

## Area 52



### Disclaimer



**[\*\*\* DISCLAIMER: To whom it may concern this by no means in any way shape or form is an officially sanctioned document in any capacity for the course ELEN 4006. All that being said the problems below examine core principles and are a good GENESIS (as in beginning, as in the start as in this is not an all-encompassing advanced screening for your exam...supplement this with other material please) for your exam preparation \*\*\*].**

### Phase I: Static

#### Question 1:

The following resistance values for Dr. Stillwell's Neogenicrecombinator's cooling chamber thermometer at various temperatures are measured and recorded as per Table 1.



**1.1:** Determine the measurement **sensitivity** ( $K$ ) of the Neogenicrecombinator's thermometer in  $[\Omega/^{\circ}C]$ .



**1.2:** Is the relationship between Resistance and Temperature in the Neogenicrecombinator a linear one? Motivate your answer


Table 1: Platinum Resistance Measurements

Resistance [Ohms]	Temperature [C]
307	200
314	230
321	260
328	290
335	330
342	360
349	390
356	420


## Question 2:

**Table 2: Displacement Vs Output Voltage**

Displacement $x[cm]$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Output Voltage $V$ [mV]	0	2.5	5	12.5	25	37.5	50	62.5	75	77.5	80

 **2.1:** Calculate the maximum Non-Linearity ( $N(I)$ ) as a percentage of Full Scale Deflection (%F.S). Assuming the steady-state sensitivity is calculated as in Bentley i.e.

$$K = \frac{O_{MAX} - O_{MIN}}{I_{MAX} - I_{MIN}}$$

 **2.2:** Is it possible to reduce the maximum Non-Linearity without adding an additional compensating element/circuitry. Motivate your answer


## Question 3:


The Neogenicrecombinator's thermocouple produces an E.M.F. for a given temperature governed by the following expression:


$$E(T) = 38.74T + 3.319 \times 10^{-2}T^2 + 2.071 \times 10^{-4}T^3 - 2.195 \times 10^{-6}T^4$$

The system exhibits the following characteristics:

- Temperature Range: 0 to 400°C
- $T = 0^\circ C \rightarrow E(T) = 0\mu V$
- $T = 400^\circ C \rightarrow E(T) = 20869\mu V$

 **3.1:** Calculate the expression for the ideal straight line relationship i.e.  $E(T) = KT$

 **3.2:** Determine the sensitivity of ( $K$ ) of the sensor.

 **3.3:** Determine the maximum Non-Linearity ( $N(I)$ ) of the system and express it as percentage Full Scale (%F.S) @  $T = 400^\circ C$

### Question 4:

As Dr. Farley Stillwell's newest lab assistant at Empire State University (ESU) you have been charged with investigating the effects of loading on the Neogenicrecombinator's firing mechanism. This is necessary as the firing mechanism has been identified as the culprit for the recent misfiring events [**Amazing Fantasy #15 (1962)**]. The simplified circuit of the Neogenicrecombinator's firing mechanism is presented in Figure 1. The Neogenicrecombinator being the sensitive device that is adequately loaded by a simple voltmeter.



**4.1:** Determine the measurement error at terminals AB caused by  $R_m$ .

$$R_1 = 400\Omega, R_2 = 600\Omega, R_3 = R_5 = 1000\Omega, R_4 = 500\Omega, R_m = 9500\Omega$$

