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(unit-4) **Special Continuous distribution**

Topic

* ★ Normal distribution and its MGF.
* Gamma distribution and its MGF.
* Exponential distribution and its MGF.

Normal dis^{tri}

* n = Large, $n \to \infty$
  for ↳ continuous data.

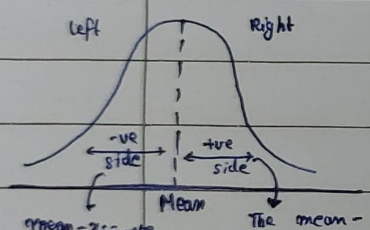
$n \to$ small ↳ B.D

$n \to$ large ↳ P.D ↓ Discrete

mean → avg value or mid value.

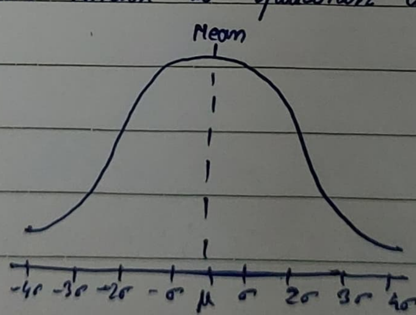
variance → Difference / deviation b/w original data and mean data.

★ it will tell you above the curve



↳ both are +ve but only for representation it is (-ve).

* Normal $dist^{ri}$ is also known as "Gaussian $dist^{ri}$" or "Bell shape curve dist"



* A normal Random $Vari^{ble}$ is said to have a normal $dist^{ri}$ with parameter μ (mean) and σ^2 (variance); if it's Pdf is given by:

$$\boxed{f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

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we denote the value of the RV 'x' as:-

$$N(\mu, \sigma^2)$$

or

$$n(x; \mu, \sigma)$$

i.e, $X \sim N(\mu, \sigma^2)$ or $X \sim n(x; \mu, \sigma)$

★ **Properties of Normal distribution.**

- The mean, median and mode are all equal or coinciding.

$\boxed{\text{mean} = \text{median} = \text{mode}}$ → equal for normal distri.

↳ arrange in ascending
↳ then the middle term.
↳ even = $\frac{\cancel{n} \; \cancel{+ n+1}}{2}$ th
↳ the avg of two middle most value
odd = $\frac{n}{2}$ → some sort of thing
↳ middle most value.

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- The curve is symmetric about the origin (μ = mean) *

- The total area under the curve and above the ~~Hozi~~ Horizontal axis is equal = 1.

i.e $\int_{-\infty}^{\infty} f(x).dx = 1$

- Area to the left and to the right of the mean (μ) is 0.5 *

- The mode, which is the point of the horizontal axis where the curve is maximum occurs at $\boxed{x = \mu(\text{mean})}$

↳

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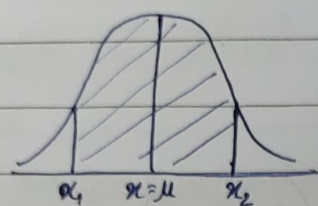
Area under the curve :-



$x_1 < x < x_2$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx$$

$$= \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

$$\boxed{= \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}.dx} \quad —①$$

it is a very lengthy approach so, we do have another method.

★ ⇒ If $x \sim N(\mu, \sigma^2)$ or $x \sim n(x; \mu, \sigma)$ for normal distri variable X with mean = 0 and var = 1 ; then the new variable Z is given by.

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

where Z is called standard normal variate

* $\mu = 0$, $\sigma^2 = 1$

whenever, X assume a value x, the corresponding value of Z is given by $\boxed{Z = \frac{x-\mu}{\sigma}}$

∴ If x takes b/w the values, $x = x_1$ and $x = x_2$, then the random variable z, will fall b/w the corresponding values,

$$Z_1 = \frac{x_1 - \mu}{\sigma} \quad \& \quad Z_2 = \frac{x_2 - \mu}{\sigma}$$

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from eq ①

$$-\frac{(x-\mu)^2}{2\sigma^2} \rightarrow -\frac{z^2}{2} \quad -②$$

↳ so from ⑪ → $P(x_1 < x < x_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} \cdot dz$

for $z \rightarrow$ mean = 0, Variance = 1

$$= \frac{1}{\underset{1}{\sigma}\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} \cdot dz$$

$$\boxed{P(x_1 < x < x_2) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} \cdot dz}$$

$$P(x_1 < x < x_2) = P(z_1 < x < z_2) \leftarrow \boxed{\int_{z_1}^{z_2} n(z; 0, 1)}$$

