COURSE NAME : DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

Time Allowed: 02:00 hrs

Max. Marks: 70

Read the following instructions carefully before attempting the question paper.

1. Match the Paper Code shaded on the OMR Sheet with the Paper code mentioned on the question paper and ensure that both are the same.
2. This question paper contains 70 questions of 1 mark each. 0.25 marks will be deducted for each wrong answer.
3. Do not write or mark anything on the question paper except your registration no. on the designated space.
4. Submit the question paper and the rough sheet(s) along with the OMR sheet to the invigilator before leaving the examination hall.

Q1. The differential equation of the form $Mdx + N\,dy = 0$ is said to be exact if

a. $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ b. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ c. $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$ d. $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} = 0$

Q2. If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, Then Integrating Factor is

a. $e^{-\int f(x)dx}$ b. $e^{\int f(x)dx}$ c. $f(x)\ e^{\int f(x)dx}$ d. $\int f(x)e^{f(x)}\,dx$

Q3. Integrating Factor of equation $(1+xy)y\,dx + (1-xy)x\,dy = 0$ is

a. $\frac{1}{xy}$ b. $\frac{1}{xy^2}$ c. $\frac{1}{x^2y}$ d. $\frac{1}{x^2y^2}$

Q4. The differential equation $x(1+y^2)dx + y(1+x^2)dy = 0$ is

a. Non-Exact Differential Eqaution b. Solvable for y c. clairaut's equation d. Exact Differential Equation

Q5. $d\left(\tan^{-1}\frac{y}{x}\right) =$

a. $\frac{xdy-ydx}{x^2+y^2}$ b. $\frac{xdy+ydx}{x^2+y^2}$ c. $\frac{ydx-xdy}{x^2+y^2}$ d. $\frac{ydx+xdy}{x^2+y^2}$

Q6. If Differential Equation $M\,dx + N\,dy = 0$ is homogeneous and $Mx + Ny \neq 0$ then I.F. is

a. $\frac{1}{Mx-Ny}$ b. $\frac{1}{Mx+Ny}$ c. $x^m y^n$ d. None of these

Q7. Which of the following function are linearly independent?

a. $\sin x, \sin 2x, \sin 3x$ b. $2x, 6x+3, 3x+2$ c. $\log x, \log x^2, \log x^3$ d. None of these

Q8. Which of the following function are linearly dependent?

a. $\log x, \log x^2, \log x^3$ b. $\sin x, \sin 2x, \sin 3x$ c. $x-2x, 3x^2+x+2, 4x^2-x+1$ d. None of these

Q9. The solution of equation $4y'' - 8y' + 3y = 0; y(0) = 1, y'(0) = 3$

a. $y(x) = e^{\frac{3x}{2}} - e^{\frac{x}{2}}$ b. $y(x) = [5e^{\frac{3x}{2}} - 3e^{\frac{x}{2}}]\ /2$ c. $y(x) = 5e^{\frac{3x}{2}} - 3e^{\frac{x}{2}}$ d. $y(x) = [e^{\frac{3x}{2}} - e^{\frac{x}{2}}]/2$

Q10. The general solution of the differential equation $y'' - 4y' - 12y = 0$

a) $y(x) = Ae^{-6x} + Be^{-2x}$ b) $y(x) = Ae^{6x} + Be^{2x}$ c) $y(x) = Ae^{-6x} + Be^{2x}$ d) $y(x) = Ae^{6x} + Be^{-2x}$

Q11. The general solution of the differential equation $9y'' - 12y' + 4y = 0$ is

a. $(A+Bx)e^{\frac{2x}{3}}$ b. $(Ax+Bx)e^{\frac{2x}{3}}$ c. $Ae^{\frac{2x}{3}} + Be^{\frac{2x}{3}}$ d. None of these

Q12. Solve $(D^2 - 6D + 18)y = 0$

a. $A\cos 3x + B\sin 3x$ b. $e^{-3x}(-A\cos 3x - B\sin 3x)$ c. $e^{3x}(A\cos 3x + B\sin 3x)$ d. $e^{3x}(A\cos 3x - B\sin 3x)$

