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Permanent Magnet Linear Motors Used as Variable Mechanical Dampers for Vehicle Suspensions

DEAN KARNOPP*

SUMMARY

Linear electrodynamic motors consisting of coils of copper wire interacting with magnetic fields produced by permanent magnets can be used to construct mechanical dampers with a damping **coefficient** which can be rapidly varied by changing the external resistance connected to the coil. Limitations to the use of these variable dampers arise due to the coil's own resistance and mass and the practically achievable magnetic field. A moving coil system is first analyzed and then modifications to the basic results when a moving magnet design is used are given. The results indicate that for the oscillation frequencies typically encountered in road vehicle **suspensions**, electrodynamic variable shock absorbers are **feasible**. A novel **coreless** design **appropriate** for use with highenergy permanent magnets is proposed which would significantly reduce the weight of the units.

INTRODUCTION

Electrodynamic linear actuators have several potential advantages for the generation of forces or motions when compared to hydraulic, pneumatic, or mechanical devices. The electrical actuators can be arranged to have very low static friction and the control of current through power electronics can be achieved rapidly and reliably so that force levels can be readily controlled. In distinction to competitive actuator types, electrodynamic actuators are inherently linear which can be an advantage.

When an electrodynamic actuator coil is shorted or connected to an external resistor, the device becomes a linear mechanical damper. When the external resistance is varied, the damping coefficient is varied. In the open circuit state the coefficient vanishes, while when the coil is shorted the coefficient reaches a maximum value. Since effective resistance can be rapidly varied electronically, an electrical actuator can function as a semi-active damper [1] - [4] in vehicle or vibration isolation suspension systems.

It is well known that electromechanical devices tend to be heavier than hydraulic devices of similar force or power level. This will probably be the case if we compare a hydraulic vehicular shock absorber with an electromechanical one of similar capabilities. Much of the weight **and** bulk of linear electric motors has to do with the iron core required, but even the weight of the coil or of the

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permanent magnets is significant. In this paper we first compute some simple performance limitations due only to the mass of the moving element (coil or magnet), the minimum resistance of the coil due to the finite conductivity of copper, and the practical magnitude of the magnetic field.

Although the basic formulas are derived for the moving coil type of motor similar to those used in electrodynamic loud speakers, similar results apply to other configurations in which the coil is stationary and the magnet moves. In the last section, a core-less design is proposed which could significantly reduce the total unit weight.

MOVING COIL DESIGN

For high frequency applications as in loudspeakers, it is common to arrange the magnet and iron core to be stationary with only the coil moving and attached to the speaker cone or other member to be controlled. This has the disadvantage that flexible electrical leads must be attached to the coil if it is to be driven by an amplifier or connected to a variable resistance. In a vehicle suspension, the heavy magnet and core would be attached to the body and the moving coil attached to the wheel.

First we study the limitations of the coil mass and resistance alone and later we will consider the effects of an added mass connected to the coil. As shown in Fig. 1, we first assume that the coil moves in a uniform field of flux density B . This means that the magnet-core design must provide a field over a length equal to the coil length L_c plus the stroke of the device.

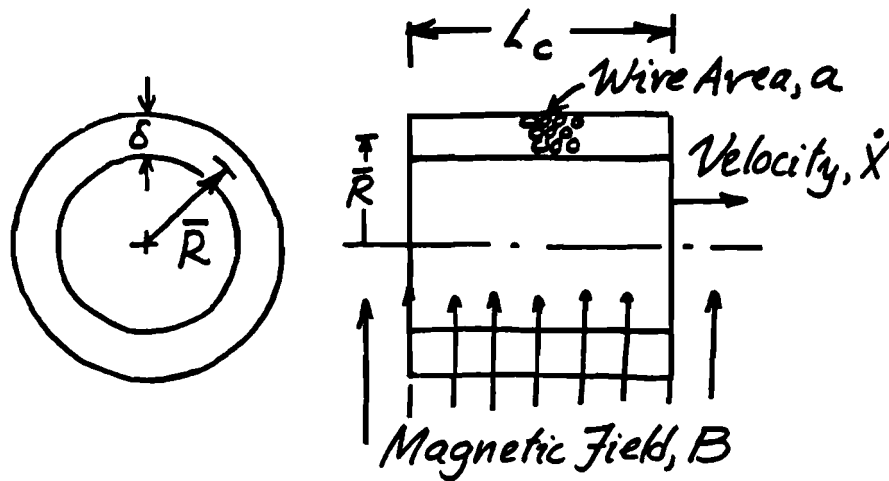


Fig. 1. Coil Dimensions.

