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B

COURSE CODE :MTH165

COURS NAME :MATHEMATICS FOR ENGINEERS

Time allowed: 2hrs Max. Marks: 70

Read the following instructions carefully before attempting the question paper.

1. Match the Paper Code shaded on the OMR sheet with the paper code mentioned on the question paper and ensure that both are the same.
2. This question paper contains 70 questions of 1 mark each. 0.25 marks will be deducted for each wrong answer.
3. Do not write or mark anything on the question paper except your registration no. on the designated space.
4. Submit the question paper and the rough sheet(s) along with the OMR sheet to the invigilator before leaving the examination hall.

Q1. Which of the following signals is not periodic:
(a) $cos3t + cos5t$ (b)$cos2t\ cos4t$ (c) $sin4t\ sin6t$ (d)$e^{-9t}\ sin10t$

Q2. Which of the following conditions must be satisfied for the existence of Fourier series of a $f(x)$:
(a) $f(x)$ is periodic, single valued and finite.
(b) $f(x)$ has a finite number of finite discontinuity in any one period.
(c) $f(x)$ has at the most finite number of maxima and minima.
(d) All of the above.

Q3. The a_n in the fourier series expansion of $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$:
(a) $\frac{\pi^2}{6}$ (b) 0 (c) $\frac{1}{n}$ (d) $\frac{1}{n^2}$

Q4. The period of the function $f(x) = sin5x$ is:
(a) 2π (b) 5π (c)$\frac{\pi}{5}$ (d) $\frac{2\pi}{5}$

Q5. The value of the Fourier coefficient $a_0$in the expansion of $x \sin x$ in the interval $0 < x < 2\pi$ is:
(a) 1 (b) 0 (c) -2 (d) -4

Q6. The complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 \le x \le 1$:
(a) $\sum_{n=0}^{\infty} \frac{(1-in\pi)}{1+n^2\pi^2} sinh1 . e^{in\pi x}$ (b)$\sum_{n=-\infty}^{\infty} \frac{(-1)^n(1-in\pi)}{1+n^2\pi^2} sinh1 . e^{in\pi x}$
(c) $\sum_{n=-\infty}^{\infty} \frac{(-1)^n(1-in\pi)}{1+n^2\pi^2} sinh1$ (d)$\sum_{n=-\infty}^{0} \frac{(-1)^n(1-in\pi)}{1+n^2\pi^2} e^{in\pi x}$

Q7. A non-constant "periodic function" is given by a function which :
(a) has a period $T = 2\pi$ (b) has a period $T$and$f(t + T) = -f(t)$
(c) has a period $T$and$f(t + T) = f(t)$ (d) has a period $T = \pi$

Q8. The Fourier series expansion of the periodic function $f(x) = x$ in the interval $-\pi \le x \le \pi$ contains:
(a) sine terms (b) cosine terms (c)sine and cosine terms (d) None of these

Q9. The value of the Fourier coefficient $b_n$in the expansion of $\sqrt{1 - cosx}$in the interval$-\pi < x < \pi$ is:
(a) $\frac{4\sqrt{2}}{\pi n} - 1$ (b) 0 (c) $-\frac{4\sqrt{2}}{\pi(4n^2-1)}$ (d) $\frac{4\sqrt{2}}{\pi}$

Q10. Which of the following is neither odd nor even function of t:
(a)t^2 (b) $5t^3 + 4t^4$ (c) $\sin 2t^3$ (d) $t^3 + 6t$

Q11. Let A = $\begin{bmatrix} a & -1 & 4 \\ 0 & b & 7 \\ 0 & 0 & 3 \end{bmatrix}$ be a matrix with real entries. If the sum and product of all the eigen values of A are 10 and 30 respectively, then $a^2 + b^2$ equals:
(a) 29 (b) 40 (c) 58 (d) 65

Q12. The rank of a matrix C=AB, where A is non-zero matrix of order 3×1 and B is non-zero matrix of order 1×3 is:
(a) 0 (b) 1 (c) 2 (d) 3

Q13. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals:
(a)19 (b) 1/19 (c) -19 (d) -1/19

Q14. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then$A^{2'} - 5A + 8$ is equal to:
(a) 2I (b) 3I (c) I (d) 4I

Q15. If A be a 4 × 4 real matrix with determinant 5. Then the determinant of adj A is:
(a) 5 (b) 25 (c) 125 (d) none of these

