# PHY109: ENGINEERING PHYSICS
# Unit I: Electromagnetic theory

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# Outline

- Vectors
- Scalar and vectors fields
- Integral calculus
- Differential calculus: Concept of gradient, divergence and curl
- Gauss theorem and Stokes theorem
- Electrostatics: Gauss law, Poisson and Laplace equations, continuity equation
- Magnetostatics and time varying fields
- Displacement current, correction in Ampere circuital law, dielectric constant
- Maxwell’s equations

# Vectors



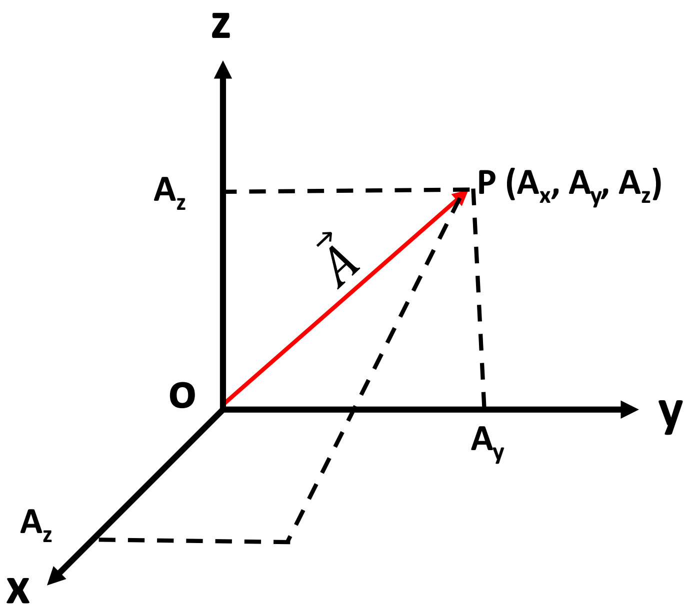
**Vectors: A physical quantity with direction and magnitude.**

$$\vec{A} = |\vec{A}|\hat{A}$$

**In component form:**

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$|\vec{A}|^2 = A_x^2 + A_y^2 + A_z^2$$




**Example:** position vector ($\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$), force, electric field etc.

- **Product of vectors:**

a) **Scalar (Dot) product:**

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|cos\theta = A_xB_x + A_yB_y + A_zB_z$$

b) **Vector (Cross) product:**

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|sin\theta\hat{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

# Field concept



**Field: A function that specifies a particular quantity everywhere in the region.**

**If the quantity is scalar=> Scalar field.**

Examples:

- Temperature distribution in a building,
- sound intensity in a theatre,
- electric potential in a region,
- refractive index of a stratified medium.

**If the quantity is vector=> Vector field.**

Examples:

- Gravitational force on a body in space,
- velocity of raindrops in the atmosphere,
- magnetic field in a region,
- electric field in a region.

# Integral calculus



- **Line integral:** Integration along a line/path *L.*

$$\int \vec{A}.\overrightarrow{dl} = \int A\cos\theta \, dl$$

where, $\overrightarrow{dl} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$ is a *differential line element of line/path L*.

- **Circulation:** Line integral in a closed path, where a closed path bounds surface.

$$\boldsymbol{circulation} = \oint \vec{A}.\overrightarrow{dl}$$

- **Surface integral:** Integration over a surface S.

$$\int \vec{A}.\overrightarrow{da} = \int A\cos\theta \, da$$

where, $\overrightarrow{da} = dxdy\hat{k}$
$= dydz\hat{\imath}$
$= dzdx\hat{\jmath}$
is a *differential area element of surface S*.

- It is a measure of flux through the surface S.

- Integration over closed surface that bounds volume is closed surface integral.

$$\oint \vec{A}.\overrightarrow{da}$$

- **Volume integral:** Integration of a scalar field over a volume *V*

$$\int \emptyset dv$$

where, $dv = dxdydz$ is a *differential volume element of volume V*.

# Differential calculus



- **Derivative:** Change of a function f(x) w.r.t. a variable x; $df = \frac{\partial f}{\partial x} dx$

- **For three variables, change of the function f(x, y, z) can be written as:**

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \qquad (1)$$

where, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ are *partial derivatives*.

- **Del operator ($\vec{\nabla}$):** Derivative vector operator.

