Fundamentals of D.C circuits

An electric circuit is a closed loop fromed by resistor, capacitor inductor , energy source in which current flow.

Circuit terminology :

Nodes: A node is a point where two branches are connected

Simple node: If there are two branches connected at a node it is called simple node.

Principle node: If there are more than 2 branches connected at a node called principle node

A node is denoted by N.

Branches

A branch is defined as a segment between two nodes containing any circuit element like resistor , capacitor inductors, voltage source & energy source.

A segment b/w two nodes can be consider as a branch if and only if it contains a circuit.

Loops:

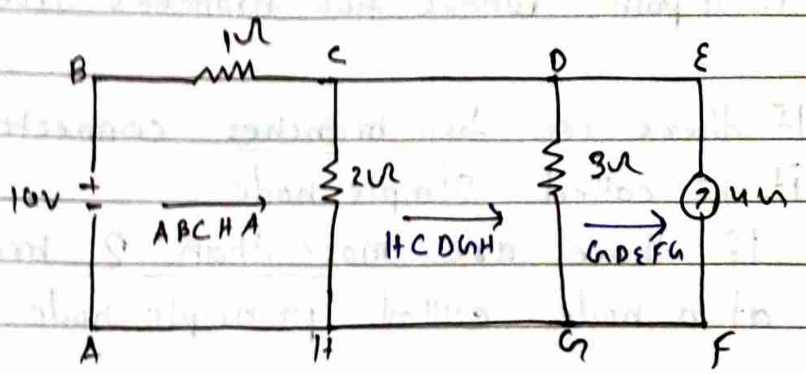
A loop is a closed path in circuit having same starting and terminating node. The loop that do not contain any other loop within it is called as independent loop or fesh.

Equation of node branches:

The nodes N, the branches B, and the independent loops M in a electric circuit must satisfy the equation of

$$\boxed{B = M+N-1}$$

Ques (i)



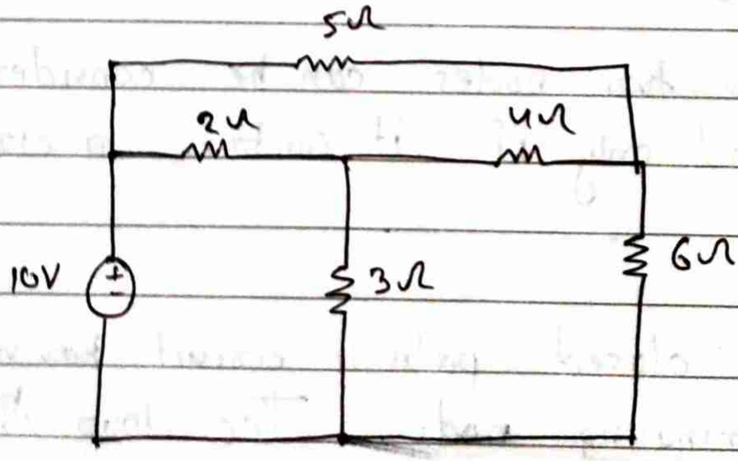
$B = M+N-1$

$5 = M+3-1$

$M = 3$.

All loops are not mesh but all mesh are loops.

Ques.



$B=6$

$M=3$

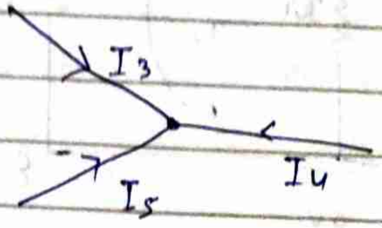
$N=4$

$B = M+N-1$

$6=6$.

Kirchoff's Current Law

In a electric circuit at a node, the sum of coming current is equal to the sum of outgoing current.



$I_4 = I_3 + I_5$.

It is also called Kirchoff's '1st' law.

$\boxed{\Sigma I = 0}$

Kirchoff's Voltage Law

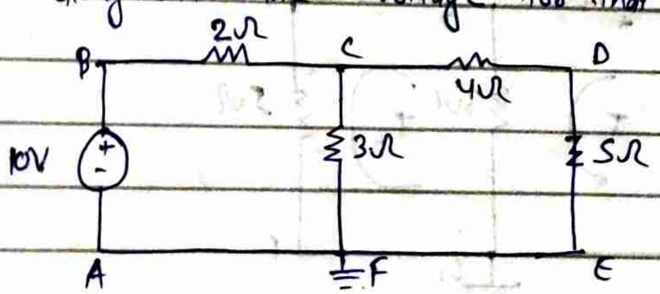
It states that in a close loop the sum of voltage is equal to zero. It is also called Kirchoff's 2nd Law.

$\boxed{\Sigma V = 0}$.

Mesh Analysis.

It uses K.V.L

Move with the current and ~~sign~~ write down the sign that comes first along with the voltage for that element

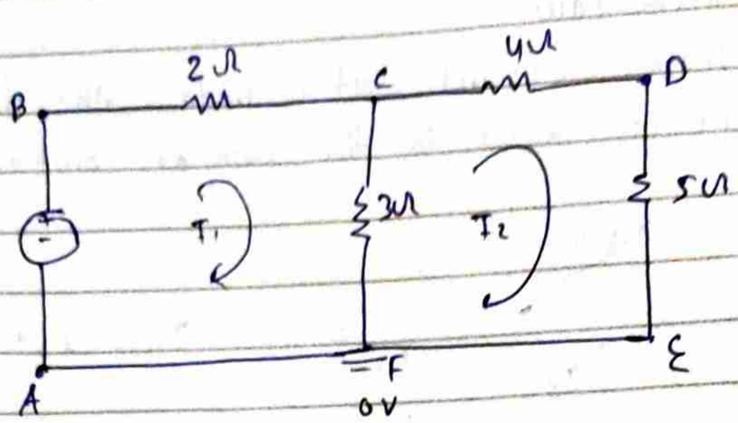


for the given circuit find out the no. of branches, loops, nodes meshes and also write down the mesh equation.

B = 5

M = 2 $\begin{cases} ABCFA \\ FCDEF. \end{cases}$ F is refrence node.

N = 4 $\begin{cases} B - \text{simple node} \\ C - \text{principle node} \\ D - \text{simple} \\ \text{[illegible]} - \end{cases}$



For loop (1) $\overrightarrow{ABCFA}$

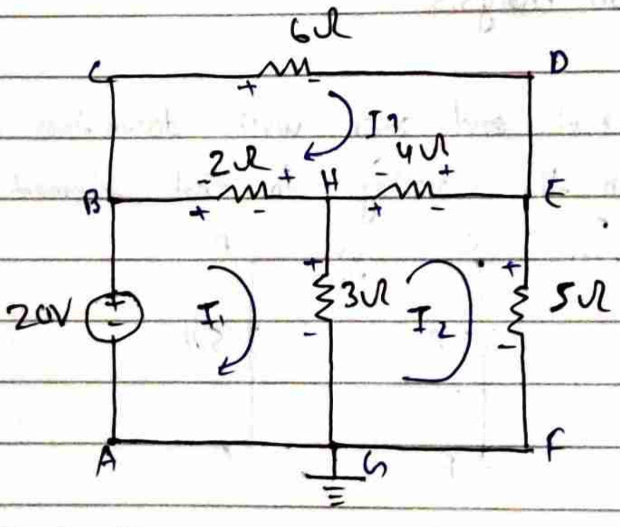
$$-10 - 2I_1 + 3(I_1 - I_2) = 0$$

For loop (2) $\overrightarrow{FCDEF}$

$$+3(I_2 - I_1) + 4I_2 + 5I_2 = 0$$

Note: For n mesh n equations.

Ques:



Write the mesh equations:

For I_1 $\overrightarrow{ABHGA}$

$$-20V + 2(I_1 - I_3) + 3(I_1 - I_2) = 0$$

For I_2.

