17.1.1 Scalar and Vector Fields

A continuous function of the position of a point in a region of space is called a *point function*. The region of space in which it specifies a physical quantity is known as a *field*. These fields are classified into two groups:

(i) *Scalar field*: A scalar field is defined as that region of space, whose each point is associated with a *scalar point function*, i.e., a continuous function which gives the value of a physical quantity as flux, potential, temperature, etc. In a scalar field, all the points having the same scalar physical quantity are connected by the means of surfaces called *equal* or *level surfaces*.

(ii) *Vector field*: A vector field is specified by a continuous vector point function having magnitude and direction, both of which change from point to point, in the given region of field. The method of presentation of a vector field is called *vector lines*, or *lines of surfaces*. The tangent at a vector line gives the direction of the vector at the point.

17.1.2 Gradient, Divergence, and Curl

In vector calculus, we study about the rate of change of scalar and vector fields. For this purpose, a common operator called *del*, or *nabla*, is used, which is written as

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

If $\phi(x, y, z)$ is a differentiable scalar function, its gradient is defined as

$$\text{grad } \phi = \nabla\phi = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\phi$$

or

$$\vec{\nabla}\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

Physically, grad ϕ is a vector whose magnitude at any point is equal to the rate of change of ϕ at a point along the normal to the surface at that point.

If $\vec{F}$ is a vector point function ($\vec{F} = F_1 i + F_2 j + F_3 k$), where F_1, F_2, and F_3 are functions of x, y, and z, then its divergence written as div F, or $\vec{\nabla} \cdot \vec{F}$, is given by

$$\vec{\nabla}\cdot\vec{F} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot(F_1 i + F_2 j + F_3 k)$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \qquad \begin{bmatrix} i\cdot i = j\cdot j = k\cdot k = 1 \\ i\cdot j = j\cdot k = k\cdot i = 0 \end{bmatrix}$$

Divergence of a vector point function, physically signifies the outward normal flux of vector field from a closed surface.

If the divergence of any vector function is zero, then the flux of vector function entering into a region must be equal to that leaving it. This vector function is called *solenoidal*.

If $\vec{F}$ is a vector point function ($\vec{F} = F_1 i + F_2 j + F_3 k$), where F_1, F_2, and F_3 are functions of x, y, and z, then its curl is defined as

$$\text{Curl } \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= i\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - j\left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) + k\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

A vector field $\vec{F}$ is called *irrotational* if curl $\vec{F} = 0$. Such fields are also known as *conservative fields*.

17.1.3 Gauss Divergence Theorem (Relation between Surface and Volume Integration)

This theorem states that the flux of a vector field $\vec{F}$, over any closed surface S, is equal to the volume integral of the divergence of that vector field over the volume V enclosed by the surface S.

$$\int_s \vec{F} \cdot d\vec{S} = \int_V \text{div } \vec{F}\, dV \tag{17.1}$$

17.1.4 Stokes Theorem (Relation between Line Integral and Surface Integration)

This theorem states that the surface integral of the curl of a vector field $\vec{A}$, taken over any surface S, is equal to the line integral of $\vec{A}$ around the closed curve forming the periphery of the surface.

$$\iint_s (\text{Curl } \vec{A}) \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$$

or

$$\iint_s (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l} \tag{17.2}$$

17.1.5 Poisson's and Laplace's Equations

Poisson's and Laplace's equations are very useful mathematical relations for the calculations of electric fields and potentials that cannot be computed by using Coulomb's and Gauss's law in electrostatic problems. These equations can be derived as follows:

Gauss law in electrostatics is given by

$$\text{div } E = \frac{\rho}{\epsilon_0}$$

Electric field and potential are related as

$$\vec{E} = -\text{grad } V = -\vec{\nabla} V$$

Thus, we obtain

$$\text{div}\,(-\,\text{grad}\; V) = \frac{\rho}{\epsilon_0}$$

or $$\vec{\nabla}\cdot(-\vec{\nabla}V) = \frac{\rho}{\epsilon_0}$$

or $$\nabla^2 V = -\frac{\rho}{\epsilon_0} \tag{17.3}$$

This equation is known as *Poisson's equation* for a homogeneous region. For a charge-free region, i.e., $\rho = 0$, Poisson's equation becomes

$$\nabla^2 V = 0$$

This is called *Laplace's equation*. This equation is applicable to those electrostatic problems, where the entire charge resides on the surface of the conductor or is concentrated in the form of point charges, line charges, or surface charges at a single position. It is also applicable in the cases, where the region between two conductors is filled with one or more homogeneous dielectrics.

