MTH174:ENGINEERING MATHEMATICS

#Zero Lecture

LTP and Credit Details

Program Name: Bachelor of Technology **Program Batch:** 2022

This Bachelor Degree program has a minimum duration of 4 years and is offered under Semester system through Regular mode.

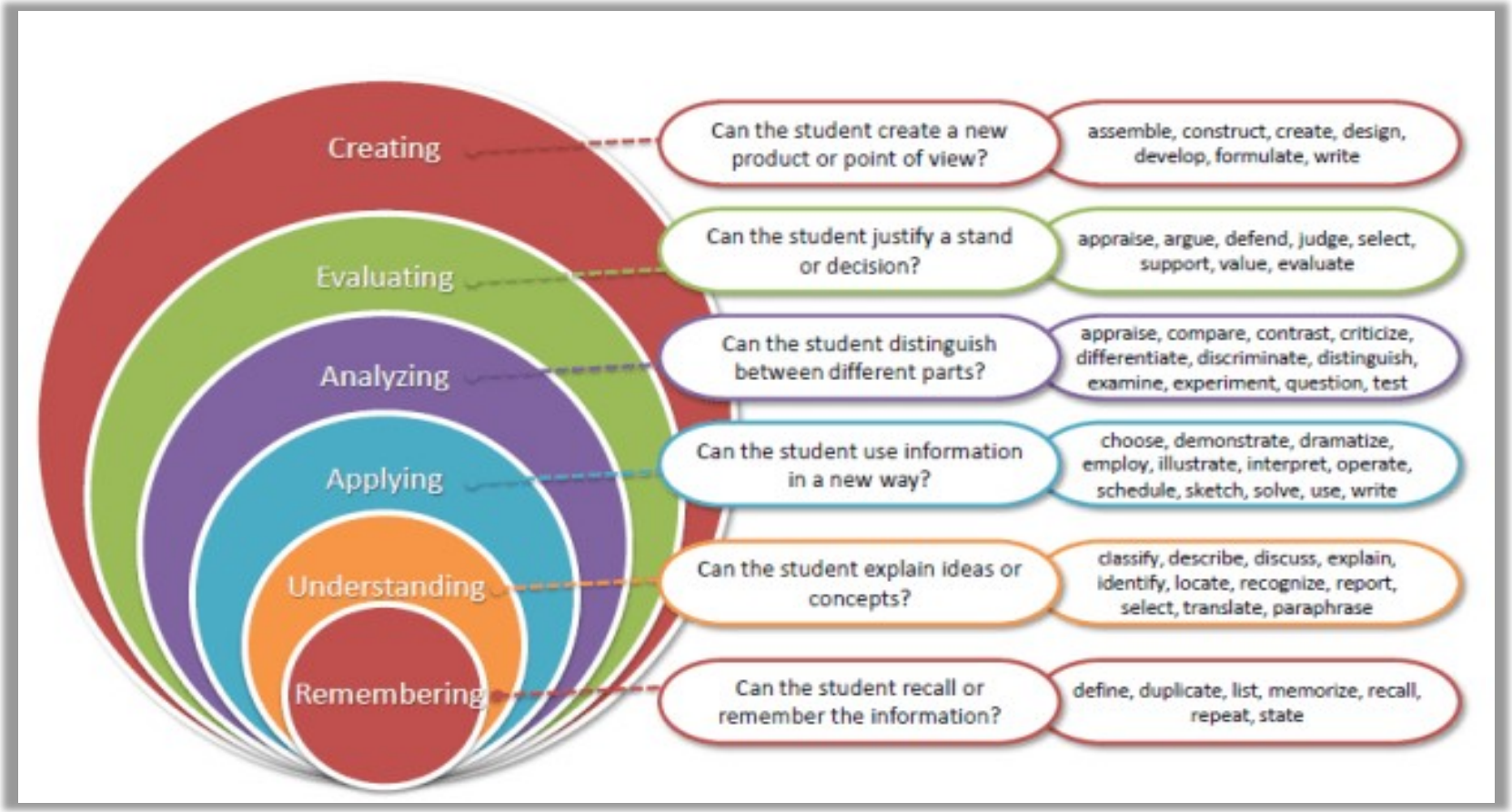
LTP and Credit Details of MTH174

Teaching Model:

L-T-P: 3-1-0 (3 Lectures, 1 Tutorial, 0 Practical)

Credit: 4

Revised Bloom's Taxonomy



Course Outcomes

Through this course students should be able to

CO1 :: recall the concepts of matrices and its application to solve the system of linear equations.

CO2 :: understand the use of different methods for the solution of linear differential equations.

CO3 :: use the concept of Fourier series for learning advanced Engineering Mathematics.

CO4 :: apply the concept of multi-variable differential calculus for solving the problems in the field of sciences and engineering.

CO5 :: analyze the surface and volume integral using various concepts of multi-variable integral calculus.

Program Outcomes

PO1 Engineering knowledge:

- Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2 Problem analysis:

- Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO12 Life-long learning

- Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course code:MTH174 **Program Name:** Bachelor of Technology **Program Batch:** 2022

1=Low :: 2=Moderate :: 3=High

Outcomes	**PO1:** Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	**PO2 :** Problem analysis::Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using basic principles of engineering and sciences.	**PO12:** Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.
CO1 :: recall the concepts of matrices and its application to solve the system of linear equations.	3	2	2
CO2 :: understand the use of different methods for the solution of linear differential equations.	3	2	2
CO3 :: use the concept of Fourier series for learning advanced Engineering Mathematics.	3	1	2
CO4 :: apply the concept of multi-variable differential calculus for solving the problems in the field of sciences and engineering.	3	2	2
CO5 :: analyze the surface and volume integral using various concepts of multi-variable integral calculus.	3	2	2
.			

Program Educational Objectives

- **Objective 1** Apply fundamentals of technical knowledge in multidisciplinary areas related to Aerospace, aeronautics, mechanical and computer systems to participate as top professionals in leading Industries.
- **Objective 2** Be sensitive to professional and ethical responsibilities, including the societal impact of engineering solutions as successful innovators, consultants and entrepreneurs.
- **Objective 3** Pursue advanced education, research and development in science, engineering, and technology, as well as other professional endeavors.

Course Content

Unit-1 :Matrix Algebra

- Elementary operations and their use in getting the rank,
- Inverse of a matrix and solution of linear simultaneous equations,
- Orthogonal, symmetric, skew-symmetric, hermitian, skew Hermitian,
- Normal & unitary matrices and their elementary properties,
- Eigen-values and eigenvectors of a matrix,
- Cayley-Hamilton theorem

Course Content

Unit-2: Linear differential equation-I

- Introduction to linear differential equation, solution of linear differential equation,
- Linear dependence and linear independence of solution,
- Method of solution of linear differential equation- differential operator
- Solution of second order homogeneous linear differential equation with constant coefficient,
- Solution of higher order homogeneous linear differential equations with constant coefficient

Course Content

Unit-3: Linear differential equation-II

- Solution of non-homogeneous linear differential equations with constant coefficients using operator method,
- Method of variation of parameters, method of undetermined coefficient,
- Solution of Euler-Cauchy equation,
- Simultaneous differential equations by operator method

Course Content

Unit-4:Fourier Series

- Euler Coefficients
- Fourier Series
- Fourier Series for Even and Odd functions
- Half range Fourier Series

Course Content

Unit-5: Multivariate Functions

- Functions of two variables
- Limits and Continuity
- Partial derivatives
- Total derivative and differentiability
- Chain rule
- Euler's theorem for Homogeneous functions
- Jacobians
- Maxima and Minima
- Lagrange method of multiplier

Course Content

Unit-6-Multivariate Integrals

- Double integrals
- Change of order of integration
- Triple integrals
- Change of variables
- Application of double integrals to calculate area and volume
- Application of triple integrals to calculate volume

What Do You Think?

What could be considered the greatest achievements of the human mind ?

It's the Greatest!

- Consider that all these things emerged because of technological advances
- Those advances relied on ALGEBRA and CALCULUS !
- ALGEBRA and CALCULUS has made it possible to:
  - Build giant bridges
  - Travel to the moon
  - Predict patterns of population change

Matrices Are The Key Elements Of Algebra:

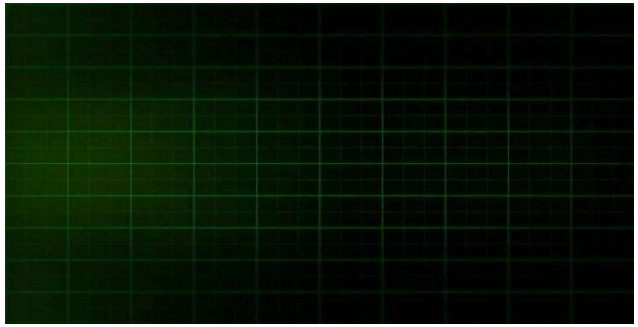
Use in Cryptography

Use in Geology

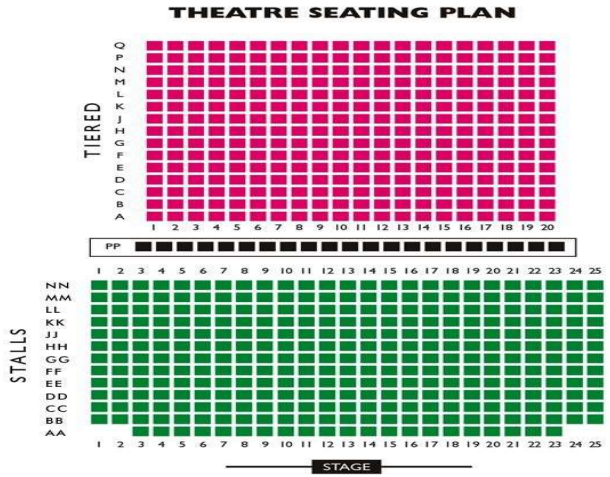
Use in Robotics

You might have observed in use of matrices in routine:

Grid of Computer Screen



Online Booking of Cinema Hall



You might have observed in use of matrices in routine:

Republic Day Parade

Matrix Movie

Uses of Matrices in Various Fields:

Encryption

Games especially 3D

Economics and business

Construction

Dance – contra dance

Animation

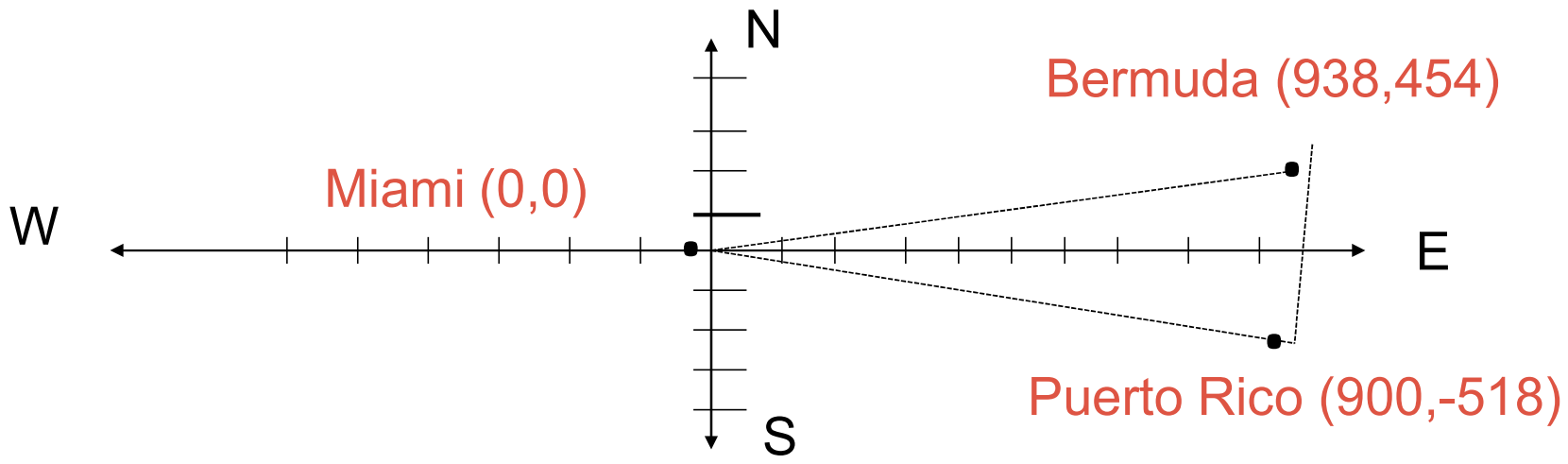
Physics

Geology

Some Practical Applications

Bermuda Triangle Mystery

The Bermuda Triangle is a large trianglular region in the Atlantic ocean. Many ships and airplanes have been lost in this region. The triangle is formed by imaginary lines connecting Bermuda, Puerto Rico, and Miami, Florida. Use a determinant to estimate the area of the Bermuda Triangle.