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} \qquad (2)$$

Using equation (2), equation (1) can be rearranged as:

$$\boxed{df = \left(\frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}\right).\left(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right) = \vec{\nabla} f . \vec{dl}}$$

# Differential calculus

- **Gradient:** operated on a scalar field.

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

- Gradient of a scalar field is a vector field.
- It measures maximum increase of the function f.
- Its magnitude gives the slope (rate of increase) along its maximal direction.
- If $\vec{\nabla} f = 0$, the function f has either a maxima, minima or saddle point.

- **Divergence:** operated on a vector field.

$$\vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

where, $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

Divergence of a vector field is a scalar field.

It is a measure of spread of vector $\vec{A}$ from a point.

If $\vec{\nabla}.\vec{A} = 0$, $\vec{A}$ is solenoidal.

- **Curl:** operated on a vector field.

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

where, $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

Curl of a vector field is a vector field.

It is a measure of circulation of vector $\vec{A}$ around a point.

If $\vec{\nabla} \times \vec{A} = 0$,
$\vec{A}$ is irrotational.
$\vec{A}$ is a conservative vector field.

# Differential calculus



- **Directional derivative:** $\vec{\nabla} f\,.\,\hat{n}$.

- Divergence of curl of a vector is zero.

$$\vec{\nabla}.\vec{\nabla} \times \vec{A} = 0$$

- Curl of gradient of a scalar is zero.

$$\vec{\nabla} \times \vec{\nabla} f = 0$$

# Fundamental theorem

- **Gauss's theorem (Divergence theorem/Green's theorem):** Total outward flux (closed surface integral) of a vector field $\vec{A}$ is the volume integral of the divergence of the vector field $\vec{A}$**.**

$$\oint \vec{A}.\overrightarrow{da} = \int (\vec{\nabla}.\ \vec{A})\boldsymbol{dv}$$

- **Stokes' theorem (Curl theorem):** The line integral of a vector field $\vec{A}$ around a closed path *L* (circulation) is the surface integral of the vector field $\vec{A}$ over the surface bound by the path *L*.

$$\oint \vec{A}.\overrightarrow{dl} = \int (\vec{\nabla} \times \vec{A}).\overrightarrow{da}$$

# Electrostatics

- **Coulomb's law:**

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

Unit of force: N

- **Electric field:**

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{\vec{F}}{Q}$$

Unit of electric field: N/C or V/m

- **Electric potential:**

$$V = -\int \vec{E}.\vec{dl} = \frac{W}{q}$$

Unit of electric potential: V or J/C

# Electrostatics



- **Properties:**
  - Electric potential is path independent i.e. it depends on initial and final positions.
  - The line integral of electric field over a closed path is zero i.e.

$$\oint \vec{E}.\overrightarrow{dl} = 0$$

  - In differential form we can write, $\vec{\nabla} \times \vec{E} = 0$. This shows $\vec{E}$ is conservative field.
  - In differential form, electric field be written as negative of potential gradient i.e.

$$\vec{E} = -\vec{\nabla}V$$

In one dimension,

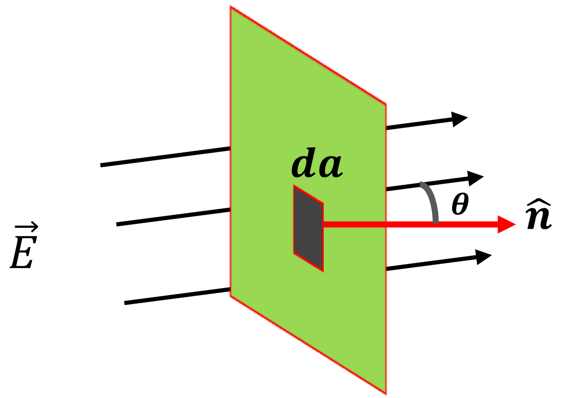
$$\vec{E} = -\frac{dV}{dx}\hat{x}$$

Thus unit of electric field is V/m.