Q. Given a standard normal distribution, find the area given under the curve, which lies to the right of z,

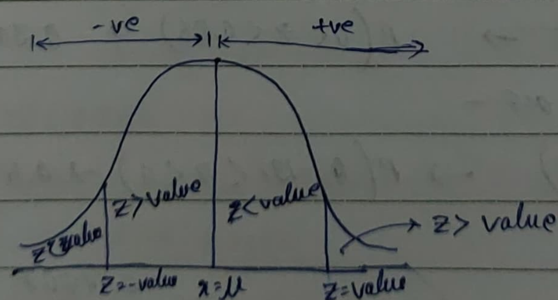
a) to the right of z = 1.84 and

b) b/w z = −1.97 and z = 0.86

< > → 1st Table

0 → value → third table.

Ex:-



↳

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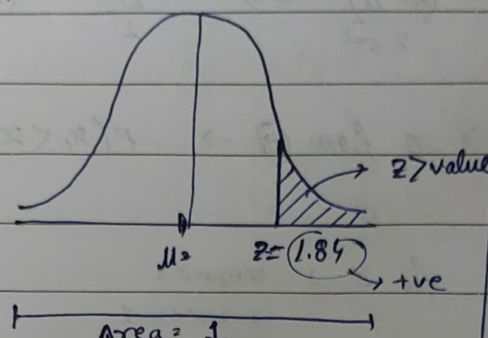
a) to the right of $z = 1.84$

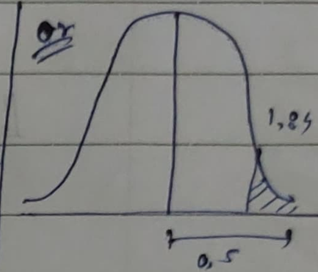


$= 1 - \underbrace{P(Z < 1.84)}$

$= 1 - 0.9671$

$= 0.0329$ A

or



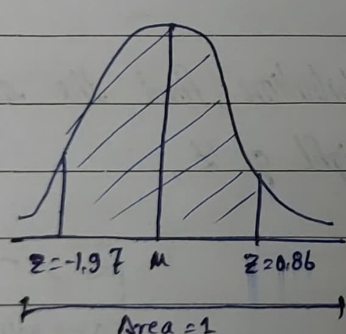
$P(Z > 1.84)$

$= 0.5 - P(0 < Z < 1.84)$

$= 0.5 - 0.4671$

$= 0.0329$

b) b/w $Z = -1.97$ and $Z = 0.86$.



2 table

$\cancel{P(Z < 0.86)} \rightarrow P(0 < Z < 0.86) \rightarrow 0.3051$

$= \cancel{0.8}$

$P(Z > 1.97) \rightarrow P(-1.97 < Z < 0) \rightarrow 0.4756$

$= 0.7807$

or

1 table

$P(Z < 0.86) - P(Z < -1.97)$

$= 0.8051 - 0.0244$

$= 0.7807$

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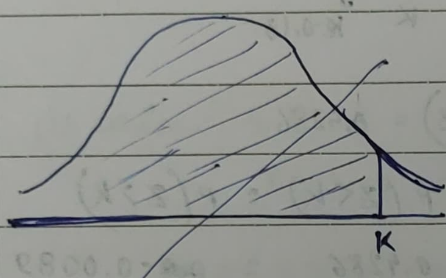
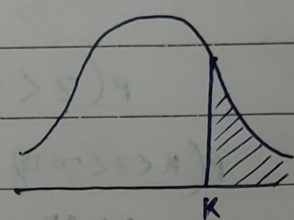
Q. Given a standard normal distribution, find the value of K such that

a) $P(Z > K) = 0.3015$

b) $P(K < Z < -0.18) = 0.4197$

a)





