Unit-V
Solid State Physics

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UNIT VI: Solid state physics

Syllabus

Free electron theory (Introduction), diffusion and drift current (qualitative);

Fermi energy, Fermi-Dirac distribution function;

Semiconductors and insulators, Fermi level for intrinsic and extrinsic semiconductors;

Band theory of solids -formation of allowed and forbidden energy bands,

Concept of effective mass -electrons and holes, Direct and indirect band gap semiconductors;

Hall effect (with derivation);

STATES OF MATTER

- **SOLID**
  - **Amorphous**
  - **Crystalline**
- Liquid
- Gas
- Plasma

1. **Metal** ($\sigma > 10^3$ S/cm)
2. **Insulator** ($\sigma < 10^{-8}$ S/cm)
3. **Semiconductor** (10^{-8} S/cm $< \sigma < 10^3$ S/cm

Conductivity of metal explained soon after the discovery of electron…

Thomson in 1897 when he was studying the properties of cathode ray. **J. J. Thomson** won **Nobel Prize** in 1906 for discovering the elementary particle electron.

Properties of metals

1. Ohm's law (1827): In steady state current density **J**, is proportional to the applied electric field **E**

$$J \,\alpha\, E \quad or\; J = \sigma E$$ **σ** Is the electrical conductivity

2. At low temperature resistivity **ρ** ($\rho = \frac{1}{\sigma}$) is proportional to the fifth power of absolute temperature

$$\rho \,\alpha\, T^5$$

3. For most of the metal, resistivity is inversely proportional to the pressure

$$\rho \,\alpha\, \frac{1}{P}$$

Properties of metals

4. Metals posses high thermal conductivity (**K**) and electrical conductivity (**σ**) and; and the ratio of these conductivities is directly proportional to the absolute temperature

$$\frac{K}{\sigma} \propto T \qquad \textit{Wiedemann-Franz Law}$$

5 . Metals have positive temperature of resistivity.. Resistance increases with temperature.. Resistance of certain metals vanishes at absolute temperature and they exhibit the phenomenon of super conductivity (**1911 ,Kamerlingh Onnes**)

All these were known.. The question 'why' was not answered until JJ Thomson discovered electron in 1897.. See superconductivity in 5, it came at a later stage and explained even much later in 1950 s.. **Cooper pairs**

Free electron theory of metals

Developed in three stages

Stage 1. Classical free electron theory

Drude and Lorentz developed this theory in 1900. According to this theory, the metals containing free electrons obey the laws of classical mechanics

Stage 2: The quantum Free Electron theory

Sommerfield developed this theory during 1928. according to this theory, free electrons obey quantum laws

Stage 3: The Band theory of solid

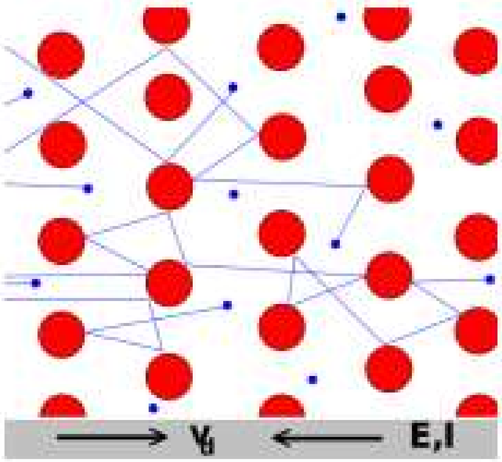
Bloch developed this theory in 1928. According to this theory, the free electrons interaction with solid lattice considered. The concept of hole, origin of band gap and effective mass of electron are the special features of this theory of metal.

Classical free electron theory

We will see how they explained many properties of metals prior to Quantum mechanics.. In fact QM was getting a shape during that period ☺

Lorentz - Drude Theory: known as the classical free electron theory (1900-1905)

Ion cores of atoms immobile (core electrons remain tightly bound to nucleus); valence electrons are loosely bound and in metal can wander around the entire metal – form the conduction electrons (referred from now on simply as electrons.)