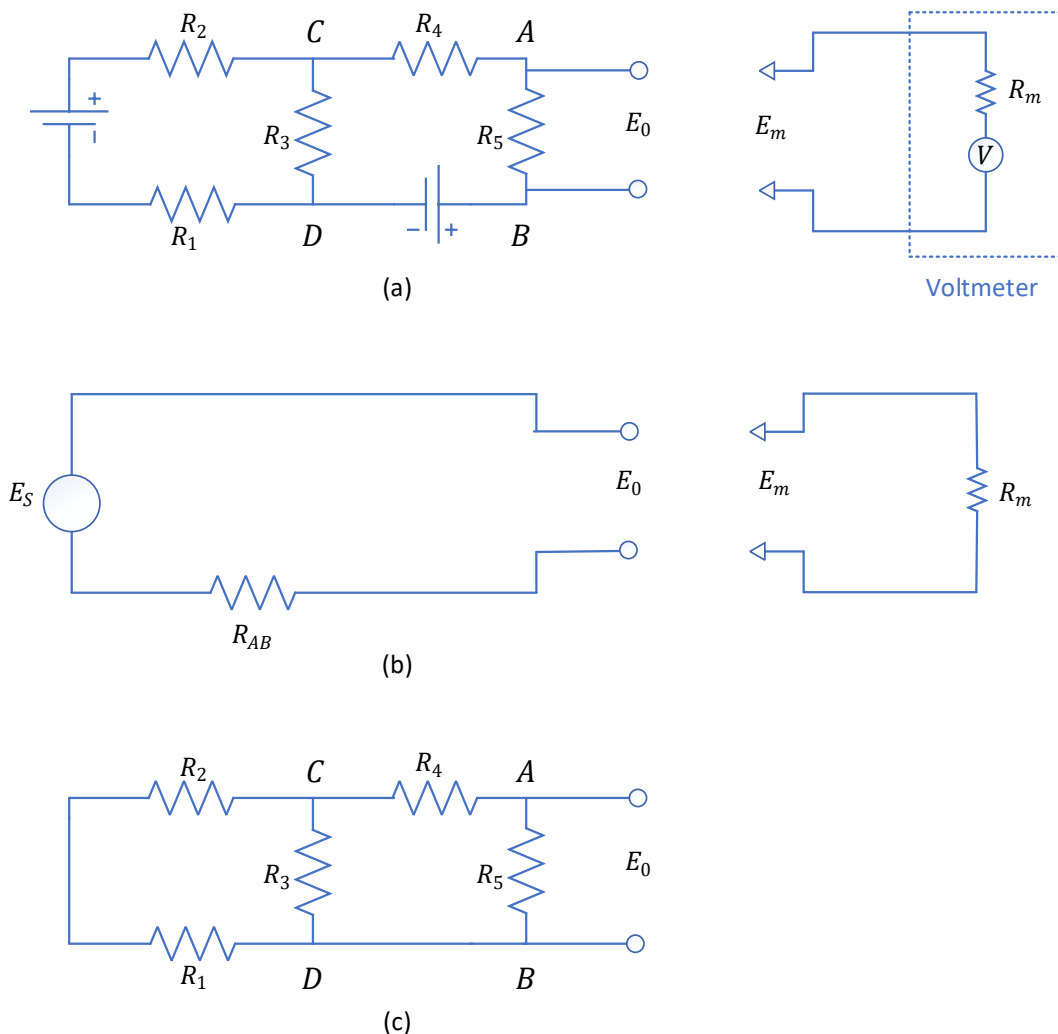


Figure 1: Measuring Circuit Illustrating Loading Effects a) Circuit measuring  $R_5$  as the output b) Thévenin Equivalent Circuit c) Circuit used to find the equivalent single resistance  $R_{AB}$

### Question 5:

Given that  $\sigma_{R_p} = 0.11$  and  $\sigma_{R_1} = \sigma_{R_2} = \sigma_{R_3} = 0.175$ , and  $\overline{R_1} = \overline{R_2} = \overline{R_3} = 500\Omega$  and the supply voltages recorded are:

$$V_{CC} = [5.02\text{ V}, 4.99\text{ V}, 5.0\text{ V}, 4.98\text{ V}, 5.01\text{ V}, 4.99\text{ V}, 5.01\text{ V}, 5.0\text{ V}, 4.98\text{ V}, 5.05\text{ V}]^T$$

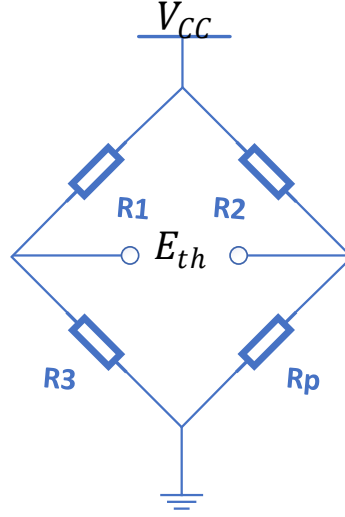


Figure 2: Whetstone (4-Elements Varying) Bridge & Differential Op-Amp

**5.1:** Determine the mean of the voltage supply ( $\overline{V_{CC}}$ ) and hence the standard deviation ( $\sigma_{V_{CC}}$ )

**5.2:** Additionally Winston would like to know the total variance ( $\sigma_{E_{th}}^2$ ) as a function of the resistance  $R_p$ , where

$$E_{th} = V_{CC} \left[ \frac{R_1}{R_1 + R_p} - \frac{R_2}{R_2 + R_3} \right]$$

**5.3:** If two standard deviations are taken ( $2\sigma_E$ ), how much of the total variance ( $\sigma_{E_{th}}^2$ ) does the supply voltage variation account for, when  $\overline{R_p} = 250\Omega$ .

