TRIGONOMETRIC IDENTITY

Basic Identities

- $\cos^2\theta + sin^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $\cot^2\theta + 1 = cosec^2\theta$

Addition Formula

- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\, co \quad + \sin\alpha\sin\beta$
- $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
- $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double Angle Formula

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
- $\cos 2\alpha = \begin{Bmatrix} \cos^2\alpha - \sin^2\alpha \\ or \quad 2\cos^2\alpha - 1 \\ or \quad 1 - 2\sin^2\alpha \end{Bmatrix}$
- $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

Triple Angle Formula

- $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$
- $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
- $\tan 3\alpha = \frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$

Half Angle Formula

- $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$
- $\cos\alpha = \begin{Bmatrix} 2\cos^2\frac{\alpha}{2} - 1 \\ or \quad 1 - 2\sin^2\frac{\alpha}{2} \end{Bmatrix}$
- $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$

t-Formula

$From;\ \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$

$let\ \ \tan\frac{\alpha}{2} = t$

$then\ \ \tan\alpha = \frac{2t}{1-t^2}$

$\Rightarrow\ \sin\alpha = \frac{2t}{1+t^2}\ \ and\ \ \cos\alpha = \frac{1-t^2}{1+t^2}$

Factor Formula

- $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$
- $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Inverse Function

- $\sin^{-1}x = (-1)^n\sin^{-1}x + n\pi$
- $\cos^{-1}x = \pm\cos^{-1}x + 2n\pi$
- $tan^{-1}x = \tan^{-1}x + n\pi$

Sum of Trigonometric Functions

Case 1: $a\cos x + b\sin x = Rcos(x-\alpha)$

Case 2: $a\cos x - b\sin x = Rcos(x+\alpha)$

Case 3: $b\sin x + a\cos x = Rsin\,(x+\alpha)$

Case 4: $b\sin x - a\cos x = Rsin\,(x-\alpha)$

Note;

$\alpha = \tan^{-1}\left(\frac{2nd\ constant}{1st\ constant}\right)$

i.e.

$for\ case\ 1\ and\ case\ 2;\quad \alpha = \tan^{-1}\frac{b}{a}$

$for\ case\ 3\ and\ case\ 4;\quad \alpha = \tan^{-1}\frac{a}{b}$

$for\ all\ four\ cases;\quad R = \sqrt{a^2 + b^2}$

Special Triangles

	Sin	Cos	Tan
$\mathbf{30^0}\left(\frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\mathbf{60^0}\left(\frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\mathbf{45^0}\left(\frac{\pi}{4}\right)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\mathbf{90^0}\left(\frac{\pi}{2}\right)$	1	0	∞

Area of a Triangle

Let the area of any triangle with sides a, b and c be represented by Δ, then;

- **For Right-angle triangle**

  $\Delta = \frac{1}{2} \times base \times height$

- **Using sine approach**

$\Delta = \frac{1}{2}(product\ of\ two\ sides)\mathbf{sin}(the\ third\ angle)$

  i.e. $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

- **Hero's Formula**

  $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

  $Where\ s = \frac{a+b+c}{2}$

Important Reduction Formulae

- $\sin(-\alpha) = -\sin\alpha$
- $\cos(-\alpha) = \cos\alpha$
- $\tan(-\alpha) = -\tan\alpha$
- $\sin(\pi \pm \alpha) = \sin\alpha$
- $\cos(\pi \pm \alpha) = -\cos\alpha$
- $\tan(\pi \pm \alpha) = \pm\tan\alpha$

Get more in your school textbook.

QUESTIONS

1. Which of the following is not a correct formula for finding the area (Δ) of a triangle with sides, a, b and c with corresponding angle A, B and C

   a) $\Delta = \frac{1}{2}ba\sin C$ b) $\Delta = [s(s-a)(s-b)(s-c)]^{\frac{1}{2}}$

   c) $\Delta = \frac{1}{2}ab\sin A$ d) $\Delta = \frac{1}{2}base \times height$

2. Which of the following is not equivalent to $\cos 2\alpha$ a) $\cos^2\alpha - \sin^2\alpha$ b) $2\cos^2\alpha - 1$

   c) $1 - 2\sin^2\alpha$ d) $\cos^2\alpha - 2\sin^2\alpha$

3. Which of the basic quantities is incorrect

   a) $\cos^2\theta + sin^2\theta = 1$ b) $1 - \sec^2\theta = -\tan^2\theta$

   c) $\sin^2\theta - \sec^2\theta = 1$ d) $\cot^2\theta + 1 = cose\ ^2\theta$

4. Find the area of a triangle with sides 5cm, 7cm and 8cm

   a) $20cm^2$ b) $17.3cm^2$ c) $16.7cm^2$ d) $10cm^3$

5. The triple angle, $sin3\alpha$ is equal to

   a) $3\sin\alpha - \sin 2\alpha$ b) $3\sin\alpha - 4\sin^3\alpha$

   c) $4\sin^2\alpha$ d) $3\sin^3\alpha - 4\sin\alpha$

6. Find the value of $\sin\theta$ if $\cos\theta = \frac{5}{12}$

   a) $\frac{\sqrt{119}}{12}$ b) $\frac{3}{12}$ c) $\frac{12}{\sqrt{199}}$ d) $\frac{5}{13}$

7. If $\tan\theta = \frac{3}{4}$ and $0^0 < \theta < 90^0$, then $\cos\theta$ =?

   a) $\frac{5}{4}$ b) $\frac{5}{3}$ c) $\frac{4}{5}$ d) $\frac{4}{3}$

8. If $\tan\theta = \frac{a^2-b^2}{2ab}$, what is the value of $\cos\theta$

   a) $\frac{ab}{2a^2+b^2}$ b) $\frac{2ab}{a^2+b^2}$ c) a^2b^2 d) $\frac{a^2b^2}{a^2+b^2}$

9. Simplify $\frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos}$

   a) 2 b) $\sin^2 x$ c) $\sin x$ d) 1

10. Find the value of θ If $\sin^2\theta - \cos^2\theta = 1$ for $0^0 \le \theta \le 90^0$ a) 0^0 b) 45^0 c) 90^0 d) 1

11. If $\tan^{-1} 3x + \tan^{-1} 2x = \frac{7\pi}{4}$, what is the value of x a) $\frac{1}{2}$ b) 2 c) 1 d) $\frac{1}{4}$

12. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, find the value of $x^2 + y^2 + z^2 + 2xy$

a) 1 b) 2 c) $\pi/2$ d) π

13. Find the value of $\frac{\sin 20^0}{\cos 70^0} - \frac{\tan 37^0}{\cot 53^0}$

a) 1 b) 0.5 c) 0 d) 2

14. Which of the following options is a correct expression for $\cos^{-1} x$

a) $\cos^{-1} x = \cos^{-1} x + n\pi$

b) $\cos^{-1} x = \pm\cos^{-1} x + 3n\pi$

c) $\cos^{-1} x = \pm\cos^{-1} x$

d) $\cos^{-1} x = \pm\cos^{-1} x + 2n\pi$

15. Without using table, evaluate $\cot\left(22\frac{1^0}{2}\right)$

a) $1+\sqrt{3}$ b) $1-\sqrt{3}$ c) $1+\sqrt{2}$ d) $1+\sqrt{5}$

16. Simplify $\frac{(\sin\theta+\cos\theta)^2}{\sin\theta\cos\theta}$

a) $\tan\theta + \cot\theta - 2$ b) $cosec^2\theta$

c) $\sec\theta\tan\theta - 2$ d) $cosec\theta\sec\theta + 2$

17. Find the value of $\sin\theta$ if $\tan\theta = 3/5$ and θ is an acute angle a) 3/2 b) 3/4 c) $\frac{3}{\sqrt{34}}$ d) $\frac{\sqrt{34}}{3}$

18. Without using mathematical tables, find the value of $\sin 75^0$ in surd form

a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ b) $\sqrt{3}+1$ c) $\frac{\sqrt{2}+\sqrt{3}}{3}$ d) $2+\sqrt{3}$

19. Which of the following is equivalent to $\sqrt{\frac{1+\cos x}{2}}$ a) $\sin\frac{x}{2}$ b) $\cos\frac{x}{2}$ c) $\tan 2x$ d) $\cos 2x$

20. Which of the following is equivalent to $\sqrt{\frac{1-\cos x}{2}}$ a) $\sin\frac{x}{2}$ b) $\cos\frac{x}{2}$ c) $\tan 2x$ d) $\cos 2x$

21. Find the value of α for $0^0 \le \alpha \le 360^0$ in the equation $\sec\alpha\cot\alpha = 1$, by using the substitution $\tan\frac{\alpha}{2} = t$ a) $45^0, 90^0, 135^0, \ldots$.

b) $90^0, 180^0, 270^0, \ldots$ c) 90^0 d) $0^0, 180^0, 360^0$

22. Express $\sin 9A\cos 2A - \sin 3A\cos 3A$ as a sum or difference of sin

a) $\frac{1}{2}[\sin 6A + \sin 2A - \sin 12A]$

b) $\frac{1}{2}(\sin 11A + \sin 7A - \sin 6A)$

c) $(\sin 14A + \sin 6A)$

d) $\frac{1}{2}(\sin 9A + \sin 2A)$

23. Express $\sin 10A - \sin 4A$ is a product of trigonometric function

a) $\sin 10A - \sin 4A$ b) $2\sin 10A\cos 4A$

c) $2\sin 3A\cos 7A$ d) $\sin A\cos 7A$

24. Given that in triangle ABC, a=8, b=6 and c=4. Find the area of the triangle and the value of angle B

a) $64\ sqr\ unit, 61.5^0$ b) $41\ sqr\ unit, 60^0$

c) $20\ sqr\ unit, 16^0$ d) $11.6\ sqr\ unit, 43.5^0$

25. Solve the equation $4\sin x + \cos x = 1\ for\ 0 \le x \le 2\pi$

a) $0, 360^0$ b) $45^0, 225^0$ c) $\frac{\pi}{2}, \pi$ d) $\pi, 2\pi$

26. What is the value of $\tan 187\frac{1^0}{2}$ in surd form

a) $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ b) $\frac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{3}+\sqrt{6}}$ c) $\frac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{3}-\sqrt{6}}$ d) $\frac{1+\sqrt{2}}{\sqrt{3}+\sqrt{6}}$

27. If $\tan\alpha = \frac{b}{a}$, find the value of $\sin\alpha$

a) $\sqrt{a^2+b^2}$ b) $\frac{b}{\sqrt{a^2+b^2}}$ c) $\frac{\sqrt{a^2+b^2}}{ab}$ d) $\frac{a}{b^2}$

28. Calculate the value of c for the triangle in which $a + b = 18.5cm, A = 72^0, B = 45^0$

a) $10cm$ b) $16cm$ c) $25cm$ d) $3cm$

29. Find the value of α for $0 \le \alpha \le 360^0$ if $\sin\alpha = \cos\alpha$ a) $0^0, 180^0, 360^0$ b) $90^0, 270^0$

c) $-45^0, -135^0$ d) $45^0, 225^0$

30. Simplify $\sqrt{\frac{1+\cot^2\theta}{1+\tan^2\theta}}$

a) $\cot\theta$ b) $\sec\theta$ c) $\tan\theta$ d) $\sin\theta$

31. Simplify $\frac{\sin 2\theta+\cos\theta}{2\sin^2\theta+\cos\theta}$

a) $\cot\theta$ b) $\sec\theta$ c) $\tan\theta$ d) $\sin\theta$

32. If $\sin\theta = \frac{\sqrt{2}}{2}$, evaluate $\frac{\tan\theta-\sec\theta}{\text{cosec}\,\theta+\cot}$

a) $2\sqrt{2}-3$ b) $2+\sqrt{2}$ c) $3-\sqrt{2}$ d) $2\sqrt{2}-2$

33. Find the value of $\cos 75^0$ in surd form

a) $\frac{1}{4}(\sqrt{6}-\sqrt{2})$ b) $\frac{1}{4}(\sqrt{3}+\sqrt{2})$ c) $\frac{1}{2}(\sqrt{6}-3)$

d) $(\sqrt{3}+2)$

34. If $\cos\theta = x$, find the expression $\sin\theta$

a) $\sqrt{1+x^2}$ b) x^2 c) $\sqrt{1-x^2}$ d) $\frac{1}{\sqrt{1-x^2}}$

35. Simplify the expression $\frac{1+\sin\theta+\cos}{1+\sin\theta-\cos\theta}$

a) $\cot\frac{\theta}{2}$ b) $\sin\frac{\theta}{2}$ c) $\tan\frac{\theta}{2}$ d) $\sec\frac{\theta}{2}$

36. $\sqrt{\frac{1-\cos x}{1+\cos}} =$ a) $1-\sec x$ b) $\operatorname{cosec} x - \cot x$

c) $\cos x - \sec x$ d) $\sin x - \cos x$

37. $(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 - 2 =$ ____

a) 1 b) 2 c) 4 d) 0

38. If $\sin\theta = \frac{a-b}{a+b}$, find the value of $1 - \tan^2\theta$

a) $\frac{a^2+2ab+b^2}{4ab}$ b) $\frac{a^2-2ab+b^2}{4a}$ c) $\frac{6ab+a^2+b^2}{4ab}$

d) $\frac{6ab-a^2-b^2}{4ab}$

39. Given that $\sin\theta = \frac{a-b}{a+b}$, what is $\sqrt{1-\cos^2\theta}$

a) $\frac{a-b}{a+b}$ b) $\sqrt{\frac{a-b}{a+b}}$ c) $\sqrt{\frac{a+b}{a-b}}$ d) $\left(\frac{a-b}{a+b}\right)^2$

40. Evaluate in surd form, the value of $\sin 15^0$

a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{\sqrt{3}-1}{2}$ c) $\frac{\sqrt{3}-1}{\sqrt{2}}$ d) $\frac{\sqrt{6}-\sqrt{2}}{2\sqrt{2}}$

41. Simplify $\frac{\cos 2\theta - 1}{\sin 2\theta}$

a) $-\tan\theta$ b) $-\cot\theta$ c) $\tan\theta$ d) $\cot\theta$

42. Given that $\operatorname{cosec}\theta + \cot\theta = \frac{17}{15}$, calculate the value of $\operatorname{cosec}\theta - \cot\theta$

a) 2/17 b) 1/25 c) 17/30 d) 15/17

43. If $\sin A = \frac{\sqrt{3}}{2}$ and $\cos B = \frac{\sqrt{2}}{2}$, where A and B are both acute, find the value of $\sin(A+B)$

a) $\frac{\sqrt{6}}{4}$ b) $\frac{1}{4}(\sqrt{6}+\sqrt{2})$ c) $\frac{1}{4}(\sqrt{6}-\sqrt{2})$

d) $\frac{1}{4}(\sqrt{6}+\sqrt{3})$

44. Which of the following is the same as $\sin(270+x)$?

a) $\sin x$ b) $\tan x$ c) $-\sin x$ d) $-\cos x$

45. If $\sin\theta = 1/2, 0^0 < \theta < 450^0$, find the possible values of θ

a) $30^0, 210^0, 390^0$ b) $60^0, 330^0, 390^0$

c) $60^0, 210^0, 450^0$ d) $30^0, 150^0, 390^0$

46. Solve for x in the equation $1 - \cos^2 x = 0$ for $0^0 \le x \le 270^0$

a) 0^0 *or* 270^0 b) 0^0 *or* 180^0 c) 180^0 d) 270^0

47. In surd form, what is the value of $\tan 105^0$

a) $-(4+\sqrt{3})$ b) $-2-\sqrt{3}$ c) $-2+\sqrt{3}$ d) $4+\sqrt{3}$

48. If θ is an obtuse angle and $\cos\theta = 1/\sqrt{2}$, find the value of $1/\tan\theta$

a) 1 b) -1 c) $\sqrt{2}/2$ d) $\sqrt{-2}/2$

49. Given that $\sin(90^0 - 5\theta) = \cos(180^0 - \theta)$, find the value of θ a) 15^0 b) 22.5^0 c) 30^0 d) 45^0

50. Express in surd form; $\frac{1+\tan 60}{1-\tan 60}$

a) $3+\sqrt{2}$ b) $3-\sqrt{2}$ c) $1-2\sqrt{3}$ d) $2-\sqrt{3}$

51. If $\sin\theta = \frac{15}{17}$, where θ is acute, find $\tan\theta$

a) $\frac{17}{15}$ b) $\frac{8}{17}$ c) $\frac{8}{15}$ d) $\frac{15}{8}$

52. If $\tan B = \frac{5}{12}$, where B is an acute angle, evaluate $\frac{\cos B}{\sin B + \cos B}$ a) $\frac{17}{13}$ b) $\frac{13}{17}$ c) $\frac{12}{17}$ d) $\frac{11}{17}$

53. Simplify $\cos^2 x\,(\sec^2 x + \sec^2 x \tan^2 x)$

a) $\tan x$ b) $\tan x \sec x$ c) $\sec^2 x$ d) $cosec^2 x$

54. If $\sin\theta = \frac{m-n}{m+n}$, find the value of $1 + \tan^2\theta$

a) $\frac{2(m^2+n^2)}{m+n}$ b) $\frac{\sqrt{2(m^2+n^2)}}{m+n}$ c) $\frac{m^2+n^2+2mn}{2mn}$

d) $\frac{m^2+n^2+2mn}{4mn}$

55. QRS is a triangle with QS=12m, <RQS=30^0 and <QRS=45^0. Calculate the length of RS

a) $18\sqrt{2}$ b) $12\sqrt{2}$ c) $6\sqrt{2}$ d) $3\sqrt{2}$

SOLUTION

1. C
2. D
3. C
4. Since we were given three sides with no angle, then the best approach is the hero's formula; Let $a = 5, b = 7$ *and* $c = 8$. Then,

$s = \frac{5+7+8}{2} = \frac{20}{2} = 10$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{10(10-5)(10-7)(10-8)}$

$\Delta = \sqrt{10 \times 5 \times 3 \times 2} = \sqrt{300} = 17.3 \ldots\ldots\ldots b$

5. B

6. $\cos\theta = \frac{5}{12}$

Since $\sin^2\theta + \cos^2\theta = 1$,

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \left(\frac{5}{12}\right)^2$

$\sin^2\theta = 1 - \frac{25}{144}$

$\sin^2\theta = \frac{119}{144}$

$\sin\theta = \sqrt{\frac{119}{144}} = \frac{\sqrt{119}}{12} \ldots\ldots\ldots\ldots a$

7. $\tan\theta = \frac{3}{4}$

$since, \quad tan = \frac{opp}{adj}, \ opp = 3 \ and \ adj = 4$

$hence, \ hyp = \sqrt{opp^2 + adj^2} = \sqrt{3^2 + 4^2} = 5$

$\therefore \cos\theta = \frac{adj}{hyp} = \frac{4}{5} \ldots\ldots\ldots c$

8. $\tan\theta = \frac{a^2-b^2}{2ab}$

Recall that $1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{a^2-b^2}{2ab}\right)^2 = \sec^2\theta$

$\sec^2\theta = 1 + \frac{a^4+b^4-2a^2b^2}{4a^2b^2}$

$\sec^2\theta = \frac{4a^2b^2+a^4+b^4-2a^2b^2}{4a^2b^2}$

$\sec^2\theta = \frac{a^4+b^4+2a^2b^2}{4a^2b^2} = \frac{(a^2+b^2)^2}{4a^2b^2}$

$\therefore \sec\theta = \sqrt{\frac{(a^2+b^2)^2}{4a^2b^2}} = \frac{a^2+b^2}{2ab}$

$Since \cos\theta = \frac{1}{\sec\theta}$

$then \cos\theta = \frac{2ab}{a^2+b^2} \ldots\ldots\ldots b$

9. $\frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos x}$

I can use whatever approach I like. See this

Recall; $\sin^2 x + \cos^2 x = 1$

Then $\sin^2 x = 1 - \cos^2 x$

Substitute this into the above expression, we have

$\frac{1-\cos^2 x}{1-\cos x} + \frac{1-\cos^2 x}{1+\cos x}$

From difference of two squares,

$1 - \cos^2 x = (1-\cos x)(1+\cos x)$

Hence, we have;

$\frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}} + \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{1+\cos x}}$

$= 1 + \cos x + 1 - \cos x = 1 + 1 = 2 \ldots\ldots\ldots a$

10. $\sin^2\theta - \cos^2\theta = 1$

Recall; $\sin^2\theta + \cos^2\theta = 1$

Then $\cos^2\theta = 1 - \sin^2\theta$

The equation becomes;

$\sin^2\theta - (1 - \sin^2\theta) = 1$

$\sin^2\theta - 1 + \sin^2\theta = 1$

$2\sin^2\theta = 2$

$\sin^2\theta = 1$

$\sin\theta = 1$

$\theta = \sin^{-1} 1 = 90^0 \ldots\ldots\ldots c$

11. $\tan^{-1} 3x + \tan^{-1} 2x = \frac{7\pi}{4}$

Let $A = \tan^{-1} 3x \implies 3x = \tan A$

Also, $B = \tan^{-1} 2x \implies 2x = \tan B$

From the main expression we were given, we can now say;

$A + B = \frac{7\pi}{4}$

$i.e. A + B = \frac{7 \times 180}{4}$

$A + B = 315$

Take tan of both sides

$\tan(A+B) = \tan 315 = -1$

$Recall \ that, \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Substituting all necessary values into the above expression (i.e. $\tan A = 3x$ *and* $\tan B = 2x$

$$\frac{3x + 2x}{1 - (3x)(2x)} = -1$$

$$\frac{5x}{1 - 6x^2} = -1$$

$$5x = -1(1 - 6x^2)$$

$$5x = -1 + 6x^2$$

$$6x^2 - 5x - 1 = 0$$

Solving the equation, we have

$$x = 1 \text{ and } x = -\frac{1}{6}$$

Taking the positive value c

12. $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$

Let $\cos^{-1} x = A, \cos^{-1} y = B$ *and* $\cos^{-1} z = C$

This means; $x = \cos A, y = \cos B, z = \cos C$

Then we have; $A + B + C = \pi$

$A + B = \pi - C$

Take cos of both sides

$\cos(A + B) = \cos(\pi - C)$

I hope you still remember this expansion

$\cos A \cos B - \sin A \sin B = \cos \pi \cos C + \sin \pi \sin C$

$\cos 180 = -1$, *and* $\sin 180 = 0$

$\cos A \cos B - \sin A \sin B = -\cos C$ i

From the expression, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A + x^2 = 1 \implies \sin^2 A = 1 - x^2$

$\therefore \sin A = \sqrt{1 - x^2}$

Also, $\sin^2 B + \cos^2 B = 1$

$\sin^2 B + y^2 = 1 \implies \sin^2 B = 1 - y^2$

$\sin B = \sqrt{1 - y^2}$

Substituting all these into equation i,

$$x \cdot y - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - y^2}\right) = -z$$

$$xy + z = \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - y^2}\right)$$

Square both sides;

$$(xy + z)^2 = (1 - x^2)(1 - y^2)$$

$$x^2y^2 + z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2$$

collect like terms

$$x^2 + y^2 + z^2 + x^2y^2 - x^2y^2 + 2xyz = 1$$

$$x^2 + y^2 + z^2 + 2xyz = 1 \ldots\ldots\ldots a$$

13. Look closely,

$$\frac{\sin 20^0}{\cos 70^0} - \frac{\tan 37^0}{\cot 53^0}$$

What did you notice?

Recall that sin and cos are complementary while tan and cot are also complementary i.e.

$\sin\theta = \cos(90 - \theta)$ *and* $\tan\theta = \cot(90 - \theta)$

The above expression becomes

$$\frac{\sin 20^0}{\sin 20^0} - \frac{\tan 37^0}{\tan 37^0} = 1 - 1 = 0 \ldots\ldots\ldots c$$

14. D

15. Questions of this form should be approached like this;

Recall; $\cot\theta = \frac{1}{\tan\theta}$

Hence, $\cot 22\frac{1}{2} = \frac{1}{\tan 22\frac{1}{2}}$

Note; $\tan 22\frac{1}{2}$ *is the same as* $\tan\frac{45}{2}$

All we need to do know is to evaluate $\tan\frac{45}{2}$

Since $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

$$\tan 45 = \frac{2\tan\frac{45}{2}}{1 - \tan^2\frac{45}{2}}$$

Let $\tan\frac{45}{2} = t$ *and not also that* $\tan 45 = 1$

Hence, the above expression becomes;

$$1 = \frac{2t}{1 - t^2} \implies 1 - t^2 = 2t$$

$$t^2 + 2t - 1 = 0$$

Solving the quadratic equation using the general formula method,

$$t = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$t = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Hence, $\tan\frac{45}{2} = \sqrt{2} - 1$

$$\therefore \cot\frac{45}{2} = \frac{1}{\tan\frac{45}{2}} = \frac{1}{\sqrt{2}-1}$$

Rationalizing,

$$\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1 \ldots\ldots\ldots c$$

16. This question looks exactly like the one in your school manual but there is a sign difference

$$\frac{(\sin\theta + \cos\theta)^2}{\sin\theta\cos\theta} = \frac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$\sin^2\theta + \cos^2\theta$

$= 1, hence\ the\ expression\ becomes$

$$\frac{1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} + \frac{2\ \cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$$

$= cose\quad \sec\theta + 2 \ldots\ldots\ldots\ldots . . d$

17. Out of the three basic identities, we need one that connects the given (tan) to the unknown (sine) which is the third one i.e. $\cot^2\theta + 1 = cosec^2\theta$

$$1 + \frac{1}{\tan^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{1}{\left(\frac{3}{5}\right)^2} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{1}{\left(\frac{9}{25}\right)} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{25}{9} = \frac{1}{\sin^2\theta}$$

$$\frac{9+25}{9} = \frac{1}{\sin^2\theta}$$

$$\frac{34}{9} = \frac{1}{\sin^2\theta}$$

$$\sin^2\theta = \frac{9}{34}$$

$$\sin\theta = \sqrt{\frac{9}{34}} = \frac{3}{\sqrt{34}} \quad \ldots\ldots\ldots\ldots . c$$

18. $\sin 75 = \sin(45 + 30)$

Recall from addition formula,

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\therefore \sin(45 + 30) = \sin 45\cos 30 + \sin 30\cos 45$

$$\sin 30 = \frac{1}{2}, \cos 30 = \frac{\sqrt{3}}{2}, \sin 45 = \frac{1}{\sqrt{2}}, \cos 45 = \frac{1}{\sqrt{2}}$$

$$\therefore \sin(45 + 30) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$\therefore \sin(45 + 30) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\therefore \sin(45 + 30) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\therefore \sin(45 + 30) = \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4} \ldots\ldots a$$

19. Very simple question

$$\sqrt{\frac{1+\cos x}{2}}$$

From half angle formula;

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1$$

$$\therefore \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1 + 2\cos^2\frac{x}{2} - 1}{2}}$$

$$= \sqrt{\frac{\cancel{2}\cos^2\frac{x}{2}}{\cancel{2}}} = \sqrt{\cos^2\frac{x}{2}} = \cos\frac{x}{2} \ldots\ldots b$$

20. Note: I didn't make mistake; the previous question and this one are not the same.

$$\sqrt{\frac{1-\cos x}{2}}$$

Also from half angle formula;

$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}$$

$$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(1 - 2\sin^2\frac{x}{2}\right)}{2}}$$

$$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1 - 1 + 2\sin^2\frac{x}{2}}{2}}$$

$$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}}}$$

$$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\sin^2\frac{x}{2}} = \sin\frac{x}{2} \ldots\ldots\ldots a$$

21. $\sec\alpha\cot\alpha = 1$

$i.e. \quad \frac{1}{\cancel{\cos\alpha}} \cdot \frac{\cancel{\cos\alpha}}{\sin\alpha} = \frac{1}{\sin\alpha}$

$If \quad \tan\frac{\alpha}{2} = t \quad \Longrightarrow \quad \sin\alpha = \frac{2t}{1+t^2}$

Hence, the expression becomes;

$$\frac{1}{\left(\frac{2t}{1+t^2}\right)} = 1$$

$$\therefore \frac{1+t^2}{2t} = 1 \quad \Longrightarrow \quad 1+t^2 = 2t$$

$$t^2 - 2t + 1 = 0$$

Solving the quadratic equation,

$t = 1 \quad twice$

$This\ means \quad \tan\frac{\alpha}{2} = 1$

$$\frac{\alpha}{2} = \tan^{-1} 1$$

$$\frac{\alpha}{2} = 45^0, 225^0$$

$$\Longrightarrow \quad \alpha = 90^0, 450^0 \ldots\ldots\ldots\ldots . c$$

22. $\sin 9A \cos 2A - \sin 3A \cos 3A$

This question is asking you transform a product form to a sum/difference form. Follow me;

All you need for this question is to remember your factor formula

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Note that the LHS has the SUM/DIFFERENCE form while the RHS has the PRODUCT form.

Note the question has a product of sin and cos that is why a picked a factor formula that has a product of sin and cos.

Now, compare the product form of the given question to the product part of the selected factor formula.

One thing you will notice is that in the given question, there is NO '2' before the product of sin and cos but we have '2' in the formula we wanna use. So, we will transfer the 2 to the left hand side. The formula becomes;

$$\frac{1}{2}\sin\alpha + \sin\beta = \sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Now, compare the question and the formula.

Case 1;

$$\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \sin 9A \cos 2A$$

$$i.e. \quad \frac{\alpha+\beta}{2} = 9A \quad and \quad \frac{\alpha-\beta}{2} = 2A$$

$$\Longrightarrow \quad \alpha+\beta = 18A \quad and \quad \alpha-\beta = 4A$$

$$Solving\ simultaneou \quad , \quad \alpha = 11A \ and \ \beta = 7A$$

Case 2;

$$\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \sin 3A \cos 3A$$

$$i.e. \quad \frac{\alpha+\beta}{2} = 3A \quad and \quad \frac{\alpha-\beta}{2} = 3A$$

$$\Longrightarrow \quad \alpha+\beta = 6A \quad and \quad \alpha-\beta = 6A$$

$$Solving\ simultaneously, \quad \alpha = 6A \quad and \quad \beta = 0$$

Joining case 1 and case 2 together into the LHS of the modified factor formula,

$\sin\alpha + \sin\beta \quad becomes$

$$\frac{1}{2}(\sin 11A + \sin 7A - \sin 6A) \ \ldots\ldots\ldots\ldots . b$$

23. $\sin 10A - \sin 4A$

Just as we did the last question but this question is asking you to transform a DIFFERENCE form to a PRODUCT form.

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

Comparing the question to the LHS of this particular factor formula;

$\alpha = 10A, \ \ \beta = 4A$

$\sin 10A - \sin 4A$

$$= 2\sin\left(\frac{10A-4A}{2}\right)\cos\left(\frac{10A+4A}{2}\right)$$

$$\sin 10A - \sin 4A = 2\sin\frac{6A}{2}\cos\frac{14A}{2}$$

$$\sin 10A - \sin 4A = 2\sin 3A \cos 7A \ \ldots\ldots\ldots . c$$

24. a=8, b=6 and c=4

Since we are given sides only, we can easily get the area using Hero's formula;

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{8+6+4}{2} = 9$$

$$\Delta = \sqrt{9(9-8)(9-6)(9-4)}$$

$$\Delta = \sqrt{9\times 1\times 3\times 5} = \sqrt{135} = 11.6\ sqr\ unit$$

To get angle B;

$\Delta = \frac{1}{2}ac\cos B$

$i.e.\ 11.6 = \frac{1}{2} \times 8 \times 4\cos B$

$11.6 = 16\cos B$

$\cos B = \frac{11.6}{16}$

$\cos B = 0.725$

$B = \cos^{-1} 0.725 = 43.5^0$ ..... d

25. $4\sin x + \cos x = 1$

Case 3: $b\sin x + a\cos x = Rsi\ \ (x + \alpha)$

Comparing our question with this case 3;

$a = 1, b = 4\ \ \alpha = \tan^{-1}\frac{a}{b} = \tan^{-1}\frac{1}{4} = 14$

$R = \sqrt{1^2 + 4^2} = \sqrt{17}$

$\Rightarrow\ \ Rsin\ (x + \alpha) becomes\ \sqrt{17} sin\ (x + 14)$

$\sqrt{17}\sin(x + 14) = 1$

$\sin(x + 14) = \frac{1}{\sqrt{17}}$

$\sin(x + 14) = 0.2425$

$x + 14 = \sin^{-1} 0.2425$

$x + 14 = 14$

$x = 0 \ldots\ldots\ldots\ldots . a$

26. $\tan 187\frac{1^0}{2} = \tan\left(247\frac{1^0}{2} - 60^0\right)$

We will have to deal with $\tan 247\frac{1^0}{2}$ first.

$247\frac{1}{2} = \frac{495}{2}$

Recall from half angle formula;

$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1 - \tan^2\frac{\alpha}{2}}$

$Henc\ \ , \tan 495 = \frac{2\tan\left(\frac{495}{2}\right)}{1 - \tan^2\left(\frac{495}{2}\right)} = \frac{2\tan 247\frac{1}{2}}{1 - \tan^2 247\frac{1}{2}}$

$Putting\ \ \tan 247\frac{1}{2} = t, then$

$\tan 495 = \frac{2t}{1 - t^2}$

Since $\tan 495 = -1\ \ we\ have$

$\frac{2t}{1 - t^2} = -1$

$2t = -1 + t^2$

$t^2 - 2t - 1 = 0$

Using the general formula method,

$a = 1\ \ b = -2\ \ \ c = -1$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2 \times 1}$

$t = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

Using $t = 1 + \sqrt{2} \Longrightarrow\ \ \ \tan 247\frac{1}{2} = 1 + \sqrt{2}$

Back to our original question;

$\tan 187\frac{1}{2} = \tan\left(247\frac{1}{2} - 60\right)$

Recall from addition formula;

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan\left(247\frac{1}{2} - 60\right) = \frac{\tan 247\frac{1}{2} - \tan 60}{1 + \left(\tan 247\frac{1}{2}\right)(\tan 60)}$

$\tan 60 = \sqrt{3}\ \ while\ \ \ \tan 247\frac{1}{2} = 1 + \sqrt{2}$

$\tan 187\frac{1}{2} = \frac{1 + \sqrt{2} - \sqrt{3}}{1 + (1 + \sqrt{2})(\sqrt{3})}$

$\tan 187\frac{1}{2} = \frac{1 + \sqrt{2} - \sqrt{3}}{1 + \sqrt{3} + \sqrt{6}}\ \ \ \ldots\ldots\ldots . b$

27. $\tan\alpha = \frac{b}{a}$

From $1 + \cot^2\alpha = cos^{\ 2}\alpha$

We have;

$1 + \left(\frac{a}{b}\right)^2 = cosec^2\alpha$

$1 + \frac{a^2}{b^2} = cosec^2\alpha$

$\frac{b^2 + a^2}{b^2} = \frac{1}{\sin^2\alpha}$

$\sin^2\alpha = \frac{b^2}{a^2 + b^2}$

$\sin\alpha = \sqrt{\frac{b^2}{a^2 + b^2}}$

$\sin\alpha = \frac{b}{\sqrt{a^2 + b^2}} \ldots\ldots\ldots\ldots\ldots\ldots . b$

28. $a + b = 18.5cm,\ A = 72^0,\ B = 45^0$

Using the expression;

$$\frac{a+b-c}{a+b+c} = \tan\frac{1}{2}A\tan\frac{1}{2}B$$

We now have;

$$\frac{18.5-c}{18.5+c} = \tan\frac{1}{2}(72)\tan\frac{1}{2}(45)$$

$$\frac{18.5-c}{18.5+c} = \tan 36^0 \times \tan 22.5^0$$

$$\frac{18.5-c}{18.5+c} = 0.3$$

$$18.5 - c = 0.3(18.5 + c)$$

$$18.5 - c = 5.55 + 0.3c$$

$$-c - 0.3c = 5.55 - 18.5$$

$$-1.3c = -12.95$$

$$c = -\frac{12.95}{-1.3} = 9.96$$

$$c \approx 10cm \ldots\ldots\ldots\ldots a$$

29. If sine and cosine of any angle have the same value, then the tangent of that angle will be equal to one i.e.

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\sin\alpha} = \frac{\cos\alpha}{\cos\alpha} = 1$$

$$i.e.\ \tan\alpha = 1 \quad \Longrightarrow \quad \tan^{-1}1 = \alpha$$

$$recall;\ tan^{-1}x = \tan^{-1}x + nx$$

Hence, $\alpha = \tan^{-1}1 + n\pi$

$$\alpha = 45^0 + 180(n)$$

$$when\ n = 0, \quad \alpha = 45^0 + 0 = 45^0$$

$$when\ n = 1, \quad \alpha = 45^0 + 180^0 = 225^0$$

$$when\ n = 2, \quad \alpha = 45^0 + 360^0 = 405^0$$

$$for\ 0^0 \le \alpha \le 360^0, \quad \alpha = 45^0, 225^0 \ldots\ldots\ldots d$$

30. This question requires fast thinking

$$\sqrt{\frac{1+\cot^2\theta}{1+\tan^2\theta}}$$

Recall from basic trig. identity;

$$1 + \tan^2\theta = \sec^2\theta \quad \text{and}$$

$$\cot^2\theta + 1 = cosec^2\theta$$

$$\therefore \sqrt{\frac{1+\cot^2\theta}{1+\tan^2\theta}} = \sqrt{\frac{cosec^2\theta}{\sec^2\theta}}$$

$$since, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad and \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\sqrt{\frac{cosec^2\theta}{\sec^2\theta}} = \sqrt{\frac{1}{\sin^2\theta}\times\cos^2\theta} = \sqrt{\cot^2\theta}$$

$$\sqrt{\cot^2\theta} = \cot\theta \ldots\ldots\ldots a$$

31. You need to think here also;

$$\frac{\sin 2\theta + \cos\theta}{2\sin^2\theta + \cos\theta}$$

Hmmmmm, I deliberately kept this question for those of you that have come across it in your university elementary mathematics II and have been trying to solve it(to get the answer in the textbook i.e. $\cot\theta$).

The expression is not correct and therefore has NO SOLUTION.

32. $\sin\theta = \frac{\sqrt{2}}{2}$

$$\frac{\tan\theta - \sec\theta}{\operatorname{cosec}\theta + \cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \cos^2\theta = 1$$

$$\frac{2}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{2}$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}}{2} \div \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{1} = \frac{2}{2} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{2}{\sqrt{2}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

$$\therefore \frac{\tan\theta - \sec\theta}{\operatorname{cosec}\theta + \cot\theta} = \frac{1-\sqrt{2}}{\left(\frac{2}{\sqrt{2}}\right)+1}$$

$$= \frac{1-\sqrt{2}}{\frac{2+\sqrt{2}}{\sqrt{2}}} = 1 - \sqrt{2}\times\frac{\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{\sqrt{2}-2}{2+\sqrt{2}}$$

Let's rationalize

$$\frac{\sqrt{2}-2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{2\sqrt{2}-2-4+2\sqrt{2}}{4-2}$$

$$= \frac{4\sqrt{2}-6}{2} = 2\sqrt{2}-3 \ \ldots\ldots\ldots\ldots a$$

33. $\cos 75^0 = \cos(45^0 + 30^0)$

From addition formula;

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos 75^0 = \cos 45 \cos 30 - \sin 45 \sin 30$

$$\cos 75 = \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$\cos 75 = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 75 = \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos 75 = \frac{\sqrt{6}-\sqrt{2}}{4} = \frac{1}{4}(\sqrt{6}-\sqrt{2}) \ \ldots\ldots\ldots\ldots\ldots a$$

34. $\cos\theta = x$

From the basic identity,

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + x^2 = 1$

$\sin^2\theta = 1 - x^2$

$\sin\theta = \sqrt{1-x^2} \quad \ldots\ldots\ldots\ldots c$

35. From the options, you will notice that you are required to introduce half angle formula;

$$\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\cos\theta = \begin{Bmatrix} 2\cos^2\frac{\theta}{2} - 1 \\ or \quad 1 - 2\sin^2\frac{\theta}{2} \end{Bmatrix}$$

$$\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$$

$$= \frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \left(2\cos^2\frac{\theta}{2}-1\right)}{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \left(1-2\sin^2\frac{\theta}{2}\right)}$$

$$= \frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 2\cos^2\frac{\theta}{2} - 1}{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 1 + 2\sin^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 2\sin^2\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$= \frac{\cancel{2}\cos\frac{\theta}{2}\left(\cancel{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}}\right)}{\cancel{2}\sin\frac{\theta}{2}\left(\cancel{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}}\right)}$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2} \ldots\ldots\ldots\ldots a$$

36. $\sqrt{\frac{1-\cos x}{1+\cos x}}$

Rationalizing

$$\sqrt{\frac{1-\cos x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}}$$

$$\sqrt{\frac{(1-\cos x)^2}{1^2-\cos^2 x}} = \sqrt{\frac{(1-\cos x)^2}{\sin^2 x}} = \frac{1-\cos x}{\sin x}$$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} = \operatorname{cosec} x - \cot x \ldots\ldots\ldots\ldots b$$

37. $(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 - 2$

Lets simply the expression one after the other

$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

Recall that $\sin^2\theta + \cos^2\theta = 1$

Hence,

$(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta$

Similarly,

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

The whole expression becomes

$1 - \cancel{2\sin\theta\cos\theta} + 1 + \cancel{2\sin\theta\cos\theta} - 2$

$2 - 2 = 0 \ldots\ldots\ldots\ldots d$

38. Recall from SOHCAHTOA

$$\sin\theta = \frac{a-b}{a+b} = \frac{opp}{hy}$$

$since, \quad hyp^2 = adj^2 + opp^2$

$adj = \sqrt{(a+b)^2 - (a-b)^2}$

$$\therefore \ 1 - \tan^2\theta = 1 - \left(\frac{opp}{adj}\right)^2$$

$$= 1 - \frac{(a-b)^2}{(a+b)^2 - (a-b)^2}$$

$$= 1 - \frac{(a-b)^2}{\cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} + 2ab - \cancel{b^2}}$$

$$= 1 - \frac{(a-b)^2}{4ab} = \frac{4ab - (a^2 - 2ab + b^2)}{4ab}$$

$$= \frac{4ab - a^2 + 2ab - b^2}{4ab} = \frac{6ab - a^2 - b^2}{4ab} \ldots\ldots\ldots d$$

39. Recall that $\sin^2\theta + \cos^2\theta = 1$

$hence, 1 - \cos^2\theta = \sin^2\theta$

$$\sqrt{1 - \cos^2\theta} = \sqrt{\sin^2\theta} = \sqrt{\left(\frac{a-b}{a+b}\right)^2}$$

$$= \frac{a-b}{a+b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots a$$

40. Think of special angles;

$\sin 15^0 = \sin(60^0 - 45^0)$

From addition formula,

$\sin(60^0 - 45^0) = \sin 60\cos 45 - \cos 60\sin 45$

$$\sin(60^0 - 45^0) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$\sin 15 = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Rationalizing the expression, we have

$$\sin 15 = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4} \ldots\ldots a$$

41. Just think of your double angle

$$\frac{\cos 2\theta - 1}{\sin 2\theta}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

Hence, the above expression becomes

$$\frac{(1 - 2\sin^2\theta) - 1}{2\sin\theta\cos\theta} = \frac{1 - 2\sin^2\theta - 1}{2\sin\theta\cos\theta}$$

$$= \frac{-\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2\sin\theta}\cos\theta} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta \ldots\ldots\ldots a$$

42. $\operatorname{cosec}\theta + \cot\theta = \frac{17}{15}$

Recall from the basic trig. identities I gave to you;

$\cot^2\theta + 1 = cosec^2\theta$

We can have;

$cosec^2\theta - \cot^2\theta = 1$

From difference of two squares, then;

$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$$\left(\frac{17}{15}\right)(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$\operatorname{cosec}\theta - \cot\theta = 1 \times \frac{15}{17}$$

$$\operatorname{cosec}\theta - \cot\theta = \frac{15}{17} \ldots\ldots\ldots d$$

43. Since angle A and B are both acute,

$$\sin A = \frac{\sqrt{3}}{2} \implies \cos A = \frac{1}{2}\ i.e.\ A = 60^0$$

$$\cos B = \frac{\sqrt{2}}{2} \implies \sin B = \frac{\sqrt{2}}{2}\ i.e.\ B = 45^0$$

Recall from addition formula

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

Hence,

$\sin(A + B) = \sin A\cos B + \sin B\cos A$

$$\sin(A + B) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\sin(A + B) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4} \ldots\ldots\ldots\ b$$

44. Note that addition or subtraction of 360^0 from a trig. function doesn't have effect.

$i.e. \sin\alpha = \sin(\alpha + 360) = \sin(\alpha - 360)$

Hence,

$\sin(270 + x) = \sin(270 + x - 360)$

$\implies \sin(270 + x) = \sin(x - 90)$

Since cos and sin have complimentary property,

$\sin(x - 90) = \cos(90 - x + 90)$

$\therefore \sin(270 + x) = \cos(180 - x) = -\cos x \ldots\ldots d$

45. $\sin\theta = \frac{1}{2} \implies \theta = \sin^{-1}\frac{1}{2} = 30^0$

Note: between 0^0 *and* 450^0, $\sin\theta$ is positive in;

- 1st quadrant ($\theta = 30^0$)
- 2nd quadrant ($\theta = 180^0 - 30^0 = 150^0$)
- Allied angle ($360^0 + 30^0 = 390^0$)

Hence, the angles are $30^0, 150^0$ *and* 390^0 $\ldots\ldots\ldots d$

46. $1 - \cos^2 x = 0$

$\cos^2 x = 1 \implies \cos x = \pm 1$

$x = \cos^{-1}(1)$ *or* $\cos^{-1}(-1)$

Hence, $x = 0^0$ *or* 180^0 $\ldots\ldots\ldots b$

47. $\tan 105^0 = \tan(60 + 45)$

$recall; \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$\tan 105^0 = \frac{\tan 60 + \tan 45}{1 - \tan 60\tan 45} = \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$$

$$\tan 105^0 = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$

Rationalize

$$\tan 105^0 = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$\tan 105^0 = \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

$\tan 105^0 = -2 - \sqrt{3} \ldots\ldots\ldots b$

48. Let's get the value θ first

$$\cos\theta = \frac{1}{\sqrt{2}} \implies \theta = \cos^{-1}\frac{1}{\sqrt{2}} = 45^0$$

For obtuse angle, $\theta = 180 - 45 = 135^0$

$Hence, \tan 135^0 = \tan(180 - 135) = 45^0$

Since tan is negative in the second quadrant;

$-\tan 45^0 = -1 \ldots\ldots\ldots b$

49. $\sin(90^0 - 5\theta) = \cos(180^0 - \theta)$

Recall by complimentary property that;

$\sin\theta = \cos(90 - \theta)$

$\therefore \sin(90^0 - 5\theta) = \cos[90 - (90 - 5\theta)]$

$= \cos[90 - 90 + 5\theta] = \cos 5\theta$

We can now say that;

$\cos 5\theta = \cos(180^0 - \theta)$

$\therefore 5\theta = 180^0 - \theta \implies 6\theta = 180^0$

$\theta = 30^0 \ldots\ldots\ldots c$

50. Recall; $\tan 60 = \sqrt{3}$

$$\frac{1 + \tan 60}{1 - \tan 60} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

Rationalizing;

$$\frac{1 + \sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1^2 - \sqrt{3}^2} = \frac{4 + 2\sqrt{3}}{1 - 3}$$

$$\frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3} \ldots\ldots\ldots d$$

51. $\sin\theta = \frac{15}{17}$ $i.e. opp = 15, hyp = 17$

$adj^2 + opp^2 = hyp^2 \implies adj^2 = 17^2 - 15^2$

$adj^2 = 289 - 225 = 64$

$adj = \sqrt{64} = 8$

$\therefore \tan\theta = \dfrac{opp}{adj} = \dfrac{15}{8} \ldots\ldots\ldots d$

52. From $\tan B = \frac{5}{12}, opp = 5,\ adj = 12$

$hyp = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

$$\cos B = \frac{adj}{hy} = \frac{12}{13}, \quad \sin B = \frac{opp}{hyp} = \frac{5}{13}$$

$$\frac{\cos B}{\sin B + \cos B} = \frac{\frac{12}{13}}{\frac{5}{13} + \frac{12}{13}} = \frac{\frac{12}{13}}{\frac{17}{13}} = \frac{12}{17} \ldots\ldots c$$

53. $\cos^2 x\ (\sec^2 x + \sec^2 x\ ta\ ^2x)$

$= \cos^2 x \sec^2 x\ (1 + \tan^2 x)$

Recall; $1 + \tan^2 x = \sec^2 x$

Hence, the expression becomes

$\cos^2 x \sec^2 x \sec^2 x$

$but \sec x = \dfrac{1}{\cos x}$

$$\cancel{\cos^2 x} \times \frac{1}{\cancel{\cos^2 x}} \times \sec^2 x$$

$= \sec^2 x \ldots\ldots\ldots c$

54. From

$\sin\theta = \dfrac{m - n}{m + n}, \quad opp = m \qquad hyp = m + n$

$adj^2 = hyp^2 - opp^2 = (m + n)^2 - (m - n)^2$

$ad\ ^2 = m^2 + 2mn + n^2 - (m^2 - 2mn + n^2)$

$adj^2 = m^2 + 2mn + n^2 - m^2 + 2mn - n^2$

$ad\ ^2 = 4mn$

$adj = \sqrt{4mn}$

$$1 + \tan^2\theta = 1 + \left(\frac{m - n}{\sqrt{4mn}}\right)^2$$

$$= 1 + \frac{(m - n)^2}{4mn}$$

$$= \frac{4mn + (m - n)^2}{4mn}$$

$$= \frac{4mn + m^2 + n^2 - 2mn}{4mn}$$

$hence, 1 + \tan^2\theta = \dfrac{m^2 + n^2 + 2mn}{4mn} \ldots\ldots d$

55. By sine rule;

$$\frac{x}{\sin 30} = \frac{12}{\sin 45}$$

$x \sin 45 = 12 \sin 30$

$$x = \frac{12\sin 30}{\sin 45} = \frac{12\left(\frac{1}{2}\right)}{\frac{1}{\sqrt{2}}} = \frac{6}{\frac{1}{\sqrt{2}}}$$

$$x = \frac{6}{1} \times \frac{\sqrt{2}}{1} = 6\sqrt{2} \ldots\ldots\ldots\ldots c$$

DIFFERENTIAL CALCULUS

STANDARD DIFFERENTIALS

y	$\frac{dy}{dx}$
$\boldsymbol{ax^n}$	$\boldsymbol{nax^{n-1}}$
$\boldsymbol{a[f(x)]^n}$	$\boldsymbol{na[f(x)]^{n-1}}$
$\boldsymbol{\sin x}$	$\boldsymbol{\cos x}$
$\boldsymbol{\cos x}$	$\boldsymbol{-\sin x}$
$\boldsymbol{\tan x}$	$\boldsymbol{\sec^2 x}$
$\boldsymbol{\sec x}$	$\boldsymbol{\sec x \tan x}$
$\boldsymbol{\cot x}$	$\boldsymbol{-\mathrm{cosec}^2 x}$
$\boldsymbol{\sin^{-1} ax}$	$\boldsymbol{\frac{a}{\sqrt{1-(ax)^2}}}$
$\boldsymbol{\cos^{-1} ax}$	$\boldsymbol{\frac{-a}{\sqrt{1-(ax)^2}}}$

GENERAL DIFFERENTIATION

$If\ y = ax^n (where\ a\ and\ n\ are\ constants)$

$$\frac{dy}{dx} = nax^{n-1}$$

Example

$If\ y = 7x^3, find \frac{dy}{dx}$

Solution

$$\frac{dy}{dx} = 3 \times 7x^{3-1} = 21x^2$$

DIFFERENTIATION OF A CONSTANT

$If\ y = a \quad (where\ 'a''is\ a\ constant)$

$$\frac{dy}{dx} = 0$$

DIFFERENTIATION OF EXPONENTIAL FUNCTION

$If\ y = e^{f(x)}$

$$\frac{dy}{dx} = \frac{d}{dx}[f(x)] \times e^{f(x)}$$

Example

$If\ y = e^{2x^3}, find \frac{dy}{dx}$

Solution

$$\frac{dy}{dx} = \frac{d}{dx}(2x^3)e^{2x^3} = 6x^2 e^{2x^2}$$

DIFFERENTIATION OF NATURAL LOGARITHM (LN FUNCTION)

If $y = \ln[f(x)]$

$$\frac{dy}{dx} = \frac{1}{f(x)} \times \frac{d}{dx}[f(x)]$$

Example

$If\ y = \ln 5x^3,\ \ find\ \frac{dy}{dx}$

Solution

$$\frac{dy}{dx} = \frac{1}{5x^3} \times \frac{d}{dx}(5x^3)$$

$$\frac{dy}{dx} = \frac{15x^2}{5x^3} = \frac{3}{x}$$

DIFFERENTIATION OF F(X) AS EXPONENT OF A CONSTANT

$If\ y = a^{f(x)}$

$Then\ \frac{dy}{dx} = a^{f(x)} \ln a$

Example

$Find\ \frac{dy}{dx}\ if\ y = 3^{2x}$

Solution

The question can be rewritten as;

$$y = 3^{2x} = (3^2)^x = 9^x$$

$$\therefore \frac{dy}{dx} = 9^x \ln 9$$

PRODUCT DIFFERENTIATION

$if\ y = uv\ \ whe\ \ \ u, v$
$= f(x)\ i.e.\ u\ and\ v\ are\ functions\ of\ x$

$then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Example

$if\ y = 2x^3e^{3x}, find\frac{dy}{dx}$

Solution

We need to split the expression for y into two.

$let\ u = 2x^3\ \ and\ \ v = e^{3x}$

$$\frac{du}{dx} = 6x^2 \quad and \quad \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (2x^3)(3e^{3x}) + e^{3x}(6x^2)$$

$$\frac{dy}{dx} = 6x^3e^{3x} + 6x^2e^{3x}$$

$$\frac{dy}{dx} = 6x^2e^{3x}(x+1)$$

DIFFERENTIATION OF QUOTIENT

$If\ y = \frac{u}{v}\ \ \ where\ u, v = f(x),$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Example

$Find\frac{dy}{dx}\ if\ y = \frac{7e^{5x^2}}{\ln 3x^4}$

Solution

$let\ u = 7e^{5x^2}\ \ and\ \ v = \ln 3x^4$

$then\ \ \frac{du}{dx} = 7 \times 10xe^{5x^2} = 70xe^{5x^2}$

$and\ \ \frac{dv}{dx} = \frac{1}{3x^4} \times 12x^3 = \frac{4}{x}$

$$\therefore \frac{dy}{dx} = \frac{(\ln 3x^4)(70xe^{5x^2}) - (7e^{5x^2})\left(\frac{4}{x}\right)}{(\ln 3x^4)^2}$$

$$\therefore \frac{dy}{dx} = \frac{(70xe^{5x^2}\ln 3x^4) - \left(\frac{28e^{5x^2}}{x}\right)}{(\ln 3x^4)^2}$$

COMPOSITE FUNCTION (CHAIN RULE/FUNCTION OF FUNCTION)

$If\ y = f(u)\ and\ u = f(x)$

i.e. if y is a function of u and u is a function of

$$Then\quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example

Differentiate $(2x^3 + 7x)^7$

Solution

$y = (2x^3 + 7x)^7$

$Let\ u = 2x^3 + 7x\quad \frac{du}{dx} = 6x^2 + 7$

$$Hence,\ \ y = u^7 \implies \frac{dy}{du} = 7u^6 = 7(2x^3 + 7x)^6$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 7(2x^3 + 7x)^6 \times (6x^2 + 7)$$

$$\frac{dy}{dx} = 7(6x^2 + 7)(2x^3 + 7x)^6$$

IMPLICIT DIFFERENTIATION

An implicit function is a function that contains product of x and y factors.

Differentiating an implicit function is quite different from what we have been doing so far.

You will differentiate a term which is a function of both x and y twice (first with respect to x, second with respect to y) writing dy/dx whenever it's with respect to y.

Example

$Differentiate\ \ x^2y^3 - 3xy^2 = 0$

Solution

The function $x^2y^3 + 3xy^2$ has two terms x^2y^3 and $3xy^2$. The first term has both x and y as factors as well as the second term.

Differentiating the first term x^2y^3;

$With\ respect\ to\ x;\quad 2xy^3$

$w.r.t\ \ y;\quad 3x^2y^2\frac{dy}{dx}$

Note the dy/dx. It is attached whenever we differentiate w.r.t **y**

Differentiating the second term $3xy^2$

$With\ respect\ to\ x;\ \ 3y^2$

$With\ respect\ to\ y;\ \ 6xy\frac{dy}{dx}$

Combining all the differentials

$$2xy^3 + 3x^2y^2\frac{dy}{dx} - 3y^2 - 6xy\frac{dy}{dx} = 0$$

$Make\ \frac{dy}{dx}\ subject\ of\ formula$

$$3x^2y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} = -3y^2 + 2xy^3$$

$$\frac{dy}{dx}(3x^2 - 6xy) = 2xy^3 - 3y^2$$

$$\frac{dy}{dx} = \frac{2xy^3 - 3y^2}{3x^2 - 6xy}$$

PARAMETRIC EQUATION

Here, y is a function of a parameter and x is also a function of that parameter.

$If\ y = f(t)\ \ and\ \ x = f(t)$

$$then\ \ \frac{dy}{dx} = \frac{dy}{dt} \times 1 \Big/ \frac{dx}{dt}\ \ or\ \ \frac{dy}{dt} \times \frac{dt}{dx}$$

Example

$If\ y = 2t^5\ \ and\ \ x = 5t^6, find\ \frac{dy}{dx}$

Solution

$$\frac{dy}{dt} = 10t^4 \frac{dx}{dt} = 50t^5$$

$$\therefore \frac{dy}{dx} = \cancel{10}t^4 \times \frac{1}{\cancel{50}t^5} = \frac{1}{5t}$$

APPLICATION OF DIFFERENTIATION

1. Tangents and Normal

Equation of tangent of a curve with gradient m is given as;

$$y - y_1 = m(x - x_1)$$

Equation of normal of a curve with gradient m is given as;

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

Example

Find the tangent as well as the normal to the curve $y = 3x^3 - 4x^2 + 6x - 1$ at $x = 1$

Solution

We need to first determine the gradient at $x = 1$. Gradient is gotten by differentiating the equation of the curve and substituting the given value of x.

$$\frac{dy}{dx} = 9x^2 - 8x + 6$$

$$m = \frac{dy}{dx}\Big|_{x=1} = 9(1)^2 - 8(1) + 6 = 9 - 8 + 6 = 7$$

Hence, the $m = 7$

Next is to determine the equivalent value of y when x is 1

$$y = 3(1)^3 - 4(1)^2 + 6(x) - 1 = 3 - 4 + 6 - 1$$
$$= 4$$

$$(x_1, y_1) = (1,4)$$

Equation of tangent

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 7(x - 1)$$
$$y - 4 = 7x - 7$$
$$y = 7x - 3$$

Equation of normal

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$(x_1, y_1) = (1,4)\ and\ m = 7$$

$$y - 4 = -\frac{1}{7}(x - 1)$$

$$7y - 28 = -x + 1$$

$$7y = -x + 29$$

$$7y = 29 - x$$

2. Rates of Change

This concept explains that if for instance, $a = f(b)$ *then* $\frac{da}{db}$ *can be interpreted as rate of change of a with respect to b.*

It also involves finding different rates from time rate and the established relationship. For example;

Area of a circle $= \pi r^2$

$$A = \pi r^2 \implies \frac{dA}{dr} = 2\pi r$$

Writing this with respect to time, we have

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \cdot \frac{dr}{dt} = \frac{\frac{dA}{dt}}{\frac{dA}{dr}}$$

Example

The volume of a sphere is increasing at the rate of $40.8cm^3s^{-1}$. Calculate, correct to three significant figures, the rate at which the surface area is increasing when the radius is 3.2cm

Solution

$$\frac{dv}{dt} = 40.8cm^3s^{-1}, r = 3.2cm$$

$$Vol\ of\ a\ sphe\ \ \ , V = \frac{4}{3}\pi r^3\ and\ area, A = 4\pi r^2$$

$$\therefore \frac{dv}{dr} = \cancel{3}\left(\frac{4}{\cancel{3}}\pi r^2\right) = 4\pi r^2$$

$$also, \frac{dA}{dr} = 8\pi r$$

We are asked to find the rate at which the area is increasing

$$i.e. \frac{dA}{dt}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} = \frac{40.8}{4\pi r^2}$$

$$also, \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi r \cdot \frac{40.8}{4\pi r^2} = \frac{2(40.8)}{r} = \frac{81.6}{r}$$

$$\frac{dA}{dt} = \frac{81.6}{3.2} = 25.5 cm^2 s^{-1}$$

3. Curve Sketching

- **Intersection with axes**

  x-intercept: this is the point where the curve touches the x-axis. To determine x-intercept, just put y to be equal to zero (i.e. y=0)

  y-intercept: this is the point where the curve touches the y-axis. To determine y-intercept, just put x to be equal to zero (i.e. x=0)

- **Turning points**

  The turning point(s) of a curve can be maximum, minimum or both.

