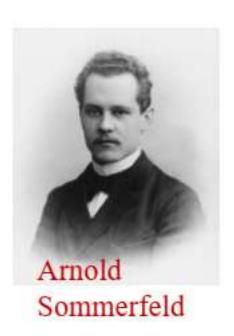
Today we will learn about....

Enrico Fermi





Sommerfeld's Quantum theory metals

Fermi energy, Fermi level
Fermi-Dirac distribution,
Fermions- any particle that obey FermiDirac distribution ..or..too deep into
particle physics!!!..spin half particles..
Eg.. Electron, proton, neutron..

Arnold Sommerfeld who combined the classical Drude model with quantum mechanical Fermi–Dirac statistics and hence it is also known as the Drude–Sommerfeld model

The main assumptions of quantum free electron theory are,

- The energy levels of the conduction electrons are quantized.
- The distribution of electrons in various allowed energy levels occur as per Pauli exclusion principle.
- However, the following assumptions of classical electron theory continue to be applicable in quantum free electron theory also.
 - The electrons travel in a constant potential inside the metal but stay confined within its boundaries.
 - b) Both the attraction between the electrons and the lattice ions, and the repulsion between the electrons themselves are ignored.

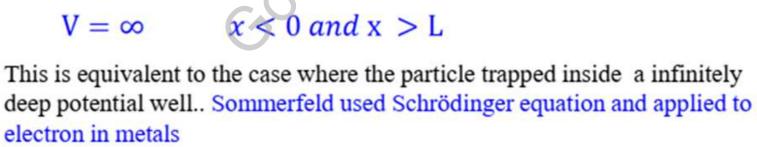
Free electron in metal

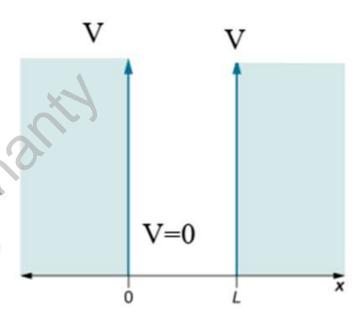
For simplicity we consider,

- 1)Electron restricted to move in the x-direction only (1 dimensional) from x=0 to x=L
- Potential energy, V of the electron is 0 inside the metal but rises to infinity out side

$$V = 0 0 \le x \le L$$

$$V = \infty x < 0 and x > L$$





$$\nabla^2 \psi + \frac{2mE}{\hbar^2} \psi = 0 \qquad \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{Eq.1}$$
 And put
$$k^2 = \frac{2mE}{\hbar^2} \quad \text{Eq.2} \qquad \frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \quad \text{Eq.3}$$

General solution for Eq.3 can be written as

$$\psi(x) = A \sin kx + B \cos kx$$
 Eq.4

Where A and B are constant. Now apply the first boundary condition. $\psi(x)=0$ at x=0

$$\psi(0) = A \sin 0 + B \cos 0 = 0$$
 \Longrightarrow B=0
$$\psi(x) = A \sin kx$$
 Eq.5 Now we will find k and E

Now apply the 2nd boundary condition. $\psi(x)=0$ at x=L, Eq.5 gives

$$\psi(L) = A \sin kL = 0$$

$$\sinh A \neq 0$$

$$\sinh kL = 0$$

$$\sinh kL = 0$$
Eq.6

Eq.6 is satisfied only when

$$kL = n\pi$$
 Where, n= 1,2,3

$$k = \frac{n\pi}{L} \qquad \text{or} \qquad k^2 = \frac{n^2\pi^2}{L^2} \qquad \text{Eq. 7}$$

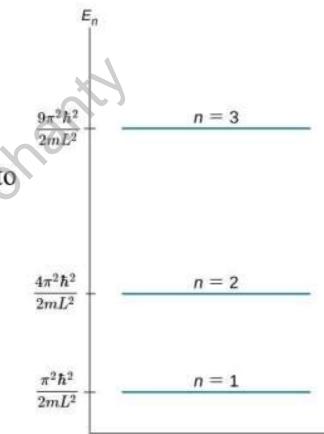
Now substitute Eq.2 in Eq.7

$$k^2 = \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$
 Eq.8

Energy of the free electron is discrete and is quantized!!

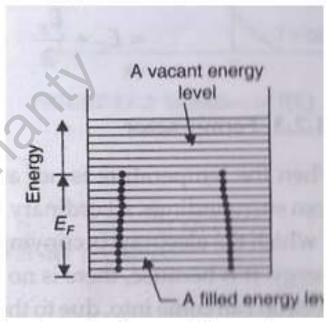
- E_n Constitute the energy level of the system
- n is the quantum number corresponds to the energy level E_n

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$



So we found out the energy (**Eigen value**) of the free electron in the metal, with the help of Schrödinger equation

For a metal containing N free electrons, there will be N such allowed energy levels, which are separated by energy differences that are characteristic of the material.



The allowed energy values for the conduction electrons in the metal are also quantized, are related to the energy levels of the metal. As per the Pauli exclusion principle, each energy level can accommodate a maximum of two electrons, one with spin up and the other one with spin down

The energy of the highest occupied level at zero degree absolute temperature is called Fermi energy, and the energy level is referred to as the Fermi level.