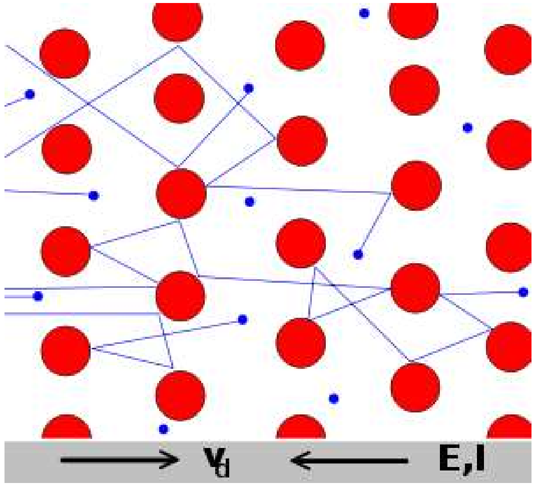
Electrons as gas molecules or electron gas and container is the metal lattice made of nucleus and core electrons- immobile positive ions

Assumptions (or salient features): Classical free electron theory

- The valence electrons of atoms are free to move about the whole volume of the metal like the molecules of a perfect gas in a container.
- The free electrons move in random direction and collide with either positive ions fixed to the lattice or the other free electrons. All the collisions are elastic in nature i.e., there is no loss of energy.
- The free electrons obey the classical Maxwell-Boltzmann distribution law.
- When the electric field is applied to the metal, the electrons are accelerated in the direction opposite to the direction of applied electric field.
- In the absence of the field. the energy associated with an electron at temperature T is given by (3/2) kT.

  It is related to kinetic energy equation $(3/2)kT = 1/2mv_{th}^2$.

  Here v_{th} is the thermal Velocity.



Classical free electron theory

1. Electrical conductivity

If we apply an electric field **E** to a metal, the force **F** experience by an electron of charge **e** and mass **m** is given by

$$F = eE = ma \qquad \therefore a = \frac{eE}{m} \longrightarrow \text{Eq.1}$$

Where **a** is the acceleration,

Now, the average velocity of the electron, called Drift velocity, with which it drift in the opposite direction of the applied electric field.

$$v_d = \frac{\sum v}{N} = \frac{\sum u+at}{N} = \frac{eE}{m}\tau \longrightarrow \text{Eq.2}$$

But, $J_d = nev_d$ where 'n' is the number electron per unit volume of the metal.

$$J_d = ne\frac{eE}{m}\tau = \frac{ne^2\tau}{m}E$$

drift Current density, Ohm's Law

Classical free electron theory

1. Electrical conductivity

(or) $J_d = \sigma E$

Where $\sigma = \dfrac{ne^2\tau}{m}$ ⟶ Eq.3

Mean free path λ

$v\tau = \lambda$

$\sigma = \dfrac{ne^2\lambda}{mv}$ ⟶ Eq.4

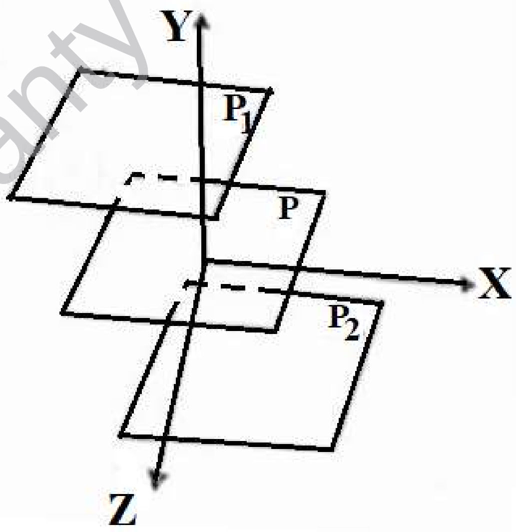
$\dfrac{1}{2}mv^2 = \dfrac{3}{2}kT$

$\sigma = \dfrac{ne^2\lambda}{3kT}v$ ⟶ Eq.5

Classical free electron theory

2. Thermal conductivity

Thermal conduction is the transfer of heat (internal energy) by microscopic collisions of particles and movement of electrons within a body.



