UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Electrical and Information Engineering
ELEN3012 Signals and Systems IIA

Tutorial 2

Ref 1: Signals, Systems and Transforms by Phillips and Parr, 2nd ed., 1999. For all problems ensure that you can solve for the problem and calculate one or two points using paper based calculations. Where appropriate, use Matlab to check your answers and to solve the problems with a reasonable number of points to obtain a good conceptual understanding.

1. Problem 5.2 from ref 1.

2. Problem 5.3 from ref 1. In addition, determine the Fourier transform using the definition.

3. Problem 5.6 from ref 1.

4. Problem 5.10 from ref 1.

5. Problem 5.11 from ref 1.

6. Problem 5.16 from ref 1. In addition determine the Bode diagrams and bandwidth for the system.

7. Problem 5.18 from ref 1.

8. Problem 5.21 from ref 1.

9. Problem 5.22 from ref 1. In addition determine the bandwidths of the input and output signals.

10. Problem 5.23 from ref 1. In addition do the analysis using Bode diagrams and determine the bandwidths of the system, input and output signals.

11. Problem 5.25 from ref 1.

12. Problem 5.26 from ref 1.

13. Problem 5.27 from ref 1.

14. An input signal $x(t) = te^{-2t}u(t)$ is passed through an LTI system with impulse response $h(t) = e^{-4t}u(t)$. $u(t)$ is the unit step.

(a) Determine the system, input and output signal, frequency response.

(b) Determine the system, input and output signal, bandwidth.

(c) Determine the system Bode diagrams and analyze the output signal arising from the system in terms of Bode diagrams.

15. An input signal

$$x(t) = \sum_{j=-\infty}^{\infty} 2u(t-2j) - 3u(t-1-2j) + u(t-2-2j)$$

is passed through an LTI system with impulse response

$$h(t) = \frac{\sin 4\pi t \sin 8\pi t}{\pi t^2}$$

Repeat parts (a) and (b) of question 14.

16. Consider the interconnection of two LTI systems S_1 and S_2 in cascade and the resultant system in parallel with S_3. S_1 is characterized by its impulse response

$$h_1(t) = e^{2t}u(-t)$$

where $u(t)$ is the unit step. S_2 is represented by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} - 0.1x(t)$$

and S_3 by the differential equation

$$\frac{dy(t)}{dt} + y(t) = -\frac{dx(t)}{dt} + 10x(t)$$

(a) Determine the system frequency response.

(b) Determine the system bandwidth.

(c) Determine the system Bode diagrams.

17. Consider an RLC series circuit with output voltage taken as the voltage across the capacitor.

(a) Prove the system is LTI.

(b) Determine the system Bode diagram for the undamped natural frequency $\omega_n = 10$ and for each damping coefficient $\zeta = 0.1; 0.5; 0.7; 1; 1.5$

18. Consider two LTI systems S_1 and S_2 in cascade. S_2 is described by

$$\frac{dy(t)}{dt} + 50y(t) = x(t)$$

Determine the differential equation of system S_1 such that the frequency response of the cascaded system $H(\omega)$ satisfies the following conditions:

(a) The magnitude of $H(\omega)$ in dB has an asymptotic slope of -40 db/decade for $\omega > 1000$ rad/sec.

(b) At d.c. the magnitude of $H(\omega)$ is between 9 and 10 dB.

(c) At $\omega = 1000$ rad/sec, the magnitude of $H(\omega)$ is between -8 and -10 dB.

(d) For all $0 \leq \omega < 1000$ rad/sec, the magnitude of $H(\omega)$ lies between ± 10 dB.

Solve the problem for both cases:

(i) S_1 and S_2 pole/zero cancellation acceptable.

(ii) S_1 and S_2 pole/zero cancellation not acceptable.

19. A system has specifications:

(i) Zero d.c. gain.

(ii) $|H(j\omega)| = 12$ dB at $\omega = 2$ rad/sec.

(iii) For $\omega_1 \leq \omega \leq 10\omega_1$ rad/sec, $20 \leq |H(j\omega)| \leq 25$ dB.

(iv) For $\omega > 10\omega_1$ rad/sec, $|H(j\omega)|$ has an attenuation of 20 dB/decade.

Determine the system transfer function, break frequencies, and Bode phase diagram.

20. A signal $x(t)$ is passed through a system to produce an output signal $y(t)$. The input signal is

$$x(t) = \frac{\sin(t/2)\sin(2t)}{t^2}$$

and the system is given by

$$\frac{dy(t)}{dt} + ay(t) = kx(t - \tau_0)$$

Determine the values of the constants a, k, τ_0 such that

(a) The bandwidth of $y(t)$ is one third the bandwidth of $x(t)$.

(b) The magnitude of $Y(j\omega)$ at $2\omega_y$ is 10 dB where $Y(j\omega) = \mathcal{F}(y(t))$.

(c) The phase of $Y(j\omega)$ at $3\omega_y$ is -75 deg.

where ω_x and ω_y are the bandwidths of $x(t)$ and $y(t)$ respectively.

21. Consider two LTI systems S_1 and S_2 in cascade to produce system S_3. S_2 is described by

$$\frac{dy(t)}{dt} + 100y(t) = \frac{dx(t)}{dt} - 2x(t)$$

Determine the differential equation of the stable system S_1 such that the frequency response $H_3(\omega)$ satisfies the following conditions:

(a) The system has infinite gain at d.c.

(b) For $0.1 \leq \omega < 50$ rad/sec, $10 \leq |H_3(\omega)| \leq 15$ dB.

(c) For $50 \leq \omega < 300$ rad/sec, $|H_3(\omega)| > 0$ dB.

(d) At $\omega = 300$ rad/sec, $|H_3(\omega)| = 0$ dB.

(e) At $\omega = 300$ rad/sec, $|H_3(\omega)|$ crosses the 0 dB line with a roll off of -20 dB/decade.

Draw the Bode magnitude and phase diagrams of system S_2 and the Bode phase diagram of system S_3.

22. For a real causal system with frequency transfer function

$$H(\omega) = P(\omega) + jQ(\omega)$$

prove that the system impulse response

$$h(t) = \frac{2}{\pi}\int_0^\infty P(\omega)\cos\omega t \, d\omega$$

23. Problem 6.1 from ref 1.

24. Problem 6.2 from ref 1.

25. Problem 6.5 from ref 1.

26. Problem 6.6 from ref 1.

27. A system is specified by

$$H(s) = \frac{-1}{-2 + j\omega}$$

Determine the system impulse response and specify whether the system is causal or not.