17.2 FUNDAMENTAL LAWS OF ELECTRICITY AND MAGNETISM

To understand Maxwell's equation, we must go through the basic laws of electricity and magnetism.

(i) *Gauss's law in electrostatics*: $\oint \vec{E}\cdot d\vec{S} = q/\epsilon_0$ (17.4)

i.e., the electric flux from a closed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

(ii) *Gauss's law in magnetostatics*: $\oint \vec{B}\cdot d\vec{S} = 0$ (17.5)

i.e., the rate of change of magnetic flux through a closed surface is always equal to zero. This also signifies that monopole cannot exist.

(iii) *Faraday's law of electromagnetic induction*: This law states that the rate of change of magnetic flux in a closed circuit induces an emf which opposes the cause, i.e.,

$$e = -\frac{d\phi}{dt} \tag{17.6}$$

(iv) *Ampere's law*: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ (17.7)

This law states that the line integral of magnetic flux is equal to μ_0 times the current enclosed by the current loop.

17.3 EQUATION OF CONTINUITY

Electric current is the rate of flow of charge. Therefore, we have

$$i = -\frac{dq}{dt} \tag{17.8}$$

If dq charge is enclosed in a volume element dV and is leaving a surface having area dS, we have

$$i = \int_s \vec{J} \cdot d\vec{S} \text{ and } q = \int_V \rho\, dV$$

where J is the current density and ρ is the volume charge density. Therefore, Eq. (17.8) becomes

$$\int_s \vec{J} \cdot d\vec{S} = -\frac{d}{dt}\int_V \rho\, dV$$

or

$$\int_s \vec{J} \cdot d\vec{S} = \int_V \frac{\partial \rho}{\partial t} dV \tag{17.9}$$

Using Gauss divergence theorem on LHS of Eq. (17.9), we get

$$\int_s \vec{J} \cdot d\vec{S} = \int_V \text{div}\, J\, dV$$

Therefore, Eq. (17.9) becomes

$$\int_V \text{div}\, \vec{J}\, dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

or

$$\int_V \left(\text{div}\, \vec{J} + \frac{\partial \rho}{\partial t} \right) dV = 0$$

Therefore, for an arbitrary surface, we have

$$\boxed{\text{div}\, \vec{J} + \frac{\partial \rho}{\partial t} = 0} \tag{17.10}$$

This expression is called *continuity equation.*

17.4 DISPLACEMENT CURRENT

According to Maxwell, it is not only the current in a conductor that produces a magnetic field. A changing electric field in vacuum or in a dielectric also produces a magnetic field. This implies that a changing electric field is equivalent to a current, which flows till the electric field is changing. This equivalent current produces the same magnetic effects as a conventional current in a conductor. This equivalent current is known as *displacement current.*

17.4.1 Modified Ampere's Law

The concept of displacement current due to the discharge of a condenser leads to the modification in Ampere's law. Consider the process of charging of a parallel plate capacitor through a series circuit as shown in Fig. 17.1.

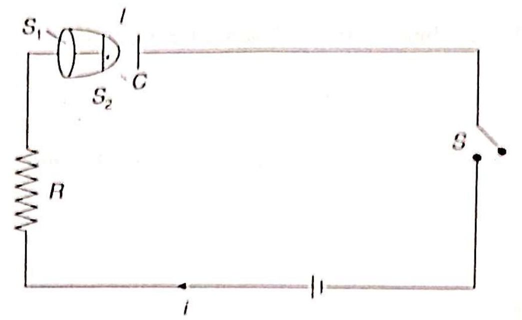

Fig. 17.1 Charging of a capacitor

Let us consider a plane surface S_1 and a hemispherical surface S_2 around the condenser plate as shown in Fig. 17.1. Let both surfaces be bounded by the same closed path l, and applying Ampere's law to the surface S_1, we get

$$\oint_{S_1} \vec{B} \cdot \vec{dl} = \mu_0 i \tag{17.11}$$

Now, during the process of charging, current i has been flowing through the plane surface S_1. If it is applied to the hemispherical surface S_2, we get

$$\oint_{S_2} \vec{B} \cdot \vec{dl} = 0 \tag{17.12}$$

(because no current is enclosed by the surface S_2).