Q16. The system of linear equations:
$x + y + z = 2,\ 2x + y - z = 3,\ 3x + 2y + kz = 4$ has a unique solution if
(a) $k = 0$ (b) -1< $k < 1$ (c) -2< $k < 2$ (d) $k \neq 0$

Q17. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{1+tanx}dx$ equals:
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

Q18. If $f'(x) = 6x^2$ and $f(1) = 2$, the value of $f(-1)$:
(a)4 (b)-2 (c) 8 (d) -1

Q19. The integral $\int \frac{2x+1}{(x+1)(x-2)}dx$ is equal to:
(a) $\frac{1}{3}\log|x-1| + \frac{1}{3}\log|x-2| + c$ (b) $\frac{5}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$
(c) $\frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$ (d) $\frac{5}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$

Q20. The integral $\int logx\,dx$ is equal to:
(a) $xlogx - x + c$ (b) $xlogx + x + c$ (c) $xlogx + c$ (d) None of these

Q21. Evaluate $\iiint_V (x^2 + y^2 + z^2)dxdydz$, where $V = \{(x, y, z): x^2 + y^2 + z^2 \le 1\}$:
(a) 4π (b) π (c) $\frac{7\pi}{2}$ (d) $\frac{4\pi}{5}$

Q22. Volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is :
(a) $\frac{\pi\, abc}{3}$ (b) $-\frac{\pi\, abc}{3}$ (c) $\frac{4\pi\, abc}{3}$ (d) $-\frac{4\pi\, abc}{3}$

Q23. If $z = x^2\tan^{-1}\frac{y}{x}, then \frac{\partial^2 z}{\partial x\partial y}$ at (1,1):
(a) $-\frac{y}{x}$ (b) 1 (c) 0 (d) $\frac{y}{x}$

Q24.Change the order of integration in $\int_0^2\int_1^{e^x} dxdy$:
(a) $\int_1^2\int_0^{e^x} dxdy$ (b) $\int_1^2\int_{e^2}^{x} dxdy$ (c) $\int_1^{e^2}\int_{logy}^{2} dxdy$ (d) $\int_0^2\int_1^{logy} dxdy$

Q25. Area enclosed using double integration by the ellipse $\frac{x^2}{1} + \frac{y^2}{9} = 1$ is:
(a) 3π (b) 5 (c) 2π (d) none of these

Q26. The value of $\int_0^1\int_x^{\sqrt{x}}(x^2 + y^2)dxdy$ equals:
(a) $\frac{1}{5}$ (b) $\frac{3}{35}$ (c) $\frac{1}{7}$ (d) $\frac{8}{35}$

Q27. Let V be the region bounded by the planes $x = 0, x = 2, y = 0, z = 0$ and $y + z = 1$. Then, the value of the integral $\iiint_V ydxdydz$is:
(a) $\frac{1}{3}$ (b) $\frac{3}{35}$ (c) $\frac{1}{7}$ (d) $\frac{8}{35}$

Q28. The volume of the sphere $x^2 + y^2+z^2 = 1$:
(a) $\frac{4}{3}\pi^3$ (b) $\frac{1}{\pi}$ (c) $\frac{4}{3}\pi$ (d) $\frac{4}{\pi}$

Q29. Change the order of integration in $\int_0^a\int_0^{\sqrt{2ay-y^2}} f(x,y)dxdy$:
(a) $\int_0^a\int_0^{\sqrt{2ax-x^2}} f(x,y)dydx$ (b) $\int_0^a\int_0^{\sqrt{2ay-y^2}} f(x,y)dxdy$
(c) $\int_0^{a\sqrt{2ay-y^2}}\int_0^a f(x,y)dxdy$ (d) $\int_0^a\int_{a-\sqrt{a^2-x^2}}^{a} f(x,y)dydx$

Q30. $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y + 2z)dzdydx$ is equal to:
(a) $\frac{2}{21}$ (b) $\frac{2}{25}$ (c) $\frac{1}{6}$ (d) $\frac{5}{3}$

Q31. If $x = acos^3\theta,\ y = asin^3\theta$, Thevalue of second order derivative of y w.r.t. x at $\theta = \frac{\pi}{6}$:
(a) $\frac{32a}{27}$ (b) $\frac{32}{27a}$ (c) $\frac{32a}{7}$ (d) $\frac{2a}{27}$

Q32. The integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7x\,dx$ equals:
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) 0