We assume that N turns of wire of cross-sectional area a are wound on the coil so that the coil nominal cross-sectional area $L_c \cdot \delta$ is filled to the fraction η with wire.

$$N = \frac{\delta \cdot L_c \cdot \eta}{a} \quad (1)$$

The length of wire ℓ is given by

$$\ell = 2\pi \bar{R} \cdot N \quad (2)$$

where \bar{R} is a mean coil radius. Many results can be conveniently expressed in terms of the total volume of copper in the coil, Ψ

$$\Psi = \ell \cdot a = 2\pi \bar{R} \cdot \delta \cdot L_c \cdot \eta. \quad (3)$$

Assuming that the B-field is perpendicular to the flow of current, i , as well as the motion of the coil, X , the force F and the induced voltage E are given by

$$F = B\ell i = \frac{B\Psi}{a} \cdot i \quad (4)$$

$$E = B\ell \dot{X} = \frac{B\Psi}{a} \cdot \dot{X} \quad (5)$$

The mass of the coil M_c is given in terms of the mass density of copper, ρ

$$M_c = \rho \Psi \quad (6)$$

while the electrical resistance R of the coil can be expressed in terms of the resistivity of copper, σ ,

$$R = \frac{\sigma \ell}{a} = \frac{\sigma \Psi}{a^2} \quad (7)$$

If the coil is used as a damper by connecting an external resistance R_{ext} , the current is related to the induced voltage. Thus

$$(R + R_{\text{ext}}) \cdot i = E \quad (8)$$

Using (4) and (5), the damper coefficient relating F and \dot{X} is

$$F = \frac{B^2 \Psi^2}{a^2} \cdot \frac{1}{(R + R_{\text{ext}})} \cdot \dot{X}. \quad (9)$$

The maximum damping effect occurs when the coil is short circuited,

$$F_{\text{max}} = \frac{B^2 \Psi^2}{a^2 R} \cdot \dot{X} = \frac{B^2 \Psi}{\sigma} \cdot \dot{X} = \frac{B^2 \cdot M_c}{\sigma \rho} \cdot \dot{X}, \quad (10)$$

upon use of (7).

LIMITATIONS OF THE DAMPING FORCE

In order to see the physical limitations of electromagnetic damping, we consider some limiting cases. First suppose only the mass of the coil is to be decelerated at the maximum possible rate by shorting the coil. Then, using (10),

$$M_c \ddot{X} = -F_{\max} = -\frac{B^2 M_c}{\sigma \rho} \dot{X}$$

$$\frac{\sigma \rho}{B^2} \ddot{X} + \dot{X} = 0. \quad (11)$$

This form leads to the definition of a mechanical time constant, τ_{mech}

$$\tau_{\text{mech}} = \frac{\sigma \rho}{B^2}. \quad (12)$$

This time constant represents the fastest rate of decay possible given the properties of copper and the value of B achievable in practice. If we assume the following values

$$\begin{aligned} \sigma &= 1.72 \times 10^{-8} \Omega \cdot m \\ \rho &= 8.9 \times 10^3 \text{ kg/m}^3 \\ B &= 0.4 \text{ T} \end{aligned} \quad (13)$$

the result is

$$\tau_{\text{mech}} = 9.57 \times 10^{-4} \text{ s} \cong 1 \text{ ms.}$$

When an externally connected mass, M_{ext} , is to be decelerated in addition to the coil mass, the time constant would be multiplied by a factor of

$$\left(1 + \frac{M_{\text{ext}}}{M_c} \right),$$

so if $M_{\text{ext}}/M_c = 100$, the time constant would be about 100 ms. Thus, there is a practical limit to the speed at which masses can be decelerated by the electromechanical damper.