28. Do problems 4.1.3; 4.2.4; 4.3.9; 4.6.5 and 4.7.1 from Lathi.

29. If one were to use a low pass RC filter for distortionless transmission of a signal within a frequency bandwidth of 2 kHz, justify the bandwidth of the filter one should use.

REF 1

PROBLEMS

5.1. Find the Fourier transform for each of the following signals, using the Fourier integral
(a) $x(t) = 2[u(t) - u(t-4)]$
(b) $x(t) = e^{-3t}[u(t) - u(t-4)]$
(c) $x(t) = 2t[u(t) - u(t-4)]$
(d) $x(t) = \cos(4\pi t)[u(t+2) - u(t-2)]$

5.2. Use the definition of the Fourier transform (5.1) to find the transform of the following time signals.
(a) $f(t) = (1 - e^{-bt})u(t)$ **(b)** $f(t) = A\cos(\omega_0 t + \phi)$

5.3. Use the table of Fourier transforms (Table 5.2) and the table of properties (Table 5.1) to find the Fourier transform of each of the following signals. *Do not* use the Fourier integral (5.1). (Note that the signals are the same as in Problem 5.1.)
(a) $x(t) = 2[u(t) - u(t-4)]$
(b) $x(t) = e^{-3t}[u(t) - u(t-4)]$
(c) $x(t) = 2t[u(t) - u(t-4)]$
(d) $x(t) = \cos(4\pi t)[u(t+2) - u(t-2)]$

5.4. Prove mathematically that the following properties of the Fourier transform described in Table 5.1 are valid.
(a) Linearity **(b)** Time shifting
(c) Duality **(d)** Frequency shifting
(e) Time differentiation **(f)** Time convolution

5.5. The Fourier transform of $\cos(\omega_0 t)$ is given in Table 5.2. Derive the Fourier transform of $\sin(\omega_0 t)$ using:
(a) The differentiation property
(b) The time-shifting property

5.6. Find and sketch the Fourier transform of the following time-domain signals.
(a) $Ae^{-\beta t}\cos(\omega_0 t)u(t)$, $\mathrm{Re}\{\beta\} > 0$ **(b)** $A\sin(\omega_1 t) + B\cos(\omega_2 t)$
(c) $6\,\mathrm{sinc}(0.5t)$ **(d)** $6\,\mathrm{rect}[(t-4)/3]$

5.7. Find the Fourier transform of the following signals.
(a) $f(t) = 5 + 7\sin(100\pi t)$ **(b)** $g(t) = 5 + 7\sin(500\pi t)$
(c) Show mathematically that the time-scaling property is (or is not) applicable to the solution of part (b).

5.8. Find the Fourier transform of the time function

$$g(t) = 162.6\cos(377t - 0.5)$$

5.9. Find the frequency spectra of the signals shown in Figure P5.9.

5.10. Given:

$$e^{-|t|} \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{2}{\omega^2 + 1}$$

Find the Fourier transform of
(a) $\dfrac{d}{dt}e^{-|t|}$ **(b)** $\dfrac{1}{2\pi(t^2+1)}$ **(c)** $\dfrac{4\cos(2t)}{t^2+1}$