SOLUTION

The approximate coordinates of the Bermuda Triangle's three vertices are: (938,454), (900,-518), and (0,0). So the area of the region is as follows:

$$Area = \pm \frac{1}{2} \begin{vmatrix} 938 & 454 & 1 \\ 900 & -518 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$Area = \pm \frac{1}{2}[(-458,884 + 0 + 0) - (0 + 0 + 408,600)]$$

$$Area = 447,242$$

Hence, area of the Bermuda Triangle is about 447,000 square miles.

Cryptography

- Cryptography is concerned with keeping communications private.
- Today governments use sophisticated methods of coding and decoding messages. One type of code, which is extremely difficult to break, makes use of a large matrix to encode a message.
- The receiver of the message decodes it using the inverse of the matrix. This first matrix is called the **encoding matrix** and its inverse is called the **decoding matrix.**

Steps to create a cryptogram

Assign a number to each letter in the alphabet with out a blank space

A = 1	E = 5	I = 9	M = 13	Q = 17
B = 2	F = 6	J = 10	N = 14	R = 18
C = 3	G = 7	K = 11	O = 15	S = 19
D = 4	H = 8	L = 12	P = 16	T = 20
Space = 27				

Steps to create a cryptogram

- To encode “CLEAR NOW”, break the message into groups of 2 letters & spaces each.

  CL EA R_ NO W_

- Convert the block of 2-letter into a 2 x 1 matrix each

$$\begin{pmatrix} 3 \\ 12 \end{pmatrix} \quad \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 18 \\ 27 \end{pmatrix} \quad \begin{pmatrix} 14 \\ 15 \end{pmatrix} \quad \begin{pmatrix} 23 \\ 27 \end{pmatrix}$$

Steps to ENCODE MESSAGES

To encode a message, choose a 2x2 matrix A that has an inverse and multiply A on the left to each of the matrices.

If $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, the product of A and the

matrices give

$$\begin{pmatrix} 6 \\ 15 \end{pmatrix} \quad \begin{pmatrix} 10 \\ 6 \end{pmatrix} \quad \begin{pmatrix} 36 \\ 45 \end{pmatrix} \quad \begin{pmatrix} 28 \\ 29 \end{pmatrix} \quad \begin{pmatrix} 46 \\ 50 \end{pmatrix}$$

The message received will appear as

6 15 10 6 36 45 28 29 46 50

ENCODING using Matrices

If you don't know the matrix used, decoding would be very difficult. When a larger matrix is used, decoding is even more difficult. But for an authorized receiver who knows the matrix A, decoding is simple. For example,

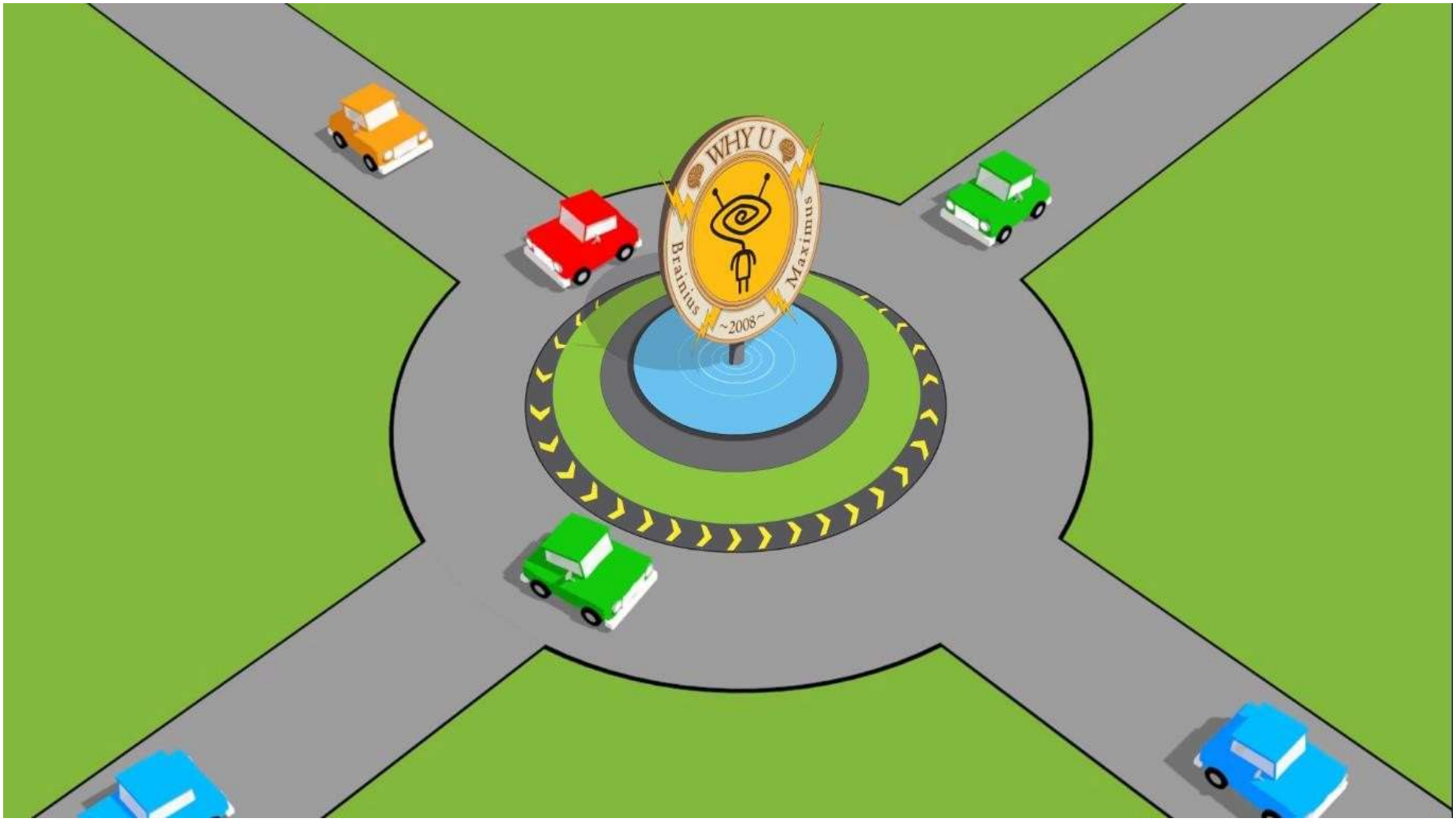
$$A^{-1} = \frac{1}{2-0}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 15 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \end{pmatrix}$$

The receiver only needs to multiply the matrices by A^{-1} on the left to obtain the sequence of numbers.

The message will be retrieved with reference to the table of letters.

Network Traffic Flow Problems



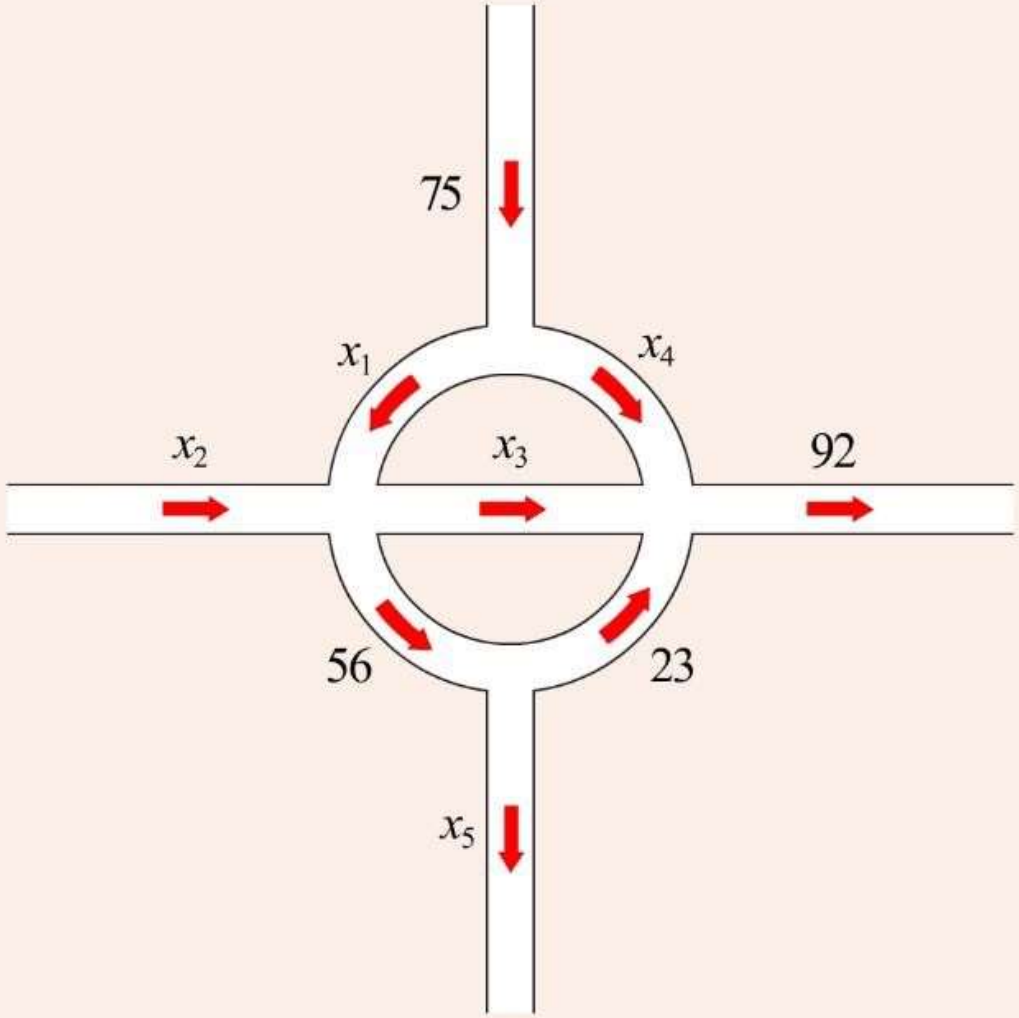
$$x_2 + 75 = x_5 + 92$$

$$x_1 + x_2 = x_3 + 56$$

$$75 = x_1 + x_4$$

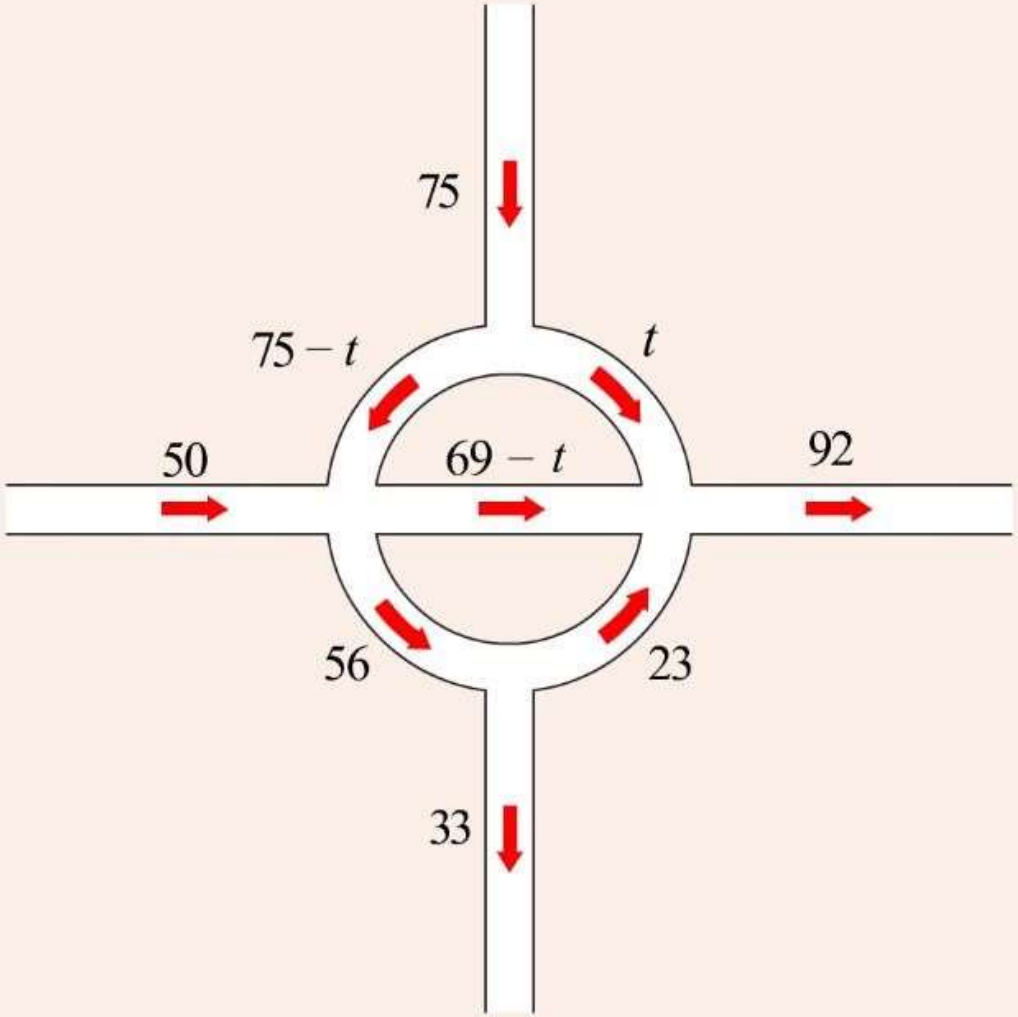
$$x_3 + x_4 + 23 = 92$$

$$56 = x_5 + 23$$



$$
\begin{aligned}
x_2 - x_5 &= 17 \\
x_1 + x_2 - x_3 &= 56 \\
x_1 + x_4 &= 75 \\
x_3 + x_4 &= 69 \\
x_5 &= 33
\end{aligned}
$$

$$
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\
\left[\begin{array}{ccccc|c}
0 & 1 & 0 & 0 & -1 & 17 \\
1 & 1 & -1 & 0 & 0 & 56 \\
1 & 0 & 0 & 1 & 0 & 75 \\
0 & 0 & 1 & 1 & 0 & 69 \\
0 & 0 & 0 & 0 & 1 & 33
\end{array}\right]
\end{array}
$$