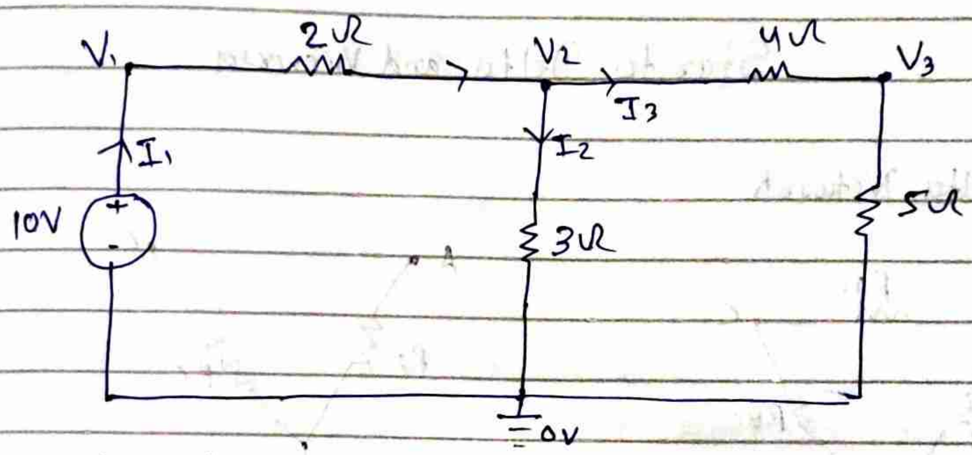
$$+3(I_2 - I_1) + 4(I_2 - I_3) + 5I_2 = 0$$

For I_3 $+6I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$

Nodal Analysis.

The nodal analysis use KCL to the node equation by which we can find the node voltages and all currents in the branches.

For the given circuit find out the number of nodes, node equations and ~~all the~~ currents in the branches



At node(2) by K.C.L

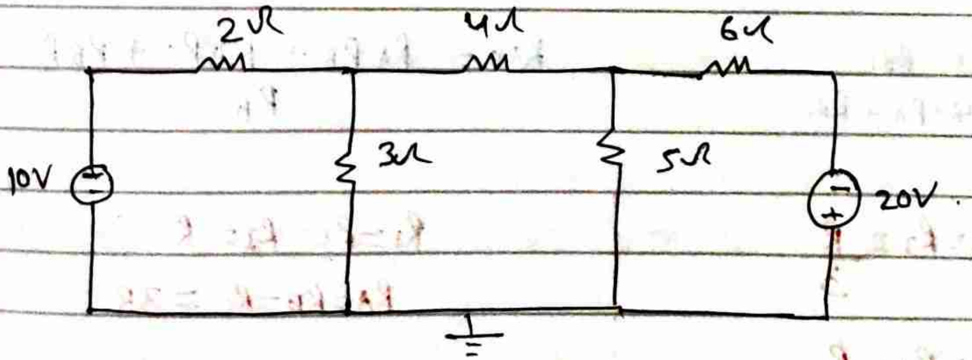
$$I_1 = I_2 + I_3$$

$$\frac{V_1 - V_2}{2} = \frac{V_2 - 0}{3} + \frac{V_2 - 0}{9}.$$

$$V_1 - 0 = 10V$$

$$V_1 = 10V.$$

Que:-



find B, m, N, L , mesh equation

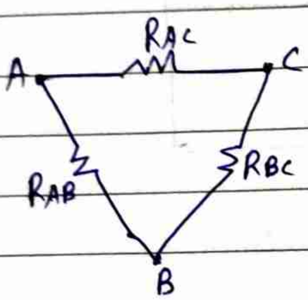
Resistance: Opposition offered by a material to the flow of electric current

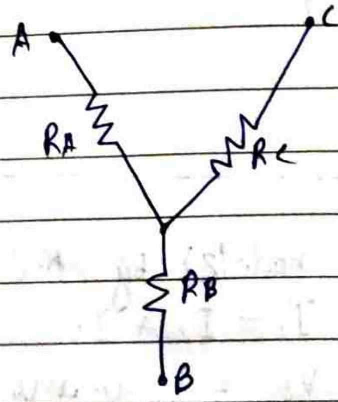
$$R = \frac{\rho l}{A}$$

If same resistance in series then $R_{eff} = nR$ and in parallel $\frac{R}{n}$.

Star to Delta and Viceversa

Delta Network.





delta to Y

$$R_A = \frac{R_{AB} \cdot R_{AC}}{R_{AB}+R_{AC}+R_{BC}}$$

$$R_{AB} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_C}$$

$$R_B = \frac{R_{AB} \cdot R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$$

$$R_{BC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_A}$$

$$R_C = \frac{R_{AC} \cdot R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$$

$$R_{AC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_B}$$

Note: $R_1 = R_2 = R_3 = \frac{R}{3}$

$R_A = R_B = R_C = R$

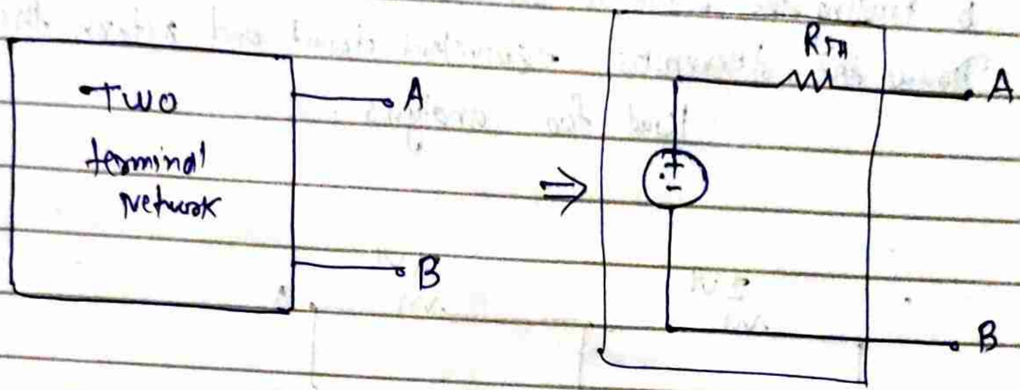
$R_1 = R_2 = R_3 = R$

$R_A = R_B = R_C = 3R$.

Network Theorem

Thevenin's theorem

Thevenin's theorem states that a circuit with two terminals can be replaced with an equivalent circuit consisting of a voltage source in series with a resistance or Impedance



The voltage source is called Thevenin's voltage source and its value is given by the voltage across the two open terminals of the circuit

The series resistance is called thevenin's resistance (R_{TH}) and it is given by looking back resistance at the two open terminal of the network. The looking back resistance is the resistance measured at the two open terminals of a circuit after replacing all the independent sources with zero value sources.

Steps to solve circuit using thevenin's theorem

Calculation of V_{TH}

1. Remove the component of interest. (Normally called the load resistor.) (R_L).
2. Mark the terminals as V_{TH} where the load was removed from.
3. Calculate the thevenin's voltage by finding the open circuit voltage across the load terminals.

Note: Remember that no current is flows on open circuit.

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Calculation of R_{TH}.