5.11. Find and compare the frequency spectra of the trapezoidal waveforms shown in Figure P5.11.

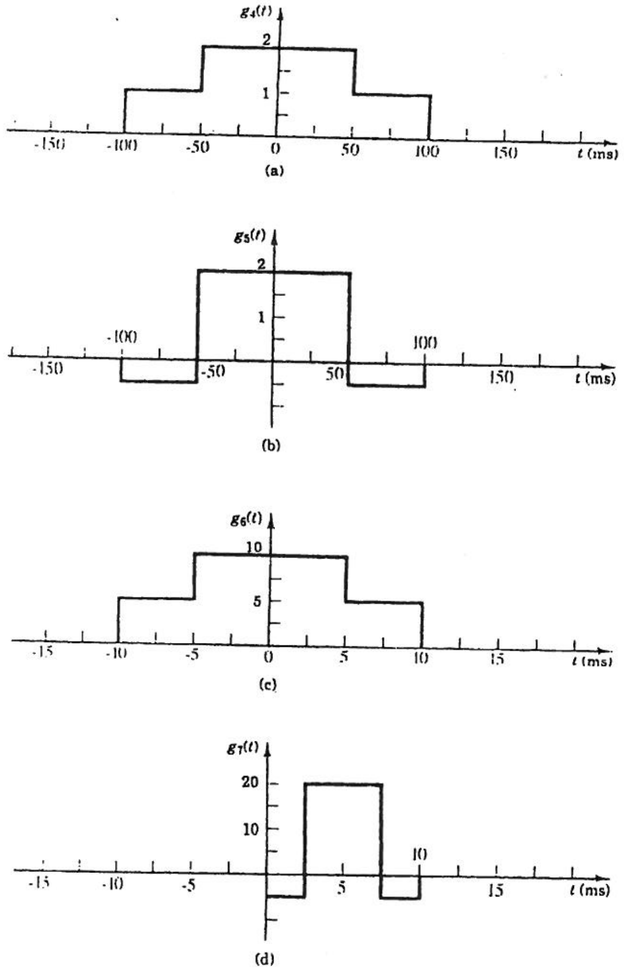

Figure P5.9

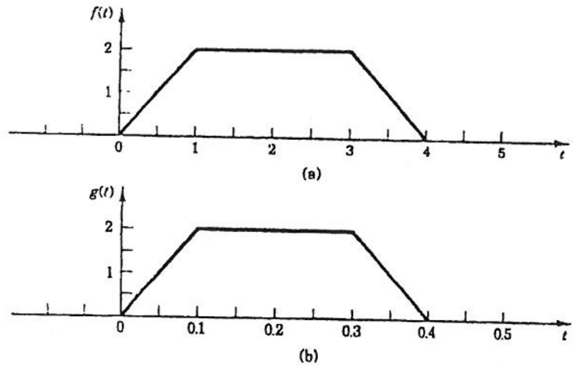

Figure P5.11

5.12. Use the time-derivative property to find the Fourier transform of th form shown in Figure P5.12.

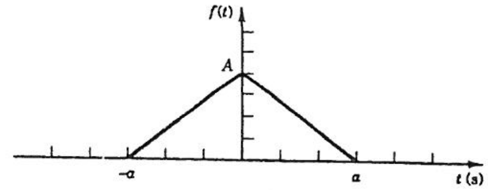

Figure P5.12

5.13. Show that the time-scaling property of the Fourier transform, with a

$$f(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{F(\omega/a)}{|a|}$$

5.14. Find the Fourier transform of the switched sine waveform shown in

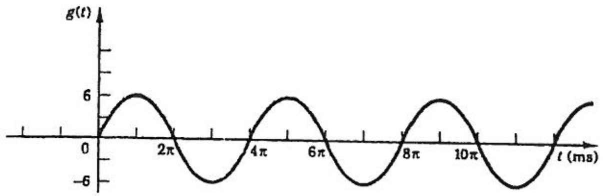

Figure P5.14

5.15. Consider a linear, time-invariant system with impulse response

$$h(t) = \frac{0.5\sin(2t)}{t}$$

(Note that the system is not *causal.*) Find the system output $y(t$ $x(t) = \cos(2t) + \sin(3t)$

5.16. For the electrical network shown in Figure P5.16:
(a) Determine the frequency response function.
(b) Sketch the magnitude and phase frequency response.
(c) Find the impulse response function for this network.

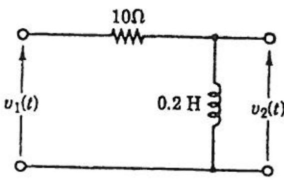

Figure P5.16

5.17. Determine the Fourier transforms of the signals shown in Figure P5.17. (USE THE
erty tables to minimize the effort.)

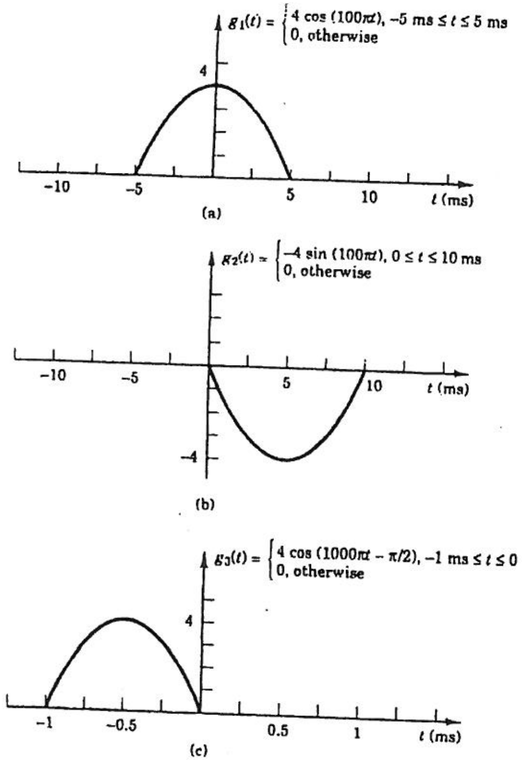

Figure P5.17

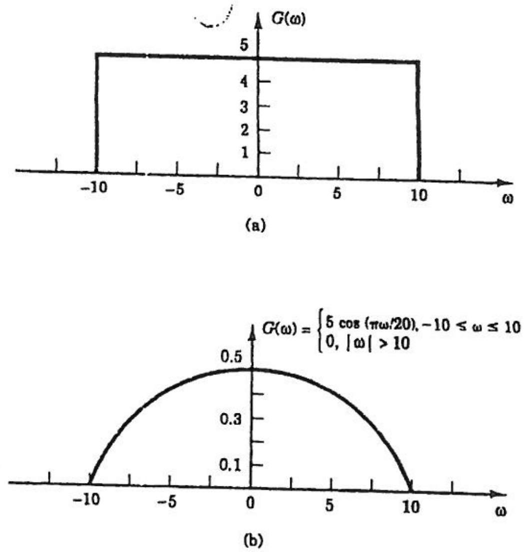

Figure P5.18

5.18. Determine the time-domain functions that have the frequency spectra shown in Figure P5.18.

5.19. The signal $g(t)$ has the Fourier transform

$$G(\omega) = \frac{j\omega}{-\omega^2 + 5j\omega + 6}$$

Find the Fourier transform of:

(a) $g(2t)$ **(b)** $g(3t - 6)$

(c) $\frac{dg(t)}{dt}$ **(d)** $g(-t)$

(e) $e^{-j100t}g(t)$ **(f)** $\int_{-\infty}^{t} g(\tau)d\tau$

5.20. **(a)** Find and sketch the frequency spectrum of the half-wave rectified cosine waveform shown in Figure P5.20(a).

(b) Find and sketch the frequency spectrum of the full-wave rectified cosine wave-form shown in Figure P5.20(b).

(c) Compare the results of parts (a) and (b).

(d) How would the frequency spectra be changed if the period of each waveform in parts (a) and (b) was halved?

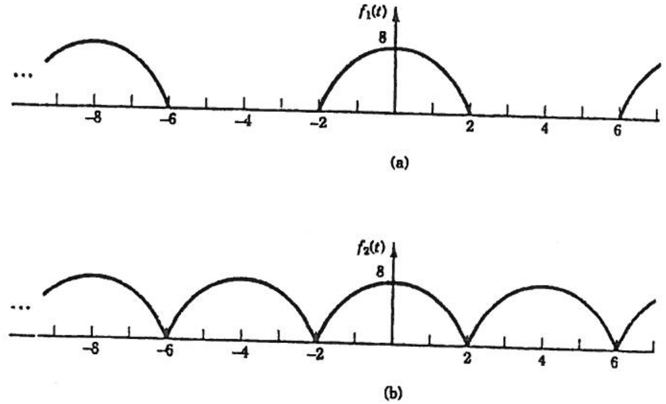

Figure P5.20

5.21. The periodic signal $g_p(t)$ is shown in Figure P5.21. Find and sketch $G_p(\omega)$.

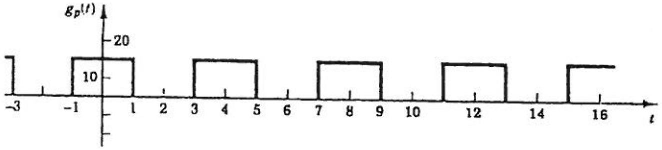

Figure P5.21

5.22. The signal $v_1(t) = \sin(50t)$ volts is applied to the input terminals of the electrical network shown in Figure P5.16. Find the frequency spectrum of the signal $v_2(t)$ that is produced at the output terminals of the circuit.