Q13. The particular integral of the differential equation $(D^3 - D)y = e^x + e^{-x}$

(a) $\frac{e^x+e^{-x}}{2}$ (b) $x\left(\frac{e^x+e^{-x}}{2}\right)$ (c) $x^2\left(\frac{e^x+e^{-x}}{2}\right)$ (d) $x^2\left(\frac{e^x-e^{-x}}{2}\right)$

Q14. The resultant second order differential equation in terms of y_2 for the two system of first order differential equations $y_1' + 2y_2 - 2y_1 - y_2 = e^{2t}$, $y_2' + y_1 - 2y_2 = 0$, is

(a) $y_2'' - 6y_2' + 5y_2 = -e^{2t}$ (b) $y_2'' + 6y_2' + 5y_2 = -e^{2t}$ (c) $y_2'' - 6y_2' + 5y_2 = e^{2t}$ (d) $y_2'' + 6y_2' + 5y_2 = e^{2t}$

Q15. The solution of differential equation $2x^2y'' + 3xy' - y = x$ which satisfies the given conditions $y(1) = 1, y(4) = \frac{41}{16}$ is

(a) $y = \frac{1}{4}\left(\sqrt{x} + \frac{1}{x}\right) + \frac{x}{2}$ (b) $y = \frac{1}{4}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) + \frac{x}{2}$ (c) $y = \frac{1}{4}\left(\sqrt{x} + \frac{1}{x}\right) + \frac{x^2}{2}$ d) None of these

Q16. If $D = \frac{d}{dx}$, then $\frac{1}{(x^2D^2+2)} 16x^3$ is equal

a) $\frac{1}{2}x^3$ b) $2x^3$ c) $\frac{1}{4}(\log x)^3$ d) $\frac{1}{4}x^3$

Q17. The solution of differential equation $x^2y'' - xy' + 2y = 6$ which satisfies the given conditions $y(1) = 1, y'(1) = 2.$

(a) $y = x[2\sin(\ln x) - 3\cos(\ln x)] + 3$ (b) $y = x[4\sin(\ln x) - 2\cos(\ln x)] + 3$

(c) $y = x[4\sin(\ln x) - 3\cos(\ln x)] + 3$ (d) $y = x[2\sin(\ln x) - \cos(\ln x)] + 3$

Q18. Which suitable transformation of independent variable should be used to covert the given differential equation $x^3y''' + x^2y'' + xy' - y = 24x^2$ into a linear differential equation with constant coefficients?

(a) $x = e^t$ (b) $x = \log t$ (c) $x = (e^t - 2)$ (d) None of these

Q19. Partial differential equation of $z = f(x + ay)$ after eliminating function 'f' will be

a. ap=q b. ap=-q c. aq=p d. aq+p=0

Q20. Degree of equation $r^2 + 2s - t^2 = 0$ is

a. One b. Two c. Three d. None of these

Q21. The partial differential equation by eliminating arbitrary constants a and b from $z = ax^2 - by^2$ *is*

a. $z = px + qy$ b. $2z = px - qy$ c. $2z = px + qy$ d. $z = px^2 - qy^2$

Q22. Equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ is of order

a. One b. Two c. Three d. None of these

Q23. Differential equation $Ar + Bs + Ct + f(x, y, z, p, q) = 0$ is hyperbolic if

a. $B^2 - 4AC < 0$ b. $B^2 - AC < 0$ c. $B^2 - 4AC > 0$ d. $B^2 - AC > 0$

Q24. Differential equation $Ar + Bs + Ct + f(x, y, z, p, q) = 0$ is parabolic if

a. $B^2 - 4AC = 0$ b. $B^2 - AC = 0$ c. $B^2 - 4AC > 0$ d. $B^2 - AC > 0$

Q25. Equation $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ *where* $(x, y) \neq (0,0)$ is

a. Parabolic b. elliptic c. hyperbolic d. None of these

Q26. Equation $2r + 4t - 2 = 0$ *is*

a. Elliptic b. Hyperbolic c. Parabolic d. None of these

Q27. P.D.E. of first order is represented as

a. $f(x, y, p, q) = 0$ b. $f(x.y.z.p, q, r, s, t) = 0$ c. $f(x, y, z, p, q) = 0$ d. $f(x, y, z, p) = 0$