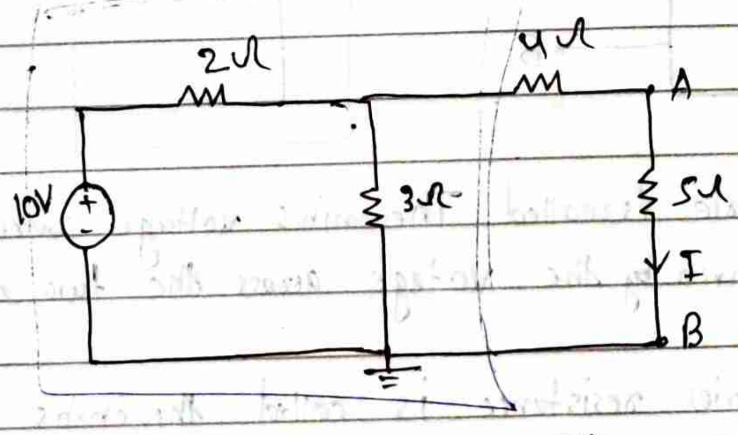
(1) Calculate the thevenins resistance R_{TH} by

a. marking the terminals as R_{TH} where the load was removed from and setting all sources to zero. (Voltage sources are replaced by a short circuit and current sources by an open circuit)

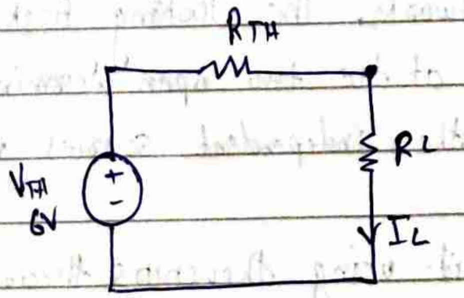
b. Finding the resistance b/w the two load terminals.

(2) Draw the thevenins equivalent circuit and return the load for analysis.

Ques:

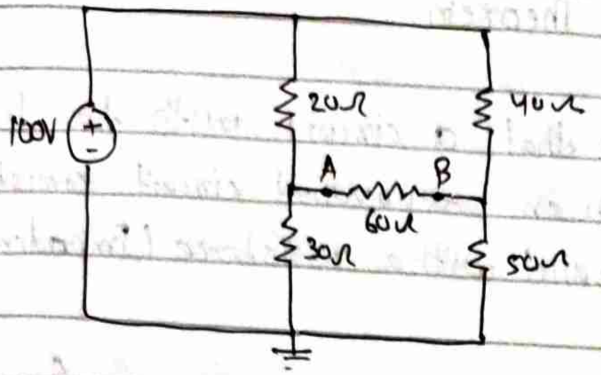


Calculate the current I by Thevenin's theorem



$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

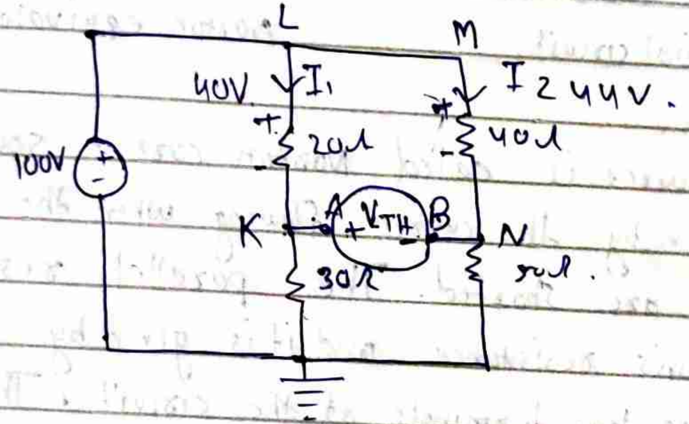
Ques':



Calculate the current in Branch AB by Thevenin's theorem

Sol^n: $R_L = 60\Omega$

Step(1) Remove Load resistance.



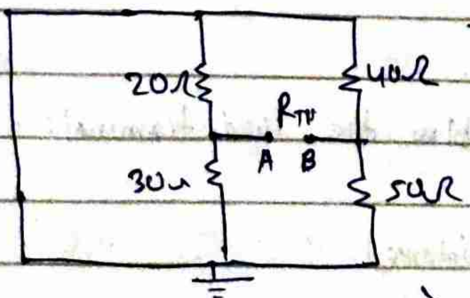
$I_1 = \frac{100}{20+30} = \frac{100}{50} = 2A$.

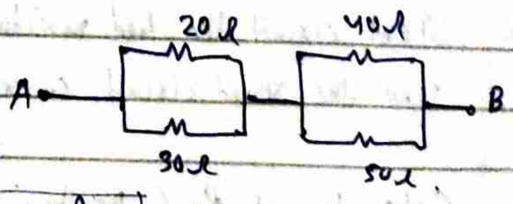
$I_2 = \frac{100}{90} = 1.1A$.

In the loop KLMNK $V_{TH} + 40 - 44 = 0$.

$V_{TH} + 40 - 44 = 0$

$\boxed{V_{TH} = 4V}$.





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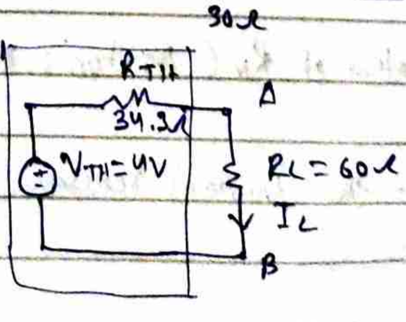
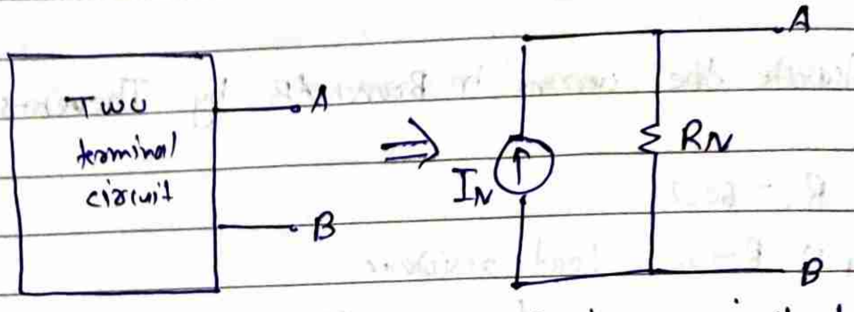


Norton's Theorem

Norton's theorem states that a circuit with two terminals can be replaced with an equivalent circuit consisting of an current source in parallel with a resistance (Impedance).



Original circuit → Norton equivalent ckt.

The current source is called Norton current source and its value is given by the current flowing when the two terminals of the circuit are shorted. The parallel resistance is called Norton's resistance and it is given by looking back resistance at the two terminals of the circuit. The looking back resistance is the resistance measured at the two opens terminals of a circuit after replacing all the independent sources by zero value sources.

Steps to solve circuit using Norton's Theorem

Calculation of I_n (Norton's Current)

a. Short circuit the load resistance.

b. Find the short circuit current b/w the load terminals.

Calculation of R_N (Norton's resistance):

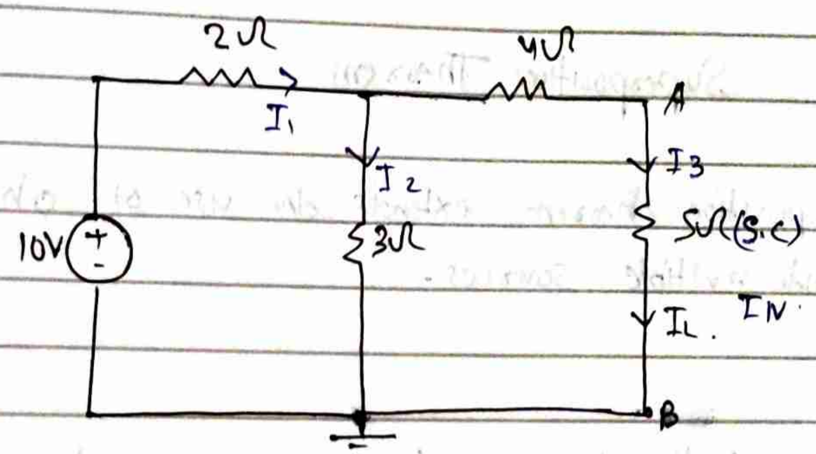
a. Calculate the Norton's resistance R_N by

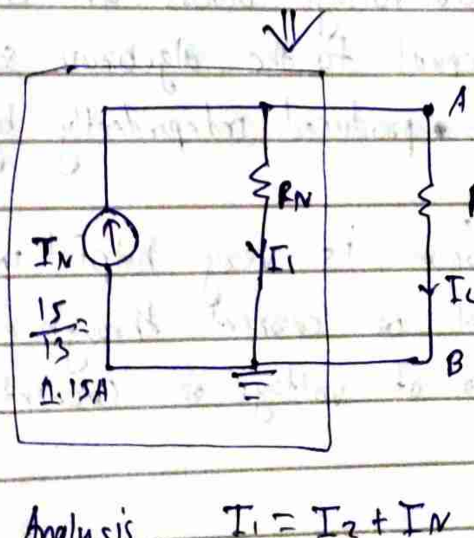
i. Marking the terminal as R_N where the load was removed from and setting all sources to zero.

ii. Finding the resistance b/w the two load terminal. This will be the same as R_{TH}

b. Draw the Norton's equivalent circuit and returned the load for analysis.