### Question 6:

Lena Oxtan (aka Tracer) experiencing some technical issues with her chronal accelerator takes it to Winston for fine tuning. The equivalent circuit of the chronal accelerator is given in Figure 3. For simplicity the chronal accelerator can be modelled as a **potentiometric displacement sensor**. In order to ensure safe operation of the chronal accelerator an acceptable error margin must always be maintained.

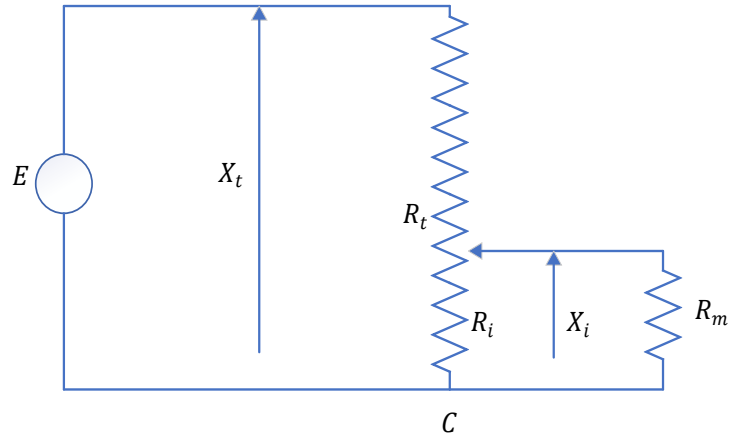


Figure 3: Chronal Accelerator (Left) Equivalent Circuit (Right)

6.1: The output of the chronal accelerator is measured using a voltmeter of internal resistance  $R_m$ .  $R_t$  is the resistance of the total length of the potentiometer  $X_t$  and  $R_i$  is the resistance between the wiper and the common point  $C$  for general wiper position  $X_i$ . Show that the **measurement error** due to the internal resistance  $R_m$  is given by the expression

$$\text{Error} = E \left[ \frac{R_i^2 (R_t - R_i)}{R_t (R_i R_t + R_m R_t + R_i^2)} \right]$$

6.2: Determine the value/position of  $X_i$  where the maximum error occurs

### Question 7:

The Shimada brothers are having a sparring session at their ancestral home in Hanamura. Genji believing to have reached the pinnacle of his training believes he can deflect an arrow fired from his older brother Hanzo in midflight. Hanzo also curious decides to indulge his younger brother request but fires four arrows instead of one channeling into a Dragonstrike. Winston wishing to measure the force of this Dragonstrike attaches 4 strain gauges to Hanzo's bow. For simplicity Hanzo's arm can be modelled as cantilever. The dimensions for Hanzo's bow (and arm) as well as the strain gauges are given in Table 3



Figure 4: Hanzo & Genji Training

Table 3: Strain Gauge & Cantilever Data

Hanzo's Bow	Strain Gauge Data
Length $l = 25\text{cm}$	Gauge Factor $G = 2.1$
Width $w = 6\text{cm}$	Unstrained Resistance $R_0 = 120\Omega$
Thickness $t = 3\text{mm}$	
Young's Modulus $E = 70 \times 10^9\text{Pa}$	
Mount Distance $x = 86\text{cm}$	