Fermi-Dirac distribution

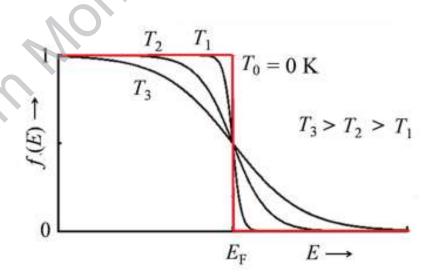
Probability of finding an electron at a particular energy level E

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$
 F(E) is the Fermi-Dirac distribution

Case 1: At absolute zero T=0

$$\frac{\mathbf{E} - \mathbf{E}_{\mathbf{F}}}{\mathbf{k}\mathbf{T}} = \begin{cases} -\infty, & \text{if } \mathbf{E} < \mathbf{E}_{\mathbf{F}} \\ +\infty, & \text{if } \mathbf{E} > \mathbf{E}_{\mathbf{F}} \end{cases}$$

$$f(E) = \begin{cases} 1, & \text{if } E < E_F \\ 0, & \text{if } E > E_F \end{cases}$$

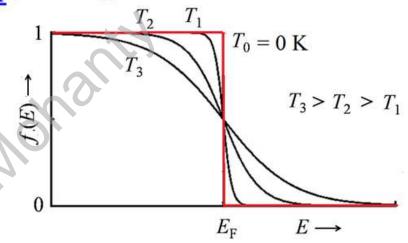


f(E) Follows the red line in figure

Fermi-Dirac distribution

Case 2: At any temperature T for $E=E_F$, the energy level corresponding to $E=E_F$ is called the Fermi level

$$f(E) = \frac{1}{e^0 + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$



So the Fermi level is the energy level at which there is half probability of finding an electron. That depends

- 1. On the number of electrons
- 2. Distribution of energy levels

low specific heat of metals is now understood

Similarities between the two theories

The following assumptions apply to both the theories:

- 1. The valence electrons are treated as though they constitute an ideal gas.
- 2. The valence electrons can move freely throughout the body of the solid.
- The mutual repulsion between the electrons, and the force of attraction between the electrons and ions are considered insignificant.

Difference between the two theories

Classical Free Electron Theory

- The free electrons, which constitute the electron gas can have continuous energy values.
- It is possible that many electron may possess same energy.
- The pattern of distribution of energy among the free electron obey Maxwell– Boltzmann statistics.

Quantum Free Electron Theory

- The energy values of the free electrons are discontinuous because of which the energy levels are discrete.
- The free-electrons obey the Pauli exclusion principle. Hence no two electrons can possess same energy.
- The distribution of energy among the free electrons is according to Fermi Dirac statistics which imposes a severe restriction on the possible way in which the electrons absorb energy from an external source.

MERITS OF QUANTUM FREE ELECTRON THEORY

- Changed the concept of free electron or conduction electrons in metals
- Successfully explained thermal and electrical conductivity of metals
- Thermionic emission from metals explained
- Temperature dependence conductivity explained
- The theory explained experimentally observed electronic specific heat of metals
- Explained paramagnetic susceptibility

But failed to Explain

Ferromagnetism in metals

Why some materials are Metal, insulator and semiconductors?

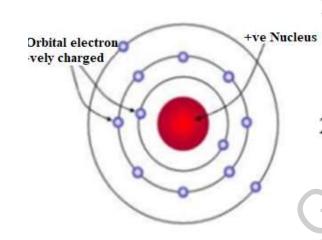
This all are explained at the third stage of development "Band theory of solid' developed by Bloch, Kroning-Penney etc.. And that will be learned from today onwards

Today we are going to learn about the band theory of solids

- 1) Electron-ion interaction
- Periodic potential function
- Schrödinger wave equations- for electron in a periodic potential
- Bloch function- solution of Schrödinger wave equation in a periodic potential- Bloch electrons
- 5) Kroning Penning model- simplification of the periodic potential
- 6) E-k diagram- deviation from the parabolic behavior
- Splitting energy levels- formation of band
- Allowed and forbidden energy concept of band gap and band overlap

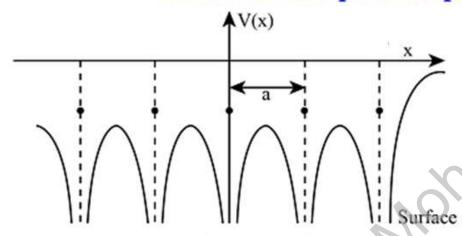
STAGE 3: Band Theory of Solid

We continue to treat valance electrons as independent (by neglecting e-e-interaction like in stage 1 and stage 2) but now we consider the electron-ion interaction. Solid is a periodic arrangement of atoms...



- Free electron: Potential energy, V=0: Total energy is just the kinetic energy of the electron
 - ✓ This is the case for electron experience between the ions position in the lattice...
- Not free when V=V₀; when electron is not in the well but at the barrier region, interaction with ion in the lattice considered.
 - ✓ This is what electron experience at the lattice point
- So electron wave experience a periodicity in the potential as it move through the lattice. .. This bring Bloch function instead Schrödinger wave function

Electrons in a periodic potential



Quantum mechanical view of the potential (V) experienced by an electron in passing through a crystalline materials

Now we go back to the Schrödinger wave equation we learned in Unit 4 for the electron of mass m and energy E

Solution for Equation 1, ψ , we have already seen and the also for the $E-V_0$

$$\psi(x) = e^{\pm ikx}$$
 Eqn.2

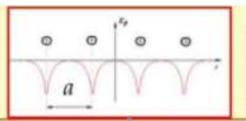
$$E - V = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} = E_k$$
 Eqn.3