Ques: Calculate the current I_L by Norton's Theorem.





Nodal Analysis $\quad I_1 = I_2 + I_N$

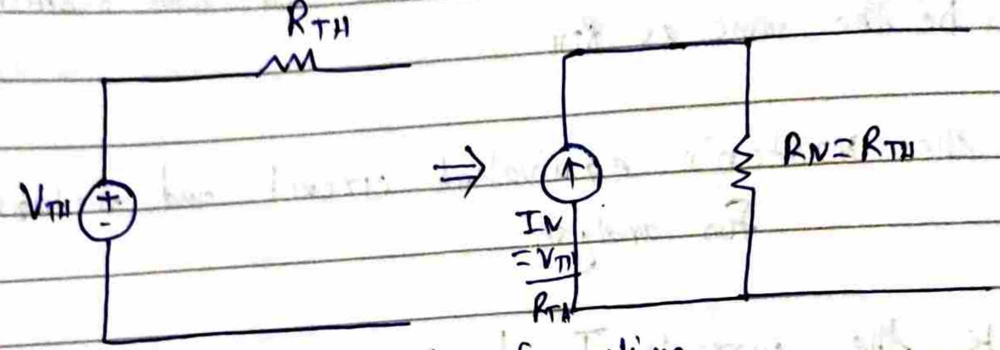
$$\frac{10-V}{2} = \frac{V-0}{3} + \frac{V-0}{4}$$

$$R_N = [(2\|3)+4] = \frac{26}{5}\,\Omega$$

$$I_N = I_1 + I_L$$

$$1.15 = \frac{V}{5.2} + \frac{V}{5}$$

* For a same circuit across the same terminal $R_N = R_{TH}$.
*. The Thevenins and Norton circuit are inter convertible.



Source transformation.

Superposition Theorem

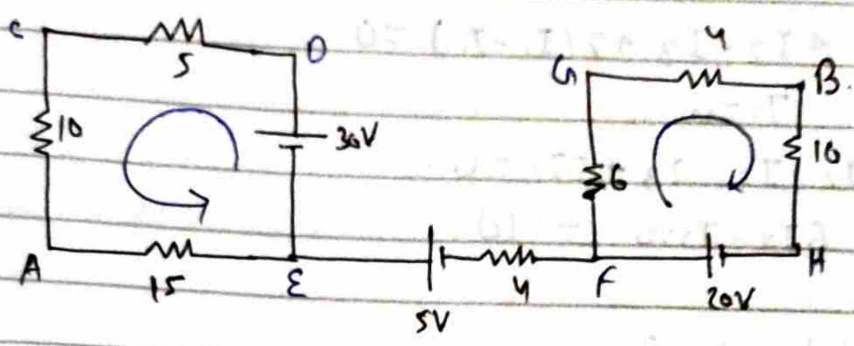
The superposition theorem extends the use of ohm's law with multiple sources.

Definitions:

The current through, or voltage across an element in a linear bylateral network is equal to the algebraic sum of the currents or voltages produced independently by each source,

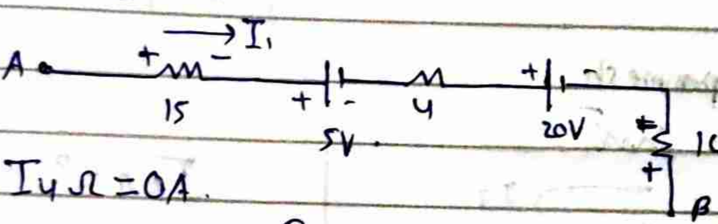
The superposition theorem is very helpful in determining the voltage across an element or current through a branch when the circuit contains multiple no. of voltage or current sources.

Ques: Find VAB using K.C.L and K.V.L.



$$I_1 = \frac{30}{5+10+15} = 1A$$

$$I_2 = \frac{20}{6+4+10} = 1A.$$



$I_{4\Omega} = 0A.$

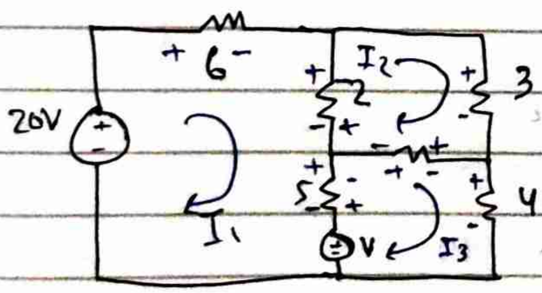
Apply K.V.L from A to B.

$V_A + I_1 \times 15 + 5 - 0 + 20 - I_2 \times 10 - V_B = 0.$

$V_A - V_B + 1 \times 15 + 5 + 20 - 10.$

$V_{AB} = 30V$

Ques:



V = ? so that $I_1 = 0A$.

Apply K.V.L to loop 1.

$6I_1 + 2(I_1 - I_2) + 5(I_1 - I_3) + V - 20 = 0$ .... (i)

$I_1 = 0.$

$-2I_2 - 5I_3 + V = 20$ ... (1)

Apply K.V.L to loop (2).

$$+3I_2 + I_2 - I_3 + 2(I_2 - I_1) = 0$$

$$I_1 = 0$$

$$3I_2 + I_2 - I_3 + 2I_2 = 0$$

$$6I_2 - I_3 = 0 \quad \text{— (2)}$$

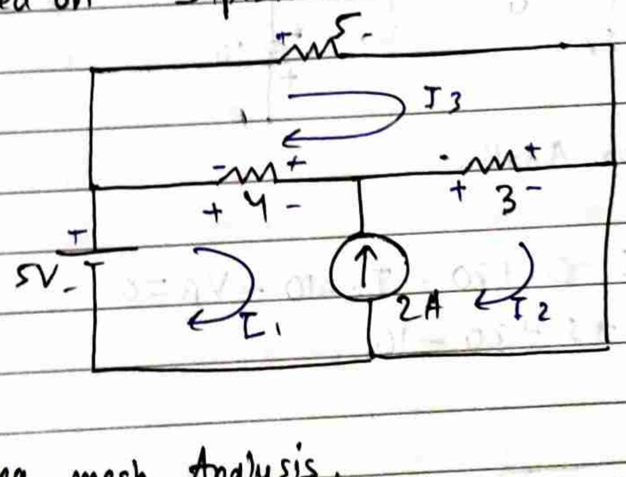
Apply K.V.L to loop 3.

$$+4I_3 - V + 5(I_3 - I_1) + I_3 - I_2 = 0$$

$$+4I_3 - V + 5I_3 + I_3 - I_2 = 0$$

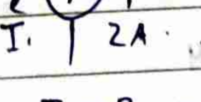
$$10I_3 - V - I_2 = 0 \quad \text{— (3)}$$