  Maximum

  A point is maximum if

  at $\frac{dy}{dx} = 0, \quad \frac{d^2y}{dx^2} < 0$

  Minimum

  A point is minimum if

  at $\frac{dy}{dx} = 0, \quad \frac{d^2y}{dx^2} > 0$

- **Inflection/ Inflection Point**

  To determine the inflection point,

  Step 1: equate the second differential to zero and get the value of x

  $i.e. \quad \frac{d^2y}{dx^2} = 0$

  Step 2: add and subtract 0.01 from the value of x you just got and insert back into the value of your $\frac{d^2y}{dx^2}$.

  If there is a change in sign, then the point is an inflection point.

- **Asymptotes**

  Vertical asymptote

  For a function $y = \frac{f(x)}{g(x)}$. The vertical asymptote is gotten at the point where y is undefined i.e. by equating the denominator {g(x)} to zero.

  Horizontal asymptote

  This is gotten by finding the value of y at the point where x tends to infinity ($x \to \infty$)

Example 1

Determine for the function $y = 2x^3 + 6x^2$

i. The maximum and/or the minimum point(s)

ii. The inflection point

Solution

$y = 2x^3 + 6x^2$

Turning points

Step 1: differentiate the function once and equate to zero

$\frac{dy}{dx} = 6x^2 + 12x$

$6x^2 + 12x = 0$

$6x(x + 2) = 0$

$x = 0 \quad or \quad x = -2$

Step 2: use these two values of x to determine the corresponding values of y

When $x = 0$

$y = 2(0)^3 + 6(0)^2$

$y = 0 + 0 = 0$

Hence, the first turning point will be $(0, 0)$

When $x = -2$

$y = 2(-2)^3 + 6(-2)^2$

$y = 2(-8) + 6(4)$

$y = -16 + 24 = 8$

Hence, the second turning point will be (-2, 8)

Step 3: to determine the nature of the points (maximum or minimum)

$From\ \frac{dy}{dx} = 6x^2 + 12x$

$\frac{d^2y}{dx^2} = 12x + 12$

For point (0, 0)

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = 12(0) + 12 = 12$

Since $12 > 0$, then the point (0, 0) is a minimum.

For point (-2, 8)

$\left.\frac{d^2y}{dx^2}\right|_{x=-2} = 12(-2) + 12 = -24 + 12 = -12$

Since $-12 < 0$, then the point (-2, 8) is a maximum

Summary;

For the curve $y = 2x^3 + 6x^2$,

The minimum point is (0, 0)

The maximum point is (-2, 8)

Point of inflexion

Step 1: equate the second differential to zero and get the value of x

$\frac{d^2y}{dx^2} = 12x + 12$

$12x + 12 = 0$

$12x = -12$

$x = -1$

From $y = 2x^3 + 6x^2$

$y = 2(-1)^3 + 6(-1)^2 = -2 + 6 = 4$

The point is (-1, 4)

Step 2: add and subtract 0.01 from the value of x you just got and insert back into the value of your $\frac{d^2y}{dx^2}$.

$x + 0.01 = -1 + 0.01 = -0.99$

$\left.\frac{d^2y}{dx^2}\right|_{x=-0.99} = 12(-0.99) + 12 = 0.12$

$x - 0.01 = -1 - 0.01 = -1.01$

$\left.\frac{d^2y}{dx^2}\right|_{x=-1.01} = 12(-1.01) + 12 = -0.12$

It is obvious that $\frac{d^2y}{dx^2}$ changes sign

Hence, the point (-1, 4) is an inflexion point

Example 2

$Given\ the\ function\ y = \frac{3x^2}{2x^2 - 8}$

Determine the vertical and horizontal asymptotes.

Solution

For vertical asymptote, equate the denominator to zero

$2x^2 - 8 = 0 \implies 2x^2 = 8 \implies x^2 = 4$

$x = \sqrt{4} = \pm 2$

For horizontal asymptote,

$y = \frac{3x^2}{2x^2 - 8}$

Divide through by the highest power of x

$y = \frac{\frac{3x^2}{x^2}}{\frac{2x^2}{x^2} - \frac{8}{x^2}}$

$y = \frac{3}{2 - \frac{8}{x^2}}$

$at\ x \to \infty$

Note: any number divided by infinity is zero

Hence,

$y = \frac{3}{2 - \frac{8}{\infty}} = \frac{3}{2}$

Hence,

Vertical asymptote is ± 2

Horizontal asymptote is $3/2$

QUESTIONS

1. Differentiate the function $y = \ln 3x^4$

   a) $\ln 12x^3$ b) $4 \ln 3x^3$ c) $4/x$ d) $x/4$

2. Find $\frac{dy}{dx}$ if $y = \frac{x^2+2}{3-x^2}$

   a) $\frac{-10x}{(3-x^2)^2}$ b) $\frac{2x}{(3-x^2)^2}$ c) $\frac{10x}{(3-x^2)^2}$ d) $\frac{4x^3}{(3-x^2)^2}$

3. Evaluate $\lim_{(n\to\infty)} \left(\frac{3}{n} - 3\right)$

   a) -3 b) -1 c) 1 d) 3

4. Evaluate $\frac{d}{dx}\cos(3x^2-2x)$

a) $-\sin(6x-2)$ b) $-\sin(3x^2-2x)$

c) $-(6x-2)\sin(3x^2-2x)$

d) $(6x-2)\sin(3x^2-2x)$

5. If $\frac{x^2}{4}+\frac{y^2}{9}=1$, find the value of $\frac{dy}{dx}$ at the point (-4,3) a) 6 b) 3 c) -1/3 d) -3/2

6. Differentiate with respect to x, $\frac{\sin x}{1-\cos}$

a) $-\frac{1}{1-\cos}$ b) $-\frac{1}{(1-\cos x)^2}$ c) $-\frac{1}{\cos x-}$ d) $\frac{1-\sin^2 x}{(1-\cos\ )^2}$

7. Differentiate the function $y=\sin^3(4x+10)$

a) $12\sin^2(4x+10)\cos^2(4x+10)$

b) $12\sin^2(4x+10)\,cos(4x+10)$

c) $12\sin^2(4x+10)$ d) $3\sin^2(4x+10)$

8. What is gradient of the curve $y=4x^2+3x-7$ at the point $x=2$ a) 12 b) -13 c) 4 d) 19

9. At the point, (1, -4), find the gradient of the curve $x^2-2xy-2y^2-2x=0$

a) -4/3 b) -4/7 c) -1/2 d) -4/9

10. The gradient of the curve $y=3x^2+11x+7$ at the point p(x, y) is -1, find the co-ordinates of P

a) $(-2,+3)$ b) $(-2,-3)$ c) $(-1,-5/2)$

d) (-3, -2)

11. Find the equation of the normal to the curve $y=2x^3+x^2-5x+2$ at the point where it cuts the y-axis

a) $x-5y+10=0$ b) $5x+y-2=0$

c) $x+5y-10=0$ d) $5x-y+2=0$

12. The distance s metres covered by a body in motion at any time t seconds is given by $s=120t-16t^2$, find, in metres, the distance covered by the body before coming to rest

a) 220 b) 225 c) 360 d) 675

13. Find the maximum value of y on the curve $y=4x^3-9x^2+6x-1$

a) -6 b) 0 c) 1/4 d) 1/3

14. Given the curve $y=\frac{2x^2-1}{x^2-5x+6}$, find the x and y intercept

a) $\frac{1}{2},\frac{1}{3}$ b) $\pm\frac{1}{2},-\frac{1}{6}$ c) $\pm\frac{1}{\sqrt{2}},-\frac{1}{6}$ d) $\pm\sqrt{2},-6$

15. From above, determine the vertical asymptote

a) 3 or 2 b) -3 or 2 c) -3 or -2 d) $\frac{1}{\sqrt{2}}or1/6$

16. What is the horizontal asymptote

a) 3 b) 6 c) 2 d) -2

Given the curve $y=\frac{x^2}{16-x^2}$, use the information to answer questions 17 to 19

17. What is the x and y intercept of the curve

a) 0, 16 b) $0,\infty$ c) 0, 0 d) 0, -4

18. The value of the vertical asymptote is _____

a) ±16 b) 8 c) ±4 d) 4

19. The horizontal asymptote is _____

a) ±2 b) ±1 c) -1 d) -2

20. A particle of mass 3kg moving along a straight line under the action of a force F N covers a distance x meters where $x=t^2+3t$. Find the magnitude of F at time t a) 0 b) 2 c) 2t d) 6

21. Differentiate $y=xe^{2x}+\sin x$ with respect to x

a) $e^{2x}(2x+1)+\cos x$ b) $2xe^{2x}+\cos x$

c) $2e^{2x}(2x+1)-\cos x$ d) $(2x+e^{2x})+\sin x$

22. Differentiate $y=\log_e x^2-3\cos x$ with respect to x; a) $\frac{2}{x^2}+3\sin 2x$ b) $\frac{2}{x}+3\sin x$

c) $\frac{2}{3x^2}+3\sin x$ d) $\frac{2}{x^2}+\cos x$

23. If $y=\frac{1}{x-1}$, then $(x-1)\frac{d^2y}{dx^2}+2\frac{dy}{dx}=$ _____

a) $x-1$ b) 2 c) 0 d) $x+1$

24. Find the coordinates of the minimum points of the curve $y=3x^5-5x^3+4$

a) 3, 5 b) 3, 5 c) 1, 2 d) 1,5

25. A particle moves such that its displacement S in metres from a fixed point after time t seconds is $S=3t^3i-5t^2j$. Find in ms^{-1}, the magnitude of its velocity when $t=4$

a) 20 b) $2\sqrt{106}$ c) $8\sqrt{106}$ d) $8\sqrt{349}$

26. Evaluate $\lim_{x\to0}\frac{\cos x-2\sin x}{3\cos x}$

a) 0 b) 1/3 c) 2/3 d) 1

27. If $f(x)=3x^2+\frac{2}{x}$, find $f'(x)$

a) $6x+2$ b) $6x+\frac{2}{x^2}$ c) $6x-\frac{2}{x^2}$ d) $6x-2$

28. If $y = e^{3x}$, find the value of radius of curvature (R) if $R = \frac{{}^{d^2y}/_{dx^2}}{\left[1+\left({}^{dy}/_{dx}\right)^2\right]^{\frac{3}{2}}}$ when $x = 2$

a) ${}^{9}/_{\sqrt{10}^3}$ b) 0 c) ${}^{9\sqrt{2}}/_{10}$ d) ${}^{4}/_{\sqrt{8}^3}$

29. Differentiate $y = 5a^3bx^2$ with respect to a

a) $10a^3bx$ b) $15a^3bx$ c) $15a^2bx^2$ d) $5a^3x^2$

30. Find $\frac{dy}{dx}$ if $x^2 - 2xy^2 = 0$

a) $\frac{x-y^2}{2xy}$ b) $\frac{x^2-y^2}{xy}$ c) $\frac{x-2y^2}{4xy}$ d) $\frac{x}{4y}$

31. If $y = \sin(2x)^0$, find $\frac{dy}{dx}$ a) $\cos(2x)^0$

b) $2\cos(2x)^0$ c) $\frac{\pi}{90}\cos(2x)^0$ d) $\pi\sin(2x)^0$

32. Let $y = \log_a x$, find $\frac{dy}{dx}$

a) $\frac{1}{a\ln x}$ b) $\frac{1}{x^2 lna}$ c) $\frac{1}{2xl\ a}$ d) $\frac{1}{x\ln a}$

33. If $y = kx^{a-2}$, find ${}^{dy}/_{dx}$ a) $k(a-2)x^{a-1}$

b) $k(a-2)x^{a-2}$ c) kax^{a-1} d) $k(a-2)x^{a-3}$

34. Differentiate with respect to x; $y = x^2 \ln x$

a) $x + 2x\ln x$ b) $x + x\ln x$ c) $x + 2\ln x$

d) $1 + 2x\ln x$

35. Find the equation of the tangent to the curve $y = 2x^3 - 6x$ at $x = 2$ a) $y = 18x - 32$ b) $y = 4x - 32$ c) $y = 8x - 16$ d) $y = 32x - 18$

36. From above, what is the equation of the normal to the curve a) $18y - x = 74$ b) $y + 18x = 54$

c) $18y + x = 74$ d) $y + x = 74$

37. Differentiate x with respect to respect to t $x = \sqrt{t - 2t^5}$ a) $\frac{1}{5}\sqrt{t-2t^5}$ b) $\frac{1}{\sqrt{t-2t^5}}$

c) $\frac{1-10t^2}{2\left(\sqrt{t-2t^5}\right)}$ d) $\sqrt{1-10t^4}$

38. If $y = 2t^3 - t^2$ and $x = t^2 + 5$, find $\frac{dy}{dx}$

a) $3t - 1$ b) $3t^2 - 2t$ c) $6t^2 - 2t$ d) $2t + 5t^2$

SOLUTIONS

1. $y = \ln 3x^4$

Remember what I told you; when differentiating '*ln*', you shouldn't have *ln* again in your answer.

$\frac{dy}{dx} = \frac{1}{3x^4} \times \frac{d}{dx}(3x^4) = \frac{12x^3}{3x^4} = \frac{4}{x} \ldots\ldots\ldots c$

2. Think of quotient rule

$y = \frac{x^2+2}{3-x^2}$

$u = x^2 + 2, \quad v = 3 - x^2$

$\frac{du}{dx} = 2x, \quad \frac{dv}{dx} = -2x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(3-x^2)(2x) - (x^2+2)(-2x)}{(3-x^2)^2}$

$\frac{dy}{dx} = \frac{2x[(3-x^2) + (x^2+2)]}{(3-x^2)^2}$

$\frac{dy}{dx} = \frac{2x(3 - \cancel{x^2} + \cancel{x^2} + 2)}{(3-x^2)^2}$

$\frac{dy}{dx} = \frac{2x(5)}{(3-x^2)^2} = \frac{10x}{(3-x^2)^2} \ldots\ldots\ldots c$

3. Think of ∞(infinity) as a very large number

$hence, as\ n \to \infty, \quad \frac{3}{\infty} \to 0$

$\left(\frac{3}{n} - 3\right) = 0 - 3 = -3 \ldots\ldots\ldots a$

4. $y = \cos(3x^2 - 2x)$

Composite;

Let; $u = 3x^2 - 2x$

$\frac{du}{dx} = 6x - 2$

$Hence, \quad y = \cos u$

$\frac{dy}{du} = -\sin u = -\sin(3x^2 - 2x)$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin(3x^2 - 2x) \times (6x - 2)$

$\frac{dy}{dx} = -(6x - 2)\sin(3x^2 - 2x) \ldots\ldots\ldots c$

5. This is an example on implicit differentiation

Hence, we have;

$\frac{2x}{4} + \frac{2y}{9}\left(\frac{dy}{dx}\right) = 0$

$\frac{2y}{9}\left(\frac{dy}{dx}\right) = -\frac{2x}{4}$

$\frac{dy}{dx} = -\frac{\cancel{2}x}{4} \times \frac{9}{\cancel{2}y} = -\frac{9x}{4y}$

At $(-4, 3)$

$-\frac{9(-4)}{4(3)} = 3 \ldots\ldots\ldots . b$

6. Quotient rule;

$y = \frac{\sin x}{1 - \cos x}$

$u = \sin x \quad \frac{du}{dx} = \cos x$

$v = 1 - \cos x \quad \frac{dv}{dx} = -(-\sin x) = \sin x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(1 - \cos x)(\cos x) - (\sin x)(\sin x)}{(1 - \cos x)^2}$

$\frac{dy}{dx} = \frac{\cos x - \cos^2 x - \sin^2 x}{(1 - \cos x)(1 - \cos x)}$

$\frac{dy}{dx} = \frac{\cos x - (\cos^2 x + \sin^2 x)}{(1 - \cos x)(1 - \cos x)}$

Recall that $\cos^2 x + \sin^2 x = 1$

$\frac{dy}{dx} = \frac{\cos x - 1}{(1 - \cos x)(1 - \cos x)}$

$\frac{dy}{dx} = \frac{-\cancel{(1 - \cos x)}}{\cancel{(1 - \cos x)}(1 - \cos x)}$

$\frac{dy}{dx} = \frac{-1}{1 - \cos x} = \frac{1}{\cos x - 1} \ldots\ldots\ldots\ldots\ a$

7. $y = \sin^3(4x + 10)$

Composite rule;

Let $u = 4x + 10$ then $\frac{du}{dx} = 4$

$y = \sin^3 u$

$let\ z = \sin u \quad then\ \frac{dz}{du} = \cos u$

Now, $y = z^3 \quad \Rightarrow \frac{dy}{dz} = 3z^2$

$y = f(z), \quad z = f(u), \ u = f(x)$

$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{du} \cdot \frac{du}{dx} = 3z^2 \cdot \cos u \cdot 4$

$\frac{dy}{dx} = 12z^2 \cos u$

$\frac{dy}{dx} = 12(\sin u)^2(\cos u)$

$\frac{dy}{dx} = 12 \sin^2(4x + 10) \cos(4x + 10) \ldots\ldots . b$

8. $y = 4x^2 + 3x - 7$

Differentiate y to get the gradient;

$\frac{dy}{dx} = 8x + 3$

$gradient, m = \left.\frac{dy}{dx}\right|_{x=-2} = 8(-2) + 3$

$m = -16 + 3 = -13 \ldots\ldots\ldots b$

9. $x^2 - 2xy - 2y^2 - 2x = 0$

Let's differentiate first

Hmmmm, implicit differentiation;

$2x - 2y - 2x\frac{dy}{dx} - 4y\frac{dy}{dx} - 2 = 0$

$-2x\frac{dy}{dx} - 4y\frac{dy}{dx} = 2 + 2y - 2x$

$\frac{dy}{dx}(-2x - 4y) = 2 + 2y - 2x$

$\frac{dy}{dx} = \frac{2 + 2y - 2x}{-2x - 4y}$

$m = \left.\frac{dy}{dx}\right|_{x=1, y=-4} = \frac{2 + 2(-4) - 2(1)}{-2(1) - 4(-4)}$

$m = \frac{2 - 8 - 2}{-2 + 16} = \frac{-8}{14} = -\frac{4}{7} \ldots\ldots\ldots\ldots b$

10. $y = 3x^2 + 11x + 7$

$gradient = \frac{dy}{dx} = 6x + 11$

The question gives the gradient as -1

$Hence, \quad 6x + 11 = -1$

$6x = -12, \quad x = -\frac{12}{6} = -2$

Get y when $x = -2$

$y = 3(-2)^2 + 11(-2) + 7$

$y = 12 - 11 + 7 = -3$

Hence, the coordinate is (-2, -3) ..... b

11. $y = 2x^3 + x^2 - 5x + 2$

$gradient, m = \frac{dy}{dx} = 6x^2 + 2x - 5$

Where it cuts the y-axis means; x=0

$\left.\frac{dy}{dx}\right|_{x=0} = -5$

$When\ x = 0, y = 2(0^3) + 0^2 - 5(0) + 2 = 2$

If the gradient of the curve is -5, then the gradient of the normal will be $\frac{-1}{-5} = \frac{1}{5}$

Hence, the equation of the normal will be;

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{5}(x - 0)$

$5y - 10 = x$

$x - 5y + 10 = 0 \ldots\ldots\ldots a$

12. $s = 120t - 16t^2$

Coming to rest means that final velocity is equal to zero.

Let us differentiate the expression for distance to get the expression for velocity and then equate it to zero.

$v = \frac{ds}{dt} = 120 - 32t$

$120 - 32t = 0$

$32t = 120 \quad \Longrightarrow \quad t = 3.75secs$

$\therefore s = 120(3.75) - 16(3.75^2)$

$s = 450 - 225 = 225m \ldots\ldots .. b$

13. $y = 4x^3 - 9x^2 + 6x - 1$

I've already given you steps the note above

$Step\ 1;\ \frac{dy}{dx} = 12x^2 - 18x + 6$

$\frac{d^2y}{dx^2} = 24x - 18$

$At\ \frac{dy}{dx} = 0; \quad 12x^2 - 18x + 6 = 0$

Solve the quadratic equation

$x = \frac{1}{2}\ or\ x = 2$

$Step\ 2; get\ the\ corresponding\ values\ of\ y$

$When\ x = \frac{1}{2}, y = 4\left(\frac{1}{2}\right)^3 - 9\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) - 1$

$y = \frac{1}{2} - \frac{9}{4} + 3 - 1 = \frac{1}{4}$

Hence, the first turning point is $(1/2, 1/4)$

$When\ x = 2, \quad y = 4(2)^3 - 9(2)^2 + 6(2) - 1$

$y = 32 - 36 + 12 - 1 = 7$

The first turning point is $(2, 7)$

Testing the value of x in the second derivative;

$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{2}} = 24\left(\frac{1}{2}\right) - 18 = 12 - 18 = -6$

$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 24(2) - 18 = 48 - 18 = 30$

Hence, the point $\left(\frac{1}{2}, \frac{1}{4}\right)$ is the maximum point.

$\therefore\ the\ maximum\ value\ of\ y\ is\ \frac{1}{4} \ldots\ldots . c$

14. x-intercept (i.e. value of x when y=0)

$\frac{2x^2 - 1}{x^2 - 5x + 6} = 0$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2} \quad \Longrightarrow \quad x = \pm\frac{1}{\sqrt{2}}$

y-intercept (i.e. value of y when x=0)

$y = \frac{2(0)^2 - 1}{0^2 - 5(0) + 6} = -\frac{1}{6} \ldots\ldots\ldots\ldots c$

15. vertical asymptote is gotten when the function is undefined i.e. when the denominator equals zero

$x^2 - 5x + 6 = 0$

$x = 3\ or\ x = 2 \ldots\ldots\ldots\ldots\ldots . a$

16. horizontal asymptote

$y = \frac{2x^2 - 1}{x^2 - 5x + 6}$

Divide through the left hand side by the highest power of x (i.e. x^2)

$y = \frac{\frac{2x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{6}{x^2}} = \frac{2 - \frac{1}{x^2}}{1 - \frac{5}{x} + \frac{6}{x^2}}$

$as\ x \longrightarrow \infty$

$y = \frac{2 - \frac{1}{\infty^2}}{1 - \frac{5}{\infty} + \frac{6}{\infty^2}} = \frac{2 - 0}{1 - 0 - 0}$

$y = \frac{2}{1} = 2 \ldots\ldots\ldots .. c$

17. x-intercept i.e. when $y = 0$

$\frac{x^2}{16 - x^2} = 0$

$x^2 = 0, \quad x = 0$

y – intercept i.e. when $x = 0$

$y = \frac{0^2}{16 - 0^2} = \frac{0}{16} = 0 \ldots\ldots\ldots c$

18. vertical asymptote (when the denominator is zero)

$16 - x^2 = 0$

$x^2 = 16, \quad x = \pm 4 \ldots\ldots . . c$

19. horizontal asymptote

$y = \dfrac{x^2}{16 - x^2}$

Divide through the left hand side by the highest power of x (i.e. x^2)

$y = \dfrac{\frac{x^2}{x^2}}{\frac{16}{x^2} - \frac{x^2}{x^2}} = \dfrac{1}{\frac{16}{x^2} - 1}$

$as\ x \longrightarrow \infty$

$y = \dfrac{1}{\frac{16}{\infty^2} - 1} = \dfrac{1}{0 - 1} = \dfrac{1}{-1} = -1 \ldots\ldots\ldots . c$

20. we were given distance to be $x = t^2 + 3t$

To get acceleration from distance, differentiate distance twice

$velocity, \quad \dfrac{dx}{dt} = 2t + 3$

$acceleration, \quad \dfrac{d^2x}{dt^2} = 2$

Since $force = mass \times acceleration$

$F = 3 \times 2 = 6N \ldots\ldots\ldots\ldots d$

21. $y = xe^{2x} + \sin x$

The first term (xe^{2x}) involves product rule while the second term is an ordinary trig. Function

$\dfrac{dy}{dx} = x\dfrac{d}{dx}(e^{2x}) + e^{2x}\dfrac{d}{dx}(x) + \dfrac{d}{dx}(\sin x)$

$\dfrac{dy}{dx} = x(2e^{2x}) + e^{2x}(1) + \cos x$

$\dfrac{dy}{dx} = 2xe^{2x} + e^{2x} + \cos x$

$\dfrac{dy}{dx} = e^{2x}(2x + 1) + \cos x \ldots\ldots\ldots a$

22. $y = \log_e x^2 - 3\cos x$

The first term involves composite rule while the second term is just a normal trig, function

Let $u = x^2 \quad \Longrightarrow \frac{du}{dx} = 2x$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(\log_e u) - 3\dfrac{d}{dx}(\cos x)$

$\dfrac{dy}{dx} = \left(\dfrac{dy}{du} \cdot \dfrac{du}{dx}\right) - 3(-\sin x)$

$\dfrac{dy}{dx} = \left(\dfrac{1}{u} \cdot 2x\right) + 3\sin x$

$\dfrac{dy}{dx} = \left(\dfrac{1}{\cancel{x}^2} \cdot 2\cancel{x}\right) + 3\sin x$

$\dfrac{dy}{dx} = \dfrac{2}{x} + 3\sin x \ldots\ldots\ldots\ldots . . b$

23. $y = \frac{1}{x-1} = (x-1)^{-1}$

$\dfrac{dy}{dx} = -1(1)(x-1)^{-2} = -(x-1)^{-2}$

$\dfrac{d^2y}{dx^2} = -(-2)(1)(x-1)^{-3} = 2(x-1)^{-3}$

To analyze the question,

$(x-1)\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx}$

$(x-1)[2(x-1)^{-3}] + 2[-(x-1)^{-2}]$

$\cancel{(x-1)}\left(\dfrac{2}{\cancel{(x-1)}^3}\right) + 2\left(\dfrac{-1}{(x-1)^2}\right)$

$\left(\dfrac{2}{(x-1)^2}\right) - \left(\dfrac{2}{(x-1)^2}\right) = 0 \ldots\ldots . . c$

24. $y = 3x^5 - 5x^3 + 4$

$gradient; \dfrac{dy}{dx} = 15x^4 - 15x^2$

$15x^4 - 15x^2 = 0$

$15(x^4 - x^2) = 0$

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

$x^2 = 0 \ or\ x^2 = 1$

$x = 0\ or\ x = \pm 1$

$x = 0\ or\ x = 1 \ or\ x = -1$

Hence, the minimum value of x, $x_{min} = 1$

The minimum value of y,

$y_{min} = 3(1)^5 - 5(1)^3 + 4 = 2 \ldots\ldots\ldots c$

25. $S = 3t^3 i - 5t^2 j$

The above equation is a displacement vector

$velocity\ vector, \quad \dfrac{dS}{dt} = 9t^2 i - 10tj$

At $t = 4$,

$velocity\ vector = 9(4^2)i - 10(4)j$

$= 144i - 40j$

Magnitude of velocity $|V|$

$|V| = \sqrt{144^2 + 40^2} = \sqrt{20736 + 1600}$

$|V| = \sqrt{22336} = 8\sqrt{349} \ldots\ldots\ldots d$

26. Just simplify the expression and make substitution for x

$$\frac{\cos x - 2\sin x}{3\cos x} = \frac{\cos x}{3\cos x} - \frac{2\sin x}{3\cos x} = \frac{1}{3} - \frac{2}{3}\tan x$$

$$\lim_{x\to 0} = \frac{1}{3} - \frac{2}{3}\tan 0 = \frac{1}{3} - \frac{2}{3}(0) = \frac{1}{3} \ldots\ldots\ldots b$$

27. $f'(x)$ means first differential i.e. $\frac{dy}{dx}$

$$f(x) = 3x^2 + \frac{2}{x} = 3x^2 + 2x^{-1}$$

$$f'(x) = 6x - 2x^{-2} = 6x - \frac{2}{x^2} \ldots\ldots\ldots c$$

28. $y = e^{3x}$

$$\frac{dy}{dx} = 3e^{3x}$$

$$\frac{d^2y}{dx^2} = 9e^{3x}$$

$$R = \frac{{}^{d^2y}/_{dx^2}}{\left[1+\left({}^{dy}/_{dx}\right)^2\right]^{\frac{3}{2}}} = \frac{9e^{3x}}{[1+(3e^{3x})^2]^{\frac{3}{2}}}$$

$$R = \frac{9e^{3(0)}}{[1+(3e^{3(0)})^2]^{\frac{3}{2}}} = \frac{9}{[1+3^2]^{\frac{3}{2}}} = \frac{9}{10^{\frac{3}{2}}}$$

$$R = \frac{9}{\sqrt{10}^3} \ldots\ldots\ldots a$$

29. You are asked to differentiate with respect to $'a'$ not the usual $'x'$ we've been doing

$$y = 5a^3bx^2$$

$$\therefore \frac{dy}{da} = 3(5a^{3-1}bx^2) = 15a^2bx^2 \ldots\ldots\ldots c$$

30. This is an implicit differentiation

$$x^2 - 2xy^2 = 0$$

$$2x - 2y^2 - 2(2xy)\frac{dy}{dx} = 0$$

$$2x - 2y^2 - 4xy\frac{dy}{dx} = 0$$

$$4xy\frac{dy}{dx} = 2x - 2y^2$$

$$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy} = \frac{x - y^2}{2xy} \ldots\ldots\ldots\ldots a$$

31. $y = \sin(2x)^0$

Note: whenever you wanna differentiate and your angle is in degree, ensure you convert it to radian

Hence,

$$y = \sin 2x\left(\frac{\pi}{180}\right)$$

$$y = \sin\frac{\pi x}{90}$$

$$\therefore \frac{dy}{dx} = \frac{\pi}{90}\cos\frac{\pi x}{90}$$

Let's take it back to degree

$$\frac{dy}{dx} = \frac{\pi}{90}\cos(2x)^0 \ldots\ldots\ldots c$$

32. Interesting question. Just write the question in form of ln (which is natural logarithm i.e. to base e).

Using the change of base concept, we have;

$$y = \log_a x = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$$

$$y = \ln x \times \frac{1}{\ln a}$$

$$\frac{dy}{dx} = \frac{1}{x} \times \frac{d}{dx}(x) \times \frac{1}{\ln a} = \frac{1}{x} \times 1 \times \frac{1}{\ln a}$$

$$\frac{dy}{dx} = \frac{1}{x\ln a} \ldots\ldots\ldots d$$

33. $y = kx^{a-2}$

$$\frac{dy}{dx} = k(a-2)x^{a-2-1} = k(a-2)x^{a-3} \ldots\ldots\ldots d$$

34. $y = x^2\ln x$

Product rule is involved

$$u = x^2 \quad \frac{du}{dx} = 2x$$

$$v = \ln x \quad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = (x^2)\left(\frac{1}{x}\right) + (\ln x)(2x)$$

$$\frac{dy}{dx} = x + 2x\ln 2x \ldots\ldots\ldots\ldots a$$

35. $y = 2x^3 - 6x$

$$gradient, m = \frac{dy}{dx} = 6x^2 - 6$$

$$at\ x = 2, \frac{dy}{dx} = 6(2^2) - 6 = 24 - 6 = 18$$

$$at\ x_1 = 2, y = 2(2^3) - 6(2) = 16 - 12 = 4$$

The equation of the tangent is;

$$y - y_1 = m(x - x_1)$$

$y - 4 = 18(x - 2)$

$y = 18x - 36 + 4$

$y = 18x - 32 \ldots\ldots\ldots a$

36. Since the gradient of the tangent is 18, then the gradient of the normal will be $^{-1}/_{18}$

The equation of the normal will then be;

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-1}{18}(x - 2)$

$18y - 72 = -(x - 2)$

$18y = -x + 2 + 72$

$18y + x = 74 \ldots\ldots\ldots\ldots c$

37. $x = \sqrt{t - 2t^5} = (t - 2t^5)^{\frac{1}{2}}$

$\frac{dx}{dt} = \frac{1}{2}(1 - 10t^4)(t - 2t^5)^{\frac{1}{2}-1}$

$\frac{dx}{dt} = \frac{1}{2}(1 - 10t^4)(t - 2t^5)^{-\frac{1}{2}}$

$\frac{dx}{dt} = \frac{1}{2}\frac{(1 - 10t^4)}{\sqrt{t - 2t^5}} \ldots\ldots\ldots c$

38. $y = 2t^3 - t^2 \quad x = t^2 + 5$

$\frac{dy}{dt} = 6t^2 - 2t \qquad \frac{dx}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dt}{dx} = \frac{6t^2 - 2t}{2t}$

$\frac{dy}{dx} = \frac{\cancel{2t}(3t - 1)}{\cancel{2t}} = 3t - 1 \ldots\ldots\ldots a$

INTEGRAL CALCULUS

GENERAL FORM OF INTEGRATION

$$\int ax^n dx = a\int x^n dx = a\left[\frac{x^{n+1}}{n+1}\right] + c$$

Where $n \neq -1$ and c is called the constant of integration.

Example;

Evaluate $\int 5x^4 dx$

Solution

$$\int 5x^4 dx = \frac{5x^{4+1}}{4+1} + c = \frac{5x^5}{5} + c = x^5 + c$$

STANDARD INTEGRALS

$\int x^n dx$	$\frac{x^{n+1}}{n+1} + c$
$\int \frac{adx}{x}$	$a\ln x + c$
$\int e^x dx$	$e^x + c$
$\int e^{f(x)}\,dx$	$\frac{1}{d/dx\,[f(x)]} \cdot e^{f(x)} + c$
$\int a^x dx$	$\frac{a^x}{\ln a} + c$
$\int \sin x\,dx$	$-\cos x + c$
$\int \cos x\,dx$	$\sin x + c$
$\int \sec^2 x\,dx$	$\tan x + c$
$\int \text{cosec}^2 x\,dx$	$-\cot x + c$
$\int \sec x \tan x\,dx$	$\sec x + c$
$\int \text{cosec}\,x \cot x\,dx$	$-\text{cosec}\,x + c$
$\int \tan x\,dx$	$\ln\lvert\sec x\rvert + c$
$\int \cot x\,dx$	$\ln\lvert\sin x\rvert + c$
$\int \text{cosec}\,x\,dx$	$\ln\lvert\text{cosec}\,x - 6 + x\rvert + c$
$\int \frac{dx}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\frac{x}{a} + c$
$\int \frac{dx}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$
$\int \frac{dx}{x\sqrt{x^2 - a^2}}$	$\frac{1}{a}\sec^{-1}\frac{x}{a} + c$

$\int \frac{dx}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\|\frac{x+a}{x-a}\right\| + c$
$\int \frac{dx}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + c$

INTEGRATION BY SUBSTITUTION

Just as we have composite/chain rule in differentiation, we have something similar in integration.

If you are making substitution for f(x), then you must make sure the whole expression is in terms of the new letter say u.

Example

Evaluate $\int \sin 5x\, dx$

Solution

$$\int \sin 5x\, dx$$

$$Let\ u = 5x \implies \frac{du}{dx} = 5 \implies dx = \frac{du}{5}$$

$$\therefore \int \sin 5x\, dx = \int \sin u \cdot \frac{du}{5} = \frac{1}{5}\int \sin u\, du$$

$$\int \sin 5x\, dx = \frac{1}{5}(-\cos u) + c = -\frac{1}{5}\cos 5x + c$$

Trigonometric Substitution

If you have the form	Substitute x with
$(a^2 - b^2x^2)$	$\frac{a}{b}\ sin\ \theta$
$(a^2 + b^2x^2)$	$\frac{a}{b}\ tan\ \theta$
$(b^2x^2 - a^2)$	$\frac{a}{b}\ sec\ \theta$

Example

$Evaluate \int x^3\sqrt{9 - x^2}dx$

Solution

I can easily re-write the equation as

$$\int x^3\sqrt{3^2 - x^2}dx$$

The expression in the square root is of the form $(a^2 - b^2x^2)$ where $a = 3\ and\ b = 1$. Hence,

$$Let\ x = 3\sin\theta \quad \frac{dx}{d\theta} = 3\cos\theta \implies dx = 3\cos\theta\, d\theta$$

The question becomes

$$\int \left[3^3\sin^3\theta\sqrt{3^2 - (3\sin\theta)^2}\right] 3\cos\theta\, d\theta$$

$$= \int \left[3^3\sin^3\theta\sqrt{3^2(1 - \sin^2\theta)}\right] 3\cos\theta\, d\theta$$

$$= \int \left[3^3\sin^3\theta \times 3\sqrt{1 - \sin^2\theta}\right] 3\cos\theta\, d\theta$$

$$= \int \left[\ 3^3\sin^3\theta \times 3\sqrt{\cos^2\theta}\ \right] 3\cos\theta\, d$$

$$= \int [\ 3^5\sin^3\theta \times \cos\theta\ ]\cos\theta\, d\theta$$

$$= 3^5\int \sin^3\theta\cos^2\theta\ d\theta$$

$$= 3^5\int \sin^2\theta\cos^2\theta\sin\theta\ d\theta$$

$$= 3^5\int (1 - \cos^2\theta)\cos^2\theta\sin\theta\ d\theta$$

$$Let\ u = \cos\theta \implies d\theta = \frac{du}{-\sin\theta}$$

$$= 3^5\int (1 - u^2)u^2\ \cancel{\sin\theta} \cdot \frac{du}{-\cancel{\sin\theta}}$$

$$= 3^5 \int (1-u^2)u^2 \cdot -du$$

$$= -3^5 \int (u^2 - u^4)\, du$$

$$= -3^5 \left[\frac{u^3}{3} - \frac{u^5}{5}\right] + c$$

Recall that $u = \cos\theta$

$$= -3^5 \left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right] + c$$

$from\ x = 3\sin\theta,\ \ \sin\theta = \frac{x}{3}$

$$\therefore \cos\theta = \frac{\sqrt{x^2-9}}{3} = \frac{(x^2-9)^{\frac{1}{2}}}{3}$$

Hence, the final answer

$$= -3^5 \left[\frac{1}{3^3}\left(\frac{(x^2-9)^{\frac{3}{2}}}{3}\right) - \frac{1}{3^5}\left(\frac{(x^2-9)^{\frac{5}{2}}}{5}\right)\right] + c$$

$$= -3(x^2-9)^{\frac{3}{2}} + \frac{1}{5}(x^2-9)^{\frac{5}{2}} + c$$

INTEGRATION OF RATIONAL FUNCTIONS

This is talking about integration involving numerator and denominator

***Case* 1**; $\int \frac{constant}{f(x)}dx$

$$\int \frac{k}{mk+n}dx = \frac{k}{m}\ln(mk+n) + c$$

Example:

$Evaluate\ i) \int \frac{5}{3x+7}dx \quad ii) \int \frac{2}{(3x+7)^4}dx$

Solution

$$\int \frac{5}{3x+7}dx = 5\int \frac{1}{3x+7}dx = \frac{5}{3}\ln(3x+7) + c$$

***Case* 2**;

$$\int \frac{2}{(3x+7)^4}dx = 5\int (3x+7)^{-4}$$

$$= 5\left(\frac{1}{3}\right)\left(\frac{(3x+7)^{-4+1}}{-4+1}\right) + c$$

$$= \frac{5}{3}\left(\frac{(3x+7)^{-3}}{-3}\right) + c = -\frac{5}{9}(3x+7)^{-3} + c$$

***Case* 2**; $\int \frac{kdx}{ax^2+bx+c}$

- If $b^2 \geq 4ac$, resolve into partial fraction and integrate separately.
- If $b^2 < 4ac$, express it using completing the square

Example

$Solve\ i) \int \frac{8x-72}{(x+1)(x-3)^2}dx\ \ ii) \int \frac{4x-1}{2x^2+5x+2}$

Solution

i) We have to first resolve into partial fraction. If you've forgotten partial fraction, go and revise it oooooooooo.

$$\frac{8x-72}{(x+1)(x-3)^2} = \frac{10}{x-3} - \frac{5}{x+1} - \frac{12}{(x-3)^2}$$

$$\int \frac{10dx}{x-3} - \int \frac{5dx}{x+1} - \int \frac{12dx}{(x-3)^2}$$

$$= 10\ln(x-3) + 5\ln(x+1) + 12(x-3)^{-1} + c$$

ii) Check the denominator

$$\frac{4x-1}{2x^2+5x+2}$$

$2x^2+5x+2,\ a=2\ \ b=5\ \ c=2$

$b^2 = 25, \quad 4ac \quad 4(2)(2) = 16$

It is obvious that $b^2 > 16$

That means we will resolve into partial fraction. If you've forgotten partial fraction, then revise it ooo.

$$\frac{4x-1}{2x^2+5x+2} = \frac{3}{x+2} - \frac{2}{2x+1}$$

$$\int \frac{4x-1}{2x^2+5x+2}dx = \int \frac{3}{x+2}dx \quad \int \frac{2}{2x+1}dx$$

$$= 3\ln(x+2) - \frac{2}{2}\ln(2x+1) + c$$

$$= 3\ln(x+2) - \ln(2x+1) + c$$

t-FORMULA

Remember the t-FORMULA I gave you under chapter one (trigonometric identity).

From $\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$let \quad \tan\frac{x}{2} = t$

$$\frac{x}{2} = \tan^{-1} t, \quad x = 2\tan^{-1} t$$

$$\frac{dx}{dt} = \frac{2}{1+t^2} \quad \Rightarrow \quad dx = \frac{2}{1+t^2}dt$$

$$then \quad \tan x = \frac{2t}{1-t^2}$$

$$\Rightarrow \quad \sin x = \frac{2t}{1+t^2} \quad and \quad \cos x = \frac{1-t^2}{1+t^2}$$

Example

$$Evaluate \int \frac{4dx}{3-5\sin x}$$

Solution

From what you have above,

$$Let \sin x = \frac{2t}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$$

$$\therefore \int \frac{4dx}{3-5\sin x} = 4\int \frac{\frac{2dt}{1+t^2}}{3-\frac{10t}{(1+t^2)}}$$

$$= 4\int \frac{2dt}{3(1+t^2)-10t} = 4\int \frac{2dt}{3+3t^2-10t}$$

$$8\int \frac{dt}{3t^2-10t+3}$$

Resolve the denominator into partial fraction

$$8\int \frac{dt}{3t^2-10t+3} = 8\left(\frac{1}{8}\right)\int \left[\frac{1}{t-3} - \frac{3}{3t-1}\right]dt$$

$$= \ln(t-3) - \ln(3t-1) + c$$

$$= \ln\left(\frac{t-3}{3t-1}\right) + c$$

$$= \ln\left[\frac{\tan\left(\frac{x}{2}\right)-3}{3\tan\left(\frac{x}{2}\right)-1}\right] + c$$

INTEGRATION BY PART

$$\int udv = uv - \int vdu$$

This concerns integration of product.

The steps are;

- Choose one of the function as **u** and the other as **dv**
- To get **v,** integrate the function **dv**
- Differentiate **u** to get **du**

Note;

✓ If you have a **log** function in the question, it should be your **u**

✓ If there is no log function, then take a factor of **x** as **u**

✓ If there is still no factor of x, then take the **exponential** function as u

Illustration

1. In $\int x^3 \ln x \, dx$

   $\ln x$ is taken as u while x^3 is taken as dv

2. In $\int x^4 \, e^{2x} \, dx$

   x^4 is taken as u while e^{2x} is taken as dv

3. In $\int e^{2x} \cos 2x \, d$

   e^{2x} is taken as u while $\cos 2x$ is taken as dv

Example

$$\int x \ln x \, d$$

You will notice there is a log function i.e. ln x

Hence,

Let $u = \ln x \quad and \quad dv = x$

$$\therefore \frac{du}{dx} = \frac{1}{x} \implies du = \frac{dx}{x}$$

Also, to get v

$$\int dv = \int x dx$$

$$v = \frac{x^2}{2}$$

We then introduce the formula

$$\int u dv = uv - \int v du$$

$$\int x \ln x \, dx = (\ln x)\left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right)\left(\frac{dx}{x}\right)$$

$$= \frac{x^2 \ln x}{2} - \frac{1}{2}\int x dx = \frac{x^2 \ln x}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + c$$

INTEGRATION OF TRIGONOMETRIC FUNCTIONS

Recall from trigonometric identity

- $\sin 2\alpha = 2 \sin \alpha \cos \alpha$
- $\cos 2\alpha = \begin{Bmatrix} \cos^2 \alpha - \sin^2 \alpha \\ or \quad 2\cos^2 \alpha - 1 \\ or \quad 1 - 2\sin^2 \alpha \end{Bmatrix}$

And from standard integrals

- $\int \sin x \, dx = -\cos x + c$
- $\int \cos x \, dx = \sin x + c$
- $\int \sec^2 x \, dx = \tan x + c$

POWERS OF SINE AND COSINE

Even powers

Example

$$\int \sin^4 x \, dx$$

Solution

Just rewrite the expression as a multiple of $\sin^2 x$

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 dx$$

Replace $\sin^2 x$ with $\frac{1}{2}(1 - \cos 2x)$

$$\int \sin^4 x \, dx = \int \left(\frac{1}{2}(1 - \cos 2x)\right)^2 dx$$

$$= \frac{1}{2^2}\int (1 - \cos 2x)^2 dx$$

$$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4}\left[\int 1 dx \quad 2\int \cos 2x \, dx + \int \cos^2 2x \, dx\right]$$

$$Recall: \cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

$$Hence, \quad \cos^2 2x = \frac{1}{2}(1 + \cos 4x)$$

The expression then becomes

$$\frac{1}{4}\left[\int 1 dx - 2\int \cos 2x \, dx + \frac{1}{2}\int (1 + \cos 4x) dx\right.$$

$$= \frac{1}{4}\left[x - 2\left(\frac{\sin 2x}{2}\right) + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\right] + c$$

$$= \frac{1}{4}\left[x + \frac{1}{2}x - \cancel{2}\left(\frac{\sin 2x}{\cancel{2}}\right) + \frac{1}{2}\left(\frac{\sin 4x}{4}\right)\right] + c$$

$$= \frac{1}{4}\left[\frac{3}{2}x - \sin 2x + \frac{\sin 4x}{8}\right] + c$$

ii. Rewrite the expression as a multiple of $\cos^2 x$

Odd powers

Example

$$\int \sin^3 x\, dx$$

Solution

You have to split the sin

$$\int \sin^3 x\, dx = \int (\sin^2 x)(\sin x) dx$$

$$= \int (1 - \cos^2 x) \sin x\, dx$$

$$Put\ u = \cos x, \frac{du}{dx} = -\sin x \implies dx = \frac{-du}{\sin x}$$

$$Hence, \int \sin^3 x\, dx = \int (1 - u^2)\, \cancel{\sin x} \times \frac{-du}{\cancel{\sin x}}$$

$$= -\int (1 - u^2) du = -\left[u - \frac{u^3}{3}\right] + c$$

$$= \frac{u^3}{3} - u + c = \frac{\cos^3 x}{3} - \cos x + c$$

APPLICATION OF INTEGRATION

Area under a curve

You will be given a function of x i.e. $f(x)$ and two ordinates of x say a and b.

$$Area = \int_a^b f(x) dx$$

Example

Find the area between the curve $y = 2x^5 + 3x^2$ and from $x = 1\ to\ x = 2$

Solution

$$Area = \int_1^2 (2x^5 + 3x^2) dx$$

$$A = \left[\frac{2x^6}{6} + \frac{3x^3}{3}\right]_1^2$$

$$A = \left[\frac{2(2)^6}{6} + 2^3\right] - \left[\frac{2(1)^6}{6} + 1^3\right]$$

$$A = \frac{512}{3} - \frac{4}{3} = \frac{508}{3}\ sqr\ unit$$

Area between two curves

You will be given two functions of x i.e. $f_1(x)$ and $f_2(x)$.

First, find their point of intersection by equating the two functions to get values for x say $a\ and\ b$.

The area we are looking for is;

$$A = \int_a^b [f_2(x) - f_1(x)] dx$$

Example

Find the area enclosed by the curves $y = x^2 + 2$ and $y = 4x - 1$

Solution

Let's find their point of intersection by equating the two equations

$$x^2 + 2 = 4x - 1$$

$$x^2 - 4x + 3 = 0$$

Solving the quadratic equation,

$x = 1 \; or \; x = 3$

$$A = \int_1^3 [(4x - 1) - (x^2 + 2)]dx$$

$$A = \int_1^3 (4x - x^2 - 3)dx$$

$$A = \left[\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right]_1^3 = \frac{4}{3} sqr \; units$$

VOLUMES OF SOLIDS OF REVOLUTION

Solids of revolution are solid shapes formed when the areas under a given function are rotated about x or y axis.

This volume can generally be gotten by;

$$V = \pi \int y^2 dx \quad or \; V = \pi \int x^2 dy$$

Where

$dx = change \; or \; rotation \; along \; x - axis \quad dy = change \; or \; rotation \; along \; y - axis$

QUESTIONS

1. Evaluate $\int_1^4 (x-1)^2 \, dx$

   a) 9 b) 10 c) 11 d) 13

2. Given that $\int_2^k (2x - 1)dx = 4$, find the value of k

   a) -2 b) 3 c) 4 d) 6

3. Evaluate $\int_0^2 \left(\frac{x^4+1}{x^2}\right) dx$

   a) 2 1/6 b) 1 1/6 c) 1 d) ½

4. Find $\int (x^2 + 1)^{1/2} \, x dx$ a) $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$

   b) $\frac{2}{3}(x^2 + 1)^{\frac{3}{2}}$ c) $\frac{2}{3}x(x^2 + 1)^{\frac{3}{2}}$ d) $\frac{1}{4}x(x^2 + 1)^{-\frac{1}{2}}$

5. Evaluate $\int \cos^5 x \sin x \, dx$

   a) $\frac{1}{4}\cos^4 x + c$ b) $\frac{1}{5}\cos^6 x + c$ c) $-\frac{1}{5}\cos^6 x + c$

   d) $-\frac{1}{6}\cos^6 x + c$

6. The gradient of a curve at any point (x, y) is $2x + 5$. If the curve passes through the point $(-2, -1)$, find the value of y when x=-4

   a) -2 b) 1 c) 30 d) 31

7. Evaluate $\int \frac{x}{x^2-3x+2} dx$

   a) $\log\left(\frac{x-2}{x-1}\right) + k$ b) $\log(x-2)^2(x-1) + k$

   c) $\log[(x-2)(x-1)] + k$ d) $\log\left[\frac{(x-2)^2}{x-1}\right] + k$

8. A curve passes through the point (3, -1). If its gradient function is 2x+5, find the equation of the curve

   a) $y = x^2 + 5x + 25$ b) $y = x^2 + 5x - 25$

   c) $y = x^2 + 4x + 20$ d) $y = x^2 + 4x + 16$

9. Find the area of the finite region enclosed by a curve $y = 2\sqrt{x}$ and the lines $x = 3 \; and \; x = 0$

   a) $4\sqrt{3}$ b) $2\sqrt{3}$ c) $16\sqrt{3}$ d) $18\sqrt{3}$

10. Find the volume generated by the curve $y = x^2$ when it rotates round the x-axis between x=0 and x=3

    a) $24.3\pi \; cubic \; units$ b) $\frac{27}{5}\pi \; cubic \; units$

    c) $\frac{81}{4}\pi \; cubic \; units$ d) $\frac{243\pi}{5} \; cubic \; units$

11. Given that $\frac{dy}{dx} = 4x - 3$ and $y = 5$ when $x = 2$, find y in terms of x a) $2x^2 - 3x + c$

    b) $2x^2 - 2x + 5$ c) $2x^2 - 3x$ d) $2x^2 - 3x + 3$

12. The gradient of a curve at any point is $2 - x$. If the curve passes through the origin, find the co-ordinates of the other point at which it crosses the x-axis

    a) (2,0) b) (-1, 0) c) (-4, 0) d) (4, 0)

13. Calculate in square units, the area of the finite region bounded by the curve $y = 1 + x - 2x^2$, the lines $x = 0, x = 1$ and the x-axis

    a) 5/6 b) 1 c) 13/6 d) 4

14. Evaluate $\int \frac{1+x}{1+x^2} dx$

    a) $\tan^{-1} x + \frac{1}{2}\ln(1 + x^2) + c$

    b) $\tan x + \frac{1}{2}\ln(1 + x^2) + c$

    c) $\tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + c$

    d) $\tan^{-1} x + 2\ln(1 + x^2) + c$

15. Evaluate $\int_{\pi/12}^{\pi/4} 2\cos 2x\, dx$

a) -1/2 b) -1 c) 1/2 d) 1

16. Find the value of $\int (x^2+3x-9)^7(2x+3)dx$

a) $\frac{1}{6}(x^2+5x-9)^6+k$

b) $7(x^2+5x-9)^6(2x+3)+k$

c) $8(x^2+5x-9)^8(2x+30)+k$

d) $\frac{1}{8}(x^2+3x-9)^8+k$

17. Evaluate $\int_2^{\pi}(sec^2x-\tan^2 x)dx$

a) $\pi/2$ b) $\pi-2$ c) $\pi/3$ d) $\pi+2$

18. Evaluate $\int \frac{x^4-1}{x^2-1}dx$

a) $\frac{x^3}{2}+x+c$ b) $\frac{x^4}{4}-\frac{x^2}{2}+c$ c) $\frac{x^3}{3}-x+c$

d) $\frac{x^3}{3}+x+c$

19. Evaluate $\int \frac{1}{16+x^2}dx$

a) $\frac{1}{4}\tan^{-1}\frac{x}{4}$ b) $4\tan^{-1}\frac{x}{4}+c$ c) $\frac{1}{4}\tan^{-1}\frac{4}{x}+c$

d) $\frac{1}{4}\tan^{-1}\frac{x}{5}+c$

20. Evaluate $\int 3e^{\sin\theta}\cos\theta\, d\theta$ a) $3e^{\cos\theta}+c$

b) $\frac{1}{3}e^{\cos\theta}+c$ c) $3e^{\sin\theta}+c$ d) $e^{3\sin\theta}+c$

21. Evaluate $\int \cos^5 x \sin^4 x\, dx$

a) $\frac{1}{5}\cos^5 x-\frac{2}{7}\sin^7 x+\frac{1}{9}\cos^9 x+c$

b) $\frac{1}{5}\sin^5 x-\frac{2}{7}\sin^7 x+\frac{1}{9}\sin^9 x+c$

c) $\cos^5 x-\cos^7 x+\cos^{11} x+c$

d) $\cos^5 x(1-\cos^2 x)^2+c$

22. Evaluate $\int e^{2x+5}dx$ a) $\frac{1}{2}e^{2x+5}+c$

b) $e^{x^2+5x}+c$ c) $e^{2x+5}+c$ d) $2e^{2x+5}+c$

23. Evaluate $\int(\sec x\tan x\sqrt{1+\sec x})dx$

a) $\frac{2}{3}\sqrt{(1+\sec x)^3}+c$ b) $\frac{2}{3}\sqrt{1+\sec x}+c$

c) $2\sqrt{(1+\sec x)^3}+c$ d) $\frac{2}{3}\sqrt{(1+\tan x)^3}+c$

24. Evaluate $\int \frac{\sin^3 x}{\cos^2 x}dx$

a) $\frac{1}{\cos x}+\cos x$ b) $-\frac{1}{\cos x}-\cos x$

d) $\sin x-\frac{1}{\cos x}$ d) $\cos x-\frac{1}{\sin x}$

25. A student blows a balloon and its volume increases at a rate of $\pi(20-t^2)cm^3/s$. If the initial volume is $0cm^3$, find the volume of the balloon after 2 seconds

a) 37.00π b) 37.33π c) 40.00π d) 43.67π

26. Find the area under the curve $y=2x^2+3x$ between the ordinates $x=1$ and $x=2$

a) $9\frac{1}{6}$ b) $5\frac{1}{3}$ c) $4\frac{1}{2}$ d) 6

27. What is the area between the line $y=x$ and the curve $y=4x^2$

a) 1/71 b) -2/37 c) -1/96 d) 3/10

28. What is the volume generated by rotating the curve $y=x+1$ from $x=1$ to $x=2$ completely round the $x-axis$

a) $\frac{7}{3}\pi\ cubic\ units$ b) $\frac{9}{13}\pi\ cubic\ units$

c) $\frac{19}{3}\pi\ cubic\ units$ d) $\frac{3}{7}\pi\ cubic\ units$

29. If $y=x(x^4+x^2+1)$, evaluate $\int_{-1}^{1} ydx$

a) 11/12 b) 11/16 c) 5/6 d) 0

30. Evaluate $\int_{\pi/4}^{\pi}(\sin x-\cos x)dx$

a) $\sqrt{2}+1$ b) $\sqrt{2}-1$ c) $-\sqrt{2}-1$ d) $1-\sqrt{2}$

31. Find the value of $\int_0^{\pi}\frac{\cos^2\theta-1}{\sin^2\theta}d\theta$

a) π b) $\pi/2$ c) $-\pi/2$ d) $-\pi$

32. Evaluate $\int x^3(1-x^4)^5dx$

a) $-\frac{1}{24}(1-x^4)^6+c$ b) $\frac{1}{12}(1-x^4)^5+c$

c) $-\frac{1}{14}(1-x^3)^6+c$ d) $\frac{1}{12}(1-x^5)^6+c$

33. Evaluate $\int \frac{2}{(t+1)^6}dt$ a) $\frac{-2}{7(t+1)^7}+c$

b) $\frac{-2}{5(t+1)^5}+c$ c) $\frac{-1}{6(t+1)^5}+c$ d) $\frac{2}{5(t+1)^7}+c$

34. Evaluate $\int \frac{\tan^{-1}x}{1+x^2}dx$

a) $\frac{1}{2}(1+x^2)^2+c$ b) $\frac{1}{2}(tan^{-1}x)^2+c$

c) $(tan^{-1}x)^3+c$ d) $\frac{1}{2}(\tan x)^2+c$

35. Evaluate $\int e^x\sin(e^x)dx$

a) $e^{x^2}\sin(e^{x^2})+c$ b) $-\cos(e^x)+c$

c) $\cos(e^{x^2})+c$ d) $2\sin(e^{2x})+c$

36. Evaluate $\int \frac{dx}{x\ln x}dx$ a) $\ln|\ln x|+c$ b) $\ln x+c$

c) $\frac{1}{\ln}+c$ d) $\ln|x^2|+c$

37. Evaluate $\int \frac{(ax+b)dx}{\sqrt{ax^2+2bx+}}$

a) $\frac{1}{\sqrt{ax^2+2bx+c}}+k$ b) $\sqrt{ax^2+2bx+c}^3+k$

c) $\sqrt{ax^2+2bx+c}+k$ d) $ax^2+2bx+c+$

38. Evaluate $\int \cos^7 x\,dx$

a) $\sin x+\sin^2 x+\frac{1}{3}\sin^3 x+\sin^4 x+c$

b) $\cos x+\sin^3 x+\frac{1}{3}\sin^5 x+\cos^7 x+c$

c) $\sin x-\sin^3 x+\frac{3}{5}\sin^5 x-\frac{1}{7}\sin^7 x+c$

d) $\sin x+\sin^2 x+\frac{1}{3}\sin^3 x+\sin^4 x+c$

39. Evaluate $\int \frac{x^2}{(x+1)}dx$ a) $\frac{x^2}{2}-1+\ln|x+1|+c$

b) $x-1+\frac{1}{x+1}+c$ c) $\frac{x^2}{2}-x+\ln|x+1|+c$

d) $\frac{(x+1)^2}{2}-2(x+1)+\ln|x+1|+c$

40. Evaluate $\int \ln x\,dx$

a) $\frac{1}{x}+c$ b)$x\ln x+c$ c) $x\ln x-x+c$

d) $\ln\left(\frac{1}{x}\right)+c$

SOLUTION

1. Let's first deal with the integration before considering the upper and lower limits.

$\int (x-1)^2\,dx$

$Let\ u=x-1 \implies \frac{du}{dx}=1 \implies dx=du$

$\int (x-1)^2\,dx=\int u^2\,du=\frac{u^3}{3}+c$

$\int (x-1)^2\,dx=\frac{(x-1)^3}{3}+c$

$hence, \int_1^4 (x-1)^2=\frac{(4-1)^3}{3}-\frac{(1-1)^3}{3}$

$\int_1^4 (x-1)^2=\frac{3^3}{3}-\frac{0^3}{3}=9 \ldots\ldots\ldots\ldots\ldots a$

2. Integrate the expression first

$\int (2x-1)dx=\frac{2x^2}{2}-x$

But the question says

$\left[\frac{2x^2}{2}-x\right]_2^k=4$

$\left(\frac{2(k)^2}{2}-k\right)-\left(\frac{2(2)^2}{2}-2\right)=4$

$(k^2-k)-(4-2)=4$

$k^2-k-2-4=0$

$k^2-k-6=0$

Solving the quadratic equation,

$k=3\ or\ k=-2$

The value of k is 3 since the upper limit of an integral must exceed the lower limit $\ldots\ldots\ldots b$

3. Factorize first

$\int_0^2 \left(\frac{x^4+1}{x^2}\right)dx=\int_0^2 \left(\frac{x^4}{x^2}+\frac{1}{x^2}\right)dx$