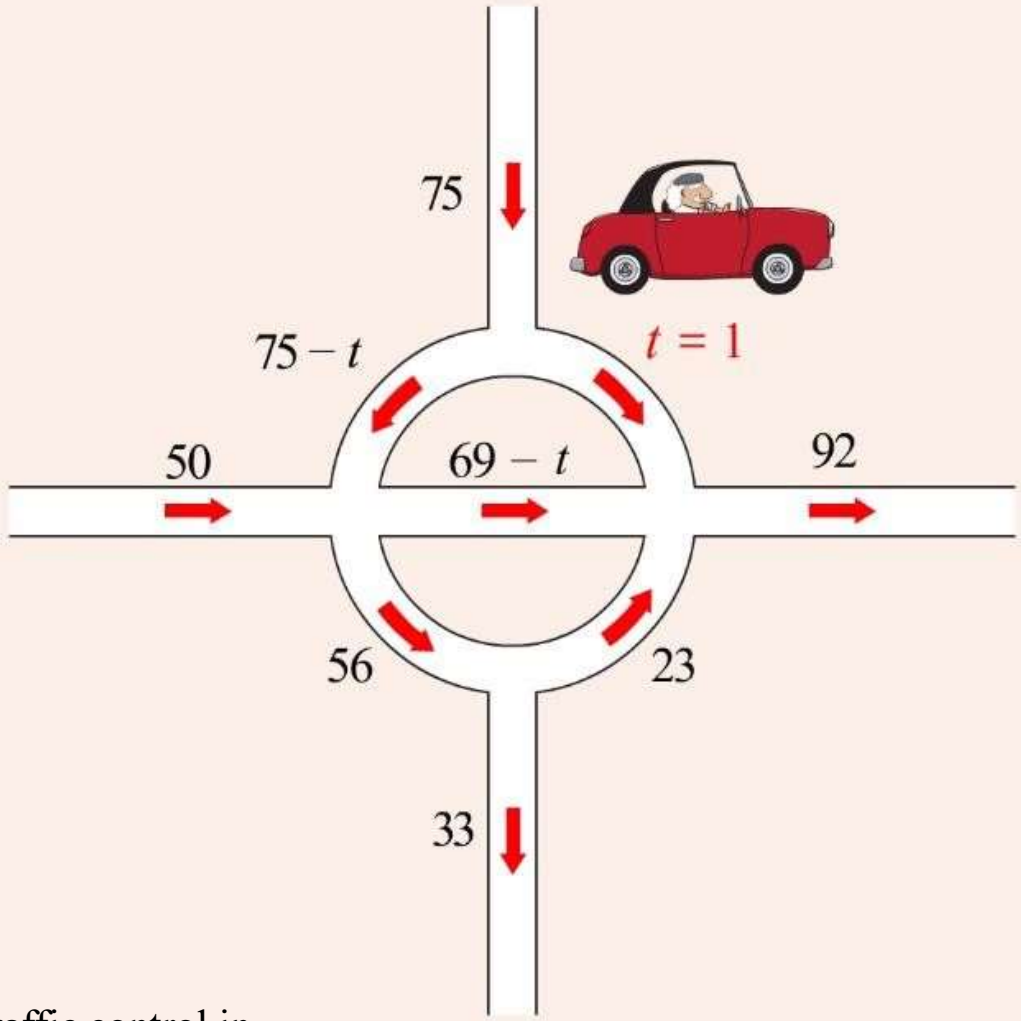
$$x_1 = 75 - t$$

$$x_2 = 50$$

$$x_3 = 69 - t$$

$$x_4 = t$$

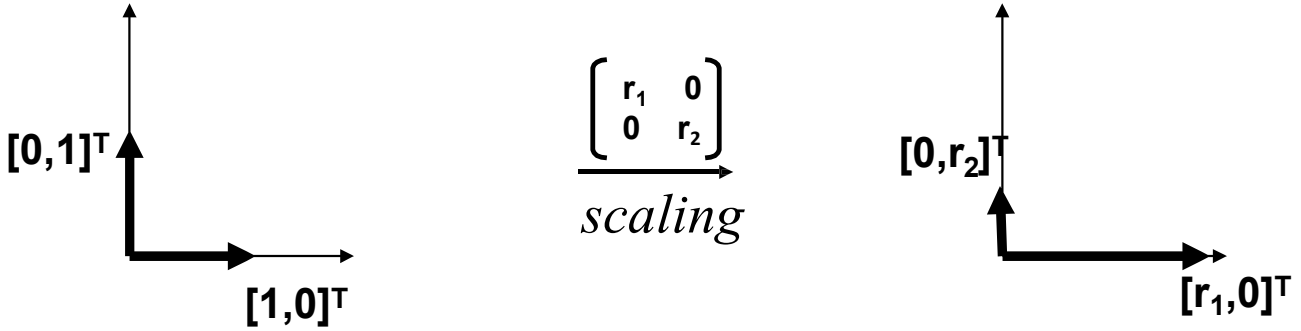
$$x_5 = 33$$

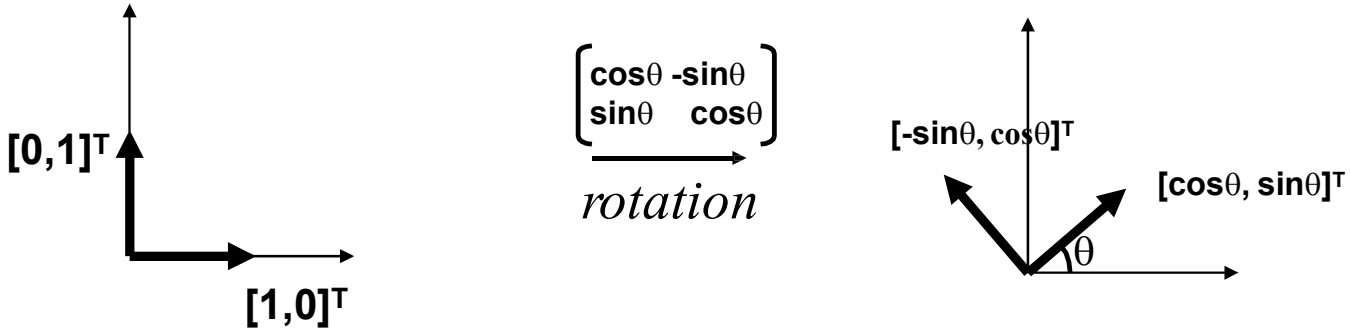


Similar way we, can control the traffic control in networking problems in computer.

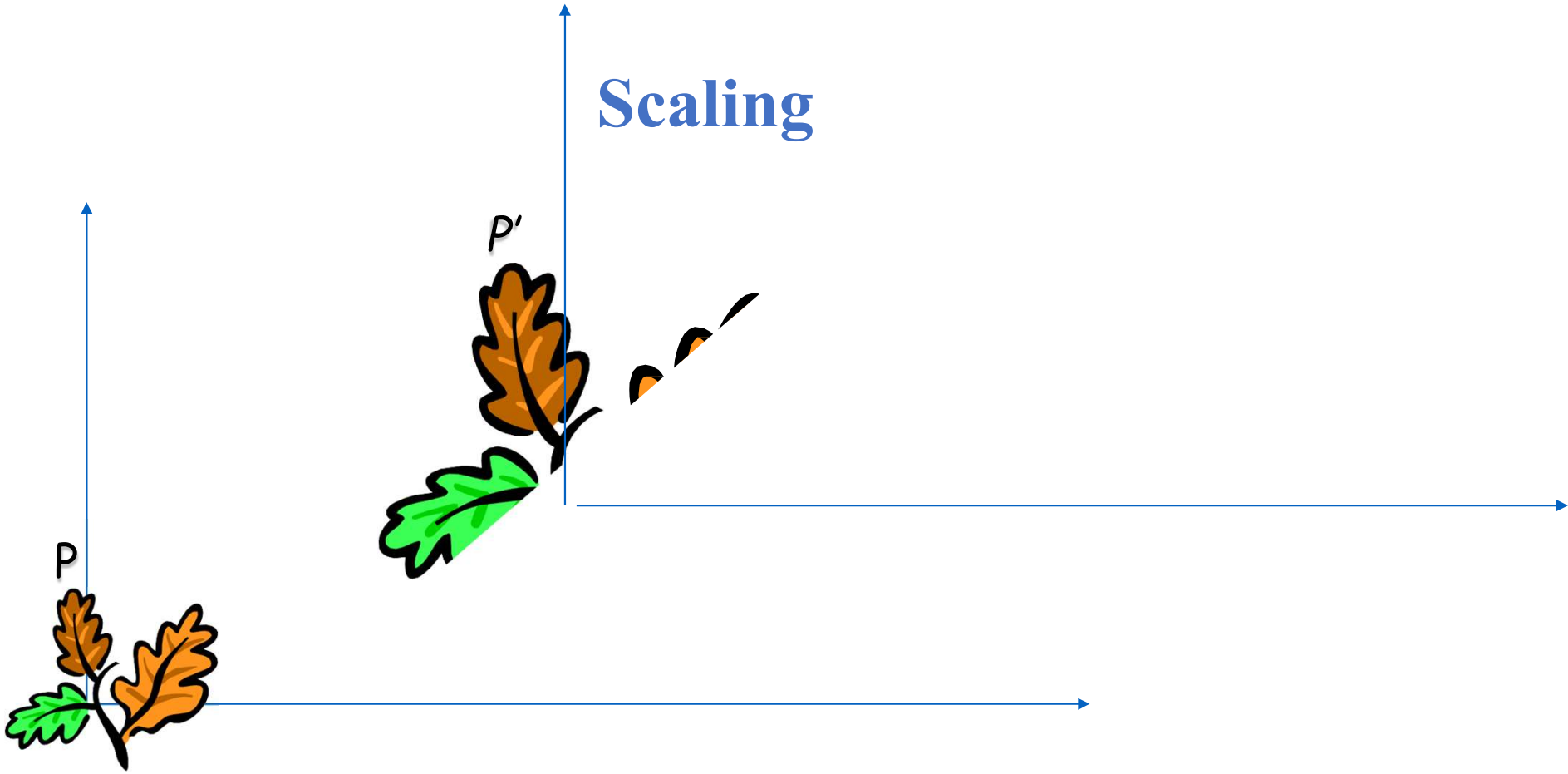
Computer graphics

- Pure scaling, no rotation => "**diagonal** matrix" (note: x-, y-axes could be scaled differently!)
- Pure rotation, no stretching => "**orthogonal** matrix" **O**
- **Identity** ("do nothing") matrix = unit scaling, no rotation!