5.23. The signal $v_1(t) = 10\,\mathrm{rect}(t)$ is applied to the input of the network with the frequency response $H(\omega) = \mathrm{rect}(\omega/4\pi)$ as shown in Figure P5.23. Determine and sketch the frequency spectrum of the output signal $V_2(\omega)$.

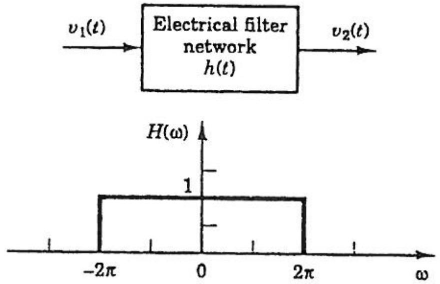

Figure P5.23

5.24. The pulsed cosine signal shown in Figure P5.24 is "on" for two cycles and then "off" for a period of time equivalent to 18 cycles of the cosine wave. The signal is periodic and the frequency of the cosine wave is 2000π rad/s. Sketch the frequency spectrum for this signal.

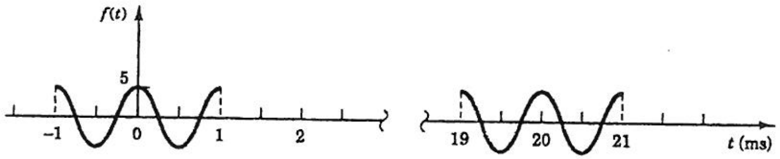

Figure P5.24

5.25. The signals with the frequency spectra shown in Figure P5.25(a) and (b) are sampled using an ideal sampler with $\omega_s = 200$ rad/s.

(a) Sketch the frequency spectra of the sampled signals.
(b) Compare and discuss the results of part (a).

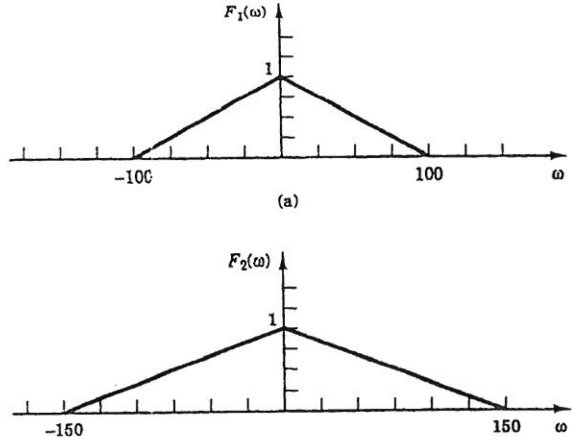

Figure P5.25

5.26. What percentage of the total energy in the energy signal $f(t) = e^{-t}u(t)$ IS CONTAINED IN the frequency band -7 rad/s $\leq \omega \leq 7$ rad/s?

5.27. A power signal with the power spectral density shown in Figure P5.27(a) IS THE INPUT to a linear system with the frequency response shown in Figure P5.27(b). CALCU AND sketch the power spectral density of the system's output signal.

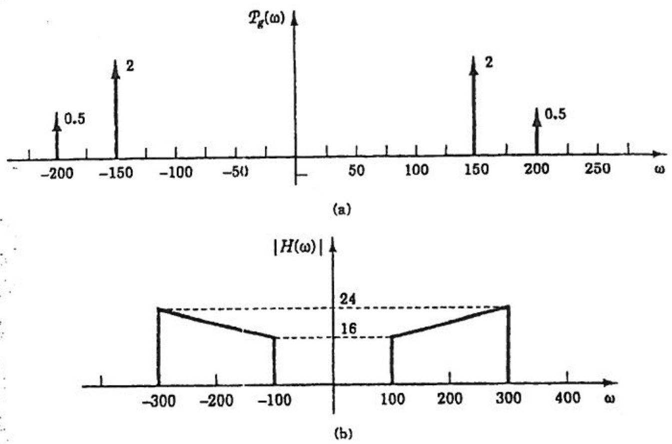

Figure P5.27

This frequency spectrum is illustrated in Figure 6.43. The reader is encouraged to compare the frequency spectrum of the *flat-top* PAM signal with that of the *natural-top* PAM signal shown in Figure 6.36(c).

Notice that the original continuous-time signal can be approximately recovered by filtering the sampled signal with a low-pass filter with cutoff frequency $\omega_M < \omega_c < \omega_s/2$.

SUMMARY

In this chapter, we have looked at several ways that the Fourier transform can be applied to the analysis and design of signals and systems. The applications considered here demonstrate use of the Fourier transform as an analysis tool.

We considered the *duration-bandwidth* relationship and found that the bandwidth of a signal is inversely proportional to its time duration. We saw that if a signal changes values rapidly in time, it has a wide bandwidth in frequency.

Four basic types of *ideal filters* were presented. Applications were shown for the concepts of the *ideal low-pass, ideal high-pass, ideal bandpass*, and *ideal bandstop* filters. Although these ideal filters are not physically realizable, it was shown that the concept of an ideal filter can simplify the early stages of a system analysis or design.

Butterworth and *Chebyschev filters* were presented as standard filter designs that provide physically realizable approximations of ideal filters. Examples were given to show how these filters can be realized by electrical circuits.

Two techniques of sinusoidal modulation (DSB/SC-AM, and DSB/WC-AM) and two types of pulse-amplitude modulation (natural and flat top) were presented to demonstrate applications of the Fourier transform to the study of communication systems and signals.

REFERENCES

1. L. W. Couch II. *Modern Communication Systems*, Upper Saddle River, NJ: Prentice Hall. 1995.
2. G. E. Carlson. *Signal and Linear System Analysis*. Boston: Houghton Mifflin. 1992.
3. International Telephone and Telegraph Corporation. *Reference Data for Radio Engineers*. 5th ed. Indianapolis. IN: Howard W. Sams, 1973.
4. C. J. Savant, Jr., M. S. Roden, and G. L. Carpenter. *Electronic Design: Circuits and Systems*, 2nd ed. Redwood City, CA: Benjamin/Cummings, 1991.
5. A. B. Williams. *Electronic Filter Design Handbook*. New York: McGraw-Hill, 1981.
6. S. Haykin. *An Introduction to Analog and Digital Communications*. New York: Wiley, 1989.
7. A. J. Jerri. "The Shannon Sampling Theorem—Its Various Extensions and Applications: A Tutorial Review." *Proceedings of IEEE*. vol. 65. pp. 1565–1596, 1977.