$\int_0^2 (x^2+x^{-2})\,dx=\left[\frac{x^3}{3}-x^{-1}\right]_0^2$

$=\left(\frac{(2)^3}{3}-2^{-1}\right)-\left(\frac{0^3}{3}-0^{-1}\right)$

$=\frac{8}{3}-\frac{1}{2}=\frac{13}{6}=2\frac{1}{6}\ldots\ldots\ldots\ldots a$

4. $\int (x^2+1)^{1/2}\,xdx$

$Let=u=\ ^2+1 \implies \frac{du}{dx}=2x \implies dx=\frac{du}{2x}$

Hence, we have;

$\int \left(u^{\frac{1}{2}}\right)\cancel{x}\cdot\frac{du}{2\cancel{x}}=\frac{1}{2}\int u^{\frac{1}{2}}\,du$

$=\frac{\frac{1}{2}\left(u^{\frac{1}{2}+1}\right)}{\frac{1}{2}+1}=\frac{\frac{1}{2}\left(u^{\frac{3}{2}}\right)}{\frac{3}{2}}=\frac{2}{3}\times\frac{1}{2}(x^2+1)^{\frac{3}{2}}$

$=\frac{1}{3}(x^2+1)^{\frac{3}{2}}+k \ldots\ldots\ldots\ldots\ldots a$

5. $\int \cos^5 x\sin x\,dx$

We've gotta make substitution

$Let\ u=\cos x$

Note: your substitute should be the trig. identity with the highest power. That is the reason I picked cos x

$\frac{du}{dx} = -\sin x \implies dx = -\frac{du}{\sin x}$

$\int \cos^5 x \sin x \, dx = \int (u^5) \cancel{\sin x} \left(-\frac{du}{\cancel{\sin x}}\right)$

$= -\int u^5 du = -\frac{u^6}{6} + c$

$= \frac{-\cos^6 x}{6} + c \ldots\ldots\ldots\ldots d$

6. Gradient means dy/dx

$i.e. \frac{dy}{dx} = 2x + 5 \implies dy \quad (2x+5)dx$

$\int dy = \int (2x+5)dx$

$y = \frac{2x^2}{2} + 5x + c$

$y = x^2 + 5x + c$

At point $(-2, -1)$

$-1 = (-2)^2 + 5(-2) + c$

$-1 = 4 - 10 + c$

$c = 5$

The relationship between x and y is now;

$y = x^2 + 5x + 5$

$\therefore when\ x = -4, \quad y = (-4)^2 + 5(-4) + 5$

$y = 16 - 20 + 5 = 1 \ldots\ldots\ldots\ldots\ldots b$

7. You have to first resolve into partial fraction

$\int \frac{x}{x^2 - 3x + 2} dx = \int \left(\frac{2}{x-2} - \frac{1}{x-1}\right) dx$

$= 2\ln(x-2) - \ln(x-1) + k$

$= 2\log_e(x-2) - \log_e(x-1) + k$

$= \log_e(x-2)^2 - \log_e(x-1) + k$

From logarithm rule;

$\log A - \log B = \log \frac{A}{B}$

Hence, we have;

$\log_e \left[\frac{(x-2)^2}{(x-1)}\right] + k \ldots\ldots\ldots\ldots\ldots\ldots\ldots d$

8. Gradient means dy/dx

$i.e. \frac{dy}{dx} = 2x + 5 \implies dy = (2x+5)dx$

$\int dy = \int (2x+5)dx$

$y = \frac{2x^2}{2} + 5x + c$

$y = x^2 + 5x + c$

At point $(3, -1)$

$-1 = (3)^2 + 5(3) + c$

$-1 = 9 + 15 + c$

$c = -25$

The relationship between x and y is now;

$y = x^2 + 5x - 25 \ldots\ldots\ldots\ldots\ldots b$

9. Hope you've not forgotten how we go about this

$A = \int_a^b y dx$

$A = \int_0^3 2\sqrt{x} dx = 2\int_0^3 2x^{\frac{1}{2}} dx$

$A = 2\left[\frac{x^{\left(\frac{1}{2}+1\right)}}{\left(\frac{1}{2}+1\right)}\right]_0^3 = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$

$A = 2 \times \frac{2}{3}\left[3^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{4}{3}\left(\sqrt{3}\right)^3 = \frac{4}{\cancel{3}}\left(\cancel{3}\sqrt{3}\right)$

$A = 4\sqrt{3} \ldots\ldots\ldots\ldots\ldots\ldots a$

10. $v = \pi \int y^2 \, dx$

$v = \pi \int_0^3 (x^2)^2 dx = \pi \int_0^3 x^4 dx$

$v = \pi \left[\frac{x^5}{5}\right]_0^3 = \pi \left[\frac{3^5}{5} - \frac{0^5}{5}\right] = \frac{243\pi}{5} \ldots\ldots\ldots\ldots d$

11. $\frac{dy}{dx} = 4x - 3 \implies dy = (4x-3)dx$

$\int dy = \int (4x-3)dx$

$y = \frac{4x^2}{2} - 3x + c$

$y = 2x^2 - 3x + c$

When x=2, y=5

$5 = 2(2)^2 - 3(2) + c$

$c = 3$

The equation is $y = 2x^2 - 3x + 3 \ldots\ldots\ldots\ldots\ldots d$

12. $\frac{dy}{dx} = 2 - x \implies dy \quad (2-x)dx$

$y = 2x - \frac{x^2}{2} + c$

At the origin , $x = 0, y = 0$

$0 = 2(0) - \frac{0^2}{2} + c$

$c = 0$

The equation becomes $y = 2x - \frac{x^2}{2}$

When it crosses $x - axis$, $y = 0$

$i.e.\ \ 0 = 2x - \frac{x^2}{2}$

$\frac{x^2}{2} = 2x \quad \Rightarrow \quad x = 4$

Hence, the co-ordinate is $(4, 0)$ ……. d

13. $y = 1 + x - 2x^2$

$A = \int y dx = \int_0^1 (1 + x - 2x^2) dx$

$A = \left[x + \frac{x^2}{2} - \frac{2x^3}{3}\right]_0^1$

$A = 1 + \frac{1^2}{2} - \frac{2(1)^3}{3} = 1 + \frac{1}{2} - \frac{2}{3} = \frac{5}{6}$ ……. a

14. Let us split the question

$\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$

First evaluate $\int \frac{1}{1+x^2} dx$

From the table of standard integral I gave you,

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

Comparing it to the question we have now,

$a = 1$

$\therefore \int \frac{1}{1+x^2} dx \quad \frac{1}{1} \tan^{-1} \frac{x}{1} + c = \tan^{-1} x + c$

Secondly, $\int \frac{x}{1+x^2} dx$

Note; if I differentiate the denominator, I will have the same power of x in the numerator

$\therefore \int \frac{x}{1+x^2} dx \quad \frac{1}{2} \ln(1 + x^2) + c$

Combining the two, we have;

$\tan^{-1} x + \frac{1}{2} \ln(1 + x^2) + c$ …… a

15. Substitution is needed

$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 2 \cos 2x \, dx$

let $u = 2x, \ \frac{du}{dx} = 2 \quad \Rightarrow \quad dx = \frac{du}{2}$

We now have, $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cancel{2} \cos u \frac{du}{\cancel{2}} = \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos u \, du$

$= \sin u = \sin 2x$

Using the limits,

$\sin 2\left(\frac{\pi}{4}\right) - \sin 2\left(\frac{\pi}{12}\right) = \sin \frac{\pi}{2} - \sin \frac{\pi}{6}$

$= \sin\left(\frac{180}{2}\right) - \sin\left(\frac{180}{6}\right)$

$= \sin 90 - \sin 30 = 1 - \frac{1}{2} = \frac{1}{2}$ …… c

16. $\int (x^2 + 3x - 9)^7 (2x + 3) dx$

Let $u = x^2 + 3x - 9$

$\frac{du}{dx} = 2x + 3 \quad \Rightarrow \quad dx = \frac{du}{2x+3}$

We now have;

$\int u^7 \cancel{(2x+3)} \cdot \frac{du}{\cancel{(2x+3)}} = \int u^7 du \quad \frac{u^8}{8} + k$

Substitute back the value of u

$= \frac{1}{8}(x^2 + 3x - 9)^8 + k$ …… d

17. Very simple question looking sooooo big

$\int_2^{\pi} (sec^2 x - \tan^2 x) dx$

From trig. identity, $\tan^2 \theta + 1 = \sec^2 \theta$

Hence, $\sec^2 x - \tan^2 x = 1$

$\therefore$ the question becomes

$\int_2^{\pi} 1 dx = [x]_2^{\pi} = \pi - 2$ …… b

18. Just simplify first

$\int \frac{x^4 - 1}{x^2 - 1} dx = \int \frac{\cancel{(x^2 - 1)}(x^2 + 1)}{\cancel{x^2 - 1}} dx$

$= \int (x^2 - 1) dx = \frac{x^3}{3} + x + c$ …… d

19. You should remember how to go about this o

$\int \frac{1}{16 + x^2} dx = \int \frac{1}{4^2 + x^2} dx$

This has to do with tan

$\int \frac{1}{4^2 + x^2} dx = \frac{1}{4} \tan^{-1} \frac{x}{4} + c$ ……. d

20. $\int 3e^{\sin\theta} \cos\theta \, d\theta = 3 \int e^{\sin\theta} \cos\theta \, d$

Hope you've not forgotten our substitution rule

Let $u = \sin\theta \quad \frac{du}{d\theta} = \cos\theta \quad \Rightarrow \quad d\theta = \frac{du}{\cos\theta}$

The question becomes;

$3\int e^u \cancel{\cos\theta}\cdot\frac{du}{\cancel{\cos\theta}} = 3\int e^u\,du$

$= 3e^u + c = 3e^{\sin\theta} + c \ldots\ldots\ldots\ldots\ldots c$

21. $\int \cos^5 x \sin^4 x\, d$

The function with highest power is $\cos x$

Hence, split the function

$\int \cos^5 x \sin^4 x\, dx = \int (\cos^2 x)^2 \cos x \sin^4 x\, dx$

$\int (1-\sin^2 x)^2 \cos x \sin^4 x\, d$

$Hence, let\ \ u = \sin x\ \ \frac{du}{dx} = \cos x\ \Rightarrow\ dx\quad \frac{du}{\cos x}$

The question becomes

$\int (1-u^2)^2 \cdot \cancel{\cos x}\cdot u^4 \cdot \frac{du}{\cancel{\cos x}}$

$= \int (1-2u^2+u^4)(u^4)du$

$= \int (u^4 - 2u^6 + u^8)\,du$

$= \frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9} + c$

$= \frac{1}{5}\sin^5 x - \frac{2}{7}(\sin^7 x) + \frac{1}{9}\sin^9 x + c \ldots\ldots\ldots\ldots b$

22. $\int e^{2x+5}dx$

$let\ \ u = 2x+5\ \ \frac{du}{dx} = 2\quad\Rightarrow\quad dx = \frac{du}{2}$

The question becomes

$\int e^u \cdot \frac{du}{2} = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c$

$= \frac{1}{2}e^{2x+5} + c \ldots\ldots\ldots\ldots\ldots a$

23. $\int(\sec x \tan x\sqrt{1+\sec x})dx$

$let\ \ u = 1+\sec x\ \Rightarrow\ \frac{du}{dx} = \sec x\tan x$

$\Rightarrow\ dx = \frac{du}{\sec x\tan x}$

The question now becomes;

$\int \cancel{\sec x\tan x}\cdot\sqrt{u}\cdot\frac{du}{\cancel{\sec x\tan x}} = \int u^{\frac{1}{2}}du$

$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}(1+\sec x)^{\frac{3}{2}} + c$

$= \frac{2}{3}\sqrt{(1+\sec x)^3} + c \ldots\ldots\ldots\ldots a$

24. Substitution is involved

$\int \frac{\sin^3 x}{\cos^2 x}dx$

$Let\ \ u = \cos x, \frac{du}{dx} = -\sin x\quad dx = -\frac{du}{\sin x}$

The question becomes

$\int \frac{\sin^3 x}{u^2}\cdot -\frac{du}{\sin x} = \int \frac{-\sin^2 x}{u^2}du$

$= -\int \frac{1-\cos^2 x}{u^2}du = -\int \frac{1-u^2}{u^2}du$

$= -\int\left(\frac{1}{u^2} - \frac{u^2}{u^2}\right)du = -\quad\left(\frac{1}{u^2} - 1\right)du$

$= -\int (u^{-2}-1)du = -\left(\frac{u^{-1}}{-1}\right) + u + c$

$= u^{-1} + u + c$

$= \cos^{-1}x + \cos x + c$

$= \frac{1}{\cos x} + \cos x + c \ldots\ldots\ldots a$

25. Increase rate of volume $= \pi(20-t^2)cm^3/s$

It simply means that;

$\frac{dv}{dt} = \pi(20-t^2)\quad\Rightarrow\quad dv = \pi(20-t^2)dt$

To get volume, just integrate;

$\int dv = \int\ (20-t^2)dt$

$v = \pi\left(20t - \frac{t^3}{3} + c\right)$

From the given information,

$when\ t = 0, v = 0$

$0 = \pi\left(20(0) - \frac{(0)^3}{3} + c\right)$

$0 = \pi c\quad\Rightarrow\quad c = 0$

After 2 seconds,

$v = \pi\left(20(2) - \frac{(2)^3}{3} + c\right)$

$v = \pi\left(40 - \frac{8}{3}\right) = 37.33\pi \ldots\ldots\ldots b$

26. $A = \int y dx$

$A = \int_1^2 (2x^2+3x)dx$

$A = \left[\frac{2x^3}{3} + \frac{3x^2}{2}\right]_1^2$

$$A = \left[\frac{2(2)^3}{3} + \frac{3(2)^2}{2}\right] - \left[\frac{2(1)^3}{3} + \frac{3(1)^2}{2}\right]$$

$$A = \left(\frac{16}{3} + \frac{12}{2}\right) - \left(\frac{2}{3} + \frac{3}{2}\right)$$

$$A = \frac{34}{3} - \frac{13}{6} = \frac{55}{6} = 19\frac{1}{6} \ldots\ldots\ldots\ldots a$$

27. $A = \int (y_2 - y_1)dx$

First equate y in the two cases to get values of x

$$x = 4x^2$$

$$4x^2 - x = 0$$

$$x(4x - 1) = 0$$

$$x = 0 \quad or \quad x = \frac{1}{4}$$

$$\therefore \int_0^{\frac{1}{4}} (4x^2 - x)dx$$

$$= \left[\frac{4x^3}{3} - \frac{x^2}{2}\right]_0^{\frac{1}{4}}$$

$$= \left[\frac{4\left(\frac{1}{4}\right)^3}{3} - \frac{\left(\frac{1}{4}\right)^2}{2}\right]$$

$$= \frac{1}{48} - \frac{1}{32} = -\frac{1}{96} \ldots\ldots\ldots c$$

28. $y = x + 1 \quad from\ x = 1\ to\ x = 2$

Since it is round the x-axis;

$$V = \pi \int y^2 dx$$

$$V = \pi \int (x + 1)^2 dx$$

$$V = \pi \int_1^2 (x^2 + 2x + 1)dx$$

$$V = \pi \left[\frac{x^3}{3} + \frac{2x^2}{2} + x\right]_1^2$$

$$V = \pi \left[\frac{x^3}{3} + x^2 + x\right]_1^2$$

$$V = \pi \left[\left(\frac{(2^3)}{3} + 2^2 + 2\right) - \left(\frac{1^3}{3} + 1^2 + 1\right)\right]$$

$$V = \pi \left[\left(\frac{8}{3} + 4 + 2\right) - \left(\frac{1}{3} + 1 + 1\right)\right]$$

$$V = \frac{19}{3}\pi \ldots\ldots\ldots c$$

29. $y = x(x^4 + x^2 + 1)$

$$y = x^5 + x^3 + x$$

$$\int y = \left[\frac{x^6}{6} + \frac{x^4}{4} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$\int y = \left(\frac{1}{6} + \frac{1}{4} + \frac{1}{2}\right) - \left(\frac{1}{6} + \frac{1}{2} + \frac{1}{2}\right)$$

$$\int y = 0 \ldots\ldots\ldots\ldots\ldots\ldots d$$

30. Very simple, but be careful

$$\int_{\frac{\pi}{4}}^{\pi} (\sin x - \cos x)dx = [-\cos x - \sin x]_{\pi/4}^{\pi}$$

$$= -[\cos x + \sin x]_{\frac{\pi}{4}}^{\pi}$$

$$= -\left[(\cos\pi + \sin\pi) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -[(\cos 180 + \sin 180) - (\cos 45 + \sin 45)]$$

$$= -\left[(-1 + 0) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right]$$

$$= -\left[-1 - \frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right]$$

$$= 1 + \sqrt{2} \ldots\ldots a$$

31. Just simplify from trig. identity

$$\int_0^{\pi} \frac{\cos^2\theta - 1}{\sin^2\theta} d\theta = \int_0^{\pi} \frac{(1 - \sin^2\theta) - 1}{\sin^2\theta} d\theta$$

$$= \int_0^{\pi} \frac{-\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}} d\theta = -\int_0^{\pi} d\theta$$

$$= -[\theta]_0^{\pi} = -\pi \ldots\ldots\ldots\ldots\ldots d$$

32. $\int x^3(1 - x^4)^5 dx$

$$Let\ u = 1 - x^4 \quad \frac{du}{dx} = -4x^3 \implies dx = -\frac{du}{4x^3}$$

The question becomes;

$$\int \cancel{x^3}(u)^5 \cdot -\frac{du}{4\cancel{x^3}} = -\frac{1}{4}\int u^5 du$$

$$= -\frac{1}{4}\left(\frac{u^6}{6}\right) + c = -\frac{1}{24}(1 - x^4)^6 + c \ldots\ldots a$$

33. $\int \frac{2}{(t+1)^6} dt$

$$let\ u = t + 1, \frac{du}{dt} = 1 \implies dt = du$$

The question becomes;

$$\int \frac{2}{u^6} du = 2\int u^{-6} du = 2\left(\frac{u^{-5}}{-5}\right) + c$$

$$= -\frac{2}{5}\left(\frac{1}{u^5}\right) + c \quad = \frac{-2}{5(t+1)^5} + c \ldots\ldots\ldots b$$

34. Substitution is involved;

$$\int \frac{\tan^{-1} x}{1+x^2}dx$$

$Let\ u = \tan^{-1} x, \frac{du}{dx} = \frac{1}{1+x^2}$

$\Longrightarrow\ dx = (1+x^2)d$

The question becomes;

$$\int \frac{u}{\cancel{1+x^2}} \cdot (\cancel{1+x^2})du =$$

$$\int udu = \frac{u^2}{2} + c = \frac{1}{2}(\tan^{-1} x)^2 + c$$

35. $\int e^x \sin(e^x)dx$

$Let\ u = e^x,\ \frac{du}{dx} = e^x,\quad dx = \frac{du}{e^x}$

The question becomes;

$$\int \cancel{e^x}\sin(u) \cdot \frac{du}{\cancel{e^x}} = \int \sin u\, du$$

$= -\cos u + c$

$= -\cos(e^x) + c \ldots\ldots\ldots . b$

36. Hope you remember our order of substitution,

$Let\ u = \ln x, \frac{du}{dx} = \frac{1}{x} \quad \Longrightarrow \quad dx = xdu$

The question becomes;

$$\int \frac{\cancel{x}d}{\cancel{x}(u)} = \int \frac{1}{u}du = \ln u + c$$

Since $u = \ln x$

$$\therefore \int \frac{dx}{x \ln x} = \ln|\ln x| + c \ldots\ldots\ldots . . a$$

37. $\int \frac{(ax+b)dx}{\sqrt{ax^2+2bx+c}}$

$Let\ u = ax^2 + 2bx + c \quad \frac{du}{dx} = 2ax + 2b$

$dx \quad \frac{du}{2ax+2b} = \frac{du}{2(ax+b)}$

The question becomes;

$$\int \frac{\cancel{(ax+b)}}{\sqrt{u}} \cdot \frac{du}{2\cancel{(ax+b)}} = \frac{1}{2}\int u^{-\frac{1}{2}}du$$

$$= \frac{1}{2}\left(\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + k = \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{1/_2}\right) + k$$

$= u^{\frac{1}{2}} + c = (ax^2 + 2bx + c)^{\frac{1}{2}} + k$

$= \sqrt{ax^2 + 2bx + c} + k \ldots\ldots\ldots\ldots\ldots\ldots\ldots d$

38. $\int \cos^7 x\, dx = \int (\cos^6 x)(\cos x)dx$

$= \int (\cos^2 x)^3 (\cos x)dx$

$= \int (1 - \sin^2 x)^3 \cos x\, dx$

$Let\ u = \sin x, \frac{du}{dx} = \cos x \quad dx = \frac{du}{\cos x}$

We now have;

$$\int (1-u^2)^3 \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$$

$= \int (1 - 3u^2 + 3u^4 - u^6)du$

$= u - u^3 + \frac{3u^5}{5} - \frac{u^7}{7} + c$

$= \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c \ldots\ldots\ldots c$

39. $\int \frac{x^2}{(x+1)}dx$

$Let\ u = x + 1,\ \frac{du}{dx} = 1 \quad dx = du$

$if\ u = x + 1, then\ x = u - 1.$

Hence, the expression becomes

$$\int \frac{(u-1)^2}{u}du = \int \frac{u^2 - 2u + 1}{u}du$$

$$= \int \left(\frac{u^2}{u} - \frac{2u}{u} + \frac{1}{u}\right)du$$

$$= \int \left(u - 2 + \frac{1}{u}\right)du$$

$$= \frac{u^2}{2} - 2u + \ln u + c$$

$= \frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + c \ldots\ldots . d$

40. $\int \ln x\, dx$

Recall integration by part;

$$\int udv = uv - \int vdu$$

$Let\ u = \ln x,\ \frac{du}{dx} = \frac{1}{x} \ \Longrightarrow\ du = \frac{dx}{x}$

$Also\ \ dv = dx \ \Longrightarrow\ \int dv = \int dx \ \Longrightarrow\ v = x$

$$\therefore \int \ln x\, dx = \ln x\,(x) - \int \cancel{x} \cdot \frac{dx}{\cancel{x}}$$

$= x \ln x - x + c \ldots\ldots\ldots\ldots c$

CO-RDINATE GEOMETRY

AND CONIC SECTION

Midpoint of a line

For a line joining the two points, (x_1, y_1) and (x_2, y_2), the co-ordinate of the midpoint, $M(x, y)$ will be gotten by;

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Gradient of a line

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Or $y - y_1 = m(x - x_1)$

When you are given the equation of a straight line, then the gradient of the line is the value attached to x when y has a coefficient of 1.

i.e. $y = mx + c$

m is the gradient while c is the y-intercept

Distance between two points

Given two $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the distance between P and Q is given as;

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Parallel lines

If two lines are parallel, their gradient will be equal. $i.e. m_1 = m_2$

Perpendicular lines

If two lines are perpendicular, the product or their gradients is equal to -1.

i. e. $m_1 m_2 = -1$ or $m_1 = -\frac{1}{m_2}$

Angle between two lines

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \; where \; m_1 > m_2$$

Circle

The general form of a circle is expressed as;

$$(x-a)^2+(y-b)^2=r^2$$

Or $x^2+y^2-2ax-2by+a^2+b^2-r^2=0$

Put $g=-a,\ f=-b\ and\ c=a^2+b^2-r^2$

We then have,

$x^2+y^2+2gx+2fy+c=0$

Where;

a, b are the centre of the circle and

r is the radius of the circle.

A circle whose centre is at the origin $(0, 0)$ will have the equation;

$$x^2+y^2=r^2$$

Note:

i. The coefficient of x^2 and y^2 must be the same
ii. The highest possible power in the equation is 2
iii. There is no term in xy

Parabola

- Eccentricity, e=1
- The equation is $y^2=4ax$when it is symmetric about x-axis (i.e. when it has equal y-coordinate)
- The equation is $x^2=4ay$ when it is symmetric about y-axis (i.e. when it has equal x-coordinate)

Ellipse

- Eccentricity, e<1
- The equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
- And $b^2=a^2(1-e^2)$

Hyperbola

- Eccentricity, e> 1
- The equation is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- And $b^2=a^2(e^2-1)$

QUESTIONS

1) What is the gradient of the line joining the points with coordinates (5, -1) and (-3, 7)
   a) 3 b) 1/3 c) -3/4 d) -1
2) What is the distance between the points (3, -2) and (8, 10) a) 5 b) 12 c) 13 d) 17
3) The midpoint of the line joining the points x(4, 2) and y(-5, 0) is
   a) (9, 2) b) (-1, 2) c) (1/2, 1) d) (-1/2, 1)
4) Given that the line $ax+4y-5=0$ is perpendicular to the line $4x-2y+6=0$, find the value of a a) -2 b) -1/2 c) 1/2 d)2
5) The equation of a straight line with gradient 1/3 which passes through the point (1, 2) is
   a) $2x-y-1=0$ b) $x-3y+5=0$
   c) $2x-3y+1=0$ d) $3x-2y+1=0$
6) What is the value of p if the gradient of the line joining $(-1, p)\ and\ (p, 4)$ is 2/3?
   a) -2 b) -1 c) 1 d) 2
7) If the lines $3y=4x-1$ and $qy=x+3$ are parallel to each other, the value of q is?
   a) -4/3 b) -3/4 c) 4/3 d) 3/4
8) Find the locus of a point that is equidistance from the points (1,2) and (3,8)
   a) $y=\frac{1}{3}(17-x)$ b) $y=\frac{1}{3}(x+13)$
   c) $y=\frac{1}{3}(x+8)$ d) $y=\frac{1}{3}(11-x)$
9) What is the distance between points (3, -2) and (8, 10) a) 5 b) 12 c) 13 d) 17
10) What is the equation of the line which makes intercepts of 2 and 3 on the x and y axes respectively a) $2x+3y=1$ b) $3x+2y=1$
    c) $3x+2x=6$ d) $3x-2y=6$
11) The equation of a straight line with gradient 1/3 which passes through the point (1, 2) is ___

a) $2x - y - 1 = 0$ b) $x - 3y + 5 = 0$
c) $2x - 3y + 1 = 0$ d) $3x - 2y + 1 = 0$

12) Find the equation of the line through the point $(1, 2)$ and parallel to the line $4x - y = 2$
a) $y + 4x - 6 = 0$ b) $y - 4x + 2 = 0$
c) $4x + y - 2 = 0$ d) $4y - x - 7 = 0$

13) Find the equation of the line perpendicular to $2y + 3x - 4 = 0$ and passes through the point (2, -5)
a) $2x - 3y + 11 = 0$ b) $3x - 2y - 16 = 0$
c) $x - 3y - 17 = 0$ d) $2x - 3y - 19 = 0$

14) If the line $ax + 4y - 5 = 0$ is perpendicular to the line $4x - 2y + 6 = 0$, find the value of a
a) -2 b) -1/2 c) 1/2 d) 2

15) The midpoint of the line joining the points $x(4, 2)$ and $y(-5, 0)$ is
a) $(9, 2)$ b) $\left(\frac{1}{2}, 1\right)$ c) $(-1, 2)$ d) $\left(-\frac{1}{2}, 1\right)$

16) What is the value of b if the gradient of line joining (-1, b) and (b, 4) is 2/3
a) -2 b) -1 c) 1 d) 2

17) $3y = 4x - 1$ and $ky = x + 3$ are equations of two straight lines. If the two lines are perpendicular to each other, find k
a) -4/3 b) -3/4 c) 3/4 d) 4/3

18) What is the value of k if the midpoint of the line joining $(1 - k, 4)$ and $(2, k + 1)$ is $(-k, k)$
a) -1 b) -2 c) -3 d) -4

19) The gradient of line joining $(n, 4)$ and $(1, 2)$ is 1/2. Find the value of n a) 3 b) -3 c) -5 d) 5

20) If (3, -4) is a point on the line $y = ax + 2$, find the value of a a) -1 b) 3 c) 2 d) 3/2

21) What is the angle of slope of the line joining points (4, -3) and (6, 8)
a) 120^0 b) 110^0 c) 85^0 d) 79.7^0

22) If the points (-3, 4), (k, -1) and (5, -6) are collinear, find the value of the constant k
a) -7 b) -5 c) 1 d) 9

23) Find the equation of the circle with centre $(-3, 4)$ and radius 4 units
a) $x^2 + y^2 + 6x - 8y - 21 = 0$
b) $x^2 + y^2 + 6x - 8y + 9 = 0$
c) $x^2 + y^2 - 6x + 8y + 9 = 0$
d) $x^2 + y^2 - 6x + 8y - 21 = 0$

24) The equation of a circle having the points $(2, 3)$ and $(-4, 5)$ as the ends of its diameter is
a) $x^2 + y^2 + 2x - 8y + 27 = 0$
b) $x^2 + y^2 + 2x - 8y + 13 = 0$
c) $x^2 + y^2 + 2x - 8y + 7 = 0$
d) $x^2 + y^2 + 2x - 8y - 3 = 0$

25) Which of the following does not represent a circle
a) $3x^2 + 3y^2 + 2xy - 7 = 0$
b) $2x^2 + 2y^2 + 7x + 3y - 1 = 0$
c) $3x^2 + 3y^2 - 5x + 4y - 5 = 0$
d) $x^2 + y^2 + 2x - 4y + 1 = 0$

26) Find the possible values of the constant m for which the curve $(m + 5)x^2 + (m^2 - 1)y^2 + 2x - 5y + 5 = 0$ is a circle
a) 2 and 3 b) 2 and -3 c) -2 and -3 d) -2 and 3

27) What is the equation of the circle, centre $(2, -3)$ which touches the x-axis
a) $x^2 + y^2 - 3x + 4y + 2 = 0$
b) $x^2 + y^2 + 2x - 4y + 1 = 0$
c) $x^2 + y^2 - 4x + 6y + 4 = 0$
d) $2x^2 + 2y^2 + 5x - 4y + 1 = 0$

28) Which of the following equations of conic represents a hyperbola
a) $x^2 + y^2 - 3x + 4y - 6 = 0$ b) $y^2 = -2x$
c) $\frac{x^2}{3} + \frac{y^2}{4} = 1$ d) $\frac{x^2}{5} - \frac{y^2}{9} = 1$

29) Find the equation of a parabola with vertex $(5, -2)$ and focus $(5, 2)$
a) $x^2 - 10x - 16y - 7 = 0$
b) $x^2 + 6x - 10y - 17 = 0$
c) $2x^2 + 16x - 10y + 17 = 0$
d) $2x^2 + x + 11y + 5 = 0$

30) Find the equation of the asymptotes of the hyperbola $4x^2 - 9y^2 = 36$ a) $y = \pm\frac{2}{3}x$ b) $y = \pm\frac{1}{3}x$ c) $y = \pm\frac{2}{9}x$ d) $y = \pm\frac{1}{2}x$

SOLUTIONS

1) Gradient, m

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - -1}{-3 - 5} = \frac{7 + 1}{-8}$

$m = -\frac{8}{8} = -1 \ldots\ldots\ldots\ldots .. d$

2) $Distance, d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(8 - 3)^2 + (10 - -2)^2}$

$d = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$

$d = \sqrt{169} = 13 \ldots\ldots\ldots\ldots . c$

3) Midpoint,

$(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$(\bar{x}, \bar{y}) = \left(\frac{4 + (-5)}{2}, \frac{2 + 0}{2}\right) = \left(-\frac{1}{2}, 1\right) \ldots\ldots . d$

4) For perpendicularity, $m_1 m_2 = -1$

For line $ax + 4y - 5 = 0, gradient, m = -\frac{a}{4}$

For line $4x - 2y + 6 = 0, grad, m = \frac{-4}{-2} = 2$

$hence, \frac{-a}{4} \times 2 = -1 \quad \Rightarrow \quad \frac{-a}{2} = -1$

$a = 2 \ldots\ldots\ldots\ldots . d$

5) The equation is $y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{3}(x - 1) \quad \Rightarrow \quad 3y - 6 = x - 1$

$x - 3y + 5 = 0 \ldots\ldots\ldots\ldots b$

6) Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(-1, p)\ and\ (p, 4)$

$\frac{4 - p}{p - (-1)} = \frac{2}{3}$

$\frac{4 - p}{p + 1} = \frac{2}{3}$

$12 - 3p = 2p + 2$

$-5p = -10$

$p = 2 \ldots\ldots\ldots\ldots . d$

7) For two lines to be parallel, $m_1 = m_2$ i.e. the gradient of the first line equals the gradient of the second.

$\therefore from\ 3y = 4x - 1$

Making y subject of formula

$y = \frac{4}{3}x - \frac{1}{3}$

$From\ qy = x + 3,$

$y = \frac{1}{q}x + \frac{3}{q}$

$recall, general\ equation\ of\ a\ line$

$y = mx +$

Hence, the gradients of the lines are

$m_1 = \frac{4}{3}; m_2 = \frac{1}{q}$

$since\ m_1 = m_2; \frac{4}{3} = \frac{1}{q}$

$q = \frac{3}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots d$

8) point that is equidistant from the points (1, 2) and (3, 8) say (x, y) will be exactly between the points i.e. distance between (1,2) and (x , y)is the same as distance between (x, y) and (3,8)

Hence,

$\sqrt{(y - 2)^2 + (x - 1)^2} = \sqrt{(8 - y)^2 + (3 - x)^2}$

$square\ both\ sides$

$(y - 2)^2 + (x - 1)^2 = (8 - y)^2 + (3 - x)^2$

$\therefore \quad \cancel{y^2} - 4y + 4 + \cancel{x^2} - 2x + 1$

$= 64 - 16y + \cancel{y^2} + 9 - 6x + \cancel{x^2}$

Collect like terms

$-4y + 16y - 2x + 6x - 68 = 0$

$12y + 4x - 68 = 0$

$3y + x - 17 = 0$

$3y = 17 - x$

$y = \frac{1}{3}(17 - x) \ldots\ldots\ldots a$

9) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(8 - 3)^2 + (10 - -2)^2}$

$d = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$

$d = \sqrt{169} = 13 \ldots\ldots c$

10) This is a case with double intercept. Hence, the equation will be;

$\frac{x}{a} + \frac{y}{b} = 1$

$We\ have\ , \quad \frac{x}{2} + \frac{y}{3} = 1 \quad \Rightarrow \quad \frac{3x + 2y}{6} = 1$

$3x + 2y = 6 \ldots\ldots\ldots . c$

11) $y - y_1 = m(x - x_1)$

$m = \frac{1}{3}, \ y_1 = 2, \ x_1 = 1$

The equation will be

$y - 2 = \frac{1}{3}(x - 1)$

$3y - 6 = x - 1$

$x - 3y + 5 = 0 \ldots\ldots\ldots b$

12) From the line $4x - y = 2$, we can rewrite it as $y = 4x - 2$. Hence, its gradient is 4.

If the two lines are parallel, there gradients will be the same.

Hence, the equation of the line through the point (1, 2) will be;

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 1$

$y - 4x + 2 = 0 \ldots\ldots\ldots b$

13) From the given line; $2y + 3x - 4 = 0$

$2y = -3x + 4$ (divide through by 2)

$y = -\frac{3}{2}x + 2$

Hence, the gradient of the line is $-3/2$

For perpendicular lines $m_1 m_2 = -1$

$m_2 = \frac{-1}{\left(-\frac{3}{2}\right)} = \frac{2}{3}$

Thus, the new line has a gradient of 2/3 and passes through the point (2, -5). Its equation will be;

$y - y_1 = m_2(x - x_1)$

$y - -5 = \frac{2}{3}(x - 2)$

$y + 5 = \frac{2}{3}(x - 2)$

$3y + 15 = 2x - 4$

$3y - 2x + 19 = 0 \ldots\ldots\ldots\ldots d$

14) Since the two lines are perpendicular, $m_1 m_2 = -1$

For the first line; $ax + 4y - 5 = 0$

$4y = -ax + 5$

$y = -\frac{ax}{4} + \frac{5}{4} \implies m_1 = -\frac{a}{4}$

For the second line; $4x - 2y + 6 = 0$

$2y = 4x + 6$

$y = \frac{4}{2}x + \frac{6}{2} \implies m_2 = 2$

$m_1 m_2 = -1$

$-\frac{a}{4} \times 2 = -1$

$a = 2$ ..... d

15) (4, 2) and (−5, 0)

$x_2 = -5, \ x_1 = 4, \ y_2 = 0, \ y_1 = 2$

$x = \frac{4 + (-5)}{2} = -\frac{1}{2}$

$y = \frac{2 + 0}{2} = 1$

Hence, the midpoint is $-\frac{1}{2} an \quad 1 \ldots\ldots\ldots d$

16) From the expression for gradient

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b - 4}{-1 - b}$

$\frac{b - 4}{-1 - b} = \frac{2}{3}$

$3b - 12 = -2 - 2b$

$5b = 10$

$b = 2 \ldots\ldots\ldots d$

17) First line $3y = 4x - 1$

$y = \frac{4}{3}x - \frac{1}{3} \implies m_1 = \frac{4}{3}$

Second line $ky = x + 3$

$y = \frac{x}{k} + \frac{3}{k} \implies m_2 = \frac{1}{k}$

Since the lines are perpendicular, $m_1 m_2 = -1$

$\frac{4}{3} \times \frac{1}{k} = -1$

$k = -\frac{4}{3} \ldots\ldots\ldots\ldots a$

18) Midpoint

$x = \frac{x_1 + x_2}{2}$

$\therefore \ \frac{1 - k + 2}{2} = -k$

$3 - k = -2k$

$k = -3 \ldots\ldots\ldots c$

19) Recall

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{1}{2} = \frac{2-4}{1-n}$

$1 - n = -4$

$n = 5 \ldots\ldots\ldots\ldots\ldots d$

20) Very simple, just substitute -3 for x and -4 for y

$-4 = a(-3) + 2$

$-4 - 2 = -3a$

$-3a = -6$

$a = 2$ c

21) Let's get the gradient first;

$m = \frac{8-3}{6-4} = \frac{8+3}{2} = \frac{11}{2} = 5.5$

$m = 5.5$

But $m = \tan\theta$

$\tan\theta = 5.5$

$\theta = \tan^{-1} 5.5 = 79.695^0$

$\theta = 79.7^0 \ldots\ldots\ldots e$

22) This means the points are on the same straight line. This implies that the gradient of the in between these points will be the same.

$\frac{-1-4}{k--3} = \frac{-6--1}{5-k}$

$\frac{-5}{k+3} = \frac{-5}{5-k}$

$5 - k = k + 3$

$2k = 2$

$k = 1 \ldots\ldots\ldots c$

23) From the equation of a circle;

$(x-a)^2 + (y-b)^2 = r^2$

$(x-3)^2 + (y-4)^2 = 4^2$

$x^2 + 6x + 9 + y^2 - 8y + 16 = 16$

$x^2 + y^2 + 6x - 8y + 9 = 0 \ldots\ldots b$

24) We can get the centre coordinates of the circle from the provided ends of the diameter

$Midpoint, (x,y) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$(x,y) = \frac{2 + (-4)}{2}, \frac{3+5}{2}$

$(x,y) = -\frac{2}{2}, \frac{8}{2} = (-1,4)$

Hence, the centre of the circle is $(-1,4)$.

The radius of the circle will be the distance between a point at the end and the centre.

That is between $(2,5)$ and $(-1,4)$

$r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$r^2 = (-1-2)^2 + (4-5)^2$

$r^2 = 9 + 1 = 10$

To get the equation of the circle,

$(x-a)^2 + (y-b)^2 = r^2$

$(x--1)^2 + (y-4)^2 = 10$

$x^2 + 2x + 1 + y^2 - 8y + 16 = 10$

$x^2 + y^2 + 2x - 8y + 7 = 0 \ldots\ldots\ldots\ldots c$

25) In the equation of a circle, there cannot be a factor of $xy \ldots\ldots\ldots\ldots a$

26) Just remember that for a circle, the coefficient of x^2 and y^2 must be the same

$\therefore m + 5 = m^2 - 1$

$m^2 - m - 6 = 0$

$m = -2 \; or \; 3 \ldots\ldots\ldots\ldots d$

27) If the circle touches the x-axis, it means that the x-axis is a tangent to this circle. Hence, with centre (2, -3), the radius will be equal to 3.

The equation of the circle will then be;

$(x-2)^2 + (y--3)^2 = 3^2$

$x^2 - 4x + 4 + y^2 + 6x + 9 = 9$

$x^2 + y^2 - 4x + 6y + 4 = 0 \ldots\ldots\ldots c$

28) Option a is a circle

Option b is a parabola

Option c is an ellipse

Option d is a hyperbola $\ldots\ldots\ldots\ldots d$

29) You will notice that the values for y in both points are constant, and the vertex is not (0, 0)

$a = (2--2) = 4$

The equation will be

$(x-5)^2 = 4(4)(y--2)$

$x^2 - 10x + 25 = 16y + 32$

$x^2 - 10x - 16y - 7 = 0 \ldots\ldots\ldots a$

30) $4x^2 - 9y^2 = 36$

Divide through by 36

$$\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

Comparing with the general equation;

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Longrightarrow a = 3 \; and \; b = 2$$

The equation of the asymptote is $y = \pm\frac{b}{a}$

$$y = \pm\frac{2}{3} \ldots\ldots\ldots a$$

STATISTICS AND PROBABILITY

MEASURE OF CENTRAL TENDENCY

- **Mean**
- **Median**
- **Mode**

Mean

This is simply the average of a set of data.

For ungrouped data

$$\bar{x} = \frac{\sum x}{N} \quad or \; A + \frac{\sum d}{N}$$

For grouped data

$$\bar{x} = \frac{\sum fx}{\sum f} \quad or \; A + \frac{\sum fd}{N}$$

Where $A = assume \quad mean$

$d = deviation \; of \; the \; assumed \; mean \; from \; the \; X$

Median

For ungrouped data

When the data is ungrouped, just arrange the data orderly and pick whichever number falls in the middle.

If two numbers are involved, add the numbers and divide by 2.

$$M_e = \left(\frac{n+1}{2}\right)^{th} or \left(\frac{\sum f + 1}{2}\right)^{th} value$$

For grouped data

$$Median = L_1 + \left(\frac{F_m - F_b}{f_m}\right) C$$

$Where$;

L_1 =lower class boundary of the median class

F_m = cumulative frequency of the median class

F_b =cumulative frequency of the class before the median class

f_m =frequency of the median class

C = class width

Mode

For ungrouped data

The mode is the value with the highest occurring frequency i.e. the number that appears most often.

Note: mode is NOT the biggest number ooooooo.

Quartiles (division into four)

1st or lower quartile $L_1 = \frac{1}{4}(n+1)^{th}\ value$

2nd quartile or median $L_2 = \frac{1}{2}(n+1)^{th}\ value$

3rd or upper quartile $L_3 = \frac{3}{4}(n+1)^{th}\ value$

Deciles (division into ten)

$1st\ \ decile, D_1 = \frac{1}{10}(n+1)^{th}\ value$

$5th\ \ decile(median),\ \ D_5 = \frac{5}{10}(n+1)^{th}$

$6th\ \ decile, D_6 = \frac{6}{10}(n+1)^{th}\ \ \ and\ \ \ so\ on.$

Percentile (division into hundred)

$1st\ percentile, P_1 = \frac{1}{100}(n+1)^{th}\ value$

$20th\ percentile, P_{20} = \frac{20}{100}(n+1)^{th}\ value$

$50th\ (\ median),\ P_{50} = \frac{50}{100}(n+1)^{th}\ value$

$70th\ percentile, P_{70} = \frac{70}{100}(n+1)^{th}\ value$

And so on.

MEASURE OF DISPERSION

- **Mean deviation**
- **Range**
- **Standard deviation**
- **Variance**

Mean deviation

For ungrouped data

$$M_d = \frac{\sum |x - \bar{x}|}{N}$$

For grouped data

$$M_d = \frac{\sum f|x - \bar{x}|}{\sum f}$$

Range

$Range = highe\quad value - lowest\ value$

Standard deviation

This is the square root of variance

$$S.D = \sqrt{\frac{\sum(x-\bar{x})^2}{N}} = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum fx}}$$

Variance

$$Variance = \frac{\sum(x-\bar{x})^2}{N} = \frac{\sum f(x-\bar{x})^2}{\sum fx}$$

QUESTIONS

1. In a soccer competition, a club had scored the following goals 2, 0, 3, 3, 2, 1, 4, 0, 0, 5, 1, 0, 2, 2, 1, 3, 1, 4, 1 and 1. What is the mean of the scores
   a) 1.5 b) 1.8 c) 1 d) 2
2. From above, what is the median of the scores
   a) 1.5 b) 1.8 c) 1 d) 2
3. The mode of the distribution is _____
   a) 1.5 b) 1.8 c) 1 d) 2
4. What is the mean deviation of 3, 5, 8, 11, 12 and 21? a) 4.7 b) 60 c) 3.7 d) 10
5. Calculate the mean, median and mode of 2, 3, 3, 3, 5 a) 3, 2, 3.2 b) 3.2, 2, 3 c) 2.3, 3, 3 d) 3, 2, 2.3
6. The mean of twelve positive numbers is 3. When another number is added, the mean becomes 5. Find the thirteenth number
   a) 29 b) 26 c) 25 d) 24
7. The median of the set of number 4, 9, 4, 13, 17, 14, 10, 17 is a) 13 b) 7 c) 19/2 d) 10
8. Below is a frequency distribution of a data which has a mean of 43/14. Find the value y

x	1	2	3	4	5
f	y+2	y-1	2y-3	y+4	3y-4

   a) 1 b) 2 c) 3 d) 4
9. Find the variance of the numbers, k, k+1, k+2
   a) 2/3 b) 1 c) $k+1$ d) $(k+1)^2$
10. The variance of the sample 6, 11, 5, 8, 6 and 9 is 5.1. What is the variance of the sample 18, 33, 15, 24, 18 and 27?
    a) 5.1 b) 8.1 c) 30.6 d) 45.9
11. The variance of $x, 2x, 3x, 4x\ and\ 5x$ is
    a) x b) $3x$ c) x^2 d) $2x^2$
12. Find the median of the distribution given below;

x	1	2	3	4	5	6	7	8	9
f	1	2	2	1	2	2	1	3	1

    a) 3 b) 4 c) 5 d) 8
13. What is the mode of the above data
    a) 3 b) 4 c) 5 d) 8
14. For the set of numbers 2, 3, 5, 6, 7, 7, 8
    a) the median is greater than the mode
    b) the mean is greater than the mode
    c) the mean is greater than the median
    d) the mean is less than the median
15. Given the set of numbers, 4, 6, 2, 10, 8 the difference between the range and the variance is _____ a) 1 b) 0 c) 2 d) 3

    Given the table,

x	0	1	2	3	4	5
f	5	8	6	6	3	2

    Use the information to answer question 16 to 20
16. What is the mean of the distribution
    a) 1/2 b) 1 c) 2 d) 3
17. What is the mode of the distribution
    a) 0 b) 1 c) 2 d) 3
18. What is lower quartile of the distribution
    a) 0 b) 1 c) 2 d) 3
19. Find the value of the 7^{th} a) 2 b) 4 c) 5 d) 3
20. What is value of the 60^{th} percentile
    a) 2 b) 4 c) 5 d) 3
21. There are 15 girls and 23 boys in a class. If a prefect is to be elected, what is the chance of a girl getting the post
    a) 15/23 b) 8/23 c) 15/38 d) 1/15
22. A box contains yellow, white and blue balls. The total number of balls in the box is 36. If a ball is chosen at random, the probability of getting a white ball is 1/18 and that of getting a yellow ball is 1/3. How many blue balls are in the box a) 12 b) 2 c) 18 d) 22
23. The table below shows the number of pupils in each age group in a class.

Age in years	10	11	12
Number of pupils	6	27	7

    What is the probability that a pupil chosen at random is at least 11 years old
    a) 27/40 b) 17/20 c) 33/40 d) 3/20

24. If the probability of one hunter hitting a target is 1/2 and the probability of another hitting is 2/3 for each shot. What is the probability that they both hit it if they each shoot one arrow
a) 7/6 b) 3/5 c) ¾ d) 1/3

25. Two numbers are chosen from at random from three numbers 1, 2, 3 and 4. What is the probability that the sum of the numbers is even
a) 2/3 b) 1/2 c) 1/3 d) 1/4

26. When two dice are thrown, what is the probability that the sum of the scores shown is divisible by 5 a) 7/36 b) 1/6 c) 1/9 d) 1/12

SOLUTIONS

1. The best way to do this is to present the data on a table

x	0	1	2	3	4	5
f	4	6	4	3	2	1
fx	0	6	8	9	8	5

$Mean = \bar{x} = \frac{\sum fx}{\sum f}$

$\bar{x} = \frac{0+6+8+9+8+5}{4+6+4+3+2+1} = \frac{36}{20} = 1.8 \ldots\ldots\ldots b$

2. The median

$M_e = \left(\frac{\sum f + 1}{2}\right)^{th} \ value$

$M_e = \left(\frac{20+1}{2}\right)^{th} \ term$

$M_e = 10.5th \ term$

Hence, the median is the value between the 10th and the 11th term

The value at the 10th term =1

The value at the 11th term =2

$Hence, the \ median, M_e = \frac{1+2}{2} = \frac{3}{2} = 1.5 \ldots\ldots a$

3. The mode of the distribution is definitely the value with the highest occurring frequency.

$Mode = 1 \ldots\ldots\ldots\ldots\ldots\ldots c$

4. $mean \ deviation = \frac{\sum|x-\bar{x}|}{n}$

$\bar{x} = \frac{\sum x}{n}$

$\bar{x} = \frac{3+5+8+11+12+21}{6} = \frac{60}{6}$

$\bar{x} = 10$

$M.D = \frac{7+5+2+1+2+11}{6} = \frac{28}{6}$

$M.D = 4.6667 \ldots\ldots\ldots a$

5. $Mean = \frac{\sum x}{n}$

$\bar{x} = \frac{2+3+3+3+5}{5} = 3.2$

Median = middle number = 3

Mode = most occurred number = 3

Hence, 3.2, 3, 3……. b

6. $Mean, \bar{x} = \frac{\sum x}{N}, \ \therefore 3 = \frac{\sum x}{12} \quad \sum x = 3(12) = 36$

When another number, say g is added,

$\sum x$ becomes $36 + g$ and the mean = 5

$\therefore \frac{36+g}{13} = 5 \quad \Longrightarrow \quad 36 + g = 65$

$\therefore p = 65 - 36 = 29$

Hence, the thirteenth number = 29 …… a

7. The number; 4, 9, 4, 13, 17, 14, 10, 17

We've got to rearrange the numbers;

4, 4, 7, 9, 10, 13, 14, 17

$M_e = \frac{9+10}{2} = \frac{19}{2} \ldots\ldots\ldots c$

8. Let's get fx

x	1	2	3	4	5
f	y+2	y-1	2y-3	y+4	3y-4
fx	y+2	2y-2	6y-9	4y+16	15y-20

$\bar{x} = \frac{\sum fx}{\sum f}$

$\bar{x} = \frac{y+2+2y-2+6y-9+4y+16+15y-20}{y+2+y-1+2y-3+y+4+3y-4}$

$\bar{x} = \frac{28y-13}{8y-2}$

$\bar{x} = \frac{43}{14}$

$\therefore \frac{43}{14} = \frac{28y-13}{8y-2}$

$43(8y-2) = 14(28y-13)$

$344y - 86 = 392y - 182$

$344y - 392y = -182 + 86$

$-48y = -96$

$y = 2 \ldots\ldots\ldots . b$

9. Just remember the formula

$Variance, \sigma = \frac{\sum(x - \bar{x})^2}{N}$

$Hence, mean,$

$\bar{x} = \frac{k + k + 1 + k + 2}{3} = \frac{3k + 3}{3} = k + 1$

Variance

$\sigma = \frac{[k-(k+1)]^2+[k+1-(k+1)]^2+[k+2-(k+1)]^2}{3}$

$\sigma = \frac{(-1)^2 + 0^2 + 1^2}{3} = \frac{1 + 0 + 1}{3} = \frac{2}{3} \ldots\ldots . a$

10. Let's differentiate the samples;

Sample X: 6, 11, 5, 8 6, 9

Sample Y: 18, 33, 15, 24, 18 and 27

You will notice that sample Y is thrice the values of sample X i.e.

$sample\ X = 3(sample\ Y)$

Thus, S.D of sample Y=3(S.D of sample X)

Since $Variance = (standard\ deviation)^2$

Hence,

$(\text{S. D of sample Y})^2 = (3 \times \text{S. D of sample X})^2$

$(\text{S. D of sample Y})^2 = 9(\text{S. D of sample X})^2$

$i.e. Variance\ of\ Y = 9\ Variance\ of\ sample\ X$

$variance\ of\ Y = 9(5.1) = 45.9 \ldots\ldots . . d$

11. Let's calculate the mean first

$\bar{x} = \frac{x + 2x + 3x + 4x + 5x}{5} = \frac{15x}{5} = 3x$

$Variance, \sigma = \frac{\sum(x - \bar{x})^2}{N}$

The difference between the variables and the mean (i.e. $x - \bar{x}$) will be;

$-2x, -x, 0, x, 2x$

$Hence, the\ variance\ will\ be;$

$\sigma = \frac{(-2x)^2 + (-x^2) + 0^2 + x^2 + (2x)^2}{5}$

$\sigma = \frac{4x^2 + x^2 + x^2 + 4x^2}{5} = \frac{10x^2}{5} = 2x^2 \ldots\ldots d$

12. $\sum f = 15$ which is an odd number

Hence, $Median = \frac{1}{2}(n + 1)th\ term$

$M_e = \frac{1}{2}(16)th\ term = 8th\ term$

$\therefore M_e = 5 \ldots\ldots\ldots\ldots\ldots . . c$

13. The mode is the value of x that has the highest frequency i.e. 8 $\ldots\ldots\ldots\ldots\ldots\ldots . . d$

14. Let's get the mean, median and mode first then start analyzing

$Mean = \frac{\sum x}{N} = \frac{2 + 3 + 5 + 6 + 7 + 7 + 8}{7}$

$= \frac{38}{7} = 5.43$

$Median = 6$

$Mode = 7$

Hence, the correct option is $\ldots\ldots\ldots . d$

15. The range is the difference between the highest and lowest value i.e. $10 - 2 = 8$

$Variance, \sigma = \frac{\sum(x - \bar{x})^2}{N}$

$\bar{x} = \frac{4 + 6 + 2 + 10 + 8}{5} = \frac{30}{5} = 6$

$\sigma = \frac{(-2)^2 + 0^2 + (-4)^2 + 4^2 + 2^2}{5}$

$\sigma = \frac{4 + 16 + 16 + 4}{5} = \frac{40}{5} = 8$

Hence, $Range - Variance = 8 - 8 = 0 \ldots\ldots . . b$

16. Let's get fx in the table

x	0	1	2	3	4	5
f	5	8	6	6	3	2
fx	0	8	12	18	12	10

$Mean, \bar{x} = \frac{0 + 8 + 12 + 18 + 12 + 10}{5 + 8 + 6 + 6 + 3 + 2}$

$\bar{x} = \frac{60}{30} = 2 \ldots\ldots\ldots\ldots . c$

17. The mode is the value with the highest frequency i.e. 1……. b

18. Try to remember that;

1st or lower quartile $L_1 = \frac{1}{4}(n + 1)^{th}\ value$

$\frac{1}{4}(30 + 1)th\ value$

$= \frac{1}{4}(31)th\ value$

$= 7.75th\ term \quad 1 \ldots\ldots\ldots b$

19. $7th\ decile, D_7 = \frac{7}{10}(n+1)^{th}\ value$

$D_7 = \frac{7}{10}(31)th\ value = 21.7th\ value$

$D_7 = 4 \ldots\ldots\ldots b$

20. $The\ 60th\ percentile, P_{60} = \frac{60}{100}(n+1)th\ value$

$P_{60} = \frac{60}{100}(31) = 18.6th\ term \quad 2 \ldots\ldots\ldots a$

21. Number of girls=15, number of boys=23

Total number of students=38

$P(girl) = \frac{number\ of\ girls}{total\ no.of\ students} = \frac{15}{38} \ldots\ldots\ldots c$

22. Total number of balls=36

If the probability of picking a white ball is 1/18,

$then; \frac{1}{18} = \frac{no.of\ white\ balls}{36}$

$no\ of\ white\ ball \quad 2$

If the probability of picking a yellow ball is 1/3,

$then; \frac{1}{3} = \frac{no.of\ white\ balls}{36}$

$no\ of\ white\ balls = 12$

Hence, the number of blue balls is;

$36 - 2 - 12 = 22 \ldots\ldots\ldots d$

23. At least 11 years means 11 years and above i.e. 11 and 12 years old. The number of pupils in this category is $27 + 7 = 34$

Total number of students =40

$Hence, P(at\ least\ 11) = \frac{34}{40} = \frac{17}{20} \ldots\ldots\ldots b$

24. From the question,

$P(hunter\ one) = \frac{1}{2}$

$P(hunt \quad two) = \frac{2}{3}$

We are required to find the probability that they hit the target (i.e. hunter one 'AND' hunter two)

$P(both) = P(hun \quad one) \times (hunter\ two)$

$P(bo \quad) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} \ldots\ldots\ldots d$

25. We need the sample space of two numbers first (1,2)(1,3)(1,4)(2,3)(2,4)(4,3). The sample space is 6

The sum of each sample is 3, 4, 5, 5,6, 7

We have 2 even sum and 4 odd sum

$Hence, P(even) = \frac{2}{6} = \frac{1}{3} \ldots\ldots\ldots c$

26. For better understanding , let's draw out the sample space for the two dice

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

As you can see, he sums that are divisible by 5 are just 7.