If the electron is moving through the periodic lattice, it experiences the periodic potential V(x) = V(x + a) Eqn.4

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} [E - V(x)] \psi = 0 \quad \text{Eqn.5}$$

Solution of equation 5 is given by Bloch theory...That is why so the called Bloch's theory of energy band in solids

Bloch Wavefunctions



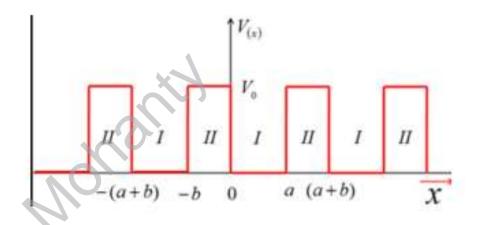
□ Bloch's Theorem states that for a particle moving in the periodic potential, the wavefunctions $\psi(x)$ are of the form

$$\psi(x) = u_k(x)e^{\pm ikx}$$
, where $u_k(x)$ is a periodic function $u_k(x) = u_k(x+a)$

- $\square u_k(x)$ has the periodicity of the atomic potential
 - The exact form of u(x) depends on the potential associated with atoms (ions) that form the solid

Kroning-Penney Model for the periodic potential

Simplified the problem by considering by considering square well potential



At the bottom of the well ($0 \le x \le a$) V is zero and electron is close to the nucleus

Out side of the well (b < x < 0) potential V is V_0 , electron is away from the nucleus

Schrodinger equations for these two cases can be written as

Region 1
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0 \qquad \text{for } (0 < x < a)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} [E - V_0] \psi = 0 \qquad \text{for } (-b < x < 0)$$

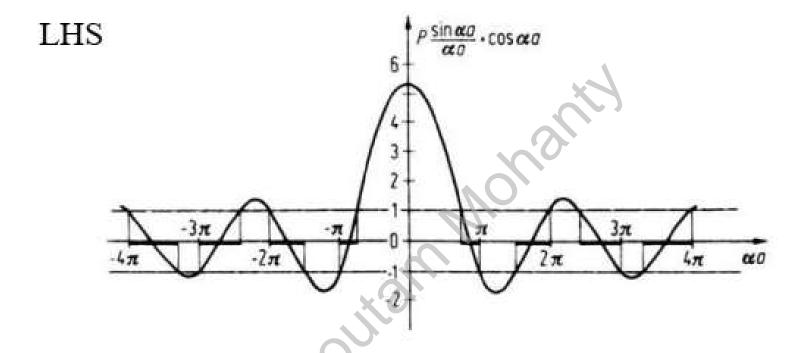
If we solve these two equations for wave function of the Bloch electron and energy of the Bloch electron, we are in the process getting all the answers In order to find out the allowed energy values of electron while moving in the lattice of periodic potentials due to the bound ionic centers Kroning and penney assumed..

- a) The energy of the electron is less than V_0
- b) Solution of the above equation is Bloch function
- c) Wave function and its derivative are continuous through out the crystal lattice
- d) Product of the width and height of the potential is finite

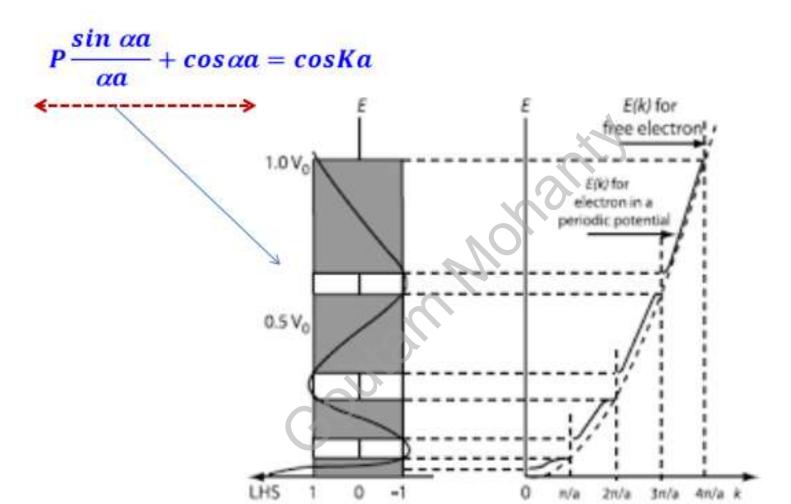
With rigorous mathematical step they obtained the solution for Eq.5 as

$$P = \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos Ka \qquad \text{Eqn.6}$$

$$P = \frac{maV_0b}{\hbar^2} \quad \text{Eqn.7} \qquad \alpha^2 = \frac{2mE}{\hbar^2} \qquad \text{Eqn.8}$$

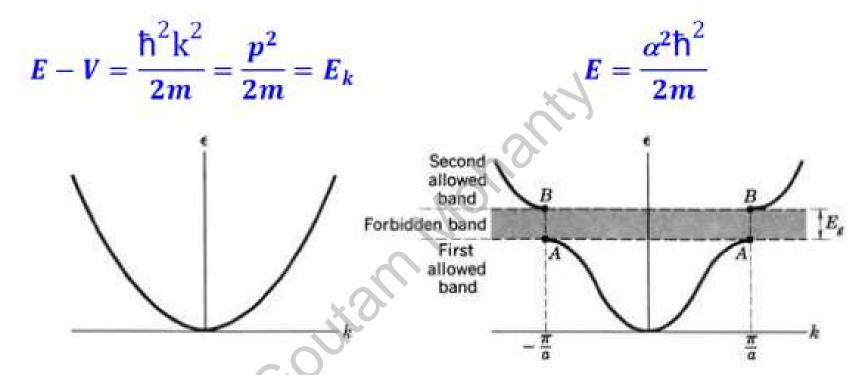


Allowed values of the quantity on the y axis is within ± 1 , that restrict the energy values allowed for the Bloch electrons.