PROBLEMS

6.1. Show mathematically that the ideal high-pass filter is not physically realizable.

6.2. Show mathematically that the ideal bandpass filter is not physically REALIZABLE

6.3. As illustrated in Figure P6.3, the periodic square wave is the input SIGNAL TO AN IDEAL

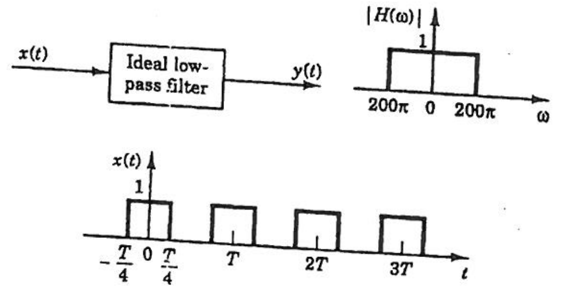

Figure P6.3

low-pass filter with the frequency spectrum shown. Use a computer ~~ex~~ CALCULATOR calculator to find the output signal if the input signal has a period of

(a) 40 ms

(b) 20 ms

(c) 10 ms

Sketch each of the output signals.

6.4. Use MATLAB and SIMULINK to show $y(t)$ if the ideal low-pass filter (replaced by an RC low-pass filter with $\omega_c = 200\pi$.

6.5. Calculate the frequency response of the circuit shown in Figure P6.5 AND DETERMI what type of ideal filter is approximated by this circuit.

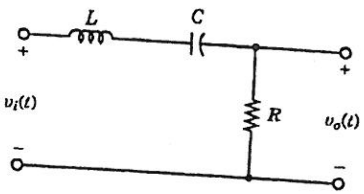

Figure P6.5

6.6. Calculate the frequency response of the circuit shown in Figure P6.6 AND DETERMINE what type of ideal filter is approximated by this circuit.

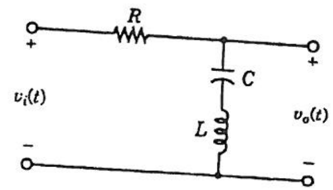

Figure P6.6

shifting of a signal does not change its amplitude spectrum, but adds a linear phase spectrum. Multiplication of a signal by an exponential $e^{j\omega_0 t}$ results in shifting the spectrum to the right by ω_0. In practice, spectral shifting is achieved by multiplying a signal with a sinusoid such as $\cos \omega_0 t$ (rather than the exponential $e^{j\omega_0 t}$). This process is known as amplitude modulation. Multiplication of two signals results in convolution of their spectra, whereas convolution of two signals results in multiplication of their spectra.

For an LTIC system with the transfer function $H(\omega)$, the input and output spectra $F(\omega)$ and $Y(\omega)$ are related by the equation $Y(\omega) = F(\omega)H(\omega)$. This is valid only for asymptotically stable systems. For distortionless transmission of a signal through an LTIC system, the amplitude response $|H(\omega)|$ of the system must be constant, and the phase response $\angle H(\omega)$ should be a linear function of ω over a band of interest. Ideal filters, which allow distortionless transmission of a certain band of frequencies and suppress all the remaining frequencies, are physically unrealizable (noncausal). In fact, it is impossible to build a physical system with zero gain $[H(\omega) = 0]$ over a finite band of frequencies. Such systems (which include ideal filters) can be realized only with infinite time delay in the response.

The energy of a signal $f(t)$ is equal to $1/2\pi$ times the area under $|F(\omega)^2|$ (Parseval's theorem). The energy contributed by spectral components within a band $\Delta\mathcal{F}$ (in Hz) is given by $|F(\omega)|^2\Delta\mathcal{F}$. Therefore, $|F(\omega)|^2$ is the energy spectral density per unit bandwidth (in Hz). The energy spectral density $|F(\omega)|^2$ of a signal $f(t)$ is the Fourier transform of the autocorrelation function $\psi_f(t)$ of the signal $f(t)$. Thus, a signal autocorrelation function has a direct link to its spectral information.

The process of modulation shifts the signal spectrum to different frequencies. Modulation is used for many reasons: to transmit several messages simultaneously over the same channel to utilize channel's high bandwidth, to effectively radiate power over a radio link, to shift signal spectrum at higher frequencies to overcome the difficulties associated with signal processing at lower frequencies, to effect the exchange of transmission bandwidth and transmission power required to transmit data at a certain rate. Broadly speaking there are two types of modulation; amplitude and angle modulation. Each of these two classes has several subclasses. Amplitude modulation bandwidth is generally fixed. The bandwidth in angle modulation, however, is controllable. The higher the bandwidth, the more immune is the scheme to noise.

In practice, we often need to truncate data. Truncating data is like viewing it through a window, which permits a view of only certain portions of the data and hides (suppresses) the remainder. Abrupt truncation of data amounts to a rectangular window, which assigns a unit weight to data seen from the window and assigns zero weight to the remaining data. Tapered windows, on the other hand, reduce the weight gradually from 1 to 0. Data truncation can cause some unsuspected problems. For example, in computation of the Fourier transform, windowing (data truncation) causes spectral spreading (spectral smearing) that is characteristic of the window function used. A rectangular window results in the least spreading, but it does so at the cost of a high and oscillatory spectral leakage outside the signal band which decays slowly as $1/\omega$. Compared to a rectangular window, tapered windows in general have larger spectral spreading (smearing), but the spectral leakage is smaller and decays faster with frequency. If we try to reduce spectral leakage by using a smoother window, the spectral spreading increases. Fortunately, the

LATHI

spectral spreading can be reduced by increasing the window width. Therefore, we can achieve a given combination of spectral spread (transition bandwidth) and leakage characteristics by choosing a suitable tapered window function of a sufficiently longer width T.