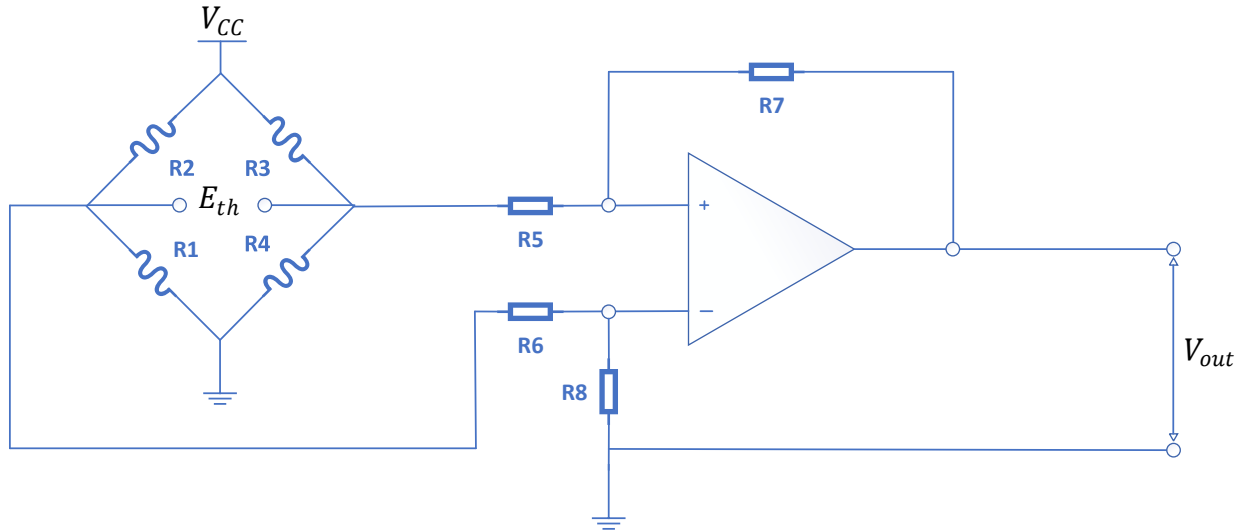


Figure 5: Strain Gauge Bridge & Differential Op-Amp

$$e = \left[ \frac{6(l-x)}{wt^2E} \right] F$$

$$\Delta R = GR_0 e$$

$$R_1 = R_3 = R_0[1 + Ge]$$

$$R_2 = R_4 = R_0[1 - Ge]$$

**7.1:** Determine the resistance of each strain gauge for  $F = 1N$  and  $F = 10N$

**7.2:** Utilizing Figure 5 design a Resistive Deflection Bridge for the force range  $1 \leq F \leq 10N$ . Clearly motivate and state all your design choices.

**7.3:** Why is it advisable to use a differential Op-Amp for the amplification of  $E_{th}$

**7.4:** Suppose due to EMI the Input Bias currents of the Op-Amp ( $I_{B+}$  &  $I_{B-} \neq 0$ ) and a common mode voltage of  $V_n$  is superimposed at the terminals  $V^+$  and  $V^-$ . Determine what the **resistance ratios**  $R_8/R_6$  and  $R_7/R_5$  need to be in order to ensure the output does not fall prey to undesirable effects.

**7.5** From the answer in 7.4 what is the minimum CMRR required to guarantee a series mode interference of no greater than 1%.

### Question 8:

When Nano-Boosted by Ana Winston's Tesla gun experiences a linearizing effect that increases its potency. For simplicity the Nano-Boosting effect can be modelled by a simple Wheatstone Bridge as seen in Figure 6.

Given that  $V_{CC} \leq 24V$  and the resistance of Winston's Tesla Gun ( $R_4$ ) when Nano-Boosted varies in the range  $1 \leq R_4 \leq 100k\Omega$

**8.1:** Design and implement a Wheatstone bridge circuit and hence calculate  $R_1, R_2, R_3$  and select  $R_5 - R_8$  such that the output voltage ranges from 0V to 10V i.e.  $0 \leq V_{out} \leq 10V$ . Assume ideal Op-Amp conditions.

**8.2:** Repeat 8.1 with an  $I_{B+} = I_{B-} = 600\mu A$  and a CMRR = 90dB

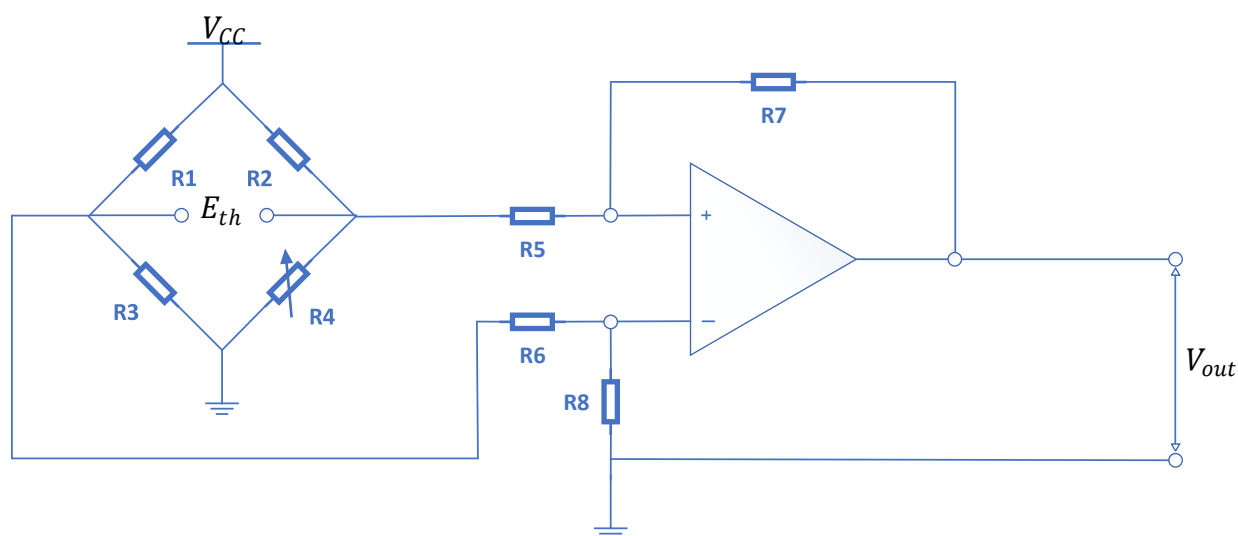


Figure 6: Wheastone (1-Elements Varying) Bridge & Differential Op-Amp

### Question 9:


Since its most recent use the Neogenicrecombinator's cooling chamber has since failed. In order to return the Neogenicrecombinator to a functional state it is decided that it's high time an error analysis of the Neogenicrecombinator's cooling chamber is conducted.