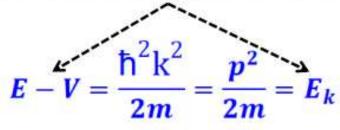
(a)

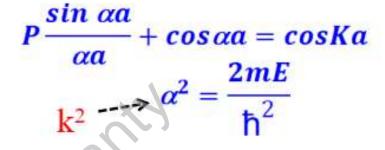
(b)

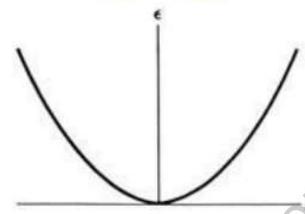


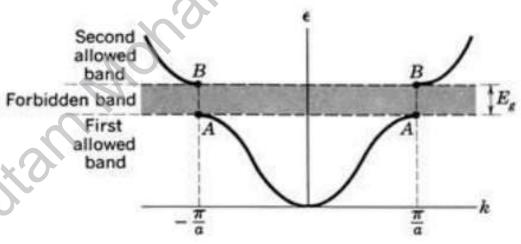
So periodic potential modulate the energy of the electron; it restrict it having a band of values and some energy is not allowed to possess in the lattice. Origin of band gap in material come to exist... now we can distinguish metal, insulator and semiconductor

Kinetic energy of electron









Quantum free electron theory

Quantum nearly free electron theory Electron-lattice interaction

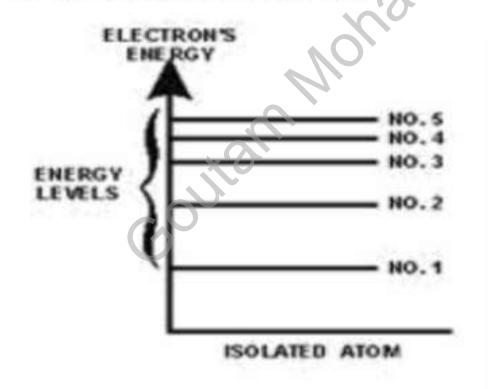
In fact the periodicity of the potential function is responsible for the gap

Today we will learn about

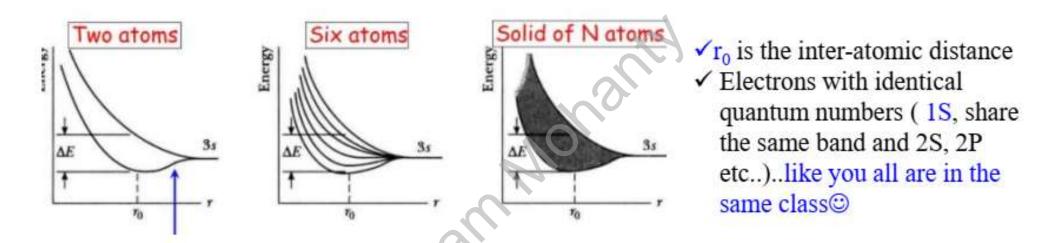
1.	Class	ification of solids with band theory concept
		Splitting of energy levels: FORMATION OF A BAND
		Merging of bands: BAND OVERLAP
		Valance and conductions bands; BAND GAP
		Conductors (Metals), Semiconductors and Insulators
2.	Semiconductors	
		Intrinsic and extrinsic semiconductors
		Fermi level for intrinsic and extrinsic semiconductor.

ENERGY BANDS IN SOLIDS

There are discrete energy levels in the case of an isolated atom.



Formation of Bands in Solids

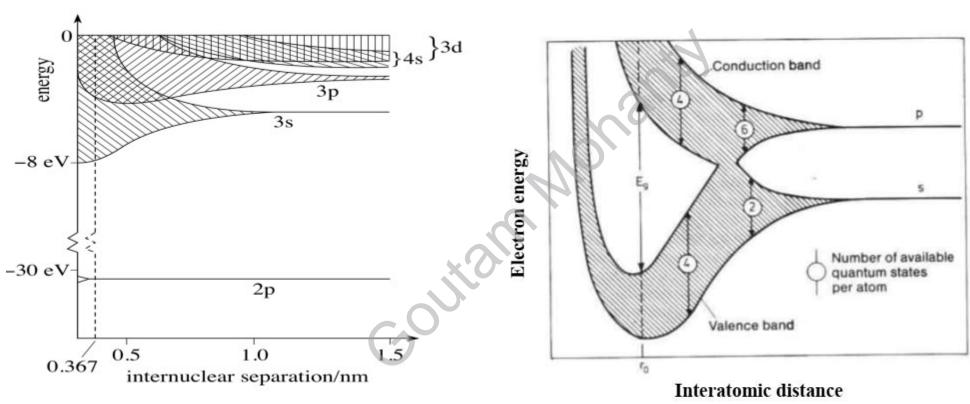


Splitting of discrete energy levels as two atoms come close to form solid due to Pauli's exclusion principle.