$Z <$

1st Table

$Z > K = 0.3015.$

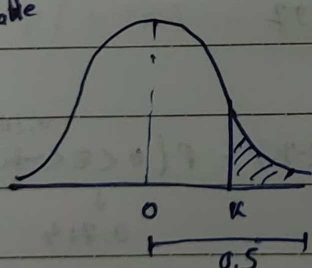
$Z < K = 1 - 0.3015$

$= 0.6985$

≈ 0.6985

$\boxed{K = 0.52}$

2nd Table



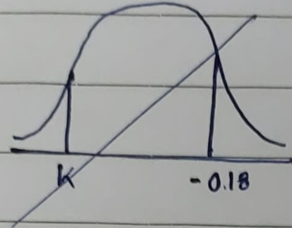
$Z > K = 0.3015$

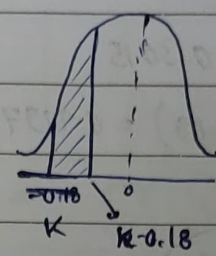
$P(0 < Z < K) = 0.5 - 0.3015$

$= 0.1985$

$\Rightarrow K = \boxed{0.52}$

b) $P(K<Z<-0.18) = 0.4197$





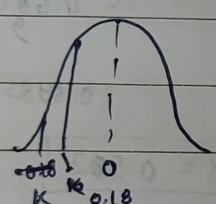
$P(Z<-0.18) = 0.4286.$

$P(K<Z<-0.18) - P(Z<K) = P(Z>K)$

$0.4197 - 0.4286 = -0.0089$

$\cancel{\boxed{K=0.1867}}$ $\boxed{K=2.37}$

Table 2



$P(K<Z<-0.18) = 0.4197$

$\cancel{P(K<0}$

$\cancel{P(K<Z<-0.18)}$ $P(0<Z<-K) - P(0<Z<\overset{-0.18}{\cancel{-K}}) = 0.4197$

0.714

$P(0<Z<-K) = 0.4197+0.0.714$

$= \cancel{0.2943}$ $\cancel{0.3483}$ (0.4911)

$K = 2.37$

Q. X is normally distributed, and mean of the $X = 12$, $\sigma = 4$.
find the Probability of the following.

(i) $X \geq 20$

(ii) $X \leq 20$

(iii) $0 \leq X \leq 12$

$\mu = 12$; $\sigma = 4$

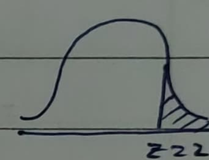
(i) ~~P(X~~ $X \geq 20$

$X = 20 \rightarrow Z = \frac{X - \mu}{\sigma} = \frac{20-12}{4} = 8/4 = 2$

$P(X \geq 20) \Rightarrow P(Z \geq 2)$

- Table 1

$P(Z \geq 2) = 1 - P(Z < 2)$



$= 1 - 0.9772$

$= 0.0228$

(ii) $P(X \leq 20) = P(Z \leq 2)$.

$= 1 - P(Z \geq 2)$

$= 1 - 0.0228$

$= 0.9772$

Table 2

$P(Z \geq 2 = 0.5 -$
$P(0 < Z < 2)$
$\downarrow$
$0.5 - 0.4772$
0.0228

(iii) $X_1 = 0$, $X_2 = 12$

$X_1 = 0 \Rightarrow Z_1 = \frac{X_1 - 12}{4} = -12/4 = -3$

$X_2 = 12 \Rightarrow Z_2 = \frac{X_2 - 12}{4} = 0/4 = 0$

$-3 < Z < 0$

Table 2

$P(-3 < Z < 0) = 0.4987$

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MGF of Normal Distribution

$$M_x t = \int_{-\infty}^{\infty} e^{tx} f(x).dx$$

$$= \int_{-\infty}^{\infty} e^{tx}. \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{(x-\mu)^2}{2\sigma^2}} .dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx}. \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}.dx$$

$$= \frac{\cancel{\sigma}}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\overset{\sigma z+\mu}{x}}. \exp\left\{-\frac{z^2}{2}\right\}.dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} . e^{-z^2/2}.dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\sigma zt - z^2/2}.dz$$

$z = \frac{x-\mu}{\sigma}$

$\sigma z = x - \mu$

$\sigma z + \mu = x$

differentiate

$\sigma dz + 0 = dx$

$\sigma dz = dx$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2\sigma z t)}.dz$$

performing, Now

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$$\Rightarrow \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2+\sigma^2t^2 - 2\sigma zt - \sigma^2t^2)}.dz$$

completeing Square

↳ coff of z^2 must be 1

↳ coff of $z = 2\sigma t$

↳ Divide by $\frac{1}{2}$

$\Rightarrow (2\sigma t)\frac{1}{2}$

$= \sigma t$

$$\Rightarrow \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}((z-\sigma t)^2 - \sigma^2 t^2)}.dz$$

$$\Rightarrow \frac{e^{t\mu}. e^{\frac{1}{2}(\sigma^2 t^2)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2}.dz$$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(b)^2} . db$$