Total sample=36

$P(div\ by\ 5) = \frac{7}{36} \ldots\ldots\ldots a$

LAUTECH 2013/2014

PAST QUESTION

1. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ where $0 < \alpha < 90$, find the value of $\frac{\sec\alpha}{\sqrt{2}}$ a) 2 b) $\sqrt{2}$ c) 1 d) none of the above

2. Find $\frac{dy}{dx}$ if $y = \frac{1+x^2}{1-x^2}$

   a) $\frac{4x}{(1+x^2)^2}$ b) $\frac{4x}{(1-x^2)}$ c) $\frac{4x}{(1-x^2)^2}$ d) $\frac{2x}{(1-x^2)^2}$

3. Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

   a) $\frac{1}{2}x^2 + 2x + \ln x + k$ b) $\frac{1}{8}x^2 + 4x + 3\ln x + k$

   c) $x^2 + 2x^3 + k$ d) $\frac{1}{2}x^2 \ln x + k$

4. Find the equation of the line through the point (2,1) and (4,5) a) $y = 3x + 5$ b) $y = 2x - 3$

   c) $y = 2x + 4$ d) $y = 3x + 2$

5. The mean of the following set of values: 33, 45, H, 36, 49 is 40, what is the median value a) 36

   b) 40 c) 37 d) none of the above

6. Find the value of a fixed angle in the trigonometric equation given by

   $2\sqrt{3}\sin\theta + 2\cos\theta = 2$

   a) 30 b) 90 c) 0 d) 60

7. Find the equation of the tangent and the normal to the curve $y = x^2 - 4$ at the point $x = 3$.

   a) $\begin{matrix} y = 6x + 13 \\ 6y + x = 33 \end{matrix}$ b) $\begin{matrix} y = 6x - 13 \\ 6y + x = 33 \end{matrix}$ c) $\begin{matrix} y = 6x - 13 \\ 6y - x = 33 \end{matrix}$

   d) $\begin{matrix} y = 6x - 13 \\ y + x = 33 \end{matrix}$

8. Evaluate $\int_1^8 \sqrt{1+3x}\, dx$

   a) 18 b) 32 c) 26 d) 12

9. Find the equation of the circle with centre (2, -1) and radius 5

   a) $x^2 + y^2 - 4x + 2y = 20$

   b) $x^2 + y^2 - 4x + 2y = 0$

   c) $x^2 + y^2 - 4x - 2y = 5$

   d) $x^2 + y^2 + 4x - 2y = 25$

10. If M represents the median and D represents the mode of this set of data: 5, 9, 3, 5, 7, 6, 8. Find M+D a) 5 b) 9 c) 10 d) 11

11. Simplify $\frac{\sin\alpha+\cos\alpha}{\cos^4\alpha-\sin^4\alpha}$ a) $\frac{1}{\sin\alpha-\cos\alpha}$ b) $\frac{1}{\sin\alpha-\cos\alpha}$

    c) 1 d) $\sin\alpha + \cos\alpha$

12. Find the derivative of $y = \cot^2(x^3)$

    a) $6x^2\,\mathrm{cosec}^2(x^3)\cot(x^3)$

    b) $-6x^2\sec^2(x^3)\cot(x^3)$

    c) $-6x^2\,\mathrm{cosec}^2(x^3)\cot(x^3)$

    d) $6x^2\sec^2(x^3)\cot(x^3)$

13. Evaluate $\int\frac{11}{22x+9}dx$

    a) $\frac{11}{9}\ln(22x+9)+k$ b) $\frac{1}{2}\ln(22x+9)+k$

    c) $\frac{11}{9}\ln(22x+9)+k$ d) none of the above

14. Find the equation of the circle with centre (2,3) which passes through the point (1, 1)

    a) $(x+2)^2 - (y-3)^2 = 5$

    b) $(x-2)^2 + (y-3)^2 = 0$

    c) $(x+2)^2 + (y+3)^2 = 0$

    d) $(x-2)^2 + (y-3)^2 = 5$

15. The scores obtained by applicants in a recruitment test are shown in the table below. If the average score is 4.7, find the value of y

Scores	2	3	4	5	6	7	8
No. of applicants	1	3	5	y	4	2	3

    a) 3 b) 4 c) 5 d) 6

16. Which of the following is equivalent to $\sec\theta\, cose\ \theta$ a) $\tan\theta + \cot\theta$ b) $cosec\,\theta\sin\theta$

    c) $\cos\theta\sec\theta$ d) none of the above

17. Find $\frac{dy}{dx}$ if $x^2y + y^2 - 4x = 1$

    a) $\frac{4-2xy}{x^2-2y}$ b) $\frac{4-2xy}{x^2+2y}$ c) $\frac{4+2xy}{x^2+2y}$ d) $\frac{4+2xy}{x^2-2y}$

18. Evaluate $\int x \cos x \, d$

a) $x\cos x + \sin x + k$ b) $x\sin x + \cos x + k$

c) $x(\cos x + \sin x) + k$ d) $x^2 \cos x + \sin x + k$

19. Find the centre of the circle

$x^2 + y^2 - 8x + 6y + 16 = 0$

a) $(-4,3)$ b) $(-4,-3)$ c) $(4,-3)$ d) $(4,3)$

20. The mean weights of three groups of students are 40kg, 45kg and 50kg respectively. If the numbers of students in the groups are 45, 40 and 25 respectively, what is the average weight of all the students

a) 33.3kg b) 45kg c) 39.2kg d) 44kg

21. Simplify $\sqrt{\frac{1+\tan^2\beta}{1+\cot^2\beta}}$

a) $\cot\beta$ b) $\tan\beta$ c) $\text{cosec}^2\beta$ d) $\sec\beta$

22. Let $y = \frac{x^2}{x^2-1}$, find the vertical and horizontal asymptotes a) $x = \pm 2; y = 1$ b) $x = \pm 1; y = 2$

c) $x = \pm 1; y = 1$ d) $x = 1; y = \pm 2$

23. Evaluate $\int \cos 7\theta \cos 5\theta \, d\theta$

a) $\frac{\sin 2\theta}{4} + \frac{\sin 12\theta}{24} + k$ b) $\frac{\sin 2\theta}{4} - \frac{\sin 12\theta}{24} + k$

c) $\frac{\cos 2\theta}{4} - \frac{\sin 12\theta}{24} + k$ d) $\frac{\cos 2\theta}{4} + \frac{\cos 12\theta}{24} + k$

24. Find the length of the length of the tangent from point (1, 2) to a circle

$x^2 + y^2 - 8x + 6y + 16 = 0$

a) 5 b) 2 c) 4 d) 3

25. The frequency table shows the marks obtained by students in a quiz. Find the mean of the marks

marks	1	2	3	4	5
frequency	2	2	8	4	4

a) 3.1 b) 3.0 c) 3.2 d) 3.3

26. Solve for θ if $8\cos^2\theta + 6\cos\theta = 5$

a) 1/2 b) 0 c) 60 d) 120

27. Find $\frac{dy}{dx}$ if $y = \log_e \sqrt{(1+x)}$

a) $\frac{1}{2(1-x)}$ b) $\frac{1}{2(1+x^2)}$ c) $\frac{1}{2(1+x)}$ d) $\frac{1}{2(1-x^2)}$

28. Evaluate $\int \frac{t^6 - t^2}{t^4} dt$ a) $\frac{t^3}{3} + \frac{1}{t} + k$ b) $\frac{t^2}{3} - \frac{1}{t^3} + k$

c) $\frac{3}{t^3} + \frac{t}{2} + k$ d) $t^3 - \frac{t^3}{2} + k$

29. Find the angle between the lines

$3y - 4x + 9 = 0$ and $y + 11 = x$

a) 2.2^0 b) 8.3^0 c) 4^0 d) 8.13^0

30. The mean of seven numbers is 10. If six of the numbers are 2, 4, 8, 14, 16, 18, find the mode

a) 16 b) 8 c) 14 d) 2

31. In a triangle ABC, a=6, c=4 and A=30, find the value of angle C a) 19.5 b) 0.3 c) 150 d) 130.5

32. The radius of a circle in increasing at the rate of 0.1cm/s. Find the rate at which the area is increasing when the radius of the circle is 10cm.

a) $6.142cm^2/s$ b) $6.214cm^2/s$ c) $6.314cm^2/s$

d) $6.284cm^2/s$

33. Find the area between the curve $y = \cos x$ and the x-axis from $x = 0 \; to \; x = \pi$

a) $\sin x$ b) 1 c) -1 d) 0

34. Find the distance between the points $(-2,4)$ and $(-5,0)$ a) 2 b) 3 c) 4 d) 5

35. If the pass mark of the following set of marks is 5, what is the percentage pass?

Marks	2	3	4	5	6	7	8
No. of students	3	1	5	2	4	2	3

a) 55% b) 45% c) 10% d) 75%

SOLUTION

1. $If \; \sin\alpha = \frac{\sqrt{3}}{2}$

$then \; \alpha = \sin^{-1}\frac{\sqrt{3}}{2} = 60^0$

$henc \;\; , \sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\cos 60} = \frac{1}{(1/2)} = 2$

$\therefore \frac{\sec\alpha}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \ldots\ldots b$

2. This involves quotient rule

$y = \frac{1+x^2}{1-x^2}$

$u = 1 + x^2 \implies \frac{du}{dx} = 2x$

$v = 1 - x^2 \implies \frac{dv}{dx} = -2x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}$

$\frac{dy}{dx} = \frac{2x - \cancel{2x^2} + 2x + \cancel{2x^2}}{(1-x^2)^2}$

$\frac{dy}{dx} = \frac{4x}{(1-x^2)^2} \ldots\ldots\ldots\ldots c$

3. Expand the bracket first

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + 1 + 1 + \frac{1}{x}\right) dx$

$= \int \left(x + \frac{1}{x} + 2\right) dx = \frac{x^2}{2} + \ln x + 2x + c \ldots\ldots a$

4. Equation of a line through (2, 1) and (4, 5)

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y-1}{x-2} = \frac{5-1}{4-2}$

$\frac{y-1}{x-2} = \frac{4}{2}$

$\frac{y-1}{x-2} = 2$

$y - 1 = 2x - 4$

$y = 2x - 3 \ldots\ldots\ldots\ldots b$

5. If the mean is 40 then;

$40 = \frac{33 + 45 + H + 36 + 49}{5}$

$163 + H = 200$

$H = 37$

Hence, the numbers are; 33, 45, 37, 36, 49

Rearranging 33, 36, 37, 45, 49

Hence, the median is 37 …….. c

6. $2\sqrt{3}\sin\theta + 2\cos\theta = 2$

Recall that;

$b\sin x + a\cos x = Rsin\,(x + \alpha)$

$b = 2\sqrt{3}, \quad a = 2$

$\alpha = \tan^{-1}\frac{2}{2\sqrt{3}} = \tan^{-1}\frac{1}{\sqrt{3}}$

$\alpha = 30^0$

$R = \sqrt{a^2 + b^2} = \sqrt{2^2 + \left(2\sqrt{3}\right)^2}$

$R = \sqrt{4 + 12} = \sqrt{16} = 4$

$hen \quad , 2\sqrt{3}\sin\theta + 2\cos\theta = 2 = 4\sin(x + 30)$

$\sin(x + 30) = \frac{2}{4}$

$\sin(x + 30) = \frac{1}{2}$

$x + 30 = \sin^{-1}\left(\frac{1}{2}\right)$

$x + 30 = 30$

$x = 0 \ldots\ldots\ldots c$

7. $y = x^2 - 4$

$\left.\frac{dy}{dx}\right|_{x=3} = 2x = 2(3) = 6$

Hence, the gradient of the tangent will be 6

Hence, $y = 3^2 - 4 = 9 - 4 = 5$

Hence, the point is (3, 5)

$m = \frac{y - y_1}{x - x_1}$

$\frac{y-5}{x-3} = 6$

$y - 5 = 6x - 18$

$y = 6x - 13$(equation of tangent)

For normal;

$\frac{y-5}{x-3} = -\frac{1}{6}$

$6y - 30 = -x + 3$

$6y + x = 33 \ldots\ldots\ldots b$

8. Integration by substitution

$\int_1^8 \sqrt{1 + 3x}\, dx$

$let\ u = 1 + 3x, \frac{du}{dx} = 3, \implies dx = \frac{du}{3}$

The question becomes

$\int_1^8 \sqrt{u}\frac{du}{3} = \frac{1}{3}\int_1^8 u^{1/2}\, du = \frac{1}{3}\left[\frac{u^{3/2}}{\frac{3}{2}}\right]_1^8$

$= \frac{1}{3}\left[\frac{(1+3x)^{3/2}}{\frac{3}{2}}\right]_1^8 = \frac{2}{9}(125 - 8) = \frac{2}{9}(117)$

$= 26 \ldots\ldots\ldots c$

9. $(x-2)^2+(y--1)^2=5^2$

$x^2-4x+4+y^2+2y+1=25$

$x^2+y^2-4x+2y=20 \ldots\ldots..a$

10. Rearranging the data 3, 5, 5, 6, 7, 8, 9

The median, M=6

The mode D= 5

$\therefore M+D=6+5=11 \ldots\ldots\ldots d$

11. No correct option

$the\ answer\ is\ \dfrac{1}{\cos^2\alpha-\sin^2\alpha}$

12. $y=\cot^2(x^3)$

$let\ u=x^3, \dfrac{du}{dx}=3x^2$

$let\ v=\cot u\ \dfrac{dv}{du}=-\operatorname{cosec}^2 u$

Hence;

$y=v^2\ \ \dfrac{dy}{dv}=2v$

$\dfrac{dy}{dx}=\dfrac{dy}{dv}\times\dfrac{dv}{du}\times\dfrac{du}{dx}$

$\dfrac{dy}{dx}=2v(-cosec^2u)(3x^2)$

$\dfrac{dy}{dx}=2\cot u\,(-cosec^2u)(3x^2)$

$\dfrac{dy}{dx}=2\cot(x^3)\times -cos\ \ ^2(x^3)\times 3x^2$

$\dfrac{dy}{dx}=-6x^2cose\ \ ^2(x^3)\cot(x^3) \ldots\ldots\ldots.c$

13. $\int\frac{11}{22x+9}dx$

$let\ u=22x+9,\ \ \dfrac{du}{dx}=22,\ \ dx=\dfrac{du}{22}$

The question becomes;

$\int\frac{11}{u}\cdot\frac{du}{22}=\frac{11}{22}\int\frac{du}{u}=\frac{1}{2}\ln u+k$

$\frac{1}{2}\ln(22x+9)+k\ \ \ldots\ldots\ldots\ldots\ldots d$

14. Let's get the radius first from the information

Using formula for distance between two points;

Radius (2, 3) and point (1, 1)

$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$r=\sqrt{(2-1)^2+(3-1)^2}$

$r=\sqrt{(1)^2+2^2}=\sqrt{1+4}=\sqrt{5}$

Hence, the equation is,

$(x-2)^2+(y-3)^2=\sqrt{5}^2$

$(x-2)^2+(y-3)^2=5 \ldots\ldots..d$

15. Let's find fx

x	2	3	4	5	6	7	8
f	1	3	5	y	4	2	3
fx	2	9	20	5y	24	14	24

$\bar{x}=\dfrac{2+9+20+5y+24+14+24}{1+3+5+y+4+2+3}=4.7$

$\dfrac{93+5y}{18+y}=4.7$

$93+5y=4.7(18+y)$

$93+5y=84.6+4.7y$

$93-84.6=4.7y-5y$

$8.4=-0.3y$

$y=-28\ \ no\ answer$

16. $\sec\theta\operatorname{cosec}\theta=\tan\theta+\cot\theta \ldots\ldots.a$

17. $x^2y+y^2-4x=1$

$2xy+x^2\dfrac{dy}{dx}+2y\dfrac{dy}{dx}-4=0$

$x^2\dfrac{dy}{dx}+2y\dfrac{dy}{dx}=4-2xy$

$\dfrac{dy}{dx}(x^2+2y)=4-2xy$

$\dfrac{dy}{dx}=\dfrac{4-2xy}{x^2+2y} \ldots\ldots\ldots.b$

18. $\int x\cos x\,dx$

Integration by part;

$\int udv=uv-\int vdu$

$let\ u=x,\ \ du=dx,$

$dv=\cos x\,dx$

$\int dv=\int\cos x\,dx$

$v=\sin x$

$\therefore\int x\cos x\,dx=x\sin x-\int\sin x\,dx$

$=x\sin x-(-\cos x)+k$

$=x\sin x+\cos x+k \ldots\ldots\ldots..b$

19. From the general equation of a circle;

$(x-a)^2+(y-b)^2=r^2$

Expanding;

$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$

Comparing to the equation given in the question;

$x^2 + y^2 - 8x + 6y + 16 = 0$

$-2ax = -8x \implies a = \frac{-8x}{-2x} = 4$

$also, -2by = 6y \implies b = \frac{6y}{-2y} = -3$

$\therefore (a, b) = (4, -3) \ldots\ldots\ldots c$

20. The average weight will be;

$$\frac{(40 \times 45) + (45 \times 30) + (50 \times 25)}{45 + 30 + 25}$$

$$= \frac{1800 + 1350 + 1250}{100} = \frac{4400}{100} = 44 \ldots\ldots d$$

21. Note that $1 + \tan^2 \beta = \sec^2 \beta$

And $1 + \cot^2 \beta = \operatorname{cosec}^2 \beta$

$$\sqrt{\frac{1 + \tan^2 \beta}{1 + \cot^2 \beta}} = \sqrt{\frac{\sec^2 \beta}{\operatorname{cosec}^2 \beta}} = \frac{\sec \beta}{\operatorname{cosec} \beta}$$

$$= \frac{\sin \beta}{\cos \beta} = \tan \beta \ldots\ldots\ldots b$$

22. $y = \frac{x^2}{x^2 - 1}$

For vertical asymptote, denominator equals zero,

$x^2 - 1 = 0$

$x^2 = 1$

$x = \sqrt{1} = \pm 1$

For horizontal asymptote,

$$y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{1}{1 - \frac{1}{x^2}}$$

$$y = \frac{1}{1 - \frac{1}{\infty^2}} = \frac{1}{1 - 0} = 1 \ldots\ldots c$$

23. Page 108 of your school textbook.....*b*

24. The equation to use is

$AB^2 = x_o^2 + y_o^2 + 2gx_o + 2fy_o + c$

$x_o = 1, y_o = 2,$

$2gx = -8x \implies g = -\frac{8}{2} = -4$

$2fy = 6y \implies f = \frac{6}{2} = 3$

$c = 16$

$\therefore AB^2 = 1^2 + 2^2 - 8(1) + 6(2) + 16$

$AB^2 = 1 + 4 - 8 + 12 + 16$

$AB^2 = 25$

$AB = 5 \ldots\ldots\ldots a$

25. Let's get fx

marks	1	2	3	4	5
frequency	2	2	8	4	4
fx	2	4	24	16	20

$$\bar{x} = \frac{2 + 4 + 24 + 16 + 20}{2 + 2 + 8 + 4 + 4} = \frac{66}{20} = 3.3 \ldots\ldots\ldots d$$

26. *let* $x = \cos \theta$

The equation becomes;

$8x^2 + 6x - 5 = 0$

$x = \frac{1}{2} \ \ or \ \ x = -\frac{5}{4}$

$hence, \frac{1}{2} = \cos \theta$

$\cos \theta = \frac{1}{2}$

$\theta = \cos^{-1} 0.5 = 60^0 \ldots\ldots\ldots c$

27. $y = \log_e \sqrt{(1 + x)}$

$y = \ln \sqrt{(1 + x)}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x}} \times \frac{d}{dx}\left(\sqrt{1 + x}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x}} \times \frac{d}{dx}(1 + x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x}} \times \frac{1}{2}(1 + x)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{(1 + x)^{1/2}}\right)\left(\frac{1}{(1 + x)^{1/2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{(1 + x)^{1/2 + 1/2}}\right) = \frac{1}{2}\left(\frac{1}{1 + x}\right) \ldots\ldots b$$

28. $\int \frac{t^6 - t^2}{t^4} dt$

$$\int \left(\frac{t^6}{t^4} - \frac{t^2}{t^4}\right) dt = \int (t^2 - t^{-2}) dt$$

$$= \frac{t^3}{3} + t^{-1} + k = \frac{t^3}{3} + \frac{1}{t} + k \ldots\ldots\ldots a$$

29. Just get the gradient of the two lines;

For first line; $3y - 4x + 9 = 0$

$y = \frac{4}{3}x - 3 \quad hence, m_1 = \frac{4}{3}$

For the second line;

$y + 11 = x$

$y = x - 11, \quad hence\ m_2 = 1$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\theta = \frac{\frac{4}{3} - 1}{1 + (1)\left(\frac{4}{3}\right)}$$

$$\tan\theta = \frac{\left(\frac{1}{3}\right)}{\left(\frac{7}{3}\right)}$$

$$\tan\theta = -\frac{1}{3} \times \frac{3}{7}$$

$$\tan\theta = \frac{1}{7}$$

$\theta = \tan^{-1}\left(\frac{1}{7}\right) = 8.13^0$ ……. d

30. Let the missing number be x

$$\frac{2 + 4 + 8 + 14 + 16 + 18 + x}{7} = 10$$

$62 + x = 70$

$x = 8$

Hence, the numbers are 2, 4, 8, 8, 14, 16, 18

The mode is 8 ……..b

31. Using sine rule

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{6}{\sin 30} = \frac{4}{\sin C}$$

$$\sin C = \frac{4}{6} \times 0.5 = \frac{1}{3}$$

$C = \sin^{-1}\frac{1}{3} = 19.47$ …… a

32. Application of differentiation

$given;\ \frac{dr}{dt} = 0.1\ cm/\quad and\ r = 10cm$

$\frac{dA}{dt} = ?$

$Area\ of\ a\ circle, A = \pi r^2 = \pi(10^2) = 100\pi$

$$\frac{dA}{dr} = 2\pi r = 2\pi(10) = 20\pi$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 20\pi \times 0.1 = 2\pi$$

$\frac{dA}{dt} == 6.284\ cm^2/s$ …… d

33. $A = \int y dx$

$$A = \int_0^{\pi} \cos x\, dx$$

$A = [\sin x]_0^{\pi} = \sin 180 - \sin 0 = 0$ …… d

34. $d = \sqrt{(-5 - -2)^2 + (0 - 4)^2}$

$d = \sqrt{(-3)^2 + (-4)^2}$

$d = \sqrt{9 + 16} = 5$ …… d

35. Percentage pass represents the percentage of those that scored 5 marks and above i.e. those that scored 5, 6, 7 and 8 which are 2, 4, 2 and 3 respectively.

$$\%pass \quad \frac{2 + 4 + 2 + 3}{3 + 1 + 5 + 2 + 4 + 2 + 3} \times 100\%$$

$\%pass = \frac{11}{20} \times 100\% = 55\%$ …… a

WAVE AND OSCILLATION

A motion is said to be simple harmonic if its acceleration is directly proportional to its displacement from a fixed point and is always directed towards that fixed point.

$$a = \omega^2 x$$

The motion of a particle undergoing simple harmonic motion can be represented in terms of sine and cosine. Any of the equations below is accepted for the motion.

$$Y = Asin(\omega t \pm kx$$

$$Y = Acos(\omega t \pm kx$$

The maximum displacement from the equilibrium position is called AMPLITUDE (A). The unit is meter (m)

The situation in which the amplitude of a wave motion drops until it comes to rest is called DAMPING.

The time taken to complete one oscillation (to and fro) is called PERIOD (T). The unit is second (s). It is also the reciprocal of frequency.

The number of oscillation completed in one second is called FREQUENCY (F). The unit of frequency is Hertz (Hz) or cycles per second (s^{-1}). It is also the reciprocal of period.

The distance between two successive crests and two successive troughs is called WAVELENGTH (λ). The unit is meter (m)

The reciprocal of wave length is called WAVE NUMBER (K).

The displacement through which a wave is propagated in one second is called VELOCITY (V)

The highest point on a wave is called CREST

The lowest point on a wave is called TROUGH.

The surface over which the disturbance has the same phase at all points is called WAVE FRONT

NOTE THE FOLLOWING FORMULA

$$Period\ (T) = \frac{time(}{number\ of\ oscillations}$$

$$Or \quad T = \frac{1}{f}$$

$Frequency\ (f)\ =\ \frac{number\ of\ oscillation(n)}{time(t)}$

(s^{-1}) or cycles per second or Hertz

Acceleration (a) $=\omega^2 A \quad or\ \ a=\omega^2 x(ms^{-2})$

Velocity (v) $=\omega A \quad or\ \ v=\omega x(ms^{-1})$

v=$\omega\sqrt{(A^2-X^2)}(ms^{-1})$

Angular frequency or angular velocity: $\omega=\frac{2\pi}{T}$

or $\omega=\ 2\pi f rads^{-1}$

Period of oscillation in a spring with force constant K and attached mass m

T=$2\pi\sqrt{\frac{m}{k}}$ (s)

Frequency (f) =$\frac{1}{2\pi}\sqrt{\frac{k}{m}}(s^{-1})$

Period of oscillation in simple pendulum

T=$2\pi\sqrt{\frac{l}{g}}$ (s)

Frequency (f) = $\frac{1}{2\pi}\sqrt{\frac{g}{l}}s^{-1}$

Wave number: k = $\frac{2\pi}{\lambda}$

Examples of simple harmonic motions are compound pendulum, simple pendulum, a loaded test tube placed vertically in water, the oscillation of liquid in a u-tube. Etc.

There are three types of oscillation; free, damped and forced oscillations.

A wave is a disturbance which travels through a medium and transfers energy from one point to another without any permanent displacement of the medium. A wave can be observed for example, when a stone is dropped from a height into a pond.

Velocity of transverse wave: V= $\sqrt{\frac{T}{\mu}}$

Doppler Effect is the change in apparent frequency as a result of relative motion between a source and an observer.

The general formula for Doppler Effect is:

$f_0=\left(\frac{v\pm u_0}{v\mp u_s}\right)f.$

Meet BRAVO for explanation on how to derive EIGHT formula from the general formula above with ease.

LAUTECH RECENT AND PAST QUESTIONS ON WAVE AND OSCILLATION.

1. The motion of a body is simple harmonic if the
   a) Path of the motion is a straight line
   b) Acceleration is directed towards a fixed point and proportional to its distance from the point
   c) Acceleration is proportional to the square of the distance from the point
   d) Acceleration is always directed towards a fixed point
2. Two waves are said to give lissajous figures when _____
   a) they have the same amplitude and the same frequency
   b) they have different amplitudes but the

same frequency

c) they have different amplitudes and different frequencies

d) they have the same amplitude but different frequencies.

3. The angular velocity in simple harmonic motion is inversely proportional to the

a) Period b) Frequency c) Amplitude

d) linear velocity

4. Calculate the angular velocity of the seconds hand of a watch

a) $0.03\pi rads^{-1}$ b) $0.03\pi radsec$

c)0.03π/radsec d) 0.03rad/sec

5. The time required for one complete oscillation(wave cycle) is called

a)Frequency b) Velocity c) Amplitude

d) Period

6. What is the frequency of a progressive wave given by $y = 4\sin(200\pi t - \frac{15\pi x}{17})$

a) 50Hz b) 100Hz c) 150Hz d) 100m

7. The change in the direction of a wave front as a result of change in velocity in another medium is called _____

a)Refraction b) Reflection

c)Diffraction d) Polarization

8. The ability of a wave to spread around corners is called

a) Polarization b)Dispersion

c) Diffraction d)Reflection

9. Which of the following is the correct expression for the period of oscillation?

a) $T=2\pi\sqrt{\frac{g}{l}}$ b) $T = 2\pi\sqrt{\frac{K}{m}}$

c)$T=2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$ d) none of the above

10. A student of LAUTECH found out from a simple pendulum experiment that 39 oscillations were completed in 78seconds.What is the period of oscillation of the pendulum?

a)200Hz b) 200sec c) 2Hz d) 2sec

11. Which of the following type of waves cannot be propagated through a vacuum

a) light b) x-ray c) water d) Radio

12. A wave transporting energy from east to west in which the particles of the medium move both northward and southward is called a) Mechanical b) Transverse

c) Electromagnetic d) Longitudinal

13. The speed in a liquid whose density is 1500 $kg\ m^{-3}$is 1836m/s. what is the bulk modulus of the liquid.

a) $2.80\times10^9\ Nm^{-2}$ b) $3.31\times10^2 Nm^{-2}$

c) $5.06\times10^9 Nm^{-3}$ d) $1.24\times10^9 Nm^{-2}$

14. Which of the following has the greatest penetrating power a) infrared ray

b) micro wave c) x-rays d) gamma rays

15. How many beats are heard assuming two vibrating strings have fundamental frequency of 184Hz and 180Hz respectively a) 3 beats b) 4 beats c) 5 beats d) 1.02 beats

16. A 3.5 kg object performs simple harmonic motion of amplitude 0.75m and period 6sec when the timing started at the Centre of the oscillation. Determine the maximum acceleration a) $0.82ms^{-2}$ b) $0.822ms^{-1}$

c) 0.28m/s d) 0.228m/s

17. Which of the following is the least energetic a) micro wave b) infrared c) visible d) ultra-violet ray

18. A wave that is both mechanical and transverse is_____

a) water b) sound c) x-ray d) radio

19. The acceleration of a moving body in SHM is given by -49x and has an amplitude of 10cm. Determine the maximum acceleration a) $490ms^{-2}$ b) $4.9ms^{-2}$ c) $4.9cms^{-2}$ d) 490cm/s

20. _____ occurs when waves of the same frequency interact a) attenuation b) interference c) damping d) antinode

21. Calculate the time taken to complete one oscillation by a fan rotating at an angular speed of 1500 rev/min for 3 sec. a) $40\pi sec$ b) 0.04 sec c)$0.04\pi sec$ d) 0.04π

22. A boat at anchor is rocked by waves whose crests are 100m apart and whose velocity is 25m/s. at what interval does the wave crest reach the boat?

a) 2500s b)75s c) 4s d) 0.24s

23. A lamp is suspended from a high ceiling with a cord 12ft long. Find its period of oscillation g=$32ft^{-2}$

a) 6.95 seconds b)3.85 seconds c) 6.95mins d)3.85mins

24. If a 1000Hz sound is blasted from a barrier , what apparent frequency is heard by a driver of a car traveling towards the barrier at 18m/s

(velocity of sound in air =330m/s)

a)500Hz b)1055Hz c) 945.5Hz d)1115Hz

25. A radio station broadcast at frequency of 300Hz .If the speed of the wave is 3×10^8, calculate the period?

a) 3.3×10^{-6}sec b) 3.3×10^{-3}sec c) 3.0×10^3 sec d) 3.3×10^6sec

26. The distance between the successive of water is 4m and its velocity is 2.78m/s find the frequency a) 1.4Hz b) 0.695sec c) 0.695Hz d)0.695Hz/sec

27. What does 0.25 represent in the wave equation Y= $0.25\sin(2\pi ft - 30\pi x)$

a)velocity b)speed c) wavelength d) amplitude

28. Which of the following is different from others? a) x-ray b)gamma rays c)cathode rays d) ultra-violet rays

29. A sound wave is a mechanical wave; not an electromagnetic wave. This means that

a) particles of the medium move perpendicular to the direction of energy transported

b) a medium is required in order for sound wave to transport energy

c) a sound wave transports its energy through a vacuum.

d) none of the above

30. A wave travels a distance of 20cm in 3 sec. the distance between successive crests of the wave is 4cm. what is the frequency of the wave?

a) 0.6Hz b) 1.67Hz c) 8Hz d)15Hz

31. Which of the following waves is an electromagnetic wave a) sound wave b) water c) heart beats d) x-ray

32. A turning fork of frequency 360Hz is sounded together with one of 364Hz. How many beats are heard
a)1 b) 2 c) 3 d) 4

33. A particle executing simple harmonic motion (SHM) has frequency of 50Hz and amplitude of 8.0cm. calculate the maximum velocity of its motion a) 2.5m/s
b) 2.51rad/s c) 25.1m/s d)25.1 rad/s

34. Infrasonic sounds are sounds whose frequencies are
a) greater than 20KHz b)less than 20KHz
c) less than 20Hz d) equal to 20KHz

35. The motion of moving speed of a talking drum can be rightly described as
a) translational b) random
c) oscillatory d) rotational

36. The reciprocal of wave length is called
a) amplitude b) wave front
c) wave motion d) wave number

37. A simple pendulum has a period of 4 sec. after shortening the pendulum by one meter. Its period is found to be 3.46 sec. calculate the value of acceleration due to gravity. a) $10m/s^2$ b) 9m/s c) $9.87m/s^2$ d) $4.95m/s^2$

38. _____ is the damping of energy
a) refraction b) reflection c) attenuation
d) wave speed

39. When the propagation of wave motion depends on the wave frequency, it is called
a) refraction b)wave front c) frequency of wave motion d) dispersion

40. The surface in which the disturbance has the phase at all points is called
a) wave number b) wave front
c) wave length d) phase.

41. A 200g mass is attached to a spring to perform (SHM) horizontally with amplitude of 4cm. the force constant of the spring is 25N/m. Determine the frequency of the oscillation
a) 1.87Hz b) 1.78 cycles per second
c) 1.7Hz d) none of the options is right.

42. A 5kg and 1kg masses rest on a horizontal frictionless surface and are joined by a spring constant of 50N/m. if the two masses oscillate with the same frequency. Determine the period of oscillation.
a) 0.811 b) 0.711 c) 0.811sec d) 0.711sec

43. All the following are simple harmonic motions except_____ a) simple pendulum
b) a vibrating cantilever c) a liquid oscillating in a u-tube d) none of the above

44. A medium in which the phase velocity is frequency dependent is known as___
a) dispersion b) wave packet
c) dispersive d) wave form

45. Which of the following waves can be polarized?
a) water b) radio c) sound d) light

46. Which of the following is required for beats to occur
a) the two pure tones must have a large difference in frequency
b) the two pure tones must differ slightly in frequency
c) the two pure tones must be of the same frequency
d) both options a & b

47. A whistle giving out 500Hz tone moves away from a stationary observer in a direction towards and perpendicular to a flat wall with a velocity of 1.5m/s. How many beats per second will be heard by the observer? (velocity of sound wave is 336m/s) a) 4 sec b) 4.4 c) 4.4Hz d) 5

48. A phenomenon which occurs when waves are obstructed is called___
a) reflection b) diffraction
c) deffraction d) refraction

49. An object is oscillating in simple harmonic motion with amplitude of 15cm and a period of 2 sec. The frequency of the oscillation is___ a) 0.5 b) 0.05Hz c) 0.5 sec d) 0.5 cycles per second

50. _____ is the deflection of light energy by fine particles of solid, liquid and gas
a) diffraction b) refraction c) scattering
d) dispersion

51. _____ is a condition for a standing wave to be produced
a) both waves must travel in opposite direction
b) both waves must travel in the same direction
c) both waves must have different amplitudes
d) both waves must have different frequencies.

52. A particle of mass 1 kg executes a SHM of its path when it is at a distance of 0.5m from the Centre of its path. Its speed is 6m/s and the force towards the Centre is 128N.Find the time of a complete oscillation
a) 0.30sec b) 0.39Hz c) 0.4sec
d) none of the above

53. A tuning fork of frequency 360Hz is sounded together with one of 364Hz. How many beats are heard?
a) 1 b) 2 c) 3 d) 4

54. The frequency of a plane progressive wave represented by the equation is
$y = 4\sin(2000\pi t - 0.5x)$ a) 100Hz
b) 100m/s c) 1000m/s d) 1000Hz

55. Which of the following characteristics of waves is used in the measurement of ocean depth a) refraction b) reflection
c) diffraction d) dispersion

56. The speed of sound travelling through various media decreases in the following order
a) water, air, brass bar
b) brass bar, water, air
c) air, water, brass bar
d)water, brass bar, air.

57. Sound waves can be diffracted because they are a) transverse b) longitudinal
c) stationary d)cannot be diffracted

58. How far from a wall should a boy stand in order to hear the echo of his clap 0.9s later
a) 36.7m b)74.2m c) 148.5m d)297.5m

59. A body moves in SHM with amplitude of 10m. If the angular speed of the body is $5rads^{-1}$, calculate its speed when it is 6m from the Centre of the motion.
a) 20m/s b) 30m/s c) 40m/s d) 50m/s

60. A vibrating driving board has a frequency of 20Hz. What is the angular velocity of the

board? a) $2\pi rad/s$ b) $20\pi rad/s$
c) 30rad/s d) $40\pi rad/s$

61. The bob of a simple pendulum of mass 0.025kg is displaced 0.1m from its equilibrium position. If the angular frequency is 4 $rads^{-1}$, calculate the energy of the system a) $5\times 10^{-4}J$ b) $2\times 10^{-3}J$
c) 5×10^{-3}J d) $7.9 \times 10 - 3\,J$

62. Which of the following processes will increase the rate of oscillation of a pendulum
a) increasing the length of the pendulum
b) decreasing the length of the pendulum
c) Increasing the mass of the pendulum
d) Decreasing the mass of the pendulum

63. Which of the following can affect the period of a simple pendulum?
i) Mass of the pendulum
ii) Length of the pendulum
iii) Acceleration due to gravity

a) I, II & III b) II & III only c) I & III only
d) I & II only

64. A simple pendulum with a period of 2 sec has its length doubled. Its new period is
a) 1s b) 1.41s c) 4s d) 2.83s

65. When the energy of vibrating skin of a talking drum is decreased, the sound waves emanating from the drum would have a corresponding decrease in their
a)Wave length b) amplitude c) quality
d) speed

66. A student found out from a simple pendulum experiment that 20 sec oscillations were completed in 38 seconds. What is the period of oscillation of the pendulum
a)8 sec b) 3.8 sec c) 2 sec d) 1.9 sec

67. The amplitude of the motion of a body performing simple harmonic motion decreases with time because
a) frictional force dissipates the energy of motion
b) the frequency of oscillation varies with time
c) the period of oscillation varies with time
d) energy is supplied by some external agencies

68. A transverse wave and a longitudinal wave traveling in the same direction in a medium differ essentially in their
a) amplitude
b) direction of vibration of the particles of the medium
c) period of vibration of the particles of the medium
d) frequency

69. Which of the following waves needs a medium for propagation a) radio waves
b) x-ray c) sound waves d) light waves

70. Which of the following is true of light and sound waves a)they both transmit energy
b) they both need a medium for propagation
c) they are both transverse waves
d) their velocities in air are equal

71. Determine the distance between the consecutive antinodes XX if the wave length is 200m. a) 10m b) 400m c) 0 m d) 100m

72. The combination of sound waves with different frequencies is known as
a) interference b) diffraction
c) superposition d) resonance

73. Two identical waves traveling in the same direction are superimposed, what should be the phase difference between the waves for maximum destructive interference to occur
a) 20 degree b) 45 degree c) 180 degree
d) 270 degree

74. Which of the following is an exclusive property of transverse waves?
a) diffraction b) reflection
c) compression d) polarization

75. During a thunderstorm the sound is heard over a long time. This phenomenon is called
a) reverberation b) superposition
c) Doppler Effect c) refraction of sound

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON WAVE AND OSCILLATION

1. Simple harmonic motion is the motion of a particle in which the acceleration is directly proportional to its displacement from a fixed point and is always directed towards that point.

a=w^2x. The correct option is……**D**

2. Lissajous figures are produced by waves of the same amplitude but different frequencies……**D**

3. $\omega = \frac{2\pi}{T}$; where ω is angular velocity and T is period……A

4. $\omega = \frac{2\pi}{T}$

T= 60 sec

$\omega = \frac{2\pi}{60} = 0.03\pi rads^{-1} \ldots\ldots\ldots\ldots\ldots A$

5. Period……D

6. Y= 4sin $(200\pi t - \frac{15\pi x}{17})$

Comparing the given wave equation with

Y= A sin $(2\pi ft - \frac{2\pi x}{\lambda})$;

$2\pi ft = 200\pi t;$

$2f = 200$

$F = 100Hz \ldots\ldots\ldots\ldots\ldots\ldots B$

7. Refraction……A

Note that velocity and wave length change but frequency remains constant when refraction occurs

8. Diffraction……C

9. C

10. Number of oscillation=39

Time=78 sec;

$Period\ (T) = \frac{time(}{num.of\ oscillation(n)}$

$T = \frac{78}{39} = 2sec \ldots\ldots\ldots\ldots\ldots D$

11. Mechanical waves are waves which require material medium for wave propagation. E.g water, sound……C

12. Transverse waves are waves in which the particles of the medium oscillate perpendicular to the direction of propagation of the wave e.g electromagnetic waves……B

13. $V = \sqrt{\frac{\beta}{\rho}}$;

$V^2 = \frac{\beta}{\rho}$; $\beta = V^2\rho = 1836^2 \times 1500$

$\beta = 5.06 \times 10^9 Nm^{-3}$……C

14. Gamma rays……D

Using

R AMI VUX G

$- - - - - - - - - - - - \rightarrow$

order of increasing penetrating power/energy
order of increasing frequency
order of decreasing wavelength

- RA….Radio wave,
- M……Microwave,
- I……..Infrared rays,
- V……..Visible light,
- U…….Ultra-violet,
- X……x-ray,
- G…….Gamma rays

15. Beats $F = f^{11} - f^1$;

$f = 184 - 180$

= Beat 4……B

16. Mass= 3.5kg, A=0.75m,

T=6sec $a_{max} = -\omega^2 A$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

$a = -\omega^2 A$

$\left(\frac{\pi}{3}\right)^2 \times 0.75 = 0.822 m/s^2 \ldots\ldots\ldots\ldots A$

17. A……Microwave

18. Water wave is both mechanical wave and transverse wave

19. $a_{max} = -\omega^2 A = -\omega^2 x$

$-49x = -\omega^2 x \implies \omega^2 = 49; \omega = 7.$

$a_{max} = -\omega^2 x$

Given x=10cm =0.1m

$a_{max} = 7^2 \times 0.1 = 4.9 m/s^2$……..B

20. Interference……B

21. Angular speed $\omega = 1500 rev/min$

Recall that $2\pi = 360^0 = 1$ rev.

$1500 rev/min = \frac{1500 \times 2\pi}{60} = 50\pi rad/sec$

$\omega = 50\pi rad/sec$

$\omega = \frac{2\pi}{T} \implies T = \frac{2\pi}{\omega} = \frac{2\pi}{50\pi}$

$= 0.04 se$ ……B

22. Distance between crests $(\lambda) = 100m$

Velocity= 25m/s

V=fλ $\implies$ $f = \frac{1}{T}$

V=$\frac{\lambda}{T}$ $\implies$ T=$\frac{\lambda}{V}$=$\frac{100}{25}$

T=4sec……C

23. Length of the cord =$12ft; \frac{32ft}{s^2}$

$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{12}{32}}$

$= 3.85 sec$……B

24. This is the case of an observer moving towards a stationary source

$f^1 = \left(\frac{v + u_0}{v}\right) f$

$f^1 = \left(\frac{330 + 18}{330}\right) 1000 = \left(\frac{348}{330}\right) 1000$

$= 1055Hz$ ... . B

25. $v = 3 \times 10^8,\ f = 300Hz$

$T = \frac{1}{f} = \frac{1}{300} = 3.3 \times 10^{-3} sec$ ... ... B

26. $v = f\lambda;\quad \lambda = 4m;\quad v = 2.78ms^{-1}$

$f = \frac{v}{\lambda} = \frac{2.78}{4} = 0.695Hz$ C

27. Amplitude.....D

28. Cathode rays.....C Other options are electromagnetic waves

29. B

30. Distance=20cm; time=3 sec

λ =4cm

$v = \frac{distance}{time} = \frac{20}{3} = 6.67m/s$

$F = \frac{v}{\lambda} = \frac{6.67}{4} = 1.67Hz$ ... ... B

31. X-ray.....D

32. Beat F= $f^{11} - f^1 = 364 - 360 = 4Hz$

Beat= 4.....D

33. $f = 50Hz, A = 8cm = 0.08m; v$ =?

$\omega = 2\pi f\pi$

$\omega = 2\pi \times 50 = 100\pi$

$\omega = 100\pi rads^{-1}$

$v = -\omega A \quad 100\pi \times 0.08 = 25.1ms^{-1}$. C

34. Infrasonic sounds are sounds whose frequencies are less than 20Hz.....C

Note: ultrasonic sounds are sounds whose frequencies are greater than 20 KHz

35. Oscillatory .....C.

note: oscillatory motion is also called planetary motion, to and fro, back and forth and not back and front.

36. Wave number.....D

37. $T_1 = 4sec; T_2 = 3.46sec; l_1 = l;$

$l_2 = l - 1$

Using, $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring both sides

$T^2 = 4\pi^2 \frac{l}{g} \implies 4^2 = 4\pi^2 \frac{l}{g}$

$16g = 4\pi^2 l$ ... ... $equ.(1)$

$From\ equation\ (1);\ l = \frac{16g}{4\pi^2} = \frac{4g}{\pi^2}$

After the length has been shortened by 1,

$3.46^2 = 4\pi^2 \left(\frac{l-1}{g}\right)$

$12g = 4\pi^2 l - 4\pi^2$ ... ... $equ\ (2)$

$substitute\ l\ into\ equ.(2)$

$12g = 4\pi^2 \left(\frac{4g}{\pi^2}\right) - 4\pi^2$

$12g = 16g - 4\pi^2$

$12g - 16g = -4\pi^2$

$-4g = -4\pi^2$

$g = \pi^2 = 9.87ms^{-2}$ C

38. Attenuation C

39. Dispersion……..D. The medium in which dispersion occurs is called dispersive medium.

40. Wave front ……..B

41. Given: mass =200g=0.2kg; k= 25N/m

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.2}}$$

$$= 1.78Hz$$

$or\ 1.78\ cycles\ per\ second \ldots\ldots B$

42. Given: $m_1 = 5kg;\ m_2 = 1kg; k = 50N/m$

Using $T = 2\pi\sqrt{\frac{\mu}{k}};$

$$where\ \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$= \mu = \frac{5 \times 1}{5+1} = \frac{5}{6}kg$$

$$T = 2\pi\sqrt{\frac{5/6}{50}}$$

$= \ 0.811\ sec$……..C

43. D

44. Dispersive medium……..C

45. Light wave(transverse wave)……..D

46. B.

47. The observer hears a note of apparent frequency f^1 from the whistle directly when moving away. And a note of apparent frequency f^{11} from the sound waves reflected from the wall which is approaching the observer.

When the Source is moving away $f^1 = \left(\frac{v}{v+u_s}\right)f$

$$f^1 = \left(\frac{336}{336+1.5}\right)500 \ = \ 497.8Hz$$

When the source is approaching $f^{11} = \left(\frac{v}{v-u_s}\right)f$

$$= \left(\frac{336}{336-1.5}\right)500$$

$$= 502.2Hz$$

Beats per second will be

$$f^{11} - f^1 = 502.2 - 497.8$$

$$beats = 4.4Hz \ldots\ldots\ldots C$$

48. Diffraction……..B

49. Given :$t = 2\ sec;$

$$f = \frac{1}{T} = \frac{1}{2} = 0.5\ cycles\ per\ second \ldots\ldots D$$

50. Scattering ……..C

51. For a standing or stationary wave to be produced, the two waves must travel in the opposite direction, must have the same frequency and same amplitude……..A

52. Given: mass = 1 kg; x=0.5m; v=6m/s F=128N;

$$K = \frac{F}{m} = \frac{128}{0.5} = \frac{256N}{m}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{1}{256}} = 0.39sec$$

$$T = 0.4sec \ldots\ldots\ldots C$$

53. $beats\ heard \quad f_2 - f_1$

$beats\ heard \quad 364 - 360$

$beats\ heard \quad 4beats\ per\ seconds$……D

54. The general equation of a wave is given by

$y = A\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right)$

Comparing with

$y = 4\sin(2000\pi t - 0.5\pi x)$

$2\pi ft = 2000\pi t$

$f = 1000Hz$……D

55. Reflection ……B

56. B

57. Longitudinal ……B

58. Given: time = 0.9s

Speed of sound (v) = 330m/s

$V = \frac{2d}{t} \implies 2d = vt$

2d=330× 0.9=297

$d = \frac{297}{2}$

d = 148.5m……C

59. A=10m, X=6m, $\omega = 5rads^{-1}$

V=$\omega\sqrt{A^2 - X^2}$

V=$5\sqrt{10^2 - 6^2}$=$5\sqrt{100 - 36}$=$5\sqrt{64}$

V=5 × 8 = =40m/s……C

60. Angular velocity, $\omega = 2\pi f$

ω=2π × 20=40$\pi rad/s$----------D

61. Mass=0.025kg, X=0.1m,$\omega = 4rad/s$

Energy in SHM= $\frac{1}{2}KA^2$=$\frac{1}{2}Kx^2$

Recall ω^2=$\frac{K}{m}$

Therefore, k=mω^2

$\frac{1}{2}Kx^2$=$\frac{1}{2}m\omega^2x^2$

$\frac{1}{2}$× 0.025 × 4^2 × 0.1^2= 2× $10^{-3}J$……B

62. Decreasing the length of the pendulum……B.

The rate of oscillation here means the number of oscillation per unit time which defines frequency. Recall that the frequency of oscillation is inversely proportional to the length. Therefore, the higher the length, the lower the number of oscillations per second (frequency) and vice versa

63. The period of oscillation is affected by the length of the pendulum (L) and the acceleration due to gravity (g)……B

64. T $\propto \sqrt{L}$

T^2= kL

Where K is constant of proportionality

$K = \frac{T^2}{L} \implies \frac{T_1^2}{L_1} = \frac{T_2^2}{L_2}$

Given:$T_1 = 2sec; L_1 = l; L_2 = 2L_1;$

$\frac{2^2}{l} = \frac{T_2^2}{2l}$

T_2^2=$\frac{8l}{l}$

$T_2 = \sqrt{8} = 2.83$sec……D

65. Amplitude ……B

66. T=$\frac{time\ taken}{number\ of\ oscillation}$=$\frac{38}{20} = 1.9\ sec$ … … D

67. A

68. The direction of vibration for longitudinal is parallel and that of transverse is perpendicular……B

69. Sound wave is a mechanical wave which requires a medium for its propagation……C

70. All waves transmit energy……A

71. Antinode=$\frac{wave\ lengt}{2}$=$\frac{\lambda}{2} = \frac{200}{2} =$ $100m$……D

72. Superposition ……C

Note; superposition occurs between waves of different frequencies while interference occurs when waves of the same frequency, wavelength and amplitude interact

73. Destructive interference occurs when waves are out of phase. It occurs at half of a circle. That is, half of wave length

Phase difference$= \frac{\lambda}{2} = \frac{360}{2} = 180^0$……C

74. Polarization is only exhibited by transverse waves……D

75. D

OPTICS

Photometry is the science of light measurement.

$$Illumination\ (E)\ = \frac{\Phi}{A}\ = \frac{luminous\ flux}{area}(lux)$$

$$Illumination\ (E)\ = \frac{I\cos\theta}{R^2}$$

$$Illumination\ (E)\ = \frac{I}{R^2}$$

$$Luminance\ (L)\ = \frac{luminous\ intensity}{Area}(cdm^{-2})$$

$$Luminous\ intensity\ = \frac{\Phi}{w}\quad candela\ (cd)$$

$$Luminous\ flux\ (\Phi\ ) = 4\pi l\quad lumen(lm$$

$$Mirror\ equation\ = \frac{1}{f}\ = \frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$

Note: wherever an objected is placed on a convex mirror, the image formed is always behind the mirror.

The image formed on a convex mirror is always VIRTUAL, ERECT AND DIMINISHED

The image formed on a concave mirror is always VIRTUAL ERECT and MAGNIFIED

The focal length of a convex mirror is negative

The focal length of a concave lens is negative.

The focal lengths of a concave mirror and a convex lens are both positive

$$Linear\ magnification\ (m) = \frac{v}{u} = \frac{h_i}{h_o}$$

$$M = \frac{v}{f} - 1$$

The Number of images formed when two mirrors are inclined at an angle is given as:

$$N = \frac{360}{\theta} - 1$$

Note that the number of images formed must be an odd number. If after calculating the number of images formed and your answer happens to be an even number like: 2, 4, 6, 8 etc. you are expected to deduct 1 from the answer to give an odd number. Take note of that!

Images formed on a plane mirror are always VIRTUAL, SAME SIZE AS THE OBJECT, SAME DISTANCE AS THE OBJECT IS FROM THE MIRROR AND ERECT

Note: The two conditions required for total internal reflection to occur are:

i) The critical angle must be exceeded.
ii) Light must travel from a denser medium to a less dense medium

LAUTECH RECENT AND PAST QUESTIONS ON OPTICS

1. The science of light measurement is called
   a) Spectrometry b) photometry
   c) geometry d) photography.
2. A lens that is thinner at the middle and thicker at the edge is
   a) diverging b) converging
   c) piano-convex d) converging meniscus
3. An object is placed in front of a mirror , if the mirror is rotated through an angle of θ, the image rotated through an angle
   a) θ b) 2θ c) $\frac{\theta}{2}$ d) $90 - \theta$
4. The unit of luminous flux is _____
   a) lumen (lm) b) candela (cd)
   c) candela per square meter(cdm^{-2})
   d) lux per square meter(lxm^{-2})
5. What is the critical angle of a medium of refractive index 1.60?
   a)52^0 b) 58^0 c) 38.7^0 d) 30^0
6. Two plane mirrors are inclined at angle 120^0 to each other face to face, the number of images formed is? a) 2 b) 4 c) 6 d) 3
7. Two plane mirrors are inclined at angle 45^0 face to face to each other, the number of images formed is a)7 b) 8 c) 9 d)6
8. The use of lens is not applicable in
   a) projector b) human eye c) periscope
   d) telescope
9. A 60^0 prism is made of glass whose refractive index for certain light is 1.80. The angle of incidence at which minimum

deviation will occur is

a) 63^0 b) 64.2^0 c) 32^0 d) 30^0

10. The image of an object in a convex mirror is 2cm from the mirror. If the mirror has a radius of 12cm, find the object position

a) -1.5cm b) 1.5cm c) 1.5m d) -1.5m

11. An object of 12cm long is placed at Centre of curvature of concave mirror whose focal is 40cm. the location of the image formed is? a) 40cm b) 20cm c)100cm d) 80cm

12. Which of the following does not describe the image formed by a plane mirror

a) enlarged b) virtual c) laterally inverted d) the same size as the object

13. An object is placed 10cm in front of a plane mirror, if the mirror is moved towards the object a distance of 3m, the distance through which the image moves is?

a) 1m b) 2m c) 3m d) 4m

14. In a pin-hole camera, when the hole is large, the image formed is? a) bright and blurred b) small and bright c) dark and sharp d) bright and sharp

15. In which of the following devices is total internal reflection not applicable

a) optical fiber b) prism binocular c) periscope d) camera

16. What part of the camera corresponds to the iris of the eye a) the film b) the shutter c)the diaphragm d) the lens

17. A fish is 8m below the surface of the pond. If the refractive index of water is 1.33, what is the apparent depth of the fish below the surface of the pond

a)10.0m b) 6.0m c)1.5m d)1.33m

18. Total internal reflection will not occur when light travels from

a) water to air b) glass to air c) glass to water d) water to glass

19. A concave mirror of focal length 10cm forms a real image with magnification 0.5. calculate the image distance

a) 1.5m b)15cm c)15m d) 0.5cm

20. A mirror that is thicker at the middle and thinner at the edge is?

a) convex b) concave c) diverging mirror d) both a & c

21. Which of the following is not correct

a) A ray parallel to the principal axis is reflected through the principal focus (F)

b) A ray through the principal focus is reflected parallel to the principal axis

c) A real image is formed by the intersection of the real rays of light

d) convex lens is the same as diverging lens

22. The focal length of a mirror is given as

a) $\frac{r}{2}$ b) $\frac{2}{r}$ c) $2r$ d) r

23. Which of the following is a condition for internal reflection to occur

a) The light must travel from a dense medium to a dense

b) the light must travel from a dense medium to a less dense medium

c) the angle of incidence must be the same as the critical angle

d)the critical angle must be greater than the angle of incidence

24. The transmission of light through glass fibres is called

a) fibre optics b) photometry c) geometry d) photography

25. A ray of light is incident at 60^0 at an air-glass plane surface. Find the angle of refraction in the glass ($\eta\ for\ the\ glass = 1.5$) a) 25.3^0 b) 35.3^0 c) 15.3^0 d) 30^0

26. Which of the following correctly describes the most suitable shaving mirror and position of the person shaving? The person stands
a) at the focus of a convex mirror
b) at the focus of concave mirror
c) within the focal length of a convex mirror
d) within the focal length of a concave mirror

27. The critical angle of glass is 42^0, find the refractive index
a) 1.50 b) 1.30 c) 1.80 d) 2.40

28. An object is located 30mm from a converging mirror with the radius of curvature of 10mm, if the objected is 4mm tall, how tall is the image
a) 6.0cm b)6.0mm c) 3.0cm d)0.8mm

29. What is the speed of light in a glass of refractive index 1.50 If the speed of the same light is given as 3.0×10^8m/s
a) 1.5×10^8m/s b)2.0×10^8m/s c)3.0×10^8m/s d) 450m/s

30. Which of the following is used for controlling the amount of light entering the eye
a) cornea b) pupil c) iris d) optic nerve

31. It is recommended that the illumination be $540lm/m^2$ for newspaper reading. How far from the paper should a 2.65cd source be placed to provide the illumination
a) 0.7m b) 1.8m c) 2.0m d) 3.0m

32. When light enters an optically denser medium, its rays _____
a) bends towards the normal
b) bends away from the normal
c) remains in the same path
d) go parallel to the normal

33. When an object is placed between the Centre of curvature and the principal focus, the image formed is
a) real, inverted and same size
b) at infinity c) real, inverted and magnified d) virtual, erect and magnified.

34. Light of frequency $6.0 \times 10^{14}Hz$ travelling in air is transmitted through glass of refractive index 1.5. Calculate the frequency of the light in the glass
a)$6.0 \times 10^{14}Hz$ b) $4.0 \times 10^{14}Hz$
c) $7.5 \times 10^{14}Hz$ d) $9.0 \times 10^{14}Hz$

35. After reflection from the concave mirror, rays of light from the sun converges
a) at the radius of curvature
b) at the focus
c) beyond the radius of curvature
d) between the focus and radius of curvature

36. A 70^0glass prism has a refractive index of 1.5. Calculate the angle of incidence for minimum deviation
a) 35^0 b) 48^0 c) 59^0 d) 45^0

37. An object placed 12cm in front of a convex lens produces a virtual image of magnification 3.0, the focal length of the lens is a) 9cm b) 12cm c) 48cm d) 36cm

38. The purpose of the condenser in a film projector is to
a) cast an image of the film on the screen
b) prevent the image from being blurred
c) invert the image laterally
d) make the image brighter

39. An object form an image three times its own size when placed 10.72cm from the pole of a spherical mirror. What is the focal length if the image is real?
a)7.5cm b) 15cm c) 80.4cm d) 9.5cm

40. The ability of the eye to focus object at different distances is called
a) power b)accommodation c) normal vision d) persistence of vision

41. For a converging lens to be used as a magnifying lens, the object must be placed
a) between C and F b) between F and lens c) at C d) beyond F

42. The number of image formed by a three mirror kaleidoscope is
a) 4 b) 5 c) 9 d) infinite

43. The image of an object on the retina of a human eye is a) erect and magnified b)inverted and virtual c) inverted and real d) virtual and diminished

44. Which of the following is not the property of the image formed by a concave lens?
a) erect image b) virtual image c) real image d) diminished image

45. Which of the following cannot be explained by the theory of light?
a) refraction b) Compton effect
c)diffraction d) interference

46. The following are all luminous bodies except a) the sun b) a candle
c) the moon d) a fluorescent body

47. A major property of light waves is that it is
a) propagated in s curved line
b) electromagnetic in nature
c) in the form of longitudinal waves
d) electrostatic in nature

48. Shadows and eclipses result from the
a) refraction of light b)rectilinear propagation of light c) diffraction of light
d) reflection of light

49. Total eclipse of the sun occurs when
a)earth is between the moon and the sun
b) sun is between the moon and the earth
c) moon is between the sun and the earth
d) ozone layer is threatened

50. The pin-hole camera produces a less sharply defined image when the
a) pin-hole is large b) illumination is less
c) screen is farther from the pin-hole
d)object is further from the pin-hole

51. A pin-hole camera is placed 300cm in front of a building so that the image is formed on a screen 5cm from the pin-hole. If the image is 2.5cm high, the height of the building will be a) 25cm b) 50cm c) 1000cm d) 150cm

52. An object is placed in front of two plane mirrors inclined at right angles to each other. The object is 1 unit distance from each other. The number of image formed is
a) 1 b)2 c) 3 d) 4

53. The plane mirror in kaleidoscope are usually placed at an angle of

a)60^0 b) parallel to each other c) 45^0 d) perpendicular to each other

54. An object placed at the Centre of curvature of a converging mirror. Its image is
a) real and diminished b) erect and virtual c) at the principal focus d) at the Centre of curvature

55. A concave mirror used for head lamps should be_____ a) small b) narrow c) wide d) thick

56. A ray of light is incident on the surface of water at an angle of30^0. Calculate the deviation suffered by the ray in the water (η_w=1.33)
a) 22.1^0 b) 30^0 c) 8^0 d) 90^0

57. A pool of water appears to be1.0m deep when viewed vertically from above. If the refractive index of water is 1.33, what is the actual depth of the pool
a) 0.75m b)1.013m c) 1.330m d) 13.33m

58. When light passes through two media X and Y with refractive indices 1.51 and 1.33 respectively, the speed of the light in
a) X is same as in Y
b) X and Y is same as in vacuum
c) X is higher than in Y
d) Y is higher than in X

59. For the correction of myopic defects in human eye we require a) a convex lens b) a concave lens c) a combination of concave and convex lenses d) a concave mirror

60. An astronomical telescope has an eyepiece of focal length 5cm. if the angular magnification in normal adjustment is 10, what is the distance between objective and eyepiece in cm
a)110cm b) 55cm c) 45cm d) 50cm

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON OPTICS

1. Photometry ……..B
2. Diverging lens……..A. note that a diverging lens is also called a convex lens and a concave lens is also called a converging lens.
3. 2θ … … … … … . B
4. Lumen (lm)…….A

Note the following units

- The unit of luminous intensity is Candela (cd)
- The unit of illumination is lux(lx)
- The unit of luminance is Candela per meter square(cds^{-2})
- The unit of luminous flux is lumen(lm)

5. Refractive index(η) $= \frac{1}{sinC}$

Given:$\eta = 1.60$

$$1.60 = \frac{1}{sinC} \implies 1.60\ sinC = 1$$

$$\sin C = \frac{1}{1.60} = 0.625$$

$$C = sin^{-1}(0.625) = 38.7^0 \text{.....C}$$

6. $N = \frac{360}{\theta} - 1$

Note: it is not in all cases that the above formula is applicable. You can use,

$$N = \frac{360}{\theta}$$

provided your answer is odd number.