Ques: Based on Supermesh

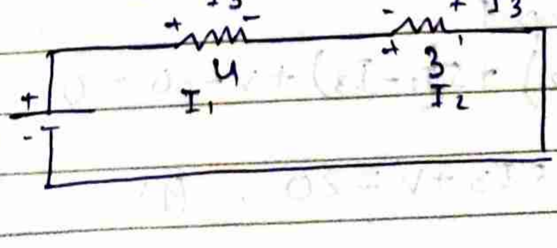


$I_{5\Omega} = ?$

Using mesh Analysis.



$$I_2 - I_1 = 2 \quad \text{--- (1)}$$



Apply K.V.L to supermesh.

$$+4I_1 - 4I_3 + 3I_2 - 3I_3 - 5 = 0$$

$$\therefore -4I_1 - 3I_2 + 7I_3 = -5 \quad --(2)$$

Apply K.V.L to loop 3.

$$+5I_3 + 3I_3 - 3I_2 + 4I_3 - 4I_1 = 0$$

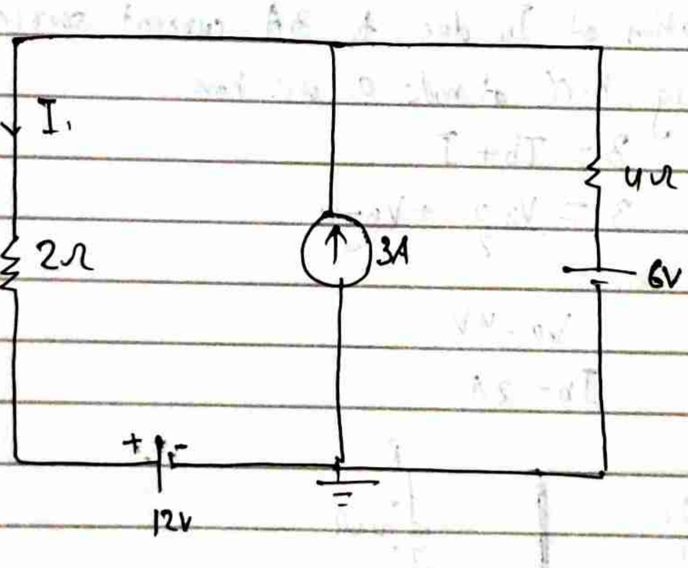
$$\therefore 4I_1 + 3I_2 - 12I_3 = 0 \quad -- (3)$$

Solving eqn (1), (2) & 3.

$$I_1 = 0.85, \; I_2 = 2.857, \; I_3 = 1A.$$

$$I_5 = I_3 = 1A.$$

#1 Problem.

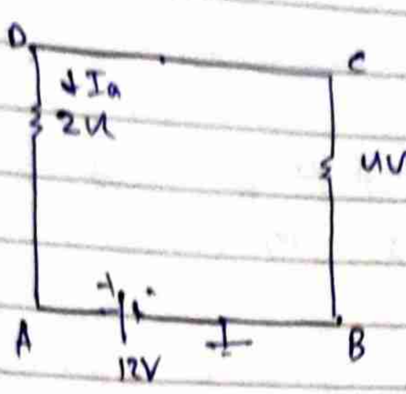


Find the current I_1 in the 2Ω resistance using superposition theorem.

Soln: Let $I_1 = I_a\big|_{12V} + I_b\big|_{3A} + I_c\big|_{6V}$.

Step 1: Calculation of I_a due to 12V source acting alone.

ckt 1.

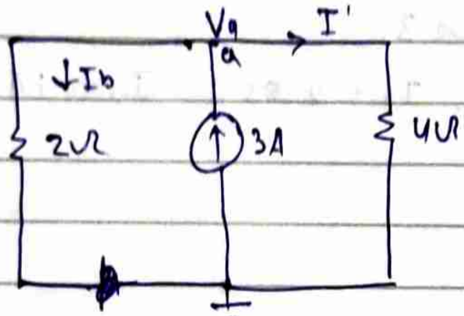


Writing K.V.L from loop ABCDA.

$$+2I_a + 12 + 4I_a = 0.$$

$$I_a = -2A.$$

ckt 2.



Step 2: Calculation of I_b due to 3A current source acting alone.

Writing K.C.L at node a we have.

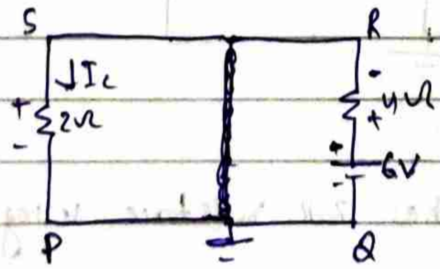
$$3 = I_b + I'$$

$$3 = \frac{V_a - 0}{2} + \frac{V_a - 0}{4}$$

$$V_a = 4V$$

$$I_b = 2A.$$

ckt 3:



Step 3: Calculation of I_c due to 6V source

Writing K.V.L in loop PQRS

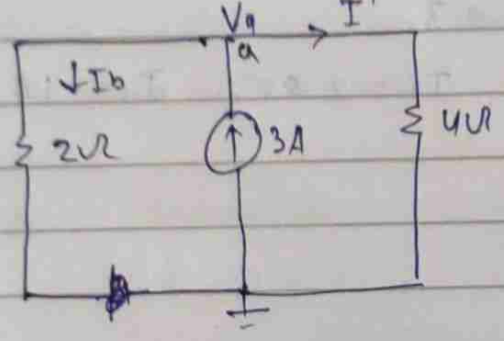
$$-6 + 4I_c + 2I_c = 0 \qquad I_c = 1A.$$

writing K.V.L from loop ABCDA.

$$+2I_a + 12 + 4I_a = 0.$$

$$I_a = -2A.$$

ckt2.



Step 2: Calculation of I_b due to 3A current source acting alone.

writing K.CL at node a we have.

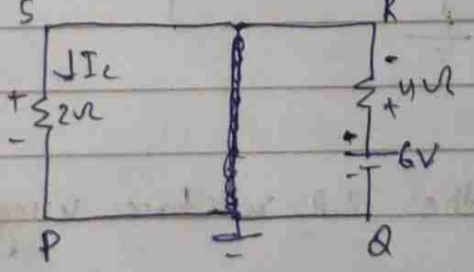
$$3 = I_b + I'$$

$$3 = \frac{V_a - 0}{2} + \frac{V_a - 0}{4}$$

$$V_a = 4V$$

$$I_b = 2A.$$

ckt3:



Step 3: Calculation of I_c due to 6V source

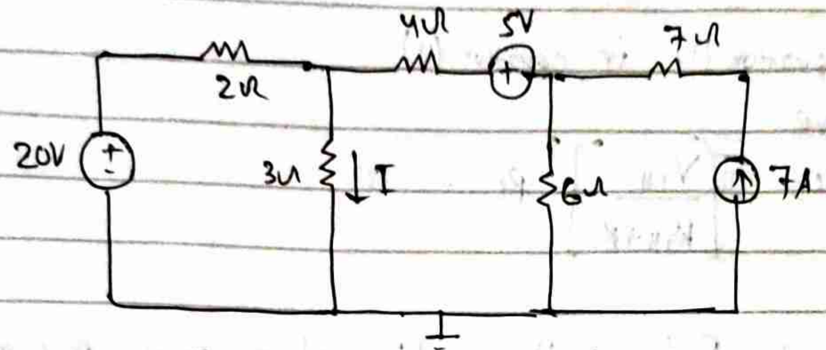
writing K.V.L in loop PQRS

$$-6 + 4I_c + 2I_c = 0 \qquad I_c = 1A.$$

$= -2\big|_{12V} + 2\big|_{3A} + 1\big|_{6V}$

$= 1A.$

2. Problem

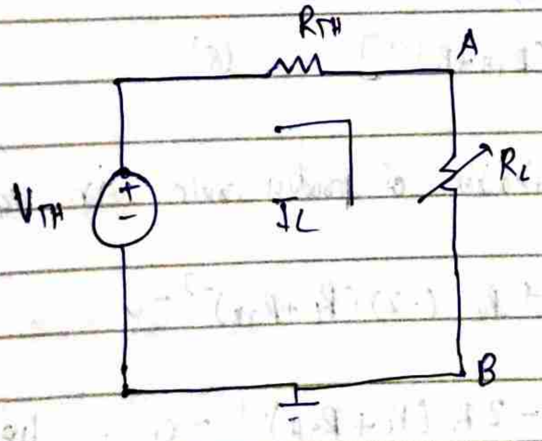


Find out the current I using superposition theorem.