If we consider the damper applied to an oscillator, we have an equation of the form

$$(M_c + M_{\text{ext}}) \ddot{X} + \frac{B^2 M_c}{\sigma \rho} \dot{X} + KX = 0 \quad (14)$$

where K is a spring constant. We define the undamped natural frequency ω_n and damping ratio ζ by the relationships

$$\omega_n^2 = \frac{K}{M_c + M_{ext}} \quad , \quad 2\zeta\omega_n = \frac{B^2}{\sigma\rho} \cdot \frac{M_c}{(M_c + M_{ext})} \quad (15)$$

Using (12), (15) and the periode T

$$T = 2\pi/\omega_n \quad (16)$$

we find

$$\zeta = \frac{I}{4\pi} \cdot \frac{T}{\tau_{mech}} \cdot \frac{M_c}{(M_c + M_{ext})} \quad (17)$$

Since we normally desire that $M_{ext} \gg M_c$, this means that the period T must be considerably greater than τ_{mech} in order to achieve reasonably large values of ζ .

As an example, consider the damping of a wheel with natural frequency $f_n = \omega_n/2\pi = 8.5$ Hz or $T = 0.1176$ s. If $\tau_{mech} \cong 1$ ms, and $M_{ext} = 20 M_c$, the damping ratio $\zeta = 0.445$. When $M_{ext} = 40 M_c$, $\zeta = 0.2283$. This indicates that at this wheel frequency, the damper coil might have to have a mass of perhaps 3 percent of the wheel to achieve reasonable damping.

If the body of the vehicle had a natural frequency of 1 Hz or $T = 1$ s, and if $M_{ext}/M_c = 200$, we find $\zeta = 0.396$, and if $M_{ext}/M_c = 400$, the result is $\zeta = 0.198$. Thus, the same damper may be able to achieve a reasonable body damping ratio because the low frequency compensates for a much greater mass ratio.

This section shows that electromechanical dampers can be effective as long as the periods of the vibration are long compared to the mechanical time constant. Since the mechanical time constant can be of the order of 1 ms, the periods encountered in typical vehicle oscillations allow reasonable damping with fairly lightweight coils.

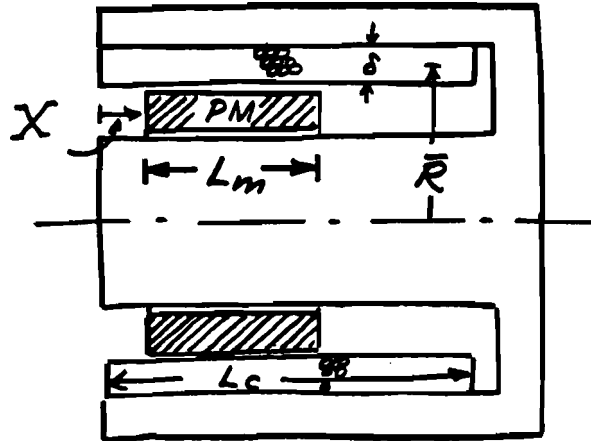


Fig. 2. Moving Magnet System.

Of course, these calculations assume a shorted coil in which all dissipation occurs in the coil internal resistance. If an external resistance is used, the damping coefficient is reduced according to (9). Then the damping ratio is reduced by the factor $1 + R_{\text{ext}}/R$.

$$\zeta = \frac{1}{4\pi} \cdot \frac{T}{\tau_{\text{mech}}} \cdot \frac{1}{(1 + R_{\text{ext}}/R)} \cdot \frac{M_c}{(M_c + M_{\text{ext}})}$$

MOVING MAGNET DESIGN

The calculations in the previous two sections can be easily modified to apply to a variety of alternative designs. In Fig. 2 for example, the coil is fixed to a soft iron core and a ring magnet is the moving element. In this case, if the magnet can produce an essentially radial B-field, Eqns. (4) and (5) must be modified since the field extends only over a fraction of the coil.

