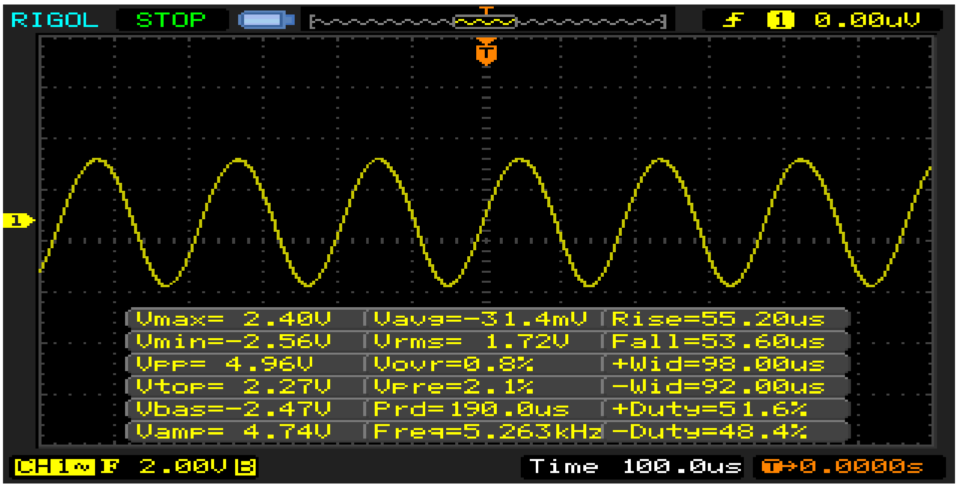

Fig. 5. Sinusoidal input at 5kHz

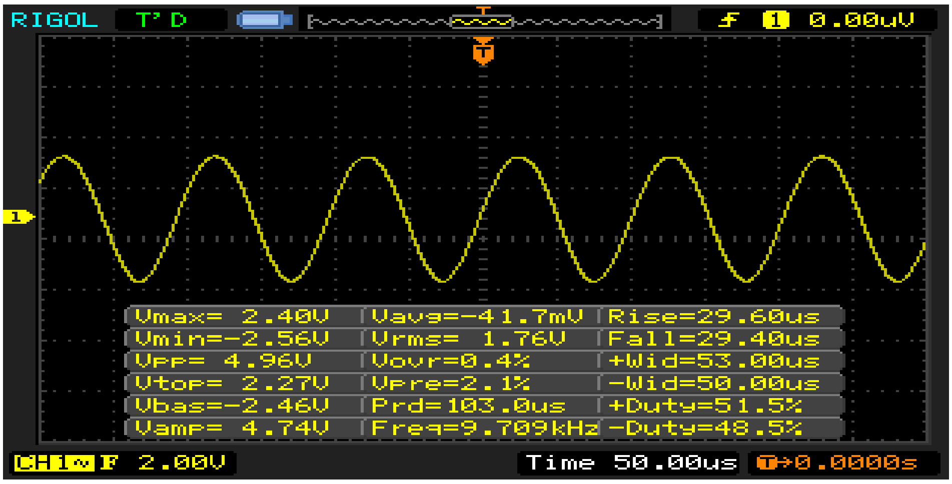

Fig. 6. Sinusoidal input at 10kHz

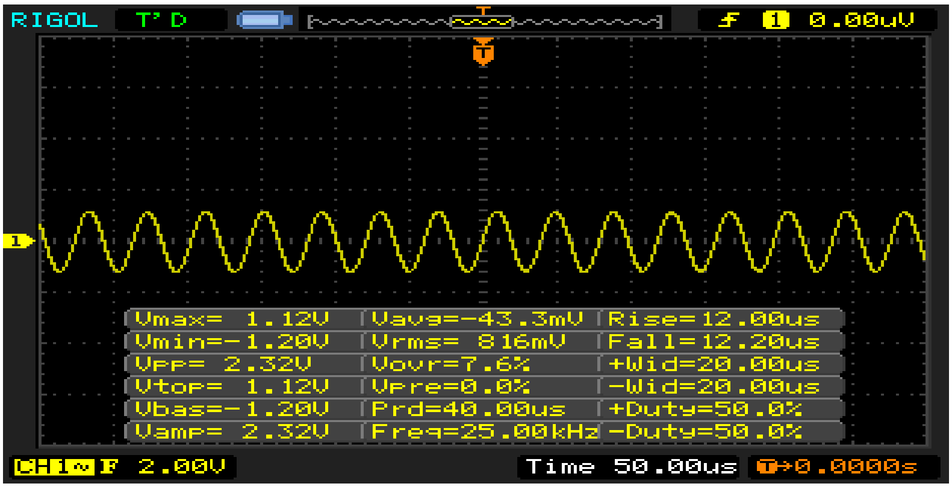

Fig. 7. Sinusoidal input at 25kHz