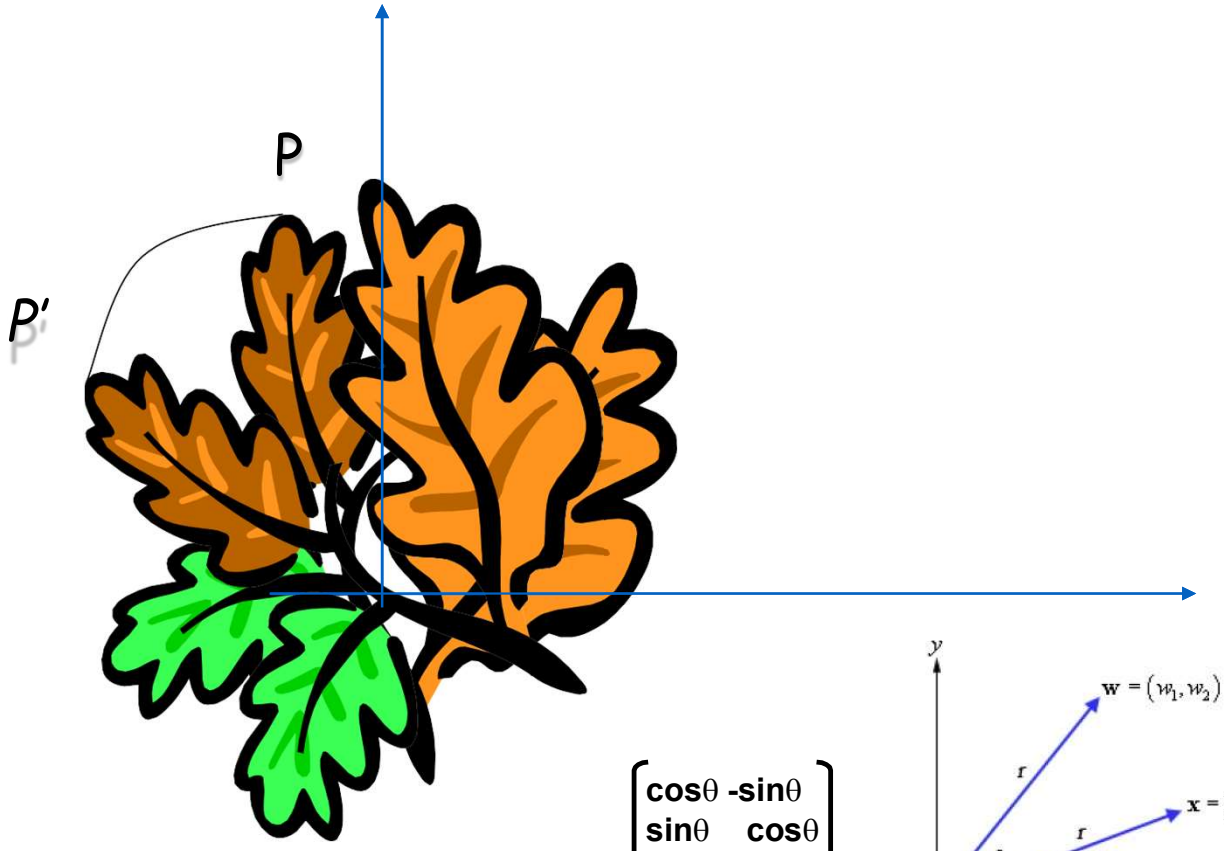
Scaling



$$\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$$

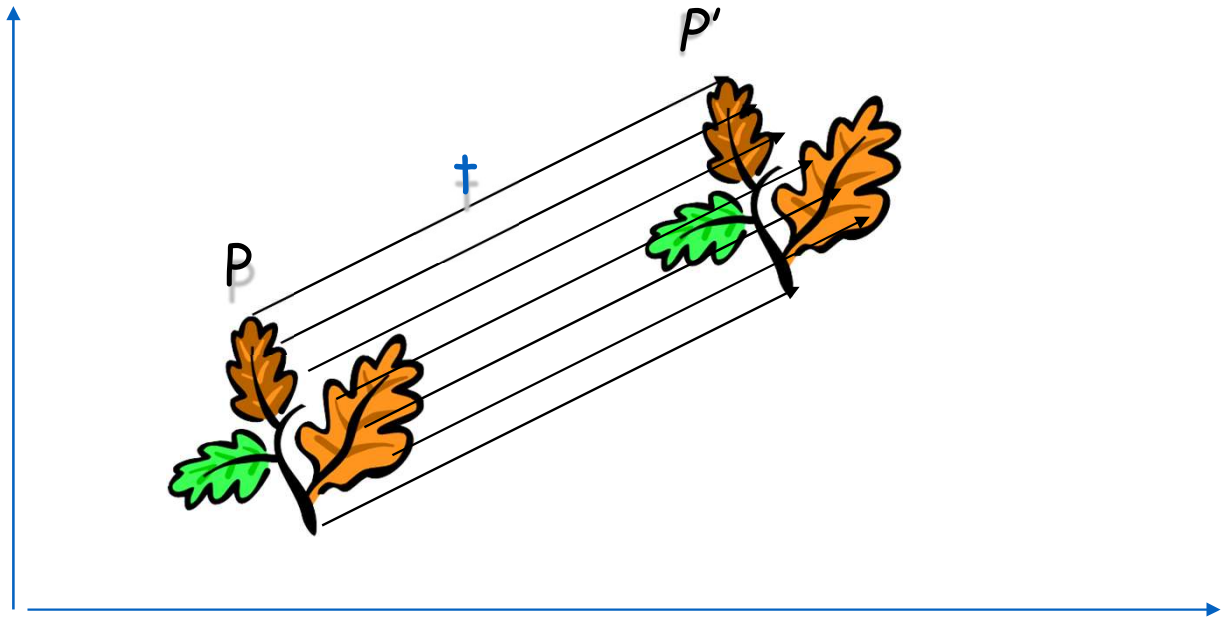
a.k.a: dilation (r >1),
contraction (r <1)

Rotation

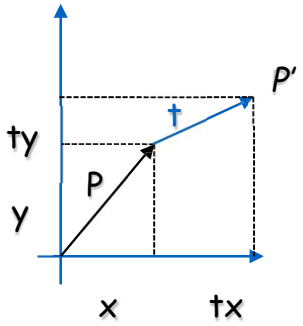


$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2D Translation



$$\mathbf{P'} = (x + t_x, y + t_y) = \mathbf{P} + \mathbf{t}$$



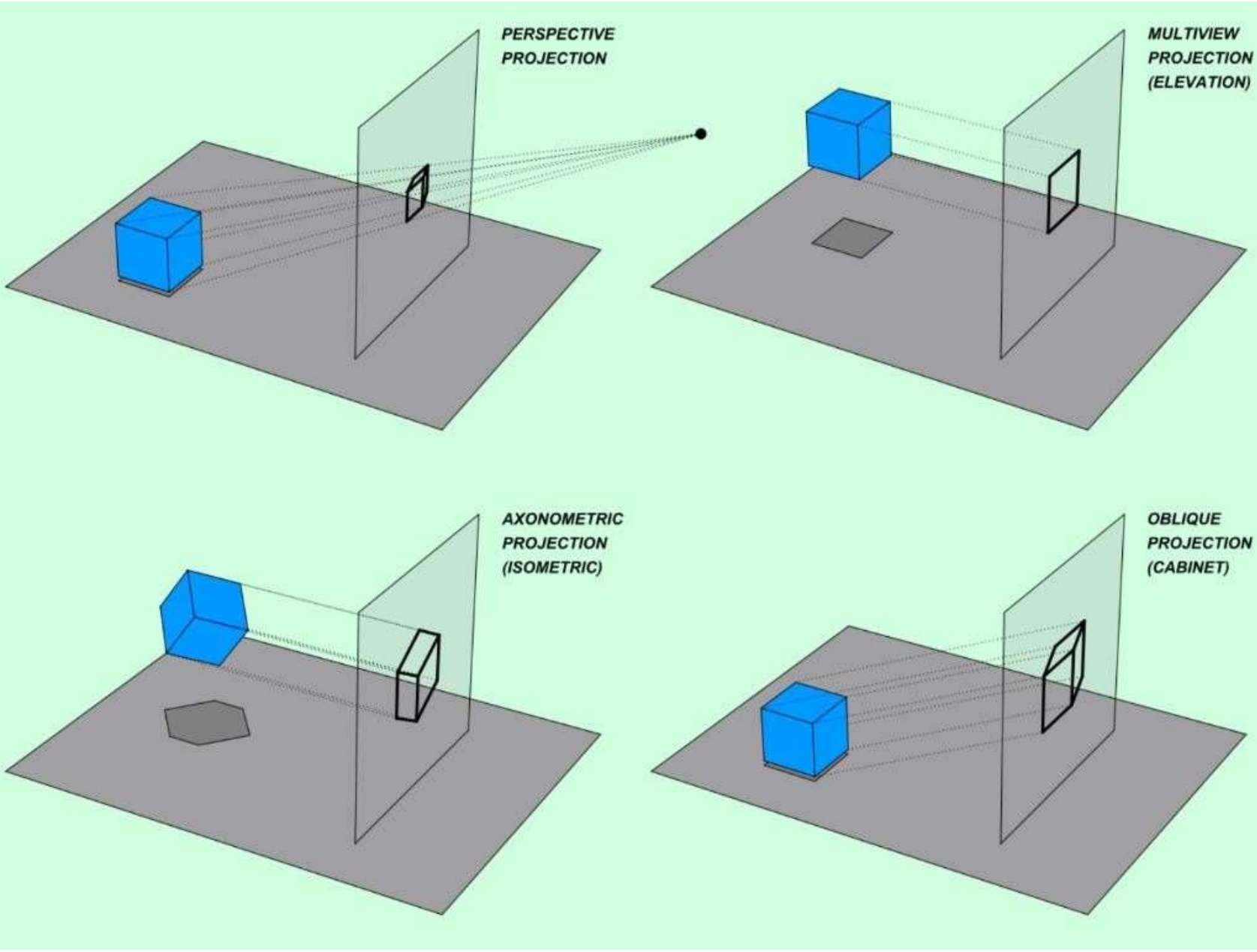

PERSPECTIVE
PROJECTION
MULTIVIEW
PROJECTION
(ELEVATION)
AXONOMETRIC
PROJECTION
(ISOMETRIC)
OBLIQUE
PROJECTION
(CABINET)

Image Processing

Before

$$*\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} =$$

Box Blur

After

Before

$$*\begin{bmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{bmatrix} =$$

Gaussian Blur

After

Before

$$*\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} =$$

Sharpen Kernel

After

Before

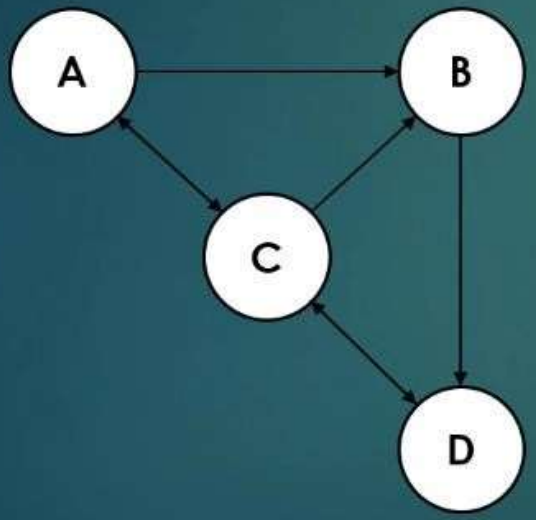
$$*\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} =$$

Edge Detection

After

Google page ranking

We can use matrix operations instead of the iterative approach
~ we updated values one by one: we can use matrix operations to do multiple calculations at the same time



$$\begin{bmatrix} 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{3} & 0 \end{bmatrix}$$

$$PR_{t+1} = \underline{\underline{H}}\ PR_t$$

„power method"

Matrix representation

We can come to the conclusion → we have to multiply the matrix with a vector on every iteration

What is the initial vector? It is the initial page rank assigned to every page

$$\underline{v} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$$

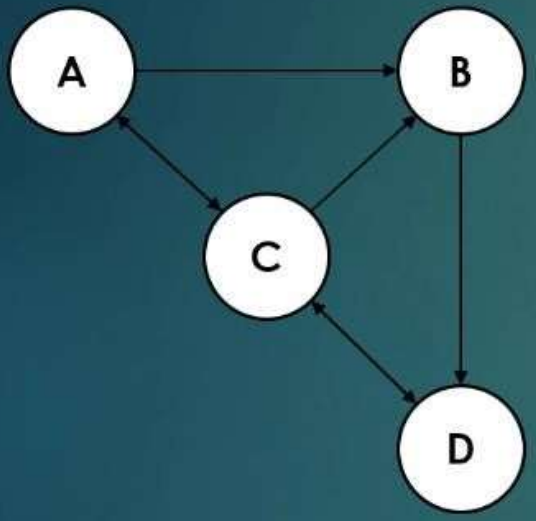
$$\underline{v}_2 = \underline{\underline{H}}\,\underline{v}$$

$$\underline{v}_3 = \underline{\underline{H}}\,\underline{v}_2 = \underline{\underline{H}}\,(\underline{\underline{H}}\,\underline{v}) = \underline{\underline{H}}^2\,\underline{v}$$

$$\underline{v}_n = \underline{\underline{H}}^n\,\underline{v}$$

If we make several iterations, again, it tends to the equilibrium value

PageRank algorithm



	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	2/12	1
B	1/4	2.5/12	15/12	4
C	1/4	6/12	4.5/12	2
D	1/4	4/12	13.5/12	3