$$\begin{aligned} F &= B\ell \cdot \frac{L_m}{L_c} \cdot i = \frac{B\mathcal{V}}{a} \cdot \frac{L_m}{L_c} \cdot i, \\ E &= B\ell \cdot \frac{L_m}{L_c} \cdot \dot{X} = \frac{B\mathcal{V}}{a} \cdot \frac{L_m}{L_c} \cdot \dot{X} \end{aligned} \quad (19)$$

The resistance of the coil is still given by (7), but the maximum damping force when the coil is short-circuited is

$$F_{\text{max}} = B^2 \left(\frac{L_m}{L_c} \right)^2 \frac{\mathcal{V}}{\sigma} \dot{X} = B^2 \left(\frac{L_m}{L_c} \right)^2 \frac{M_c}{\sigma \rho} \cdot \dot{X}. \quad (20)$$

Letting the magnet mass be M_m we have, for the case in which the magnet alone is decelerated by F_{max} ,

$$\begin{aligned} M_m \ddot{X} &= -F_{\text{max}} = -B^2 \left(\frac{L_m}{L_c} \right)^2 \frac{M_c}{\sigma \rho} \dot{X} \\ \frac{\sigma \rho \left(\frac{L_c}{L_m} \right)^2 M_m}{B^2} \ddot{X} + \dot{X} &= 0. \end{aligned} \quad (21)$$

This yields a mechanical time-constant of

$$\tau_{\text{mech}} = \frac{\sigma \rho \left(\frac{L_c}{L_m} \right)^2 M_m}{B^2} \quad (22)$$

For long-stroke designs, $L_c \gg L$, and the mechanical time constant could be considerably longer than with a moving coil system in which the B-field is uniform over the entire stroke of the coil. On the other hand, $M_m < M_c$ for this case which tends to reduce τ_{mech} . For samarium cobalt, for example, the mass

density is somewhat less than the density of copper, and the magnet radial thickness would be approximately the same as the gap (or coil thickness) 6. Thus, $M_m/M_c \cong L_m/L_c$ and τ_{mech} would be increased over the value given in (12) by only the ratio L_c/L_m , not the square of this ratio.

The damping ratio for the moving coil system with external resistance and mass is a simple modification of (17)

$$\zeta = \frac{1}{4\pi} \cdot \frac{T}{\tau_{\text{mech}}} \cdot \left(\frac{1}{(1 + R_{\text{ext}}/R)} \right) \cdot \left(\frac{M_m}{(M_m + M_{\text{ext}})} \right) \quad (23)$$

in which the τ_{mech} is evaluated as in (22).

These considerations indicate that although a moving coil in a uniform B-field stretching over the entire range of travel of the coil is the most effective scheme for generating damping forces, the moving magnet system with high energy product magnets can be almost equally effective unless very long strokes are required. The fixed coil system has several advantages: flexible-wires or sliding contacts are not required and heat energy dissipated in the coil can be easily conducted to the core and the case of the device.

DYNAMICS OF THE ELECTRICAL CIRCUIT

As is clear from Fig. 2, a current in the coil will tend to induce a magnetic field. Generally, the flux lines will tend to encircle the coil, lying mainly in the central core, the external shell and connecting at the two ends. At the left end, the flux lines can follow the core from the inner core to the outer cylindrical shell, while at the left end the flux line will have to pass from the inner core to the outer shell through the air (or non-magnetic material). Similar flux paths would apply for the moving coil case in which the roles of magnet and coil are interchanged from the case of Fig. 2.

The result is that the coil will possess self inductance L_e as well as resistance R so that an electrical time constant, τ_{elec} can be defined

$$\tau_{\text{elec}} = \frac{L_e}{R} \quad (24)$$

In order for the results of the previous sections to be meaningful, the electrical time constants should be short compared to the period of oscillations to be damped so that the current and damping force are essentially proportional to the relative velocity.