$z - \sigma t = b$

diff

$dz - 0 = db$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\sqrt{2\pi}} . 2\int_{0}^{\infty} e^{-\frac{b^2}{2}} . db$$

$$\int_{-a}^{a} f(x).dx = 2\int_{0}^{a} f(x).dx$$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\sqrt{2\pi}} . 2\int_{0}^{\infty} e^{-c} . \frac{1}{\sqrt{2c}} . dc$$

$\frac{b^2}{2} = c$

$\cancel{\frac{2b\,db}{2} = dc}$ $\quad \frac{\cancel{2}b.db}{\cancel{2}} = dc$

$db = \frac{dc}{b}$

$\sqrt{2c}$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\sqrt{2}\pi} \frac{2}{\sqrt{2}} \int_{0}^{\infty} e^{-c} . c^{-\frac{1}{2}} . dc$$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\sqrt{\pi}} \int_{0}^{\infty} \cancel{e^{-c}}\, c^{-\frac{1}{2}} . e^{-c} . dc$$

$\alpha = -\frac{1}{2} + 1$

$= \frac{1}{2}$

$$\Gamma\tfrac{1}{2} \Rightarrow \int_{0}^{\infty} c^{-\frac{1}{2}} . e^{-\frac{1}{2}} . dc$$

$= \Gamma\frac{1}{2}$

$$\Rightarrow \frac{e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}{\cancel{\sqrt{\pi}}} . \Gamma\tfrac{1}{2} \rightarrow \cancel{\sqrt{\pi}}$$

$$\boxed{M_x(t) = e^{\left(tu+\frac{1}{2}\sigma^2t^2\right)}}$$

Gamma function.

$\Gamma\frac{1}{2} = \sqrt{\pi}$

value of Gamma $\frac{1}{2} = \sqrt{\pi}$

$$\boxed{\Gamma\alpha = \int x^{\alpha-1} . e^{-\alpha} . dx}, x>0$$

ex:- $\Gamma 3 = \int x^2 . e^{-3} . dx$

Normal Distribution as Limiting case of Binomial Distribution.

↳ Normal Distribution is the another Limiting case of Binomial Distribution just like poison's Distribution.

↳ The conditions are :-

** (i) n, the no. of trials is indefinately very large, (ie $n \to \infty$)

(ii) neither p nor q is small. $(p, q \not\to 0)$

If ~~capital~~ "X" is a binomial r.v with mean $(\mu) = np$ and variance $(\sigma^2) = npq$, then the Limiting form of the distribution is $\boxed{Z = \frac{X - np}{\sqrt{npq}}}$ *

as $n \to \infty$, is the standard Normal Distribution, $n(Z; 0, 1)$

Remark :- The ND with $\mu = np$ and var $= npq$ not only provides a very accurate approximation to the binomial distribution, when n is Large, and p is not extremely close to 0 or 1, but also provides a fairly good approximation, even when n is small and p is close to $\frac{1}{2}$.

↳

Q The prob that a patient recovers from a rare blood disease. (0.4)
If 100 people are known to have contacted this disease, what is the prob that fewer than 30 patient survive.

→ $n = 100$

$p = 0.4$ $\qquad q = 0.6$

possion → (statement) ↓ on an averag

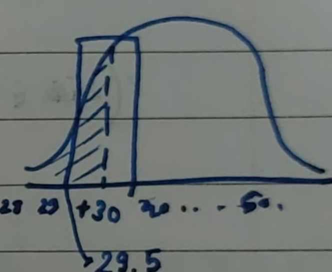
we can convert all Dist to Normal Dis

$P(x < 30)$

$np = 100 \times 0.4 = 40$

$\sqrt{npq} = \sqrt{100 \times 0.4 \times 0.6} = 2\sqrt{6}$

$Z = \dfrac{x - np}{\sqrt{npq}}$ $\quad x = ? = \cancel{30}\ 29.5$

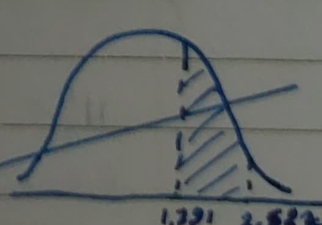


$Z = \dfrac{29.5 - 40}{2\sqrt{6}} = -2.14$

$P(x \leq 30) = P(Z \leq -2.14)$

$= 0.0162$

Q A MCQ Quiz has 200 Question each with 4 possible answer of which only one is correct. What is the prob. of sheer (guesswork) yeilds from 25→30 correct answer for the 80 of the 200 Q problems about which the player has no. Knowledge.