The number of images formed must be an odd number

$N = \frac{360}{\theta}$; $N = \frac{360}{120} = 3 \ldots\ldots\ldots\ldots D$

7. $N = \frac{360}{\theta} - 1$; $\frac{360}{45} - 1 = 7 \ldots\ldots\ldots A$

8. A periscope uses two plane mirrors and not lens……C

9. Given: $\eta = 1.80$; $A = 60^0$

$$\eta = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Let $\sin\left(\frac{A+D}{2}\right) = sini$

$$\eta = \frac{\sin i}{\sin\left(\frac{A}{2}\right)} \implies 1.80 = \frac{\sin i}{\sin\left(\frac{60}{2}\right)}$$

$$1.80 = \frac{\sin i}{sin30} \implies 1.80 sin30 = sini$$

$0.9 = \sin i \implies i = sin^{-1}(0.9) = 64.2^0$

$i = 64.2^0 \ldots\ldots\ldots\ldots B$

10. $Given\ R = 12cm;\ f = \frac{R}{2}$;

$$f = \frac{12}{2} = 6cm.$$

$v = -2cm$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \implies \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$;

$\frac{1}{u} = \frac{1}{6} + \frac{1}{2}$

$\frac{1}{u} = \frac{1+3}{6} = \frac{4}{6} = 1.5cm \ldots\ldots\ldots B$

11. You have to be careful with a question like this

Given: $h_0 = 12cm; f = 4cm$

Note: when an object is placed at the Centre of curvature, the image has the same size as the object

Therefore $M = 1; m = \frac{v}{f} - 1$

$1 = \frac{v}{40} - 1$

$2 = \frac{v}{40}$; $v = 80cm \ldots\ldots\ldots\ldots D$

12. Enlarged ……A

13. C……3m. Any image formed on a plane mirror is the same distance as the object.

14. A

15. D

16. C

17. $\eta = \frac{real\ depth}{apparent\ depth}$

$$1.33 = \frac{8}{apparent\ depth}$$

$$apparent\ depth = \frac{8}{1.33} = 6.0m \ldots\ldots\ldots B$$

18. Total internal reflection will only occur when light travels from a denser medium to a less dense medium……D

19. Given: $f = 10cm$; $M = 0.5$

Using $m = \frac{v}{f} - 1$

$0.5 = \frac{v}{10} - 1 = 0.5 + 1 = \frac{v}{10}$

$1.5 = \frac{v}{10}$

$V = 1.5 \times 10 = 15cm \ldots\ldots\ldots\ldots B$

20. Convex mirror……D.

Note: A convex mirror is also called a diverging mirror while a concave mirror is also called a converging mirror.

A convex lens is also called a converging lens while a concave lens is also called a diverging lens.

21. D

22. A……$\frac{r}{2}$

23. B

24. A……fibre optics

25. Given :$i = 60^0; \eta = 1.5\,; r$ =?

Refractive index$(\eta) = \frac{sini}{sinr}$

$1.5 = \frac{sin60}{sinr} => 1.5sinr \quad 0.866$

$sinr = \frac{0.866}{1.5}$

$r = sin^{-1}(0.5773)$

$= 35.3^0 \ldots\ldots\ldots\ldots . B$

26. Make up, Shaving, dressing mirrors are made of concave mirrors. The person using the mirror must stand with the focal length. That is, between f and the pole so that an image that is erect and magnified can be produced……D

27. Refractive $(\eta) = \frac{1}{sinC}$

$\eta = \frac{1}{sin42} \implies \eta sin42 = 1$

$\eta = \frac{1}{0.669} \implies \eta = 1.50 \ldots\ldots\ldots . . A$

28. Given u=30mm; R=10mm;

$f = \frac{R}{2} \implies f = \frac{10}{2} = 5mm$

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \implies \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \implies \frac{1}{5} - \frac{1}{30}$

$\frac{1}{v} = \frac{1}{6} \implies v = 6mm$

$\frac{h_i}{h_0} = \frac{v}{u} \implies \frac{h_i}{4} = \frac{6}{30}$

$h_i = \frac{24}{30} \implies h_i = 0.8mm$

29. $\eta = \frac{speed\ of\ light\ in\ air}{speed\ in\ medium}$

$1.5 = \frac{3 \times 10^8}{speed\ of\ light\ in\ medium}$

$Speed\ of\ ligh\ \ in\ medium\ = \frac{3\ \times\ 10^8}{1.5}$

$=2.0\times 10^8 ms^{-1}$……B

30. Iris ……C

31. $Given: illumination(E) =; 540lm^{-2}$

$intensity(I) = 265cd;$

$E = \frac{I}{r^2} \implies r = \sqrt{\frac{I}{E}}$

$r = \sqrt{\frac{265}{540}} \implies r = 0.7m \ldots\ldots\ldots . A$

32. When light enters an optically denser medium, its rays bend towards the normal……A

33. C

34. Refractive index

$(\eta) = \frac{frequency\ of\ light\ in\ air}{frequency\ of\ light\ in\ glass}$

$1.5 = \frac{6.0 \times 10^{14}}{freq.of\ light\ in\ glass}$

$Freq\ .of\ light\ in\ glass = \frac{6.0 \times 10^{14}}{1.5} Hz$

…… …. . B

35. At the focus……B

36. $Refractive\ index(\eta) = \frac{\sin(\frac{A+D}{2})}{sin(\frac{A}{2})};$

$let \frac{A+D}{2} = i$

$\eta = \frac{sini}{sin\frac{A}{2}} \quad \Rightarrow \quad 1.5 = \frac{sini}{sin\left(\frac{70}{2}\right)}$

$sin35 \times 1.5 = sini$

$i = \sin^{-1}(0.8604)$

$i = 59^0$.....C

37. $u = 12cm$

$m = \frac{v}{u} \quad \Rightarrow \quad 3 = \frac{v}{u}$

$v = 3u \quad \Rightarrow \quad v = 36cm$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{36} + \frac{1}{12} = \frac{1}{f}$

$\frac{1+3}{36} = \frac{1}{f}$

$4f = 36; \ f = 9cm$.....A

38. It makes the image brighter.....D

The condenser also called 2 piano convex lenses provides a strong illumination for the slide to produce a brighter image.

39. $v = 3u$

From sign convention, real images have positive image distance.

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$u = 10.72cm$

$v = 3 \times 10.72 = 32.16cm$

$\frac{1}{f} = \frac{1}{32.16} + \frac{1}{10.72}$

$\frac{1}{f} = 0.03109 + 0.09328$

$\frac{1}{f} = 0.12437; f = 8.04cm$.....C

40. Accommodation.....B

41. Between F and lens.....B

42. Infinite.....D

43. Inverted and real.....C

44. Real C

45. Compton effect.....B

46. Moon..... C

47. Electromagnetic in nature.....B

48. Rectilinear propagation of light.....B

49. Eclipse of the sun occurs when the moon is between the sun and the earth.....C

50. A

51. Given , u=300cm, v=5cm, H_0 =?, $H_i 2.5cm$

$\frac{V}{U} = \frac{H_I}{H_0} = \frac{5}{300} = \frac{2.5}{H_0}$

$H_0 = \frac{300 \times 2.5}{5} = 150cm \ldots\ldots\ldots\ldots\ldots\ldots D$

52. At right angle, the angle between them is90^0.

$N = \frac{360}{\theta} - 1 = \frac{360}{90} - 1$

$= 4 - 1 = 3 \ldots\ldots\ldots\ldots\ldots C$

53. 45^0.....C

54. At the Centre of curvature.....D

55. $wide$.....C

56. Refractive index $(\eta) = \frac{sini}{sinr}$

$1.33 = \frac{sin30}{sinr} \quad \Rightarrow \quad sinr = \frac{sin30}{1.33}$

$\sin r = 0.3759$

$r = Sin^{-1}(0.3759) = 22.1^0$

$Deviation = i - r$

$d = 30 - 22.1 = 7.9 = 8^0$.....C

57. Refractive index $= \frac{real\ depth}{apparent\ depth}$

$$1.33 = \frac{real\ dept}{1.0}$$

Real/actual depth= 1.33× 1.0= 1.33m……..C

58. Light moves faster in a less dense medium (with a small refractive index) than a dense medium (with a large refractive index)……..D

59. For the correction of myopic (short sightedness) eye defect, a concave lens is used. For long sightedness, a convex lens should be used……A

60. In Normal adjustment,

$M = \frac{f_0}{f_e}$

$f_0 = Mf_e = 10 \times 5 = 50cm$

The distance between the lenses=$f_e + f_0$

$= 50 + 5 = 55cm \ldots\ldots\ldots\ldots\ldots\ldots\ldots B$

ELECTROSTATICS

TIPS:

Coulomb's law

$F \propto \frac{q_1 q_2}{r^2} \implies F = \frac{Kq_1 q_2}{r^2}$

$K = \frac{1}{4\pi\varepsilon_0}$

$K = 9 \times 10^9 Nm^2/C$

ε_0=$8.85 \times 10^{-12} Fm^{-1}$

Charge of an electron= -$1.6 \times 10^{-19} C$

Electric field intensity (E) =$\frac{F}{Q}$ (NC^{-1})

V= Ed (volt)

E =$\frac{Kq}{r^2}$

LAUTECH RECENT AND PAST QUESTIONS ON ELECTROSTATICS

1. Permittivity of a material has the unit of
   a) Coulomb/m b) farad/m c) farad/m^2
   d) Coulomb/m^2
2. The electric potential at a point, (d)from an electron of charge (e) placed in a medium of permittivity (ε_0) is given by
   a) $\frac{q^2}{4\pi\varepsilon_0 r}$ b) $\frac{q^2}{4\pi\varepsilon_0 r^2}$ c) $\frac{e}{4\pi\varepsilon_0 d}$ d) $\frac{e}{4\pi\epsilon_0 r}$
3. Determine the magnitude of the strength of a field if a point of charge $2\times10^{-7}C$ experiences a force of 0.02N in this uniform electric field.
   a) $1.0\times10^{-9}NC^{-1}$ b)$4\times10^{5}NC^{-1}$
   c)$1\times10^{4}NC^{-1}$ d)$1\times10^{5}NC^{-1}$
4. A potential difference of 3.6V is maintained between two plates which are 20cm apart. Calculate the electric field intensity between the plates a) 16.0V/m b) 18.0V/cm c) 18.0V/m d) 16.0V
5. A capacitor consists of two parallel plates separated by a layer of air 0.4cm thick, the area of each plate being 202cm^2. Compute its capacitance C
   a) $4.47\times10^{-12}F$ b) $447\times10^{-12}F$
   c) 44.7×10^{-12} d) 44.7×10^{-1} F
6. If two charged plates are maintained at a p.d of 3KV, find the work done in taking a charge of 600μC across a field.
   a) 0.8J b) 1.0J c)1.8J d) 2.0J
7. Find the work done in moving a 2C charge between two points X and Y in an electric field, if the potential difference is 100V
   a)50J b) 400J c) 200 J d) 100J
8. Which of the following is not a dielectric material a) Paraffin wax b) glass
   c)ebonite d) none of the above
9. Gauss's law states that
   a)the total flux through a closed surface is equal to the net charge enclosed by the surface divided by the permittivity
   b) the electric potential in a surface is one
   c) the net charge on a surface can be zero or one
   d) none of the above
10. A 6μF and 12μF capacitors are connected in series. What is their equivalent capacitance
    a) 0.25μF b) 3μF c) 4μF d) 18μF
11. The capacitance of a capacitor is not affected by a) area of plate b) relative permittivity c) thickness of the plates
    d) distance between the plates.
12. The force between two charged particles q_1 equals 2μC *and* q_2 equals -6μC separated by a distance of 20cm is
    a) 2.7N attractive b) 2.7N repulsive
    c)3.5N repulsive d) 2.85 attractive
13. The equivalent capacitance of three capacitors of 5μF each connected in parallel is a)5μF b) 1.67μF c) 0.6μF d)15μF
14. When capacitors are connected in series
    a) the charges on each are different
    b) the charges on each capacitor are equal
    c) the charges on each capacitor cannot be determined
    d) none of the above
15. Calculate the force acting on an electric charge 1.6×10^{-19}C placed in an electric

field of intensity $10^3 V/m$

a)$16 \times 10^{10} N$ b)1.6×10^{-10}

c) 1.6×10^{-16}N d)1.6×10^{-16}

16. Two parallel plates at a distance of $8 \times 10^{-3} m$ apart are maintained at a potential difference of 600V with negative earthed to the ground. What is the electric field strength?

a) 65000N/C b)$7.5\times 10^4 N/C$

c)$7.510^{-4} N/C$ d) 7500N/C

17. 30J of work was done in transferring $5\mu C$ of charges from a point B to A. find the potential difference. a) $6\times 10^6 V$

b) $6\times 10^6 V/m$ c) $6\times 10^6 N$ d) $6\times 10^3 V$

18. _____ is the unit of electric field intensity a) V/m b) V c) N/C d) a & c

19. Which of the following is not an effect of dielectric material on a capacitor

a) separate the plate b) reduce the electric break down c) keep the plate together

d) raise the capacitance of capacitor relative to air

20. _____ is not one of the uses of capacitors a) time base b) ignition system c) telephone d) none of the above

21. The electric field intensity of a point 4cm away from a charge of magnitude $1\mu C$ is

a) $5.62\times 10^{-6} N/C$ b) $5.62\times 10^6 N/C$

c) $5 \times 10^5 NC$ d) $5\times 10^{-5} NC$

22. A 240V A.C supply is supplied to operate electric kettle that has a resistance of 30Ω. Calculate the maximum instantaneous current passing through the electric kettle.

a) 8.00A b) 11.31A c) 0.80A d) 1.13A

23. Which of the following is a characteristic of pure capacitive circuit a) current lags voltage by 180 degree b) current leads voltage by 90 degree c)voltage leads current by 180 degree d) voltage leads current by 90 degree

24. Which of the following is true of resistive A.C circuit a)current and voltage are in phase b) current and voltage are out of phase c)current leads voltage by 45 degree d) Voltage leads current by 45 degree

25. The simplest instrument used in detecting electric charges is the a) electroscope b) ammeter c) voltmeter d) galvanometer

26. The sign of charge on a charged glass rod may be determined with a)a charged electroscope b) an uncharged electroscope c) a galvanometer d) as electrometer

27. The distance between two stationary charged particles is doubled, the magnitude of the electrostatic force between them will be a) doubled b) halved c) a quarter of its former value d) four times the original value.

28. Which of the following is not true?

a) electrical charging can be done by induction

b) the human body is not a conductor of electricity

c)electric field exists around metals which carry electric charges

c) a pointed charged conductor has a high density of charge at the pointed end.

29. If the force on a charge of 0.2C in an electric filed is 4N, then the electric field intensity of the field is a) 0.8N/C b)0.08N/C c) 20.0N/C d) 4.2N/C
30. Why is it impossible for the lines of force of an electric field to cross one another
    a) like charges repel each other
    b) a charge particle is deflected by an electric field
    c) likes charges are arranged on the same lines of force
    d) a positively charged particle can travel in only one direction at any time
31. The instrument used for securing a large number of similar charges by induction is called a)electroscope b) proof plane c) capacitor d)electrophorus
32. Capacitors are used in the induction coil to
    a) dissipate energy b) prevent electric sparks c) prevent distortion of electric fields d) control circuits
33. Calculate the capacitance of a capacitor which stores 2.5×10^{-3}J of energy when a potential difference of 500V is applied across it.
    a) $1\mu F$ b) $5\mu F$ c) $100\mu F$ d)$1000\mu F$

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON ELECTROSTATICS

1. Farad/m……B
2. C
3. Given: charge=$2 \times 10^{-7}C$
   Force=$0.02N$
   Recall; E =$\frac{F}{Q}$ *or*
   $E = \frac{V}{d}$ *or* $E = \frac{Kq}{r^2}$
   Here we have charge and force
   E=$\frac{F}{Q}$
   E=$\frac{0.02}{2\times10^{-7}} = 1 \times 10^5 N/C$……D

4. Given: v=3.6; d=20cm =0.2m
   $E = \frac{V}{d}$
   $E = \frac{3.6}{0.2} = 18.0\, V/m$ … … … … … … . C

5. Given: distance=0.4cm=0.410^{-2}=0.004m
   Area= $202cm^2 = 202 \times (10^{-2}m)^2$
   $= 202 \times 10^{-4}m^2$
   $Capacitance = \frac{E_0 A}{d}$
   $C = \frac{8.85 \times 10^{-12} \times 202 \times 10^{-4}}{0.004}$
   $C = 44 \times 10^{-12}F$ … … … … … … … . D
6. Given: V= 3KV= $3\times1000 = 3000V$
   Q=$600\mu C = 600 \times 10^{-6} = 6 \times 10^{-4}C$
   W= $QV = 6 \times 10^{-4} \times 3000$= 1.8J…….C
7. Given: Q=2C; V=100V
   W= $QV = 2 \times 100$ =$200J$……C
8. All the given materials are dielectric materials ……D
9. A
10. _____|$\frac{6\mu F}{.}$|_____|$\frac{12\mu F}{.}$|_____

$Capacitance\ in\ series = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C} = \frac{1}{6\mu F} + \frac{1}{12\mu F} = \frac{3}{12} = \frac{1}{4}$

C=$4\mu F$ … … … … … C

11. Thickness of the plates……C

12. Given: $q_1 = 2\mu C$; $q_2 = -6\mu C$

$r = 20cm = 0.2m$

$$F = \frac{Kq_1q_2}{r^2}$$

$$F = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 6 \times 10^{-6}}{(0.2)^2}$$

F= 2.7N Attractive……A

Note: The reason it is 2.7N ATTRACTIVE is because the charges were in the opposite directions. One was positive while the other was negative. If they were both positively charged or both negatively charged, the answer would have been REPULSIVE.

13. Given: three capacitors of $5\mu F$ each connected in parallel

$C_T = C_1 + C_2 + C_3$

$= 5\mu F + 5\mu F + 5\mu = 15\mu F$……D

14. From the diagram, C and C are connected in parallel,

$$C_T = C + C = 2C$$

The capacitance is then connected in series with $4\mu F$

Recall that the effective capacitance is $2\mu F$

$$\therefore \frac{1}{2} = \frac{1}{4} + \frac{1}{2C}$$

$$\frac{1}{2C} = \frac{1}{2} - \frac{1}{4} \implies \frac{1}{2C} = \frac{2-1}{4}$$

$$\frac{1}{2C} = \frac{1}{4} \implies 2C = 4$$

$C = 2$ … … … … … … … … A

15. When capacitors are connected in series, the charges on each capacitor are equal……B

16. Given: electric field intensity =1000V/m

Q= $1.6\times 10^{-19}C$; force=?

Recall; $E = \frac{F}{q}$

$F = Eq = 1.6 \times 10^{-19} \times 1000$

$F = 1.6 \times 10^{-16}N$ … … … … … C

17. Given: distance= $8\times 10^{-3}m$; V= 600V;

E=?

V=Ed;

$$E = \frac{V}{d} = \frac{600}{8 \times 10^{-3}}$$

E=$7.5 \times 10^4 N/C$

18. Given: work done=30; q=$5\mu C$

W=QV

$$V = \frac{30}{5 \times 10^{-6}} = 6 \times 10^6 V \text{ … … … … } A$$

19. D

20. Keep the plate together……C

21. D

22. Given: q=$1\mu F = 1 \times 10^{-6}F$

R=4cm= 4/100= 0.04m

$$E = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{(0.04)^2}$$

=$5.62 \times 10^6\ N/C$……B

23. $V = IR$

$V = 240V$

$R = 30\Omega$

$$I = \frac{V}{R} = \frac{240}{30} = 8A \text{ … … … … … } A$$

24. B

25. A

26. Electroscope measures charges. Galvanometer is simpler than an electroscope for charge measurement……D

27. A charged electroscope……A

28. f $\propto \frac{1}{r^2}$

$$f = \frac{1}{2^2} = \frac{1}{4}$$

The force is a quarter of its former value……C

29. B

30. Electric field intensity, E=$\frac{F}{q}$

$E = \frac{4}{0.2} = 20N/C \ldots\ldots\ldots\ldots C$

31. D

32. Electrophorus ……..D

Electrophorus is a machine for transferring and storing charges by induction

Proof plane is used to transfer charges

Capacitor stores charges when a battery is connected across its terminals

Electroscope is used to detect, test and compare charges.

33. B

34. w=QV

$2.5 \times 10^{-3} = Q \times 500$

$Q = \frac{2.5 \times 10^{-3}}{500} = 5 \times 10^{-6} = 5\mu F \ldots\ldots\ldots B$

LAUTECH RECENT AND PAST QUESTIONS ON DIRECT CURRENT ELECTRICITY

1. A 6V battery of internal resistance of 0.5Ω is connected across a high bulb of 2 resistances. What is the reading of a voltmeter connected across the bulb?
   a) 4.80V b) 4.90V c) 4.98V d) 5.08V
2. The primary use of a Wheatstone bridge is
   a) measurement of current
   b) measurement of resistance
   c) measurement of voltage
   d) all of the above
3. The circuit of bulbs with switches in a building having different rooms is usually in
   a)series b)interface c)parallel d)serial
4. The sensitivity of a galvanometer can be increased by the use of
   a) less turn on the coil b) heavy pointer
   c) weaker hairspring d) weaker magnet
5. The difference between a generator and a motor is that
   a) a motor converts electrical energy to mechanical energy whereas a generator converts electrical energy to mechanical energy
   b) the motor uses Fleming's right hand rule whereas a generator uses Fleming's left hand rule
   c) a motor converts electrical energy to mechanical energy whereas a generator converts mechanical energy to electrical energy
   d) a motor uses a split ring as well as a generator
6. The following are examples of an insulator except a) rubber b) germanium c) mica d) glass
7. Find the power consumed by an electric heater of resistance 20Ω for a current of 13A a) 3.38KW b)2.38KW c) 1.38KW d) 2.50KW
8. A material that obeys ohm's law is _____
   a) dry wood b) glass c) Aluminium d) mica

9. The following affects the electrical resistance of a wire except
   a) mass b) temperature c) length
   d) cross sectional area
10. What resistance must be placed in parallel with a 20Ω resistor to obtain an equivalent 5Ω resistance a) 7.66Ω b) 6.67Ω
    c) 5.6Ω d) 4.5Ω
11. A wire of length 90cm and diameter of 0.3mm has resistivity of 11×10^{-6}. Calculate its resistance
    a) 0.022Ω b) 22Ω c) 0.033Ω d) 140Ω
12. If two charged plates are maintained at potential difference 3000V, the work done in taking a charge of 600μC across the field is a) 1.8J b) 1.6J c) 1.4J d) 1.2J
13. The mains voltage suitable for a 60W filament lamp with resistance 735Ω is
    a) 120V b) 210V c)240V d)110V
14. When resistors are connected in parallel, the voltage across each resistor are
    a) increased as the resistors increase
    b) the same c) decreased as the resistors decrease d)different
15. A lamp bulb is rated 60W for 240V supply. What is the resistance?
    a) 690Ω b) 609Ω c) 906Ω d)960Ω
16. Which of the following can be used to measure resistance a) Ohmmeter b) Ohm
    c) volt d) voltmeter
17. Ohm's law states that
    a)the current through a metallic conductor is zero
    b)the current through a metallic conductor is directly proportional to the p.d across its end provided the temperature and other physical conditions vary
    c) the current through a metallic conductor is directly proportional to the p.d across its end provided the temperature and other physical conditions are constant
    d)none of the above
18. A copper wire has a resistance of 10.0Ω at20^0C. What will be its resistance at80^0C?($\propto = 0.0004$)
    a) 12Ω b)12.4Ω c) 13.4Ω d) 5Ω
19. The sensitivity of a galvanometer is increased by the following except
    a) more turns on the coil b) a weak magnet c) weaker hairspring d) a light pointer
20. Two 240Ω light bulbs are connected in series with a 120V power source. What is the current in each bulb?
    a) 0.2A b) 0.25A c) 0.52A d)250A
21. _____ is an instrument with the highest resistance a) Ammeter b) galvanometer
    c) voltmeter d) millimeter
22. Electrical resistance is a property of an electrical conductor that causes electrical energy to be converted into _____
    a) chemical energy b) heat energy
    c) magnetic d) solar energy
23. Kirchhoff's law of current states that
    a) the sum total current across anode is zero
    b) the sum of the current across anode is less than zero
    c) the current at a particular point is equal

to 1

d) the sum total across anode is not zero

24. An avometer is also called

a) multiplier b) ammeter

c) galvanometer d) voltmeter

25. Which of the following is not an application of a potentiometer

a) the measurement of internal resistance

b) measurement of E.M.F c) measurement of current d) none of the above

26. The resistance of an open series circuit is

a) zero b) half the normal resistance of the circuit c) double the normal resistance of the circuit d) infinity

27. A 24V potential difference is applied across a parallel combination of four 6 ohms resistors. The current in each resistor is a) 1A b) 4A c) 16A d) 8A

28. A cell of unknown EMF and of internal resistance 2Ω is connected to a 5Ω. If the terminal potential difference of the cell is 1.0V, determine the value of the EMF

a) 0.4V b) 0.6V c) 1.0V d) 1.4V

29. A cell of EMF 2V is connected to a potentiometer and the balance length obtained is 30cm. when a cell X is connected to the same potentiometer, the balance length becomes 45cm. calculate the EMF of X a)3V b) 4V c) 5V d) 6V

30. Resistance is a measure of_____

a) Volume b) resistor c) temperature d) time

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON DIRECT CURRENT ELECTRICITY

1. Given: $v = 6v; r = 0.5\Omega\,; R = 2\Omega$

$$I = \frac{V}{R+r} \implies I = \frac{6}{2+0.5}$$

$$= 2.4\,A$$

$recall\ that\ v = IR = 2.4 \times 2 = 4.8\,V \ldots\ldots A$

2. Measurement of resistance……B

3. Series……A

4. Weaker hairspring……C

5. C

6. Germanium ……B

7. Given: $R = 20\Omega\,; I = 13A$

Recall; $P = I^2R$

$P = 13^2 \times 20$

Power=3380=3.38KW……A

8. Aluminium ……C

Note: it is only metals that obey ohm's law…

9. Mass ……A

10. Resistance in parallel can be arranged as thus;

$$\frac{1}{R} = \frac{1}{R} + \frac{1}{R} \implies \frac{1}{5} = \frac{1}{20} + \frac{1}{R}$$

$$\frac{1}{R} = \frac{1}{5} - \frac{1}{20} \implies \frac{1}{R} = \frac{4-1}{20}$$

$$\frac{1}{R} = \frac{3}{20} \implies 3R = 20$$

$R = 6.67\Omega$……B

11. Given: $l = 90cm = 0.9m;$

$$d = 0.3mm = 0.0003m$$

Recall; $area(A) = \frac{\pi d^2}{4}$

$A = \frac{3.142 \times (0.0003)^2}{4}$

$A = 7.07 \times 10^{-8}$

$$R = \frac{\rho l}{A}$$

$$R = \frac{11 \times 10^{-6} \times 0.9}{7.07 \times 10^{-8}}$$

$R = 140\Omega$……D

12. Given: p.d=3000V; q=600$\mu C = 600 \times 10^{-6}$

q= 6×10^{-4}

Recall; work done (W) = qv

W= $6 \times 10^{-4} \times 3000$

W=1.8J……A

13. Given: power=60W; R=735Ω

$P = \frac{V^2}{R}$

$v = \sqrt{PR} = \sqrt{60 \times 735}$

V=210V……B

14. The same……B

15. Given: V=240V; power=60W

Recall; p=$\frac{V^2}{R}$

$R = \frac{V^2}{P}$

$R = \frac{(240)^2}{60} = 960\Omega \ldots\ldots\ldots\ldots\ldots\ldots D$

16. Ohmmeter……A. Ohm is the unit of resistance.

17. C

18. Given: R=10.0Ω ; $\theta = 20^0C$

Using; ∝= $\frac{\Delta R}{R\Delta T}$

$\Delta R = \propto R\Delta T = 0.0004 \times 10 \times (80 - 20)$

$= 0.0004 \times 10 \times 60$

$\Delta R = 2.4\Omega$

$resistance\ at\ 80^0C = R + \Delta R$

$= 10.0 + 2.4 = 12.4\Omega$……B

19. Weak magnet……B

20. Two 240Ωresistors connected in series has equivalent resistance as:

R=$R_1 + R_2 = 240 + 240$

R=480Ω; V=120V; I=?

Recall; I=$\frac{V}{R} = \frac{120}{480}$

$= 0.25A$……B

21. Voltmeter……C

22. Heat energy……B

23. Kirchhoff's law of current states that the sum total current across a node is ZERO……A

24. An avometer is called a multiplier……A

25. None of the above……D

26. An open circuit delivers no current. From ohm's law, R =$\frac{V}{I} = \frac{V}{0} = infinity$.

Therefore, the resistance in an open circuit is INFINITY……D

27. The four resistors were connected in parallel,

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1+1+1+1}{6}$$

$\frac{4}{6} = \frac{2}{3}\Omega \quad \Longrightarrow \quad R = \frac{3}{2}\Omega$

$from\ ohm's\ law, I = \frac{V}{R}$

$I = \frac{24}{2/3} = \frac{24}{3} \times 2 = 16A \ldots\ldots\ldots\ldots\ldots C$

28. R=5Ω, r=2Ω, V=1.0V

Recall, I=$\frac{V}{R+r}$

V= I(R+r)

Find the current in the circuit.

I=$\frac{V}{R} = \frac{1}{5}$

I= 0.2A

V=I(R + r)

V=0.2(5+2)

V=0.2(7) =1.4V……D

29. In potentiometer, EMF $(E) \propto L$

E=K L

K=$\frac{E}{L}$

$\frac{E_1}{L_1}=\frac{E_2}{L_2}$

$\frac{2}{30}=\frac{X}{45}$

E_2=$\frac{90}{30}=3V$……A

30. Temperature ……C

SIMPLE A.C CIRCUIT AND ELECTROMAGNETIC INDUCTION

A current which varies periodically is called an alternating current (A.C). The electric circuit in which the A.C flows is called an A.C circuit.

The average power dissipated in an A.C circuit is given as $P_{av} = {}^1\!/_2\, I_0^2 R$

Root means square (rms) current is defined as the steady or direct current which produces the same heating effect per second in a given resistor.

Root means square current:

I_{rms}=$\frac{I_0}{\sqrt{2}}$=$0.7071I_0$

Similarly, $v_{rms} = \frac{V_0}{\sqrt{2}} = 0.7071V_0$

In a pure resistive A.C circuit, current and voltage are in phase.

In a capacitive circuit, current leads voltage by 90^0

In an inductive circuit, current lags voltage by 90^0

When a circuit contains both resistor and capacitor, the impedance (resistance) is given as: Z=$\sqrt{R^2 + X_C^2}$

When a circuit contains a resistor and an inductor, the impedance (resistance) is given as: Z=$\sqrt{R^2 + X_L^2}$

When a circuit contains an inductor and a capacitor, the impedance (resistance) is given as: Z= $\sqrt{(X_L^2 - X_C^2)}$

When a circuit contains a resistor, a capacitor and an inductor, the impedance (resistance) is given as: Z = $\sqrt{R^2 + (X_L - X_C)^2}$

LAUTECH RECENT AND PAST QUESTIONS ON SIMPLE A.C CIRCUIT AND ELECTROMAGNETIC INDUCTION

1. Average power dissipated in RLC current is given by a) I^2X b) I^2X_C c) I^2R d) I^2V
2. Current lags behind the voltage in
   a) an R-C circuit b) a pure inductor current
   c) a pure capacitor current d) R-L-C circuit
3. A wire carrying current of 10A and 2.0m length is placed in a field of flux density $1.5 \times 10^{-1}T$. What is the force on the wire if it is placed at 45^0 to the field
   a) 12N b) 1.2N c) 21.2N d) 2.12N
4. Tesla is a unit of a) weber b) flux
   c) magnetic field d) EMF
5. An A.C generator consists of the following except_____ a) magnet b)one slip ring
   c) an armature d) two slip rings
6. The process of converting alternating current to direct current by p-n junction diodes is called
   a)amplification b) clipping
   c) rectification d)transformation
7. A 40m long wire carrying a current 10A is placed in a field of a flux density 0.012T, what is the force experienced when two wires are placed parallel to the field?
   a) 36.4N b)0 N c) 4.8N d) 0.0048N
8. Which of the following is an example of a ferromagnetic materials
   a)nickel b)copper c) iron d) a & c
9. Which of the following equations gives the correct relationship between the peak value I_0 and root mean square I_{rms} of an alternating current?
   a)$I_o = \sqrt{2I_{rms}}$ b) $I_o = \frac{I_{rms}}{\sqrt{2}}$
   c)$I_o^2 = \sqrt{2I_{rms}}$ d) $I_0 = \sqrt{2} \times I_{rms}$
10. The following are basic elements of transformer except a) core b) primary winding c) secondary winding d) mutual flux
11. What will be the resonant frequency of an RLC series circuit whose inductance and capacitance are 0.12H and 12μF respectively? a) 120Hz b) 125Hz
    c)133Hz d) 240Hz
12. Which of the following is a feature of an ideal transformer a) the primary resistance is one (1) b) the primary resistance is zero c) the secondary is on close circuit d)there is flux leakage
13. A transformer connected to a 120V A.C power line has 200 turns in its primary winding and 50 turns in secondary. The voltage across the secondary is
    a) 45V b) 60V c) 30V d)15V
14. Current must be maximum for resonance in an alternating current circuit because
    a) impedance is maximum b) impedance is minimum c) impedance is unity d) zero
15. A 240V A.C supply is used to operate electric kettle that has a resistance of 30Ω. The maximum instantaneous current

passing through the electric kettle is?
a) 8.01A b)10.52A c) 11.23A d)11.32A

16. Which of the following is not one of the ways of increasing an A.C current
a)increase the number of turns in the coil
b)increase the flux c) winding the coil on soft iron d) none of the above

17. The following are not active devices except a) resistor b) transistor c) rectifier d) diode

18. In a RLC circuit, power is dissipated in
a) resistor b)ammeter c) capacitor
d) inductor

19. If a household voltmeter delivers a voltage of 240V to 300W hot wire, the peak voltage is____ a) 33.94 V b) 3.399V c) 339.4V d)3394V

20. The negative sign in faraday's law of induction is an expression of a)Lenz's law b) newton's law c)kepler's law d)Boyle's law

21. An alternator is referred to as
a) DC motors b)DC generators
c) A.C generator d)diode

22. Inductance has the unit of
a) volt b) Tesla c) weber d) Henry

23. What is the energy stored in a coil of 15×10^{-3}H which carries 0.1A
a)7.5×10^{-5}J b)$7.5 \times 10^{-5}J$
c)$5.710^{-5}J$ d) $7.5 \times 10^{5}J$

24. The production of electric current by moving a conductor in a magnetic field is
a) magnetism b) electric conduction
c) magnetic induction d)electromagnetic induction

25. The energy stored in an inductor is _____
a) chemical b) electrical c)solar d)heat

26. Which of the following is not part of an A.C generator a) split ring b)slip ring
c)carbon brushes d) field magnet

27. A step up transformer increases
a) power b)current c)voltage d)period

28. A step down transformer is used on a 2200V line to deliver 110V. How many turns are on the primary winding if the secondary has 25 turns?
a) 500 b)250 c)700 d) 1000

29. The total resistance of a mixed circuit containing any two or all the circuit elements is called a)impedance
b)inductance c) capacitance d)capacitor

30. Which of the following has the same unit as resistance? a) resistivity b) inductance
c)impedance d)voltage

31. A sinusoidal, 50Hz AC voltage is read to be 110V by an ordinary voltmeter. What is the peak value the voltage takes during the cycle?
a) 176V b) 156V c) 651 V d) 165 V

32. ……..is a ferromagnetic material
a) rubber b)Silver c) iron d) wood

33. Which of the following exhibits a unidirectional behavior a) resistor
b)capacitor c) diode d)inductors

34. Some metals and alloys that obey ohm's law are called a)ohmic conductors
b) non ohmic conductors c) alloy conductors d)rear alloy conductors

35. Conductance of a conductor can be referred to as

a)the reciprocal of the resistance

b) the distance between two conductors

c) the resistance of a diode

d)the reciprocal of temperature

36. The process in which an E.M.F is induced in a circuit whenever there is a change in magnetic flux is

a) electromagnetic field

b) electromagnetic circuit

c) electromagnetic induction

d) induction

37. When the magnetic flux threading a circuit is changing, an E.M.F is induced in the circuit. This is a statement of_____

a) Faraday's first law of electromagnetic field b) Faraday's first law of electromagnetic induction c) Faraday's second law of electromagnetic field

d) Faraday's second law of electromagnetic induction.

38. In Fleming right hand rule, the thumb points in the direction of_____

a) the field b) motion c)induced current

d) none of the above

SOLUTIONS TO QUESTIONS ONSIMPLE A.C CIRCUIT AND ELECTRIMAGNETIC INDUCTION

1. Power dissipated in RLC=I^2R.....C
2. Current lags behind the voltage in a pure inductor circuit .....B
3. Given: l=2m; flux density(B)= $1.5\times 10^{-1}T$

   $\theta = 45^0$;

   $current(I) = 10A$F=?

   The above given parameters are related by the formula below

   $F = BILsin\theta$

   $F = 1.5 \times 10^{-1} \times 10 \times 2 \times sin45$

   $2.12N$ ..... D
4. Tesla is the unit of MAGNETIC FIELD.....C
5. One slip rings.....B
6. Rectification.....C
7. Given: l=40m; flux density= 0.012T, I=10A; F=?

   NOTE: since the wires are placed parallel to the field, the angle between the wires and the field is ZERO. $\theta = 0$

   $F = BILsin\theta$

   $F = 0.012 \times 10 \times 40sin0$

   $F = 0\ Newton$ .....B
8. D
9. D
10. Mutual flux.....D
11. Given: L=0.12H ;

    C=12$\mu F = 12 \times 10^{-6}$

$$F_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$F_0 = \frac{1}{2\pi\sqrt{0.12 \times 12 \times 10^{-6}}} = 133Hz \ldots \ldots C$$

12. The primary resistance is zero.....B
13. Given; $V_0 = 120V$; $N_P = 200$

    $N_S = 50; V_S$ =?

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

$$V_S = \frac{V_P \times N_S}{N_P} = \frac{120 \times 50}{200} = 30V$$

    The voltage across the secondary is 30V.....C
14. Impedance is minimum.....B

15. $I_{rms} = \frac{V_{rms}}{R}$

$I_{rms} = \frac{240}{30} = 8\ A$

$I_0 = I_{rms}\sqrt{2} = 8 \times \sqrt{2} = 11.32\ A \ldots\ldots D$

16. None of the above……D

17. Transistor……B

18. Resistor……A

19. Given: $V_{rms} = 240V$

V_{max} =?

$V_{max} = V_{rms}\sqrt{2}$

$V_{max} = 240\sqrt{2} = 339.4\ V \ldots\ldots\ldots\ldots C$

20. Lenz's law……A

21. A.C generator……C

22. Henry ……D

23. E=$\frac{1}{2} = I^2L$

$E = \frac{1}{2} \times 0.1^2 \times 15 \times 10^{-3}$

$= 7.5 \times 10^{-5} J \ldots\ldots\ldots\ldots\ldots A$

24. Electromagnetic induction ……D

25. Heat ……D

26. Split ring……A

27. Voltage ……C

28. Given: $V_P = 2200V;\ V_S = 110V$

$N_S = 25;\ N_{p=}$?

$\frac{V_P}{Vs} = \frac{N_P}{N_S}$

$N_P = \frac{V_P \times N_S}{V_S}$

$= \frac{2200 \times 25}{110}$

$N_P = 500 \ldots\ldots\ldots\ldots\ldots A$

29. Impedance……A

30. Impedance has the same unit as resistance which is OHM(Ω)……C

31. Given: $f = 50Hz;\ V_{rms} = 110V$

$V_{rms} = \frac{V_0}{\sqrt{2}}$

$v_0 = 110\sqrt{2} = 156V$

The peak voltage is $156V \ldots\ldots\ldots\ldots B$

32. Iron……C

33. Diode ……C

34. A

35. A

36. Electromagnetic induction……C

37. Faraday's first law of electromagnetic induction……B

38. Motion……B

ATOMIC AND NUCLEAR PHYSICS

Half-life is the time taken for a radioactive element to disintegrate to half its original mass.

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

N=$N_0 e^{-\lambda t}$

Where N_0 *is* the initial mass

N is the mass final mass.

N_0 is always greater than N.

λ= radioactive constant.

Average life or mean life $(t_m) = \frac{1}{\lambda}$

$t_m = \frac{t_{\frac{1}{2}}}{0.693}$

$\frac{N}{N_0}$= The fraction or mass left/remained after the decay.

$1 - \frac{N}{N_0}$= The fraction or mass that decayed

Alpha (α) particle is 7500 times heavier than beta (β) particle and beta particle is 7500 times less than alpha particle.

Alpha particle (α) is slightly deviated by magnetic and electric field. it is positively charged.

Beta particle (β) is strongly deviated by both magnetic and electric field. it is negatively charged.

Gamma Ray (γ) is not deflected by both magnetic and electric field. it is uncharged.

LAUTECH RECENT AND PAST QUESTIONS ON ATOMIC AND NUCLEAR PHYSICS

1. Which of the following radiations cannot be deflected by an electric field or magnetic field a) alpha and beta rays b) gamma rays only c) alpha and gamma rays d) beta rays only
2. Which of the following radiations is non - ionizing? a) neutron radiation b) ultra-violet c) radio frequency(micro wave) radiation d) laser radiation
3. Which of these radiations has a strongly ionizing power? a) β b)α c) γ d) σ
4. A specimen of element X of mass 1.0g was left for two weeks and then analyzed. It was found to contain only 0.25g of X. calculate its half-life a) 48 hrs.b)168.0hrs c) 150 hrs. d) 761 hrs.
5. which of the following is positively charged a) proton b) neutron c) electron d) gamma ray
6. Which of these radiations has a very high penetrating power? a) alpha particle b) beta particle c) gamma ray d)none
7. Which of these has the highest mass a) alpha particle b) positron c) negatron d) gamma ray
8. Which of the following contains electromagnetic rays a) alpha particle b) beta particle c) gamma d) none
9. A radio-active nuclei has 9600 counts per minute at 8:00am. When will the count rate decrease to 300 counts per minute if its half-life is 15 minutes? a) 1:15am b) 1:15pm c) 9: 15am d) 9:15pm

10. When atoms have the same neutrons but different atomic number, they are referred to as a) isotopes b) isobars c) isotones d)Isotopy
11. 1 g of a radio-active radium 226 decays with a half-life of 1620 years. calculate the disintegration constant
a)$1.36 \times 10^{-11}s^{-1}$ b) $1.36 \times 10^{-11}\text{m}^{-1}$
c) $5.36\times 10^{-11}hr^{-1}$ d)$1.36 \times 10^{-1}\ yr^{-1}$
12. _____occurs when two light nuclei are fused together a) fission b) fussion
c) fusion d) radioactivity
13. X-rays are produced by energy changes in
a) the nucleus b) electrons far from the nucleus c) electrons close to the nucleus
d) electrons and protons
14. By whom was natural radioactivity discovered a) Fermi Becquerel b) Joseph John Thompson c) Henri Becquerel
d) Wilhelm Conrad Roentgen
15. Find the radioactive constant when the half-life is 36mins
a) 0.0193/sec b)0.0193min
c) 0.0193sec d)0.0193/min
16. The reciprocal of half-life is called
a)mean life b) average life c) average mean life d) decay constant
17. Which of the following is not of the four basic types of ionizing radiation?
a) alpha particles b) radio waves
c)neutrons d) gamma rays
18. The mechanism that causes damage to cells from radiation exposure is
a)excitation b) cancer c) ionization
d)tumor
19. Which of the following is not part of the fundamental particles from which all atoms are constructed? A) protons b)photons
c) neutrons d) electrons
20. Which of the following determines the chemical behavior of element? a)isotone
b) isomers c) isobar d)isotope
21. The three methods by which the external radiation hazards occur include the following except a) time b) distance c) acceleration d) shielding
22. It is known that a neutron exist in a light atomic nucleus, which of the following also exist in the nucleus a) an electron b) an x-ray c) a proton d) an atom
23. The half-life of radium is 10days. After how many days will only $\frac{1}{16}$ of radium sample remain? a) 10 days b) 20 days c) 30 days d) 40 days
24. The half-life of a radioactive substance is 6 days. If we have 1.28g of X initially, what is the mass of X after 30 days?
a)0.32g b) 0.16g c) 0.08g d) 0.04g
25. If a radioactive atom emits a beta particle, its mass number
a) increases by 1 b) remains the same
c) decreases by 1 d) increases by 2
26. A nucleus has a proton number of 84. It emits an alpha particle and then a beta particle to achieve stability. What is the proton number of the product a) 81 b) 82 c) 83 d)84
27. The work function of a mental is 4.65eV and the metal is illuminated with a radiation of 6.86eV. What is K.E of the electron ejected

from the surface of the mental?

a) -11.51eV b) 11.51eV c)-2.21eV d) 2.21eV

28. Radium has a work function of 1.6eV. Its threshold wavelength is?

a)$777\times 10^{-9}m$ b) $7.29\times 10^{-2}m$

c)$723\times 10^{4}m$ d) $777m$

29. A base ball of mass 0.154g is thrown with a speed of 45.0m/s. What is the de Brogile wavelength of the ball

a) 9.6×10^{-35} b) $6.9\times 10^{-35}m$

c) $9.6\times 10^{-30}m$ d)$9.6\times 10^{-35}m$

30. The process through which electrons are ejected from a metal surface when light is incident on it is called a) thermionic emission b) photo electric effect c) photon electron d)thermo effect

31. The energy gained by proton accelerated through a p.d of 0.21V is a)0.12eV b)0.123eV c) 1.02eV d) 0.21eV

32. Which of the following statements is correct a) alpha is negatively charged b) the mass of gamma is one c) beta is heavier than alpha d) none of the above

33. Which of the following is NOT correct

a) radiation is of two sources: natural and man-made b) alpha particles are a major source of natural background radiation c) proton is positively charged d) gamma has high mass number

34. Man-made sources of radiation include the following except a) thorium b) uranium radium c) oxygen d) actinium

35. The work function of lithium is 2.3ev. calculate the maximum energy in joules of photoelectrons liberated by light of wavelength $3.3\text{x}10^{-7}$m

a) $2.32\text{x}10^{-16}$J b)$2.32\text{x}10^{-17}$J

c) $2.32\text{x}10^{-18}$J d)$2.32\text{x}10^{-19}$J

36. Which of the following is not a danger of radiations

a) protection from cancer b)Leukemia

c) genetic mutation d) eye cataracts

37. Which of the following is correct?

a) $N = N_o e^{\mu}$ b) $N = N_o e^{-\mu}$

c) $t_1 = \frac{-0.693}{\mu}$ d) $A = A_o e^{\mu 1}$

38. The half-life of a radioisotope is 92seconds. The N/N_o left after 2hrs is

a) 54.22 b) 0.368 c) 650.59 d) $2.8\text{ x }10^{-24}$

39. If a radioactive material has a half-life of 18 days, what fraction of the radioactive nuclei will remain after 54 days?

a) 1/3 b) 3 c)1/8 d) 8

40. The portion of the spectrum which produces most sensation of heat is the portion where we have

a) x-rays b) infrared c) ultraviolet d) gamma Rays

41. The instantaneous rate of decay per unit atom of a substance is called?

a) decay rate b) decay constant c) half-life d) radioactivity

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON ATOMIC AND NUCLEAR PHYSICS

1. Gamma rays……..B. Alpha and beta particles are deflected by both electric and magnetic field.

2. Neutron radiation…. ……A

3. $\alpha\ (alpha)$……..B.

Note: Alpha particles are the most strongly ionized particles but they have the least penetrating power.

Beta particles are more ionized than gamma rays but not as ionized as the alpha particles. Beta particles have higher penetrating power than the alpha particles but not as great as that of gamma rays.

Gamma rays are the least ionized but the rays with the highest penetrating powers

4. It is advisable to always check the answer before solving in order to know the unit to work towards.

2 weeks $=2 \times 7 \times 24 = 336hrs$

Recall;

$$N = N_0 e^{-\lambda t}$$

Note that N_0 is the initial mass and is always greater than N

$$\frac{N}{N_0} = e^{-\lambda t} \implies \frac{0.25}{1} = e^{-336\lambda}$$

$0.25 = e^{-336\lambda}$

Apply log_e to both sides

$log_e 0.25 = log_e e^{-336\lambda}$

$log_e = \ln and\ \ log e_e = 1$

$ln 0.25 = -336\lambda$

$-1.3863 = -336\lambda$

$\lambda = 4.126 \times 10^{-3}/hrs$

Recall;

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$t_{\frac{1}{2}} = \frac{0.693}{4.126 \times 10^{-3}}$$

$= 168hrs$……B

5. Proton ……..A
6. Gamma rays……..C
7. Alpha particles have the highest mass among the options given……..A
8. Gamma contains electromagnetic waves……..C
9. Given: $N_0 = 9600\ ; N = 300; t_{\frac{1}{2}} = 15mins$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} \implies \lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

$$= \frac{0.693}{15} = 0.0462/min$$

N=$N_0 e^{-\lambda}$

$300 = 9600e^{-0.0462}$

$$\frac{300}{9600} = e^{-0.0462t} \implies 0.03125 = e^{-0.0462t}$$

Apply $log_e$to both sides.

$log_e 0.03125 = log_e e^{-0.0462t}$

$ln 0.03125 = -0.0462t$

$-3.4657 = -0.0462t$

$t = 75\ mins = 1\ hr: 15\ mins$

Recall that previous time was 8:00am

:. Add 1hr: 15mins to it.

8:00am + 1: 15=9:15am……..C

10. Isotones ……..C

11. Given: $t_{\frac{1}{2}} = 1620$ years

Convert 1620 years to seconds

$1620\times 365 \times\times 24 \times 60 \times 60$

$= 5.1088 \times 10^{8} sec$

$$t_{\frac{1}{2}} = \frac{0.693}{5.1088 \times 10^{8}}$$

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} \implies \lambda = \frac{0.693}{5.1088 \times 10^{8}}$$

$\lambda = 1.36 \times 10^{-11}/sec$……..A

12. Fusion ……C

13. Electrons close to the nucleus ……C

14. Henri Becquerel……C

15. $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$$\lambda = \frac{0.693}{36} = 0.0193/mins \ldots\ldots\ldots\ldots\ldots\ldots D$$

16. Average mean life……C

17. Radio waves……B

18. Ionization ……C

19. Photons ……B

20. Isotopes ……D

21. Acceleration……C

22. Proton……C

23. $t_{\frac{1}{2}} = 10$ days

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} \implies \lambda = \frac{0.693}{10}$$

$\lambda = 0.0693/days$

$$\frac{N}{N_0} = \frac{1}{6} \implies \frac{1}{6} = -e^{\lambda t}$$

$$log_e \frac{1}{6} = log_e e^{-\lambda t}$$

$$-2.773 = -0.0693t$$

$t = 40\ days$……D

24. $t_{\frac{1}{2}} = 6\ days\ ; N_0 = 1.28g \quad t = 30\ days$

$recall\ t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$$\lambda = \frac{0.693}{6} \implies \lambda = 0.1155/days$$

$$N = N_0 e^{-\lambda t} \implies N = 1.28 \times e^{-0.1155\times 30}$$

$N = 0.04g \ldots\ldots\ldots D$

25. Beta particle is represented $with\ \frac{0}{-1}e$. The mass of the new radioactive element formed remains the same but the atomic number increases by 1 ……B

Note: If an alpha particle is emitted, the mass of the new radioactive element formed decreases by 4 and its atomic number decreases by 2 since an alpha article is represented with a helium atom, $\frac{4}{2}He$

26. $Z_{84} \longrightarrow Y_{83} + \frac{4}{2}He + \frac{0}{-1}e$……B

27. Given: W=6.86eV ; $W_0 = 4.65eV$

K.E=W $- W_0$

K.E=6.86eV $-4.65eV$

K.E=2.21eV……D

28. $Given: \phi = 1.6eV;\ \lambda_0 = ?$

$1eV \quad 1.602 \times 10^{-19} J$

$\phi = 1.6 \times 1.602 \times 10^{-19}$

$= 2.56 \times 10^{-1}\ J$

$$\lambda_0 = \frac{hc}{\phi} = \frac{6.63 \times 3 \times 10^{-34} \times 3 \times 10^{8}}{2.56 \times 10^{-9}}$$

$= 7.77 \times 10^{-7} m$

$\lambda_0 = 7.77 \times 10^{-7}\ or\ 777 \times 10^{-9} m \ldots\ldots\ldots\ldots A$

29. Given: mass=0.154 kg; v=45m/s;

De Brogile wavelength is given as;

$\lambda = \frac{h}{mv}$

$h = i\ \ called\ planck\ constant$

$\lambda = \frac{6.63 \times 10^{-34}}{0.154 \times 45}$

$= 9.6 \times 10^{-3}\ m \ldots\ldots\ldots\ldots\ldots\ldots D$

Unit is very important as far as physics is concerned.

30. Photoelectric effect……B

31. Energy gained = eV

=0.21eV……D

32. None of the above……D

33. Gamma has the lowest mass number……D

34. Oxygen ……C

35. $work\ function = 2.3eV.$

$1eV = 1.602 \times 10^{-19} J$

$W_o = 2.3 \times 1.602 \times 10^{-19}$

$= 3.685 \times 10^{-19} J$

$Kinetic\ Energy = W -\ _o$

$W = hf = \frac{hc}{\lambda}$

$h = 6.626 \times 10^{-34} Js^{-1}$

$c = 3 \times 10^8 ms^{-1} (speed\ of\ lig\ \ )$

$\lambda = 3.3 \times 10^{-7} m$

$\therefore W = \frac{6.626 \times 10^{-3}\ \times 3 \times 10^8}{3.3 \times 10^{-7}}$

$W = 6.02 \times 10^{-19} J$

$KE\ \ \ 6.02 \times 10^{-19} - 3.685 \times 10^{-19}$

$KE = 2.32 \times 10^{-19} J \ldots\ldots\ldots\ldots\ldots\ldots D$

36. Protection from cancer……A.

Regular exposure to radiations causes cancer. It doesn't protect one from getting cancer

37. Remember the radioactive decay formula

$N = N_o e^{-\mu}$ where $\mu = \lambda$ ……B

38. Given : half-life=92 seconds

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{92} = 7.532 \times 10^{-3} s^{-1}$

$also, \lambda = \frac{-2.303}{t} \log \frac{N}{N_o}$

$t = 2hrs\ \ \ 7200secs$

$7.532 \times 10^{-3} = \frac{-2.303}{7200} \log \frac{N}{N_o}$

$7.532 \times 10^{-3}\ \times 7200 = -2.303 log \frac{N}{N_0}$

$\log \frac{N}{N_o} = \frac{54.2304}{-2.303}$=-23.5477

$\frac{N}{N_o} = 10^{-23.5477}$

$= 2.8 \times 10^{-24} \ldots\ldots\ldots\ldots\ldots\ldots D$

39. Given: half-life = 18 days

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{18} = 0.0385 days^{-1}$

$also, \lambda = \frac{-2.303}{t} \log \frac{N}{N_o}$

$t = 54 days$

$0.0385 = \frac{-2.303}{54} \log \frac{N}{N_o}$

$\log \frac{N}{N_o} = -0.9027$

$\frac{N}{N_o} = 10^{-0.9027} = 0.125 = \frac{1}{8}$ ……C

40. Infrared ……B

41. Decay constant……B

LAUTECH RECENT AND PAST QUESTIONS ON SEMICONDUCTORS

1. N –type extrinsic semiconductor is formed when a semiconductor is doped with
   a) gallium b) boron c) lithium d) arsenic
2. Which of the following is a typical application of semiconductor diode
   a) smoother b) charge storer
   c) modulator d) clipper
3. Which of the following is not a semiconductor material
   a) Si b) Ge c) Zn d) GaAs
4. P-type extrinsic semiconductor is formed when a conductor is doped with
   a) phosphorus b)arsenic c) indium d)antimony
5. Another name for common collector is
   a) voltage follower b) transformation device c)emitter restorer d) none
6. Which of the following is true
   a) semiconductor has large energy gap
   b)conductor has no energy gap
   c) insulator has little or no energy gap
   d) the energy gap of insulator is 1eV
7. Which of the following is not a doping agent? a) indium b) gallium c) helium d)phosphorus
8. The process of converting alternating current of direct current by P-N junction diode is called a) rectification b) gravitation c) multiplication d) oscillation
9. The random motion of holes and free electrons due to thermal agitation is called
   a) hybridization b) random motion
   c)diffusion d) conversion
10. When a trivalent impurity is added, the semiconductor becomes a) extrinsic
    b) P-type c) neutral d) positive
11. Which of the following is not a semiconductor a) carbon b) germanium
    c) silicon d) silicon-carbide
12. Which of the following is not the application of P-N junction diode?
    a) clipper b) rectifier c) transformer
    c) clamper
13. The charge carriers in semiconductors are
    a) holes and free electrons b) electrons only c) holes and electrons d) holes only
14. The process of adding impurities to a pure semiconductor is called a) clipping
    b)rectification c) doping d) dropping of electrons
15. _____ is used for measuring the conductivity of a semiconductor a)semen per meter b) Siemen c) Siemen per meter square d) Siemen per meter
16. In common emitter, the signal enters at ___ a) collector b) base c) collector and base d) emitter
17. _____ is not a property of common emitter a) high voltage gain b) suffer phase change c) low voltage gain d) none
18. Which of the following is a disadvantage of half wave rectification a) high efficiency
    b) low efficiency c) over efficient d) under efficient

19. Another name for clipper is a) d.c restorer b)wave shaper c)rectifier d)voltage regulator
20. Which of the following is true about FET
    a) it operates with minority carrier only
    b)it operates with majority carrier only
    c)it operators with minority and majority carriers
    d) It does not operates with carriers
21. N-type extrinsic semiconductor is formed when a semiconductor is doped with
    a)Gallium b) Boron c)lithium d)arsenic
22. When p is connected to the negative terminal of the battery and n is connected to the positive terminal, it is called
    a) forward bias b) potential barrier
    c) positive bias d) reverse bias
23. The following are advantages of semiconductor transistor except
    a) they may be operated at a very low voltages
    b)they consume low power resulting in higher efficiency
    c)they resist damage from shock and vibration
    d) they produce little fluctuation
24. The following are characteristics of common-base connection except
    a) it has high voltage gain
    b)no signal phase change
    c) it suffers fluctuation
    d)it is not useful in multistage amplification
25. Which of the following is not a transistor
    a)BET b) FET c) MOSFET d) SECFET
26. Which of the following is used for smoothing out the large fluctuations in full-wave rectifications a) reservoir capacitor b) reservoir inductor c) uprising diode d) transistor

COMPREHENSIVE SOLUTIONS TO QUESTIONS ON SEMICONDUCTORS

1. D……..Arsenic
2. D……..Clipper
3. C……..Zn
4. C……..Indium
5. Another name for common –collector is voltage follower……..A
6. B
7. Helium……C
8. Rectification ……A
9. Diffusion ……..C
10. P-type……B
11. Carbon ……A
12. Transformation ……C
13. Holes and free electrons……A
14. Doping……C
15. Siemen per meter…….. D
16. Base……B
17. Low voltage gain…….. C
18. Low efficiency……B
19. Wave shaper……B
20. B.
21. Arsenic……D
22. Reverse bias……..D
23. They produce little fluctuation……..D
24. it suffers fluctuation……C
25. SECFET……D

26. Reservoir capacitor……A

CHEMICAL BONDING

The chemical behavior of an atom to a large extent is basically determined by the number and arrangement of electrons on the outer orbitals (shell) of the atom

Electrons on the outer most orbital are called the valence electrons. Atoms combine or bond together to attain a stable electronic configuration state as in noble (group VII) elements e.g. Helium (He) $1s^2$

Note: for an element to be stable, it means it has no unpaired electrons on the outermost shell. Therefore, bonding takes place to share the unpaired electron with another atom and then attaining a stable state.

We have various types of bonds but the most common are IONIC (Electrovalent) bond: This is a type of bond formed by the electro static attraction between positive and negative ions.

It involves the transfer of one or more electron from a metal (electron donor) to the nonmetal (electron acceptor) with the formation of ions. The metal form cation (i.e. +ve ions) having low ionization energy and the nonmetal anion (-ve ions) having large electron affinity value.

Example; The bond between Na and Cl

$$\overset{Na}{1s^2 2s^2 2p^6 3s^1} + \overset{cl}{1s^2 2s^2 2p^6 3s^2 3p^5} \longrightarrow$$

$$\overset{Na^+}{1s^2 2s^2 2p^6} + \overset{cl^-}{1s^2 2s^2 2p^6 3s^2 3p^6}$$

Using Lewis standard

$$Na\cdot + \cdot\overset{\cdot\cdot}{\underset{\cdot\cdot}{Cl}}: \longrightarrow Na^+ + \left[:\overset{\cdot\cdot}{\underset{\cdot\cdot}{Cl}}:\right]$$

Properties of Ionic Compound

They are crystalline solid at ordinary temperature

They have high melting and boiling point due to strong electron static attraction between the oppositely charged ions.

They conduct electricity in molten form therefore are used as electrolyte.

They are soluble in H_2O and other polar solvent due to the electro static attraction between the ions and the polar molecules of the solvent.

COVALENT BOND: This involves the sharing of one or more electrons between two atoms of either the same or different elements. After bonding, the atoms involved obtain the electronic configuration of the nearest noble gas.

The electron pair shared by two atoms are called bonding pair while those that remain on the atom are lone pairs

PROPERTIES OF COVALENT BOND

They exist as separate or big grant molecules

They are at room temperature gases, liquid or solid of low melting points.

They do not conduct electricity therefore are non- electrolyte

They are soluble in non-polar solvent e.g. Benzene

Reaction between them is very slow

COVALENT BOND is special amidst all bonds in the sense that it can form polar and non-polar bond.