Eigenvectors and Eigenvalues

Definition-A non –zero vector x is said to be **Eigen vector** of square matrix A of order n if there exist some scaler λ such that

$$Ax = \lambda x$$

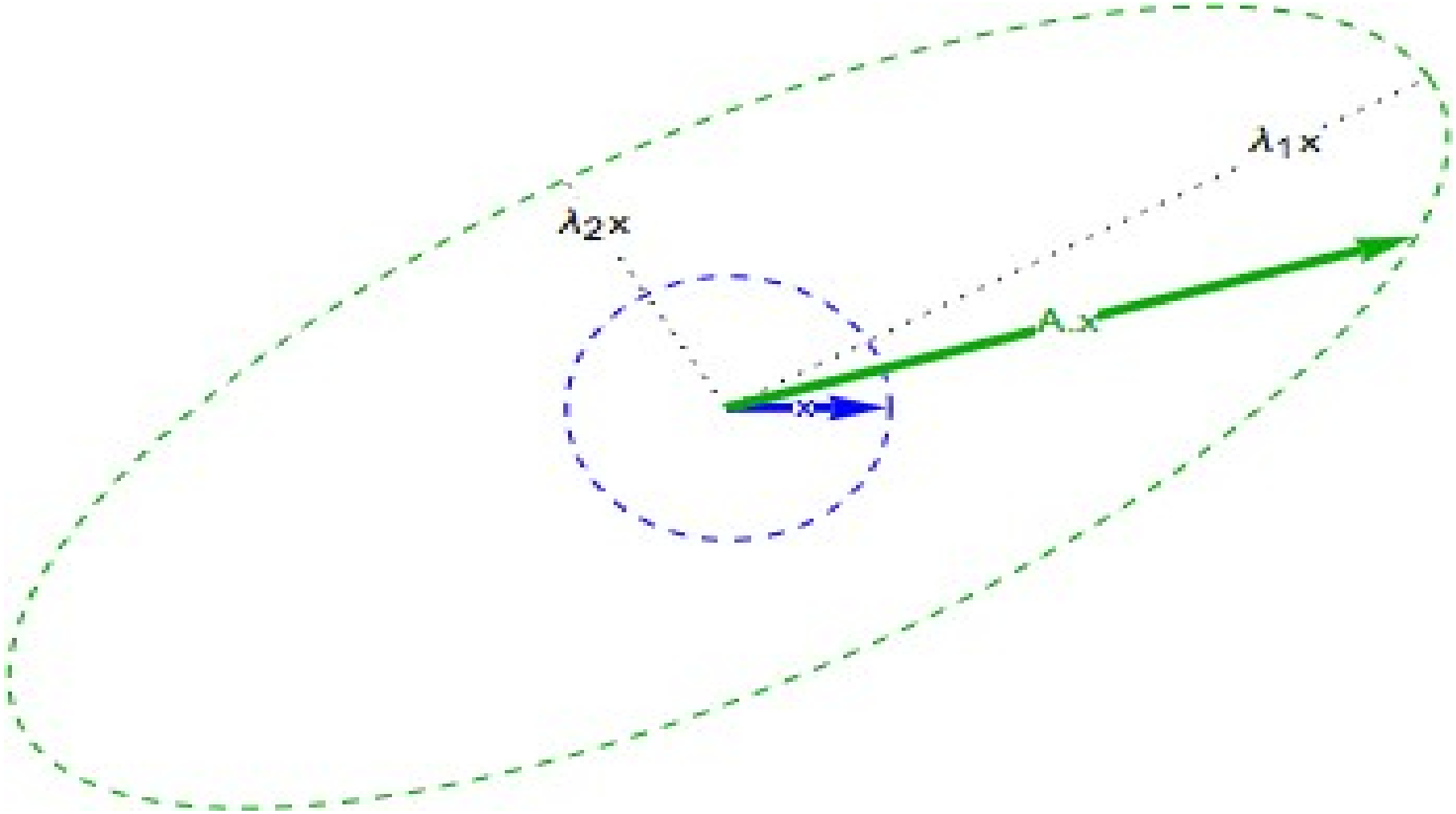
and this scalar λ is called an **Eigenvalue** of A

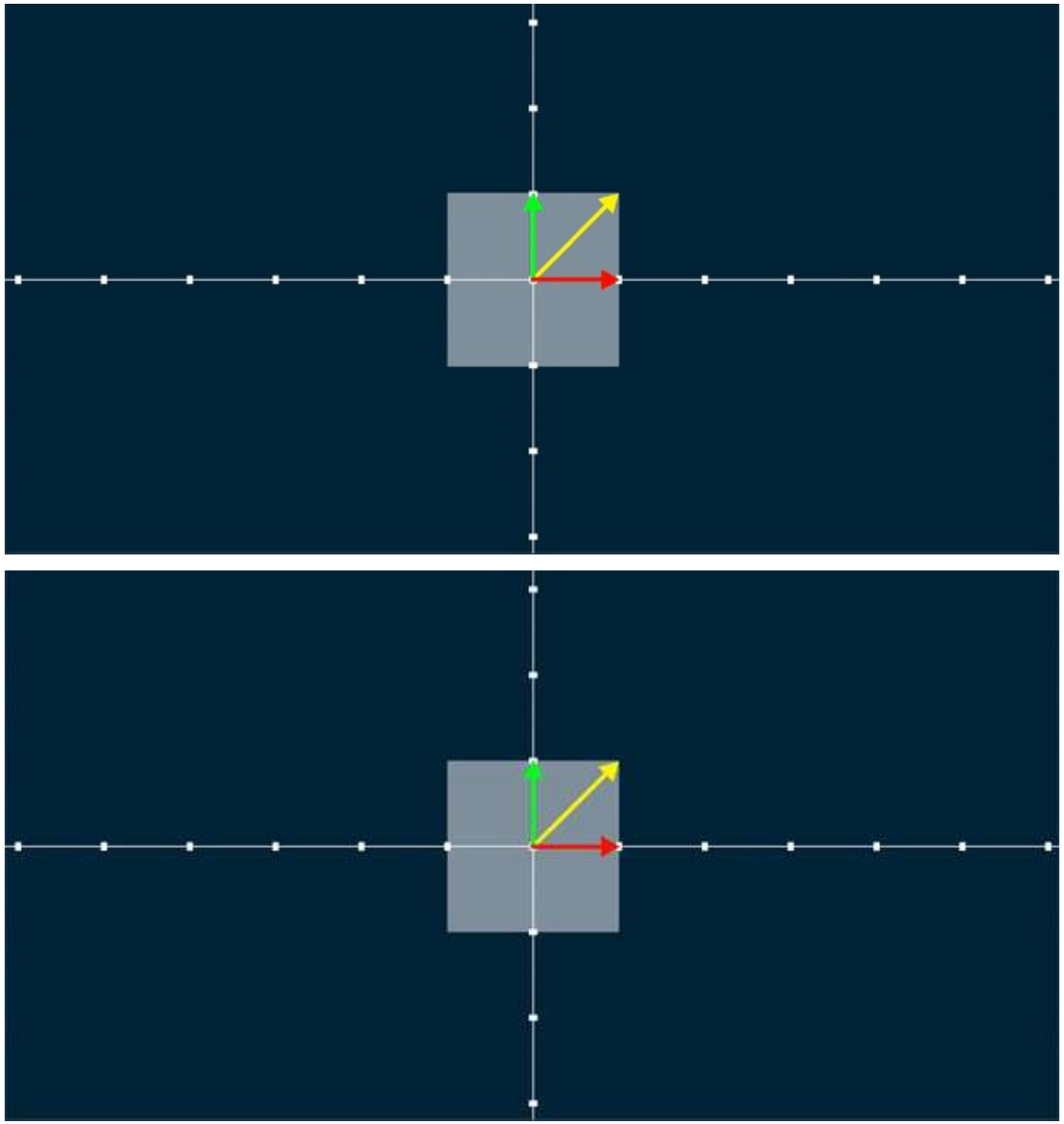
$$Ax = \lambda x \Rightarrow Ax - \lambda x = 0 \Rightarrow (A - \lambda I)x = 0$$

$$(A - \lambda I)x = 0 \qquad (1)$$

It is a homogeneous system of equations and it will have a non-zero solution iff

$$|A - \lambda I| = 0 \qquad (2)$$


A_1x
A_2x
Ax
x



$$A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix},\ X_1=\begin{bmatrix}1\\ 0\end{bmatrix},\ X_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}=1\begin{bmatrix}1\\ 0\end{bmatrix}$$

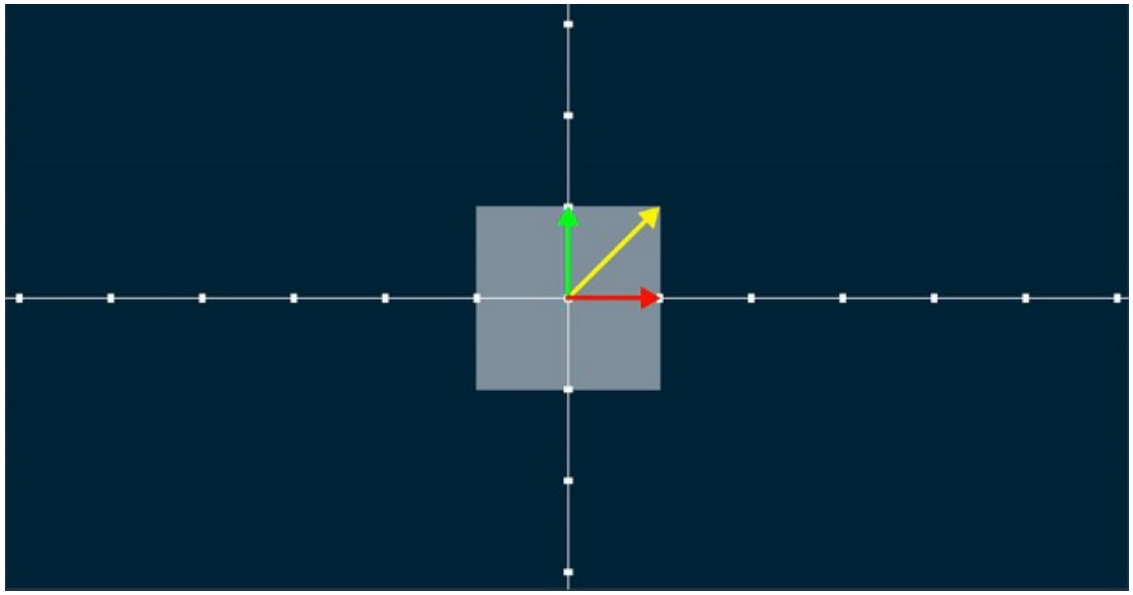
$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0\\ 2\end{bmatrix}=2\begin{bmatrix}0\\ 1\end{bmatrix}$$

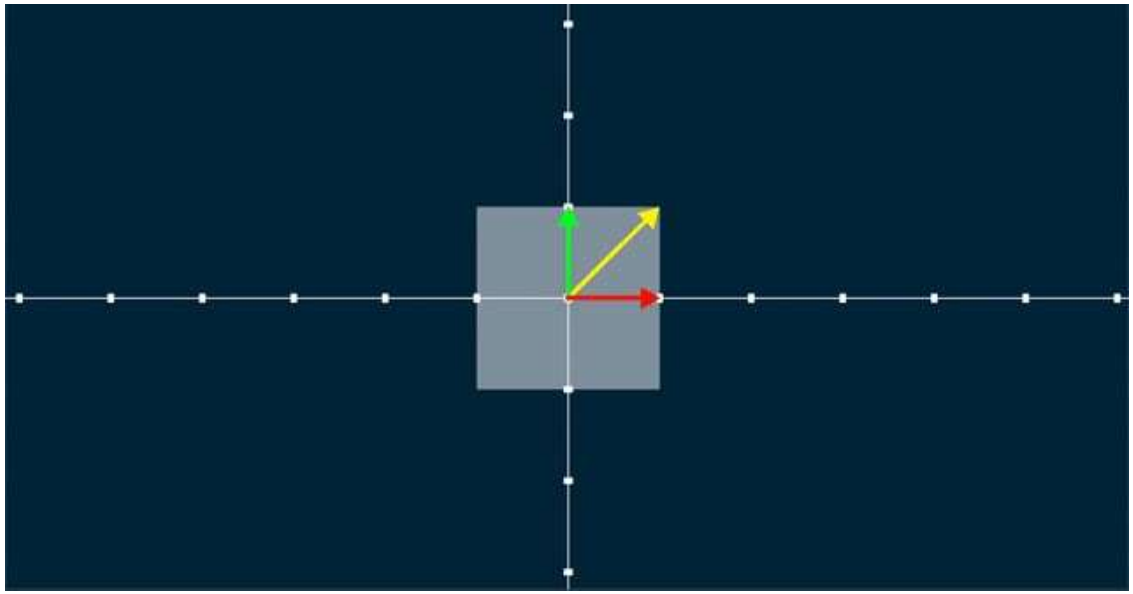
$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix},\ \theta=180^{\circ},$$

$$X_1=\begin{bmatrix}1\\ 0\end{bmatrix},\ X_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}-1\\ 0\end{bmatrix}=-1\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0\\ -1\end{bmatrix}=-1\begin{bmatrix}0\\ 1\end{bmatrix}$$





$$A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix},\ X_1=\begin{bmatrix}1\\ 0\end{bmatrix},\ X_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}2\\ 0\end{bmatrix}=2\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0\\ 2\end{bmatrix}=2\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$A=\begin{bmatrix}1 & \tan\theta\\ 0 & 1\end{bmatrix},\ \theta=15^{\circ},\Rightarrow A=\begin{bmatrix}1 & 0.269\\ 0 & 1\end{bmatrix}$$

$$X_1=\begin{bmatrix}1\\ 0\end{bmatrix},\ X_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0.269\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}=1\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0.269\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0.269\\ 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0.269\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}1.269\\ 1\end{bmatrix}$$