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Q33. If $4a + 2b + c = 0$, the equation $2ax^2 + 2bx + c = 0$ has atleast one real root lying in the interval:
(a) $(0, 2)$ (b) $(1, 2)$ (c) $(0, 1)$ (d) none of these

Q34. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3 y \frac{dy}{dx}$ equals:
(a) 0 (b) 1 (c) -1 (d) none of these

Q35. The limit $\lim_{t\to\infty}\left(1 + \frac{1}{t^2+2t+1}\right)^{t^2+3t+1}$:
(a) e-1 (b) e (c) 1 (d) 1/e

Q36. If the cone of maximum volume is inscribed in a given sphere, then the ratio of the height of the cone to the diameter of the sphere is:
(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

Q37. The coefficient of $(x-1)^2$ in the Taylor series expansion of $f(x) = xe^x$ about the point x=1:
(a) $\frac{e}{2}$ (b) $\frac{e}{4}$ (c) $1.5e$ (d) $3e$

Q38. A jet of an enemy is flying along the curve $y = x^2 + 2$. The shortest distance between the jet and the soldier where soldier is placed at the point (3, 2) is:
(a) $\sqrt{7}$ (b) 8 (c) 2 (d) $\sqrt{5}$

Q39. The limit $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$:
(a) -1 (b) 2 (c) 0 (d) 1

Q40. If $f: \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$ then choose the correct option:
(a) $f_x(0,0) = 0$ and $f_y(0,0) = 0$ (b) $f_x(0,0) = 1$ and $f_y(0,0) = 0$
(c) $f_x(0,0) = 0$ and $f_y(0,0) = 1$ (d) $f_x(0,0) = 1$ and $f_y(0,0) = 1$

Q41. If $z = \tan^{-1}\frac{y}{x}$, then the value of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$:
(a) $-\frac{y}{x}$ (b) 1 (c) 0 (d) $\frac{y}{x}$

Q42. If z is homogeneous function of x and y of degree n, then $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y}$:
(a) $\frac{\partial z}{\partial x}$ (b) $n\frac{\partial z}{\partial x}$ (c) $(n-1)\frac{\partial z}{\partial x}$ (d) $n(n-1)\frac{\partial z}{\partial x}$

Q43. For the function $f(x,y) = 3x^2 + 4xy + x^3 + y^2$, comment about the point (0,0):
(a) point of maxima (b) point of minima (c) neither maxima nor minima (d) not a critical point

Q44. The degree of the homogenenous function $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$:
(a) -1 (b) 1 (c) -3 (d) -2

Q45. If $z = xyf\left(\frac{x}{y}\right)$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$:
(a) 0 (b) -2z (c) 2z (d) $-z$

Q46. Change the order of integration $\int_0^1 \int_{4y}^4 e^{x^2}\, dxdy$:
(a) $\int_0^1 \int_x^{4x} e^{x^2}\, dydx$ (b) $\int_0^4 \int_0^{\frac{x}{4}} e^{x^2}\, dydx$ (c) $\int_0^2 \int_0^x e^{x^2}\, dydx$ (d) $\int_0^4 \int_{4y}^4 e^{x^2}\, dydx$

Q47. Evaluate $\iint xydxdy$ over the region A=$\{(x,y): 0 \le x \le 1, 1 \le y \le 2\}$:
(a) 0.75 (b) 2.5 (c) 2.25 (d) 3.5

Q48. Area lying inside the cardioid $r = 2(1 + cos\theta)$ and outside the circle $r = 2$:
(a) $\pi + 1$ (b) $\pi + 8$ (c) π (d) $\pi - 2$

Q49. If the triple integral over the region bounded by the planes $3x + y + z = 7,\ x = 0, y = 0, z = 0$ is given by $\int_0^7 \int_0^{\beta(x)} \int_0^{\alpha(x,y)} dzdydx$, then the function $\beta(x) - \alpha(x,y)$ is:
(a) $x + y$ (b) $x - y$ (c) x (d) y.

Q50. The value of $\int_0^\pi \int_0^x xsiny\, dxdy$:
(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi^2}{2} - 1$ (c) $\frac{\pi^2}{2} + 2$ (d) None of these

Q51. Minimum value of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$ is:
(a) 350 (b) 100 (c) 20 (d) 70

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Q52. Which of the following statement is not true:
(a) Extremum is a value which is either a maximum or minimum.
(b) Saddle point is a point, where the function is neither maximum nor minimum.
(c) Every stationary point is an extrema.
(d) Extrema occurs only at stationary points.