POLAR BOND: When the bond is between two atoms of different elements e.g. (C-Cl, H_2–Cl_2) and therefore is called DIPOLE

NON POLAR: When the bond is between atoms of same element. Atoms of same element have the same electrons which are shared equally between the atoms e.g. (Cl-Cl, N-N, O-O)

NOTE: Electronegativity is the ability of atom to attract electron to itself.

COORDINATE COVALENT (DATIVE) BOND

This is a very sensitive case of a covalent bond in which the paired electron share is denoted by one of the atoms only.

e.g.

METALLIC BOND: The attraction between the positive nuclei of all the closely packed atom in lattice and the electron cloud.

This kind of bond occurs between metal and alloys

INTERMOLECULAR FORCES

The weak hydrogen bond is formed by the interaction between polar covalent compounds

Vander Waal's forces is formed by interaction between either polar molecules or non-polar

Polar molecules = dipole-dipole,

Non – polar =induced dipole – induced dipole, polar and non-polar = instantaneous dipole – induced dipole

PAULING ELECTRONGATIVITY SCALE

This is the measure of the ability of an atom on a molecule to draw bonding electrons to itself and produce an inductive effect

The higher the difference in electronegativity value, the greater the percentage of ionic bond character. When the value is zero, pure covalent, if >0 but < 2.1= polar covalent and value of 2.1 and above are almost entirely ionic

QUESTIONS

1. The type of bond that occurs between non polar molecule is
   a) dipole – induced dipole

b) induced dipole – induced dipole
c) polar bond
d) dipole – dipole

2. The ability of atom is attract electron to itself is called
a) electron affinitive b) electronegativity
c) electron attraction d) none

3. Using electronegativity value of 0.5 predict the nature of bond In CCl_4
a) ionic b) polar covalent c) purely covalent d) pole

4. _____ is the force of attraction that bonds atoms of an element to one another to form a more complete substance
a) dative bond b) metallic bond
c) chemical bond d) coordinate covalent bond

5. The bond between LiF is a) ionic b)purely covalent c)dative d) ionic dative

6. The type of bond where the shared pair of electron is donated by one atom is
a) coordinate covalent b) metallic c) ionic
d) intermolecular

7. Hydrogen bond is formed by interaction between
a) polar covalent compounds
b) non-polar covalent compounds
c) polar and non-polar compounds d) none

8. Which of the following bonds carries partial +ve and –ve charges no the opposite ends
a) polar covalent bond b) dative bond
c) ionic bond d) metallic bond

9. Which of the following is not a property of ionic compounds
a) they do not conduct electricity on solid state
b) they are soluble in polar solvents
c) reaction between them is very slow
d) they have high melting and boiling points

10. When is a set of elements said to be isoelectronic
a) they have same electronic configuration
b) when the number of electrons at the outer shell is equal
c) when the electronic configuration are not the same
d) none of the options

11. All these are isoelectronic except
a) O^{-2} b) F^{-} c) Mg^{2+} d) Ar

12. When the bonding pairs of electron is denoted by the atom, the bond is called
a) covalent b) ionic c) hydrogen bond
d) coordinate covalent

13. Which of the following is also called a dipole bond a) polar covalent bond
b) coordinate covalent c) dative d) ionic

14. Every covalent bond is characterized by _____and _____
a) bond and bond energy
b) bond length and one energy
c) bond angle and strength d) none

15. What is the no of electrons transferred from Na to Cl in the formation of NaCl
a) one b) three c) two d) zero

16. What type of bond is presented in NH^{4+}
a) coordinate covalent b) hydrogen
c) ionic d) metallic

17. Which type of bond involves the transfer of an electrons a) ionic b) metallic
c)dative d) chemical

18. Using the shape of atom, which is the odd one out a) Bf_3 b) CCl_4 c) NaCl d) CH_4

19. _____ is an example of molecule with triple bond a) N_2 b) CO_2c) SO_4 d) NH_3

20. HCl has _____ bond a) ionic b) metallic c) polar covalent d) dative

21. Complete the table below

Molecule	Electronegative difference	nature of bond
PCl_5		
NaF		
H_2O		
$CsCl_2$		
I_2		

Where p=2.1, Cl=3.0, Na=0.9, F=4.0, H=2.1, O=3.5, C_S=0.7, I=2.5

22. Pure covalent bond is observed on following except a) Cl_2 b) N_2 c) Hl_2 d) HCL

ANSWERS

1. B
2. B
3. B
4. B
5. A
6. A
7. C
8. C
9. C
10. A
11. D
12. D
13. A
14. D
15. A
16. A
17. C
18. ..... A
19. ..... D
20. ..... A
21. $pcl_5 = 3.1 - 2.0 = 1.0$ (polar covalent)

$Naf = 4.0 - 0.9 = 3.1(ionic)$,

$H_2O = 3.5 - 2.1 = 1.4\ (polar\ covalent)$

$C_5Cl_2 = 3.0 - 0.7 = 2.3(ionic)$

$I_2 = 2.5 - 2.5 = 0$(Purely covalent)

22. Pure covalent can only occur between element of same electronegativity value ..... D

COORDINATION CHEMISTRY

Coordination chemistry deals with the naming of compounds

The following are IUPAC laid down rules for naming coordination compounds

1. In naming coordination compounds, cations are always named before the anions
2. In naming, negative ligands end with the suffix "O" example, fluorine becomes floro, cyanine becomes cyano, nitrate becomes nitrato etc.
3. If in a compound we have several ligands, the naming takes the order neutral ligands e.g. (NH_3ammine), H_2O (aqua) are named before negative (Cl^-=chloro, Br^{-1}=Bromo). Then the ligands (NO^+_2, H_2 NH_3^+) and the naming take alphabetical order. The prefixes ligands di=2, tri=3, tetra=4etc. are used to specify the number of ligands present. dis=2, tris=3 and tetrakis=4 and used for complicated ligands
4. We have neutral ligands, the common examples are NH_3= ammine, pH_3= phosphine, H_2O= aqua, CO=carbonyl and NO=nitrosyl of all these only H_2O aqua changes to **aquo**, others retain their names
5. The oxidation number of metals is designated by Roman numeral in a bracket followed by the name of the complex ion
6. ligands are to be named before the negative ions
7. The suffix used for negative ions(i.e. anion) the positive ions (cations) carries no suffix

Some metal bear their latin name of the suffix **ate** is used e.g. CU=cup rate, Fe=ferrate etc.

QUESTIONS

1. The central ion in $Ni(CO)_4$ is
   a) CO b) C c) Ni d) $(CO)_4$
2. Which is a neutral ligand?
   a) hydroxo b)chloro c) aqua d) cyano
3. _____ is the neutral molecule with the lone pair used to form bond to a metal?
   a) cation b) ligands c) chelates d) ions
4. In naming ligands, we name in the order
   a) positive ⟶ negative ⟶ neutral
   b) neutral ⟶ positive ⟶ negative
   c) neutral ⟶ negative ⟶ positive
   d) negative ⟶ positive ⟶ neutral
5. What is the positive ion on $Na[Au(CN)_2]$
   a) Au b) Na c) $Au(CN)_2$ d) CN
6. A ligand with more than one lone pair that can be in bonding with a metal is called

a) hexadentate b) tridentate c) bidentate d) chelate

7. Ligands that can form more than two bonds with a metal are called a) Polydentate b) tridentate c) tetradentate d) none

8. Transition metals are characterized by the following **except**
a) incompletely filled d-shell
b) incompletely filled d-orbital
c) none d) they are used as catalyst

9. Paramagnetic moment is represented by
a) $\sqrt{n}(n+2)$ b) $\sqrt{n}(n-2)$ c) $n\sqrt{n+2}$ d) none

10. One of the following is not a complex ion
a) $[cr(H_2O)_6]cl_3$ b) $[Cu(CN)_4]^{2-}$
c) $[C_oF_6]^{3-}$ d) $[Ni(H_2O)]^{2+}$

11. The metallic ion in a complex is referred to as the a) ion b) central metal ion c) central donor d) central ligand

12. Unpaired electron increases paramagnetic effect of _____ set of metal a) transition b) alkali c) alkali earth d) non

13. Aminepentachlorocuprate (ii) ion has the formula a)$[NH_3CuCl_5]^{2-}$ b) $[CuNH_3Cl_5]^{2-}$ c) $[CuCl_5NH_3]$ d) none

14. Covalent compounds are made up of
a)molecules b) ions c)atoms d) ligands

15. _____ is responsible for the colouring of transition metals a) completely filled d-orbital b) incompletely filled d - 0rbital c) no idea d) presences of f – orbital

16. Transition metals have what type of bond between atoms a) metallic b) ionic c) covalent d) dative

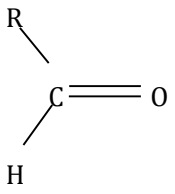
17. Complex ion is made up of a) $metal\ ion + ligands$ b) $metal\ ion\ alone$ c) $ligands + ligands$ d) $none$

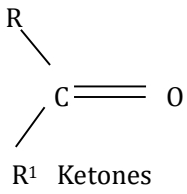
ANSWERS

1. Nickel is the central ion …….. C
2. Aqua …….. C
3. …… b
4. …….. C
5. Na …. B
6. Chelate …… D
7. Polydentate …. A
8. There is nothing like d-shell……. A
9. $\sqrt{n}(n+2)$
10. ……. A
11. ……. B
12. ……. A
13. …… B
14. Molecules…….. A
15. …….. B
16. …….. A
17. ……A

CARBONYL COMPOUND

These are compounds that contain carbonyl group -C=O as the only functional group. The group is made up of two classes ALDEHYDES AND KETONES. The two have a general molecular formula $C_nH_{2n}O$. Aldehydes possess a hydrogen atom attached to the Carbonyl carbon while ketones contain two hydrocarbon groups





Ketones

R= alky group

R,R¹= Alkyl group

ALDEHYDE

In naming aldehydes, the following rules must be obeyed.

1. Identify the longest continuous carbon chain containing the carbonyl group
2. Numbering starts with the aldehyde because it is at the beginning of the chain
3. The last "e" on the parent alkane is replaced with "al"to derive the name

E.g. HCHO methanal

$CH_3CH_2CH_2CH(CH_3)CHO$2-methylpentanal

$CH_3CH_2C{\equiv}C\ CH_2CHO$ hex-3-ynal

$(CH_3)_3CCH{=}CHCH_2CHO$ 5,5-dimethylhex-3-enal

KETONES

In naming, numbering must starts from the end that is closer to the carbonyl group and the last "e" in the parent alkane is replaced with "one" e.g.

CH_3COCH_3 propanone or propan-2-one

$CH_3COCH_2CH(CH_3)_2$ 4-methylpentan-2-one

$CH_3CH(CH_3)COCH_3$3methylbutan-2-one

Note: Aldehydes are readily oxidized and therefore strong reducing agents but ketones are very difficult to oxidize therefore, a test is used to distinguish between aldehydes and ketones.

QUESTIONS

1. One of the following is reducing in nature
   a) ketones b) ester c) alcohol c) aldehyde
2. Which of the carbonyl compounds has its functional group in between the chain
   a) aldehyde b) ketone c) alkane d) none
3. Methanal is otherwise called a) methane
   b) formaldehyde c) carboxylic d) no option
4. A chain that contains maximum number of hydrogen is said to be a) saturated
   b) unsaturated c) aldehyde d) aromatic
5. Hectaldehyde also has the name?
   a) ethanal b) methanal c) butan-2-one
   d) alkanone
6. Butoxymethane ($CH_3\ CH_2CH_2\ CH_2)_2O$ is also called _____
   a) dibutylether b) ethoxyethane
   c) diethylether d) methanol
7. Which of the industrial processes yields methanol in large quantity
   a) catalytic dehydrogenation of methanol
   b) hydrogenation of alkanes
   c) oxidation of alkenes
   d) none of the above
8. Dehydrogenation of ethanol over a copper catalyst yields_____ industrially a)ethanal
   b) butanal c) butan-2-al d) ethene
9. In the lab, controlled oxidation of primary alcohols using milder oxidizing agent gives
   a) aldehydes b) ketones c) alkanal
   d) esters
10. In the lab, controlled oxidation of secondary alcohols using the same process above gives a) aldehydes b) alkanal
    c) esters d) ketones
11. Alcohols react with aldehydes and ketones in the presence of a catalyst to produce one of the following
    a) hemiacetals b) benzene c) no idea
12. The second product of the reaction of alcohols with aldehydes and ketones is
    a) acetal b) hemiacetal c) hemiketals
    d) ketal
13. One of the following aliphatic aldehydes does not react with ammonia a) ethanal
    b) methanal c) buthanal d) acetal
14. The general molecular formula of aldehyde and ketone is a) $CnH_{2n}O$
    b)CnH_{2n+1} c)$C_nH_{2n}R$ d) C_nH_n+O
15. The carbonyl compound that has one hydrogen atom attached to the carbonyl carbon is _____ a) ketone b) alkanal
    c) alcohol d) aldehyde
16. In naming aldehydes the last "e" in the parent alkane is replaced with
    a) 'al' b) '–one' c)'-yne' d) none
17. When naming aldehydes with double bond we recognize the _____ before _____
    a) methyl group/double bond
    b) double bond / methyl group
    c) parent chain/ methyl group
18. One important use of polyvinyl chloride is in the production of _____
    a) paper b) ink c) pipes d) rods
19. _____ solely determines the chemical properties of aldehydes and ketones
    a) functional group b) double bond
    c) single bond d) the name
20. The reaction test used to distinguish between aldehyde and ketone

a) they chemical properties

b) ability to oxidize them c) they colour

d) none of the above

ANSWERS

1. …… D
2. ……. B
3. ….. B
4. ……. A
5. …… A
6. …….. A
7. ……
8. …… A
9. ……. A
10. …… D
11. …… A
12. ….. C
13. …… B
14. …… A
15. ……. D
16. ……. A
17. …… A
18. ……. C
19. …….. A
20. …….B

ELEMENT AND THEIR COMPOUNDS

GROUP I

The group 1 elements are also known as ALKALI METALS. They are the most reactive of all elements. They have one electron in their outer most shell and are therefore characterized by ns^1 (where the n could be 1,2,3,4,5, or 6) electronic configuration. They form univalent ion by losing their ns^1. Majorly, they are ionic and soluble in water.

Occurrence

They occur naturally as compounds and never as free metals

They are obtainable from aluminosilicate minerals except francium.

PROPERTIES OF GROUP I ELEMENT

1. All group I elements are silvery white metallic solids. But caesium will be liquid on a hot day which melts at 28°C
2. They show metallic character therefore good conductors of heat and electricity
3. Due to their low densities, they are soft metals. Na can be cut with a knife
4. They impact a characteristics colour to the Bunsen flame; this property is associated with the outermost 's' electron
5. They are associated with a larger radius than any other atoms in the same period
6. There is a progressive increase in atomic radius and number down the group
7. As we move from lithium to caesium, atomic radius increases, melting point decreases and bonding strength decreases
8. The first ionization energy of alkali metals is the lowest amongst the element in their respective periods and there is a progressive decrease in the number of nuclear shells.

USES OF GROUP I ELEMENT

1. Li (lithium) is used in the production of specialized electrochemical cells and batteries
2. Because of its strong reducing power Na is used in the production of other metals and in the preparation of organic compounds
3. Na is used to produce tetraethyl lead "an anti-knock" addictive to gasoline.

ANORMALOUS PROPERTIES OF LITHIUM

Lithium shows some similarities to magnesium and is therefore said to exhibit a diagonal relationship with the second element in the next group

1. It has small atomic and ionic radii
2. It burns in air to form normal oxides and nitrides; others do not
3. Li reacts with carbon directly to form ionic carbide
4. Hydroxides of lithium and magnesium are not deliquescent and are much less soluble in water
5. The fluorides, carbonates and phosphate of lithium are sparingly soluble in water while that of other group I elements are soluble.
6. The carbonates, hydroxides and nitrate of lithium decompose at a high temperature to their oxide and others are stable to heat.

QUESTIONS

1. The nitrate of _____ element decomposes at high temperature
   a)Na b) Cesium c) lithium d) sodium
2. Lithium has a diagonal relationship with
   a) aluminum b) magnesium c) silicon
   d) rubidium

3. All these are group I elements except
   a) beryllium b) lithium c) francium
   d) potassium
4. All group I elements are obtainable from aluminosilicate except a) magnesium
   b) potassium c) francium d) sodium
5. Alkali metals form univalent ion by_____
   a) gaining one electron b) changing their name c)losing their ns^1 valence electron
   d) none of the options
6. The atomic radii of alkali metal
   a) increases down the group
   b) increases across the period
   c) reduces across the period
   d) deduces down the group
7. Caesium melts at _____ therefore liquid at hot temperature
   a) 25°C b) 30°C c) 22°C d) 28°C
8. The alkali metals are soft due to their
   a) low densities b) melting point
   c) colour d) ionization potential
9. Alkali metals reacts with halogens to form _____ a) oxides b) halides c) molecules
   d) monoxide
10. Which of the following is used in the production of electrochemical cells
    a) lithium b) sodium c)manganese
    d) caesium
11. Alkali metal used in the production of anti-knock additive to gasoline is
    a) francium b) sodium c) silicon
    d) potassium
12. The alloys of _____ is used in air craft construction a) lead b) francium
    c) lithium d) sodium
13. _____ is used as coolant in nuclear reactors a) Na b) Li c) Mn d) Mg
14. _____ is principally a water solution alkali metals a) lime water b) rock water c) sea water d) soda water
15. All these impact a characteristic colour to the Bunsen flame except a) the boron
    b) sodium c) francium d) caesium
16. The purple colour of $KMnO_4^-$ is due to _____ a) the colour of MnO_4^-
    b) presence of k c) colour of O_4 d) none
17. Na is used as coolant due to _____
    a) its colour b) its high thermal
    c) because it's always cool
    d) its availability
18. An experiment indicating a yellow colour indicates the presence of
    a) Li b) Na c) Fr d) K
19. Li has _____ colour
    a) red b) violent c) crimson d) blue
20. One of these colours is not associated with alkali metals
    a) lilac b) crimson c) green d) blue
21. One of these does not react with Nitrogen to form nitrides a) Fr b) Na c) Cs d) Rb
22. Sodium is kept in oil to stop it from
    a) evaporating b) flowing
    c) reacting with air d) mixing with others
23. One of these is not an oxide of alkali metal
    a) super oxides of alkali metals
    a) super oxides b) peroxides c) dioxides
    d) monoxides
24. All group I elements below have a commercial use except
    a) Fr b) Cs c) Rb d) K

25. The halogen that does not react with alkali metals is

a) F b) Cl c) Br d) no option

26. Caesium (Cs) has the colour of

a) blue b) yellow c) lilac d) violet

ANSWERS

1. C
2. B
3. A
4. ……C
5. ….. C
6. …… A
7. …… D
8. …… A
9. …….. B
10. …… A
11. ….. B
12. ….. C
13. …… A
14. …… C
15. …… A
16. ….. A
17. ….. B
18. … B
19. ….. C
20. ….. C
21. …… A
22. …… B
23. …… C
24. ….. C
25. Because it's very rare …. .. A
26. All halogens react with alkali metals ….. D
27. ….. A

GROUP II (ALKALI EARTH METALS)

These elements are chemically reactive but less reactive than group I

They have two electrons in their outermost shell and therefore characterized by ns^2 electronic configuration. They form divalent bond by losing their ns^2valence electrons. Like alkali metals, alkali earth metals also occur naturally as compounds

PROPERTIES OF GROUP II ELEMENT

1. They are harder than alkali earth metals but their hardness decreases down the group as barium is very soft

2. Beryllium has a grey colour and is as hard as an iron
3. Melting and boiling point decrease down the group but greater than that of alkali metals
4. Higher melting points and greater hardness are characterized by increased bonding strength from two valence electrons

ANOMALOUS PROPERTIES OF BERYLLIUM

- Beryllium shows similarities with aluminum as well is said to exhibit a diagonal relationship
- The ratio of ionic charge to ionic radius is very similar
- Be and Al have same electronegativity values
- Both are rendered passive by nitric acid
- Both form an acid film at room temperature which protects them against corrosion by water and further oxidation at moderate temperature
- The hydroxides of both can react with acids as well as bases and so are said to be Amphoteric
- Their sulphates are readily soluble in water
- The carbides of both react with water to give methane and hence are referred to as methides

QUESTIONS

1. _____ element has a diagonal relation with Al a) Be b)Ba c) Ca d) Sr
2. Be is referred to as methide because
   a) its carbide reacts with water to give methane
   b) it can turn to methane
   c) it has the properties of methane
   d) no idea
3. Be as well as Al is rendered passive by nitric _____
   a) water b) solution c) acid d) minerals
4. The melting and boiling points of alkali earth metals _____
   a) increase across the group
   b) decrease down the group
   c) are same across the period
   d) are equal
5. All alkali earth metals react with hydrogen to form hydride except
   a) Ba b) Ca c) Be d) St
6. Which of the following group II elements does not react with halogens_____
   a) Be b) Mg c) Sr d) no option
7. _____ does not react with water either hot or cold a) Ca b) MgC c) Be d) Ba
8. Hydroxide of all alkali earth metals are soluble in water except that of ____
   a) calcium b) magnesium c) beryllium
   d) barium
9. _____ is used in making aluminium alloy
   a) Mg b) Ca c) Sr d) Ba and Be
10. The two elements used as reducing agents to separate other metals from their compounds are a) Mg and Sr b) Mg and Cr c) Ca and Sr d) Ba and Be
11. _____ is used because of its transparency to make windows for x –ray tubes

a) calcium b) barium c) magnesium d) beryllium

12. In dehydrating alcohol and in manufacture of hydrolith _____ is used a) calcium b) sodium c) barium d) none
13. The cans used in holding uranium in nuclear reactors are made of a) calcium b) magnesium c) aluminum d) iron
14. One of these is used as a weapon in nuclear reactors
    a) Be b) Ba c) Mg d) iron
15. _____ is used in the manufacture of television and vacuum table to remove traces of air _____
    a) Sr b) Ba c) Be d) Ca
16. Which of the following will not form a nitrite a) Li b) Na c) Be d) Mg
17. Be reacts with NaOH to give H_2 and _____ a) sodium hydroxide b) sodium berrylate c) beryllium oxide d) none
18. When alkali earth elements react with hydrogen, they form halides of the formula a) M_2H b) MH c) MH_2 d) none
19. One of these oxides is not soluble on water a) BeO b) BaO c) MgO d) CaO
20. In metallurgy, all these are referred to as deoxidant except a) Ca b) Mg c) Ba d) Sr
21. Oxides of group II elements are not freely found naturally except that of
    a) MgO b) Ca0 c) SrO d) BaO
22. Calcium acts as _____ agent in the preparation of some common metals such as thorium a) drying b) reducing
    c) oxidation d) additive
23. One of these is not a group II element
    a) indium b) barium c) calcium
    d) magnesium
24. Radium as group II element is extremely scarce and is also a _____ element
    a) radioactive b) radiating c) x-ray
    d) none
25. All group II element form lodides on reaction with iodine except
    a) Ba b) Ca c) none d) Be

ANSWERS

1. ……A
2. ……A
3. ……C
4. ……B
5. ……C
6. ……D. All react with halogens
7. …… C
8. …….. C
9. …….. A
10. …… B
11. …… D
12. ……A
13. …… B
14. …… A
15. …… B
16. …… B
17. …….B
18. ……..B
19. ……..A
20. ……..C
21. …….. A
22. …….. B

23. …….. A
24. …….. A
25. All from iodides …. C

GROUP III

They have three electrons in their outermost shell and are characterized by $ns^2\ np^1$ electronic configuration therefore there they are called p-block they metallic character increases down the group. Boron is a metalloid (semi metal and others are metals)

OCCURENCE

Boron occurs as borax ($Na_2B_4O_7.10H_2O$) though not an abundant element. Kernite ($Na_2B_4O_74H_2O$) colemanite ($Ca_2B_6O_{11}.5H_2O$). Al is the third most abundant elements on earth. Others are relatively rare.

PREPARATION

Boron is extracted from borax by treatment of borax with hydrochloric acid to give boric acid and dehydration of boric acid gives anhydride boric oxide which is reduced to boron by heating with magnesium

$$B_2O_3 + 3Mg \xrightarrow{\Delta} 2B_{(s)} + 3MgO_{(s)}$$

Aluminium is gotten from the ore by electrolysis of the fused salt with cryolite (Na_3Alf_6) which is added to lower the melting point.

OXIDATION STATE

All group III exhibit variable oxidation state except boron which shows valency of +3 at the excited state because one electron in s-orbital is promoted to empty p- orbital.

PROPERTIES OF GROUP III

- There is a great decrease in hardness down the group. Boron being as hard as diamond but thallium is very soft
- Melting point decreases from boron to gallium then increases from gallium to thallium
- Atomic radius increases down the group therefore reducing bond strength anomalous properties of boron
- Boron and silicon occur naturally as oxo–compounds i.e. borates and silicates indicating that B.O is as stable as Si-O bond
- Both are solids of high melting point and law density, hard and brittle with low electricity conductivity
- They both have giant molecular structures
- Oxides of boron and silicon are weakly acids solids showing no amphoteric tendency
- Chlorides are hydrolyzed to form boric and orthosilicic acid along with hydrochloric acid

CHEMICAL PROPERTIES

All group iii reacts with O_2 to form oxide in the +3 oxidation state of the elements

$$4Al_{(s)} + 3O2_{(g)} \longrightarrow 2Al_2O_{3(s)}$$

Thallium also form +1 metal oxide Tl_2O

They do not react with water except boron that reacts readily with steam at red heat to boric acid and hydrogen

$$2B_{(s)} + 6H_2O_{(l)} \longrightarrow 2H_3BO_3 + 3H_{2(g)}$$

Al, Ga and In reacts directly with halogens to give the metal (iii) halides. Thallium also form the +1 metal halide Ti Cl

USES

1. Gallium and Indium have been used to make semi-conductors for solid state electronics
2. Al is used for surfacing mirrors of large telescope
3. Boron is used as deoxidizer in manufacturing some metals
4. Al is used as foil in wrapping foods and confectionery and for milk bottle tops
5. Where lightness is of paramount importance, Al is used to replace Cu

QUESTIONS

1. Because of its high electrical and thermal conductivity _____ is used in making pans, kettles and pots a) silicon b) thallium c) aluminum d) indium
2. One of this is used as deoxidizer in metal production
   a) boron b) indium c) gallium d) none
3. All group iii element react with halogen to give metal (iii) halides but _____ also form metal (i) halide a) indium b) thallium c) gallium d) aluminum
4. Water has no effect on group iii except
   a) Be b) B c) Ga d) In
5. _____ dissolve in fused strong base to form metaborate ion and hydrogen
   a) Cl b) B c) Si d) none
6. Group III show valiancy of _____ at excited state a)+3 b) +1 c) +2 d) +4
7. Which of the group III elements is in period 2 of the periodic table
   a) All b) none c) Al d) B
8. Boron has a diagonal relationship with
   a) all group Iv b) group II c) silicon d) none
9. _____ is the third most abundance element on earth a) Al b) B c) Ti d) In
10. Boron is extracted from borax by treatment with a) AlO b) hydrochloric acid c) silicon (ii) chloride d) magnesium
11. Hydrites of ___ and ___ are spontaneously inflammable a) silicon and aluminum b) thallium and indium c) boron and silicon d) gallium and aluminum
12. Trioxonitrate (v) acid renders _____ unreactive a) all group III elements b) boron (v) c) Aluminum d) thallium
13. _____ is used as replacement for copper where light weight is required a) boron b) gallium c) no option d) aluminum
14. Boron is used as addictive to the semi-conductors silicon and _____
    a) Al b) Ga c) In d) Si
15. Group iii has electronic configuration of
    a) Ns_2Np_1 b) Ns_1Np_2 c) ns^3 d) ns^1np^1

16. Group iii elements include all except
a) gallium b) radium c) indium d) boron
17. Mono-valency in group III is explained by _____ a) p-block b) s-block c) s-electron d) none
18. The third most abundant element occurs as a) kermite b) bauxite c) borax d) cryolite
19. Boric acid is heated with magnesium to give a) bromine b) beryllium c) boron d) silicon
20. In the production of nuclear energy _____ is used as neutron absorber
a) silicon b) beryllium c) boron d) none
21. _____ act as oxidizing agent with group III element
a) H_2O_4 b) HCl c) HNO_3 d) K_2SO_4
22. _____ is used as foil in wrapping of food
a) Al b) Ga c) In d) Ti
23. In the removal of O_2 and N_2 from steel _____ is used a) In b) Ga c) B d) Al

ANSWERS

1. …… C
2. Boron …..A
3. ….. B
4. …. B
5. …… B
6. +3 …… A
7. ….. D
8. …. C
9. …. A
10. …. B
11. ….. C
12. ….. C
13. ….. D
14. ….. B
15. ….. A
16. … B
17. … C
18. … B
19. …. C
20. … C
21. …. A
22. ….. A
23. Al ….. D

GROUP IV ELEMENTS

They are characterized by ns^2np^2 electronic configuration. Also called p-block element, their outermost electron is contained in p sub-shell. On descending the group, there is this smooth trend from non-metal $\Longrightarrow$ metalloid $\Longrightarrow$ metal and this is accounted by change in electronegativity due to increasing atomic radius

OCCURRENCE

Carbon is the only element in the group that occurs freely in coals and anthracites. Germanium is a rare element. Tin occur as tin-stone or cassiterite (SnO_2). Lead chiefly as galena (PbS).

PREPARATION

Carbon black is produced in large amount by incomplete combustion of hydrocarbon from nature gas and oil in presence of limited air supply. Other members of the group are prepared by reducing their oxides e.g. silicon can be gotten by reducing its oxide with magnesium powder.

GROUP V ELEMENTS

The elements of the group have five electrons in their outermost shell and are therefore characterized by ns^2np^3 electronic configuration. Their metallic characteristic increase down the group with nitrogen and phosphorus as non-metal, arsenic and antimony as metalloids and bismuth is a metal. Also they are p-blocks elements

OCCURRENCE

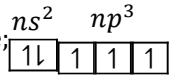
Phosphorous is the most abundant in the group and occurs in phosphate mineral.Other members except nitrogen are much less abundant and occurs as oxides and sulphide ores

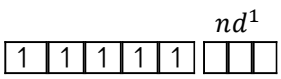
PREPARATION OF GROUP V ELEMENT

The important process of producing nitrogen in large quantity is liquefaction and fractional distillation of air. White phosphorus is a major industrial chemical and is prepared by heating phosphate rock (fluorapatite $3Ca_3(PO_4)_2$. CaF_2 with coke (C) and sand (SiO_2) in an electric furnace. Others may be obtained from their ores by roasting their sulphide ores to the oxide in air followed by reduction of oxide with coke.

OXIDATION STATE OF GROUP V ELEMENT

They exhibit variable oxidation state

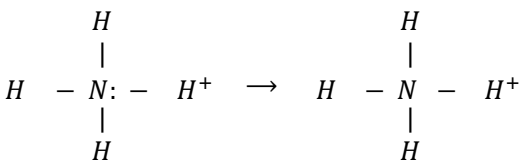
Ground state; ns^2 np^3  +3 oxidation state

+5 oxidation state; nd^1 

In order to attain the +5 electronic structure, the five electrons would have to be lost and the energy requirement for the loss is enormous and the process never occurs. The inert effect is very strong in the heavier members which leads to the formation of M^{3+}ion, the strength increases from arsenic to bismuth. All members except nitrogen are capable of showing covalency of 5

REASONS FOR UNIQUENCESS OF NITROGEN

1. It has a very small atomic radius
2. In ability to extends its coordination number beyond for due of the absence of d-orbital



3. Great stability of the free element
4. It is the only member that forms bonds with hydrogen
5. Ability to form multiple bonds with itself or with other element e.g. $N \equiv N$; $C \equiv N$

PROPERTIES OF GROUP V ELEMENTS

Allotropic form

Nitrogen has two stable isotopes ^{14}N and ^{15}N with relative abundance 272:1. Phosphorus has two major allotropes: red and white phosphorus. White phosphorus is a waxy white solid. It is very reactive and poisonous. Red is less reactive and not poisonous. Arsenic is normally a brittle grey solid. Antimony is a silvery lustrous solid. Bismuth is a reddish white lustrous metal. Density of members increases down the group.

CHEMICAL PROPERTIES

1. **With oxygen**: All members react with O_2 to form oxides of different oxidation states. Nitrogen reacts to form seven types of oxides with oxidation states ranging from +1 to +6
2. **With halogens:** Nitrogen is not affected by halogens, phosphorus reacts to give penta halides as well as trihalides
3. **With hydrogen:** In the presence of a catalyst, nitrogen is sufficiently reactive at elevated temperature to combine directly with hydrogen. Other members do not combine directly with hydrogen.

USES OF GROUP V ELEMENT

1. White phosphorus is used to manufacture phosphoric acid
2. Nitrogen is used in the manufacture of fertilizer, nitric acid and Nylon
3. Antimony is used in combination with lead for strong battery plate
4. Antimony Bismuth and arsenic are used as alloy
5. Red phosphorous is used in making matches by heating at 240°C in an inert atmosphere.

QUESTIONS

1. Arsenic, bismuth together with _____ is used to make alloys a) nitrogen b) antimony c) phosphorus d) sulphur
2. Nitrogen when reacting with oxygen is capable of forming _____ different oxides a) 14 b) 8 c) 7 d) 4
3. Hydrogen has no easy effect on the following except a) bismuth b) nitrogen c) arsenic d) antimony
4. The most abundant of group V elements is a) nitrogen b) potassium c) phosphorus d) carbon
5. Group 5 elements are also called _____ block elements a) b b) s c) d d) p
6. Nitrogen is produced in large quantity by fractional distillation of air and _____ a) purification b) liquefaction c) filtration d) oxidation
7. Nitrogen has two stable isotopes _____ and _____ a) ^{14}N and ^{7}N b) ^{7}N and ^{14}N c) ^{14}N and ^{15}N d) ^{6}N and ^{14}N

8. Inert effect is not seen in one the following
a) hydrogen b) nitrogen c) bismuth
d) phosphorus
9. The group V element(s) that is/are gas at room temperature is/are _____a) N b) N and P c) Bi and As d) Sb and Bi
10. Nitrogen acts as _____ by displacing H_2O from aquo complex
a) acid b) base c) ligand d) none
11. One of this is not a property of nitrogen monoxide
a) it is a colourless gas
b) it is insoluble in water
c) it is neutral oxide d) none
12. Chemically, nitrogen reacts with halogens to form a) pentahlides b) trihalides
c) both d) none
13. Red phosphorus is used in making
a) paint b) acid c) matches d) battery
14. Lend is combined with _____ to make storage battery plate a) antimony b) arsenic c) phosphorus d) bismuth
15. Lead is hardened for lead shot using _____ a) bismuth b) arsenic c) antimony d) nitrogen
16. One of these is not an oxide of nitrogen
a) nitrogen (ii) oxide b) nitrogen (vii) oxide c) nitrogen (v) oxide d) nitrogen (vi) oxide
17. The _____ of the group V element increases down the group
a) density b) ionic potential
c) boiling point d) symbol
18. Group v elements include all the following except a) arsenic b) antimony
c) unstable
19. The d-orbital of nitrogen is
a) very reactive b) unreactive c) absent
d) unstable
20. In the production of fertilizer _____ is used a) N b) P c) Bl d) Sb
21. Which of these is called a laughing gas
a) N_2O_4 b) HNO_3 c) N_2O d) NO-
22. Nitrogen cannot extend its coordination number beyond a) five b) four c) three
d) two
23. The two isotopes formed by nitrogen are
a) relatively unstable b) stable c) rare
d) absent
24. The relative abundance of the isotopes is
a) 37:2 b) 27:2 c) 272:1 d)97
25. Group V elements are characterized by _____ electronic configuration a) ns^4np^5
b) ns^2np^3 c) np^3ns^2 d) none

ANSWERS

1. ..... B
2. C
3. ..... B
4. C
5. ..... D
6. ..... B
7. C
8. ..... B
9. ..... A
10. C
11. ..... D

12. Nitrogen is not affected by halogens …… D
13. …… C
14. …….. A
15. ……... B
16. …….. B
17. …….. A
18. ……. C
19. Nitrogen has no d- orbital …… C
20. …….. A
21. ……... C
22. ……... B
23. ……... B
24. ……... C
25. ……... B

GROUP VI ELEMENTS

They have six electrons in their outermost shell therefore are characterized by ns^2np^4 electronic configuration. They show trend from non-metal to metal. The elements are highly electronegative and reactive

OCCURRENCE

Oxygen is the most abundant and occurs freely in the atmosphere. Hydrosphere and lithosphere combined as oxides. Sulphur occurs in sulphide minerals which areimportant metal ores(sulphide minerals). Also present in coal and petroleum as organism sulphur compound and in natural gas as hydrogen sulphide. Selenium and tellurium occur in mixed form with sulphide ores.

PREPARATION OF GROUP VI ELEMENT

O_2 is produced in large quantity by fractional distillation of liquid air. Thermal decomposition is a pure method of controlled preparation of oxygen.

Free sulphur is mould by the fraschprocess, where underground deposit of solid sulphur are melted in place with superheated water and the molten sulphur is forced upward as a froth using air under pressure.

OXIDATION STATE

They exhibit variable oxidation state as in have their compounds mainly in -2 oxidation states, other members also have -2 oxidation but +4 and +6 are common

Ground state; ns^2 np^4 | 1↓ | 1↓ | 1 | 1 |

1st excited state | 1↓ | 1 | 1 | 1 | 1 | | | | | nd^1 → +4 valency

2nd excited state | 1↓ | 1 | 1 | 1 | 1↓ | | | | | nd^2 → +6 valency

Selenium and tellurium in the +6 oxidation are stronger oxidation agents showing the greats stability of the +4 oxidation state in the heavier group VI element due to inert pair effect

PROPERTIES OF GROUP VI ELEMENTS

1. At ordinary temperature, O_2 is a gas
2. Sulphur is a solid with m.pt o 113^oC
3. O_2 and S are slightly soluble in water but readily soluble in organic solvent
4. O_2 has two allotropes ; oxygen and ozone
5. Sulphur has four main allotropes; three are crystalline and one amorphous. The stable form of sulphur is called rhombic sulphur
6. Selenium also exists in several allotropic form
7. Selenium also exists in silvery solid metallic luster. Polonium is a metal with a simple lattice

CHEMICAL PROPERTIES

With oxygen: Sulphur burns in air with a characteristic blue flame to sulphur (iv) oxide selenium and tellurium also burns in air to form the dioxides which are solids

With halogens: Sulphur, selenium and thallium react directly with halogens

With metal: Sulphur reacts with most metals to form sulphides and in some cases to form disulphides also.

With H_2O and dilute acids:

No group VI elements react with H_2O or dilute acids

CLASSIFICATION OF OXIDES

They may be classified as

Acidic: e.g. most oxides of non-metallic elements (Co_2, No_2, P_4O_{10} , So_3 etc.)

Basic: oxides of electro positive elements (Na_2O, CaO, etc.)

Amphoteric:oxides of less electropositive elements (BeO, $B_{12}O_3$e.t.c)

Neutral: oxides that do not interact with water or aqueous acids or bases e.g (CO, NO etc)

DIFFERENCE BETWEEN SULPHUR AND OXYGEN

Oxygen shows valency of 2 and rarely 4 but sulphur shows 4 and 6 as its maximum

- Atomic radius of O_2 is much smaller than that of sulphur which makes it a more electronegative element
- O_2 form more stable double or multiple bond with itself and other elements more readily then sulphur
- Sulphur has a much stronger tendency to form chains with itself then oxygen

QUESTION

1. The oxy-acetylene flame used in welding and cutting of proper is made of _____
   a) selenium b) sulphur c) oxygen
   d) nitrogen
2. Sulphur burns in air with a characteristics flame of _____ colour
   a) yellow b) violent c) blue d) white
3. Which of these is formed when selenium and tellurium burns in air a) peroxides
   b) trioxides c) pentaoxides d) dioxides
4. One of these does not react directly with halogens a) oxygen b) sulphur
   c) selenium d) tellurium
5. In reaction with halogens, sulphur monochloride formed can react with chlorine at _____ temperature to form

sulphur dichloride

a) 20^oC b) O^oC c) 45^oc d) 10^0c

6. Sulphides and disulphides in some cases are formed when sulphur reacts with _____

a) oxygen b) metal c) non-metals d) acids

7. Liquid oxygen is used as _____ fuel

a) aero plane b) trailer c) rocket d) ship

8. _____ is used in high pressure gasification of coal a)selenium b) oxygen

c) polonium

9. In alloys, one of these is used a) tellurium

b) polonium c) selenium d) sulphur

10. In steel making, _____ is used in large quantity

a) boron b) oxygen c) selenium d) none

11. All group VI elements have allotropes except qellurium and _____ a) sulphur

b) polonium c) oxygen d) selenium

12. The two allotropes of oxygen are oxygen and

a) oxone b) ozone c) white oxygen d) none

13. H_2S has the odour of a _____

a) ginger b) rotten egg c) gasoline d) none

14. Hydrogen sulphide has a _____ colour

a) yellow b) colourless c) green colour

d) white

15. Sulphur forms four main allotropes, the first three are crystalline and the fourth is

a) amorphous b) rhombic c) precipitate

d) none

16. Sulphur has the tendency to form chain with itself more than _____

a) selenium b) oxygen c) sulphur

d) polonium

17. _____ form stable double or multiple bond with itself a) sulphur b) O_2

c) polonium d) selenium

18. Oxygen shows maximum covalency of _____ and sulphur shows element of _____ a) 4 and 3 b) 6 and 4 c) 4 and 4

d) 4 and 4

19. At ordinary temperature, _____ is the only gas in group VI elements a) sulphur

b) selenium c) oxygen d) polonium

20. Using fractional distillation of air, _____ is produced in large quantity

a) all group vi element b) all except O_2

c) sulphur d) oxygen

21. The most abundant of the elements is

a) sulphur b) oxygen c) polonium

d) selenium

22. All these are classes of oxide formed by groups VI except

a) acidic b) basic c) neutral d) none

23. The photoelectric cells and rectifiers in electronic are made up of _____ a) sulphur

b) polonium c) selenium d) tellurium

24. Oxygen occurs in all except

a) atmosphere b) lithosphere

c) hydrosphere d) hydrosphere

ANSWERS

1. C
2. C
3. D
4. A
5. B
6. B
7. C
8. B

9. A
10. Polonium and tellurium do not have allotropes…….. B
11. ….. B
12. ….. B
13. ….. B
14. ….. B
15. …… A
16. …… B
17. …… B
18. …… C
19. ….. C
20. ……. D
21. ….. B
22. ….. D
23. ….. C
24. D

GROUP VII ELEMENTS

This group is also called the halogens. They have seven electrons in their outermost shell and are characterized by ns^2np^5 electronic configuration. All are non-metals and electronegative. Members form diatomic molecules in gaseous state and the two atoms joined with a single bond

OCCURRENCE

These elements are too reactive to be found free in nature; fluorine is widely available in fluorapatite, bromine occurs as bromide in certain brines and ocean water at a recoverable concentration whileiodine occurs as sodium iodide

PREPARATION OF GROUP VII ELEMENT

Fluorine can only be prepared by electrolysis of fused potassium fluoride dissolved in liquid hydrogen fluoride.

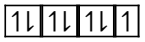
Iodine is produced from natural brines by oxidizing iodide ion with chlorine.

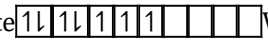
Chlorine is prepared by electrolysis of aqueous sodium chloride.

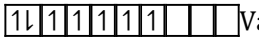
Bromine is obtainable from sea water or brine oxidation of bromide ion in solution with chlorine.

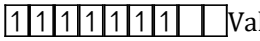
OXIDATION STATE

All except fluorine exhibit variable oxidation state

Ground state ns^2 np^5  valency of 1

1st ex state  Val of 3

2nd ex state  Val of 5

1st ex state  Val of 7

Oxidizing power decreases down the group

CHEMICAL PROPERTIES

1. **With water:**Iodine is sparingly soluble in water. Chlorine and bromine are moderately soluble. Fluorine reactsvigorously with water to literate oxygen gas

$$2F_{2(g)} + 2H_2O_{(l)} \longrightarrow 4H^+_{(ag)} + 4F^-_{(ag)} + O_{2(g)}$$

2. **With metals**: All halogens react with metals to form salt of the metal

$$2k_{(g)} + F_{2(g)} \longrightarrow 2KF$$

$$H_{(g)} + 12_{(g)} \longrightarrow HgI_2$$

3. **With non-metal:** Halogens react with all non-metals except oxygen, nitrogen and carbon but fluorine alone attacks carbon.
4. Oxidizing power increases as we ascend the group (i.e. the electronegativity increases upward from Astatine to fluorine.

PHYSICAL PROPERTIES

All halogens exist as diatomic molecules Fe, Cl_2, Br_2, I_2.

Fluorine is pale yellow while chlorine is greenish yellow. Bromine is a volatile reddish brown liquid. Iodine is a shiny black solid. Melting and boiling point increases down the group.

Fluorine is used to express the ionic character of metals. All halogens are oxidizing agents. Fluorine, in reacting with oxygen containing compounds, behaves differently from other halogens because it is more electronegative than oxygen.

USES OF GROUP VII ELEMENTS

1. Iodine is used in the production of dyes and in photographic industry
2. As a powerful germicide, iodine is also used medically
3. Chlorine is used to bleach paper pulp and textile and also to disinfect water

QUESTIONS

1. _____ is used to test the ionic character of metals a) bromine b) fluorine c) chlorine d) astatine
2. One of these is sparingly soluble in water a) iodine b) chlorine c) bromine d) fluorine
3. All of these do not react directly with oxygen except a) iodine b) bromine c) fluorine d) none
4. Halogen reacts with hydrogen to produce halides of colourless _____ a) water b) solution c) solids d) gases
5. _____ does not exhibit variable oxidation state a) bromine b) fluorine c) iodine d) chloride
6. Group VII elements exhibit up to _____ excited state a) 2 b) 4 c) 1 d) 5
7. Which of the group VII elements occurs freely in nature a) all b) none c) astatine d) fluorine
8. Fluorine can only be prepared by electrolysis of fused _____ fluoride dissolve in liquid hydrogen fluoride a) aluminum b) potassium d) calcium d) none
9. _____ is obtained from natural brines by oxidizing iodide ion with chlorine a) bromine b) chlorine c) iodine d) astatine
10. Oxidizing ability increases as we ascend the group due to increasing a) electron affinity b) boiling point c)electro nativity d) penetrating power
11. We use _____ in the bleaching of paper pulp a) bromine b) chlorine c) iodine d) no idea
12. Halogens exist as _____ a) tri-atomic molecule b) mono-atomic molecules

c) di-atomic molecules

d) none of these

13. The two atoms formed by the halogens in their gaseous state is joined by ___ bond(s)

a) single b) a double c) triple d) multiple

14. One of these can combine directly with carbon a) fluorine b) bromine c) iodine d) astatine

15. In the halogen group, only _____ exhibits monovalency a) chlorine b) bromine c) iodine d) fluorine

16. Halides formed by halogens with hydrogen have a sharp penetrating _____

a) power b) odour c) ability d) no idea

17. The strongest amidst the halides formed by the halogens is _____

a) HCl b) H_1 c) HF d) HBr

18. HF has a boding point of 20^{o}C and its easily _____ a) solidifies b) liquefied c) turned gas d) condensed

19. The reaction of ____ and water is vigorous

a) chlorine b) fluorine c) astatine d) none

20. Which of the acids below is the weakest

a) hydrofluoric b) hydrochloric

c) hydroiodic d) hydrobromic

ANSWERS

1. B

2. A

3. D

4. **D**

5. **B**

6. **D**

7. **B**

8. **B**

9. **C**

10. **C**

11. **B**

12. **C**

13. **A**

14. **A**

15. **D**

16. **B**

17. **B**

18. **B**

19. **B**

20. **A**

GROUP VIII ELEMENTS

They have (8) eight electrons in their outermost shell therefore are characterized by ns^2np^6 electronic configuration.

OCCURRENCE

Except radon, all other are found in air. Argon is the most abundant and others are relatively scarce

PREPARATION

Neon, argon, krypton and xenon can be gotten from fractional distillation of liquid air.

PROPERTIES

1. All noble gases are colourless, odourless gases with low B.pt
2. Their solubility in water is relatively high and this character increases down the group
3. Because of weak intermolecular force, noble gases vaporize easily.
4. Increase in boiling point down the group explains the increase in VanderWaal's forces between the molecules.

Xenon react with fluorine to form three different fluorides (difluoride, tetrafluoride & hexafluoride)

QUESTIONS

1) All group 8 element are scarce except____which is most abundant.
a) argon b)xenon c) krypton d)helium.
2) The other name for a group 8 element is.
a) rare gas b) noble gas c) rare noble gas d) stable gas.
3) Fractional distillation of liquid gas yields all these except. a)radon b)krypton c)neon d)argon.
4)Noble gases have ____ colour a)yellow b)bluish yellow c)brown d)colorless.
5) The solubility of water____down the group a)reduces b)increases c)decreases d)no idea.
6)_____is the intermolecular force in group 8 element a)VanderWaal's force b)hydrogen bond c)ionic bond d)covalent bond
7)They have electronic configuration of ns^2np^6..only_____-has ns^2 a)neon b)krypton c)helium d)radon
8)Xenon react with fluorine to form_____different fluorides a)2 b)3 c)4 d)8
9) The fluorides formed from the reaction of xenon and fluorine includes all except a)difluorine b)trifluorine c)tetrafluorine d)hexafluorine.
10)At room temperature the fluorides are white_____solids a)metallic b)metalloid c)crystal d)none
11) _____is the least soluble in water in the group 8 elements a)radon b)helium c)krypton d)xenon.
12)Controlled hydrolysis of XeF_6gives____ a)hexafluoride b)oxofluoride c)trifluoride d)hydrogen fluoride
13)Xenon difluoride is a _____agent a)fluorinating b)oxidizing c)reducing d)defluorinating.
14)_____has the highest ionization potential a)helium b)radon c)xenon d)krypton.
15)The last member of the noble gas is a)radon b)xenon c)helium d)neon.

ANSWERS

1) A……
2) B…..
3) A……
4) D……
5) B…..
6) A……
7) C…….
8) B…….
9) B……
10) C……
11) B……
12) B……
13) A……
14) A……
15) A……..

ALKANES

Alkanes are hydrocarbons having SP^3 carbon atoms, that is, alkanes contain only single bonds in the molecules. Alkanes and cycloalkanes (incycloalkanes, the carbon are joined in rings) are saturated hydrocarbons, which means that their carbon are saturated with hydrogen. This also means that they cannot accommodate any extra hydrogen atom

The general formula for the alkane family is given by $C_nH_{2n} + 2$

When $n = 1$, we have CH_4, which is the simplest member of the alkanes called methane

The first four members of the alkane family are gases, then from C_5 to C_{20} are liquid and the rest are solid

Isomerism and nomenclature

Isomerism occurs in alkane family form the fourth member called butane

Number of carbon	Total possible isomers
1,2,3	1
4	2
5	3
6	4
7	5
8	9
9	18
10	35
20	75

NOMENCLATURE

The following rules should be followed when naming alkanes:

Rule 1: choose the longest chain

Rule 2: name the compound as a derivative of the alkane represented by the longest continuous chain of the carbon atoms

Rule 3: locate the substituents (CH_3, CHC_2H_5, CL, OH) the longest carbon chain are numbered in such a way as to give the carbon atom bearing the substituent the lowest possible number

Rule 4: name each substituent and specify its position on the longest carbon chain based on the numbering

Rule 5: when there are two or more different kinds of alkyl group present as substituent, these names are usually placed in alphabetical order but without regard to hyphenated prefixes such as tert, sec

Rule 6: the number designating the position of the substituent is placed before the name. Note also two, three and four identical substituents are specified as di, tri and tetra respectively

Rule 7: the name of the alkane is written as are ward with the name of the substituent and its number position serving as a prefix

Note the following substituents

- NO_2------- nitro
- F----- Fluoro
- I----- iodo
- BR--- Bromo
- CL ---- Chloro
- CH_3 --- Methyl

METHODS OF PREPARING ALKANES

1. From unsaturated hydrocarbons;

Alkenes or alkynes in the presence of nickel or plantinum catalyst can be hydrogenated to give alkanes

ii) from alkyl halides

iii) from alcohols

iv) from acids

ALKYL GROUP

This is formed by removing one atom hydrogen from alkane

QUESTIONS

1. The product of incomplete combustion of alkanes are
   a) constituents of car exhaust
   b) useful in the manufacture of ink
   c) applicable in the formation of water
   d) none of the above
2. The presence of weak VanderWaal's forces in alkanes makes the compound to have
   a) low boiling point
   b) moderate boiling point
   c) high melting point
   d) moderate boiling point
3. Butane has _____ isomers
   a) 1 b) 2 c) 3 d) 4
4. Which of the following is a member of alkane family
   a) C_2H_5 b) C_3H_7 c) C_4H_9 d) C_9H_{20}
5. Homologous series members are prepared From a) different methods b) the same method c)a&b d) none of the above
6. Wurtz reaction is a method of preparing
   a) higher alkanes b) lower alkanes
   c) lower alkenes d) higher alkenes
7. The general formula for Grignard reagent is
   a) R-Mg b) R-MgX c) H-Mg d) H-MgX
8. Decarboxylation occurs when sodium salts of alkanoic acids are heated with _____
   a) NaOH b) Na_2CO_3 c) CO_2 d) H_2O
9. The lowest member of the alkane family has _____ hydrogen atoms attached to the carbon atom a) 2 b) 3 c) 4 d) 5
10. Any alkane from C_{21} is _____
    a) gas b) solid c) liquid d) ice
11. Which of the following is a feature of the alkane
    a) they are highly reactive
    b) undergo combustion
    c) members of the alkane family are all liquid d) they have double bonds

12. Methane is often called _______ a) oleum fio b) phenol c) marsh gas d) beehive gas
13. Which of the following is not one of the products of the substitution reaction of Methane
a) $CHCL_3$ b) CCL_4 c) CH_2CL d) CH_3CL
14. _____ is used in dry cleaning
a) CH_3CL_3 b) CCL_4 C) CH_2CL_2 d) CH_3CL
15. The name of the organic compound below is _____

$$\begin{array}{c} H \\ | \\ CH_3CHC\ \ {}_2CH_3 \end{array}$$

a) 2-methyl propane b) 3-methyl butane
c) 2-methyl butane d) 3-methyl propane
16. The general formula of the alkane family is
a) $C_n H_n+2$ b) $C_n H_{2n}$ c) $C_nH_{2n}+2$ d) CnH_{2n} -2
17. The alkanes are called ______
a) paraffin b) oleumfio c) glycol
d) Sabatier
18. Alkanes burn in air to give ___ and ___
a) CO and H_2O b) CO_2 and O_2 c) H_2O and O_2
d) CO_2 and H_2O
19. In the preparation of alkanes, sodium is preferred to ordinary sodium hydroxide pellets because
a) it is deliquescent
b) it is not deliquescent
c) it is a dry element
d) it dissolves when exposed to air
20. The prefix "iso" is used for those alkanes which have a methyl group attached to _____ a) first carbon atom b) second to the last carbon on a continuous chain
c) third carbon in a continuous chain
d) last carbon
21. The prefix tert is used for alkanes which have _____ a) 4 methyl groups attached to the carbon atom b) 3 methyl groups attached to the carbon atom c) 2 methyl groups attached to the carbon atom
d) 1 methyl group is attached to the carbon atom
22. The inter molar force responsible for the physical for the physical properties of the alkanes is a) strong covalent bonds
b) weak VanderWaal's forces c) hydrogen bond d) ionic bonds
23. Alkanes are generally known to be _____
a) polar b) non-polar c) crystalline
d) cyclo alkane
24. Alkanes have ______ bond
a) single b) double c) triple d) half

25.
$$\begin{array}{l} \qquad\qquad\qquad\quad Cl \\ \qquad\qquad\qquad\quad | \\ CH_3CH_2CH_2CH_2CH_3CH_3CH_3. \\ \qquad\ | \\ \qquad CH_3 \end{array}$$

The above structure is
a) 2- methyl, 3- chloro hexane
b) 3- chloro, 2- methyl hexane
c) 3- chloro heptane
d) 3, chloro, 2, methyl hexane

ANSWERS
1) A.........
2) C.........
3) B.........
4) D.........
5) B.........

6) ……..D
7) B……
8) A……..
9) C……
10) B……..
11) B…….
12) C…….
13) C…….
14) B…….
15) C…….
16) C…….
17) A……..
18) D……..
19) B…….
20) B……
21) B……
22) B……
23) B……
24) A…..
25) B……

ALKENES

These are hydrocarbons that contain C=C. the bond in the alkenes is one sigma and one π bond .the sigma bond is stronger than the π bond the general formula is C_nH_{2n}

Functional groups

These are groups that are responsible for the chemical property of the family. C=C bond is responsible for most of the chemical characteristics of the alkanes family
Alkenes are more reactive than the alkanes. This is due to the availability of the more exposed π electrons. The bonds that bear the double bonds are called olefins
In the past, alkenes were referred to as olefinic because most of them and their derivatives had oily appearance.
Alkenes undergo geometric isomerism

QUESTIONS

1. The simplest member of the alkene family is a) methane b) ethene c) ethyne d) ethane
2. Alkenes exhibit a) sp hybridization b) sp^3 hybridization c) sp^2 hybridization d) sp^4 hybridization
3. Alkenes were formally called _____ a) olefins b) olefleum c) oleumfio d) a and c
4. The functional group formed when an atom of hydrogen is removed from alkenes is called a) alkyl b) alkynyl c) alkenyl d) none of the above
5. Alkenes have _____ appearance a) soapy b) oily c) solid d) simple
6. The double bonds between the carbons of alkenes consist of a) one sigma and one π bond b) half sigma and one π bond c) 2 sigma bonds d) 2 pie bonds
7. Which of the following is not a method of preparing alkenes
   a) dehydrohulogenation of alkyl halides
   b) dehydration of alkynes
   c) reduction of alkynes
   d) reduction between alkanes and and alcohol
8. Per acids react with olefins to form cyclic ether called a) ethose b) epoxide c) ozonolyte d) ozonide

9. Alkenes have _____ bonds
a) single b) double c) triple d) 2 sigma
10. Trans alkene is formed by the reduction of alkyne with _____ in liquid ammonia
a) sodium b) hydrogen c) oxygen
d) sulphur
11. _____ is formed when ozone reacts with alkanes a) ozonide b) ozonolyte
c) ozonolysis d) none of the above
12. Dehydrohalogenation of alkyl halides occurs in the presence of _____
a) alcoholic solution b) hot conc. alcoholic solution c) alkane d) hydrogen
13. Which of the following is true
a) the π- bond in alkenes is stronger than the sigma bond
b) the sigma bond in alkanes is stranger than π- bond
c) alkenes have two bonds which are sigma bonds
d) both sigma and pi bonds have the same strength
14. $CH_3CHCH = CH_2$ (with CH_3 below the second CH)
The nomenclature of the above structure is
a)2-methyl butane b) 2-methyl but-1-ene
c) 3-methyl but-1-ene d) 3-methyl butane.
15. Which of the following exhibits geometrical isomerism a) alkanes
b) alkenes c) alkynes d) alkenyl
16. Which of the following is a method of determining the stability of alkenes
a) heat of combustion data
b) heat of evaporation data
c) heat of halogenation data
d) ozonoly
17. The process of producing ozonide from ozone is called a) ozonology b) ozonidity
c) ozonolysis d) ozonility
18. Which of the following is true
a) cis- isomer gives off more heat than trans
b) cis- isomer gives less heat than trans
c) trans – non cyclic alkenes are less stable
d)the stability of alkenes does not depend on the position of the double bonds
19. Alkenes react with cold alkaline solution of KM_nO_4 a) 1,2 triol b) glycol c) 1,2 diol
d) both b and c
20. Alkenes are polymerize readily under the influence of a) basic catalyst b) salt
c) organic catalyst d) acidic catalyst

ANSWERS

1. …… B
2. …… C
3. ….. D
4. ……. C
5. ……..B
6. ….. A
7. ….. D
8. ……B
9. …..B
10. ….. A
11. ….. A
12. …. B
13. … B
14. ….C
15. …B
16. …. A

17. C
18. .... A
19. ...D
20. ....D

ALKYNES

The alkynes are unsaturated hydrocarbons with the general molecular formula C_nH_{2n}-2. The alkynes have triple bond(-C≡C-). The simplest member of the alkynes is ethyne. Alkynes are classifiedinto terminal and non-terminal alkynes

ISOMERISM IN ALKYNES

The first two members of the alkyne family (ethyne and propyne) do not exhibit any type of isomerism, but other higher alkynesexhibit various types of isomerism

METHOD OF PREPARATION

i) Action of water on calcium
ii) dehydrohalogenarion of dihaloalkanes
iii) dehalogenation of tetrahaloalkanes

QUESTIONS

1. The general formula of alkyne family is _____ a) C_nH_{2n}+2 b) C_nH_{2n} c) C_nH_{2n}-2 d) C_nH_{2n}+1
2. Alkynes have _____ bonds a) single b) double c) triple d) 1 sigma bond and 1 pi bond
3. The characteristic reaction of alkynes is electrophilic due to the availability of ____ a) mobile sigma electron b) mobile sigma – pi electron c) mobile pi electrons d) static pi electrons
4. Alkynes are rapidly hydrogenated with hydrogen gas to give _____ a) alkanes b) alkenes c) alkyne d) cyclo alkyne
5. Controlled hydrogenation to alkene can be achieved by using a) platinum catalyst b) lindlar's catalyst c) lithium catalyst d) helium catalyst
6. Positional isomerism occurs due to the difference in a) position of the double bond b) position of the triple bond c) position of the single bond d) none of the above
7. Which of the following exhibits isomerism a) ethyne b) propyne c) methane d) butane
8. Which of the following is a) terminal alkynes are less acidic than non-terminal alkynes b) terminal alkynes have double bond within a chain c) terminal alkynes are more acidic than the non-terminal alkynes d) alkynes have one of the members to be Hexane
9. Compounds having many triple bonds are known as _____ a) alkapolynes b) alkanes c) alkynes d) alkyl
10. Ethyne can be prepared from a) the action of water in calcium carbide b) oxidation of alkane c) reaction between alkane and alkanoic acid d) benzene
11. The hybridization of alkynes is _____ a) sp^2 b) sp c) sp^3 d) sp^4
12. Terminal alkynes are weak acids when compared to water TRUE or FALSE

13. Which of the following is true a) the boiling point of an alkyne is higher than that of the related alkanes b) the boiling point of alkane is higher than that of the related alkyne c) alkynes have double bonds d) they are less reactive than alkane
14. Alkynes undergo isomerism in the presence of traces of
a) base b) alkali c) salt d) hydrogen
15. Depending on the temperature or catalyst used, alkynes polymerizes to give linear or _____ a) cyclic compounds b) salt c) cyclo alkanes d) alkene
16. _____ is used in the manufacture of artificial rubber a) vinyl acetylene b) ozonide c) polythene d) polymers
17. The reduction of the alkyne with sodium in liquid ammonia yields a) basic oxide b) trans alkene c) acidic salt d) basic salt
18. Unlike the alkenes,alkynes polymerize to give a) a high molecular weight polymerb) a low molecular weight polymer c)alkynes d) alkenyl
19. Alkynes when treated with acidic $K_2Cr_2O_7$ give a) carboxylic acid b) a basic salt c) an acidic salt d) green colouration
20. The electrons in ethanide are held in an/a
a) Sp orbital b) sp^2 orbital c) sp^3 orbital d) sp^4 orbital

ANSWERS

1. ….. C
2. ….. C
3. …. C
4. ….. B
5. …… B
6. …… B
7. ……..D
8. …… C
9. …….. A
10. …… A
11. ……..B
12. ….. TRUE
13. …. A
14. ……..B
15. …… A
16. ….. A
17. ….. B
18. ….. A
19. …. A
20. Sp^3 orbital……. C

TYPES OF ORGANIC REACTION

We have four (4) main types of organic reaction: they are

1. Substitution or displacement reaction
2. Addition reaction
3. Elimination reaction
4. Re arrangement reaction

Note: all may be initiated by electrophilic, nucleophilic or free radical attack on the substrate.