# Electrostatics

- **Electric flux: Number of electric field lines passes through a surface S.**

$$\emptyset = \int \vec{E}.\overrightarrow{da} = \int E cos\theta \, da$$




Unit of electric flux: $Nm^2/C$

- **Gauss's law: The total flux through closed surface is the $1/\varepsilon_0$ times total charge enclosed in a volume bounded by the surface.**

$$\oint \vec{E}.\overrightarrow{da} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E}.\overrightarrow{da} = \frac{1}{\varepsilon_0}\int \rho dv$$

**Maxwell equation in integral form.**

$$q = \int \rho dv$$

$$\vec{\nabla}.\,\vec{E} = \frac{\rho}{\varepsilon_0},$$

**Maxwell equation in differential form.**

$where, \rho\ is\ volume\ charge\ density\ and\ unit\ is\ C/m^3.$

# Electrostatics

- **Poisson's equation:**

$$\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0} \qquad where, \vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- **If volume charge density, $\rho = 0$,**

$$\vec{\nabla}^2 V = 0$$

**Laplace equation**

- **Continuity equation**

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t}$$

If volume charge density $\rho$ is constant with time then $\vec{\nabla}.\vec{J} = 0$, which is the steady state condition.

# Magnetostatics (when E and B are independent)

- **Lorentz force:** Force on charge (Q) while moving with velocity ($\vec{v}$) in electric ($\vec{E}$) and magnetic ($\vec{B}$) fields:

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

- Magnetic force on current carrying conductor ($I\overrightarrow{dl}$) can be expressed as:

$$\vec{F}_{mag} = I(\overrightarrow{dl} \times \vec{B})$$

where, $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\overrightarrow{dl} \times \hat{r}}{r^2}$ is the magnetic field due to steady current in line (**Biot-Savart's Law**). The direction of the field is around the circular loop.
The unit of magnetic field: T or $N(Am)^{-1}$.

- If current between two parallel wires flows in **same direction**, magnetic ($\vec{F}_{mag}$) will be **perpendicular** to the wires and **opposite in direction** i.e. **attract**.

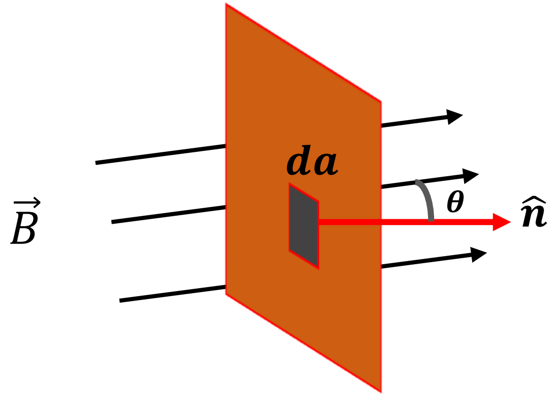
- If current between **two parallel wires** flows in **opposite direction**, magnetic ($\vec{F}_{mag}$) will be **perpendicular** to the wires and **same in direction** i.e. **repel**.

# Magnetostatics



- **Magnetic flux: Number of magnetic field lines passes through a surface S.**

$$\emptyset = \int \vec{B}.\overrightarrow{da} = \int B cos\theta\, da$$




Unit of magnetic flux: Wb or $Tm^2$.
The unit of magnetic field (B) also written as $Wb/m^2$.

- The net magnetic flux **through a closed surface is zero** i.e. the number of magnetic field lines entering the closed surface is equal to the number of lines coming out of the surface.

$$\oint \vec{B}.\overrightarrow{da} = 0$$ **Maxwell equation in integral form.**

$$\vec{\nabla}.\vec{B} = 0$$ **Maxwell equation in differential form.**

The magnetic field ($\vec{B}$) is called **solenoidal**.

# For time varying fields

- **Faraday's law:** work done in moving a test charge around a closed loop equals the rate of decrease of the magnetic flux through the enclosed surface.

$$\varepsilon = \oint \vec{E}.\overrightarrow{dl} = -\frac{d\emptyset}{dt}$$

$$\oint \vec{E}.\overrightarrow{dl} = -\int \frac{\partial \vec{B}}{\partial t}.\overrightarrow{da}$$

Maxwell equation in integral form.

