Numericals on Laser

Find the intensity of a laser beam of 10 mW power and having a diameter of 1.3 m. Assume the intensity to be uniform across the beam.

Solution. Power $P = 10 \text{ mW} = 10 \times 10^{-3}$ watt

Diameter $d = 1.3 \text{ mm} = 1.3 \times 10^{-3}$ m

Intensity $= \dfrac{\text{Power}}{\text{Area}} = \dfrac{4P}{\pi d^2}$

$$I = \frac{4 \times 10 \times 10^{-3} \text{ watt}}{3.14 \times (1.3 \times 10^{-3} \text{ m})^2 \text{ m}^2} = 7537 \text{ W/m}^2 = \mathbf{7.5 \text{ kW/m}^2}.$$

A laser beam can be focused on an area equal to the square of its wavelength λ^2. For a He-Ne laser $\lambda = 6328$ Å. If the laser radiates energy at the rate of 1 mW, find out the intensity of focused beam.

Solution. $\lambda = 6328 \text{ Å} = 6328 \times 10^{-10}$ m

$P = 1 \text{ mW} = 10^{-3}$ W

$A = \lambda^2 = (6328 \times 10^{-10})^2 \text{ m}^2 = 40 \times 10^{-14} \text{ m}^2$

Intensity $I = \dfrac{P}{A} = \dfrac{10^{-3} \text{ Watt}}{40 \times 10^{-14} \text{ m}^2} = \mathbf{2.5 \times 10^9 \text{ W/m}^2}.$

A certain ruby laser emits 1.00 J pulses of light whose wavelength is 6940 Å. What is the minimum number of Cr^{3+} ions in the ruby ?

Solution. Power $= \dfrac{nhc}{\lambda}$

$$1.00 \text{ J} = \frac{n \times 6.62 \times 10^{-34} \text{ J-sec} \times 3 \times 10^8 \text{ m/sec}}{6940 \times 10^{-10} \text{ m}}$$

$$n = \frac{1.00 \text{ J} \times 6940 \times 10^{-10} \text{ m}}{6.62 \times 10^{-34} \text{ J-sec} \times 3 \times 10^8 \text{ m/sec}} = \mathbf{3.49 \times 10^{18} \text{ ions.}}$$

A laser beam has a power of 50 mW. It has an aperture of 5×10^{-3} m and wavelength 7000 Å. A beam is focused with a lens of focal length 0.2 m. Calculate the areal spread and intensity of the image.

Solution. Given $\lambda = 7000$ Å $= 7000 \times 10^{-10}$ m, $d = 5 \times 10^{-3}$ m,

$f = 0.2$ m

$$\therefore \text{Angular spread } d\theta = \frac{1.22\,\lambda}{d} = \frac{1.22 \times 7 \times 10^{-7}}{5 \times 10^{-3}} = 1.708 \times 10^{-4} \text{ radian}$$

$$\text{Areal spread} = (d\theta \times f)^2 = (1.708 \times 10^{-4} \times 0.2)^2 = \mathbf{0.584 \times 10^{-8}\ m^2}$$

$$\text{As intensity} = \frac{\text{Power}}{\text{Area}} = \frac{50 \times 10^{-3}\ \text{watt}}{0.4 \times 10^{-8}\ \text{m}^2} = \mathbf{125 \times 10^5\ watt/m^2.}$$

Calculate the coherence length for CO_2 laser whose line width is 1×10^{-5} nm at IR emission wavelength of 10.6 μm.

Solution. Coherence length $= \dfrac{\lambda^2}{\Delta\lambda} = \dfrac{(10.6 \times 10^{-6})^2\ \text{m}^2}{10^{-5} \times 10^{-9}\ \text{m}} = \mathbf{11.2\ km.}$

Numericals on Fiber Optics

An optical fibre has a NA of 0.20 and a cladding refractive index of 1.59. Determine the acceptance angle for the fibre in water which has a refractive index of 1.33.