Electrophilic is an electron deficient group.

Nucleophilic electron rich group.

Free radicals are atoms or group of atom possessing an unpaired electron.

SUBSTITUTION REACTION (DISPLACEMENT)

This takes place when an atom or group of radicals attached to a carbon are replaced

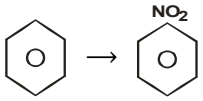
by another atom or group of atoms in a compound without any change in the type of bonding or degree of unsaturation.

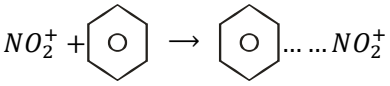
Electrophilic substitution

This involves the attack of the electron rich portion of the substrate by an electron deficient atom or group of atom e.g

Nitration of benzene

$HNO_3 + 2H_2SO_4 \rightarrow NO_2^+ + 2HSO_4 + H_3O^+$





Nucleophilic substitution

Here a nucleophilic replaces the nucleophile in the substrate

$R{-}X + OH^- \rightarrow R{-}OH + X^-$

Nucleophilic substitution reaction is of two types

Unimolecular and Bimolecular

FREE RADICAL SUBSTITUTION REACTION

In this, homolytic bond fission occurs; where by a covalent bond between two atoms is broken so that each atom acquires one of the bonding electrons

$$A + B \longrightarrow A^\circ + B^\circ$$

$$A^\circ B \longrightarrow A^\circ + B^\circ$$

Where A° and B° are the free radicals

Example is the halogenations of Alkane

$$CH_4 + Cl^\circ \longrightarrow \dot{C}H_3 + HCl$$

$$\dot{C}H_3 + Cl_2 \longrightarrow CH_3Cl + Cl^\circ$$

ADDITION REACTION

In this type an extra atom or group are attached to an sp or sp^2 hybridized carbon(i.e. unsaturated carbon) and the molecule becomes more or completely saturated

ELECTROPHILIC ADDITION REACTION

Here the electrophile attacks first example

Addition of HBr to ethene

$$CH_2 = CH_2 + HBr \longrightarrow \overset{H}{\underset{|}{CH_2}} - \overset{Br}{\underset{|}{CH_2}}$$

Step 1: $HBr \longrightarrow H^+ + Br^-$

Step 2: H^+(electrophile) attacks the pi-bond to give carbonium ion.

$H^+ + CH_2 = CH_2 \longrightarrow CH_3 + CH_2^+$

Step 3: the Br^{-1} (nucleophile) attacks the carbonium ion to give next product

$Br^- + CH_3CH_2^+ \longrightarrow CH_3CH_2Br$

NUCLEOPHILIC ADDITION REACTION

Usually involves aldehydes and ketone compound with C=C due to their high polar nature

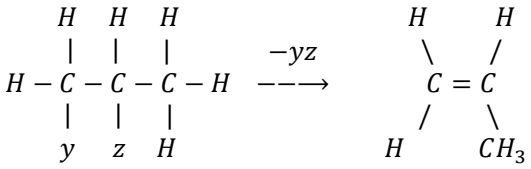
$\rangle C = O \leftrightarrow C^+ - O^-$

e.g. addition of HCN to acetone (propane)

$$CH_3\overset{\overset{O}{||}}{C} - CH_3 + HC \longrightarrow CH_3 - \overset{\overset{OH}{|}}{\underset{\underset{CN}{|}}{C}} - CH_3$$

ELIMINATION REACTION

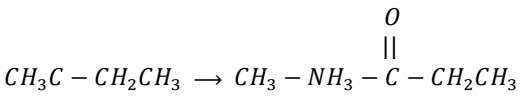
This reaction involves the removal of a group or an atom from sp^3hybridized carbon(i.e. saturated) to give unsaturated. It is one of the methods used for the synthesis of alkenes

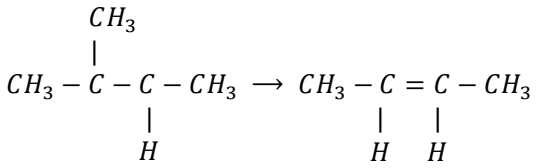


Elimination also has unimolecular and bimolecular.

REARRANGEMENT REACTION

This involves the migration of an atom from one side to another within the same molecule. The result always gives a structural isomer of the original compound. Example;





QUESTIONS

1. All these are types of reaction except____
   a) displacement b) replacement
   c) addition d) none
2. An electron deficient atom can be called
   a) nucleophile b) electrophile c) free radical d) no option
3. Nucleophile is an electron _____ atom
   a) reach b) rich c) empty d) no option
4. In which of the reactions does the electrophile attack before the nucleophile
   a) rearranging b) elimination c) addition d) displacement
5. _____ reaction is one of the methods used for the synthesis of alkane
   a) elimination b) replacement
   c) displacement d) addition
6. In which of the types of reaction do unsaturated molecules become saturated
   a) addition b) saturation c) displacement d) elimination
7. In _____ free radical, homolytic bond fission occurs a) addition b) substitution c) displacement d) none
8. One of the following is not a factor that influences reaction
   a) inductive effect b) steric effect
   c) carbanion d) electromagnetism
9. Nucleophilic reaction of _____ reaction involves aldehydes and ketone
   a) substitution b) addition
   c) displacement d) re arrangement
10. _____ is also called 1,2- elimination
    a) elimination reaction b) addition reaction c) substitution reaction d) none
11. ___ reaction involves the removal of atoms from two adjacent carbon atoms
    a) addition b) substitution c) displacement d) elimination

12. $H^+ + CH_2=CH_2 \rightarrow CH_3-CH_2$ in the reaction , H^+ is called a a) free radical b) nucleophile c) electrophile d) electron

13. A+B ⇨A^0+B^0. A^0&B^0 are called a) electrophile b) free radical c) nucleophile d) none

14. Electrophile substitution involves the attack of electron-rich portion by atom a) deficient b) supplement c) reach d) none

15. Which of the effect is due to size and shapes of molecules a) steric b) electronic c) resonance d) inductive

16. Additional reaction does not show one of the following a) nucleophilic b) electrophilic c) free radical d) termination

17.____ determines the reactivity of the atom in organic reaction a) nature of substituent b) Nature of bond c) type of reaction d) name of substituent

ANSWERS

1. …… D
2. ……. B
3. ……. B
4. Addition ….. C
5. …… A
6. ……. A
7. ……. B
8. …… D
9. ….. B
10. …… A
11. ….. D
12. ….. C
13. ……. B
14. ….. A
15. …. A
16. Termination is shown in displacement reaction ….. D
17. …. A

The following are factors that can influence organic reaction

1. Partial ionic character of covalent bond
2. Inductive effect
3. Electromeric effect
4. Mesomeric effect
5. Steric effect etc.

PHENOLS

Phenols are compounds containing hydroxyl(-OH) group attached directly to an aromatic carbon. They have general formula ArOH they may be classified as monohydric or polyhydric, depending on the number of hydroxyl group they contain.

PROPERTIES OF PHENOLS

At room temperature phenols are white solid or colourless liquid due to effect of intermolecular hydrogen bonding, phenols have high melting, boiling point and high density

SOLUBILITY

Phenols show a high solubility in water than alcohols. Polyhydric phenols due to high intermolecular hydrogen bonds are more soluble in water than monohydric

CHEMICAL REACTION

Phenols undergo two main types of reaction

1. Side chain substitution (involving replacement of the acidic or the hydroxyl group
2. Electrophilic substitution in the ring

DISPLACEMENT OF THE HYDROXYL GROUP

Phenols do not undergo any form of reaction with hydrogen and react only slowly with phosphorous penta-chlorides to give a poor yield of chloro-benzene.

Phenols generally are stronger acids than alcohol because of their ability to form resonance.

QUESTIONS

1. On reacting with ferric chloride, phenols give _____
   a) yellow b) red c) violet d) blue
2. The intermolecular hydrogen bonding accounts for _____ a) solubility
   b) volatility c) visibility d) no idea
3. The OH group in phenols is directly attached to ____
   a) aromatic carbon b) the double bond
   c) the single bond d) hydrogen
4. Phenols are classified based on number of _____ a) hydrogen atoms b) hydroxyl group c) carbon atom d) hydrogen group
5. At room temperature the liquid phenol has _____ colour a) white b) violet
   c) colourless d) green
6. The Solubility of phenols is a) highly in water than alcohol b) higher in alcohol than water c) equal in both liquids d) none
7. Phenols undergo these reactions which are electrophilic substitution and _____ substitution a) nucleophilic b) side chain
   c) free radical d) double chain
8. Generally phenols are stronger than alcohol due to their ability to form
   a) long chain b) resonance
   c) branched chain d) compound
9. Arranging meta, ortho, para amino phenol in the order of increasing acidic strength, the correct order is
   a) ortho> para> meta
   b) meta>ortho> para
   c) meta> para>ortho
   d) ortho> para > meta
10. Phenols react with hydrogen halides to form ____ a) none b) acid c) base d) salt of the halide
11. The mixing of the phenol with dilutetrioxonitrate(v) acid to form mixture of 2- and 4- nitrophenol is called _____ of phenol a) hydrogenation b) nitration
    c) oxidation
12. Phenol reacts with acetone to form
    a) bi-phenol A b) tris-phenol A
    c) bi-phenol d) phenol acetone
13. Phenols from aldehyde is widely used in the making of _____
    a) plastics b) paper c) PUIP d) cupper
14. The most important reaction of phenol is
    a) reaction with acetone b) condensation with formaldehyde c) condensation with ketoned) no idea
15. Phenol is oxidized by chromic acid to give a a) quinone b) oxide phenol c oxidized phenol d) ester
16. Phenols are reduced to cyclohexananol by hydrogen using_____ as catalyst
    a) calcium b) nickel c) lead d) arsenic
17. Reaction of phenol with electropositive metal liberates

a) hydrogen ion b) hydrogen molecule
c) hydroxyl ion d) acidic salt

18. The test for phenolic compounds in the laboratory is
a) reaction with alcohol
b) reaction with alkali
c) reaction with acid d) none

19. Acids react with phenol to form
a) alkanes b) esters c) phenol esters
d) alkenes

20. One of these is not a method of preparing phenol
a) alkali fusion of sodium aryluphonates
b) esterification of alcohol
c) hydrolysis of diazonium salt d) none

21. Trihydric phenol contains_____ molecule of hydroxyl group
a) four b) three c) four d) no idea

ANSWERS

1. …… C
2. ….. A
3. ….. A
4. ….. B
5. …… C
6. ……. A
7. …… B
8. …… B
9. Ortho<para<meta …… D
10. Phenols do not react with hydrogen halides … A
11. ……. B
12. ……. A
13. …….. A
14. ……. B
15. ……. A
16. ……. B
17. …… B
18. …… B
19. ……. B
20. …… B
21. ……. B

NOMENCLATURE

This is the method by which organic compounds are named. It is synonymous to binomial nomenclature in biology (although a lot more technical) and is essential because of the ability of carbon to form bonds with itself (catenation), resulting in a lot of organic compounds, hence the need to accurately name these compounds; in earlier times, when relatively few pure organic chemicals were known, new compounds were named at the discretion of their discoverer. Thus, **urea** (CH_4N_2O) is a crystalline substance isolated from urine; **morphine** ($C_{17}H_{19}NO_3$) is an analgesic (painkiller) named after Morpheus, the Greek god of dreams; and **barbituric acid is** a tranquilizing agent said to be named by its discoverer in honor of his friend Barbara (hmmmm).

The system of nomenclature that is widely accepted, is that devised by the **International Union of Pure and Applied Chemistry** (IUPAC, usually spoken as I-U-PACK).

Every chemical name has four parts in the IUPAC system of nomenclature namely: **prefix, locant, parent, and suffix.**

The prefix identifies the location and identity of the substituent groups in a molecule, the locant gives the location of the primary (most important) functional group, the parent name illustrates the main part of the molecule and tells how many carbon atoms are in that part, while the suffix as the name implies is the last portion of the name that identifies the primary functional group responsible for its chemical properties. The

name of compounds that are derived using the IUPAC nomenclature is the **systematic name.**

PRFEIX -Where and what are the substituent's?

LOCANT -Where is the primary functional group?

******Note:** A functional group is a group of atoms within a larger molecule that has a characteristic chemical reactivity. Because functional groups behave in approximately the same way in all molecules where they occur, the chemical reactions of an organic molecule are largely determined by its functional groups present in it.

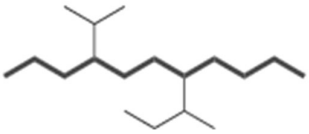
PARENT-How many carbons?

SUFFIX-What is the primary functional group?

As new functional groups are covered in this series, their applicable IUPAC rules of nomenclature will be discussed accordingly, let's see how to name general organic compounds and learn some general naming rules that are applicable to all compounds. All but the most complex branched-chain alkanes can be named by following the rules illustrated below

1. **Find the parent hydrocarbon.**

(a) Identify the longest continuous chain of carbon atoms in the molecule, and use the name of that chain as the parent name.

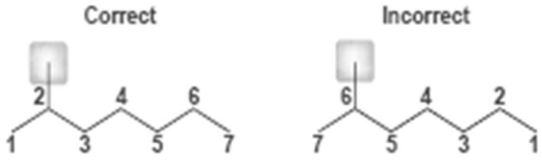


It should be noted that the longest chain may not always be apparent from the manner of writing of the compound; you may have to **"turn corners."**

$$CH_3CH_2CH_2\underset{\displaystyle CH_2CH_3}{\underset{|}{CH}}-CH_3 \qquad \textit{named as a substituted hexane}$$

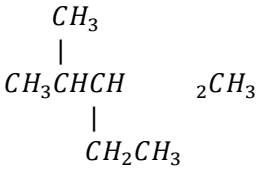
$$CH_3-\underset{\displaystyle CH_2CH_2CH_3}{\underset{|}{\underset{\displaystyle CH_2}{\underset{|}{CHCH}}}}-CH_2CH_3 \qquad \textit{named as a substituted heptane}$$

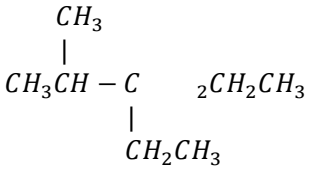
(b)If two different chains of equal length can be identified, choose the one that gives the substituent groups the lowest possible number(s).



As illustrated above, the substituent group in the above compound will have the locant **6-** due to naming from the right hand side, which is incorrect because that is not the lowest number possible; if we number the parent chain from the left hand side, the locant will have the value of **2-**

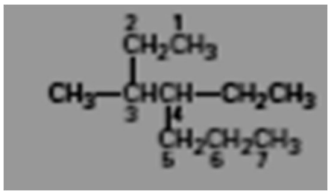
Another example is shown below using the first method of numbering opposed to the second.



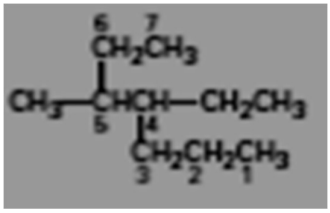


2. **Number the atoms in the main chain.**

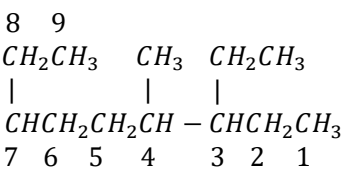
(a) Beginning at the end that is closest to the first branch point, number each carbon atom in the parent chain.



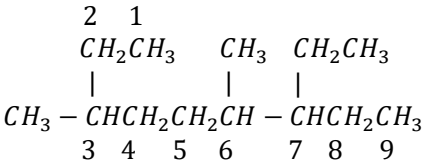
NOT



(b) If there is branching an equal distance away from both ends of the parent chain, begin numbering at the end that is closest to the second branch point. Example is shown below.

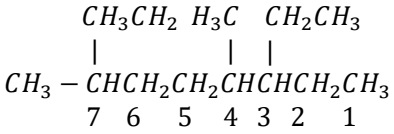


NOT



3. **Identify and number the substituents.**

(a) Assign a number, referred to a ***locant,*** to each substituent present in the compound to determine its point of attachment to the parent chain.



Hence in the above compound, we have 3-ethyl, 4-methyl, 7-ethyl

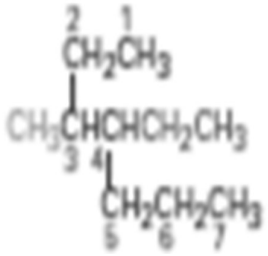
***note I'm not naming the compound yet, just illustrating the substituent groups by their locants.

(b) In situations where two substituents are present on the same carbon, give both the same number and indicate the prefix *di-*. In some situations, we use the prefix bis-, tris. But that is out of the scope of this book

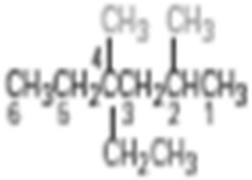
4. Write the name as a single word. Hyphens are used to separate the different prefixes, and commas should be used in separating numbers. When two or more different substituents are present (as stated earlier), name them in alphabetical order ignoring the multiplier prefixes *di-*, *tri-*, *tetra-* e.t.c . When two or more identical substituents

are present in the parent chain, the prefixes *di-*, *tri-*, *tetra* should also be used.

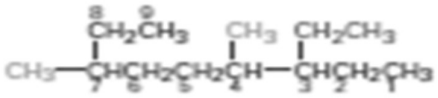
Full names for some compounds are as follows.



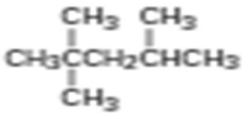
4-Ethyl-3-methylheptane



4-Ethyl-2,4-dimethylhexane

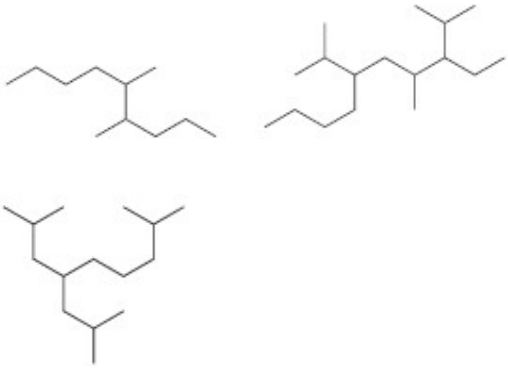


3-Ethyl-4,7-dimethylnonane

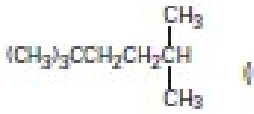


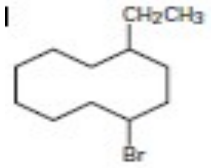
2,2,4-Trimethylpentane

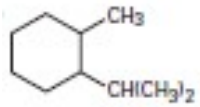
1. Identify the longest carbon chain in the following compounds.



1. Name the following compounds







If u still have challenges with these, lets meet @ nurudeen grammar school and they would be solved for you.

HYBRIDIZATION

The concept of hybridization explains *how* carbon forms four equivalent tetrahedral bonds but not *why* it does so.

It was described 1931 by Linus Pauling, who showed how an *s* orbital and three *p* orbitals on an atom can combine mathematically, or *hybridize,* to form four equivalent atomic orbitals with tetrahedral orientation.

Hybridization is the phenomenon by which new atomic orbital are formed by the mixing of orbitals. There are various forms or types of hybridization; they include sp^3, sp^2,sp

sp^3	sp^2	Sp
Formed by the mixing of 2s and three 2p orbitals	Formed by the mixing of one S with two P orbitals	Formed by the mixing of one S with one P orbitals
25% and 75 % S and P character	50% and 50% S and P character	75% and 25% S and P character
Found in alkanes, bond angle of 109^0 28'	Found in alkenes, bond angle of 120^0	Found in alkynes, bond angle of 180^0
Sigma bonds are formed by the overlap of hybridized orbitals with hydrogen orbital.	One sigma and one pi bond	Made up of one sigma and two pi bonds

QUESTIONS

1. The higher the bond energy of an organic compound, the ___ the bond length
   a) shorter b) longer c) lower d) higher
2. _____is the type of bond that results from side to side overlap of p-orbitals
   a)pi bond b) sigma c)sigma pi bond d) none
3. A sample of methane 96.67mg on combustion produces 26.53mg of CO_2 and 21.56mg of H_2O. Calculate the % composition of C and H in CH_4 respectively
   a)25%,75% b) 75%,25%
   c) 60%,40% d)40%,60%

Use the following information to answer question 4 and 5

	HYBRIDIZATION	BOND ANGLE	BOND SHAPE
C-C	V	W	TETRAHEDRAL
C=C	S	T	U

4. In the above table, V and W represent _____ (a)sp^2,180 ^{o} (b)sp^3 109.5^0 (c)$sp^3$and 120^o
5. S, T and U represent _____and _____ respectively in the table above?
   a)sp,180 ^{o} and triangular
   b) sp^2, 180^o and triangular
   c) sp^2, 120^o
   d) sp^2, 180^o and triangular
6. CH_3 - CH=CH_2.From the left, the type of hybridization of C_1 and C_3 is ___
   a)sp and sp^3 b)$sp^3$and sp^2
   c) $sp^2$and sp^3 d)sp and sp^2
7. Determine the molecular compound Y from the following result. Combustion of 3.54mg of Y gave 8.03mg of CO_2 and 3.34mg of H_2O. The molecular mass of Y is 116.
   a) $C_2H_{12}O_2$ b) C_3H_3O
   c) $C_6H_{12}O_2$ d)$C_3H_6O_2$
8. _____ is the energy level in hybridization.
   a)Sub-secondary energy level
   b)primary –energy level
   c)secondary energy level
   d)main energy level
9. The overlap of the 1s hydrogen electron and $2p_x$ carbon electron produces _____

a) sp b)sp^2 c) sigma bond
d) one sigma and two pi

10. The triple bond consists of ___bonds
a) three pi b) two sigma and one pi
c)one sigma and one pi
d)one sigma and two pi

11. _____ is why the double and triple bonds are shorter than single bonds.
a)decreased 'p' character
b) increased 'p' character
c) decreased 's' character
d) increased 's' character

12. _____ is why sigma bonds are stronger than pi bonds.
a) overlap of 's' orbitals
b) overlap of 's' and 'b' orbitals
c) overlap of 'p' orbitals
d) hybridization of orbitals

13. As the 's' character increases, _____ becomes closer to the nucleus.
a)bond pair orbitals
b)bond pair electron
c)bond pair length
d)bond pair strength

14. Which of the following orbital of carbon are mixed during hybridization?
a)1s and 2s b)1s and 2p c)2s and 2p
d)2s and 3p

15. Which of the following is true
i) Unhybridized are more effective than hybridized orbitals
ii) Hybridized orbitals form the sigma bond
iii) Unhybridized orbitals for the sigma bond
iv) Unhybridized orbitals form only pi bonds
a) i and ii(b)ii and iv(c)I and iv (d)ii and iii

SOLUTION

1. D. this is because the closer the bond of the C-C atom , the higher the bond energy
2. D See table given
3. B
4. B. Refer to the table given.
5. C
6. C
7. C
8. C
9. C refer to the table given
10. D
11. C
12. D
13. B
14. C
15. B

ISOMERISM

The term *isomers* come from the Greek words *isos* and *meros*, meaning "made of the same parts". That is, isomers are compounds that are constructed from the same atoms (same molecular formula) but that still differ from each other. Hence, isomerism is the occurrence of two or more compounds with the same molecular formula but having different chemical properties. There are various types of isomerisms

Constitutional or structural isomerism this is the type of isomerism exhibited by compounds having the same molecular formula but different constitution (order of

connectivity of atoms)/arrangement of atoms in the compound. Examples include Dimethyl ether and ethanol, 2-methylpropane and butane.

Stereoisomerism this is the type of isomerism exhibited by compounds same molecular and structural formula and constitution but different spatial arrangement of atoms in the compound.

There various types of **structural isomerisms,** they include;

- **Positional isomerism**-found in compounds within a homologous series having the same molecular formular but their functional groups are at different positions. It is exhibited in ortho, meta and para isomers of a compound.
- **Chain isomerism-**exhibited by compounds that have the same molecular formular but differ in the arrangement of carbon atoms in either straight or branched chains.
- **Tautomerism** this is a form of structural isomerism that is exhibited by compounds in equilibrium.
- **Functional group isomerism** this is illustrated in compounds that have different functional group but same molecular formular.

Stereoisomerism is divided into two parts **geometric** and **optical isomerism**. One particular **difference** between them is that while optical isomers (**enatiomers**) are mirror images of each other (**non-super imposable**), geometric isomers aren't ; under geometric isomerism we have the **cis- and trans- isomer.**

ALKANOLS

Alcohols also referred to as alkanols can be thought of as organic derivatives of water in which one of the water hydrogen atom is replaced by an organic group (either by an alkyl or aryl group); H-O-H as against **R-O-H (ROH)**or **Ar-O-H**. Alcohols have also been described as hydroxyl derivatives of alkanes; In practice, the group name *alcohol* is restricted to compounds that have their -OH group bonded to a saturated, sp^3-hybridized carbon atom, while compounds with their -OH group bonded to a vinylic, sp^2-hybridized carbon are called *enols.*

Nomenclature of alkanols

There are two basic methods of naming alkanols they are

1. IUPAC NAME
2. COMMON NAME

Simple alcohols are named by the IUPAC system as derivatives of the parent alkane, using the suffix –*ol* in replacing the -*ane*.

THE FOLLOWING RULES ARE SPECIFIC TO THE NAMING OF ALKANOLS.

Rule 1 Select the longest carbon chain containing the hydroxyl group, and derive the parent name by replacing the -*e* ending of the

corresponding alkane with -*ol*. The -*e* is deleted to prevent the occurrence of two adjacent vowels: hexanol rather than hexaneol, for example.

Rule 2 Number the alkane (parent chain) beginning at the end nearest to the carbon attached to the hydroxyl group.

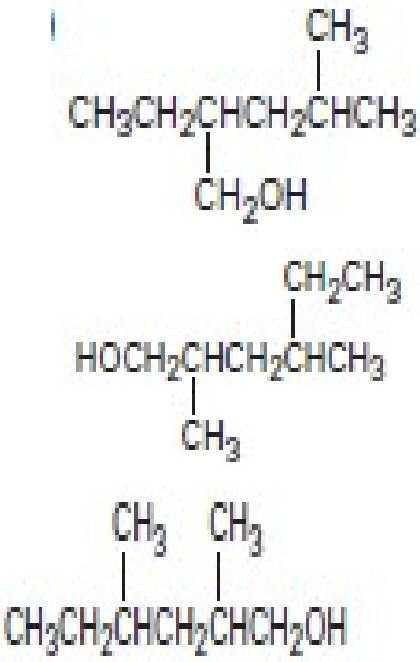
Rule 3 Number the substituent's according to their position on the chain, and write the name listing the substituent's in alphabetical order and identifying the position to which the -OH is bonded.

The position of the hydroxyl group is indicated using a locant. E.g **2**-butanol OR butan-**2**-ol.The IUPAC rules published in 1979 dictate that this locant be placed immediately before the parent, while the IUPAC recommendations released in 1993 and 2004 allow for the locant to be placed before the suffix "ol." Both names are acceptable IUPAC names

Rule 4 Cyclic alcohols are numbered starting at the position bearing the hydroxyl group, so there is no need to indicate the position of the hydroxyl group; it is understood to be at C–1

******Note that in naming *cis*-1,4-cyclohexanediol, the final -*e* of cyclohexane is not deleted because the next letter, *d*, is not a vowel, that is, cyclohexanediol rather than cyclohexandiol**.

Let's try naming the following simple alcohols following the rules given above.



Did u get them correctly, if yes legooooo!!!!!!!!!! Look for more compounds to name and you are on your way to becoming a guru in naming compounds. If not, get someone to do so for you or add 08094551078 on whatsapp and I would gladly help you out with.

COMMON NAME OR TRIVIAL NAME.

Alcohols are named as derivatives of the alkyl group bonded to the hydroxyl functional group. Examples include;

CH_3OH- Methyl Alcohol

CH_3CH_2OH-Ethyl alcohol

Physical Properties Of Alcohols

The physical properties of alcohols are quite different from the physical properties of alkanes or alkyl halides.

1. The boiling point of ethanol is much higher than boiling points of alkanes with

about the same molecular mass, this is as a result of the high energy that must be applied in breaking the hydrogen-bonding interactions that occur between molecules of alkanols. These interactions are fairly strong intermolecular forces, and they are also critical in understanding how alcohols interact with water. For example, methanol is **miscible** with water, which means that methanol can be mixed with water in any proportion (they will never separate into two layers like a mixture of water and oil). However, not all alcohols are miscible with water. To understand why, we must realize that every alcohol has two regions. The **hydrophobic** region does *not* interact well with water, while the **hydrophilic** region *does* interact with water via hydrogen bonding. This is because in the case of methanol, ethanol and propanol, the hydrophobic end of the molecule is fairly small but it is not true of pentanol. In a molecule of pentanol, the hydrophobic end of the pentanol molecule is large enough to prevent miscibility. Water can still be mixed with pentanol, but not in all proportions. In other words, pentanol is considered to be soluble in water, rather than miscible. The term **soluble** means that only a certain volume of a compound will dissolve in a specified amount of water at room temperature. As the size of the hydrophobic region increases, solubility in water decreases. For example, octanol exhibits extremely low solubility in water at room temperature. Alkanols with more than eight carbon atoms, such as nonanol are considered to be insoluble in water. It should also be noted the boiling point of alkanols increases with a corresponding increase in molecular mass and decreases with an increase in degree of branching due to the shape which is assumed by the alcohol molecule, causing lesser contact between neighboring molecules. Relationship between alcohols and their boiling point can be represented as $1^0 > 2^0 > 3^0$. Also, volatility is inversely relational to boiling point i.e it increases with an increase in branching and vice versa.

2. Alkanols are as described above can be soluble in water, it should also be mentioned that the degree of solubility of alcohols in water increases with an increasing –OH group content. i.e monohydric alcohols are less soluble than dihydric alcohols and they in turn are less soluble than trihydric and polyhydric alcohols respectively. This is illustrated in the high solubility of glucose in water.
3. Lower members of the alcohol family are liquids, while the higher members are solids.

CHEMICAL PROPERTIES

1. Alkanols are weak acids, weaker acids than water. They react with electropositive metals such as K, Na, Al e.t.c to yield a class of compounds known as ALKOXIDES.

$$ROH + X \rightarrow ROX + H^+$$

where ROX is an alkoxide. Also, as noted earlier, alkyl groups have an inductive effect (+I),hence the acidity of alcohols decrease with an increase along the homologous series.

2. Alcohols react with acid halides or anhydrides to form esters

$$CH_3CH_2COCl + C_2H_5OH \rightarrow CH_3CH_2COOC_2H_5$$

3. Alcohols also react with alkanoic acids to form esters

$$RCOOH + R'OH \rightarrow RCOOR'$$

4. ALCOHOLS are also dehydrated upon heating, or by conc. H_2SO_4 to alkenes.

Questions

1) The reason why the boiling point of alkanols decreases with increased branching is ? (a) the molecule cools easily (b)the molecule assumes a circular shape (c) the molecule assumes a tetrahedral shape due to hydrogen bonding (d) the molecule assumes a spherical shape

2) When Na is added to propan-1-ol,the products formed are ? (a) sodium propoxide and water (b) sodium propoxide and hydrogen (c) sodium hydroxide and water (d)NaOH and H

3) Which of the following alkanols will not yield alkanoic acids on reacting with excess acidified $K_2Cr_2O_7$? (a) $(CH_3)_3COH$ (b) $(CH_3)_2CHCH_2OH$(c) $(CH_3)_2CCH_2OH$ (d) CH_3CH_2OH

4) The raw material for a large scale production of ethanol ? (a) starch(b) C_2H_6 (c) C_2H_2 (d) C_2H_6

5) A secondary alkanol reacts with acidified $KMnO_4$ solution to form a ……..? (a) Alkene(b) Alkanone (c) Alkanoic acid (d) Alkanal

6) Alkanols form hydrogen bonding with…? (a) another alkanol (b) H_2O (c) carboxylic acid (d) Ester

7) Dehydration of propan-1-ol produces……? (a) propanoic acid(b) propanol (c) propene (d) propan-1,2,3-triol

8) ……..is called alkaline hydrolysis of esters (a) esterification (b) saponification (c) dehydroalkylation (d) fermentation

9) Alkanols are more soluble than the corresponding hydrocarbons because of what? (a)ability to form intermolecular bonds with themselves (b) ability to form hydrogen bond with water molecules (c) being acidic more than them (d) being basic more than them

10) Complete oxidation of butan-1-ol gives ……..? (a)butanal(b) butanone (c) butanoic acid (d) butan-2-ol

11) One of the products of the combustion of ethanol in excess air is …..? (a) O_2(b) H_2 (c) CO_2 (d) C

12) Ethanol is obtained from sugars by ….? (a)fermentation (b) saponification (c) dehydration (d) esterification

13) Which of the following has the highest boiling point is ? (a)1^0(b)2^0 (c) 3^0(d) polyhydric alcohol

14) Which of the following can be used to differentiate alkanals from alkanones? (a) HCN(b) $NaHSO_3$ (c) fehlings reagent (d) 2,4-dinitrophenylhydrazine

15) Which of the following forms the strongest hydrogen bond? (a) -OH(b) -COOH (c) –CHO (d) -CO

16) Which of these is the most reactive, inspite of the alkyl hydrocarbon groups? (a) primary alcohol (b) tertiary alcohol (c) polyhydric alcohol (d) secondary alcohol

17) Which of these is formaldehyde? (a) HCHO (b) RCHO (c) CH_3CH_3CHO (d) –CHO

18) A liquid that is neutral to litmus paper but gives a colourless gas with metallic sodium must be ....? (a) alkene(b) alkane (c)alkanoate (d) alkanol

19) Why does palmwine get sour with time? (a) it is usually aldulterated by the sellers (b) it is naturally acidic(c) microbial activity results in the production of ethanoic acid within it (d) it is an acid anhydride

20) The gas evolved when alkanols react with sodium is .....? (a) O (b) N (c) H_2 (d) CO_2

21) A dihydric alcohol is .....? (a) ethanol(b) glycerol (c) phenol (d) glycol

22) An example of secondary alcohol is ? (a)CH_2=CH-CH_2OH(b)$CH_3CH(OH)CH_3$ (c) $CH_2(OH)CH_2$ (d) CH_3CH_2OH

23) Which of these is the byproduct of the fermentation of sugar to ethanol? (a)propanol(b)propane-1,2,3-triol (c) CO_2 (d) CO

24) The method used to separate ethanol from the mother liquor obtained in fermentation of simple sugars is …..... is ? (a)fractional crystallization (b)fractional distillation (c) filtration (d) centrifugation

25) The ease of dehydration of monohydric alcohols in ascending order is …... is ? (a)$3^o > 2^o > 1^o$(b) $1^o < 2^o < 3^o$ (c) $3^o < 1^o < 2^o$ (d) $3^o < 2^o < 1^o$

26) Which of the following has the highest solubility_____(a)secondary alcohol (b)polyhydric alcohol(c)tertiary alcohol(d)dihydric alcohol

27) .The alcohol group that will be oxidized by the iodoform test to give a positive test is _____(a)CH_3ROH(b)CH_3CHROH(c)CH_3CH_2ROH(d)all.

28) A compound that reacts readily with sodium to liberate hydrogen gas is _____(a)$CH_3COCH_2CH_3$(b) $CH_3CH_2CH_2CH_3$ (c)CH_3CH_2 CH_2CHO(d) $CH_3CH(OH)CH_2CH_3$

29) Which of these is the most acidic ? (a)$3^o > 2^o > 1^o$(b) $1^o < 2^o < 3^o$ (c) $3^o < 1^o < 2^o$ (d) $3^o < 2^o < 1^o$

SOLUTION

1. D. this has been discussed in the lecture note.
2. B
3. A. it should be noted that tertiary alkanols can only be oxidized by acidic oxidizing agent such as conc HNO_3.
4. A. ethanol is prepared on large scale (industrial) by the fermentation of starch.
5. B. when alkanols are reacted with oxidizing agents, secondary alcohols yield

ketones upon initial oxidation and carboxylic acids upon further oxidation. Primary alkanols yield alkanols

6. B. alkanols form hydrogen bonds with water easily, which determines their solubility.
7. C. the dehydration of alkanols yield alkenes. It can be done by heating or addition of conc. H_2SO_4
8. B. saponification; important in the soap industry
9. B. see lecture note
10. C. this has been explained in Q3
11. C. the combustion of ethanol yields water and carbon (iv) oxide
12. A. explained earlier
13. D. due to increased –OH groups. see lecture note.
14. C
15. B
16. C
17. A the first member of the alkanol family 'methanol'
18. D. the gas released is H_2 with the corresponding formation of an alkoxide
19. C
20. C. discussed in Q18 above
21. D
22. B
23. C.
24. D this is due to their different boiling points
25. D
26. B. discussed earlier
27. B
28. D. see Q18
29. D

FOOD CHAIN

The sun is the source of all energy on earth. The transfer of this energy (food energy) from the source in plants through a series of organisms with repeated stages of eating and being eaten is known as food chain

Examples of food chain in aquatic habitat

a) Spirogyra ⟶ tadpole ⟶ fish snake

b) phytoplanlaton ⟶ zooplankton ⟶ whale ⟶ bacteria

example in terrestrial habitat

a)Grass ⟶ Grasshopper ⟶ lizard ⟶ hawk

FOOD WEB

This is a series of food chains interlocking together and it actually exists in nature. E.g, most herbivores eat many types of plants while most carnivores eat several types of herbivorous and other carnivores

TROPHIC LEVEL

Organism feeding at the same number of step in a food chain or web from the autotrophs are said to be at the same tropic level. It therefore means feeding level

ECOLOGICAL PYRAMIDS

When materials pass through a food chain, only a small proportion of the energy taken up by each link is transferred to the next step. This is because at each transfer, most of the energy is lost as heat. Only 20% of the energy contained in the food material becomes incorporated into the body of consumer, the rest is lost as heat, used up in respiration or lost

TYPES OF HABITAT

1. AQUATIC HABITAT :

a) Marine habitat: Make up of seas and oceans

b) Estuarine habitat: Estuary is the point of transition between the sea and river. It is the tidal mouth of the river where there is a mixture of salt and fresh water

c) Fresh water habitat: includes rivers, ponds and lakes

2. TERRESTRIAL HABITAT :

a) Marsh: A low–lying wet area. Vegetation consists of few species of grasses, sledges and lichens

b) Forest: is a covered with trees and occupies about two –thirds of the world's land surface.

Animals in forest include mammals, birds, reptiles, amphibians

Stratification is a common feature of the forest and includes

A ➡ Upper Story Trees

B ➡ Middle Story Trees

C ➡ Lower Story Trees

D ➡ Shrubs Story Trees

E ➡ Herb Story Trees

c) GRASS LAND: Is an extensive habitat found in Africa, South America, north Australia and Asia consisting predominantly, of grasses usually burnt annually also known savannah

d) Arid lands (i) exerts: they are characterized by low precipitation or rainfall and are found in Africa, central Asia, Australia, Mexico and the Kalahari

PAST QUESTIONS

1. The interaction within an eco system regulate which of the following a) primary productively b) energy flow c) cycling of nutrients d) energy flow and cycling of nutrient
2. The association between the root nodule of a leguminous plant and rhizobium is called a) commensalism b) parasitism c) mutualism d) saprophytism
3. A group of individual of a single specie living together is known as _____ a) community b) association c) population d) ecotypes
4. The following are examples of interactions within a community except a) commensalism b) predation c) mutualism d) socialism
5. Predation is an example of which factors? a) abiotic b) biotic c) edaphic d) societal
6. The relationship between egret bird and cattle is a) neutral b) symbiotic c) parasitic d) commensalism
7. Regulation of energy flow and cycling of nutrient are phenomena common in _____ a) community b) xerarch succession c) ecology d) ecosystem
8. The energy system flowing through an ecosystem is derived from a) national grid b) the 133KVA substation at new bussa c) sun d) the moon
9. One of these is important in the early stages of succession on a rock a) symbonts b) commensals c) parasites d) predators
10. All ecosystem consist of two kinds of living organisms ____ and_____ a) chemolythotrophs and ologotrophs b) autotrophs and parasites c) chemo organotrophes and auxotroph d) autotrophs and heterotrophs
11. _____ is referred to as a community and it a factional unit of ecology a) ecosystem b) population c) diversity d) environment
12. _____ refers to all biotic and abiotic conditions that surround living organisms a) population b) environment c) habitat d) factors

13. Physical variables that influence the behavior of organisms are termed _____ a) abiotic factor b) biotic factors c) niche d) predation
14. The specific role a specie In its environment is a) co-evolution b) predation c) biotic roles d) niche
15. _____ is when individuals of different species complete for limited resources a) predation b) competition c) niche d) habitat
16. _____ is the flow of energy from the sun, passing from one organism to another a) energy flow b) food web c) trophic level d) food chain
17. A step in the movement of energy in an eco-system step in the transfer of energy is called ____ a) trophic level b) phyramid c) energy flow d) niche
18. Parasitic microorganisms are called _____ a) pathogens b) parasites c) symbiosis d) predation
19. Soil conditions are called a) edaphic factors b) soil factors c) soil nutrient d) soil type
20. Nitrogen constitutes _____ % of the earth's surface a) 78% b) 1% c) 2% d) 50%

ANSWERS

1. ……. D
2. ……. C
3. ……. C
4. ……. D
5. ……. B
6. ……. B
7. …….. D
8. …….. C
9. ……. A
10. ……. D
11. ……. A
12. ……. B
13. ……. A
14. …… D
15. ……..B
16. …….. A
17. …….. A
18. …….. A
19. …….. A
20. ……. A

TAXONOMY

This is the science of classification and identification of organism. It's also known as SYSTEMATICS

Taxonomy deals with the classification, identification, description and nomenclature of living organisms and each level of classification is known as a TAXON or TAXA (plural)

CLASSIFICATION

a) CRITERIAL: We consider three features of an organism to determine the class of that organism. They are;

i. Morphological features

ii. physicalfeatures

iii. Phylogenetic relationship

b) ORDER OF CLASSIFICATION: The order in descending order; kingdom division /

phylum → class →order→ family→ genus species

c) REASONS FOR CLASSIFICATION

i. for easy and effective identification

ii. for effective study

iii. for effective communication

iv. To reflect / show evolutionary relationship

SYSTEM OF CLASSIFICATION

1) Two kingdom system:

a – kingdom Animalia b – kingdom plantae

The plant kingdom contains 4 divisions

- Thallophyta
- Bryophata
- Pteridophyta
- Spermatophyte

2. Three kingdom system:

A – Kingdom Protista; containing all single forms of life e.g. bacteria, algae, fungi, protozoa

B – KingdomPlantae; all plants

C – kingdom Animalia ; all animals

3. Four kingdom system;

A – kingdom monera; all proccaryotes e.g bacteria and blue green algae

B – Kingdom Protista ; algae, protozoa and fungi

C – KingdomPlantae; all green plants

D – kingdom Animalia ; all animals

4) Five kingdom system :

A – Kingdom monera, prokaryote unicellular e.g virus, bacteria in blue – green algae

B – KingdomProtista; eukaryotic, both plants and animal characteristic e.g., protozoa, algae

C – Kingdom mycota; eukaryotic, lack chlorophyll, extracellular digestion e.g. fungi, mushroom

D – kimgdom plantae ; malticelluler , immotile , autotrophice.g. plants

E – Kingdom Animalia, motile, heterotrophic e.g. animals

BINOMIAL SYSTEM OF MOMENCLATURE

This system ensures that every organism is given two names:

1. First name ⇨ genetic name: starts with capital letter and is a noun usually. It is underlined separately from the second name. It's also a Latin word
2. Second name ⇨ specific epithet: starts with small letters and usually an adjective. It is also underlined separately from the first name and a Latin word

The two names together are called "species name" e.g.: Dioscorea alata and aspilia Africa

⇩ ⇩

Generic specific epithet

The binomial system is the most widely accepted system of nomenclature discovered by the father of taxonomy – (Carolus Linnaeus)

PAST QUESTIONS

1. Taxonomy is a) the science of classification and identification of organisms b) the science of identification and nomenclature organism c) the science of gathering and assessment of classification data d) the science of classification and nomenclature of organism
2. The practice of naming individual taxonomic group plant and animal using certain rules is called a) systematics b) nomenclature c) classification d) epithet
3. _____ is known as the author of the binomial system of nomenclature a) whit taker b) clarles lewis c) Charles Darwin d) carolus Linnaeus
4. _____ is the grouping of organisms according to their relativeness and affinities a) classification b) nomenclature c) identification d) description
5. A taxonomist a) identifies , groups, describes and name b) survey, name, inform, reproves c) classify, inform, store, name d) categorize, preserve, inform storage and name
6. A biological organism is given two names a) generic and specific b) genus and species name c) generic name and specific d) generic name and species name
7. The organized technique for studying the world of plants or animals in order to establish their relationship is a) botany/zoology b) systematic c) taxonomy d) systematic/taxonomy
8. The _____ system is the widely acceptable method of scientific naming of organisms a) binomial b) unicellular c) trinomial d) polynomial
9. Taxonomy is synonymous with a) systematic b) systemic c) statistics d) steromatics
10. A generic name and its specific epithet make up a _____ a) species name b) specific name c) genera name d) genetic name
11. Who is believed to be father of taxonomy a) Karl max b) carolus Linnaeus c) john ray d) caessssalpinus
12. Which of the following statements is correct a) the lowest taxonomic category is the generic name b) the highest unit of classification is the phylum c) a collection of related families o organism forms an order d) members of a family of plants share more characteristics than these in a genus
13. The smallest unit by which an organism can be ranked is a) specific epithet b) specie c) species d) specific name
14. Identification means _____ a) the determination of a name for an unknown organism b) the study of parts an organism
15. Each grouping by which a set of related organisms can be recognized is referred to as a) taxos b) taxon c0 taxo c) taxonomy
16. why do we need to classify plants? a) to establish the criteria for their classification

b) to arrange them for easy study a) to provide information on possible lines of evolution d) all of the above

17. Which of the following statement is correct? a) the lowest taxonomic unit is the genus b) the highest unit of classification is the phylum c) a collection of related families forms an order d) members of a family of plant are closer than those of the genus
18. The technique whereby an organism is scientifically given two names is known as _____ a) nomenclature b) binomial nomenclature c) binomial naming d) nomenclature
19. Modern Biological classification according to carolus Linnaceus is based on a) shared anatomical characteristics b) shared morphometric characteristics c) shared physical characteristics d) shared phenotypic and genotypic characteristics
20. Which of these is not a rule of binomial nomenclature a) two names to an organism b) internationally recognized names c) practically Latin language is used d) names are italicized

ANSWERS

1. …….. A
2. ……. B
3. ……D
4. ……. A
5. ……. A
6. ……C
7. …….. D
8. …….A
9. ……. A
10. ……. A
11. ……. B
12. ……. A
13. …….C
14. …….. D
15. …….. B
16. …… D
17. ……C
18. ……. C
19. …….. C
20. ……. B

DIVERSITY OF PLANT

The simplest classification of plants is the 2-kingdom classification that divides plants into four.

1. division thallophyta
2. division bryophyta
3. division pteridophyta
4. division spermatophyte

Thallophyta	bryophyta	pteridoptyta	spermatophyte
undifferentiated body (no root) stem or leaf	Differentiated body into root-like, stem- like and leaf- like	Body differential into root , stem, leaf	Body differential into root, stem, leaf
female gamete is oorgonium	Female gamete is archegonium	Female gamete is archegonium	Female gamete is archegonium
gamete borne in unicellular game-tangra	Gamete borne in multicalluler gametangra	Gamete borne in multicellular gametangra	Gamete borne in multicellular gametangra
no alternation of generation	Alternation of generation(g-ametophyte dominate)	Alternation of generation (sporophyte dominate)	Alternation of generation(sporohyte dominate)
no embryo	Form embryo	Form embryo	Form embryo

	(embryophytes)		
zygote undergo resting stage	No resting stage	No resting stage	No resting stage
non- vascular plants	Non- vascular plants	Vascular plants(tracheophyte)	Vascular plants
flowersless (cryptogams)	Flowerless (cryptogam)	Flowing plants(phaneragams)	Flowing plants (phaneragams)
Water is needed for fertilization	H2O is needed for fertilization	H2O is needed for fertilization	H2O is not needed for fertilization
reproduce by syngamy (conjugation)	Reproduce by oogamy	Reproduce by spore	Reproduce by seed
E.g is bacteria algae,fungi and lichens	E.g , liverwort and mosses	E.g ferns and dryopteris	E.g is Gymnosperm

BACTERIA - THATALLOPHYTE

1. they are minute organisms that cannot be seen with the naked eyes but under light microscope
2. some occur everywhere, in air, water soil and inside other organisms
3. they live mostly under warm condition but some survive in the cold
4. their shapes vary from coccus to biccillus, vibrio and spirillum
5. some bacteria can survival aerobically or anaerobically
6. some can feed autotrophically or heterotrophically
7. they reproduce by repeated binary fission

SPERMATOPHYTA

1. Class gymnospermae
2. class angiospermae

class gymnospermae are the non-flowing plants. They have naked seed e.g cycas

class angiospermae are the flowing plant possessing protected seed. They are further divided into two: monocotyledons and dicotyledons

PAST QUESTIONS

1. Living things are made up chiefly of _____, a substance in which all life processes taken place. it is contained in compartment called _____ a) protoplasm cell b) stem, root c) cytoplasm, ER d) cell, cell wall
2. plants are classified according to their physiology, morphology and _____ characterized a) behavioral c) phylogenetic relationship c) genetic similarities d) phenotic
3. the ____ is the dominant generation of bryophyte a) ametophyte b) spermatophyte c) thallophyte d) pteridophyle
4. the five kingdom of living organisms include monera, protista, phantae, animalia and ____ a) bryophyte b) pteridophta c) mycot d) plant
5. bacteria exists in four body forms which are coccus, baecillus, spirillium and _____ a) bent b) ovoid c) rod-like d) vibrio
6. sexual reproduction between identical motile gametes is _____ a) conjugation b)isogamy c) anisogamy d) apolanogametes

7. fusion of identical aplanogametes is called_____ a) anisogemy b) ocgamy c) isogamy d) conjugation
8. bacterial cell wall consists of ____ and _____ a) cell membrane and cell cellulose b) peptodoglycam and nucleic acid c) peptidoglycan and murein d)none
9. the _____ of a plant is the key to its literature a) class b) identity c) description d) name
10. the stamen is _____ while the pollen sacs are the _____ a) microsporophyll, microsporangia b) pericarpm, mestocarp c) mesocarp, enduocarp, epical
11. the _____ form the fruit white the _____ form the seed a) ovary, ovule b) ova, ovide c) ovule, egg d) stamen, pollen
12. the brilliant colours characteristics of the leaves of many plants are caused by presence of _____ a) pigments b) chrolophyll c) chloroplast d) accessory pigments
13. _____ and _____ do not undergo modification a) leaf blade , petiole b) leaf blade, petiole c) petiole, stamen d) stamen, pollen
14. _____ is the principal food making organ of a plant a) leaf base b) leaf c) stem d) root
15. _____ is a part of a vascular plant that commonly bears leaves and buds a) root b) shoot c) stem d) leaf
16. _____ is the reproductive organs of certain plants a) flowers b) seed c) shoot d) stem
17. the four types of modified leaves in a flower are; sepals, petals, ovary and _____ a) ovules b) stamen c) pollen d) sporangia
18. A matured ovary in flowing plants is _____ a) flower b) fruit c) root d) short
19. type of fruits include, nut ash and _____ a) maple b) drupe c) ash d) all of the above

ANSWERS

1. ..... a
2. ..... b
3. ..... a
4. c
5. ..... d
6. ..... a
7. c
8. c
9. ..... d
10. ..... a
11. ..... a
12. ..... d
13. ..... b
14. ..... b
15. c
16. ..... a
17. ..... a
18. ..... b
19. .....d

DIVERSITY OF ANIMALS

The most commonly used instrument of animals classification is their morphology and anatomy

Animals can either be invertebrates or vertebrate

Invertebrate animalsinclude: protozoa, annellida, Athropoda, Mollusca, Echinodermata, Platyhelminthes (flat worms) and coelenterate (cniodaria)

Vertebrate animals include :*pisces, amphibian, reptilia, aves and mammalia* known as "CHORDATA"

INVERTEBATA

1. PROTOZOA: Futher classified into phylum, sarcomastigophora, ciliophara, apicomplexa, myxozoa, microspora, labyrinthomorpha and phylum asceptospora

a. **PHYLUMSARCOMASTIGOPHORA**

This phylum comprises three(3) sub phyla : they are sub – phyla sarcodina, mastigopora and opalinata. They possess single type of nucleus.Theirlocomotory organelles are flagella, pseudopodia or filopodia that also serve for food collection purposes. All members are both parasitic and free – living organisms

b. PHYLUM APICOMPLEXA

All members are parasitic (endoparasites) and incapable of active life outside their host

They have no external locomotory device e.g. is plasmodium specie. The cycle is indirect and it employs an invertebrate host – anopheles mosquito

c. **PHYLUM CILIOPHARA**

All members possess cilia at least in juvenilestages either as a complete covering over the whole body surface or restricted to one surface only. They are both parasitic and free –living. They undergo asexual reproduction by transverse binary fission

e.g.Paramecium anurelia

2. CNIDARIA (coelenterate)

It is divided into 3 classes namely: hydrozoa e.g. hydra, scyphozoan e.g aurelia and anthozoaeg metricium /sea anemone

3. NEMATODA: is further divided into 2 classes:

- Aphasmidiae.g. trichinella

- phasmidiae,gascaris specie

4. ANNELIDA: Divided into four(4) classes namely;

- polychaetae.g tubeworm

- oligochaetae.g earthworm

- hirudineae.g. leeches

- archiannelidae.g. dinophilus, polygrordius, etc

5. ARTHROPODA: divided into four sub – phyla

- crustacean

- insecta

- arachnida

- myriapoda

6. MOLLUGSCA: Divided into seven classes;

- polyplatcophora (chitons)

- aplacophora (solenogasters)

- scaphopeda (tooth shells)

- Bivalvia (bivalves)

- gastropoda (snail and slugs)

- Cephalopoda (squids and octopuses)

- monoplacophora (neopilina)

7. ECHINODERMATA: Divided into two sub – phyla

- pelmatozoa

- eleutherozoa : divided into –

class asteroidean(starfish)

- class ophiaroidea (brittle fish)

- class enchinoodea(sea urchines)

- classholothuridae (sea cucumber)

8. **PLAYHELIMIHTHES:** Examoles include;

Planaria, liverfluke, tapeworm, amphilolina, polystoma

THE CHORDATES

1. SUB – PHYLUM HEMICHORDATE:

Known as false characters because they have indirect development form a tomaria larva which is similar to echinoderm larvae. Examples are balanoglossus and (cephalodiscus

2. SUB – PHYLUM UROCHORDATE

They are unsegmented marine chordates that feed by filtering sea water and differ from other chordates in that their notochord is present only in the larval form confirmed to tail and posterior regions

In most of them, the bulk of the chordate characters are lost in the adult but are found in the free swimming larval states.E.g. gona intestinalis (sea squirt)

3. SUB-PHYLUM (EPHATOCHORDATE) :

Also known as lancelets, are small fish- like animals that possess most of the main chordate characters inhabit sandy beaches

Sexes are separate but males and females are indistinguishable apart from the nature of the gonads

4. SUB – PHYLUM VERTEBRATE:

They are chordates with a series of cartilaginous or bony vertebrae which either replace or re – in force the notochord i.e. possess vertebral columns or back bones:

Include class Pisces, amphibian, reptilian, aves and mammalia

PAST QUESTIONS

1. The body divisions in animals like annelids is known as a) repetition segmentation b) metameric segmentation c) serial segmentation d) cephalization
2. The class of coelenterates which exhibits alternating medusoid and polypoid form is a) hydrozoa b) metazoan c) scyphozoan d) anthozoa
3. Asexual reproduction in protozoans does not involve one of the following a) schizogamy b) spotogony c) syngamy d) endodyogeny
4. The alimentary carnal of cockroaches is divided in order the of a) stomodaeum, protodaeum, mesenteron b) mesenteron, protodaeum and stomaeum c) stomodaeum, mesenteron and protodaeum d) protodaeum, stomodaeum and mesentron
5. The main organelle of excretion in protozoa is _____ a) lysosome, b) contractile vacuole c) cell membrane d) semi – permeable membrane
6. Flame __cell system functions primarily in the regulation of a) metabolic wastes b) water balance c) CO_2 d) no answer
7. The beginning of tubular excretory system was first seen in ____ a) coelenterates b) flatworms c) flagellates d) all of the above
8. The level stage in annelids is _____ a) plannula b) redia c) trocophore d) blastula
9. A unique characteristic feature of flatworms is _____ a) cephalization b) strobilization c) segmentation d) triploblastic
10. The segments of the leg of cockroach include the following except _____ a) tibia b) coxa c) femur d) fibula
11. Molluscs are thougth to have evolved from a phylum known as a) nematode b) annelid c) platyheluinthes
12. Earthworms excretory organ is known as a) nephridium b) flame cell c) kidney d) urethra
13. The four major sub-phyla of phylum arthopoda are crustacea, myriapoda, inscta and ____ a) arachnida b) xyphosura c) onycophora d) crustacean
14. One of the following organelles is not unique to protozoa a) nematocysts b) trochocysts c) tosocysts d) contractile vacuole
15. A peculiar alternation of generation encountered in cnidarian can be described as ___ a) metamerism b) metagenesis c) morphogenesis d) mutagenesis
16. A known function of tentacles in coelenterates is not one of these a) food capture b) ingestion c) defense d) secretion
17. Cephalization process starts from the _____ a) annelid b) anthrozoan c) protozoa d) arthropoda
18. The exosteleton has the following discernible parts, tergum, pleuron and _____ a) sternum b) sterna c) plural d) extensor