Given that  $\theta$  is the true temperature and  $\theta_M$  is the measured temperature (in Kelvin). The model equations and the corresponding uncertainties are given in Table 4.



Table 4: Temperature Sensor Model Equations

	Thermistor	Deflection Bridge	Recorder
Model Equations	$R_\theta = K_1 e^{\frac{\beta}{\theta}}$	$V_0 = V_s \left[ \frac{1}{1 + \frac{3.3}{R_\theta}} - a_1 \right]$	$\theta_M = V_0 K_2 + a_2$
Mean Values	$\overline{K_1} = 5 \times 10^{-4} k\Omega$ $\overline{\beta} = 3 \times 10^3 K$	$\overline{V_{CC}} = 5.00V$ $\overline{a_1} = 0.77$	$\overline{K_2} = 50.0 kN$ $\overline{a_2} = 300K$
Standard Deviations	$\sigma_{K_1} = 0.5 \times 10^{-4}$ $\sigma_B = 0$	$\sigma_{V_{CC}} = 0.03$ $\sigma_{a_1} = 0.01$	$\sigma_{K_2} = 0$ $\sigma_{a_2} = 3.0$

 **9.1:** Determine the mean output  $\overline{\theta_M}$  and the mean error  $\overline{E} = \overline{\theta} - \overline{\theta_M}$  for an input temperature of 320K.

 **9.2:** Calculate the standard deviation of the output error  $\overline{E}$  for an input of 320K

## Phase II: Dynamic

### Question 10:

Dr Curt Connors fresh from being returned to his bipedal Homosapien self after Dr. Stillwell's latest mishap with the Neogenicrecombinator. Decides to take matters into his own hands (quite literally as he pries the Neogenicrecombinator from Dr. Stillwell's hands) and decides to

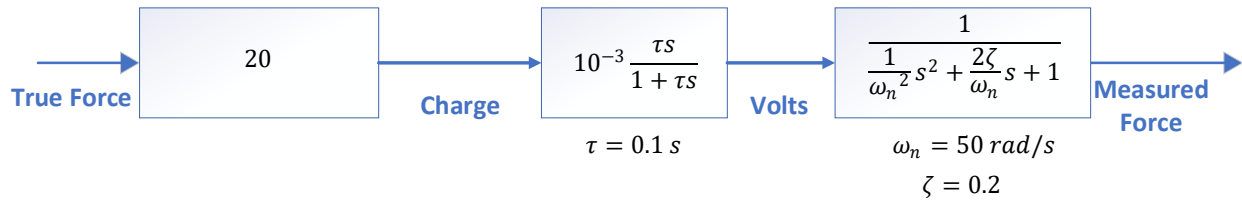





Figure 7: Force Measurement System


$$F_1(t) = 50 \left( \sin(10t) + \frac{1}{3} \sin(30t) + \frac{1}{5} \sin(50t) \right)$$

$$F_2(t) = 50u(t)$$

 **10.1:** Calculate the output of the system in response to the inputs  $F_1(t)$  and  $F_2(t)$  and hence determine the dynamic error of the system:

 **10.2:** From your answers in **10.1** identify which term(s) contribute significantly to the dynamic error.

 **10.3:** Considering  $F_1(t)$  &  $F_2(t)$  which input is more representative of the worst case dynamic error. Motivate your answer.

 **10.4:** Explain with a sketch/diagram or otherwise what modifications are necessary to reduce the dynamic error of  $F_1(t)$  &  $F_2(t)$ .

### Question 11:

Dr Curt Connors fresh from being returned to his bipedal Homosapien self after Dr. Stillwell's latest mishap with the Neogenicrecombinator. Decides to take matters into his own hands (quite literally as he pries the Neogenicrecombinator from Dr. Stillwell's hands) and decides to perform his own dynamic analysis on the troublesome Neogenicrecombinator cooling chamber.

