Exercise 5.7

Find the general solution of the following differential equations.

find the general solution of the following differential equations.
1.
$$(D^2 + 5D + 4)y = 18e^{2x}$$
.
2. $(D^2 - 1)y = 8e^{3x}$.

3.
$$(D^2 + 3D + 4)y = 18e^{2x}$$
.
 $(D^2 - 3D - 4)y = e^x + 6e^{5x}$.

5.
$$(D^2 - 3D - 4)y = e^x + 6e^{3x}$$
.
5. $(D^2 + 3D + 3)y = 7e^x$.

$$-3D + 3)y = 7e^x$$
.
-6D + 1)y - 4e^x

7.
$$(9D^2 - 6D + 1)y = 4e^{-x}$$
.

$$D + 1)y = 4e^{-x}.$$

$$D - 6)y = e^{2x}$$

$$1)y = 6xe^x.$$

$$(9D^2 + 6D + 1)y = e^{-x/3}$$

13.
$$(9D^2 + 6D + 1)y = e^{-x/3}$$
.
15. $(D^3 + 2D^2 - 5D - 6)y = 4e^x$.

9.
$$(D^2 + D - 6)y = e^{2x}$$
.
11. $(D^2 - 1)y = 6xe^x$.

17. $(D^3 - 2D^2 - D + 2)y = e^{3x}$.

6.
$$(D^2 - 2D + 1)y = 5e^{4x} + 4e^{2x}$$
.

8.
$$(D^2 - 6D + 9)y = 14e^{3x}$$
.

10.
$$(2D^2 - 3D - 2)y = xe^{-x/2}$$
.
12. $(4D^2 + 9D + 2)y = -2x$

4. $(D^2 + D + 2)y = e^{x/2}$.

$$0^2 + 9$$

$$^{2} + 9D + 2$$

12. $(4D^2 + 9D + 2)y = xe^{-2x}$. 14. $(2D^2 + 7D - 4)y = xe^{-4x}$.

$$D+2)y=$$

16. $(2D^3 + 3D^2 - 3D - 2)y = 10e^{2x}$.

18. $(D^3 - 6D^2 + 12D - 8)y = 18e^{2x}$.

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19.
$$(2D^3 - 3D^2 + 1)y = 16e^x$$
.

21.
$$(D^2 + 16)y = \cos 2x$$
.

23.
$$(3D^2 - 7D + 2)y = \sin x + \cos x$$
.

25.
$$(D^2 + D + 1)y = 16 \cos x$$
.

27.
$$(D^2 + 9)y = \sin 3x$$
.

29.
$$(D^2 + 2D + 5)y = e^{-x} \cos 2x$$
.

31.
$$(D^2 - 6D + 13)y = 28e^{3x} \sin 2x$$
.

33.
$$(D^3 - 3D^2 + D - 3)y = 6 \cos x$$
.

35.
$$(D^3 - 4D^2 + 9D - 10)y = 24e^x \sin 2x$$
.

37.
$$(D^4 + 5D^2 + 4)y = 16 \sin x + 64 \cos 2x$$
.

38.
$$(D^2 + 25)y = 9x^3 + 4x^2$$
.

40.
$$(D^2 - 2D - 3)y = 2x^2 + 6x$$
.

42.
$$(D^2 + D - 2)y = x^2 \sin x$$
.

44.
$$(D^2 + 7D + 12)y = e^x \sin 2x$$
.

46.
$$(D^2 + 3D + 4)y = e^x \cos(\sqrt{7}x/2)$$
.

48.
$$(D^2 + 9)y = xe^{2x}\cos x$$
.

50.
$$(D^4 + 3D^2 + 2)y = 16x^2 \cos x$$
.

20.
$$(D^3 + 3D^2 - 4D - 12)y = 12xe^{-2x}$$
.

22.
$$(2D^2 - 5D + 3)y = \sin x$$
.