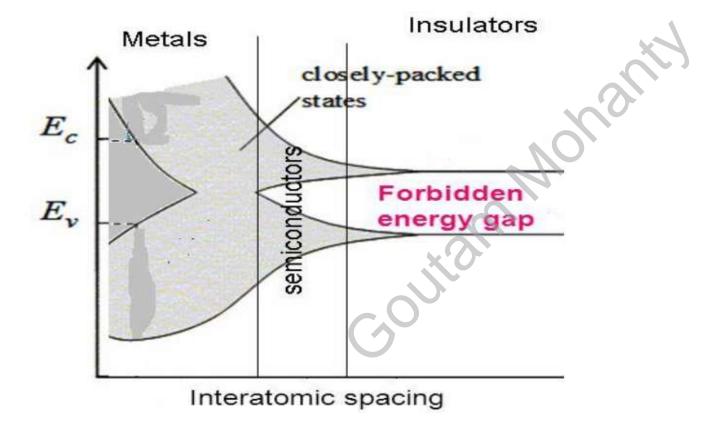
All discrete energy levels opens up.. But width ΔE increases towards higher levels (quantum numbers, or K,L,M,N etc.. Or s,p,d,f)

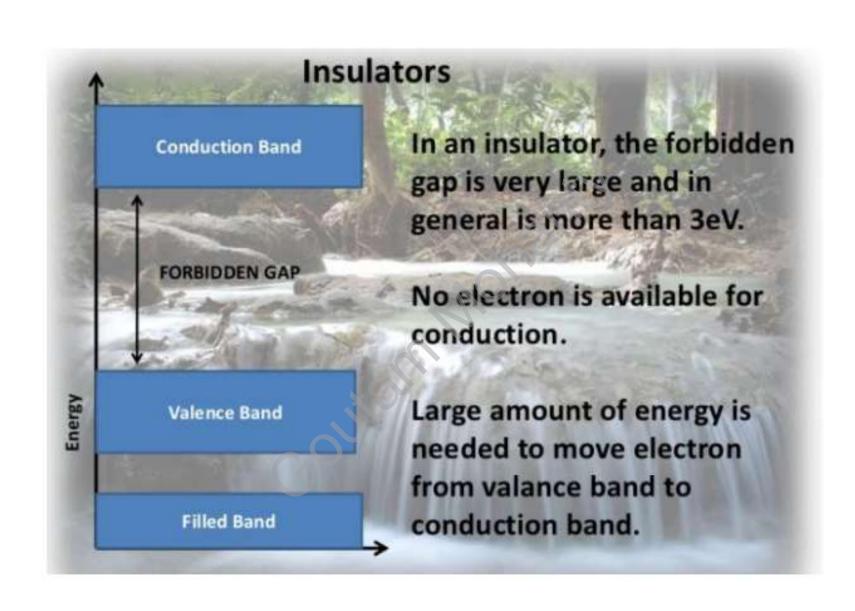
Merging of bands

Formation of Band Gap



Insulator or semiconductor





Semiconductors **Conduction Band** FORBIDDEN GAP Around 0.7eV (Ge) and 1.1 eV (Si) Energy Valence Band

Filled Band

In the case of semiconductors the forbidden gap is very small.

At OK the conduction band is empty and the valence band is completely filled.

When a small amount of energy is supplied, the electrons can easily jump the forbidden gap.

The conductivity of a semiconductor is of the order of 10 ²mho m⁻¹

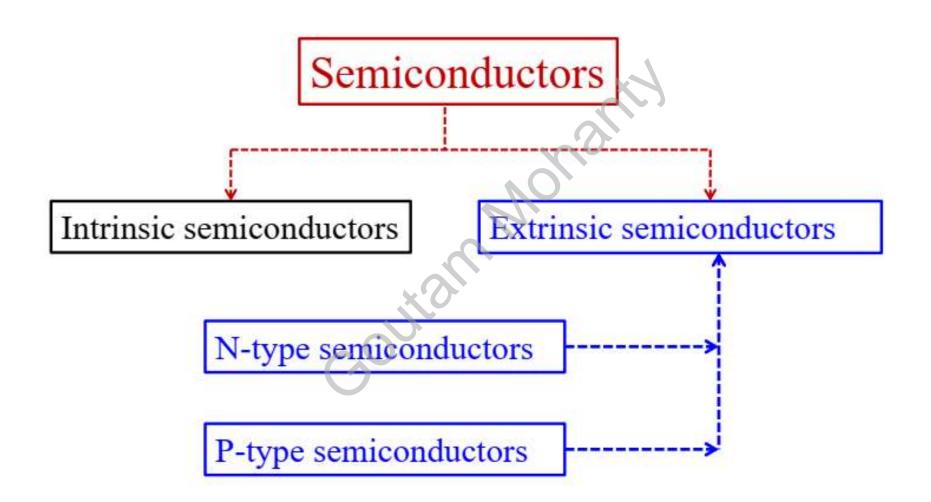
Conductors **Conduction Band** Energy No forbidden gap Valence Band

In conductors there is no forbidden gap.

The valence band and the conduction band overlap.

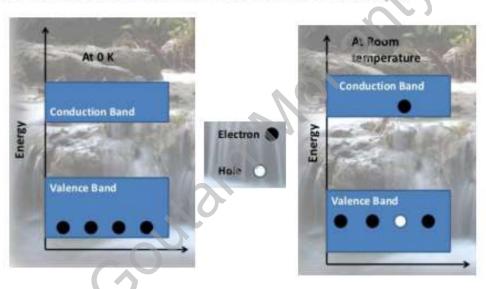
The electrons from valence band freely enter into the conduction band due to overlapping of bands.