Q53. If $f(x,y) = xy\left(\frac{x^2-y^2}{x^2+y^2}\right)$, where $(x,y) \neq (0,0)$ and $f(0,0) = 0$, then choose the correct statement:
(a) $f_{xy}(0,0) = 1$ (b) $f_{xy}(0,0) = f_{yx}(0,0)$ (c) $f_{yx}(0,0) \neq f_{yx}(0,0)$ (d) $f_{yx}(0,0) = -1$

Q54. Evaluate $\frac{dz}{dt}$ at t=1, where $z = xy^2 + x^2y, x = at^2, y = 2at$:
(a) 26 (b) 26a (c) $36a^3$ (d) $26a^3$

Q55. If $f(x,y) = \frac{x+y}{x-y}$, find $\frac{\partial f}{\partial x}$ at (2,-1):
(a) 2 (b) -2 (c) 0 (d) 3

Q56. For the function $f(x,y) = x^2 + xy$, comment about the point (0,0):
(a) point of maxima (b) point of minima (c) neither maxima nor minima (d) not a critical point

Q57. Consider a function defined by $(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & otherwise \\ 0, & (x,y) = (0,0) \end{cases}$, then
(a) $f(x,y)$ is continuous at (0,0). (b) $\lim_{(x,y)\to(0,0)} f(x,y)$ exists.
(c) $f(x,y)$ is not differentiable at (0,0). (d) $f(x,y)$ is continuous but not differentiable.

Q58. If $z = e^x y^2, x = tcost, y = tsint$, compute $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$:
(a) $1 - \frac{\pi}{2}$ (b) $\pi^2 - \frac{\pi}{2}$ (c) $\pi^2 - \frac{\pi^3}{2}$ (d) $\pi - \frac{\pi^3}{8}$

Q59. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x^2-y^2}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$:
(a) $\sec u$ (b) 1 (c) 0 (d) $\tan u$

Q60. If $z = x^3 - xy + y^3$ and $x = rcos\theta, y = rsin\theta. Evaluate \frac{\partial z}{\partial \theta}$ at $\theta = \frac{\pi}{2}$:
(a) r^2 (b) $3r - 1$ (c) $-r^2$ (d) $r^2 - 2$

Q61. The area between the parabolas $y^2 = 4ax \, and \, x^2 = 4ay$ is:
(a) $\frac{16}{3}a$ (b) $\frac{16}{3a}$ (c) $\frac{16}{3}a^2$ (d) $\frac{a^2}{3}$

Q62. Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = 4$:
(a) 2 (b) 2.25 (c) 1.5 (d) 3

Q63. The value of the fourier coefficient b_n in half range sine series of $f(x) = x$ in $0 < x < 2$ is:
(a) $\frac{(-1)^n 4}{n^2\pi}$ (b) $\frac{4}{n^2\pi^2}[(-1)^n - 1]$ (c) $-\frac{4}{n\pi} - 1$ (d) $-\frac{(-1)^n 4}{n\pi}$

Q64. The Fourier series for the function $f(x) = x - x^2$ in the interval $-\pi < x < \pi$ contains:
(a) sine terms (b) cosine terms (c) sine and cosine terms (d) None of these

Q65. Using the concept of change of interval, the value of a_0 in the Fourier series expansion of
$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$:
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{2}$

Q66. Which of the following is an "odd" function of t:
(a) t^2 (b) $5t^2 + 4t^4$ (c) $\sin 2t + 3t$ (d) $t^3 + 6$

Q67. The behaviour of the Fourier series near a point of discontinuity is called:
(a) Gibbs phenomenon (b) frequency spectrum (c) superposition principle (d) D' Alembert phenomenon

Q68. The Fourier coefficient b_n in the expansion of x^2 in the interval (-p,p) is:
(a) 0 (b) 2 (c) $\frac{8}{3}$ (d) $\frac{1}{3}$

Q69. Choose the function $f(t), -\infty < t < \infty$, for which a Fourier series cannot be defined:
(a) 3 (b) $cos3t + 2$ (c) $e^{-3t}\sin(5t)$ (d) $sint$

Q70. The Fourier coefficient a_0 for the function given by :
$f(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 4, & 2 \le x \le 4 \end{cases}$:
(a) 8 (b) $\frac{1}{3}$ (c) $\frac{8}{3}$ (d) $\frac{16}{3}$

End of the Paper