Solution. Numerical aperture

$$(\text{NA}) = \sqrt{n_1^2 - n_2^2} \quad \text{when } n_0 = 1 \text{ (air)}$$

$$0.20 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad n_1 = \sqrt{(0.20)^2 + (1.59)^2} = \mathbf{1.6025}$$

In water $\quad n_0 = 1.33$

$$\text{NA} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{\sqrt{(1.6025)^2 - (1.59)^2}}{1.33} = \mathbf{0.15}$$

∴ Acceptance angle

$$\theta_m = \sin^{-1}(\text{NA}) = \sin^{-1}(0.15) = \mathbf{8.6°}$$

An optic fibre is made of glass with a refractive index of 1.55 and is clad with another glass with a refractive index of 1.51. Launching takes place from air :

(a) What numerical aperture does the fibre have ?

(b) What is the acceptance angle ?

Solution. (*a*) The normalized difference between the indices is

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.55 - 1.51}{1.55} = 0.0258$$

The numerical aperture is

$$\text{NA} \approx n_1\sqrt{2\Delta} = 1.55\sqrt{2 \times 0.0258} = \mathbf{0.352}$$

(*b*) The acceptance angle is

$$\theta_m = \sin^{-1}(\text{NA}) = \sin^{-1}(0.352) = \mathbf{20.6}$$

An optics fibre is made of glass with refractive index 1.55 and is clad with another glass with a refractive index 1.51. The fibre has a core diameter of 50 μm and is used at a light wavelength of 0.8 μm.

(i) What is the numerical aperture does the fibre have ?
(ii) What is the acceptance angle ?
(iii) Find V-number for the fibre and
(iv) Approximate number of modes it will support.

Solution. *(i)* Numerical aperture (NA) is given by

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.55)^2 - (1.51)^2}$$

$$= \sqrt{2.4025 - 2.2801}$$

$$= \sqrt{0.1224} = 0.3498$$

(ii) Acceptance angle

$$\theta_m = \sin^{-1}(NA) = \sin^{-1}(0.3498)$$

$$\theta_m = 20.47°$$

(iii) *V*-number is given as

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda}(NA)$$

Here $a = \frac{50}{2} = 25\ \mu m,\ \lambda = 0.8\ \mu m$

$$V = \frac{2 \times 3.14 \times 25\ \mu m}{0.8\ \mu m} \times 0.3498$$

$$= \frac{54.9186}{0.8} = 68.64$$

(iv) As $V > 2.405$

∴ The number of modes is given as

$$N \simeq \frac{V^2}{4} = \frac{(68.64)^2}{4} = 1178.14$$

The core diameter of multimode step index fibre is 60 μm. The difference in refractive indices is 0.013. The core refractive index is 1.46. Determine the number of guided modes when the operating wavelength is 0.75 μm.

Solution. As V-number is given by

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$= \frac{2 \times 3.14 \times 30\,\mu m}{0.75\,\mu m}\sqrt{(1.46)^2 - (1.447)^2}$$

$$= \frac{188.4 \times 0.0378}{0.75} = 9.493$$

Number of guided modes :

$$N \simeq \frac{V^2}{2} = \frac{90.118}{2} = \mathbf{45.06.}$$

Determine the normalized frequency for a step-index fibre having a 25 μm core radius, $n_1 = 1.48$ and $n_2 = 1.46$. How many modes propagate in this fibre at 0.82 μm ?

Solution. As $V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$

Given : $a = 25\ \mu m$, $\lambda = 0.82\ \mu m$

$n_1 = 1.48$ and $n_2 = 1.46$

$$V = \frac{2 \times 3.14 \times 25\,\mu m}{0.82\,\mu m}\sqrt{(1.48)^2 - (1.46)^2}$$

$$= 46.45$$

The number of modes that propagate through the fibre are

$$N = \frac{V^2}{2}$$

$$\therefore \quad N = \frac{(46.45)^2}{2} = \mathbf{1079}$$