19. Arthropods' blood opens into a space called _____ a) haemocytes b) haemocoels c) haemoglobin d) haemalaya
20. Nematocysts in coelenterates function for all but one of these a) posion injection b) prey hodding c) prey adhesum d) prey sucking

ANSWERS

1. …… B
2. ……. A
3. ……. C
4. …… C
5. …… B
6. …….B
7. …… B
8. ……. C
9. ……. B
10. …… D
11. …… B
12. …….. A
13. ……. A
14. ……. A
15. ……. B
16. …….. D
17. …… A
18. …… A
19. …..B
20. ……. D

INTRODUCTION ECOLOGY

CBT PRACTICE QUESTION

1. Bacteria reproduce by ____ when conditions are suitable
a) binary fusion b) binary fission
c) buddy d) karyogenesis
2. Desert plants are called _____
a) hydrophytes b) mesophyte
c) xerophytes d) cactus
3. _____ is characterized by low precipitation a) savanna b) arid land
c) marsh d) forest
4. Stratification is a common feature of the _____ a) desert b) forest c) savanna
d) marsh
5. Low- lying wet area is called _____
a) grassland b) forest c) arid land
d) marsh
6. Trophic level is synonymous to feeding level. TRUE or FALSE
7. A series of transfer of energy by organism form each trophic level feeding on one another is known as? a) food chain b) food web c) pyramid of number
8. When competition occurs among individuals of the same species, it is termed _____ competition a) specific b) interspecific c) intraspecific d) none
9. Which of the following is odd?
a) humidity b) light c) Predation
d) temperature
10. _____ is the specific role a species play in its environment a) habitat b) niche
c) population d) feeding
11. The branch of biology in which the relations of organism to the physical and biotic environment are studied is

a) ecology b) ecosystem c) community d) biosphere

12. The major driving force in an ecosystem is a) chemical energy b) solar energy c) lalater d) nutrient

13. One of these is not among the major parts of an ecosystem a) decomposes b) competitors c) producer d) consumer

14. The functional unit of ecology is a) niche b) habitat c) autecology d) ecosystem

15. A unit that is composed of all living populations in a habitat a) population b) ecology c) community d) niche

16. The sum total of the biotic and abiotic condition in which an organism lives is? a) ecology b) ecosystem c) environment d) community

17. The sub- phylum vertebrate is classified into two super classes and these are ___ and ___ a) echinoidea and asteroidean b) holotharoidea and echinodea c) agnathans and gnathostomata d) larvacea and ascidiacea

18. The developmental stage in the cucumaria is known as ____ a) auricularia larvae b) introvert anus c) corona d) none of the above

19. Which of the following is true of the mode of fertilization in the echinoderms a) all animals undergo external type of fertilization b) all animals are viviparous with no exception c) all animals undergo external type of fertilization with no exception while few are viviparous d) all are viviparous

20. Respiration in the star fishes occurs mainly through _____ a) cloaca b) genital bursea c) papulae d) perristomial gill

21. Rapid method of locomotion in hydra is _____ a) creeping b) gilding c) looping d) somersaulting

22. The only parasitic class of annelids is _____ a)oligochaetea b) polychaetea c) archiannnelida d) hirudinea

23. The relationship between egret bird and cattle is a) neutral b) symbiotic c) parasitic d) commensal

24. Phylum which are most closely related to the chordates are a) Mollusca b) echinodermata c) arthropoda d) annelid

25. The lower jaw of the mammal is compound of a) dentary bone b) none c) bone d) teeth

26. The inner surface of a protozoan body which is fluid – like is ____ a) endocrine b) ectoplasm c) endoplasm d) endolymph

27. Which of these descriptions best defines protozoan a) they are uninuclate accellular b) they are single – celled unicellular c) forming animals d) they are non cellular animals

28. In chronological order, the three eras in the geological time scale are a) Cenozoic, paleozoix and mesozoic b) Cenozoic, Mesozoic, Paleozoic c) Mesozoic, Cenozoic, Paleozoic d) Paleozoic, Mesozoic, Cenozoic

29. Which of the following pairs is correct? a) cenozoic era – green algae b) Paleozoic

era – blue green algae c) Mesozoic era – bacteria d) Pleistocene era – fungi

30. The best quality of pearls is known as a) real moti b) sweta moti c) lingha moti d) none
31. Blood of the Mollusca contains a) RBS b) WBC c) amoebocytes d) none
32. The lower jaw of class mammalian are the following a) bone b) dentary bone c) double bone d) none of the above
33. The following are types of fruit except a) sweet chest nut b) sweet pea c) pineapple plant d) fruit pea
34. Fungal cell consist of a) cellulose b) chitin and cellulose c) glycogen d) chitin
35. The following are floral of paleozoie except a) marine algae b) green algae c) age of ferns d) bacteria
36. Factors that may be external, internal, physical or biological that affects organisms in its environment are called _____ a) total factors b) indigenous factors c)ecological factors d) environmental factors
37. The following factors are physical environmental factors a) light, water, competition b) P^H, humidity, salinity c) water current, topography, predation d) mineral salts, wave action, parasitism
38. The factor that distinguishes marine water, brackish water, and fresh water is a) light b) temperature c) humidity d) salinity
39. A system where a series of food chains interlocks together naturally in nature is _____ a) inter locking b) food web c) trophic levels d) predation
40. When competition exists between individuals of different species, it is called _____ a) competition b) intraspecific competition c) interspecific competition d) predation
41. When an organism used another organism as a habitat, this kind of relationship is called _____ a) parasitism b) competition c) predation d) food chain
42. The sum total of physical variables that influence the behavior of an organism in an ecosystem is known as _____ a) biotic factors b) abiotic factors c) climatic factors d) edaphic factors
43. The association between the root modules of a leguminous plant and rhizobium is called _____ a) commercialism b) parasitism c) mutualism d) saprophytism
44. The feeding of **taenia saginata**on its host digested food is termed a) commensalism b) autotropism c) mutualism d) parasitism
45. The following are examples of interaction within a community except a) commensalism b) predation c) mutualism d) socialism
46. The community interaction that occurs when a shared resource is in limited supply is known as _____ a) co – evolution b) association c) competition d) parasitism
47. Which of these oxidizes ammonia formed during the decay of animal and plant

protein a) nitrobacter b) hydrogen bacteria c) iron bacteria d) nitrcoccus

48. The energy flowing through an ecosystem is derived from a) national grid b) the 133KvA succession c) sun d) moon

49. Regulation of energyflow and cycling of nutrient are common in _____ a) community b) xerarch succession c) ecology d) ecosystem

50. Which of these is true in a competition a) a stored resource is in short supply b) all species have enough to shared c) one specie is consumed by another in a community d) all the species involved are not affected in any way

ANSWERS

1. .... B
2. C
3. .....B
4. ..... B
5. C
6. ..... TRUE
7. ..... A
8. C
9. C
10. ..... B
11. ..... A
12. .... B
13. .... B
14. ..... D
15. C
16. C
17. C
18. ..... A
19. C
20. C
21. ..... D
22. ..... D
23. .... B
24. ..... B
25. ..... A
26. C
27. ..... D
28. ..... B
29. ..... B
30. C
31. C
32. ..... B
33. ..... D
34. ..... D
35. C
36. C
37. ..... B
38. ..... D
39. ..... B
40. C
41. .... A
42. .... B
43. C
44. ..... D
45. ..... D
46. C
47. .... A
48. C
49. ..... D
50. C

PLANT DIVERSITY

CBT QUESTION

1. The part of a vascular plant that commonly bear leaves and bonds are _____ a) rhizoid b) root c) stem d) shoot
2. Eucalyptus leaf is an example of _____ a) succulent leaf b) ever green leaf c) dimorphic leaf d) carnivorous leaf
3. If successive pairs of leaves are arranged at right angles, the leaves are said to be _____ a) phyllotaxy b) verticillate c) latioled d) decussate
4. In angiosperm, the ovary develops to the fruit wall which is otherwise known as _____ a) epicarp b) pericarp c) mesocarp d) endocarp
5. One of the following statements is not true? a) calyx and corolla of a flower are called perianth b) a fruit is a mature ovule c) a seed develop from a mature ovule d) maize is a caryopsis
6. _____ is a simple dry dehiscent fruit which when ripens, dehisces(split) longitudinally to one side only to release the seeds inside it? a) legume b) capsule c) follicle d) siliqua
7. All the following fruit are hesperidia except a) grape fruits b) tomato c) orange d) lemons
8. _____ is the term used when matured of fruit occurs without fertilization? a) parthenogenesis b) parthenostamen c) parthenocarpy d) none of the above
9. Some flowers give off putrid odour to attract their pollinators and are called _____ a) carrion flowers b) odourish flowers c) scenty flowers d) nectarish flowers
10. The inner most part of a flower are the a) stamens b) petals c) carpels d) sepals
11. The embryo, food reserve (endosperm) and the protective coats all together are called _____ a) seed b) fruit c) seed coast d) testa
12. Carpels frequency fused to form a) gymnoecium b) stigma c) pistil d) ovary
13. A seed is said to be _____ if by the time the seed enter dormancy, all the endosperm is not yet fully absorbed into the embryo? a) exospermous b) endospermus c) exalbuminous d) non-endospermic
14. The endosperm nucleus that is formed after fertilization in angiosperms is _____ a) hapoid b) triploid c) diploid d) tetraploid
15. Vessels are completely absent in the xylem of gymnosperms except those of _____ a) cycadales b) gnetales c) ginkagoales d) taxales
16. The part of the head of a sporangium that is thin – walked constitutes the _____ a) annnules b) rhlzome c) stomium d) annulus
17. Vegetative reproduction in ferns is by means of _____ a) clusters b) spores c) sori d)rhizome
18. _____ are sometimes named “the vascular cryptogams”? a)pteridophyte b)gymnosperms c)bryophytes d) thallophytes
19. While the young gametophyte in ferns is called prothallus, that of a moss plant is

called _____ a) ramenta b) peristome c) protonema d) dryopteris

20. The body of the capsule of the sporophyte in moss plant is differentiated into how many? a) 4 b)2 c)3 d)5

21. The antheridia of moss plant are intermixed with a large number of sterile structures called _____ a) elaters b) paraphyses c) spoeogonium d) antherozoide

22. The body o thallus of marchantia polymorpha (liverwort) is dorsiventrally _____ a) flattened b) curved c) straightened d) oblique

23. _____ are widely used in the production of litmus and in manufactory of cosmetic? a) fungi b) mushrooms c) algae d) lichens

24. The kind of association that exists in lichens between fungi and an algae is _____ a) saprophytic b) mutualistic c) commensal

25. The following are diseases caused by fungi except ____ a) ring worm b) aspergillosis c) athlete's foot d) poliomyelitis

26. Of economic importance of fungi, ____ is used in the flavouring cheese a) Agaricus campestries b) Penicillum roguerfortii c) Amanita muscaria d) neuroapora

27. Which of the following saccharomyces is extensively used in bread making and beer – brewing a) S. carevisiac b) neurospora c) S. cllipsoids c)aspergillus

28. The nutritive hyphae in rhizopus nigricans is called _____ a) stolon b) sporangiophore c) columella d) rhizoids

29. Deuteromycetes are said to be imperfect because of _____ a) their lifecycle lacks sexual phase b) they lack chitinous cell wall c) they lack imperfect nucleus d) they are false fungi

30. The feeding stage in myxomycetes (slime moulds) is called ____ a) plasmodium b) oogonium c) myxoamoebae d) trophozoid

31. Cryptograms are ___ ,___ , and ___ a) thallopyta, spermatophata, pteridophyta b) spermatophyta, pteridophyta, bryophyta, c) bryophyta, thallophyta, spermatophytad) thallophyta, pteridophyta, bryophyta

32. _____ is the term used in fertilization when water is prerequisite? a) isogamy b) zooidogamy c) oogamy d) hydrogamy

ANSWERS

1. C
2. C
3. ..... D
4. ..... B
5. .... B
6. ..... B
7. ..... B
8. C
9. ....A
10. C
11. ....A
12. C

13. ……B
14. ….B
15. …..B
16. …..C
17. …..D
18. ……A
19. …C
20. ….C
21. ….. B
22. ….A
23. …..D
24. ….B
25. ….D
26. ……B
27. ….A
28. ….D
29. ……A
30. …A
31. …..D
32. …..B

ANIMAL DIVERSITY

CBT QUESTIONS

1. Which of the habitats below is not occupied by protozoans? a) water b) salts water c) brackish water d) sea water
2. The chlorates are divided into four (4) main sub – phyla namely hemichordate, urochordata, cephalochordate and _____ a) vertebratab) aotochord c)prochordata d) prrotochordata
3. The phylum orthropoda is divided into the following sub – phyla crustacean, myriupoda, insect, _____ a) arachnida b) peripation c) xiphosuran d) aranea
4. The exoskeleton of the limb in insert form a serves of tubular segment called_____ a) podomeres b) cuticle c) sternum d) extensors
5. The proctodaeum is a coiled tube divisible into the _____ a) ileum, colon, rectum b) ileum, malphigian c) colon, rectum, pharynx d) gizzard, crop, thorax
6. Torsion in gastropoda is the anticlockwise rotation of the visceral mass and shell through the angle of 180^{o} so that the manle cavity becomes anterior. TRUE OR FALSE
7. Insect mouth part is made up of the following mandible, maxillae, hypopharynx and _____ a)labrum b) mentum c) submentum d) babium
8. Red water fever is caused by important parasites such as ___ a) trypanosome b) plasmodium c) Babesia bigemina d) Theileria parum
9. The most complex and highly organized protozoa are the _____ a) apicomplexa b) sarcomastigophora c) asceeptospora d) ciliophora
10. Which of the following classes of the phylum Platyhelminthes contain free – living flatworm with a leaf -like body a) cestoda b) cestodaria c) turbellaria d) trematoda
11. Endoskeleton in annelids is aided by means of _____ a) cuticle b) chitin c) cartilages d) coelomic fluid

12. The most successful of the invertebrate in their conquest of the kind of land habitat belongs to the phylum _____ a) anneloda b) nematode c) coelentrate d) arthropoda
13. In insects, _____ are generally regarded as the first pair of appendages a) antennae b) mouth parts c) walking legs d) ocelli
14. Inserts that have their wing developed primarily as in growths of the ectoderm are the _____ a) apterygota b) holometabola c) hemimetabola d) exporterygto
15. Complete metamorphosis is a characteristic feature of inserts belonging to the division ____ a) endopterygota b) exoptoygota c) hemimetabola d) ametabola
16. Carnivorous forms of molluscs such as those that prey upon shrimps, fishes and crabs belong to the class _____ a) cephaloda b) polyplacorphora c) bivalia d) gastropoda
17. The chitonous exoskeleton in some arthropods is impregnated with mineral matter which is chiefly _____ a) calcium sulphate b) calcium nitrate c) calcium carbonate d) none of the above
18. Which of the following is an example of sarcodina a) parameoeium b) leptomouas c) amoeba d) euglena
19. The thin and long locomotory organelle used by phystomella is _____ a) filopodia b) axopida c) lobopodia d) reticulopodia
20. The cell which separate the outer and inner layer in forifera is _____ a) choanocytes b) gramulocytes c) pinacocytres d) amoebocytes
21. Exeretion and respiration in porifera is by means of ___ a) osomosis b) expanded body c) diffusion d) evaporation
22. The infective stage of phylum apicomplexa is _____ a) schizogony b) sporozoite c) sporogony d) merozoite
23. Locomotory organelle of Actinomorpha is _____ a) filopodia b) lobopodia c) reticulopodia d) axopodia
24. In cridaria, the outer epidermis and the inner gastrodermis are being separated by the non – cellular fibrous _____ a) tentacles b) mesoglea c) enteron d) enoplasm
25. The tubular part in alternation of generation in cnideria is _____ a) medusa b) mesoglea c) polyp d) enteron
26. _____ is referred to as the umbrella shaped part of alternation of generation in coelenterate a) medusa b) polup c) gastrodermis d) epidermis
27. Which of the following class is not found in phylum chnidaria a) hydrozoa b) scyphozoan c) mesozoa d) anthozoa
28. The body wall of the annelid is covered with ____ a) chaetae b) siphon c) glandular epidemis d) scolex
29. The lerva stage of phylum platyhelminthes is _____ a) haptors b) claspas c) exacanth d) lycophore

30. The class aplacophora of phylum molluca is also called a) solenogasters b) chitones c) dentslium d) gtops
31. In annelids, excretion is by nephridium while it is by _____ in arthropods a) tentacle b) body surface c) malphigian tubule d) malphigran layer
32. The distinct regoin of both division in class insert are head, thorax and ____ a) sternum b) labium c) abdomen d) mesenteron
33. Amphilina is an example of a) cestoda b) cestoderia c) trematodn d) monogeriea
34. Which of the following is an example of sarcodina a) paramecium b) leptomonas c) amoeba d) euglena
35. Amorbocytes with pigment are _____ a) choanocyte b) archeocyte c) chromocytes d) scleroblast

ANSWERS

1. ..... A
2. ...A
3. ....A
4. ....A
5. .....A
6. ..... TRUE
7. .....A
8. C
9. .....D
10. C
11. .....D
12. ....D
13. ....A
14. ....B
15. ....A
16. ...A
17. ...C
18. C
19. ...A
20. ....D
21. ...A
22. ....B
23. ...D
24. ....B
25. C
26. ...A
27. C
28. C
29. ...D
30. ....A
31. ...C
32. ...C
33. ....B
34. C
35. C

THE COMPUTER SYSTEM

1. The computing system is made up of _____, _____ and _____
   a) input unit, processing unit and output unit b) computer system, the user and the environment
   c) monitor, CPU and keyboard
   d) operating unit, firmware and peripherals
2. The characteristics of a computer includes the following except a) accuracy
   b) automation c) reliability d) availability
3. The computer hardware comprises the _____ a) input unit, processing unit and output unit b) computer system, the user and the environment
   c) monitor, CPU and keyboard
   d) operating unit, firmware and peripherals
4. Computer _____ is the series of instructions that enable the computer to perform a task or group of tasks a) software b) hardware c) firmware
   d) filmware

5. Computer programs could be categorized into _____ and _____ programs
   a) software and hardware
   b) input and output
   c) system software and application
   d) environment and users
6. The categories of computer users are _____ and _____ users
   a) casual and expert
   b) original and expert
   c) brilliant and intelligent
   d) available and scarce
7. The first, third and fifth generation of electronic computers are between _____, _____ and _____ respectively
   a) 1937-1953, 1954-1962, 1963-1972
   b) 1937-1953, 1963-1972, 1984-1990
   c) 1937-1953, 1954-1962, 1973-1990
   d) 1963-1972, 1984-1990, 1990-date
8. In what generation was vacuum tubes used
   a) first b) third c) fifth d) sixth
9. The first generation purpose programmable electronic computer was the _____ a) EDVAC b) UNIVAC c) FORTRAN d) ENIAC
10. FORTRAN, ALFOL and COBOL which are high level programming languages were introduced in the _____ generation of computer
    a) first b) second c) third d) fourth
11. FP and PROLOG which are very high level programming languages were developed in what generation of computer
    a) first b) second c) third d) fourth
12. The parameters for classifying computers are _____, _____ and _____
    a) size, accuracy, density
    b) automation, flexibility, capacity
    c) signal type, purpose, capacity
    d) automation, purpose, density
13. Digital, Analog and Hybrid computers are classification of computers based on _____ a) density b) signal type c) capacity d) purpose
14. The analog _____ rather than _____
    a) measures, count b) count, measures
    c) analyze, count d) measures, analyze
15. The central processing unit(CPU) is made up of the _____ and _____
    a) ALU, PC b) ALU, CU c) CU, PC d) CU, RAM
16. The _____ performs addition, subtraction, multiplication, division, logic and comparison a) ALU b) CU c) PC d) RAM
17. Random Access Memory (RAM) is an example of _____ memory a) tertiary b) secondary c) primary d) intermediate
18. Your normal flash drive is an example of a _____ memory a) tertiary b) secondary c) primary d) intermediate
19. The following are secondary storage devices except a) RAM b) magnetic tape c) floppy disk d) magnetic disk
20. The special memory units used by the CPU (especially the ALU) is the _____
    a) registers b) files c) storex d) firmware

COMPUTER SOFTWARE

1. A _____ is a series of coded instructions written in a computer programming language showing the logical steps that the computer follows to solve a given problem
   a) software b) program c) system d) firmware
2. The computer software can be grouped into _____ and _____ software
   a) system, application b) firm, operating
   c) operating, system d) application, firm
3. System software can be divided into _____ and _____
   a) application, operating
   b) operating, program c) program, application d) firmware, operating
4. The name given to the combination of persistent memory, program code and data stored in an electronic device is known as _____ a) operation b) program
   c) application d) firmware
5. The BIOS is an example of _____
   a) operation b) program c) application
   d) firmware
6. The program the acts as an interface between a user of a computer and the computer hardware is _____
   a) operating system b) firmware
   c) program d) interfacial contact
7. The basic resources of a computer system are its _____, _____ and _____
   a) data, program, wares
   b) hardware, software, data
   c) remote, program, data
   d) software, program, firmware
8. The operating system is a _____ program
   a) user b) interface c) control d) data
9. _____ is large array of words or bytes
   a) data b) remote c) memory d) process
10. A _____ is a collection of related information defined by its creator
    a) program b) file c) memory d) film
11. The command _____ is the primary interface between the user and the rest of the system a) program b) interface
    c) data d) interpreter
12. The suites of programs that translate other programming languages to machine language are called language _____
    a) translators b) interface c) data
    d) interpreters
13. The initial program written in a programming language different from machine language is called _____ while its equivalent in machine language is called _____

a) source program, object code
b) object code, suites
c) suites, source program
d) object code, source program

14. _____ is a program that accepts a source program in assembly language program, reads and translate into object code a) interpreter b) complier c) assembler d) compiler
15. A /An_____ accepts, reads and translates high level language to machine language at once a) interpreter b) complier c) assembler d) compiler
16. A /An_____ accepts, reads and translates high level language to machine language line by line a) interpreter b) complier c) assembler d) compiler
17. _____ is designed basically for numerical figures and reports
a) word processors b) spreadsheets
c) graphics packages d) database packages
18. _____ are software used for designing, setting up and managing an organized collection of data that allows for modification, taking care of different user views a) word processors b) spreadsheets
c) graphics packages d) database packages
19. _____ software are designed specifically for a particular company or organization
a) word processing b) special
c) user-defined d) database
20. _____ are programs or packages that perform a variety of different processing operations that perform a variety of different processing operations using data that is compatible with whatever operation is being carried out
a) word processors b) spreadsheets
c) integrated packages d) database packages

COMPUTER NETWORKS AND NETWORK SECUTIRY

1. A group of interconnected computers refers to computer _____ a) connection b) interconnection c) networks d) juxtapose
2. For there to be a network, there must be at least two computer devices called _____
a) connection creator b) network computers c) workstations d) connectors
3. Typical parameters for classifying networks include the following except a) transmission medium b) functional relationship c) network topography d) network types
4. _____ is the connection of all networks
a) internet b) extranet c) overall connection d) intranet

5. The set of interconnected networks using the internet protocol (IP) and using IP based tools a) internet b) extranet c) overall connection d) intranet
6. A network _____ is a network device that connects multiple network segments
a) connector b) interface c) bridge d) input
7. A general name for all programs that are harmful to the computer system is
a) virus b) malware c) spyware d) threat
8. Threats to computer include all except
a) viruses b) Trojans c) spatians d) worms
9. Trojan horses are not viruses because
a) they do not reproduce and spread
b) they not affect files
c) they affect programs not files
d) they do not infect the system
10. A Trojan horse is similar to a _____
a) logic bomb b) Trojan donkey c) virus d) bacteria
11. _____ is a program which spreads over network connections
a) viruses b) Trojans c) spatians d) worms
12. A component or set of components that restricts access between protected network and the internet or between other sets of networks is called _____ a) a Trojan b) a firewall c) firmware d) an anti-virus
13. In which network are the end user computers tied to the server which controls the transmission of all other work stations
a) star topology b) bus topology c) ring topology d) local topology
14. The differences between the characteristics of the Local Area Networks (LANs) and the Wide Area Networks (WANs) include the following except
a) their higher data transfer rates
b) smaller geographical range
c) lack of firewall actuators
d) lack of need for leased telecommunication lines
15. A computer network used for communication among computer devices close to one person is called _____
a) personal area network
b) local area network
c) wide area network
d) campus area network
16. Two or more networks or network segments connected using devices such as a router is called _____
a) a connectwork b) an internetwork
c) an extranetwork d) a widework
17. It is compulsory for intranet and extranets to have connections to the internet a) true b) false
d) I don't know d) maybe
18. Service providers and large enterprises exchange information about the reachability of their address ranges through the _____ a) boundary gateway protocol b) exchange gateway protocol c) reachability gateway protocol
d) border gateway protocol

19. A network _____ is a piece of computer hardware designed to allow computers to communicate over a computer network
a) card b) hardware c) software d) access
20. Unlike bridging, _____ allows the networks to communicate independently as separate networks a) disbridging b) routing c) communicating d) secluding

COMPUTER ARITHMETIC

1. The two major methods of representing real numbers are the _____ and _____ point
a) imaginary, decimal b) fraction, decimal
c) fixed, floating d) submerged, variable
2. The general form of real number representation is $N = m^* b^e . m, b, e$ respectively are the _____, _____ and _____
a) main, big number, fixed number
b) mantissa, big number, exponent
c) mantissa, base, expression
d) argument, radix, exponent
3. In the _____ representation, the mantissa is always between 0.1 and 0.999…
a) floating point b) variable c) submerged
d) fixed point
4. In the conversion from base 10 to base 2, the _____ remainder is taken as the most significant bit (MSB)
a) first b) last c) most frequent d) largest
5. Convert 45_{10} to binary a) 101101
b) 100011 c) 111001 d) 101010
6. Convert 0.375_{10} to base two
a) 0.1011 b) 0.011 c) 0.1010 d) 0.1111
7. What is 101.0111_2 in decimal
a) 3.435 b) 12.543 c) 5.375 d) 7.135
8. Convert 111011110110_2 to base 8
a) 7366 b) A237 d) 1475 d) 4533
9. Convert 1475_8 to binary
a) 111011110110_2 b) 1000100011_2
c) 1100111101_2 d) 1100011101_2
10. Convert 111011110110_2 to hexadecimal
a) $EF6_{16}$ b) $AF6_{16}$ c) $DA3_{16}$ d) $BD6_{16}$
11. Convert $AB6_{16}$ to binary
a) 111011110110_2 b) 1100011101_2
c) 1000100011_2 d) 101010110110_2
12. Convert 538_{10} to 8421 BCD
a) $11000111000_{8421\ BCD}$
b) $10100111000_{8421\ BCD}$
c) $10111111000_{8421\ BCD}$
d) $101100111000_{8421\ BCD}$
13. Convert 378_{10} to 2421 BCD
a) $1101111110_{2421\ BCD}$
b) $1101111111_{2421\ BCD}$
c) $11010001110_{2421\ BCD}$
d) $1000111110_{2421\ BCD}$
14. Convert 625_{10} to XS3 BCD
a) 100000111000_{XS3} b) 1000110011000_{XS} c) 100101011011_{XS}
d) 100101011000_{XS3}
15. What is the one's complement of 1001_2
a) 0101_2 b) 1011_2 c) 0110_2 d) 1100_2
16. What is the two's complement of 1001_2
a) 0111_2 b) 1011_2 c) 0110_2 d) 1100_2
17. If B=false, C=true, D=false, E=true, evaluate C AND D OR E a) true b) false
c) NOR d) none of the above

18. From above, evaluate C AND D AND B OR E a) true b) false c) NOR d) none of the above
19. Evaluate D NAND B NOR E
a) true b) false c) NOR d) none of the above
20. Evaluate NOT D NAND B a) true b) false c) NOR true d) none of the above

PROGRAMMING THE COMPUTER

1. The language that allows the programmer to direct the activities of the computer is known as _____ language
a) directing b) programming c) computer d) high
2. The computer instruction has two parts namely, _____ and _____
a) operation code, operand(s)
b) instructor, instructee
c) binary simulation, operand
d) operation code, instructor
3. _____ tells the computer what to do while _____ tell(s) the computer the items involved a) operation code, operand(s)
b) instructor, instructee
c) binary simulation, operand
d) operation code, instructor
4. The following are the basic types of instruction except
a) arithmetic instructions
b) logic instructions
c) branching instructions
d) root instructions
5. An arithmetic operation on the computer must include at least _____ operands
a) one b) two c) three d) none
6. A _____ statement will cause the computer to branch to a statement only when certain conditions exist
a) logic branch b) conditional branch
c) diversion branch d) serious branch
7. _____ allow the computer to change the sequence of execution of instruction, depending on conditions built into the program by the programmer a) arithmetic instructions b) logic instructions c) branching instructions d) root instructions
8. _____ cause the computer to alter the sequence of execution of instruction within the program a) arithmetic instructions
b) logic instructions c) branch instructions
d) root instructions
9. The effective utilization and control of a computer system is primarily through _____ a) software b) program c) logic
d) language
10. The primary means of human-computer communication is the _____ language

a) communicating b) programming
c) primary d) high level

11. The following languages are the classes of computer programming language except
a) machine language b) high level language
c) very high level language d) highest level language

12. In assembly language, the operation code is expressed as a combination of letters rather than binary numbers, sometimes called _____ a) mnemonics b) alterna c) harmonics d) idomanics

13. The translation of the assembly language program into a machine language is accomplished by a _____ a) translator b) accomplisher c) assembler d) languager

14. The very high level language is a _____ generation language
a) first b) third c) fourth d) last

15. The following are characteristics of a good computer program except a) accuracy b) availability c) usability d) robustness

16. Which of the following stages is not involved in the preparation of a computer program a) problem definition b) debugging the program c) removing the potential viruses d) testing the program

17. A step by step method of solving a problem is _____ a) common sense b) algorithm b) flow chart d) functioning

18. On a flowchart, the rhombus shape is used for _____ a) decision making b) begin/end c) module d) connector

19. On a flowchart, the oval shape is used for _____ a) decision making b) begin/end c) module d) connector

20. _____ is a informal high-level description of the operating principle of an algorithm a) syntax codes b) pseudo codes
c) basic codes d) high level codes.

INTRODUCTION TO BASIC PROGRAMMING LANGUAGE

1. The term B-A-SIC stands for
a) Best Available Symbolic Instruction Code
b) Beginners Available Symbolic Instruction Code
c) Beginners All-purpose Symbolic Instruction Code
d) Best Available System Instruction Code

2. Which of the following is not an advantage of BASIC

a) easy to use
b) its compiler can be installed on any computer
c) its interpreter needs very nice and sophisticated modifications on other computer
d) debugging is simple

3. The initial MS-DOS came with a basic interpreter called
a) BASICA b) BASICS c) BASE d) BASESA

4. The _____ BASIC has provision for more than one statement in the same line a) Dell b) Computer c) Microsoft d) System

5. REM is short form for _____
a) REMEMBER b) REMEDIAL c) REMARKS d) REMAP

6. The constants allowed by BASIC are
a) all constants
b) numeric and string constants
c) numeric and alphabetic constants
d) alphabetic and string constants

7. Which of the following is incorrect about BASIC
a) BASIC doesn't allow for constants
b) BASIC does not distinguish between integer and real numbers
c) commands are not allowed in numerical terms
d) the limit on the number of digits that can be used varies from computer to computer

8. The two types of BASIC variables are _____ and _____
a) loop and Boolean variables
b) numeric and string variables
c) numeric and alphabetic variables
d) alphabetic and string variables

9. In BASIC arithmetic operations, which of the following hierarchy of operations is correct a) exponent⟶addition ⟶ division
b) division ⟶addition ⟶exponent
c) exponent⟶ division ⟶ addition
d) addition ⟶ division⟶exponent

10. Which of the following is not correct about the rules to apply in arithmetic expressions
a) string constants and variables should not be used in arithmetic expressions
b) denominator of an expression should add up to zero
c) two operators must not appear together
d) when brackets are used, they must be used in pairs

11. BASIC equivalent of $D^3 + 2BC$ is
a) 3D+2*B*C b) D^3+2B*C c) 3^D+2*BC
d) D^3+2*B*C

12. In Algebraic expression, the BASIC expression 5*A − B^2 is
a) $5A - 2B$ b) $A^5 - 2B$ c) $5A - B^2$
d) $5A - B^2$

13. Which of the following set of relational operators correctly represent 'less than or equal to, not equal to and equal to
a) <=, <>, = b) <, =/=, = c) >=, ==, =
d) <<, <>, =

14. _____ expressions are used in conditional statements to determine the course of action of a running program a) logical b) sequential c) relational d) conditional

15. _____ operators are used to connect two or more relations and return a TRUE or FALSE value to be used in a decision a) logical
b) sequential c) relational d) conditional
16. Conjunction, logical negation and disjunction are represented by _____, _____ and _____ respectively
a) NOT, AND, OR
b) OR, AND, NOT
c) AND, NOT, OR
d) AND, OR, NOT
17. In BASIC programming, the word 'library' stands for _____ a) references b) stationery c) books functions d) collection
18. The three kinds of input statements offered by BASIC are
a) LET, WRITE, READ-DATA
b) LET, INPUT, READ-DATA
c) WRITE, READ-DATA, INPUT
d) INPUT, INSERT, USE
19. A READ statement will always have a _____ statement along with it
a) WRITE b) INPUT c) DATA d) INSERT
20. If Besty needs a string that can cater for the input; '21.34', the following format strings can be used except
a) '##.##' b) "###.##" c) "##.##"
d) "####.##"

ANSWERS

THE COMPUTER SYSTEM

1) b	6) a	11) d	16) a
2) d	7) b	12) c	17) c
3) a	8) a	13) b	18) b
4) a	9) d	14) a	19) a
5) c	10) b	15) b	20) d

COMPUTER SOFTWARE

1) b	6) a	11) d	16) a
2) a	7) b	12) a	17) b
3) d	8) c	13) a	18) d
4) d	9) c	14) c	19) c
5) d	10) b	15) b	20) d

COMPUTER NETWORKS AND NETWORK SECURITY

1) c	6) c	11) d	16) b
2) c	7) b	12) b	17) b
3) d	8) c	13) a	18) d
4) a	9) a	14) c	19) a
5) d	10) a	15) a	20) b

COMPUTER ARITHMETIC

1) c	6) b	11) d	16) a
2) d	7) c	12) b	17) a
3) a	8) a	13) a	18) a
4) b	9) c	14) d	19) b
5) d	10) a	15) c	20) a

5) 45_{10} to base two

2	45
2	22 r 1
2	11 r 0
2	5 r 1
2	2 r 1
2	1 r 0
2	0 r 1

$45_{10} = 101101_2$ …… b

6) 0.375_{10} to base two

To do this, multiply the number by the required base and retain the whole number while you continuously multiply the decimal part by the required base

	375×2
0	750
1	500
1	000

$0.375_{10} = 0.011_2$

7) $101.011_2 = (1 \times 2^2) + (0 \times 2^1) +$
$(1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$
$= 4 + 0 + 1 + 0 + 0.25 + 0.125$
$= 5.375_{10}$

8) Break the binary number into groups of 3 bits and using the TABLE OF NUMBER CONVERSION in your school textbook.

$\underset{7}{111}\ \underset{3}{011}\ \underset{6}{110}\ \underset{6}{110} = 7366_8$ …….. a

9) Just like doing the reverse of what we just did

$1475_8 = \underset{001}{1}\ \underset{100}{4}\ \underset{111}{7}\ \underset{101}{5}$

$1475_8 = 001100111101_2$

$1475_8 = 1100111101_2$ …… c

10) This time around, you break the number into 4 bits

$111011110110_2 = \underset{E}{1110}\ \underset{F}{1111}\ \underset{6}{0110}$

$111011110110_2 = EF6_{16}$ …… a

11) $AB6_{16} = \underset{1010}{A}\ \underset{1011}{B}\ \underset{0110}{6}$

$AB6_{16} = 101010110110_2$ …… d

12) $538_{10} = \underset{0101}{5}\ \underset{0011}{3}\ \underset{1000}{8}$

$538_{10} = 010100111000_2$

$538_{10} = 10100111000_{8421\ BCD}$ …… b

13) $378_{10} = \begin{matrix} 3 & 7 & 8 \\ 0011 & 0111 & 1110 \end{matrix}$

$376_{10} = 1101111110_{2421\ BCD}$

14) This one requires that you add 3 to each digits before or after converting

$$625_{10} = \begin{matrix} 6 & 2 & 5 \\ +3 & +3 & +3 \\ \hline 9 & 5 & 8 \\ 1001 & 0101 & 1000 \end{matrix}$$

$625_{10} = 100101011000_{XS3}$

15) Just invert the digits

$1001_2 = 0110_2$

16) Just add 1 to its one's complement

i.e. $1001_2 = 0110_2 + 1 = 0111_2$

17) B=false, C=true, D=false, E=true

C AND D OR E

=true AND false OR true

=false OR true=true …….. a

18) C AND D AND B OR E

=true AND false AND false OR true

=false AND false OR true

=false OR true

=true …….. a

19) D NAND B NOR E

=false NAND false NOR true

=true NOR true

=false ……. b

20) NOT D NAND B

=NOT false NAND false

=true NAND false

=true …….. a

PROGRAMMING THE COMPUTER

1) B	6) b	11) d	16) c
2) A	7) b	12) d	17) b
3) A	8) c	13) c	18) a
4) D	9) a	14) c	19) b
5) B	10) b	15) b	20) b

INTRODUCTION TO BASIC PROGRAMMING LANGUAGE

1) C	6) b	11) d	16) c
2) C	7) a	12) d	17) d
3) A	8) b	13) a	18) b
4) C	9) c	14) a	19) c
5) C	10) b	15) a	20) a

QUESTIONS

1. $111110111011101111100011_2$ to hexadecimal;

Split into 4 bits

$$\begin{matrix} 1111 & 1011 & 1011 & 1011 & 1110 & 0011 \\ F & B & B & B & E & 3 \end{matrix}$$

$= FBBBE3_{16}$

2. Convert 294+328 form Decimal (i.e. base 10) to BCD

294 + 328

2	9	4	+	3	2	8
0010	1001	0100		0011	0010	1000

We have

$$\begin{array}{r} 0\,0\,1\,0\,1\,0\,0\,1\,0\,1\,0\,0 \\ +\ 0\,0\,1\,1\,0\,0\,1\,0\,1\,0\,0\,0 \\ \hline 0\,1\,0\,1\,1\,0\,1\,1\,1\,1\,0\,0 \\ \hline \end{array}$$

$\therefore 294 + 328 = 010110111100_2$

3. Convert 13.75_{10} to base 2

Convert the whole part and decimal part separately

$13.75_{10} = 13_{10} + 0.75_{10}$

2	13
2	6 r 1
2	3 r 0
2	1 r 1
2	0 r 1

$=1011_2$

	75×2
1	50×2
1	0

$=0.11_2$

Hence, $13.75_{10} = 1011 + 0.11 = 1011.11_2$

4. Evaluate $3_{10} - 7_{10}$ in 1's complement;

$M = 3_{10} = 0011_2$

$S = 7_{10} = 0111_2$

$1's\ complement\ of\ S = 1000$

$0011 + 1000 = 1011_2$

$\therefore 3_{10} - 7_{10} = 0100_2 = -4_{10}$

$2's$ complement

$M = 3_{10} = 0011_2$

$S = 7_{10} = 0111_2$

$1's\ complement\ of\ S = \ 1000$

$Add\ one\ to\ it \qquad +1$

$\therefore 2's\ complement\ of\ S = 1001$

Add M to the 1001

$$\begin{array}{r} 0011 \\ +1001 \\ \hline 1100 \\ \hline \end{array}$$

1's complement of the result= 0011

Add one to it

$$\begin{array}{r} 0011 \\ +\ 1 \\ \hline 0100 \\ \hline \end{array}$$

$\therefore 3_{10} - 7_{10} = 0100_2 = -4_{10}$

5. If A=TRUE, B=FALSE, C=FALSE, D=TRUE

Evaluate; NOT (C AND D) OR NOT (A OR B)

Answer

=NOT (F AND T) OR NOT (T OR F)

=NOT (FALSE) OR NOT (TRUE)

=TRUE OR FALSE

=TRUE

6. From above, evaluate

D OR B AND NOT B OR NOT A AND D

Answer

=T OR F AND NOT F OR NOT T AND T

T AND T OR F AND T

T OR F

TRUE

7. Give the full meaning of the following

a) ASCII: American Standard Code for Information Interchange

b) EBCDIC: Extended Binary Coded Decimal Interchange Code

8. Write the following in fixed point format

$a) 0.130 \times 10^6 = 130600$

$b) 0.5 \times 10^{-9} = 0.0000000005$

9. "-11"

a) As sign magnitude: using 8 bits,

$-11 = 10001011_2$

b) As one's complement:

$-11 = -1011_2 = -0100_2$

10. Convert 78_{10} *to* 8421 *BCD XS*3

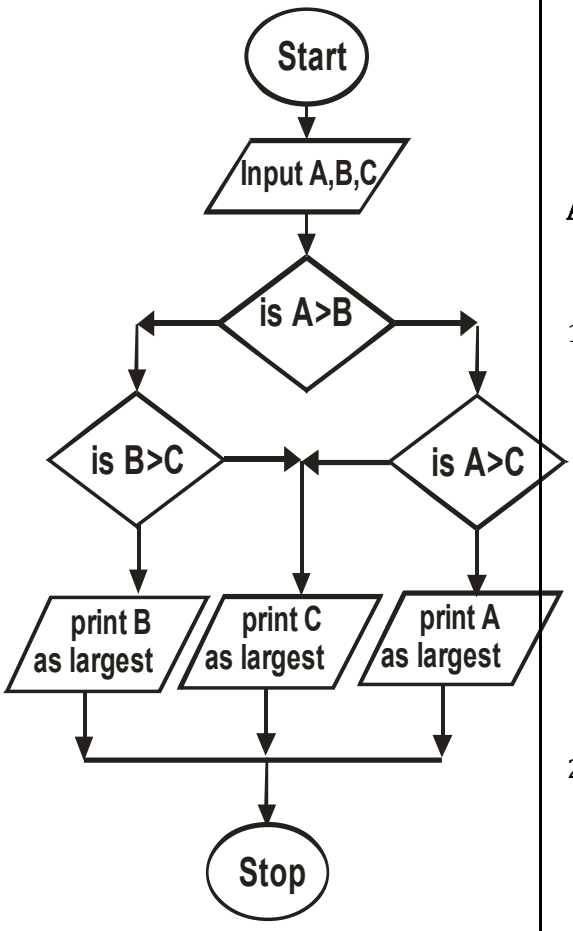
Add 3 to each digit before or after converting to binary. I will do mine before

$$78_{10} = \begin{array}{cc} 7 & 8 \\ +3 & +3 \\ \hline 10 & 11 \end{array}$$

$$\begin{array}{cc} 10 & 11 \\ 1010 & 1011 \end{array}$$

$$\therefore\ 78_{10} = \ 10101011_{XS3}$$

11. Draw a Flowchart to find the largest of three numbers



12. Write an algorithm to calculate the area of a sphere using the formula (Area=$4\pi r^2$)

Step 1: start

Step 2: obtain r

Step 3: Area=4*3.142*r*r

Step 4: print Area

Step 5: stop

13. Full meaning of the following

i. ROM: Read Only Memory

ii. RAM: Random Access Memory

iii. EPROM-Erasable Programmable Read Only Memory

iv. EEPROM: Electronic Erasable Programmable Read Only Memory

v. ENIAC: Electronic Numeric Integrator

GENERAL PRACTICAL

PRACTICE QUESTION

UNIT 1

ASPECTS OF GRAMMATICAL STRUCTURES IN SCIENCE EXPRESSIONS IN ENGLISH

1. English language in modern times has been pressed into diverse functions of human engagements such as _____ and _____ a) serious academic discourse and technology pedagogy b) serious academic discourse and diurnal usage c) serious academic discourse and technological usage d) serious academic discourse and scientific usage

2. Functions performed by scientists include the following except _____ a) definition b) classification c) analysis d) generalization

3. The word "definition" is from the Latin word ___ a) definer b) defined c) defunct d) defines

4. fixing bond or limit of words is called _____ a) dictionary b) rhetorical c) definition d) generalizing
5. All these explain the term "definition" except_____ a) used to achieve rhetorical end b) establishes the major idea of a write up c) sets limits to an idea d) generalize an idea
6. _____ is the recommended method used in defining a topic a) eclectik b) ecletec c) eclectics d) ecclectic
7. Definitions based on class consideration are called _____ a) idiosyncrasy b) logical c) rational d) pedagogy
8. All these are types of definitions except a) explanation b) example c) metaphorical d) description
9. "Thermal conductivity is the rate of passage of heart from face to face area per difference of temperature between faces when one is think". Definition of this type is by _____ a) metaphorical b) description c) example d) comparative
10. One major quality of definition by synonyms is _____ a) brevity b) instantiation c) logic d) clarification
11. "as dark as coal is an example of _____ a) metaphor b) personification c) simile d) oxymoron
12. The expression "volcanoes are windows through which the scientist look into the bowels of the earth ....." is an example of _____ type of definition a) example b) metaphorical c) logical d) formal
13. Clarification of items depends on _____ and _____ a) similarities and definition b) result and differences c) similarities and difference d) classifier and nature
14. The interest of the classifier, the nature of the data and the specifications of certain fields are referred to as _____ a) guidance to classification b) speculation to classification c) similarities to classification d) limit to classification
15. A proposition assumed for the sake of an argument that is subject to approval or disapproval is called _____ a) research b) hypothesis c) beacon d) experiment
16. How many types of reasoning do we have _____ a)4 b) 3 c) 5 d) 2
17. Make a generalization on these:
    I. All good lecture are handsome
    II. Face cap is a good lecturer
    a) Face cap is not handsome b) face cap is handsome c) face cap is a good lecturer d) face cap is not a lecturer
18. Either the student or the tutor ____ here a) live b) lived c) lives d) leaved
19. I, alongside my teachers, parents, friends and uncle_____ happy a) is b) are c) am d) were
20. Which law of concord dictates that the verb operating context, coordinated by "or", "and" etc. should agree with the nearest subject ____ a) accompaniment concord b) proximity concord c) relative concord d) mandative concord

ANSWERS

1. **B**
2. **D**
3. **A**
4. **C**
5. **D**
6. **C**
7. **B**
8. **A**
9. **B**
10. **A**
11. **C**
12. **B**
13. **C**
14. **A**
15. **B**
16. **B**
17. **B**
18. **C**
19. **C**
20. **B**

UNIT 2

THE USE OF CONNECTIVES IN SCIENTIFIC REPORT WRITINGS

1. All these are communication skills except _____ a) learning b) writing c) reading d) speaking
2. _____ will be needed when the initial point is to lead us accurately to further points a) classifier b) jointers c) connectives units except ____ a) paragraph b) clauses c) phrases d) morphemes
3. These are grammatical units except _____ a) paragraph b) clauses c) phrases d) morphemes
4. We use the AND connectives for all these except _____ a) for summing – up idea b) for movement from idea to idea c) for idea multiplying c) for perfect re – statement of ideas
5. These expressions "to summarize", in conclusion", "therefore"etc. are used to _____ a) sum – up idea b) show contrast c) represent idea d) make motive
6. To change a meaning in the initial idea first presented is called _____ a) or – connectives b) but – connectives c) and - connectives d) as – connectives
7. And, or, but are classified under ____ a) enumerating connectors b) logical connectors c) contrast connectors d) apposition connectors
8. Classification such as: enumeration, addition, transition, summation, apposition, inference and result are discovered by _____ a) quirk and green Baum b) winter hood and Murray c) duduyemi d) osuala
9. Phonic reference are also called ____ a) discourse reference b) alter – native reference c) logical reference d) initial reference
10. How many types of signal marks do we have ____ a) 2 b) 3 c) 4 d) 1
11. When the signal marks point forward, it is called _____ a) phobic reference b) anaphoric reference c) cataphoric reference d) ellipsis reference

12. All these are also connectives except ____ a) ellipsis b) substitution c) phonic reference d) transistors

Read this expression and answer the questions under

Carbohydrates are substances that conform to the general formula $C_x(H_2O)$ y. they contain hydrogen and oxygen in the same proportion as in water

13. The pronoun "THEY" points back to ____ a) hydrogen b) oxygen c) carbohydrates d) formula
14. From the expression, water contains _____ and _____ a) substances b) formula and carbon c) hydrogen and oxygen d) hydrogen and formula
15. When grammatical parts are deliberately to as _____ a) bond b) repetition c) ellipsis d) substitution
16. A writer need to be careful when using repetition in report writing so that it will not be ____ a) heteronous b) pointful c) monotonous d) ambiguous
17. Repetition of the key word in the write – up provides ____ a) meaning b) understanding c) emphasis d) coherence
18. This punctuation mark (^) is called _____ a) hyphen b) caret c) Capet d) parenthesis
19. These words: "to this end", in the light of this', from the foregoing …. Are examples of ____ a) cataphoric reference b) anaphoric reference c) projected reference d) concluding reference
20. All these function alongside " BUT" except ___ a) so b) although c) nevertheless d) however

ANSWERS

1. A
2. C
3. D
4 .C
5. A
6. B
7. B
8. A
9. A
10. A
11. C
12. D
13. C
14. C
15. C
16. B
17. C
18. B
19. B
20. A

UNIT 3

OUTLINING

1. ___ is the art of writing a structured detailed statement of the essential contents and representation of the Organisation pattern of a spoken or written text a) description b) outlining c) patterning d) incongruting
2. All these are feature of a good outlining except _____ a) ease of recall b) accuracy c) brevity d) coherence
3. Making the choice of words simple and easy to understand is a feature of outlining called _____ a) orderliness b) brevity c) clarity d) accuracy
4. Uses of outlining include the following except____ a) stylistic Organisation of information b) organizing and planning of the writing c) reviewing and revision of materials d) hormonal coordination of write up
5. How many types of outlining do we have _____ a) 4 b) 3 c) 2 d) 5
6. The most commonly used producer of outlining is the _____ a) roman numerals b) Greece numerals c) Arabic numerals d) French numerals
7. A common term used in outlining is _____ a) liners b) sub division c) marking d) explicators
8. Outlining literally represents an _____ version of our original texts a) abridged b) elongated c) numbered d) logical
9. Give a good structured outline of these topics and sub –topic
   Gigantism
   Causes of gigantism
   Signs and symptoms
   a) 4.1, 4.1.1, 4.1.2 b) 4.1.1, 4.1.2, 4.1.0 c) 4.0.1, 4.1.1, 4.1.2
10. All these can be outlined except _____ a) thesis b) essays c) projects d) sentences
11. Which of these is correct with reference to the definition of outlining a) gbemileke, 1988 b) adegbija, 1987 c) osuale, 1990 d) aladeyomi, 1987
12. To be able to outline with the use of a sketh, the writer must have a good knowledge of he _____ a) main idea b) topic c) classes d) analysis

ANSWERS

1. **B**
2. **A**
3. **C**
4. **D**
5. **C**
6. **C**
7. **B**
8. **A**
9. **A**
10. **D**
11. **B**
12. **B**

UNIT 4
TENSE AND ASPECT IN SCIENTIFIC WRITING

1. In technical term, the name given to the variety of language distinguished according to use is called _____ a) expression concepts b) general vocabulary c) register d) scientific writing
2. All these are subsumes is the field of science except ____ a) applied economics b) engineering c) pure and applied chemistry d) applied statistics
3. The tense of the action shows _____ a) the duration of the action b) the usage of the action c) the form of the action d) the type of the action
4. These are three major types of tenses except ____ a) simple present, simple past and simple future b) simple present , simple past, simple progressive c) simple present, simple particular , simple future d) simple present, simple continuous, simple future
5. Auxiliary verbs used to denote future actions are ____ and _____ a) may and shall b) will and can c) will and shall d) will and ought to
6. An action that is done regularly is called _____ a) express action b) habitual action c) present action d) regular action
7. The vice chancellor _____ for Maiduguri on Monday next week , a) leaves b) leave c) leaved) left
8. All these are variant s of auxiliary verbs except _____ a) has b) is c) were d) so
9. There are ___ aspects of verb in English language a) 3 b) 4 c) 2 d) 7
10. ____ aspect shows that an action or activity describe has been completed a) the perfective b) the regressive c) the progressive d) the reflexive
11. The expression, 'tom is reading a book' is a _____ tense a) present perfect b) future perfect c) present progressive d) past progressive
12. We _____ the matter soon a) shall be investigating b) will be investigation c) must be investigate the matter soon d) be investigating the matter soon
13. All of these are aspect and tense except _____ a) past perfect tense b) present perfect tense c) future progressive tense d) past present progressive
14. 'I shall have been cleaning the floor by this time tomorrow' is a ____ tense a) future perfect progressive b) past perfect progressive c) present perfect progressive d) past future continuous
15. The expression, "she has been reading English" is an example of _____ tense a) future b) past c) future perfect progressive d) present perfect progressive
16. Classification of scientific writings for our purpose include all of these except ____ a) expressing a scientific fact b) reporting the

result of an experiment c) relating hypothsis d)generalizing principle

17. Which of these classification involves the use of simple present tense only ___ a)reporting the result of an experiment b) describing the result d) relating hypothesis d) giving instructions
18. Sun drying the tomatoes _____ to 100% preservation of their contents a) lead b) leads c) led d) leading
19. A parasite ____as an organism that lives within or on another living organism called the host, from where it obtains food and shelter a) has being defined b) have been defined c) was defined d) has been defined
20. When you ___ talking, hang up a) have finished b) be finished c) are finished d) finished
21. This result _____ the desirability of these fragrance plants by residents in VG a) show b) show c) showed d) has showed
22. What is the correct plural from of 'hypo thesis' a) hypothesis b) hypotheses c) hypotheses d) hyphotheses
23. All these are parts of instruction giving in science except ____ a) medical b) education c) safety d) operational
24. How many types of definition done have _____ a) 4 b) 5 c) 2 d) 3
25. The rate of diffusion of gas _____ inversely proportional to the square root of its vapor density a) are b) were c) has d) is

ANSWERS

1. C
2. A
3. A
4. A
5. C
6. B
7. A
8. D
9. C
10. A
11. C
12. A
13. D
14. A
15. D
16. D
17. C
18. B
19. D
20. A
21. A
22. B
23. B
24. B
25. D

UNIT 5

WORD FORMATION

1. Word formation mean ____ a) process of making words meaningfully b)Process by which words are structured, made, produced or organized c) process by which words are generated, formed and used d)

process by which words are formulated, utilized and expressed

2. All these are importance of word formation except _____ a) enhance spoken abilities b) aids writing skills c) assists student s in generalizing information not supplied d) enable students to understand the texts they read
3. All these are classification of word formation except _____ a) blending b) clipping c) acronym d) acronym
4. There are ____ word formation process discussed a) 7 b) 8 c) 9
5. A _____ is the smallest meaningful unit of language a) morpheme b) word c) letter d) compound word
6. ' prefixation and 'suffixation' are examples of _____ word formation process a) clipping b) affixation c) neologism d) acronym
7. Which prefix will you add to this add to his word 'teeth'to form a new word a) Co- b) mis- c) pseudo- d) pro-
8. 'drive _drives _ driving _ drove' is an example of _____ affixation a) derivational b) in flexional c) inflectional d) deviation
9. The word ' radioactive' is a ____ a) verb b) adverb c) adjective d) pronoun
10. A _____ is a shortenend tag of a longer word a) clip b) blend c) morpheme d) acrons
11. The combination of transfer resistor will form___ a) transfer - resistor b) transition c) transistor d) transfustor
12. The root word in 'Amplification' is _____ a) amply b) ample c) plify d) phificate
13. Expressions O.A.U, LAUTECH, LIFO etc. are example of _____ a) selection b) blending c)arouynms d) abronyms
14. Creating new word or using existing word with new meanings is called ___ a)generalizing b) coinage c) idiomaticity d)affixation

WHICH WORD FORMATION PROCESS IS USED IN THE FOLLOWING WORDS

15. 'psychosexual' a) affixation b) suffixation b)neologism d) clipping
16. 'flu' a) clipping b) idiomatic c) prefixation d) abronymy
17. 'alphalocholism' a) compounding b) blending c) clipping d) acronymy
18. 'do - or - die a) clipping b) blending c) coinage d) compounding
19. 'subnorrmality' a) affixation b) coinage c) compounding d) idiomaticity
20. ' a bird in hand is worth two in the bush' a) neologism b) I diomaticity c) compounding d) abronymy

ANSWERS

1. **B**
2. **C**
3. **D**
4. **A**
5. **A**
6. **B**
7. **C**

8. B
9. C
10. A
11. C
12. B
13. C
14. B
15. B
16. A
17. B
18. D
19. A
20. B

UNIT 6
WRITING SCIENTIFIC PROJECT

1. The art of rendering one's thought process and bodies of knowledge acquired through careful studies into the scripted form is _____ a) reading b) writing c) studying d) hypothesizing
2. Science specifically thrives on _____ and ____ a) new people and environment b) new research and conclusion c) new discoveries and innovations d) new hypothesis and inventions
3. An inter disciplinary academic engagement which involves extensive research into specific areas of knowledge with the aim of finding out omissions which leads to new discoveries is called _____ a) project formation b) project development c) project research d) project writing
4. The first step in project writing is _____ a) identify the problem b) get the supervisor c) select a topic d) go to the field
5. Picking only one topic in project writing and research is tantamount to _____ a) academic variety b) academic Luke warmness c) academic versatility d) academic activities
6. There are _____ steps of project writing and research a) 7 b) 5 c) 8 d) 4
7. All these are steps to project writing except _____ a) writing the first draft b) the final draft c) the second draft d) description of apparatus
8. Research work is majorly experimented _____ a) on the field b)in the laboratory c) on paper d) in perti - dishes
9. All these are reasons why a candidate must handle his apparatus carefully except _____ a) avoid mistakes b) wrong observation c) miscalculations c) ultimate conclusions
10. The art of reading existing materials on the topic before present research starts is called _____ a) methodology b) literature review c) introduction d)drafting
11. All these make up the draft in project writing except _____ a) conclusion b) the post firs draft activities c) data analysis d) data collection
12. Data collection, presentation, analysis and conclusion is a major part of ____ in project

writing a) mathematics b) statistics c) interpretation d) aim

13. The 'MLA' and' APA' styles are classified under_____ in project writing a) bibliography portion b) ending portion c) drat portion d) referential portion
14. Which of these is a correct adopt _____ a) gbemilekeogunkoya, COGENT ENGLISH, OGBOMOSO: 8760 NIG LTD .P.176 b) gbemilekeogunkoya, ogbomoso, English: 1179 LTD.P. 121 c) gbemilekeogunkoya, COGENT ENGLSH: 8760 NIG. LTD. P.173 d) gbemilekeogunkoya, COGENT ENGLISH, Ogbomoso: 8760 NIG LTD. P.112
15. Which of these part can be in a bracket a) the name of the author b) the company and year of publication c) page of the quoted part d)title of work
16. The last stage of project writing is the _____ a) the draft ending b) the final draft c) the last draft d) the ending draft
17. Which of these is not a scientist a)Galileo b) boyles c) Mendel d) Karl marx
18. Which of these is not a technical term a) power b) lymphocyte c) fossil d) spiral
19. Project writing and research also lead to ____ a) expansion of resource b)expansion of knowledge c) expansion f activities of knowledge d) expansion of experts of field
20. All these are materials needed to text for the presence of starch in a piece of cassava except____ a) petri- dishes b) iodine c) a piece of cassava d) chlorine

ANSWERS

1. **B**
2. **C**
3. **C**
4. **C**
5. **B**
6. **C**
7. **C**
8. **B**
9. **D**
10. **B**
11. **D**
12. **B**
13. **A**
14. **A**
15. **B**
16. **B**
17. **B**
18. **A**
19. **B**
20. **D**

UNIT 7

WRITING A CURRICULUM VITAE

1. _____ highlights the information that an applicant for a job needs to supply to the employer a) curriculum vital b) curriculum vitum c) curriculum vitae d) curricullum vitaee

2. There are ___ major part in '1' above a) 5 b) 4 c) 3 d) 6
3. These are parts of a C.V except _____
   a) references b) co - curricular activities
   c) personal details d)employer's reasons
4. _____ must go with the C.V to assist the employer a) a long application letter b) an informal letter c) a short notice letter d)an abridged application letter
5. In the personal data section of a C.V we have all these except _____ a) sex b) data of birth c) hobbies d) nationality
6. The educational background section must be arranged _____ a) as endingly b) descending c) accordingly d) pictorially
7. The working experience is the _____ section in the C.V a) 2nd b)3rd c) 4rd d) 6th
8. The name and address f the next of kin must be in the _____ section of the C.V a) educational b) personal data c) reference d) experience
9. All these need a C,V except _____ a) technologist b) engineer c) apprehentic d) practioner

ANSWERS

1. C
2. A
3. D
4. D
5. C
6. A
7. C
8. B
9. D