(11) $x(1-x)y'' - 3xy' - y = x$

$a_0(x)y'' + a_1(x)y' + a_2(x)y = X$ is said to be normal in I

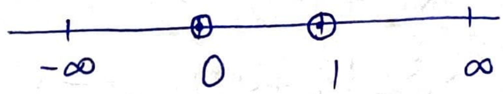
if (i) $a_0(x), a_1(x), a_2(x), X$ are continuous in I.

(ii) $a_0(x) \neq 0$ in I.

Here, $x(1-x), -3x, -1$ and x are continuous everywhere in $(-\infty, \infty)$.

$x(1-x) \neq 0$ everywhere in $(-\infty, \infty)$ except 0 and 1.

$\therefore$ $I = (-\infty, 0), (0, 1), (1, \infty)$.



(d) is correct.

(12) $2y'' - 3y' - y = \log x$

(i) $2, -3, -1$ are continuous everywhere in $(-\infty, \infty)$.

$\log x$ is continuous for $x > 0$.

(ii) $2 \neq 0$ everywhere in $(-\infty, \infty)$.

$\therefore$ $I = (0, \infty)$

(b) is correct.

(13) $\{k, e^k\}$

Result: A set containing zero is LD.

(a) $\{0, e^0\} = \{0, 1\} \rightarrow$ Set containing 0.

$\therefore$ (a) is correct.

(14) Sols : e^{2x}, xe^{2x}

$y_1 = e^{2x}, \ y_2 = xe^{2x}$

$\therefore y = C_1 e^{2x} + C_2 e^{2x} x$, C_1 and C_2 are constants.

$= (C_1 + x C_2) e^{2x} \rightarrow$ Repeated roots.

Roots are 2, 2.

$(m-2)^2 = 0 \rightarrow$ Auxiliary equation

$\Rightarrow \quad m^2 + 4 - 4m = 0$

$(D^2 - 4D + 4)y = 0 \rightarrow$ Eq in operator form.

$y'' - 4y' + 4y = 0$

(a) is correct.

(15) $y'' + 2y' - 3y = 0$

$m^2 + 2m - 3 = 0$

$(m+3)(m-1) = 0$

$m = 1, -3$

$y(x) = Ae^{-3x} + Be^{x}$; A, B are constants.

(a) is correct.

(16) $y'' - 4y' + 4y = 0.$

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2.$

$y(x) = \cancel{C}\,(A + Bx)e^{2x}.$

(d) is correct.

(17) $y'' - 2y' + 10y = 0$

$m^2 - 2m + 10 = 0$

$m = \dfrac{2 \pm \sqrt{4-40}}{2} = \dfrac{2 \pm 6i}{2} = 1 \pm 3i$

$y(x) = e^x[C_1 \cos 3x + C_2 \sin 3x]$

or $y(x) = e^x[A \cos 3x + B \sin 3x]$

(a) is correct.

(18) $y' - 3y = 0,\ y(0) = 1$

$m - 3 = 0$

$m = 3.$

$y(x) = Ae^{3x}$

$y(0) = 1 \Rightarrow A = 1$

$y(x) = e^{3x}$

(C) is correct.

(19) $\cancel{m^2 - 2m + 2 = 0}$ $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1.$

$y(x) = (A + xB)e^x$

$y(1) = 0 \Rightarrow (A+B)e^1 = 0 \Rightarrow (A+B)e = 0$

$\cancel{(A+B)e \neq}$ Since $e \neq 0$

$\therefore A + B = 0.$

(a) is correct.

(20) $m^2+2m-3=0$

$(m+3)(m-1)=0$

$m=1,-3$

$y(x)=C_1e^{x}+C_2e^{-3x}$ or $y(x)=Ae^{x}+Be^{-3x}$

$y(0)=6 \Rightarrow A+B=6$

(d) is corect.

(21) $\underline{PI}$ $\dfrac{1}{D^2+D-2}e^{x}$

$= e\dfrac{1}{1+1-2}e^{x} = \dfrac{1}{0}e^{x}$ $= x.\dfrac{1}{2D+1}e^{x}$

$\downarrow$

case of failure

$= \dfrac{xe^{x}}{3}$

(C) is corect.

$(1+x)^{-2}=1-2x+3x^2-4x^3-$

(22) $\underline{PI} = \dfrac{1}{D^2+4D+4}4x^2+1$

$= \dfrac{1}{(D+2)^2}(4x^2+1) = \dfrac{1}{4}\dfrac{1}{(1+\frac{D}{2})^2}(4x^2+1)$

$= \dfrac{1}{4}\left(1-\dfrac{2D}{2}+\dfrac{3D^2}{4}---\right](4x^2+1)$

$= \dfrac{1}{4}\left[4x^2+1-D(4x^2+1)+\dfrac{3}{4}D^2(4x^2+1)\right]$

$= \dfrac{1}{4}\left[4x^2+1-8x-0+\dfrac{3}{4}(8)\right] = \dfrac{1}{4}\left[4x^2-8x+7\right]$

(b) is corect.