Given that the Neogenicrecombinator(sensing element) is connected to a perfectly linear deflection bridge designed by ESU's latest graduate student that results in an output voltage

$$\Delta V(t) = K \cdot \Delta T_t(t)$$

Dr. Connors fearful of the reemergence of his reptilian alter ego is insistent on a compensating circuit being attached to the output of the deflection bridge.

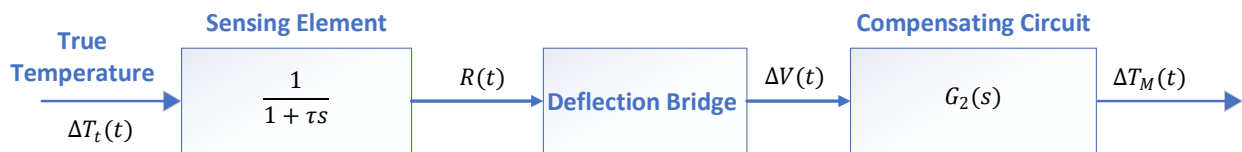





Figure 8: Neogenicrecombinator Cooling Chamber System

 **11.1:** If  $\tau = 0$ , determine  $K$  and  $G_2(s)$  such that the system experiences minimal/no dynamic error

 **11.2:** Determine the dynamic error of the system when  $\tau = 7.125s$ , and  $G_2(s) = 1$  when the system experiences a **sudden** (some might even say **instantaneous**) change in temperature of  $20^\circ C$ .

 **11.3:** If  $G_2(s) = 0.5 \left[ \frac{1+0.5\tau s}{1+\tau s} \right]$ . Give an expression for the complete transfer function (factored pole-zero form is fine) and calculate the dynamic error of the now complete/compensated syst

### Phase III: The Other Evolve Or...

#### Question 12:

Hobgoblin deputizing for the injured Banshee agrees to help Phoenix (a.k.a Jean Grey) in measuring the velocity of travelling meteors in the ionosphere [ *X-Men Inferno Crossover (1988)* ] thrown by one Madelyn Pryor. A sonic pulse transmitter ( $T_X$ ) (fitted to Hobgoblin's facemask) transmits sonic pulses that refract off the meteor surface and are received by Phoenix (Acting as the  $R_X$ ). The experimental setup is shown in Figure 9. According to Dr. Hank McCoy the  $T_X$  transmits continuous sound waves at a **frequency**  $f = 1\text{MHz}$  and the **velocity** of the meteors is in the dynamic range  $5\text{ms}^{-1} \leq v \leq 20\text{ms}^{-1}$ .

The angle  $\theta = 30^\circ$  and the speed of sound in the fluid  $v_f \equiv c = 1485\text{m.s}^{-1}$ .