But Eqs. (17.11) and (17.12) show contradiction to each other. Hence, Maxwell introduced the idea that a changing electric field is a source of magnetic field in the gap between the capacitor plates (during charging) and is equivalent to the displacement current devalued by i_d. If ϕ_E is the electric flux, then from equation of continuity, i_d should be equal to $\epsilon_0\, d\phi_E/dt$. Therefore, if along with an electric current, there exists a magnetic field, the modified Ampere's law becomes

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(i + \epsilon_0 \frac{d\phi_E}{dt} \right) = \mu_0 (i + i_d) \tag{17.13}$$

Now, the electric field for the charge q developed at the plates of a parallel plate capacitor, each having an area A is given by

$$E = \frac{q}{\epsilon_0 A}$$

or $$\frac{dE}{dt} = \frac{1}{\epsilon_0 A}\frac{dq}{dt} = \frac{i}{\epsilon_0 A}$$

or $$i = \epsilon_0 A \frac{dE}{dt} \tag{17.14}$$

$$= \epsilon_0 \frac{d(EA)}{dt} = \epsilon_0 \frac{d\phi_E}{dt} = i_d \qquad (\phi_E = EA)$$

Thus, the displacement current in the gap is identical to the conduction current in the connecting wires.

From Eq. (17.14), we can write

$$i_d = A\frac{d(\epsilon_0 E)}{dt} = A\frac{dD}{dt} \qquad (\because \vec{D} = \epsilon_0 \vec{E})$$

or $$\frac{i_d}{A} = \frac{dD}{dt}$$

or $$J_d = \frac{dD}{dt}$$

Hence, modified Ampere's law becomes

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i + \epsilon_0 \frac{d\phi_E}{dt}\right) \tag{17.15}$$

17.5 MAXWELL'S ELECTROMAGNETIC EQUATIONS

Maxwell's equations are based on the fundamental laws of physics, which we have already discussed in previous articles. With the help of these equations, one can analyse time-varying fields.

17.5.1 Maxwell's Equations in Differential Form

(i) $\vec{\nabla}\cdot\vec{D} = \rho$ or Div $\vec{D} = \rho$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$ or Div $\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or Curl $\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ or Curl $\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

where

$\vec{D}$ = Electric displacement vector, $\vec{B}$ = Magnetic flux density

$\vec{E}$ = Electric field intensity, $\vec{H}$ = Magnetic field intensity

$\vec{J}$ = Current density (conventional)

ρ = Charge density

17.5.2 Maxwell's Equations in Integral Form

(i) $\int_S \vec{D}\cdot\vec{dS} = \int_V \rho dV$ or $\oint_S \vec{E}\cdot\vec{dS} = q$

(ii) $\oint_S \vec{B}\cdot\vec{dS} = 0$

(iii) $\oint \vec{E}\cdot\vec{dl} = -\dfrac{\partial}{\partial t}\int \vec{B}\cdot\vec{dS}$.

(iv) $\oint \vec{H}\cdot\vec{dl} = \int_S \left(\vec{J} + \dfrac{\partial \vec{D}}{\partial t}\right)\cdot\vec{dS}$

Symbols used have the same meaning, as given in Section 17.5.1.