Q28. The solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 0$ by separation of variable, we assume

a) $z = X(x) + Y(y)$ b) $z = X(x) - Y(y)$ c) $z = X(x).Y(y)$ d) $z = X(x).Y(t)$

Q29. The solution of the P.D.E. $U_{tt} = U_{yy}$ is

a) $\sin(x - t)$ b) $\sin(x - \pi t)$ c) $\sin(x + t)$ d) None of these

Q30. The solution of the Laplace equation by separation of variables is

a) $(c_1\cos kx + c_2\sin kx)(c_3\cos ckt + c_4\sin ckt)$ b) $(c_1\cos kx + c_2\sin kx)(c_3e^{ky} + c_4e^{-ky})$

c) $(c_1\cos kx + c_2\sin kx)c_3e^{-k^2t}$ d) $(c_1x + c_2)c_3$

Q31. Which of the following is the solution of $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$

a) $u(x, y) = (2x + y)^3$ b) $u(x, y) = (y - 2x)e^{y-2x}$ c) $u(x, y) = (y + 2x)e^{y-2x}$ d) $u(x, y) = e^{y-2x}$

Q32. Which one of the following is correct?

a) Wave equation is elliptic and Laplace equation is Hyperbolic
b) Wave equation is Hyperbolic and Laplace equation is elliptic
c) Wave equation and Laplace equation are elliptic
d) Wave equation and Laplace equation are Hyperbolic

Q33. The general solution of the equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, $u(0,t) = 0$, $u(l,t) = 0$, $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$, $u(x,0) = f(x)$ is

a) $\sum b_n \sin\frac{n\pi x}{l} e^{c^2n^2\pi^2t/l^2}$ b) $\sum b_n \sin\frac{n\pi x}{l} e^{-c^2n^2\pi^2t/l^2}$

c) $\sum b_n \sin\frac{n\pi x}{l} e^{-n^2\pi^2t/l^2}$ d) None of these

Q34. Let u(x, t) satisfy $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $u(0,t)=0$, $u(1,t)=0$,, $u(x,0)=\sin \pi x$ than

a) $u(x,t)=\sum \sin(\pi x)\, e^{-n^2\pi^2 t}$

b) $u(x,t)=\sum \sin(\pi x)\, e^{-n\,\pi^2 t/l^2}$

c) $u(x,t)=\sin(\pi x)\, e^{-\pi^2 t}$

d) None of these

Q35. The second order P.D.E. $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y^2}=0$ is

a) Elliptic if x<0,y>0 b) Hyperbolic if x<0,y<0 c) Elliptic if x>0,y>0 d) Hyperbolic if x>0,y>0

Q36. If A is constant vector and $R=3xI+3yJ+3zK$ then $grad(3A.R)=$

(a) A (b) 27A (c) 9A (d) 3A

Q37. If A is constant vector and $R=xI+yJ+zK$ then $A\times Curl\,R=$

(a) A (b) 0 (c) 1 (d) None of these

Q38. If A is constant vector and $R=xI+2yJ-2zK$ then $Curl\,R=$

(a) A (b) 2 (c) 1 (d) None of these

Q39. The unit normal vector to the surface $x^2-y^2+z=0$ at the point (1, 1, -1) is

(a) $(2i-2j+1k)/\sqrt{9}$ (b) $(2i+2j+1k)/\sqrt{9}$ (c) $(2i-2j+1k)/\sqrt{19}$ (d) $(2i-2j-1k)/\sqrt{9}$

Q40. The normal vector to the surface $x^2-2y^2+2\log z=0$ at the point (1, -1, 1) is