The law of heat conduction, also known as Fourier's law,

Net transfer of heat energy, Q from the plane 'P_1' to 'P_2' through the plane 'P' per unit area and unit time is

$$Q = \frac{T_1 - T_2}{2\lambda} K \quad \text{Eqn.(a)}$$

Where **K** is the thermal conductivity of metal and λ is the mean free path. (distance between P_1 to P or P to P_2) . Let see whether we can find relation for **K** like we did for **σ**

Classical free electron theory

2. Thermal conductivity

If '**n**' is the total number of electron per unit volume in the metal.. Then 'n/6' electrons moving in a particular direction say + y as in figure

Energy of each electron = 3/2 kT, where k is the Boltzman's const. and T is the temperature

Heat transfer rate from P_1 to P_2 = $\frac{nv}{6} \times \frac{3}{2} kT_1$

Heat transfer rate from P_2 to P_1 = $\frac{nv}{6} \times \frac{3}{2} kT_2$

Classical free electron theory

2. Thermal conductivity

Net transfer of energy from P_1 to P_2 through P

$$Q = \frac{nv}{6} \times \frac{3}{2} k\,(T_1 - T_2)$$

Eqn.(a)

$$Q = \frac{nvk}{4}\,(T_1 - T_2) \longrightarrow \text{Eqn.(b)}$$

$$Q = \frac{T_1 - T_2}{2\lambda} K$$

$$\frac{T_1 - T_2}{2\lambda} K = \frac{nvk}{4}\,(T_1 - T_2)$$

$$K = \frac{nvk\lambda}{2}$$

Relation for the thermal conductivity of a metal

Classical free electron theory

3. Ratio of K to σ

We already have the relation for thermal conductivity and electrical conductivity using classical free electron theory (Drude-Lorentz theory)

$$K = \frac{knv\lambda}{2} \qquad \sigma = \frac{ne^2\lambda}{3kT}v$$

Take the ratio

$$\frac{K}{\sigma} = \frac{\frac{knv\lambda}{2}}{\frac{ne^2\lambda v}{3kT}} = \frac{knv\lambda}{2} \times \frac{3kT}{ne^2\lambda v}$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k}{e}\right)^2 T = \text{constant}$$

Nothing but the Wiedemann-Franz law

DIFFUSION CURRENT

Diffusion currents : charge moving from higher concentration to lower concentration

Can diffusion current occur in conductor?

Diffusion currents occur in metals. There are also contact potentials (charge redistribution from metals with a high Fermi energy to those with a lower Fermi energy). The balance of these currents is temperature-sensitive, which gives us the property known as the **Seebeck effect**, and useful devices called thermocouples.

But we have to learn Fermi level and quantum free electron theory

Failures of Classical free electron theory

1. Completely failed to explain the heat capacity and the paramagnetic susceptibility of conduction electrons and its temperature dependence
2. Could not explain the long mean path at low temperature
3. Unable to predict the correct dependence of resistance on the temperature
4. Could not explain why only some materials are metallic
5. Why resistivity metal increases with temperature while it decreases in semiconductor and insulator? No answer with Classical free electron theory
6. Why radiation does not affect the resistivity of metals but the resistivity of semiconductor decreases?
7. Why resistivity of metal increases with impurity while that of semiconductor decreases?

Furthermore it uses Maxwell-Boltzmann statistics which assume all free electron participate in the thermal conduction…. not right..

The one near Fermi level participate and Fermi-Dirac statistics must be applied.. And so quantum theory come to play its part

2nd stage... Sommerfeld free electron theory of metals…Used quantum mechanics and Fermi-Dirac statistics