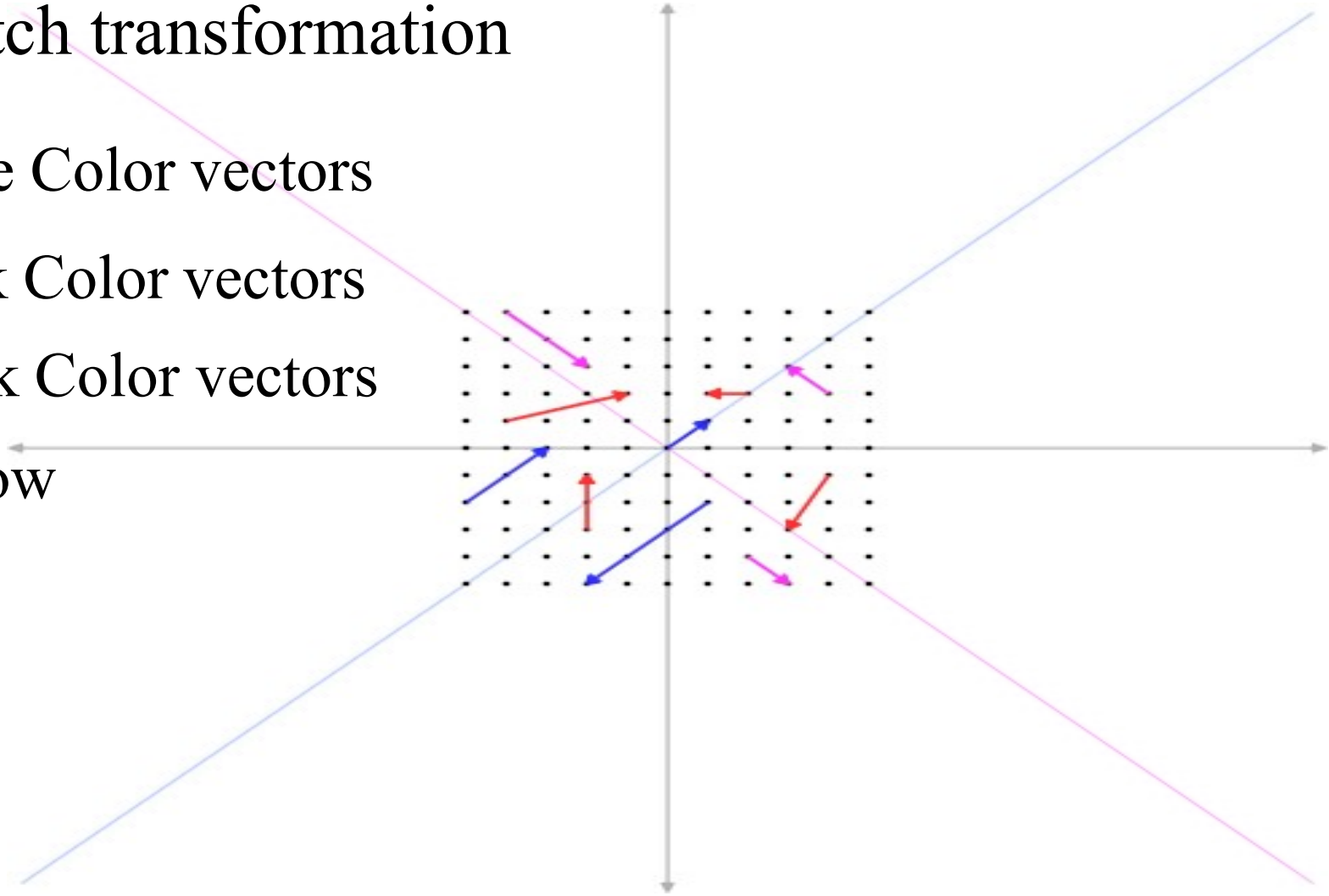
Which of the following Vectors are the Eigen vectors under this stretch transformation

A) Red and Blue Color vectors

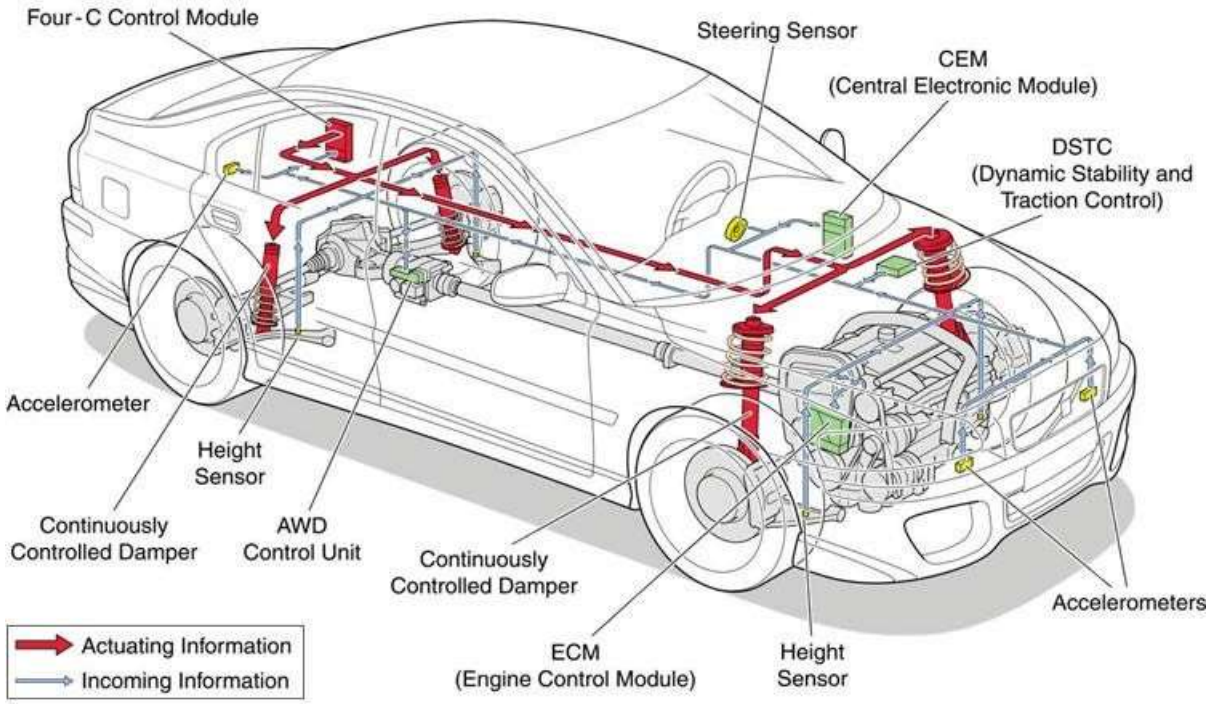
B) Red and Pink Color vectors

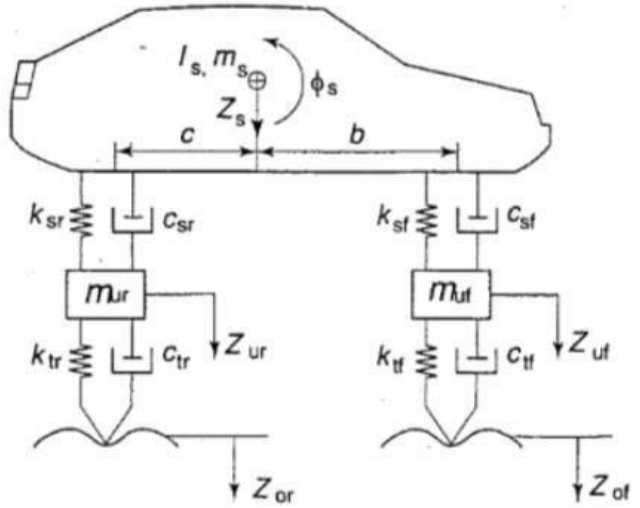
C) Blue and Pink Color vectors

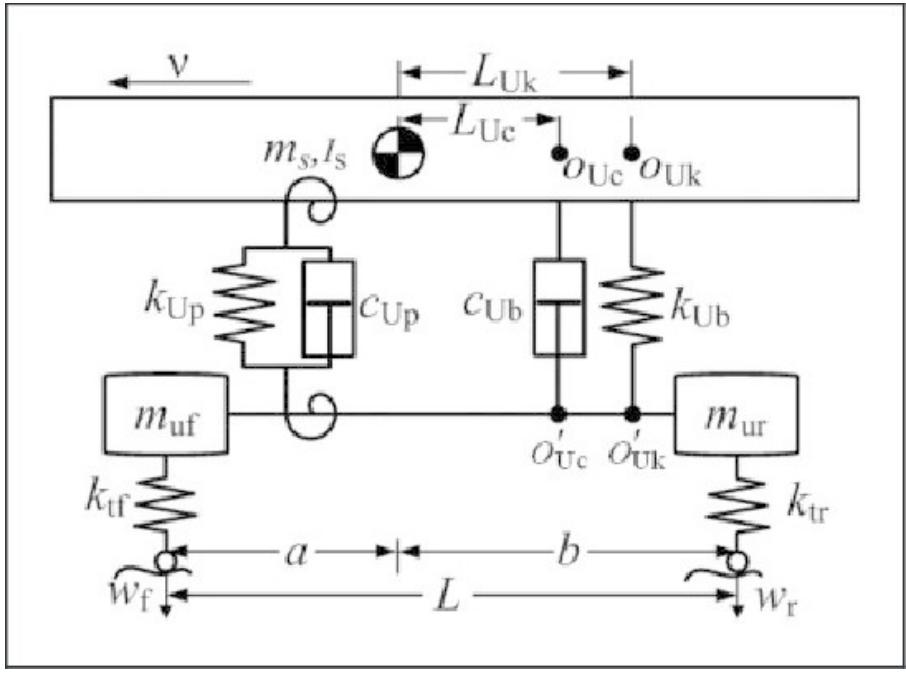
D) I Don’t Know



Correct Position of Rear and Front shocker





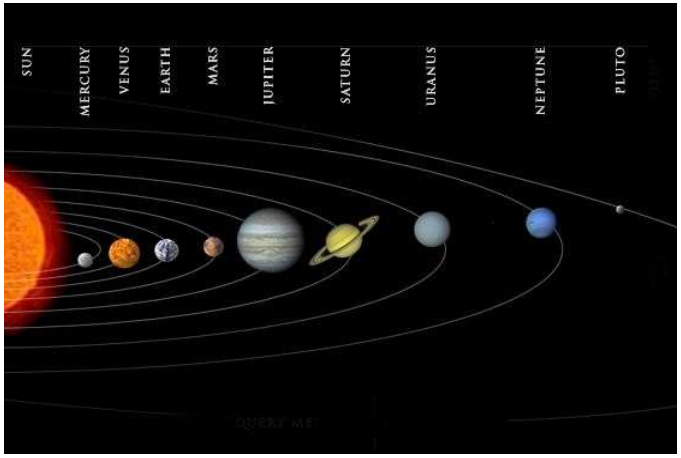


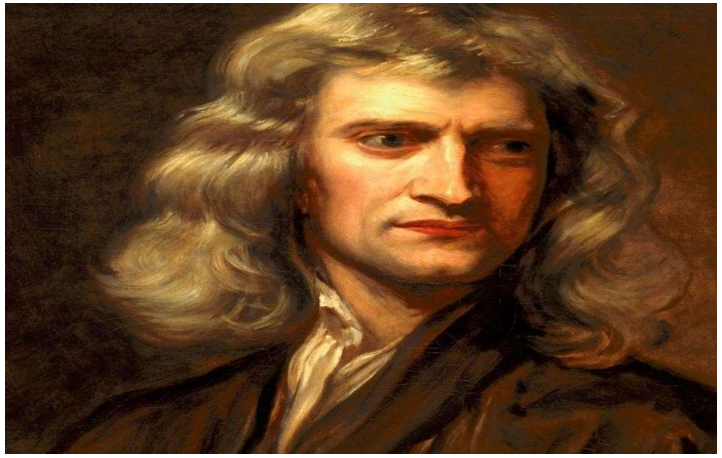
Eigenvector centrality in Networks

Google page rank Algorithm

Uses of Calculus:

Sir Isaac Newton used calculus to solve many physics problems such as the problem of planetary motion, shape of the surface of a rotating fluid etc. – recorded in Principia Mathematica



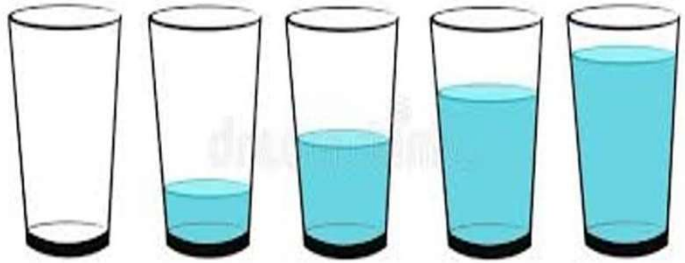


Uses of Calculus:

Gottfried Leibniz developed calculus to find area under curves

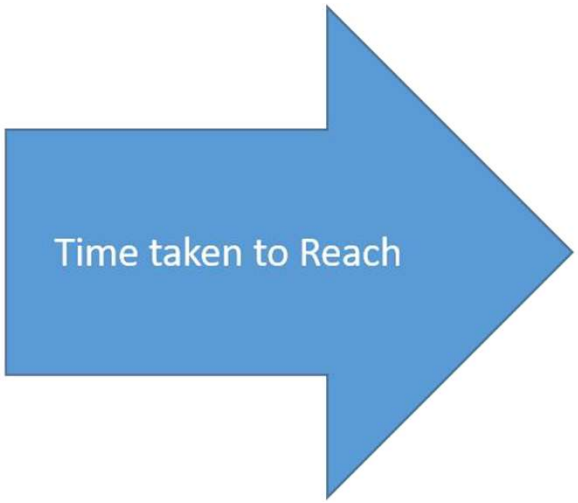
Rate of change is everywhere….