APPENDIX B

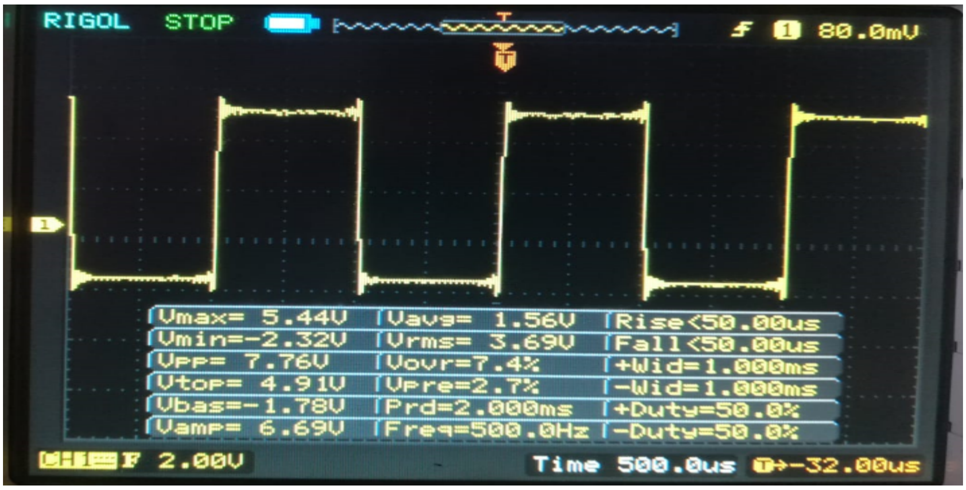

Fig. 8. Sinusoidal input at 25kHz

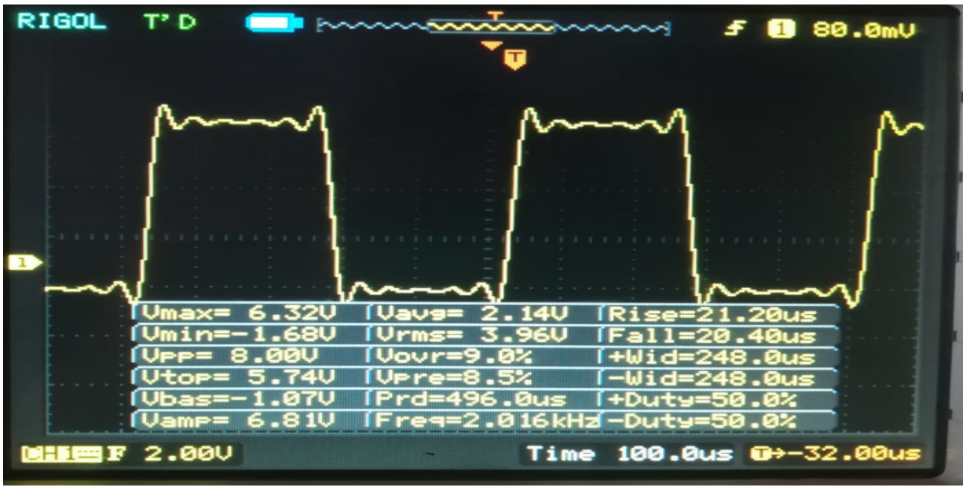

Fig. 9. Sinusoidal input at 5kHz

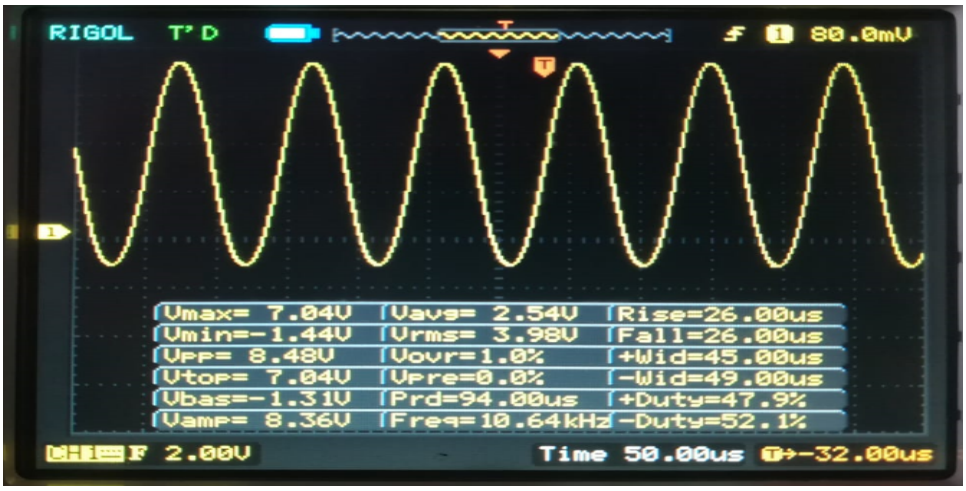

Fig. 10. Sinusoidal input at 10kHz