Therefore very low potential difference can cause continuous flow of current.



Intrinsic semiconductors

In a semiconductor ejection of electron to the conduction band from the valance band is purely by thermal excitation is an intrinsic semiconductor



- > The effect is temperature dependent
- Produce equal number of holes and electrons
- ➤ Electrons or holes are the intrinsic carriers
- ➤ Conductivity is then called the intrinsic conductivity

Intrinsic semiconductors

Concentration of electron (with effective mass m_e) in an intrinsic semiconductor is given by

$$n_e = 2 \left[\frac{2\pi m_e kT}{h^2} \right]^{3/2} e^{(E_F - E_c)/kT}$$
 Eq.1 $n_e = n$

Concentration of holes (with effective mass m_h) in an intrinsic semiconductor is given by

$$n_h = 2 \left[\frac{2\pi m_h kT}{h^2} \right]^{3/2} e^{(E_v - E_F)/kT}$$
 Eq.1 $n_h = p$

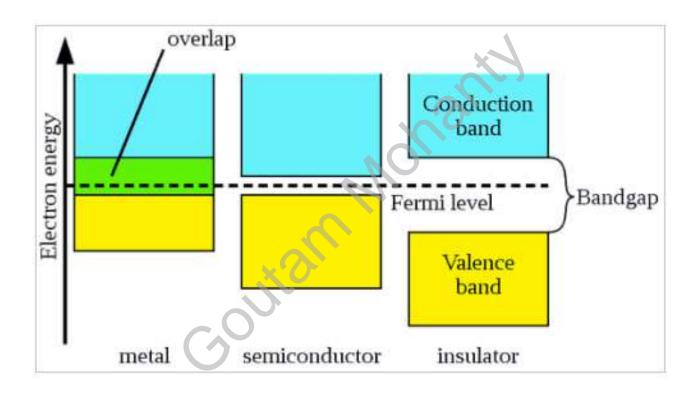
And the product of the two carriers concentration is $n_e n_h = AT^3 e^{(E_v - E_c)/kT}$; a constant for given material and a given temperature

For an intrinsic semiconductor $n=p=n_i$

$$np = n_i^2 = AT^3 e^{(E_v - E_c)/kT}$$

Very important relation called law of action Hold good for extrinsic semiconductors as well

Fermi level for semiconductor



We defined the Fermi level of a metal as the energy of the top most filled level at absolute zero. Or the Energy of the electron for which probability of finding an electron at any temperature is half at Fermi level.

Fermi level of intrinsic semiconductor

$$n_{e} = 2 \left[\frac{2\pi m_{e}kT}{h^{2}} \right]^{3/2} e^{(E_{F} - E_{c})/kT} \qquad n_{h} = 2 \left[\frac{2\pi m_{h}kT}{h^{2}} \right]^{3/2} e^{(E_{v} - E_{F})/kT}$$

$$2 \left[\frac{2\pi m_{e}kT}{h^{2}} \right]^{3/2} e^{(E_{F} - E_{c})/kT} = 2 \left[\frac{2\pi m_{h}kT}{h^{2}} \right]^{3/2} e^{(E_{v} - E_{F})/kT}$$

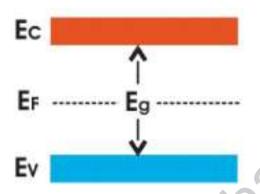
$$e^{(2E_{F} - E_{c} - E_{v})/kT} = \left(\frac{m_{h}}{m_{e}} \right)^{3/2}$$

$$E = \frac{E_{c} + E_{v}}{2} + \frac{3}{4} kT \ln \frac{m_{h}}{m_{e}}$$

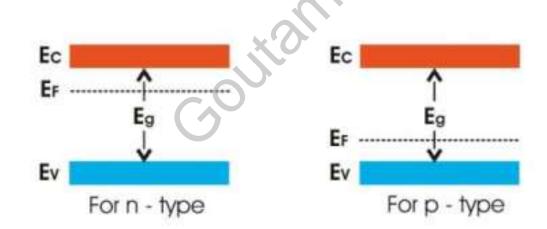
If m_e and m_h are same then $E_F = \frac{E_c + E_v}{2}$ Fermi level is at the center of the

band gap Eg for an intrinsic semiconductor

$$E_F = \frac{E_c + E_v}{2}$$

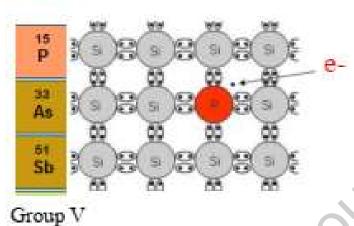


Intrinsic semiconductor



Extrinsic semiconductors

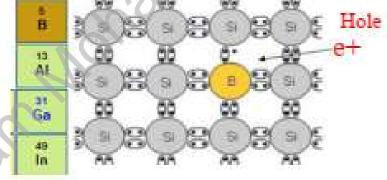






N-Type semiconductor

- By doping Si or Ge with Group V elements
- Majority carriers: electrons



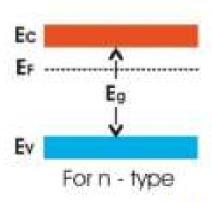
Group III

- P-Type semiconductor
- √By doping Si or Ge with Group III
 elements
- √Majority carriers: holes