References

1. Churchill, R.V., and J.W. Brown, *Fourier Series and Boundary Value Problems, 3d ed.*, McGraw-Hill, New York, 1978.
2. Bracewell, R.N., *Fourier Transform and Its Applications, revised 2nd Ed.*, McGraw-Hill, New York, 1986.
3. Guillemin, E.A., *Theory of Linear Physical Systems*, Wiley, New York, 1963.
4. Lathi, B.P., *Modern Digital and Analog Communication Systems, 3rd ed.*, Oxford University Press, New York, 1998.
5. J. Carson, "Notes on Theory of Modulation" *Proc. IRE*, vol 10, February 1922, pp. 57-64.
6. J. Carson, "The Reduction of Atmospheric Disturbances" *Proc. IRE*, vol 16, July 1928, pp. 966-975.
7. Armstrong E.H., "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation", *Proc. IRE*, vol. 24, May 1936, pp. 689-740.
8. Hamming, R.W., *Digital Filters, 2nd Ed.*, Prentice-Hall, Englewood Cliffs, N.J. 1983.
9. Harris, F.J., "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", *Proc. IEEE*, vol. 66, No. 1, January 1978, pp. 51-83.

Problems

4.1-1 Show that if $f(t)$ is an even function of t, then

$$F(\omega) = 2\int_0^\infty f(t)\cos \omega t\, dt$$

and if $f(t)$ is an odd function of t, then

$$F(\omega) = -2j\int_0^\infty f(t)\sin \omega t\, dt$$

Hence, prove that if $f(t)$ is a real and even function of t, then $F(\omega)$ is a real and even function of ω. In addition, if $f(t)$ is a real and odd function of t, then $F(\omega)$ is an imaginary and odd function of ω.

4.1-2 Show that for a real $f(t)$, Eq. (4.8b) can be expressed as

$$f(t) = \frac{1}{\pi}\int_0^\infty |F(\omega)| \cos[\omega t + \angle F(\omega)]\, d\omega$$

This is the trigonometric form of the Fourier integral. Compare this with the compact trigonometric Fourier series.

4.1-3 A signal $f(t)$ can be expressed as the sum of even and odd components (see Sec. 1.5-2):

$$f(t) = f_e(t) + f_o(t)$$

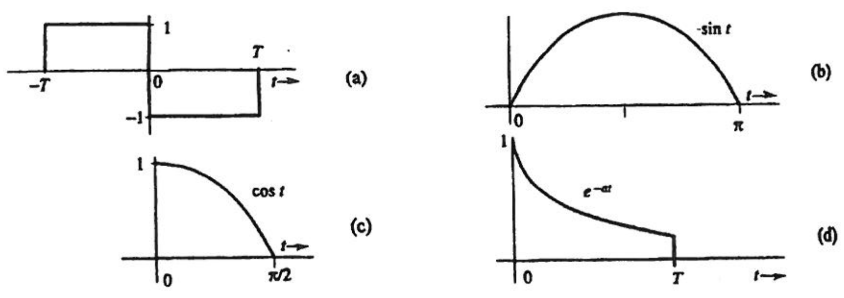

Fig. P4.3-3

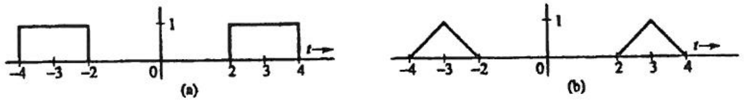

Fig. P4.3-4

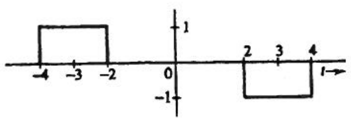

Fig. P4.3-5

Hint: Signals in Figs. b, c, and d can be expressed in the form $f(t)[u(t) - u(t-a)]$.

4.3-4 Using the time-shifting property, show that if $f(t) \Longleftrightarrow F(\omega)$, then

$$f(t+T) + f(t-T) \Longleftrightarrow 2F(\omega)\cos T\omega$$

This is the dual of Eq. (4.41). Using this result and pairs 17 and 19 in Table 4.1, find the Fourier transforms of the signals shown in Fig. P4.3-4.

4.3-5 Prove the following results, which are duals of each other:

$$f(t)\sin\omega_0 t \Longleftrightarrow \frac{1}{2j}[F(\omega-\omega_0) - F(\omega+\omega_0)]$$
$$\frac{1}{2j}[f(t+T) - f(t-T)] \Longleftrightarrow F(\omega)\sin T\omega$$

Using the latter result and Table 4.1, find the Fourier transform of the signal in Fig. P4.3-5.

4.3-6 The signals in Fig. P4.3-6 are modulated signals with carrier cos 10t. Find the Fourier transforms of these signals using the appropriate properties of the Fourier transform and Table 4.1. Sketch the amplitude and phase spectra for parts (a) and (b).

4.3-7 Using the frequency-shifting property and Table 4.1, find the inverse Fourier transform of the spectra depicted in Fig. P4.3-7.

4.3-8 Using the time convolution property, prove pairs 2, 4, 13 and 14 in Table 2.1 (assume $\lambda < 0$ in pair 2, λ_1 and $\lambda_2 < 0$ in pair 4, $\lambda_1 < 0$ and $\lambda_2 > 0$ in pair 13, and λ_1 and $\lambda_2 > 0$ in pair 14). Hint: You will need partial fraction expansion. For pair 2, you need to apply the result in Eq. (1.23).

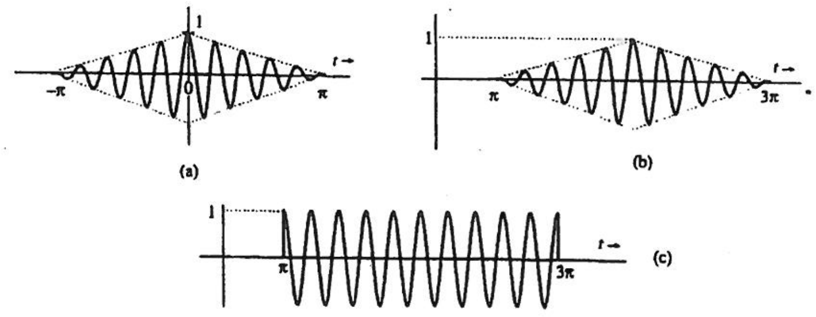

Fig. P4.3-6

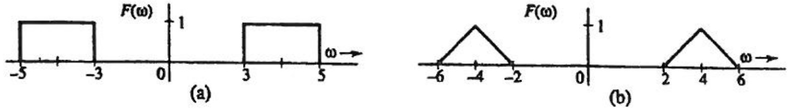

Fig. P4.3-7

4.3-9 A signal $f(t)$ is bandlimited to B Hz. Show that the signal $f^n(t)$ is bandlimited to nB Hz. Hint: Start with $n = 2$. Use frequency convolution property and the width property of convolution.