17.5.3 Derivation of Maxwell's Equations

1. ***Maxwell's first equation, div D = ρ or*** $\vec{\nabla}\cdot\vec{D} = \rho$:

When a dielectric is placed in a uniform electric field, its molecules get polarised. Thus, a dielectric in an electric field contains two types of charges—free charges, which are embedded, and polarisation charges or bound charges. If ρ and ρ_P are the free and bound charge densities, respectively, at a point in a small volume element dV, then for such a medium, Gauss's law may be expressed as

$$\int_S \vec{E}\cdot\vec{dS} = \frac{1}{\epsilon_0}\int_V (\rho + \rho_P)\, dV \qquad (17.16)$$

$\because D = \rho + \rho_p$

$\epsilon_0 E = (\rho + \rho_p)$

$E = \left(\dfrac{\rho + \rho_p}{\epsilon_0}\right)$

where ϵ_0 is the permittivity of the free space.

Now, the bound charge density

$$\rho_P = -\operatorname{div}\vec{P}, \text{ where } \vec{P} \text{ is electric polarisation.}$$

div D = ρ

div D = $(\rho + \rho_p)$

Therefore, $\displaystyle\int_S \vec{E}\cdot\vec{dS} = \frac{1}{\epsilon_0}\int_V (\rho - \operatorname{div}\vec{P})\, dV$

Using Gauss divergence theorem on left-hand side of the above expression, we get

$$\int_s \vec{E}\cdot d\vec{S} = \int_v \text{div}\, E\, dV = \frac{1}{\epsilon_0}\int_v \rho\, dV - \frac{1}{\epsilon_0}\int_v \text{div}\,\vec{P}\, dV$$

or
$$\int_v \epsilon_0\, \text{div}\,\vec{E}\, dV + \int_v \text{div}\,\vec{P}\, dV = \int_v \rho\, dV$$

$$\int_v \text{div}\epsilon_0\,\vec{E}\, dV + \int_v \text{div}\,\vec{P}\, dV = \int_v \rho\, dV$$

$$\int_v \text{div}(\epsilon_0\,\vec{E} + \vec{P})\, dV = \int_v \rho\, dV$$

But $\epsilon_0\vec{E} + \vec{P} = \vec{D}$ is the electric displacement vector.

Thus,
$$\int_v \text{div}\,\vec{D}\, dV = \int_v \rho\, dV$$

or
$$\int_v (\text{div}\,\vec{D} - \rho)\, dV = 0$$

Therefore, for an arbitrary surface, we have

$$\text{div}\,\vec{D} - \rho = 0$$

or
$$\text{div}\,\vec{D} = \rho$$

or
$$\vec{\nabla}\cdot\vec{D} = \rho$$

This is the required Maxwell's first equation.

In free space, volume charge density $\rho = 0$.

Therefore, Maxwell's first equation in free space is reduced to

$$\text{div}\,\vec{D} = 0 \text{ or } \vec{\nabla}\cdot\vec{D} = 0$$

or
$$\text{div}\,\epsilon_0\,\vec{E} = 0$$

or
$$\epsilon_0\,\text{div}\,\vec{E} = 0$$

or
$$\text{div}\,\vec{E} = 0 \text{ or } \vec{\nabla}\cdot\vec{E} = 0$$

2. *Maxwell's second equation, div $\vec{B} = 0$ or $\vec{\nabla}\cdot\vec{B} = 0$:*

It has been experimentally observed that the number of magnetic lines of force entering any closed surface enclosing a volume is exactly the same as that leaving it, i.e., the net magnetic flux through any closed surface is always zero.

Hence,

$$\phi_B = \oint_S \vec{B} \cdot \vec{dS} = 0 \tag{17.17}$$

The above expression implies that a monopole or an isolated magnetic pole cannot exist to serve as a source or sink for the line of magnetic induction $\vec{B}$. This expression is also known as *Gauss's law in magnetostatics*.

Using Gauss divergence theorem in Eq. (17.6), we have

$$\oint_S \vec{B} \cdot \vec{dS} = \int_V \operatorname{div} \vec{B}\, dV = 0$$

where V is the volume enclosed by surface S.