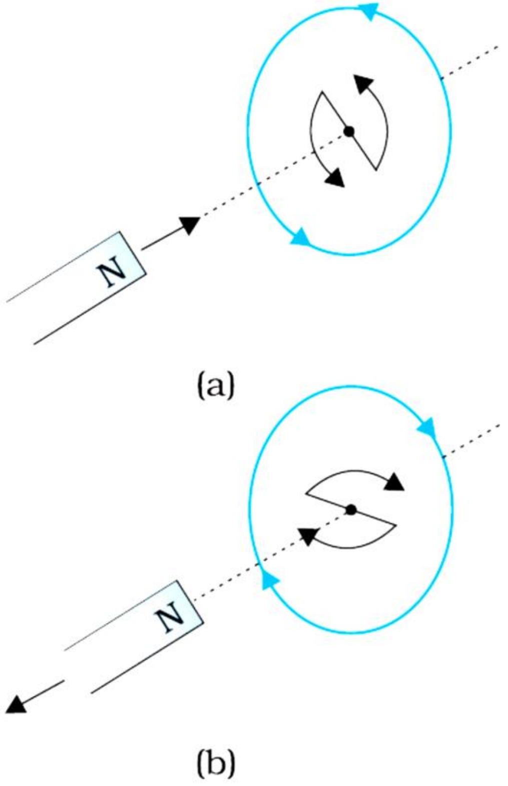
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Maxwell equation in differential form.

- For steady state condition i.e. $\vec{B}$ is constant, last equation reduces to:

$$\vec{\nabla} \times \vec{E} = 0$$

which is for electrostatic condition.




**Lenz's rule**

# For time varying fields

- **Ampere's circuital law:** Magnetic field ($\vec{B}$) in a loop due to a current carrying conductor is $\mu_0$ times of current ($I_{enc}$) enclosed by the loop.

$$\oint \vec{B}\,.\,\overrightarrow{dl} = \mu_0 I_{enc}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}, \qquad where, \mathrm{I} = \int \vec{J}\,.\,\overrightarrow{da}$$

- Ampere's circuital law fails to explain in time varying fields. For example charging/discharging of capacitor connected in a circuit.

- The equation is modified by Maxwell by introducing **displacement current** $\left(\vec{J}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$:

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_D)$$ **Maxwell equation in differential form.**

$$\oint \vec{B}\,.\,\overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E}\,.\,\overrightarrow{da}$$ **Maxwell equation in integral form.**

# Dielectric constant

- **For parallel plate capacitor, capacitance can be expressed as:**

$$c = \varepsilon_0 \frac{A}{d}, \qquad where, \varepsilon_0\ is\ permitivity\ of\ free\ space.$$

- Introducing a dielectric material the capacitance is:

$$c = \varepsilon \frac{A}{d}, \qquad where, \varepsilon\ is\ permitivity\ of\ dielectric\ material.$$

$$and, \varepsilon = \varepsilon_r \varepsilon_0$$

$\varepsilon_r$ is diectric constant of the material.

- The displacement current for dielectrics can be written as:

$$\vec{J}_D = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

# Maxwell's equation

## In differential form

$$\vec{\nabla}.\ \vec{E} = \frac{\rho}{\varepsilon_0}.$$

*where* $\rho$ *is volume charge density,* $q = \int \rho dv$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \vec{J}_D)$$

*where* $\vec{J}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ *is displacement current density*

*where* $\vec{J}$ *is current density,* $\mathrm{I} = \int \vec{J}.\overrightarrow{da}$

## In integral form

$$\oint \vec{E}.\overrightarrow{da} = \frac{1}{\varepsilon_0}\int \rho dv$$

$$\oint \vec{B}.\overrightarrow{da} = 0$$

$$\oint \vec{E}.\overrightarrow{dl} = -\int \frac{\partial \vec{B}}{\partial t}.\overrightarrow{da}$$

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \frac{d}{dt}\int \vec{E}.\overrightarrow{da}$$

# Maxwell's equation in free space

Maxwell's equations in free space where free charge density and current is zero; $\rho = 0$ & $J = 0$.

$$\vec{\nabla}.\vec{E} = 0 \qquad \vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

By decoupling the above linear differential equations for $\vec{E}$ and $\vec{B}$ we get a second order differential wave equation:

$$\vec{\nabla}^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \vec{\nabla}^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \times 10^8\ m/s \textit{ is speed of electromagnetic waves.}$$

**The solution of the wave equations is:**

$$\vec{E} = E_0 e^{i(kz - \omega t + \varphi_1)} \hat{n} \qquad \vec{B} = B_0 e^{i(kz - \omega t + \varphi_2)} \hat{m}$$

where *k*, *w*, $\varphi$ is wave vector, angular frequency and phase of the waves. z is the direction of propagation of the em wave. Unit vectors *n* and *m* are the polarization direction of electric and magnetic field.
$E_0$ and $B_0$ are the amplitude of the wave; **$E_0 = cB_0$; $c = w/k$.**

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