Maximum Power Transmission Theorem
Transfer.

Definition: Maximum power transfer theorem states that "Maximum power is transferd from the source to the load when the load resistance is equal to the thevenins equivalent resistance"

Let us consider a thevenins equivalent circuit as shown



From thevenins equivalent circuit it is clear that

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} \quad \text{---(1)}$$

Let P_L be the power dissipated at the load resistance, so

$$P_L = I_L^2 R_L \quad - (2)$$

Putting Equation (1) in equation (11)
we have

$$P_L = \left[\frac{V_{TH}}{R_{TH}+R_L}\right]^2 . R_L \quad --- (3)$$

In order to find out the condition of maximum power transfer from source to load we differentiate eqⁿ (3) with respect to R_L and equate it to zero. Therefore we have

$$\frac{dP_L}{dR_L} = 0 \quad --- (4)$$

$$\frac{d}{dR_L}\left[\frac{V_{TH}}{R_{TH}+R_L}\right]^2 R_L = 0 \quad -- (5)$$

or, $$V_{TH}^2 \frac{d}{dR_L} \frac{R_L}{(R_{TH}+R_L)^2} = 0 \quad -- (6)$$

or. $$\frac{d}{dR_L} \frac{R_L}{(R_{TH}+R_L)^2} = 0 \quad -- (7)$$

or $$\frac{d}{dR_L}\left[R_L (R_{TH}+R_L)^{-2}\right] \quad -- (8)$$

Using the differentiation of product rule in eq. 8.

or $$(R_{TH}+R_L)^{-2} + R_L (-2)(R_L+R_{TH})^{-3} = 0 \quad -- 9.$$

or. $$(R_{TH}+R_L)^{-2} - 2R_L (R_L+R_{TH})^{-3} = 0 \quad -- (10)$$

or. $$\frac{1}{(R_{TH}+R_L)^2} = \frac{2R_L}{(R_L+R_{TH})^3} \quad -- (11)$$

or $R_L + R_{TH} = 2R_L$ ...(12)

or $\boxed{R_L = R_{TH}}$

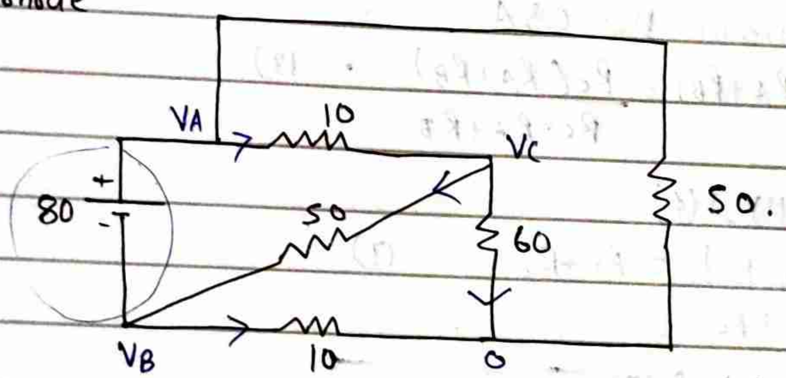
In order to write down maximum power we substitutes the value of R_L in equation (3) as R_{TH}

$$P_L = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} \cdot R_L$$

$$= \frac{V_{TH}^2}{4R_{TH}^2} \cdot R_{TH}$$

$$\boxed{P_{Lmax} = \frac{V_{TH}^2}{4R_{TH}}}$$

Ques: Supernode



$I_{60} = ?$

Nodal Analysis

Eq^n of voltage Source

$V_A - V_B = 80$ ---- (1)

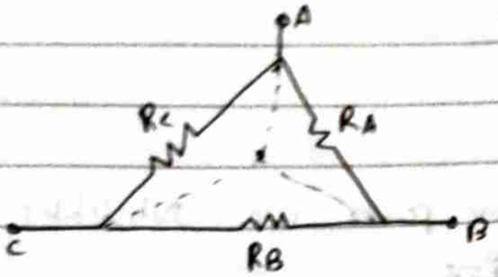
Apply K.CL to supernode

$$\frac{V_A - 0}{50} + \frac{V_A - V_C}{10} + \frac{V_B - 0}{10} = \frac{V_C - V_B}{50} \quad \text{.. (2)}$$

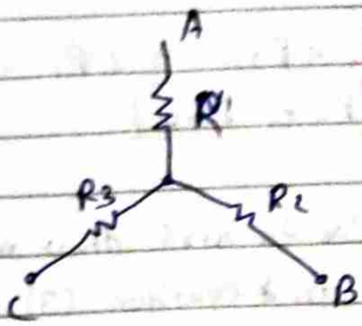
Apply K.C.L to node C

$$\frac{V_A - V_C}{10} = \frac{V_C - V_B}{50} + \frac{V_C - 0}{60} \quad \text{-- (3)}$$

Derivation of a delta to Star



Delta



Star.

Eq. Resis. b/w A & B:

$$R_A \parallel (R_B+R_C) = \frac{R_A(R_B+R_C)}{R_A+R_B+R_C} \quad \text{-- (1)}$$

Similarly Eq. resis b/w B & C

$$R_B \parallel (R_C+R_A) = \frac{R_B(R_C+R_A)}{R_B+R_C+R_A} \quad \text{-- (2)}$$

& Eq. resistance b/w C & A

$$R_C \parallel (R_A+R_B) = \frac{R_C(R_A+R_B)}{R_C+R_A+R_B} \quad \text{-- (3)}$$

Eq. resis b/w A & B

R_1+R_2 --(4)

b/w B & C

$=R_2+R_3$ -- (5)

b/w C & A

R_3+R_1 -- (6)

Equating (1) & (4)

$$\frac{R_A(R_B+R_C)}{R_A+R_B+R_C} = R_1+R_2 \quad \text{-- (7)}$$

Equating (2) & (5)

$$\frac{R_B(R_C+R_A)}{R_A+R_B+R_C} = R_2+R_3 \quad \text{-- (8)}$$

Equating 3 & 6.

$$\frac{R_C(R_A+R_B)}{R_A+R_B+R_C} = R_3+R_1 \quad \text{-- (9)}$$

Equation (7) - (8)

$$\frac{R_A(R_B+R_C) - R_B(R_C+R_A)}{R_A+R_B+R_C} = R_1 - R_3 \quad \text{-- (10)}$$