24.
$$(2D^2 - 7D + 3)y = \sin 2x$$
.

26.
$$(8D^2 - 12D + 5)y = 16 \sin x$$
.

28.
$$(D^2 + 3)y = \cos \sqrt{3}x$$
.

30.
$$(D^2 - 4D + 5)y = 24e^{2x} \sin x$$
.

32.
$$(D^2 - 2D + 10)y = 16e^x \cos 3x + 24e^x \sin 3x$$
.

34.
$$(D^3 - D^2 + 9D - 9)y = 30 \cos 3x$$
.

36.
$$(4D^3 - 12D^2 + 13D - 10)y = 16e^{x/2} \cos x$$
.

39.
$$(D^2 + 6D + 9)y = 4x^2 - 1$$
.

41.
$$(D^2 - 5D + 6)y = x \cos 2x$$
.

43.
$$(D^2 - D - 6)y = xe^{-2x}$$
.

45.
$$(D^2 + 4D + 3)y = e^{2x} \cos x$$
.

47.
$$(D^2 + 3D + 2)y = x e^x \sin x$$
.

49.
$$(4D^2 + 8D + 3)y = xe^{-x/2}\cos x$$
.

Exercise 5.7

1.
$$Ae^{-x} + Be^{-4x} + e^{2x}$$
.

3.
$$Ae^{-x} + Be^{4x} + e^{5x} - (e^x)/6$$
.

2.
$$Ae^x + Be^{-x} + e^{3x}$$
.

4.
$$e^{-x/2} [A \cos (\sqrt{7}x/2) + B \sin (\sqrt{7}x/2)] + \frac{4}{11} e^{x/2}$$
.

5.
$$e^{-3x/2}[A\cos(\sqrt{3}x/2) + B\sin(\sqrt{3}x/2)] + e^x$$
.

6.
$$(A + Bx)e^x + 4e^{2x} + (5e^{4x})/9$$
.

8.
$$(A + Bx)e^{3x} + 7x^2e^{3x}$$
.

10.
$$Ae^{2x} + Be^{-x/2} - e^{-x/2} (4x + 5x^2)/50$$
.

9.
$$Ae^{2x} + Be^{-3x} + (xe^{2x})/5$$
.

7. $(A + Bx)e^{x/3} + (e^{-x})/4$.

11.
$$Ae^x + Be^{-x} + [3e^x(x^2 - x)]/2$$
.

12.
$$Ae^{-2x} + Be^{-x/4} - \frac{1}{98} (7x^2 + 8x)e^{-2x}$$
.

13.
$$(A + Bx)e^{-x/3} + (x^2e^{-x/3})/18$$
.

14.
$$Ae^{x/2} + Be^{-4x} - e^{-4x} (9x^2 + 4x)/162$$
.

15.
$$Ae^{-x} + Be^{2x} + Ce^{-3x} - (e^x)/2$$
.

17.
$$Ae^x + Be^{-x} + Ce^{2x} + (e^{3x})/8$$
.

19.
$$(A + Bx)e^x + Ce^{-x/2} + (8x^2e^x)/3$$
.

21.
$$A \cos 4x + B \sin 4x + (\cos 2x)/12$$
.

23.
$$Ae^{2x} + Be^{x/3} + (3\cos x - 4\sin x)/25$$
.

16.
$$Ae^x + Be^{-2x} + Ce^{-x/2} + (e^{2x})/2$$
.

18.
$$(A + Bx + Cx^2)e^{2x} + 3x^3e^{2x}$$
.

20.
$$Ae^{2x} + Be^{-2x} + Ce^{-3x} - 3e^{-2x} (2x^2 - 3x)/4$$
.

22.
$$Ae^x + Be^{3x/2} + (\sin x + 5\cos x)/26$$
.

24.
$$Ae^{3x} + Be^{x/2} + (14\cos 2x - 5\sin 2x)/221$$
.