(a) $(2i-2j+1k)/\sqrt{18}$ (b) $(2i+4j+1k)/\sqrt{24}$ (c) $(2i-2j+1k)/\sqrt{19}$ (d) $(2i+4j+2k)/\sqrt{24}$

Q41. Unit normal vector to the surface $A=x-y-z$ at (1, 1, 1)

(a) $(i+j-k)/\sqrt{3}$ (b) $(i+j+k)/\sqrt{3}$ (c) $(i-j+1k)/\sqrt{3}$ (d) None of these

Q42. Unit normal vector to the surface $A=x^2+y^2-3e^z$ at (2, -1, 0)

(a) $(i+j+k)/\sqrt{3}$ (b) $(i+j+1k)/\sqrt{2}$ (c) $(4i-2j-3k)/\sqrt{23}$ (d) None of these

Q43. If A and B are two constant vectors then div(A x B) =

(a) 0 (b) 2(A+B) (c) A.B (d) A+B

Q44. If A and B are two constant vectors then Curl (A x B)+ A x B =

(a) 0 (b) 2(A+B) (c) A.B (d) A x B

Q45. The length of the Helix traced by $r(t)=a\cos t\ I+a\sin t\,J+ctK, a>0, 0\le t\le\pi$ is

(b) $\pi(a^2+c^2)^{1/2}$ (b) $2\pi ac$ (c) $4\pi(a^2+c^2)^{1/2}$ (d) πac

Q46. The length of the curve traced by $r(t)=a\cos t\ I+a\sin t\,J,\ a>0, 0\le t\le\frac{\pi}{4}$ is

(a) $2\pi a^2$ (b) $\frac{\pi a}{2}$ (c) $4\pi a^2$ (d) $\frac{\pi a}{4}$

Q47. The parametric representation of the curve x+z=3, y-z=0 is

(a)x=3-2t , y=t, z=t (b) x=0 , y=3/2, z=3/2 (c) x=3-t, y=t, z=t (d) 3x=y=z

Q48. The parametric representation of the curve y=3x+5 is

(a)x=3-2t , y=t (b) x=t , y=5t+3 (c) x=1, y=8 (d) none of these

Q49. The position vector of a moving particle is $\vec{r}(t)=(\cos t+\sin t)\hat{\imath}+(\sin t-\cos t)\hat{\jmath}+2t\hat{k}$. Its speed is

(a) $\sqrt{6}$ (b) t (c) $\sqrt{3}$ (d) 1

Q50. If the unit tangent vector of the curve $x=t, y=t^2, z=t^3$, at $t=1$ is $\alpha\hat{\imath}+\beta\hat{\jmath}+\gamma\hat{k}$ then $\beta=$

(a)0 (b) 2 (c) $2/\sqrt{14}$ (d) $1/\sqrt{14}$

Q51. If the equation of the normal to the surface $2x^2+y^2+2z=3$ at $(2,1,-3)$ is $\frac{x-2}{l}=\frac{y-1}{m}=\frac{z+3}{n}$ then choose the right option.

(a)$l=4, m=2, n=2$ (b) $l=2, m=0, n=1$ (c) $l=2, m=0, n=4$ (d) $l=4, m=1, n=1$

Q52. The directional derivative of $\emptyset(x,y,z)=x^2yz+4xz^2$ at the point (1,0,-1) in the direction of PQ where P=(1,2,-1) and Q=(-1,2,3) is

(a) $\frac{12}{\sqrt{5}}$ (b) $\frac{-28}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-25}{\sqrt{5}}$

Q53. If C is curve x=3cost , y=3sint. ds is

a. 2dt b. 3dt c. 4dt d. 5dt

Q54. If C is x^2=4y, z=x and 0≤ $x \leq 2$, parametric equation of C is

a. x=t,y=t^2/4,z=t b. x=t,y=t^2,z=t c. x=t,y=t,z=t d. x=t^2,y=t^2/4,z=t

Q55. If F is conservative vector field, F= gradf, then which of the following is correct

a. work done is independent of path of integration. b. work done is dependent of path of integration.
c. work done is constant d. Work done is zero.