(23) PI $\frac{1}{D^2+2D+3}\sin x$ Case 2

$= \frac{1}{-1+2D+3}\sin x$

$= \frac{1}{2+2D}\sin x$

$= \frac{1}{2}\frac{1}{D+1}\frac{D-1}{D-1}\sin x$

$= \frac{1}{2}\frac{(D-1)}{D^2-1}\sin x = \frac{1}{2}\frac{1}{-1-1}[\cos x - \sin x]$

$= -\frac{1}{4}(\cos x - \sin x)$

$= (\sin x - \cos x)/4.$

(C) is correct.

(24) $y'' - 3y' + 2y = xe^{3x}$ Case 4.

PI $\frac{1}{D^2-3D+2}xe^{3x}$

$= e^{3x}\frac{1}{(D+3)^2-3(D+3)+2}x$

$= e^{3x}\frac{1}{D^2+3D+2}x$ Case 3

$= e^{3x}\frac{1}{2}\left[1+\frac{D^2}{2}+\frac{3D}{2}\right]^{-1}x$

$= \frac{e^{3x}}{2}\left[1-\frac{D^2}{2}-\frac{3D}{2}+\ldots\right]x = \frac{e^{3x}}{2}\left[x-0-\frac{3}{2}\right] = e^{3x}\left(\frac{x}{2}-\frac{3}{4}\right)$

(C) is correct

(25) $PI \ \frac{1}{D^2+3D+2} e^x \cos x$ $\rightarrow$ case 4.

$= e^x \frac{1}{(D+1)^2+3(D+1)+2} \cos x$

$= e^x \frac{1}{D^2+5D+6} \cos x$ case 2

$= e^x \frac{1}{-1+5D+6} \cos x$

$= e^x \frac{D-1}{5(D+1)} \frac{1}{D-1} \cos x$

$= \frac{e^x}{5} \frac{(D-1)}{D^2-1} \cos x = \frac{e^x}{5} \frac{[-\sin x - \cos x]}{-2}$

$= \frac{e^x}{10} (\sin x + \cos x)$

(b) is correct.

(26) $PI = \frac{1}{D^2+3D+2} e^{2x}$ ~~$= \frac{1}{4+6-2}$ or case of failure~~

$= \frac{e^{2x}}{4+6-2}$ Case 1

$= \frac{e^{2x}}{8}$.

(b) is correct.

~~26~~

27 $m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2, 3.$

$y_c(x) = C_1 e^{2x} + C_2 e^{3x}.$

<u>Result</u> If $X = x^m$

Trial sol is $y = C_0 x^m + C_1 x^{m-1} + C_2 x^{m-2} + \text{---} + C_{m-1} x + C_m$

Here $m = 1$.

$y(x) = C_0 x + C_{\cancel{m}1} = Ax + B.$

<u>Note</u>: If any term is repeated in C.F., multiply trial sol by x^k, k represents the number of times the term is repeated.

(Check the notes)

(a) is correct.

28 $y'' - 4y' + 4y = e^{2x} \Rightarrow y_c(x) = (C_1 + xC_2) e^{2x}$

<u>Result</u>: If $X = e^{ax}$, trial sol $y_p(x) = ke^{ax}$

Since e^{2x} is repeated two times in C.F..

$\therefore$ Trial sol is $y_p(x) = Ax^2 e^{2x}$.

(b) is correct.

(29) and (30)

By Variation of parameter,

$$y_p(x) = -y_1 \int \frac{y_2\, g(x)}{W}\, dx + y_2 \int \frac{y_1\, g(x)}{W}\, dx.$$

In question, it is given that $y = A(x)\, y_1 + B(x)\, y_2$.

$$\therefore\ y_p(x) = y_1 \underbrace{\left(-\int \frac{y_2\, g(x)}{W}\, dx\right)}_{A(x)} + \underbrace{\left(\int \frac{y_1\, g(x)}{W}\, dx\right)}_{B(x)} y_2$$

$$W(x) = \begin{vmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{vmatrix} = \cancel{-\frac{2}{x} - \frac{2}{x}} = -\frac{2}{x}$$

$$g(x) = \frac{x^3}{x^2} = x$$

(29) $$A(x) = -\int \frac{\frac{1}{\cancel{x}} \cdot \cancel{x}}{-\frac{2}{x}}\, dx = \frac{1}{2}\int x\, dx = \frac{x^2}{4} \quad \text{(a)}$$

(30) $$B(x) = \int \frac{x \cdot x}{-\frac{2}{x}}\, dx = -\frac{1}{2}\int x^2\, x\, dx = -\frac{1}{2}\,\frac{x^4}{4} = -\frac{x^4}{8} \quad \text{(C)}.$$