Time Taken to Fill



Time Taken to Grow into

Rate of change is everywhere….



Rate of change is everywhere….

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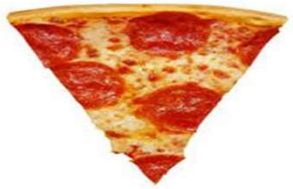
Differentiation and Integration are Inverse of each other…

$$\frac{d(\text{pizza})}{d(Pizza)} = Pizza\ Slices$$



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Differentiation and Integration are Inverse of each other…

$$\int_{Slices=1}^{8}$$  =

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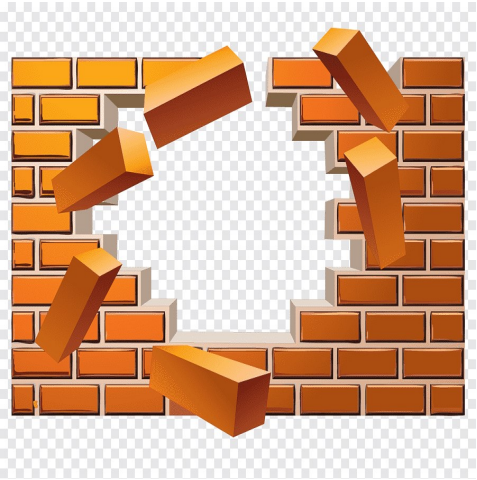
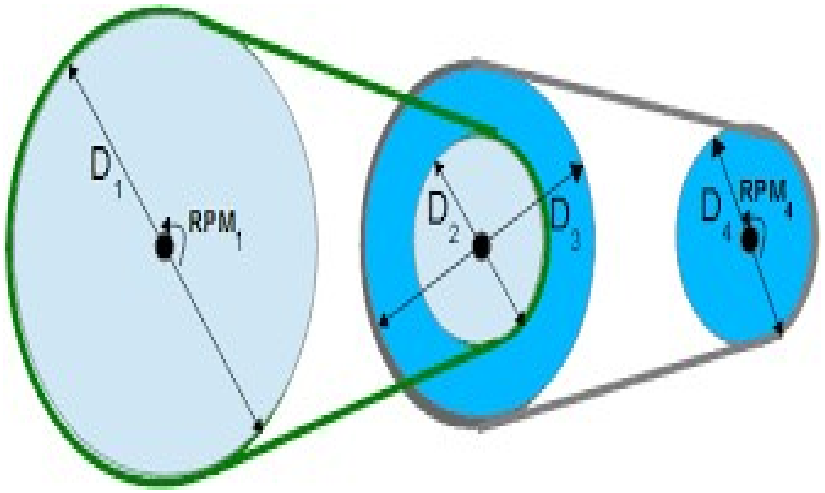
Differentiation and Integration are Inverse of each other…

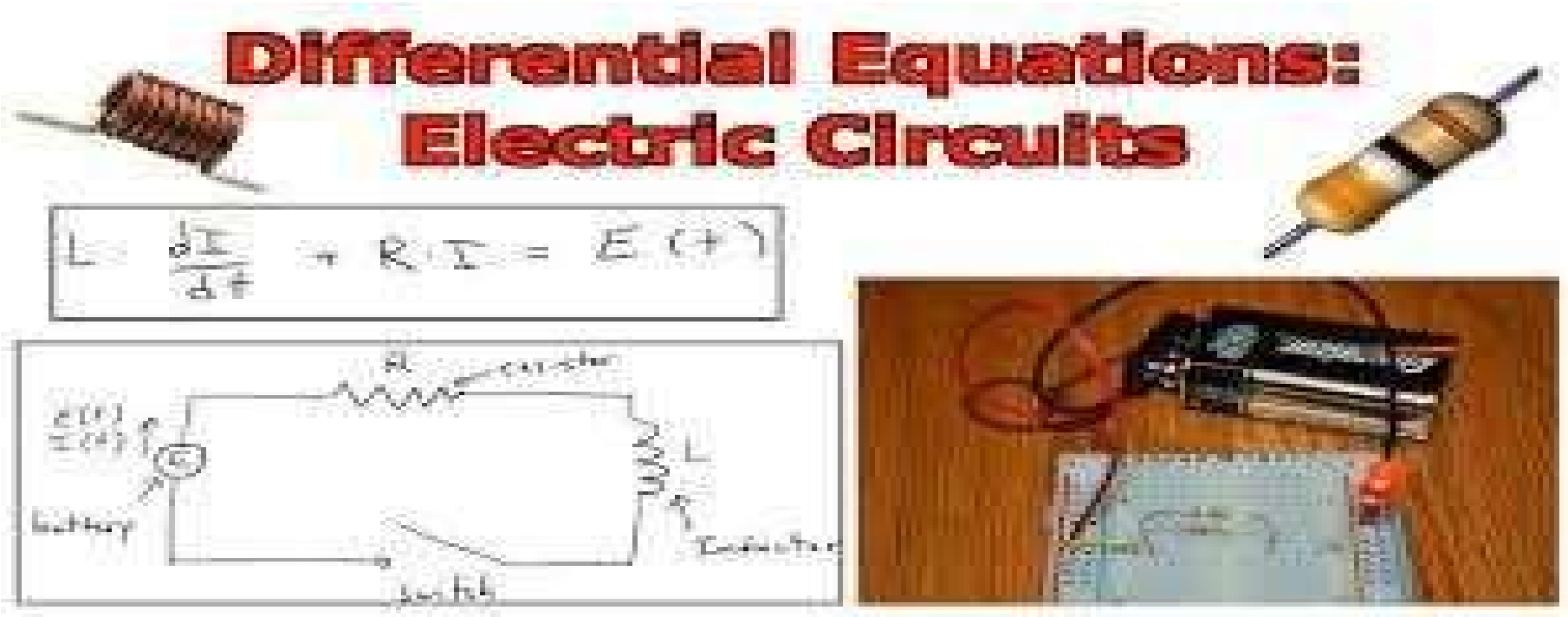


Differentiation Integration

The best example of use of chain rule in differentiation, is the working of pulleys of different sizes with same belt to reduce the effort and optimize the output.



The various kinds of LCR circuits can not be solved without differential equations and Ohm's law which is dependent on derivative of voltage.



Development of different kinds of computer languages such as C, C++, Java, Linux, Python and development of various mobile apps has a great reliance on Calculus.

Multivariate Calculus

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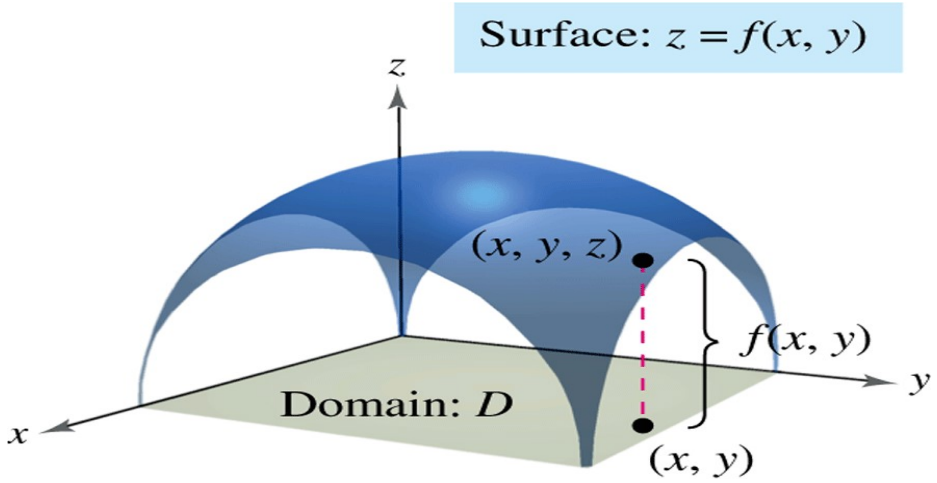
Definition of a Function of Two Variables

Let D be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in D there corresponds a unique real number $f(x, y)$, then f is called a **function of x and y.** The set D is the **domain** of f, and the corresponding set of values for $f(x, y)$ is the **range** of f.



Multivariate Calculus

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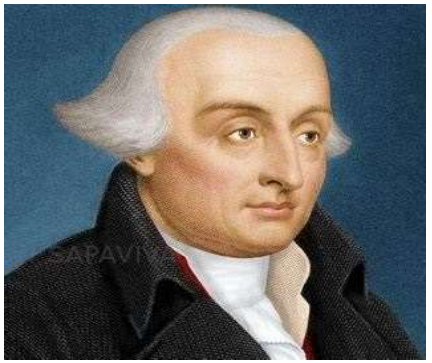
Major Contributors are:

Leibnitz

Lagrange



Multivariate Calculus

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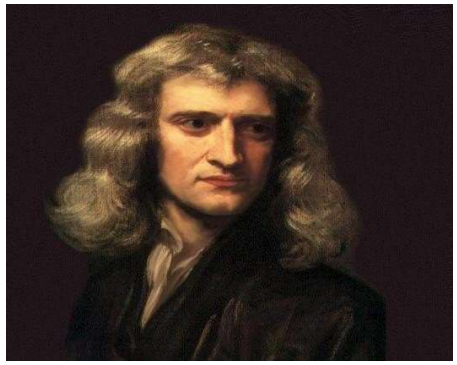
Major Contributors are:

Newton



Newton

Difference:

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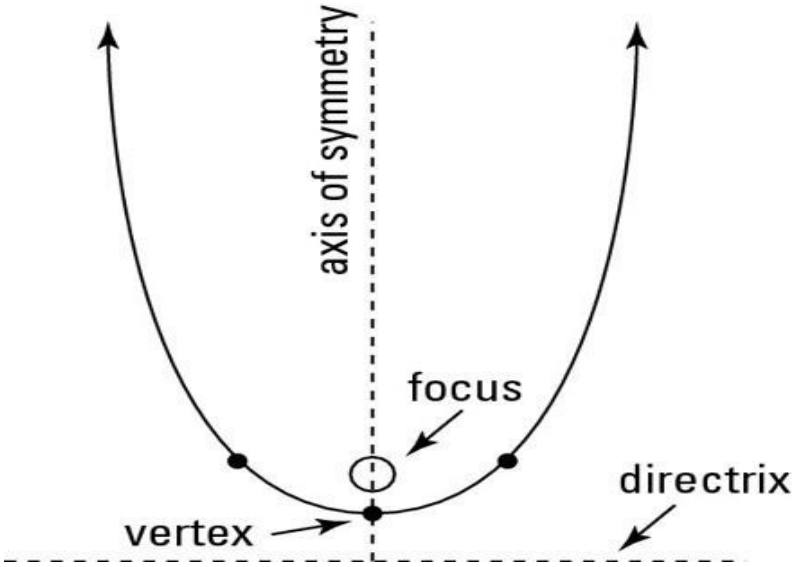
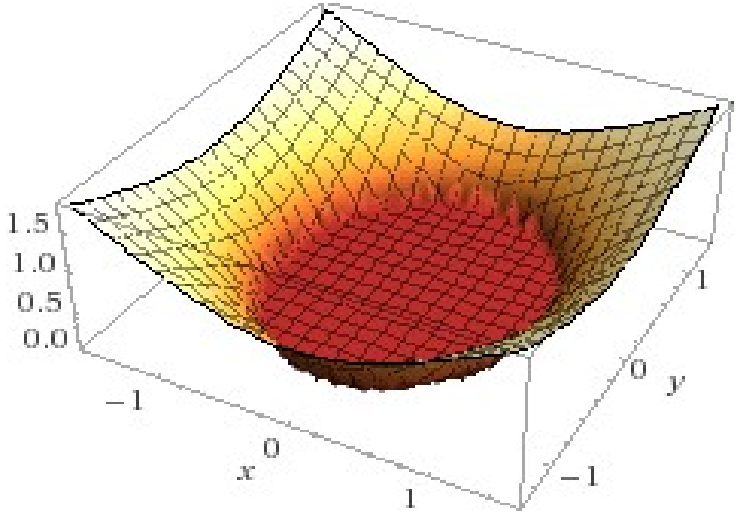
Single variable calculus