A rough idea of the electrical time constant in terms of the coil parameters may be derived as follows: The inductance L_e may be expressed in terms of the turns, N and the permeance of the path of the coil-induced flux, P

$$L_e = N^2 \cdot P \quad (25)$$

Using Eqn. (1) for N, (3) and (7) for R, and substituting in (24) we find

$$\tau_{\text{elec}} = \frac{L_c \cdot \delta \cdot \eta \cdot P}{\bar{R} \cdot 2\pi \cdot \sigma} \quad (26)$$

The permeance is hard to estimate, but since in most designs such as that shown in Fig. 2, the flux induced by the coil must pass through air or nonmagnetic material over a significant length assuming the core material is not saturated, the low permeance of the air part of the path is dominant. For order of magnitude calculations we define an equivalent air gap of area A_g , length ℓ_g , and with the permeability of free space μ_o . Then

$$P = \frac{A_g \mu_o}{\ell_g} \quad (27)$$

where

$$\mu_o = 4\pi \cdot 10^{-7} \text{ H/m} \quad (28)$$

The electrical time constant is then

$$\tau_{\text{elec}} = \frac{L_c \delta A_g \eta \mu_o}{\bar{R} \ell_g 2\pi \sigma} \quad (29)$$

If we assume for the configuration of Fig. 2 that $A_g \cong \pi \bar{R}^2$, then another form is

$$\tau_{\text{elec}} = \frac{L_c \cdot \delta \cdot \bar{R} \cdot \eta \cdot \mu_o}{\ell_g \cdot 2 \cdot \sigma} \quad (29a)$$

Using the parameter values' in (13) and (27), and assuming $\eta = 0.7$, $L_c = 5 \times 10^{-2}$ m, $\delta = 1 \times 10^{-2}$ m, $R = 2 \times 10^{-2}$ m, $\ell_g = 5 \times 10^{-2}$ m, the result is $\tau_{\text{elec}} \cong 5.11$ ms. In this calculation the length of the effective air gap is hard to estimate, but it is hard to imagine the result being sufficiently in error to yield a τ_{elec} as large as the typical wheel hop period of over 100 ms. This could happen only for very large coil dimensions and small ℓ_g . In all probability, response delay due to the self inductance effect will not be a limitation at the typical vibration frequencies encountered in vehicle suspensions. Of course, if external resistance is used in addition to the coil internal resistance, the time constant is even smaller as may be seen from (24).

MINIMUM WEIGHT DESIGN USING HIGH ENERGY MAGNETS

A typical voice coil actuator is sketched in Figs. 1 and 2. The radial magnetic field in the cylindrical air gap is produced by the permanent magnet through the use of iron pole pieces. If high energy magnetic material is used, only a short magnet length is necessary. However, as can be seen, even when the magnet is

quite short, the pole pieces remain large, and the weight and size of the complete actuator are mainly a function of these iron pieces. The volume of the iron cannot be decreased arbitrarily since the entire flux of the magnet must flow through the iron and iron saturates at a certain flux density. If high energy permanent magnets are used, the contribution to the weight and size of the device from the magnets themselves becomes small compared to the contribution of the pole pieces. Thus an effective use of such magnets is only achieved by designs which minimize or eliminate the use of pole pieces.