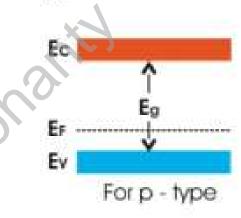
Extrinsic semiconductors

N-Type semiconductor

P-Type semiconductor



$$E_F = E_c - kT \ln \frac{N_c}{N_d}$$



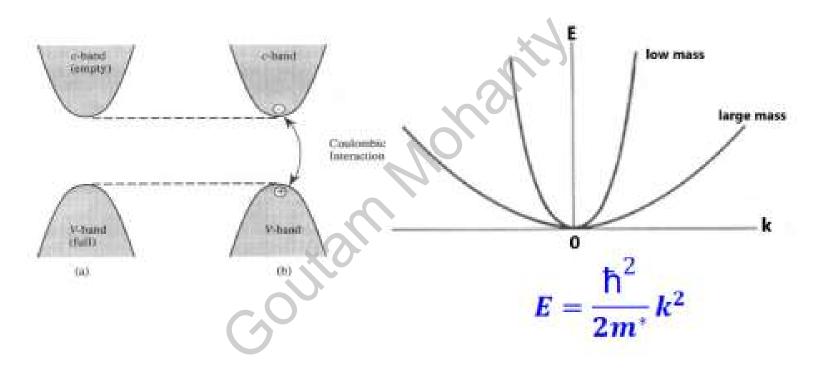
$$E_F = E_V + kT \ln \frac{N_V}{N_a}$$

k-Boltzmann's constant, T-temperature, N_a and N_d density of acceptor and donor atoms, Nv and Nc density of holes in the valance band and density of electrons in the conduction band

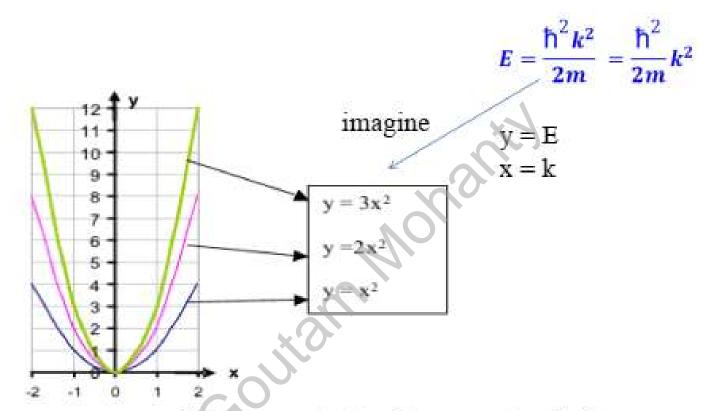
Fermi level lies just below the conduction band Fermi level lies just above the valence band



Effective Mass (m*) electrons and Holes



The concept of effective mass (m*) introduced to explain the curvature of the E-k diagram -



If the same electron in energy bands shows different shape then the mass must be different??

Effective mass concept formulated to explain it...

$$E = \frac{\hbar^2}{2m!} k^2$$
 For free electron theory m=m*

Differentiating the above equation we get

$$\frac{dE}{dk} = \frac{\hbar^2}{2m^*} 2k$$

$$\therefore m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

Effective mass is inversely proportional to the curvature of the band

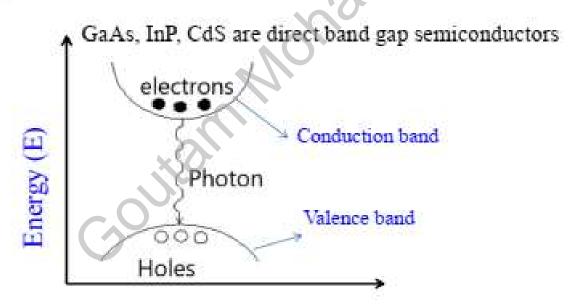
There are crystals in which the effective mass of the carriers is much larger or much smaller than m_0 .

The effective mass may be anisotropic, and it may even be negative.

The important point is that the electron in a periodic potential is accelerated relative to the lattice in an applied electric or magnetic field as if its mass is equal to an effective mass.

Direct Band gap semiconductors

In E-k diagram, when the lowest-energy point of the conduction band lies directly above the highest-energy point of the valence band in a semiconductor, the movement of a electron across the band gap conserves momentum and the gap is classified as "direct"



optical transitions more likely in materials with direct band gap compared to indirect bandgap.

k-Vector

Used in solar cell, LED etc

Indirect Band gap semiconductors

When the highest-energy point of the valence band is not directly below the lowestenergy point in the conduction band, a phonon must carry away the momentum offset if a transition is to occur between the valence and conduction band.