25.
$$e^{-x/2}[A\cos(\sqrt{3}x/2) + B\sin(\sqrt{3}x/2)] + 16\sin x$$
.

26.
$$e^{3x/4}[A\cos(x/4) + B\sin(x/4)] + 16(4\cos x - \sin x)/51$$
.

27.
$$A \cos 3x + B \sin 3x - (x \cos 3x)/6$$
.

28.
$$A\cos(\sqrt{3}x) + B\sin(\sqrt{3}x) + (x\sin(\sqrt{3}x)/(2\sqrt{3})$$
.

29.
$$e^{-x}(A\cos 2x + B\sin 2x) + (xe^{-x}\sin 2x)/4$$
.

30.
$$e^{2x}(A\cos x + B\sin x) - 12x\cos xe^{2x}$$
.

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31.
$$e^{3x}(A\cos 2x + B\sin 2x) - 7x\cos 2x e^{3x}$$
.

32.
$$e^x[A\cos 3x + B\sin 3x + x(8\sin 3x - 12\cos 3x)/3].$$

33.
$$Ae^{3x} + B\cos x + C\sin x - 3x(\cos x + 3\sin x)/10$$
.

34.
$$Ae^x + B\cos 3x + C\sin 3x - x(3\cos 3x + \sin 3x)/2$$
.

35.
$$Ae^{2x} + e^x(B\cos 2x + C\sin 2x) - 6xe^x(2\sin 2x - \cos 2x)/5$$
.

36.
$$Ae^{2x} + e^{x/2}(B\cos x + C\sin x) - 4xe^{x/2}(2\cos x + 3\sin x)/13$$
.

37.
$$A \cos x + B \sin x + C^* \cos 2x + D^* \sin 2x - 8x(\cos x + 2 \sin 2x)/3$$
.

38.
$$A \cos 5x + B \sin 5x + (225x^3 + 100x^2 - 54x - 8)/625$$
.

39.
$$(A + Bx)e^{-3x} + (12x^2 - 16x + 5)/27$$
.

40.
$$Ae^{-x} + Be^{3x} - (18x^2 + 30x - 8)/27$$
.

41.
$$Ae^{2x} + Be^{3x} + [(52x + 25)(\cos 2x - 5\sin 2x) - 21(5\cos 2x + \sin 2x)]/2704$$
.

42.
$$Ae^x + Be^{-2x} - [(25x^2 + 5x - 9)(3 \sin x + \cos x) + (35x + 12)(3 \cos x - \sin x)]/250$$
.

43.
$$Ae^{3x} + Be^{-2x} - e^{-2x}(5x^2 + 2x)/50$$
.

44.
$$Ae^{-3x} + Be^{-4x} + e^x(8 \sin 2x - 9 \cos 2x)/290$$
.

45.
$$Ae^{-x} + Be^{-3x} + e^{2x}(7\cos x + 4\sin x)/130$$
.

46.
$$e^{-3x/2}[A\cos p + B\sin p] + 4e^{x}(25\cos p + 10\sqrt{7}\sin p)/1325$$
, $p = \sqrt{7}x/2$.

47. Write
$$xe^x \sin x = \text{Im} \left[xe^{(1+i)x} \right]$$
, $Ae^{-x} + Be^{-2x} + e^x [5(1-x)\cos x + (5x-2)\sin x]/50$.

48. Write
$$xe^{2x} \cos x = \text{Re}\left[xe^{(2+i)x}\right]$$
, $A \cos 3x + B \sin 3x + e^{2x}\left[(30x - 11)\cos x + (10x - 2)\sin x\right]/400$.

49.
$$Ae^{-x/2} + Be^{-3x/2} - e^{-x/2}[(x-2)\cos x - (x+1)\sin x]/8$$
.

50.
$$A \cos x + B \sin x + C^* \cos \sqrt{2}x + D^* \sin \sqrt{2}x - 4[9x^2 \cos x - (2x^3 - 51x) \sin x]/3$$
.