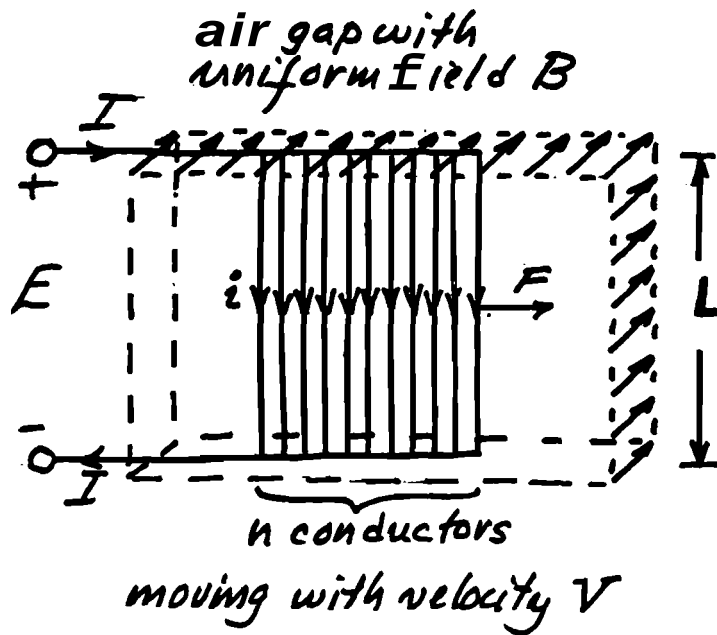


Fig. 3. Grid of Individual Conductors Moving in a Uniform Magnetic Field.

Figure 3 shows an air gap with a uniform flux density B in which a moving grid of conductors moves with velocity V . A total current I flows over a length L and generates a force F on the grid. Assuming the vector directions of B , I and V are mutually perpendicular, the induced voltage E is

$$E = BL \cdot V, \quad (30)$$

and the force F is

$$F = BL \cdot I, \quad (31)$$

In this scheme, it is assumed that n individual conductors remain in the B -field as the grid moves through its useful stroke and each individual conductor carries a current i so that

$$I = ni. \quad (32)$$

Two disadvantages of this scheme are that a long magnetic field must be provided and it is only partially utilized and, because of the parallel connection of the individual conductors, the external current is high and the voltage is low.

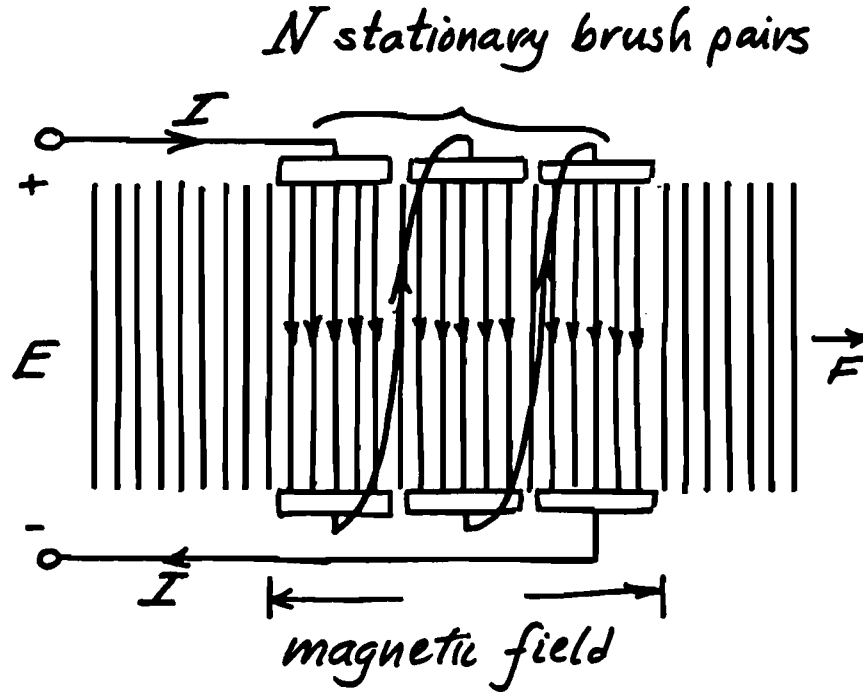


Fig. 4. Commutation of Individual Conductors Using Stationary Brushes.

Figure 4 shows that through the use of N pairs of stationary brushes, a short magnetic field can be fully utilized by a long grid of conductors and groups of conductors can be connected in series. If the individual conductors carry the current i , the total current is reduced from the value given by Eq. (32) to

$$I = \frac{ni}{N}. \quad (32a)$$

The external voltage is then

$$E = BNL \cdot V, \quad (30a)$$

and the force is

$$F = BNL \cdot I, \quad (31a)$$

Using Eqs. (31) and (32) and (31a) and (32a) one can see that both Eq. (31) and (31a) yield the same result:

$$F = BLni \quad (33)$$

so that no matter what series-paralleled combination is used, it is the number of active conductors and the current per conductor which determine the force. The use of brushes allows the complete utilization of the magnetic field (important for long stroke applications) and adjustment of the external impedance.