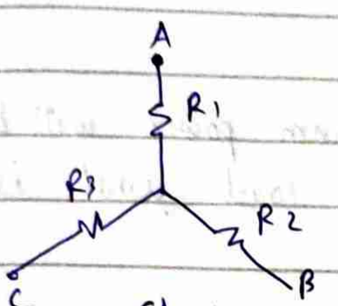
Adding Eq^n. (9) & (10)

$$\frac{R_C.R_A + R_CR_B + R_AR_B + R_AR_C - R_BR_C - R_BR_A}{R_A+R_B+R_C} = 2R_1$$

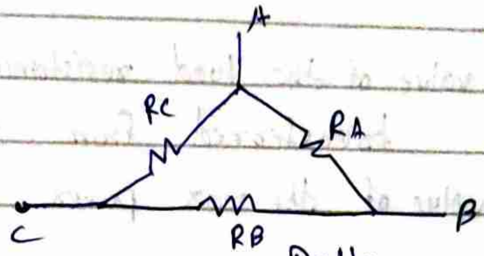
$$\frac{2R_AR_C}{R_A+R_B+R_C} = 2R_1$$

$$\boxed{R_1 = \frac{R_A.R_C}{R_A+R_B+R_C}}$$

Star to delta Conversion



Star



Delta

$$R_1 = \frac{R_AR_C}{R_A+R_B+R_C} \; ; \; R_2 = \frac{R_AR_B}{R_A+R_B+R_C} \; ; \; R_3 = \frac{R_B.R_C}{R_A+R_B+R_C}.$$

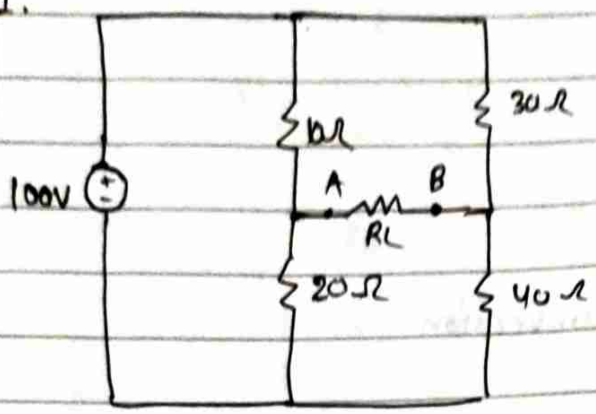
$$R_1.R_2+R_2.R_3+R_3.R_1 = \frac{R_A^2R_B.R_C + R_A.R_B^2.R_C + R_AR_B.R_C^2}{(R_A+R_B+R_C)^2}$$

$$= \frac{R_A.R_B.R_C\,\cancel{(R_A+R_B+R_C)}}{(\cancel{R_A+R_B+R_C})^2}$$

$$= \frac{R_A.R_BR_C}{(R_A+R_B+R_C)} = R_A.R_3.$$

$$\boxed{R_A = \frac{R_1R_2}{R_3} + R_2 + R_1}$$

Problem#1.



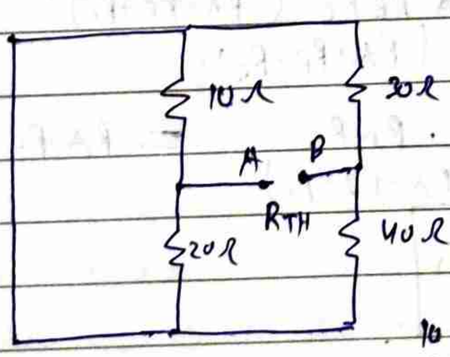
What value of the load resistance maximum power will be transferred from source to load what is the value of the max. power.

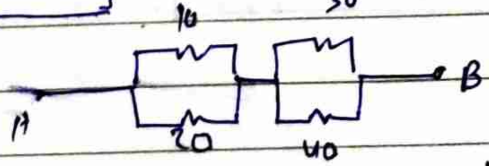
Solⁿ:- For maximum power transfer we have,

$R_L = R_{TH}$;

Thavenising the circuit across the terminal AB for the value of Rth and Vth

Calculation of R_{TH}.

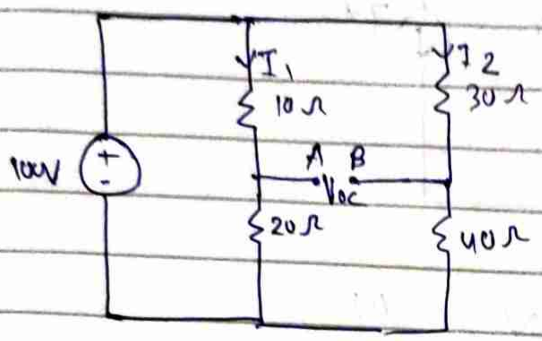




$$= \frac{1}{R_{th}} = \left(\frac{1}{10} + \frac{1}{20}\right) + \left(\frac{1}{30} + \frac{1}{40}\right)$$

$R_{TH} = 23.8\Omega$.

Calculation of V_{TH}:



$I_1 = \frac{100}{30} A$; $V_1 = \frac{100}{3} V$.

$I_2 = \frac{100}{70} A$. ; $V_2 = \frac{300}{7} V$.

Writing K.V.L
for KLMNK;
$V_{oc} + V_1 - V_2 = 0$. $V_{TH} = 9.5V$

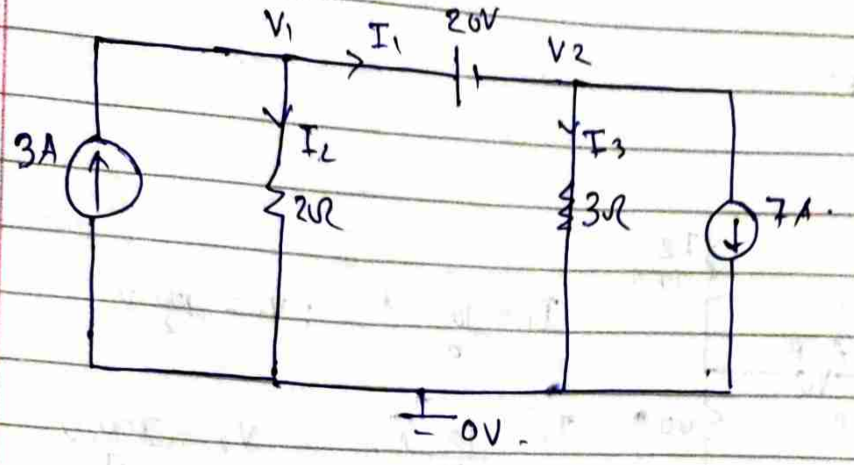
Calculation of maximum power

$$= \frac{V_{TH}^2}{4R_{TH}}$$

Supermesh and Supernode Analysis.

Supernode: A supernode is formed when we have voltage source connected b/w two non-reference node. In order to analyze such circuit for the value of node voltages we express the voltage source connected b/w two non-reference nodes as the difference of two node voltages of the non-reference nodes.

We can write down node equation's for the two non-reference node by taking the current as it is flowing through the voltage source. Adding the two node equation we got an equation in terms of ~~adding~~ the node voltages at the two non-reference node.



$$V_1 - V_2 = 20V \quad - (1)$$

At node (1)

$$3 = I_1 + I_2 \quad -- (2)$$

$$3 = \frac{(V_1 - 0)}{2} + I_1 \quad -- (3)$$

A node (2) $I_1 = I_3 + 7 \quad -- (4)$

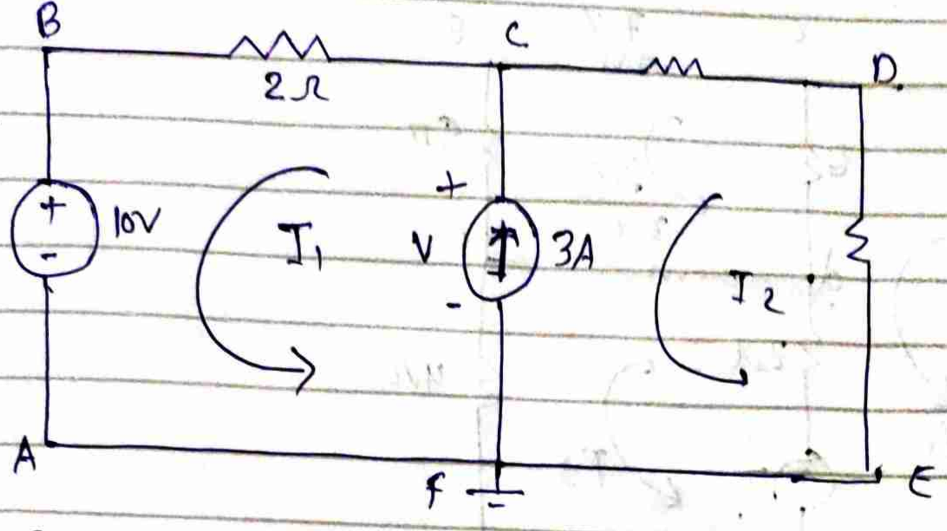
$$I_1 = \frac{V_2 - 0}{3} + 7 \quad -- (5)$$

$$3 = \frac{V_1}{2} + \frac{V_2}{3} + 7 \quad -- (6)$$

Supermesh Analysis

A Supermesh is formed when we have a current source common b/w two meshes. In order to analyze a supermesh we express the common current source as the difference of the two currents in the meshes for which the current source is common.