$$y = x^2$$



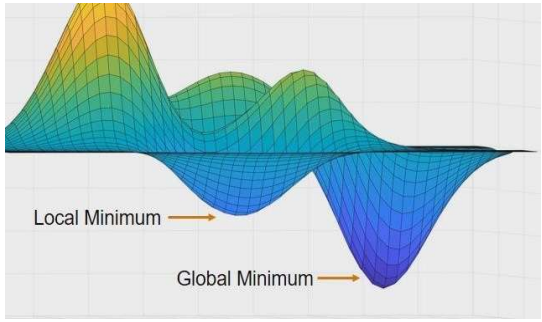
Multivariable Calculus

$$z = x^2 + y^2$$

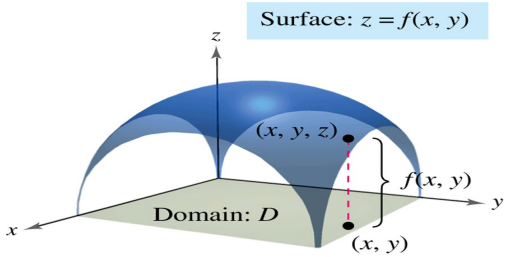


Uses of Multivariate Calculus:

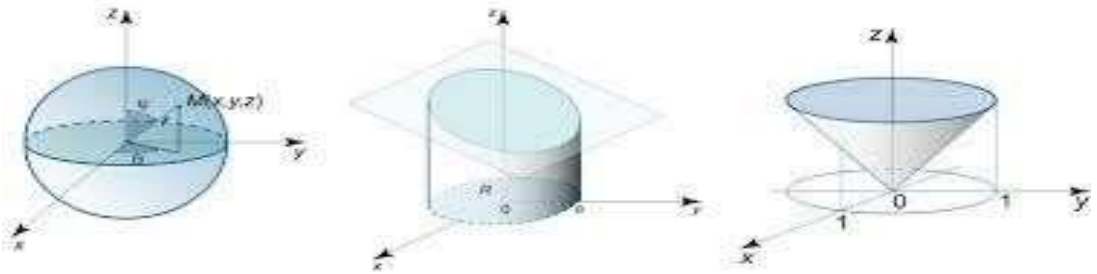
Maxima and Minima



Area under curve



Volume



Uses of Fourier Series:

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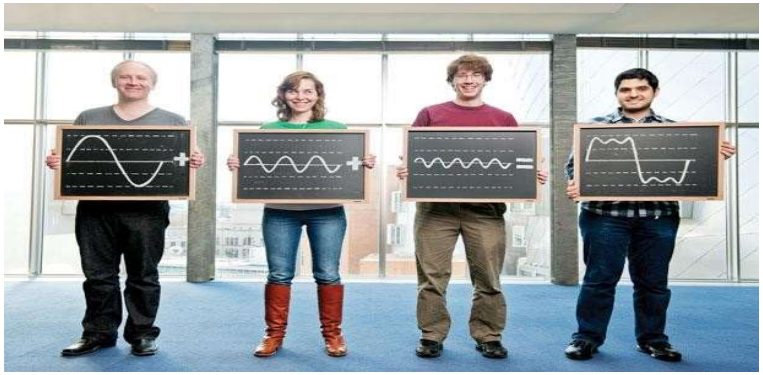
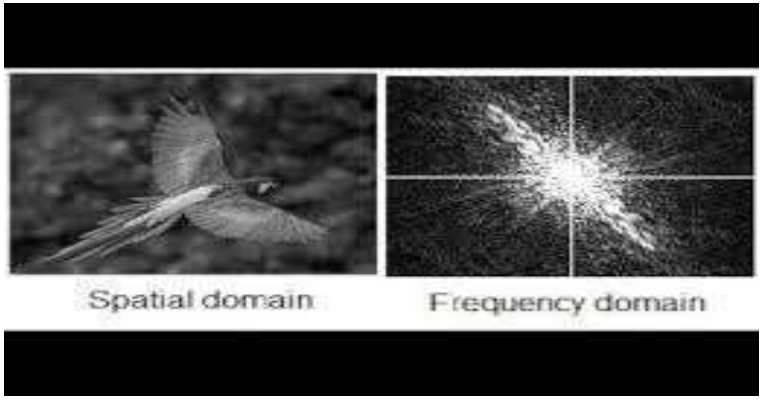
In Signal Processing



In Image Processing



Course Assessment Model

Teaching Model:

L-T-P: 3-1-0 (3 Lectures, 1 Tutorial, 0 Practical)

Marks Breakup:

Attendance	5
CA (2 best out of 3 Tests) (Subjective)	25
MTE (MCQ)	20
ETE (MCQ)	50
Total	**100**

Relevant Resources Which Could Be Used for Better Understanding of the Course.

Audio Visual Aids (AV)		
Sr No	**(AV aids) (only if relevant to the course)**	**Salient Features**
AV-1	https://www.khanacademy.org/math/linear-algebra/	Video lecture on Type of matrices, inverse of matrices and solution of system of equations.
AV-2	https://www.khanacademy.org/math/multivariable-calculus	Video lectures on multivariable-calculus
AV-3	https://www.youtube.com/watch?v=Ld9AtgPmyvM&list=UUMMt9zLt3UojrK2z52E5vng&index=41	NPTEL Video lectures on Multivariable calculus by IIT Roorkee
AV-4	https://www.youtube.com/watch?v=vA9dfINW4Rg	video lectures on Fourier series

Relevant Websites (RW)		
Sr No	**(Web address) (only if relevant to the course)**	**Salient Features**
RW-1	http://math.stackexchange.com/	A platform for students and teachers to discuss any topic
RW-2	http://tutorial.math.lamar.edu/Classes/CalcIII/PartialDerivatives.aspx	Lecture notes on Partial Derivatives
RW-3	http://tutorial.math.lamar.edu/Classes/CalcIII/MultipleIntegralsIntro.aspx	Lecture notes on multiple integral
RW-4	https://www.digimat.in/nptel/courses/video/109104124/L01.html	Video lectures on calculus of one variable

Other Reading (OR)	
Sr No	**Journals articles as Compulsary reading (specific articles, complete reference)**
OR-1	https://ncert.nic.in/textbook.php?lemh1=5-6 ,
OR-2	https://ncert.nic.in/textbook.php?lemh2=1-7 ,

Books Required

Text Book:

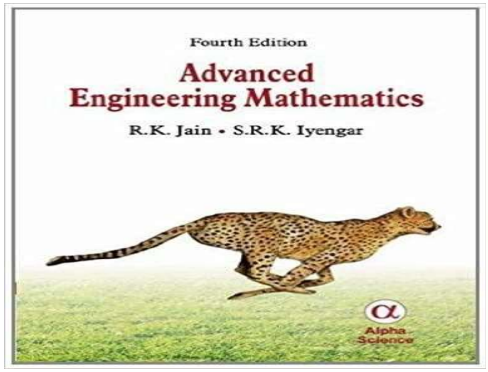
ADVANCED ENGINEERING MATHEMATICS BY JAIN AND IYENGAR

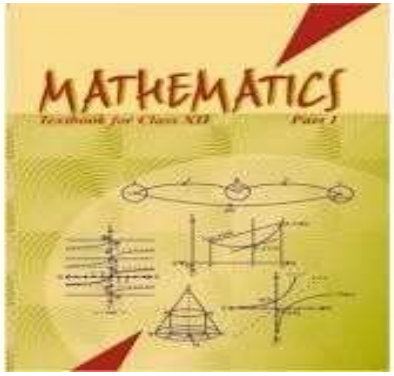
References Books:

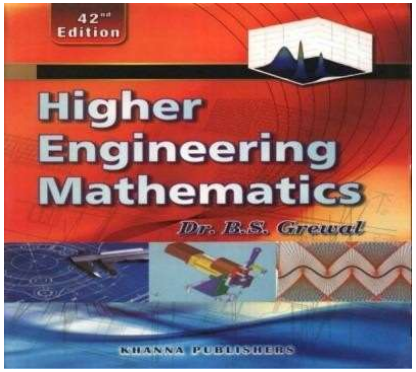
HIGHER ENGINERING MATHEMATICS BY B.S GREWAL

Other Readings

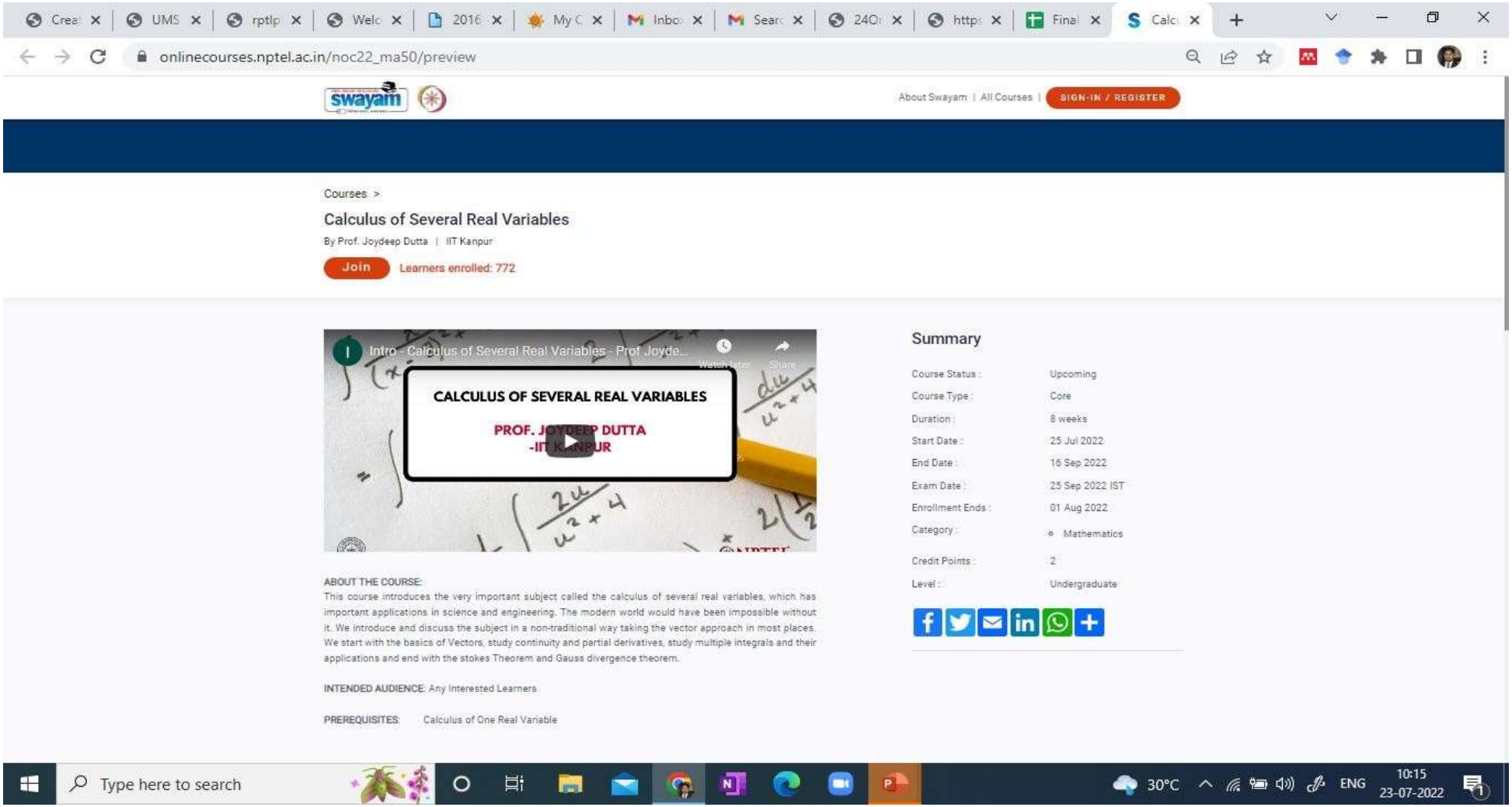
MATHEMATICS FOR CLASS 12 PART 1-2 BY NCERT







MOOC Associated With the Course



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Next Class: **Matrix Algebra**

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