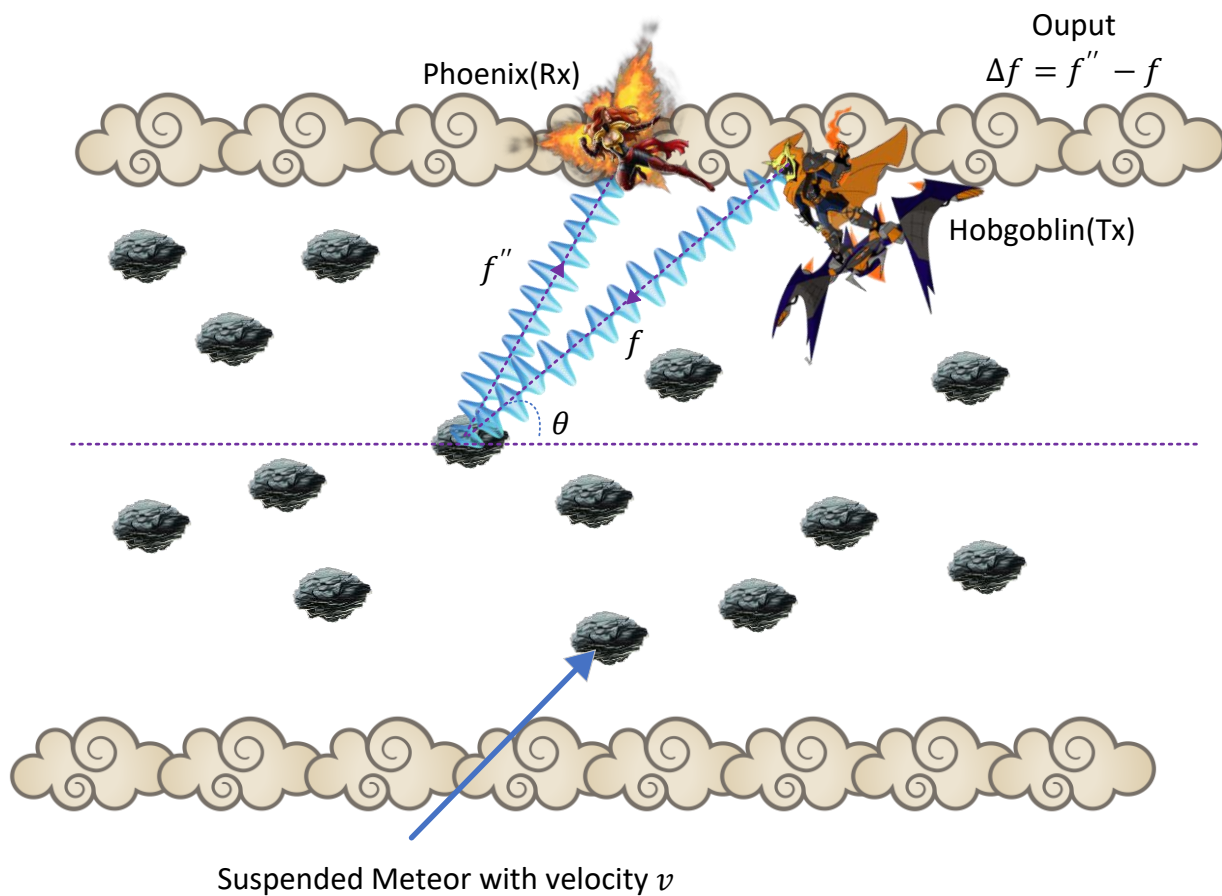






Figure 9: Inferno Meteor Velocity Measurement

 **12.1:** Explain the working principle that the experimental setup in Figure 9 is based on (\*cough\* Doppler), hence give an appropriate **expression for the flow velocity** ,  $v$  , in terms of the Doppler shift  $\Delta f$ . Upon further consultation with Dr. McCoy it is agreed that only the frequency of the Doppler signal is used for calculations. **\*Additionally you may assume  $v \ll c$  is really tiny\***

 **12.2:** Dr McCoy wanting to digitize the Doppler shift ( $\Delta f$ ) information wants to know the **minimum required sampling frequency**  $f_s$  that will be required by the **ADC**.

 **12.3:** Hobgoblin (of all people) suggests to Dr. McCoy that he should in fact increase the sampling frequency  $f_s$  to a value much higher than the minimum required value...in essence oversampling. What if any are the **benefits of oversampling** the Doppler shift  $\Delta f$  information

 **12.4:** Madelyn Pryor enraged by the sight of Phoenix causes the angle to vary randomly around  $\theta = 30^\circ$ , so that the true angle is Gaussian distributed around the mean  $\bar{\theta} = 30^\circ$  and a standard deviation  $\sigma_\theta = 1^\circ$ . Determine a 95% confidence interval (corresponding to  $2\sigma$ ) for the velocity,  $v$ . What is the **maximum error** due to the misalignment caused by Madelyn Pryor as a percentage of maximum flow velocity?

### Question 13:

Ana wanting to test the average projective velocity of her Biotic rifle enlists the help of her twin daughters Pharah A and Pharah B. This is achieved by the subsequent fire of short ultrasonic rocket pulses first from Pharah A (Transducer A) to Pharah B (Transducer B) and then from Pharah B to Pharah A.. The recorded data produces the cross-correlation data present in Figure 12.