4.3-10 Find the Fourier transform of the signal in Fig. P4.3-3a by three different methods:
(a) By direct integration using the definition (4.8a).
(b) Using only pair 17 Table 4.1 and the time-shifting property.
(c) Using the time-differentiation and time-shifting properties, along with the fact that $\delta(t) \Longleftrightarrow 1$.
Hint: $1 - \cos 2x = 2\sin^2 x$.

4.3-11 (a) Prove the frequency differentiation property (dual of the time differentiation):

$$-jtf(t) \Longleftrightarrow \frac{d}{d\omega}F(\omega)$$

(b) Using this property and pair 1 (Table 4.1), determine the Fourier transform of $te^{-at}u(t)$.

4.4-1 For an LTIC system with transfer function

$$H(s) = \frac{1}{s+1}$$

find the (zero-state) response if the input $f(t)$ is (a) $e^{-2t}u(t)$ (b) $e^{-t}u(t)$ (c) $e^{t}u(-t)$ (d) $u(t)$
Hint: For part (d), you need to apply the result in Eq. (1.23).

4.4-2 A stable LTIC system is specified by the transfer function

$$H(\omega) = \frac{-1}{j\omega - 2}$$

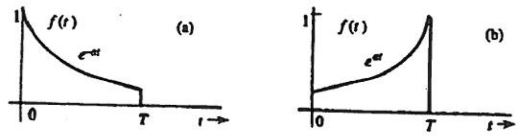

Fig. P4.1-4

Fig. P4.1-5

Fig. P4.1-6

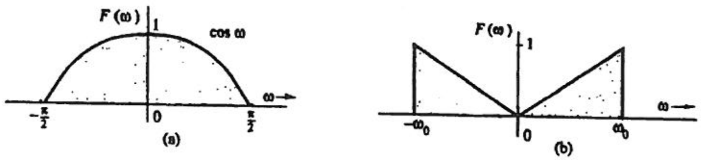

Fig. P4.1-7

(a) If $f(t) \Longleftrightarrow F(\omega)$, show that for real $f(t)$,

$$f_e(t) \Longleftrightarrow \mathrm{Re}[F(\omega)] \quad \text{and} \quad f_o(t) \Longleftrightarrow j\,\mathrm{Im}[F(\omega)]$$

(b) Verify these results by finding the Fourier transforms of the even and odd components of the following signals: (i) $u(t)$ (ii) $e^{-at}u(t)$.

4.1-4 From definition (4.8a), find the Fourier transforms of the signals $f(t)$ in Fig. P4.1-4.

4.1-5 From definition (4.8a), find the Fourier transforms of the signals depicted in Fig. P4.1-5.

4.1-6 Using Eq. (4.8b), find the inverse Fourier transforms of the spectra in Fig. P4.1-6

4.1-7 Using Eq. (4.8b), find the inverse Fourier transforms of the spectra in Fig. P4.1-7.

4.2-1 Sketch the following functions:
(a) $\mathrm{rect}\left(\frac{t}{2}\right)$ (b) $\Delta\left(\frac{3\omega}{100}\right)$ (c) $\mathrm{rect}\left(\frac{t-10}{8}\right)$ (d) $\mathrm{sinc}\left(\frac{\pi\omega}{5}\right)$ (e) $\mathrm{sinc}\left(\frac{\omega-10\pi}{5}\right)$
(f) $\mathrm{sinc}\left(\frac{t}{5}\right)\mathrm{rect}\left(\frac{t}{10\pi}\right)$. Hint: $f\left(\frac{x-a}{b}\right)$ is $f\left(\frac{x}{b}\right)$ right-shifted by a.

4.2-2 From definition (4.8a), show that the Fourier transform of $\mathrm{rect}\,(t-5)$ is $\mathrm{sinc}\left(\frac{\omega}{2}\right)e^{-j5\omega}$. Sketch the resulting amplitude and phase spectra.

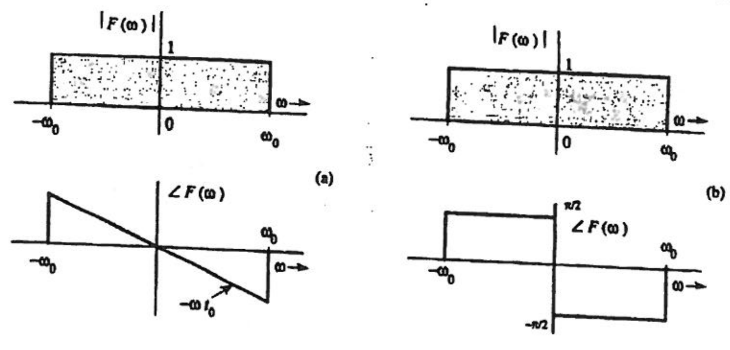

Fig. P4.2-4

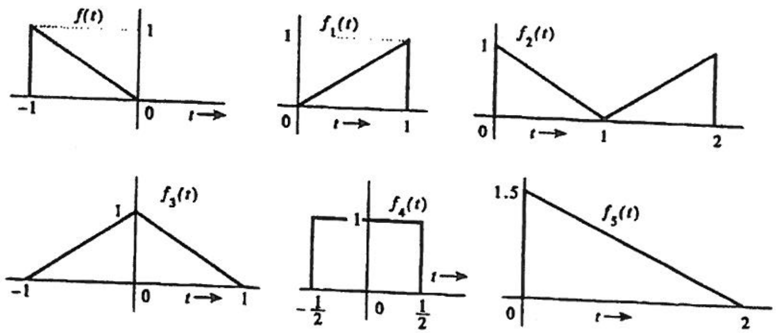

Fig. P4.3-2

4.2-3 From definition (4.8b), show that the inverse Fourier transform of $\mathrm{rect}\left(\frac{\omega-10}{2\pi}\right)$ is $\mathrm{sinc}\,(\pi t)\,e^{j10t}$.

4.2-4 Find the inverse Fourier transform of $F(\omega)$ for the spectra illustrated in Figs. P4.2-4a and b.
Hint: $F(\omega) = |F(\omega)|e^{j\angle F(\omega)}$. This problem illustrates how different phase spectra (both with the same amplitude spectrum) represent entirely different signals.