Hence, for an arbitrary surface,

$$\operatorname{div} \vec{B} = 0$$

or

$$\vec{\nabla} \cdot \vec{B} = 0$$

3. *Maxwell's third equation (Faraday's law of electromagnetic induction)*:

According to Faraday's law of electromagnetic induction, the induced emf around a closed circuit is equal to the negative time rate of change of magnetic flux linked with the circuit, i.e.,

$$e = -\frac{d\phi_B}{dt} \tag{17.18}$$

If $\vec{B}$ is the magnetic induction, then the magnetic flux linked with an area $\vec{dS}$ is

$$\phi_B = \int_S \vec{B} \cdot \vec{dS} \tag{17.19}$$

On combining Eqs. (17.18) and (17.19), we get

$$e = -\frac{d}{dt} \int_S (\vec{B} \cdot \vec{dS})$$

or

$$e = -\int_S \frac{\partial}{\partial t} (\vec{B} \cdot \vec{dS}) \tag{17.20}$$

According to definition, the induced emf is related to the corresponding electric field as

$$e = \int_C \vec{E} \cdot \vec{dl} \tag{17.21}$$

Equations (17.20) and (17.21) will give

$$\int_C \vec{E} \cdot \vec{dl} = -\int_S \frac{\partial}{\partial t} (\vec{B} \cdot \vec{dS})$$

$$= -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Now, using Stoke's theorem on left-hand side, we get

$$\int_c \vec{E} \cdot d\vec{l} = \int_s \text{curl } \vec{E} \cdot d\vec{S}$$

Thus, we have

$$\int_s \text{curl } \vec{E} \cdot d\vec{S} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

or
$$\int_s \left(\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S} = 0$$

For any arbitrary surface dS, we will have

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or
$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

i.e.,
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. *Maxwell's fourth equation (modified Ampere's law)*:

In Section 17.2, Ampere's law is given as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Using formula $I = \oint_s \vec{J} \cdot d\vec{S}$ $\left(\text{using } J = \frac{I}{A} \right)$

we get

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \oint \vec{J} \cdot d\vec{S}$$

Using Stoke's theorem on the left-hand side of the above expression, we get

$$\oint_s \text{curl } \vec{B} \cdot d\vec{S} = \mu_0 \oint_s \vec{J} \cdot d\vec{S}$$

$$\frac{1}{\mu_0}\oint_s \text{curl}\,\vec{B}\cdot d\vec{S} = \oint_s \vec{J}\cdot d\vec{S}$$

$$\oint_s \text{curl}\,\frac{\vec{B}}{\mu_0}\cdot d\vec{S} = \oint_s \vec{J}\cdot d\vec{S}$$

Now, from dielectric properties, we have

$$\frac{B}{\mu_0} = H$$

$$\therefore \quad \int_s \text{curl}\,\vec{H}\cdot d\vec{S} = \int_s \vec{J}\cdot d\vec{S}$$

or

$$\int_s (\text{curl}\,\vec{H} - \vec{J})\cdot d\vec{S} = 0$$

For an arbitrary surface, we have

$$\text{curl}\,\vec{H} - \vec{J} = 0$$

or

$$\text{curl}\,\vec{H} = \vec{J} \tag{17.22}$$

Taking divergence on both sides, we get

$$\text{div curl}\,\vec{H} = \text{div}\,\vec{J}$$

But div curl $\vec{H} = 0$ (From vector calculus)

$$\therefore \quad \text{div}\,\vec{J} = 0$$

From continuity equation, we have

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Hence,

$$\frac{\partial \rho}{\partial t} = 0$$

or

$$\rho = \text{constant (static)}$$

This implies that Ampere's law is applicable only for static charges. However, for time-varying fields, Maxwell suggested that Ampere's law must be modified by adding a quantity having dimension as that of current and produced due to polarisation of charges. This physical quantity is called *displacement current* (J_d). Thus, modified Ampere's law now becomes

$$\text{curl}\,\vec{H} = \vec{J} + \vec{J_d}$$

Taking divergence on both sides, we get

$$\text{div curl}\,\vec{H} = \text{div}\left(\vec{J} + \vec{J_d}\right) \qquad (\text{div Curl}\,\vec{H} = 0)$$

$$0 = \text{div } \vec{J} + \text{div } \vec{J}_d$$

or $$\text{div } \vec{J} = -\text{div } \vec{J}_d$$

But $$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \text{ (Continuity equation)}$$