Q56. By using Green's theorem $\oint(x^2+y^2)dx+(y+2x)dy$ over curve C is same as

a. $\iint(2-2y)dxdy$ over the region R b. $\iint(2+2y)dxdy$ over the region R
c. $\iint(2-y)dxdy$ over the region R d. $\iint(2+y)dxdy$ over the region R

Q57. Evaluate $\oint(e^{-x}(\sin y\, dx+\cos y\, dy))$ over curve line joining (0,0) and (π/2,0)

a. 0 b. 1 c. -1 d. 2

Q58. Which of the following is correct for a piecewise simple closed curve C bounding a region R

a. $\oint f(x,y)dx+g(x,y)dy=\iint(\frac{\partial g}{\partial x}-\frac{\partial f}{\partial y})dxdy$ b. $\oint f(x,y)dx+g(x,y)dy=\iint(\frac{\partial g}{\partial x}+\frac{\partial f}{\partial y})dxdy$
c. $\oint f(x,y)dx+g(x,y)dy=\iint(\frac{\partial f}{\partial x}-\frac{\partial g}{\partial y})dxdy$ d. $\oint f(x,y)dx+g(x,y)dy=\iint(\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y})dxdy$

Q59. Evaluate surface integral for v=3x^2i+6y^2j+zk over surface z=4 if D is the region bounded by x^2+y^2=16, z=0 z=4.

a. 64 b. π c. 64π d. 0

Q60. If v=2y^3i+x^3j+zk, curl V is

a. (3x^2-6y^2)i b. (3x^2-6y^2)j c. (3x^2-6y^2)k d. (3x^2+6y^2)i

Q61. Which of the following is correct

a. If F is conservative field, work done along any curve is zero.
b. If F is conservative field, work done along simple closed curve is zero.
c. If F is conservative field, surface integral over any surface is zero.
d. If F is conservative field, volume integral over any region is zero.

Q62. Evaluate surface integral of v=yzi+zxj+xyk over the surface of sphere $x^2+y^2+z^2=a^2$

a. 1 b. 0 c. 3a d. $3a^2$

Q63. Evaluate line integral of v=(2x-y)i-yz^{2}j-y^{2}zk over upper half of $x^2+y^2+z^2=1$ bounded by its projection o plane, given curl v=k.

a. a^2 b. πa^2 c. π d. 1

Q64. Evaluate line integral $\int(x^2+yz)dz$, over C given by x=t, y=t^2, z=3t and 1≤ $t \leq 2$.

a. 163/4 b. 163 c. 4 d. none of these

Q65. If S is any closed surface enclosing a volume V and F=axi+byj+czk, $\int F.n\, ds$ over S is

a. (a+b+c)V b. V c. (abc)V d. none of these

Q66. Evaluate $\int[x(y-z)i+y(z-x)j+z(x-y)k].ds$ over any closed surface S ,

a. 0 b. 3 c. 1 d. none of these

Q67. Evaluate $\int 3x^2dx+(2zx-y)dy+zdz$ over C given by x=2t,y=t,z=3t ,0≤ $t \leq 1$

a. 16 b. 6 c. 11 d. 4

Q68. The parametric equation of the curve x^2=4y, 3x^2=8z is

a. x=t, y=t^2/4, z=3t^2/8 b. x=t, y=t^2, z=3t^2 c. x=t, y=t/4, z=3t/8 d. x=t+1, y=t^2/4, z=3t^2/8

Q69. Which of the following is correct for Divergence theorem of Gauss

a. It gives a relation between surface integral and volume integral.
b. It gives a relation between line integral and volume integral.
c. It gives a relation between surface integral and line integral.
d. None of these

Q70. The Vector $\vec{V}=e^x\sin y\,\hat{\imath}+e^x\sin y\,\hat{\jmath}$ is

(a) Solenoidal (b) irrotational (c) rotational (d) none of these

-- End of Question Paper --