→ $n = 80$

x : correct Q/A.

$p = 1/4$, $q = 3/4$

$np = 80 \times 1/4 = 20$

$\sqrt{npq} = \sqrt{80 \times 1/4 \times 3/4} = \sqrt{15} = 3.872$

$P(25 \leq x \leq 30)$

$= P(1.291 \leq Z \leq 2.582)$

$Z_1 = \dfrac{25 - 20}{\sqrt{15}} = 5/\sqrt{15} = 1.291$

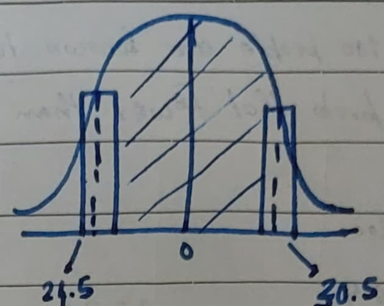
$Z_2 = \dfrac{30 - 20}{\sqrt{15}} = 10/\sqrt{15} \approx 2.582$

$\Rightarrow n = 80$

$p = 1/4 \ , \ q = 3/4$

$np = 20$

$\sqrt{npq} = \sqrt{15}$



$X_1 = 24.5 \qquad Z_1 = \frac{24.5 - 20}{\sqrt{15}} = \frac{4.5}{\sqrt{15}} = \frac{4.5}{3.872} \approx 1.162$

$X_2 = 30.5$

$Z_2 = \frac{30.5 - 20}{\sqrt{15}} = \frac{10.5}{\sqrt{15}} = \frac{10.5}{3.872} \approx 2.711$

$P(25 \leq x \leq 30) = P(1.162 \leq Z \leq 2.71)$

$= P(0 \leq Z \leq 2.71) - P(0 \leq Z \leq 1.162)$

$= 0.4966 - 0.3770$

≈ 0.1196

* **Mean and Variance of Normal Distribution using MGF.**

↳ **Derivation of Mean and Variance.**

$\mu_2' = E[x^2] = \frac{d^2}{dx^2} M_x t \Big|_{t=0.}$

(i) **Mean**

$\mu = E(x) = \frac{d}{dt} M_X(t) \Big|_{t=0}$

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$$Var,\ \sigma^2 = E[x^2] = \frac{d^2}{dx^2} M_x t \Big|_{t=0.}$$

(i) Mean $\Rightarrow$ $E[x] = \frac{d}{dt} M_x t$

$$= \frac{d}{dt}\left[e^{\mu t + \frac{1}{2}\sigma^2 t^2}\right]_{t=0.}$$

$$= e^{(\mu t + \frac{1}{2}\sigma^2 t^2)} \cdot \left(\mu + \frac{\cancel{2}}{\cancel{2}}\sigma^2 t\right)_{t=0.}$$

$$= e^0 . \mu$$

$$\boxed{= \mu}$$

(ii) Variance

$$\sigma^2 = E[x^2] = \frac{d^2}{dx^2} M_x t \Big|_{t=0}$$

$$\Rightarrow \frac{d^2}{dt^2}\left[e^{\mu t + \frac{1}{2}\sigma^2 t^2}\right]_{t=0.} = \frac{d}{dt}\left(e^{(\upsilon t + \frac{1}{2}\sigma^2 t^2)} \cdot \left(\mu + \frac{1}{2}\sigma^2 t\right)\right)$$

$$\cancel{\Rightarrow \left(\mu + \frac{1}{2}\sigma^2 t\right)\left\{\frac{d}{dt}\left(e^{\mu t + \frac{1}{2}\sigma^2 t^2}\right).\right\}}$$

$$\Rightarrow \left(e^{\upsilon t + \frac{1}{2}\sigma^2 t^2}\right) \cdot \left(\frac{\cancel{2}}{\cancel{2}}\sigma^2\right) + \left(\mu + \frac{\cancel{2}}{\cancel{2}}\sigma^2 t^2\right) \cdot \left\{e^{(\mu t + \frac{1}{2}\sigma^2 t^2)} \cdot \mu + \frac{\cancel{2}}{\cancel{2}}\sigma^2 t^2\right\}_{t=0}$$

$$= e^0 . \sigma^2 + e^0 . \mu^2 = \boxed{\sigma^2 + \mu^2.}$$

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Gamma and Exponential Distribution.