Indirect band gap: Transition assisted with phonon (quantized lattice vibrations)

Direct band gap

- Bottom of conduction band(CB) lies directly above top of the valance band(VB)
- Electron recombines with holes gives photon
- The photon have energy equal to the band gap
- It is radiative recombination
- It is used to build light emitting devices
- Eg) GaAs

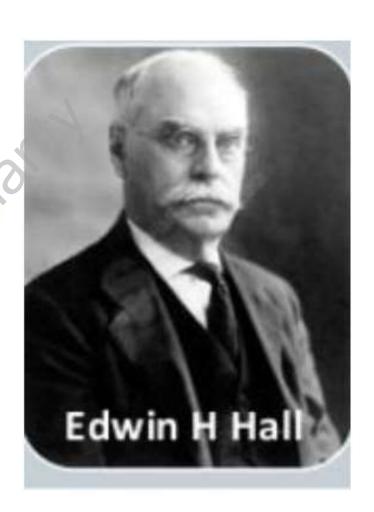
Indirect band gap

- Bottom of CB not lies directly above the top of VB
- So for the conservation of momentum electron losses its energy by interacting with phonons
- The electron and hole recombines getting energy as form of heat
- It is Non radiative recombination
- Eg) Si,Ge

Today we are going to learn about

HALL EFFECT AND ITS APPLICATIONS

- ☐ Hall coefficient
- ☐ Hall voltage
- ☐ Applications of Hall effect

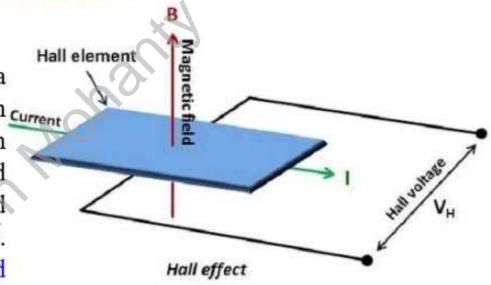


Hall effect

It was discovered by Edwin Herbert Hall in 1879.

The Hall effect is the production of a voltage (the Hall voltage) across an current electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current.

Combined effect of Electric field and magnetic field on the flow of charges



☐ Observed much before the discovery of electron!. However, Could not be explained by the Classical free electron theory (1900) and Quantum free electron theory (1928). Positive and negative value of Hall coefficient

HALL COEFFICIENT AND HALL VOLTAGE Derivation--

Force on the charge carriers due to the produced Hall electric field $(\mathbf{E}_{\mathbf{H}})$ is given by

$$\mathbf{F}_E = \mathbf{q}\vec{\mathbf{E}}_H \longrightarrow Eq.1$$

Force on the charge carriers due to the applied magnetic field (B) is given by

$$\mathbf{F}_{M} = \mathbf{q} \left(\vec{\mathbf{v}}_{\mathbf{d}} \times \vec{\mathbf{B}} \right) \longrightarrow \mathbf{Eq.2}$$

Where v_d is the drift velocity and q is the electric charge of the carrier.

At equilibrium/stationary state these two forces must be equal, so from Eq.1 and Eq.2 we get

$$\mathbf{F}_E = \mathbf{F}_M = \mathbf{q}\vec{\mathbf{E}}_H = \mathbf{q}(\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}) \longrightarrow \vec{\mathbf{E}}_H = \vec{\mathbf{v}}_d \times \vec{\mathbf{B}} \longrightarrow \underline{\mathbf{E}}_{q.3}$$

If we talk about the magnitude Eq.3 can be written as

$$E_{H} = v_{d}B \sin\theta$$

In the present case 0 is 90°, so we get

$$E_{\rm H} = v_{\rm d}B \longrightarrow Eq.4$$

HALL COEFFICIENT AND HALL VOLTAGE Derivation--

Consider n is the number of charge carriers per unit volume, then the current density can written in terms of the drift velocity (vd) as follows

$$J = nqv_d \longrightarrow Eq.5$$
Substitute $Eq.5$ in Eq.4
$$E_H = v_d B$$

$$E_H = \frac{JB}{nq} \longrightarrow Eq.66$$

Substitute Eq.5 in Eq.4

If b is the width of the block, then hall electric field E_H can be related potential as

$$E_{H} = \frac{V_{H}}{b}$$

V_H is known as the hall voltage and written as

$$V_{H} = E_{H}b = \frac{J B b}{nq} \longrightarrow Eq.6$$

HALL COEFFICIENT AND HALL VOLTAGE Derivation--

Coefficient of proportionality 1/nq is called the Hall coefficient and is denoted by R_H . So we have

$$R_{H} = \frac{1}{nq} \longrightarrow Eq.7$$

R_H will be negative if the charge carriers are electrons and will be positive if the charge carriers are holes. So can be used to distinguish the type of semiconductors

To relate the Hall electric field (E_H) to the applied electric field (E_x) we use the relation for J, $I = \sigma E_x = \frac{E_x}{I}$ in $E_H = \frac{JB}{I}$

Where are σ and ρ are the electrical conductivity and resistivity, respectively of the slab under consideration

$$\frac{\mathbf{E}_{\mathbf{H}}}{\mathbf{E}_{\mathbf{x}}} = \frac{\mathbf{B}}{\rho \mathbf{n} \mathbf{q}} \longrightarrow \mathbf{E}_{\mathbf{q}}.$$

So can be used as an electric and magnetic field sensors. Also can be used to measure the resistivity and conductivity of the semiconductors/materials

APPLICATIONS OF HALL EFFECT

- 1. To determine the resistivity and conductivity of semiconductors
- 2. To determine the Type of Semiconductors
- 3. To calculate the Carrier Concentration
- 4. To determine the Mobility (Hall Mobility)
- 5. To measure Magnetic Flux Density

Commercially available devices

- 1. Position and motion sensors
- 2. IC switch
- 3. GPS
- 4. Fuel injector
- 5. Sensors to detect rotation speed

In the presence of large magnetic field strength and low <u>temperature</u>, one can observe the quantum Hall effect, which is the quantization of the Hall resistance.

This is now the official standard for electrical resistance.