We write mesh equations by assuming a voltage across a current source and on adding this two equations the assumed voltage variable is eliminated and we get single equation in terms of mesh current for which the current source is common.



Find out the mesh current for the two meshes.

$$I_1 - I_2 = 3A \quad \text{---- (1)}$$

For mesh AFCBA.

$$-V + 2I_1 + 10 \quad \text{---- (2)}$$

For mesh FEDCF

$$+V + 5I_2 + 4I_2 \quad (3).$$

Adding (2) and (3) we get.

$$2I_1 + 9I_2 + 10 = 0 \quad \text{-- (4)}.$$

$$2(3+I_2) + 9I_2 + 10 = 0.$$

$$\Rightarrow 6 + 2I_2 + 9I_2 + 10 = 0$$

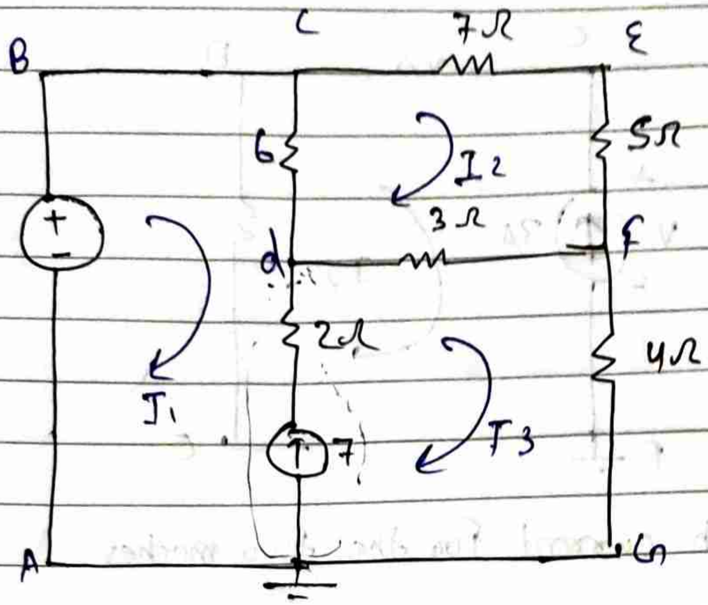
$$\Rightarrow 11I_2 = -16$$

$$I_2 = \frac{-16}{11} = -1.45A$$

$$I_1 = 3A - 1.45A$$

Second way of analyzing a supermesh is to remove the branch containing a common current source and to write the mesh equation using the two mesh current.

Ques:-



$I_3 - I_1 = 7A$ . . . (1).

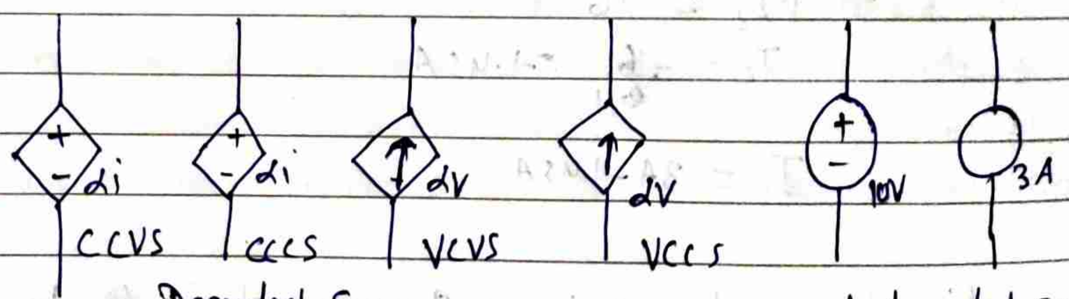
For loop ABCDFGA.

$-10 + 6(I_1 - I_2) + 3(I_3 - I_2) + 4I_3 = 0$ . . . (2)

For loop CEFDC.

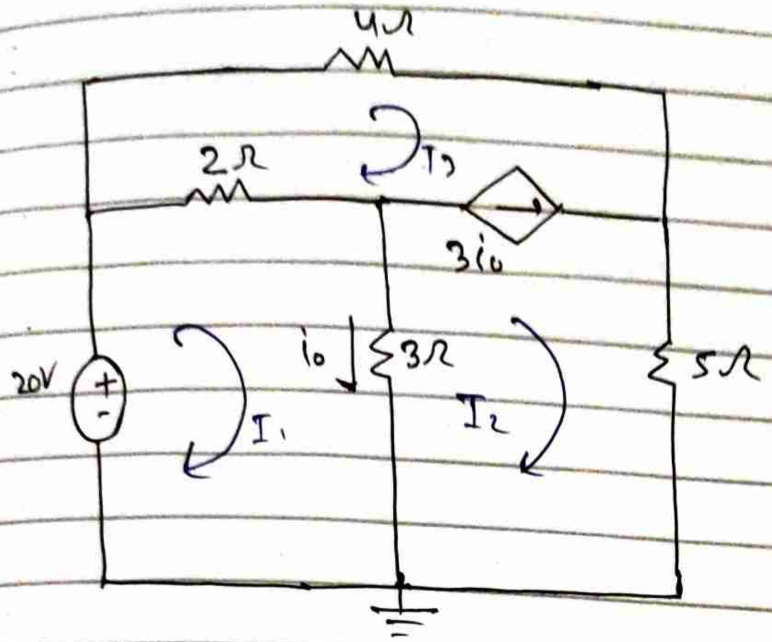
$+6(I_2 - I_1) + 7I_2 + 5I_2 + 3(I_2 - I_3) = 0$ . . . (3)

Dependent Sources.



Dependent Source. Independent source

Ans:



Find out the node voltage and the current I_0.

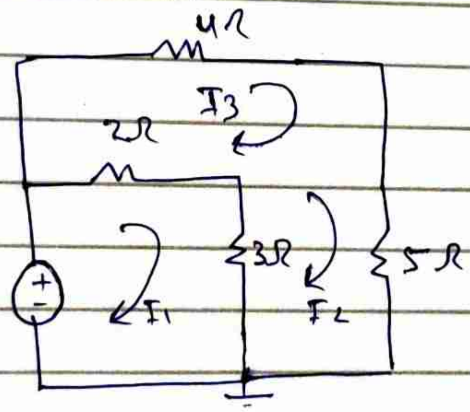
$V_1 = 20V$ — (1)

Super mesh

$I_2 - I_3 = 3i_0$

$\boxed{Key = I_1 - I_2 = i_0.}$

Remove the mesh.



$4I_3 + 5I_2 + 3(I_2 - I_1) + 2(I_3 - I_1) = 0$ -- (2)

$-20 + 2(I_1 - I_3) + 3(I_1 - I_2) = 0$ (3)