Discrete Data | Continuous Data

B.D	NGD	Geo	Posi	N.D	Gamma	Exp
n = trial ↳ small, $p \to$ given ↳ small	check for k^{th} ~~trial~~ success in n^{th} trial	check for 1st success	"on an avg term" given question	$n \to$ large, $p \to$ Large, "Normally distributed term"	K^{th} event occurs in what time = !	after K^{th} event the time period for the $K+1^{th}$ event

K^{th} event occurs in what particular time = ??

* Gamma Distribution is used to find time until K events occur.

Ex:- The time until K customers arrived in a particular restaurant.

Ex:- The time you have been invited to cake party.

Ex:- The time when 50% student will pass probability and statistics.

Gamma function is defined.

** $$\boxed{\Gamma\alpha = \int_0^\infty x^{\alpha-1} \cdot e^{-x} \cdot dx}, x>0$$

properties.

i) $\Gamma n = (n-1).(n-2).\ldots.1.$ → for +ve int n.

ii) $\Gamma n = (n-1)!$

iii) $\Gamma n = \Gamma 1 = 1$ or $0!$

iv) $\Gamma_{1/2} = \sqrt{\pi}$

Defination :- The continuous RV 'X' has gamma distribution with parameters α and β,

if its PDF is given by :-

where $\alpha > 0$, $\beta > 0$ **

$$\boxed{f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma\alpha} x^{\alpha-1} \cdot e^{-x/\beta} & , x>0 \\ 0, & \text{elsewhere} \end{cases}}$$

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Remark :- The relation b/w λ and β (Possion distribution.)

$\Rightarrow \boxed{\lambda = 1/\beta}$ where λ = Average no. of events per unit time.

α = No. of Events.

β = Avg time b/w no. of events.

Exponential Distribution. :- The continous RV 'X' has an exp. distribution with parameter "β", if its pdf is given by.

It occurs when $\boxed{\alpha = 1}$

$$\boxed{f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x>0 \\ 0, & \text{elsewhere} \end{cases}}$$ ★★

	Mean	Variance.
Gamma distribution	$\alpha\beta$	$\alpha\beta^2$
Exponential distribution	β	β^2.

MGF of Gamma distribution.

$$\boxed{M_x t = E[e^{tx}] = \int_0^\infty e^{tx} f(x).dx}$$

$$M_x t = E[e^{tx}] = \int_0^\infty e^{tx} f(x).dx$$

$$= \int_0^\infty e^{tx} \left(\frac{1}{\beta^\alpha \Gamma\alpha} x^{\alpha-1} . e^{-x/\beta} \right).dx$$

$$= \frac{1}{\beta^\alpha \Gamma\alpha} \int_0^\infty e^{-(1/\beta - t)x} . x^{\alpha-1} . dx.$$

Put $\left(\frac{1}{\beta} - t\right) x = u$

$$x = \frac{u}{1/\beta - t}$$

$$dx = \frac{du}{1/\beta - t}$$

$$= \frac{1}{\beta^{\alpha}\Gamma\alpha} \int_0^{\infty} e^{-u} \left(\frac{u}{1/\beta - t} \right)^{\alpha-1} \cdot \frac{du}{1/\beta - t}$$

$$= \frac{1}{\beta^{\alpha}\Gamma\alpha} \cdot \frac{1}{(1/\beta - t)^{\alpha-1}} \cdot \frac{1}{(1/\beta - t)} \cdot \int_0^{\infty} e^{-u} \cdot u^{\alpha-1} \cdot du$$

$$= \frac{1}{\beta^{\alpha}\Gamma\alpha} \cdot \frac{\beta^{\alpha-1}}{1-\cdot}$$

$$\boxed{M_x t = \frac{1}{(1-\beta t)^{\alpha}}}$$

13/11/23

Derivation of Mean and Variance of Gamma distribution by using MGF.