4.3-1 Apply the symmetry property to the appropriate pair in Table 4.1 to show that
(a) $\frac{1}{2}[\delta(t) + \frac{j}{\pi t}] \Longleftrightarrow u(\omega)$ (b) $\delta(t+T) + \delta(t-T) \Longleftrightarrow 2\cos T\omega$
(c) $\delta(t+T) - \delta(t-T) \Longleftrightarrow 2j\sin T\omega$.

4.3-2 The Fourier transform of the triangular pulse $f(t)$ in Fig. P4.3-2a is expressed as

$$F(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$$

Using this information, and the time-shifting and time-scaling properties, find the Fourier transforms of the signals $f_i(t)$ ($i = 1, 2, 3, 4, 5$) shown in Fig. P4.3-2.
Hint: See Sec. 1.3 for explanation of various signal operations. Pulses $f_i(t)$ ($i = 2, 3, 4$ can be expressed as a combination of $f(t)$ and $f_1(t)$ with suitable time shift (which may be positive or negative).

4.3-3 Using only the time-shifting property and Table 4.1, find the Fourier transforms of the signals depicted in Fig. P4.3-3.

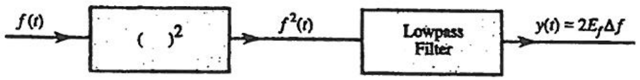

Fig. P4.6-3

Hint: If $f^2(t) \Longleftrightarrow A(\omega)$, then show that $Y(\omega) \approx [4\pi A(0)\Delta\mathcal{F}]\delta(\omega)$ if $\Delta\mathcal{F} \to 0$. Now, show that $A(0) = E_f$.

1.6-4 Generalize Parseval's theorem to show that for real, Fourier transformable signals $f_1(t)$ and $f_2(t)$

$$\int_{-\infty}^{\infty} f_1(t)f_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(-\omega)F_2(\omega)\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)F_2(-\omega)\,d\omega$$

4.6-5 For the signal

$$f(t) = \frac{2a}{t^2 + a^2}$$

determine the essential bandwidth B Hz of $f(t)$ such that the energy contained in the spectral components of $f(t)$ of frequencies below B Hz is 99% of the signal energy E_f. Hint: See Exercise E4.5b.

4.7-1 For each of the following 3 baseband signals (i) $m(t) = \cos 1000t$ (ii) $m(t) = 2\cos 1000t + \cos 2000t$ (iii) $m(t) = \cos 1000t \cos 3000t$

(a) Sketch the spectrum of $m(t)$.
(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10{,}000t$.
(c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.
(d) Identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB and LSB spectra. Explain the nature of frequency shifting in each case.

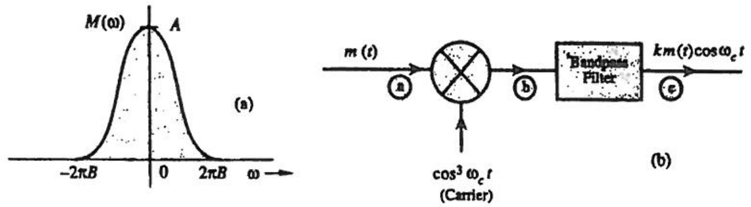

Fig. P4.7-2

4.7-2 You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos\omega_c t$, where $m(t)$ is a signal bandlimited to B Hz (Fig. P4.7-2a). Figure P4.7-2b shows a DSB-SC modulator available in the stock room. The bandpass filter is tuned to ω_c. The carrier generator available generates not $\cos\omega_c t$, but $\cos^3\omega_c t$.
(a) Explain whether you would be able to generate the desired signal using only this equipment. If so, what is the value of k?
(b) Determine the signal spectra at points b and c, and indicate the frequency bands occupied by these spectra.
(c) What is the minimum usable value of ω_c?
(d) Would this scheme work if the carrier generator output were $\cos^2\omega_c t$? Explain.
(e) Would this scheme work if the carrier generator output were $\cos^n\omega_c t$ for any integer $n \geq 2$?

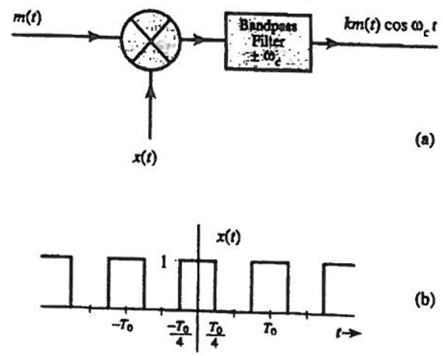

Fig. P4.7-3

4.7-3 In practice, the analog multiplication operation is difficult and expensive. For this reason, in amplitude modulators, it is necessary to find some alternative to multiplication of $m(t)$ with $\cos\omega_c t$. Fortunately, for this purpose, we can replace multiplication with switching operation. A similar observation applies to demodulators. In the scheme depicted in Fig. P4.7-3a, the period of the rectangular periodic pulse $x(t)$ shown in Fig. P4.7-3b is $T_0 = 2\pi/\omega_c$. The bandpass filter is centered at $\pm\omega_c$. Note that multiplication by a square periodic pulse $x(t)$ in Fig. P4.7-3b amounts to periodic on-off switching of $m(t)$. This is a relatively simple and inexpensive operation.
Show that this scheme can generate amplitude modulated signal $k\cos\omega_c t$. Determine the value of k. Show that the same scheme can also be used for demodulation provided the bandpass filter in Fig. P4.7-3a is replaced by a lowpass (or baseband) filter.

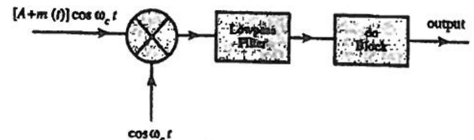

Fig. P4.7-4

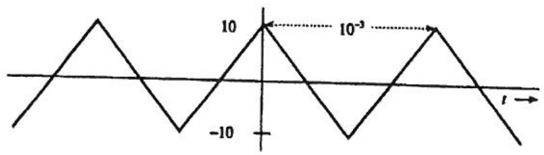

Fig. P4.7-5

4.7-4 Figure P4.7-4 presents a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal $[A + m(t)]\cos\omega_c t$ regardless of the value of A.

4.7-5 Sketch the AM signal $[A+m(t)]\cos\omega_c t$ for the periodic triangle signal $m(t)$ illustrated in Fig. P4.7-5 corresponding to the modulation index: (a) $\mu = 0.5$, (b) $\mu = 1$, (c) $\mu = 2$, and (d) $\mu = \infty$. How do you interpret the case $\mu = \infty$?