Some aspect of the device have little to do with the series and parallel connections. For **example**, if the device is short circuited, so that the current is determined by the resistance of the conductors, and if each of the n active conductors has a resistance r then the total resistance with N brush pairs (neglecting the resistance of the inter-brush connectors) is approximately

$$R = rN^2/n \quad (34)$$

The force-velocity relationship using Eqs. (31a), and (34) is

$$F = BNL \cdot I = BNL \frac{E}{R} = \frac{nBNL \cdot BNL}{N^2r} V = \frac{B^2L^2n}{r} V \quad (35)$$

Thus B^2L^2n/r is the maximum damping coefficient attainable independent of the number of brush pairs used. The number of brush pairs used would be determined by power conditioning components used in configuration with the device.

The problem of achieving locally perpendicular current, magnetic field and motion directions as shown in Figs. 3 and 4 is that both the current path and the magnetic flux path must be closed circuits. In the conventional voice coil configuration of Figs. 1, and 2, the toroidal air gap allows a very simple current path circuit while the flux circuit requires heavy pole pieces. The use of modern high energy permanent magnets reduces magnet length for a given air gap width but does not reduce the total amount of iron required to lead the flux to and from the air gap very much because of the basic geometry of the magnetic circuit.

Ideally, the magnetic circuit should consist mainly or exclusively of permanent magnet material and air gaps for the conductors with little or no iron. This leads inevitably to the idea of a toroidal path for the magnetic circuit rather than for the electrical circuit. A sketch of a toroidal magnetic circuit is shown in Fig. 5. If wedge shaped magnets are used, this configuration consists only of magnets and airgaps. The optimum number of air gaps depends on magnet properties. Magnetic circuits are usually designed to supply the greatest energy in the air gap. if B_m and H_m are values of flux density and field strength for the magnet for which the product $B \cdot H$ is maximum and if L , and L_g are lengths of the magnet and gap, and if S_m and S_g are areas of the magnet and gap, then the following relationships will provide optimum use of the magnet [5],

$$\frac{S_g L_m}{S_m L_g} = \frac{B_m}{\mu_0 H_m} \quad (36)$$

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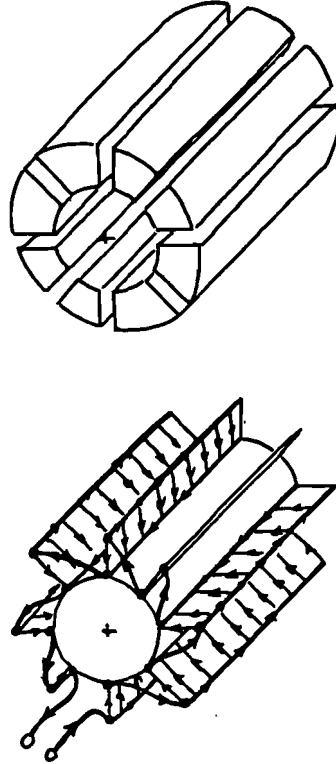


Fig. 5. Magnetic Circuit and Armature with Conducting Grids as Radial Fins.

where μ_o is the permeability in air. Since $S_g = S_m$ in this design the desired ratio of magnet length to gap length is simply given by the right hand term of Eq. (36). For modern high energy magnets, this ratio is near unity, so a fairly large number of sizeable gaps may be used.

The current must flow radially on grids inserted into the air gaps so the armature would be cylindrical with grids such as those in Figs. 3 and 4 extending radially outward. Both brushless and brush-type designs are possible and, as shown in Fig. 5, even for the brushless design, the individual grids could be connected in series although within a single grid all conductors would be in parallel.