$$\mu_r' = E[x^r] = \frac{d^r}{dt^r} M_x(t) \Big|_{t=0}$$

$$\mu = E[x] = \frac{d}{dt} M_x(t) \Big|_{t=0}$$

$$\Rightarrow \frac{d}{dt} \left[\frac{1}{(1-\beta t)^{\alpha}} \right] \Bigg|_{t=0}$$

$x' = 1$

$1/x = x^{-1}$ or $= -1 x^{-2}$

$$\Rightarrow \cancel{\left(\frac{\frac{1}{(1-\beta t)^{\alpha-1}}}{\alpha - 1} \right)}$$

$$\Rightarrow \frac{d}{dt} \left[(1-\beta t)^{-\alpha} \right] \Big|_{t=0}$$

$$\Rightarrow (-\alpha) . (1-\beta t)^{-\alpha-1} . (0-\beta) \Big|_{t=0}$$

$\Rightarrow (-\alpha).(1-0).(-\beta)$

$$\boxed{\mu = \alpha\beta}^{**}$$

Variance by MGF

$\sigma^2 = E[x^2] - (E[x])^2$

$= \frac{d^2}{dt^2}\left[\frac{1}{(1-\beta t)^\alpha}\right]\Bigg|_{t=0} \Rightarrow \frac{d^2}{dt^2}\left[(1-\beta t)^{-\alpha}\right]\Bigg|_{t=0}$

$\Rightarrow \frac{d}{dt}\left[\alpha\beta(1-\beta t)^{-\alpha-1}\right]\Bigg|_{t=0}.$

$\Rightarrow \alpha\beta.\left((-\alpha-1)(1-\beta t)^{-\alpha-2}.(0-\beta)\right)$

$\Rightarrow \alpha\beta.\left((-\alpha-1)(1-0).(-\beta)\right)$

$\Rightarrow \alpha\beta[\alpha\beta+\beta]$

$\Rightarrow \alpha^2\beta^2+\alpha\beta^2$

Now, $\sigma^2 = \cancel{\alpha^2\beta^2} + \alpha\beta^2 - \cancel{\alpha^2\beta^2}$

$$\boxed{\sigma^2 = \alpha\beta^2}^{**}$$

MGF of Exponential Distribution.

$\Rightarrow M_x(t) = E[e^{tx}] = \int_0^\infty e^{tx}.f(x).dx$

$\Rightarrow \int_0^\infty e^{tx}.\left(\frac{1}{\beta}.e^{-x/\beta}\right).dx \Rightarrow \frac{1}{\beta}\int_0^\infty e^{tx}.e^{-x/\beta}.dx \Rightarrow \frac{1}{\beta}\int_0^\infty e^{-(1/\beta-t)x}.dx$

put $\Rightarrow \left(\frac{1}{\beta}-t\right)x = u$

$x = \frac{u}{1/\beta - t}$

$dx = \frac{du}{1/\beta - t}$

$\Rightarrow \frac{1}{\beta}\int_0^\infty e^{-u}.\frac{du}{1/\beta - t}$

$\Rightarrow \frac{1}{\beta}.\frac{1}{\frac{1}{\beta}-t}\int_0^\infty u^{\alpha-1}e^{-u}.du$ (Since in Expo ($\alpha=1$) we insert $u^{\alpha-1} \Rightarrow u^{1-1} = u^0$)

$$\Rightarrow \frac{1}{\beta} \cdot \frac{\beta}{\beta 1-\beta t} \cdot \underbrace{\int_0^{\infty} e^{-u} \cdot u^{\alpha-1} \cdot dx}_{\Gamma 1}$$

$$\Rightarrow \boxed{M_x(t) = \frac{1}{1-\beta t}} \star\star$$

Mean of Exponential by using MGF

$$\mu_1' = E[x] = \frac{d}{dt} M_x t \Big|_{t=0}$$

$$= \frac{d}{dt}\left(\frac{1}{1-\beta t}\right)\Big|_{t=0} = \frac{d}{dt}\left((1-\beta t)^{-1}\right)\Big|_{t=0}$$

$$= \left((-1).(1-\beta t)^{-1-1}(-\beta)\right)\Big|_{t=0}$$

$$\Rightarrow (-1)(-\beta)$$

$$\boxed{\mu = \beta} \star\star$$

Variance of Exponential by using MGF

$$\sigma^2 = E[x^2] - (E(x))^2$$

$$= \frac{d^2}{dt^2} M_x(t)\Big|_{t=0} = \frac{d^2}{dt^2}\left[\frac{1}{1-\beta t}\right]\Big|_{t=0}$$