$\therefore$ $$\text{div } \vec{J}_d = \frac{\partial \rho}{\partial t}$$

But $$\rho = \text{div } \vec{D}\,*$$

$\therefore$ $$\text{div } \vec{J}_d = \frac{\partial}{\partial t}(\text{div } \vec{D})$$

$$\text{div } \vec{J}_d = \text{div}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

or $$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Therefore, modified Ampere's law now becomes

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

~~17.6~~ PHYSICAL SIGNIFICANCE OF MAXWELL'S EQUATIONS

(i) Maxwell's first equation $\oint \vec{E}\cdot d\vec{S} = q/\epsilon_0$ or div $\vec{E} = \rho/\epsilon_0$ represents the Gauss' law in electrostatics for the static charges, which states that the electric flux through any closed surface is equal to $1/\epsilon_0$ times the total charge enclosed by the surface.

(ii) Maxwell's second equation $\oint \vec{B}\cdot d\vec{S} = 0$ or div $\vec{B} = 0$ expresses Gauss's law in magnetostatics, which states that the net magnetic flux through any closed surface is zero. Since a magnetic monopole does not exist, any closed volume always contains equal and opposite magnetic poles (north and south poles), resulting in the net magnetic pole strength becoming zero. It also signifies that magnetic lines of flux are continuous, i.e., the number of magnetic lines of flux entering into a region is equal to the lines of flux leaving it.

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* $\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}$ $\oint \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0}\int \rho\, dV$ $\oint \epsilon_0 \cdot \vec{E}\cdot d\vec{S} = \int \rho\, dV$

$\oint \vec{D}\cdot d\vec{S} = \int \rho\, dV \int_V \text{div}\vec{D}\, dV = \int_V \rho\, dV$ div $\vec{D} = \rho$ (Gauss law)

(iii) Maxwell's third equation is the Faraday's law of electromagnetic induction, i.e., $\left(\text{curl } \vec{E} = -\partial \vec{B}/\partial t\right)$. It states that the induced electromotive force around any closed surface is equal to the negative time rate of change of the magnetic flux through the path enclosing the surface. This signifies that an electric field can also be produced by a changing magnetic flux.

(iv) Maxwell's fourth equation curl $\vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \, \partial \vec{E}/\partial t$ represents the generalised form of Ampere's law as extended by Maxwell to account for the time-varying magnetic fields. It is valid for both steady (electrostatic) and non-steady (charges are in motion) states. It states that the magnetomotive force around a closed path is equal to the sum of conduction current ($\mu_0 i$) and displacement current ($\epsilon_0 \, \partial\phi_E/\partial t$) through the surface bounded by that path. This signifies that a conduction current or a changing electric flux produces a magnetic field.

A close scrutiny of Maxwell's third and fourth equations reveals that the modified form of Ampere's law contains a term $\mu_0 i$ while Faraday's law does not. The absence of this term in Faraday's law signifies the absence of magnetic monopole. The term $\epsilon_0 \, \partial\phi_E/\partial t$, i.e., the displacement current, signifies that magnetic field can also be produced by a changing electric field. Since the quantity $\mu_0 \epsilon_0 \approx 10^{-17}$, therefore, the term $\mu_0 \epsilon_0 \, \partial\phi_E/\partial t$ will not contribute significantly unless $\partial\phi_E/\partial t$ is extremely large, i.e., the displacement current can be detectable only when electric flux changes very rapidly. Hence, Maxwell's fourth equation gives the generation of magnetic field by displacement current.

17.7 ELECTROMAGNETIC ENERGY (POYNTING THEOREM)

This theorem analyses the transportation of energy in the medium from one place to another due to the propagation of electromagnetic waves.

Maxwell's third and fourth equations in differential form are as follows:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{17.23}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{17.24}$$

Taking scalar product of Eq. (17.23) with $\vec{H}$ and of Eq. (17.24) with $\vec{E}$, we get

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \tag{17.25}$$

and
$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \tag{17.26}$$

Subtraction of Eq. (17.26) from Eq. (17.25) gives

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = -\vec{H} \cdot \frac{\partial \vec{B}}{dt} - \vec{E} \cdot \vec{J} - E \cdot \frac{\partial \vec{D}}{dt}$$