Figure 6 shown a sketch of a design in which simple rectangular magnets are used with wedges of iron to yield parallel sided air gaps. For this design, a rough comparison of the size of this device compared to the conventional voice coil can be made. Assume that in Fig. 1, the cylindrical air gap has length L, radius R and width $\delta \ll R$ so that its volume V_g is

$$V_g = 2\pi RL\delta \quad (37)$$

For the configurations of Figs. 5 and 6, assume there are M gaps of width δ ,

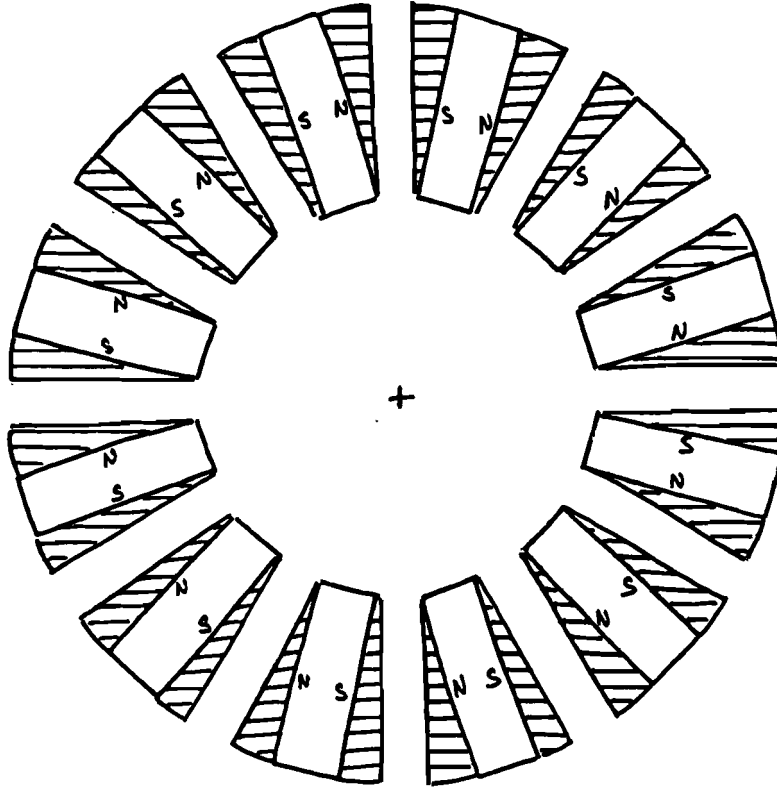


Fig. 5. Magnetic Circuit Design with Rectangular Magnets and Wedge-Shaped Pole Pieces.

length L and inner and outer radii of R_i and R_o respectively. Then the total gap volume is

$$V_g = M(R_o - R_i)L \cdot \delta. \quad (38)$$

If we assume as in Fig. 5 that $R_i \approx R_o/2$, $M = 12$ and equate Eqs. (37) and (38) assuming L and δ are equal, we find that

$$R_o = R \cdot \frac{\pi}{3} \approx R. \quad (39)$$

This means that in the proposed design the entire actuator is the same size and shape as the voice coil alone in the conventional design. Obviously this design has the potential of drastically reducing the size and weight of voice-coil type of actuators although in some applications the added complexity, particularly if brushes are used, may preclude its use.

CONCLUSIONS

The basic calculation of the mechanical time constant for a moving shorted coil in a uniform field is geometry independent. With the properties of copper, and assuming reasonable values for the magnetic field, the time constant is in the millisecond range. This indicates that an electromechanical damper can be used to damp oscillations with periods longer than the time constant even when the coil is used with an external mass. Assuming a typical vehicle wheel resonant frequency has a period of over 100 ms, the coil mass needs to be only a small fraction of the wheel mass. The calculations can be readily extended to moving magnet systems. In such a design, the performance is somewhat degraded if a long stroke compared to the magnet length is required.

Very rough calculations of the coil self-inductance time constant indicates that for reasonable sized coils and for the vibrational periods encountered in vehicle suspensions, the electrical time constant will produce only small delays in the damper response.

The optimum use of high energy permanent magnets requires rethinking basis designs. For linear motion actuators, many designs require heavy pole pieces which can be eliminated by using a simple toroidal magnetic flux path with multiple radial air gaps. Actuators based on this principle could be much smaller and lighter than conventional actuators.

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