Lecture-3

Topic Inverse of a matrix and solution of linear system of equations.

Learning outcomes

1. Finding inverse using Gauss- Jordan Method
2. Linear system of equations - Homogeneous and Non- Homogeneous
3. Sol of linear system of equations using Cramer's Rule (Determinant Method)
4. Sol of linear system of equations using Gauss- Elimination Method (Rank).

Inverse of a matrix

Let A be non singular matrix $(|A| \neq 0)$,

then $A^{-1} = \dfrac{\text{adj}\, A}{|A|}$. (Only use for MCQ, if you want)

Gauss Jordan Method

$$[A \mid I] \xrightarrow[\text{operations}]{\text{apply row/ column}} [I \mid A^{-1}].$$

I is the identity matrix.

① Using Gauss Jordan Method, find the inverse of the following:

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 3R_1} = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -10 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{-1}{10} R_2} = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & +\frac{3}{10} & \frac{-1}{10} \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - 4R_2} = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & \frac{-2}{10} & \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & \frac{-2}{10} & \frac{4}{10} \\ 0 & 1 & \frac{3}{10} & \frac{-1}{10} \end{array}\right] = [I \mid A^{-1}]$$

$$A^{-1} = \begin{bmatrix} \frac{-2}{10} & \frac{4}{10} \\ \frac{3}{10} & \frac{-1}{10} \end{bmatrix}.$$

(2) Using Gauss Jordan method, find the inverse of the following

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Sol

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 3 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2R_2, \; \cancel{R_1 \to R}$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & \cancel{3}\ 1 & \cancel{0}\ -2 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_3, \quad R_2 \to R_2 + 2R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & \cancel{0}1 & 0 & 1 & -3 & 2 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right]$$

$R_3 \to \frac{R_3}{-1}$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right]$$

$= \cancel{A}\,[I \mid A^{-1}]$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

(3) Using Gauss Jordan method, find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Sol: $[A \mid I]$

$$= \left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & -3 & -5 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & -3 & -5 & 1 & -2 & 0 \end{array}\right]$$

$R_3 \to R_3 + 3R_2,\ R_1 \to \cancel{R_1 + R_2}\ R_1 - 3R_2$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & -3 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -2 & 3 \end{array}\right]$$

$R_1 \to R_1 + 3R_3, \quad R_2 \to R_2 - 2R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & 6 \\ 0 & 1 & 0 & -2 & 4 & -5 \\ 0 & 0 & 1 & 1 & -2 & 3 \end{array}\right]$$

$$= [I \mid A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 3 & -5 & 6 \\ -2 & 4 & -5 \\ 1 & -2 & 3 \end{bmatrix}.$$