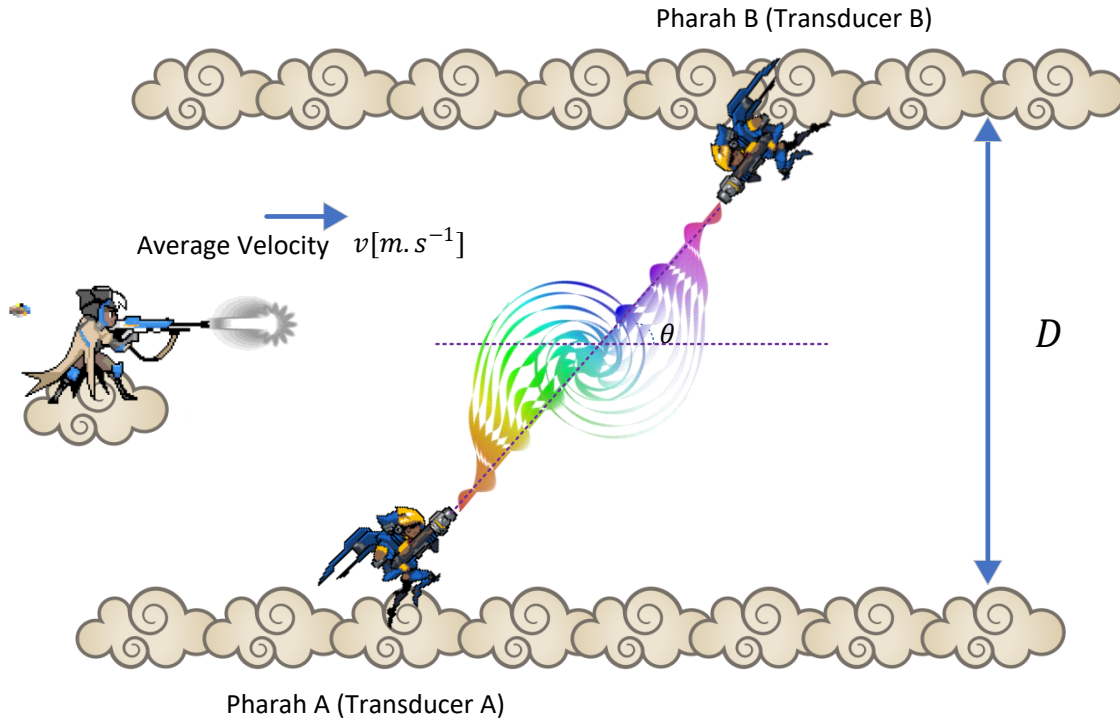


Figure 10: Biotic Rifle Average Velocity Experimental Setup

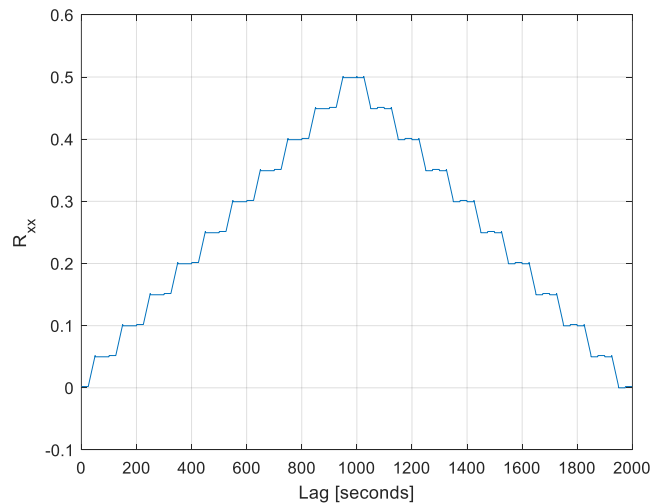


Figure 11: Cross Correlation of Ultrasonic Rocket Pulses

According to Winston the average projectile velocity of the Biotic rifle is given by

$$v = \frac{Lag_{max} \cdot c^2}{2D \cot \theta}$$

Where  $\theta$  is the angle between Pharah A & Pharah B and the center axis of the flow. The speed of sound in the fluid is given by  $c$  and  $D$  is the separation distance.

Given that  $\theta = 30^\circ$ ,  $c = 1480 \text{ m} \cdot \text{s}^{-1}$ ,  $D = 30 \text{ cm}$  and  $f_s = 10 \text{ MHz}$

**[Note  $Lag_{max}$  refers to the point at which  $R_{xx}$  max occurs]**



**13.1:** Determine the average Biotic rifle projectile speed given by  $v$



**13.2:** Winston informs you that the speed of sound  $c$  is in fact not invariant to temperature changes, hence changes in temperature should be accounted for. Assuming that the changes in temperature can be accurately modelled by a **2<sup>nd</sup> order polynomial** make the necessary **changes** to average projectile velocity of the Biotic rifle equation.



**13.3:** Given that there's some uncertainty when mounting the transducer, modeled as  $\sigma_\theta = 0.573^\circ$ . What effect does this have on the overall uncertainty of the measured average projectile velocity of the Biotic rifle.

## Additional Problems (Bentley)

- Chapter 2: 2.1-2.4, 2.7
- Chapter 3: 3.1, 3.3, 3.7
- Chapter 4: 4.1-4.9 (Yes the whole thing)
- Chapter 6: 6.5
- Chapter 8: 8.1-8.2, 8.7-8.8, 8.10
- Chapter 9: 9.2-9.4, 9.8, 9.16, 9.20

Please remember to see the Disclaimer up top these problems in my **OPINION** assess fundamental concepts by no measure am I saying do **ONLY** these.

## Additional Helpful Text?

[https://www.all-electronics.de/wp-content/uploads/2016/11/linearization-of-wheatstone-bridge\\_AN6144.pdf](https://www.all-electronics.de/wp-content/uploads/2016/11/linearization-of-wheatstone-bridge_AN6144.pdf)

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