$$= \frac{d}{dt}\left((-1)(1-\beta t)^{-2}.(-\beta)\right)\Big|_{t=0} = \frac{d}{dt}\left(\beta.(1-\beta t)^{-2}\right)\Big|_{t=0}$$

$$= -2\beta.(1-\beta t)^{-3}(-\beta)\Big|_{t=0}$$

$$= -2\beta.(1).(-\beta)$$

$$E[x^2] = \boxed{2\beta^2}$$

$$\boxed{\sigma^2 = 2\beta^2 - \beta^2} \star\star$$

$$\boxed{\sigma^2 = \beta^2}$$

16/10/23

Q Suppose that telephone calls arriving at a call centre with an avg of 5 calls coming per minute. What is the prob that up to a minute will ellapse by the time two calls have come in to the telephone calls.

→ $\lambda = 5$

$\alpha = 2$

$\beta = 1/5$

X: Time will elapse before 2 calls.

$P(X \leq 1) = ?$

$\int_0^{1} \frac{1}{\beta^\alpha \Gamma\alpha} x^{\alpha-1} . e^{-x/\beta} . dx$

$F2 = 1!$

$= 1$

$\Rightarrow \int_0^{1} \frac{1}{(\frac{1}{5})^2 \Gamma(2)(1)} . x^1 . e^{-5x} . dx$

$\Rightarrow \frac{1}{(\frac{1}{25})} \int_0^{1} x^{\alpha-1} . e^{-5x} . dx$

function by part. ILATE

$\int I.II = I\int II - \int[\frac{d}{dx} I \int II] dx$

$I\int II - \int[\frac{d}{dx} I \int II] . dx$

$\boxed{e^0 = 1}$

$\cancel{\Rightarrow 25 \left[x^1 . \frac{e^{-5x}}{-5} \Big|_0^1 - \int_0^1 I. \right]}$

$\Rightarrow 25 \left[x^1 . \frac{e^{-5x}}{-5} \Big|_0^1 - \int_0^1 1.e^{-5x} . dx \right]$

$\Rightarrow 25 \left[-5 \left[e^{-5} \right] - \frac{1}{25} . e^{-5x} \Big|_0^1 \right\}$

Ans = 0.96

Q In a bio-medical study with rats are those responds investigation is used to determine the effect of the dose of the toxicants on their survival. The toxicant is one that is frequently discharge into the atmosphere from the jet fuel. for a certain dose of the toxicants the study determine that the survival time in week as a gamma distribution $\alpha = 5$, $\beta = 10$.

What is the prob that a rat survives no longer that 60 weeks.

⇒ $\alpha = 5$, $\beta = 10$. $P(X \leq 60\text{week}) = ?$

$$\int_0^{\infty \to 60} \frac{1}{\beta^\alpha \Gamma\alpha} \cdot x^{\alpha-1} \cdot e^{-x/\beta} \cdot dx$$

$$P(X \leq 60) = \int_0^{60} \frac{1}{(10)^5 \cdot (24)} \cdot x^4 \cdot e^{-x/10} \cdot dx$$

$$= \frac{1}{(10^5) \cdot (24)} \int_0^{60} \underbrace{x^4}_{\text{diff}} \cdot \underbrace{e^{-x/10}}_{\text{integrate}} \cdot dx$$

technique when e is a variable

$$= \frac{1}{(10^5) \cdot 24} \left[\frac{x^4 \cdot e^{-x/10}}{-1/10} - \frac{4x^3 \cdot e^{-x/10}}{-1/10^2} + \frac{12x^2 \cdot e^{-x/10}}{-1/10^3} - \frac{24x \cdot e^{-x/10}}{-1/10^4} + \frac{24 \cdot e^{-x/10}}{-1/10^5} \right]_0^{60}$$

$$\Rightarrow \frac{1}{10^5(24)} \left[-10x^4 \cdot e^{-x/10} - 400 \cdot x^3 \cdot e^{-x/10} + 12000 \cdot x^2 \cdot e^{-x/10} - 240000 \cdot x \cdot e^{-x/10} + 2400000 \cdot e^{-x/10} \right]_0^{60}$$

⇒

Ans = 0.72