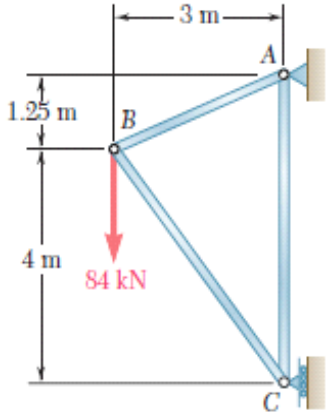
Tutorial sheet 4

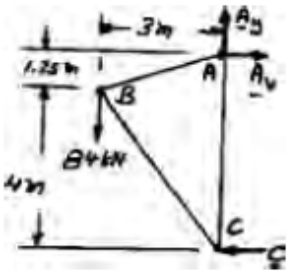
1. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[R_c=48 KN, A_y=84 KN, A_x=48 KN]**
   [AB=52 KN (T), AC=64 KN (T), BC=80 KN (C)]



SOLUTION

$$AB = \sqrt{3^2 + 1.25^2} = 3.25 \text{ m}$$

$$BC = \sqrt{3^2 + 4^2} = 5 \text{ m}$$



Reactions:

$$+\circlearrowleft \Sigma M_A = 0: \quad (84 \text{ kN})(3 \text{ m}) - C(5.25 \text{ m}) = 0$$

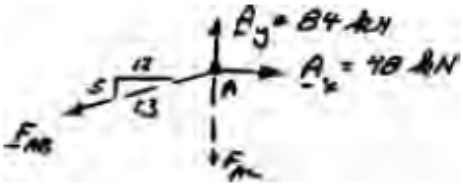
$$\mathbf{C} = 48 \text{ kN} \leftarrow$$

$$\pm \Sigma F_x = 0: \quad A_x - C = 0$$

$$\mathbf{A}_x = 48 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y = 84 \text{ kN} = 0$$

$$\mathbf{A}_y = 84 \text{ kN} \uparrow$$



Joint *A*:

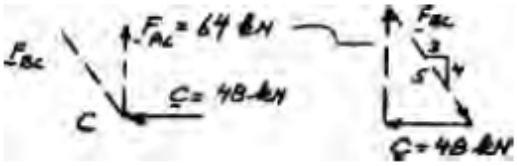
$$\pm \Sigma F_x = 0: \quad 48 \text{ kN} - \frac{12}{13} F_{AB} = 0$$

$$F_{AB} = +52 \text{ kN} \qquad F_{AB} = 52 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 84 \text{ kN} - \frac{5}{13}(52 \text{ kN}) - F_{AC} = 0$$

$$F_{AC} = +64.0 \text{ kN} \qquad F_{AC} = 64.0 \text{ kN} \quad T \blacktriangleleft$$

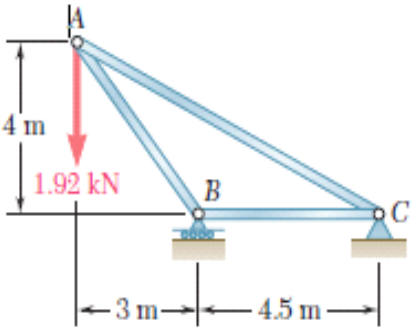
Joint *C*:



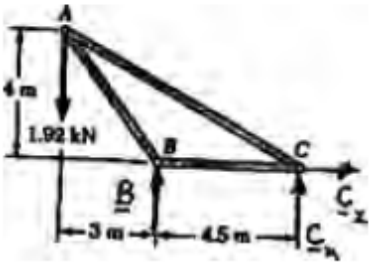
$$\frac{F_{BC}}{5} = \frac{48\ \text{kN}}{3}$$

$$F_{BC} = 80.0\ \text{kN} \quad C \blacktriangleleft$$

2. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[C_X=0, C_Y=1.28 KN, R_B=3.2 KN]**
[AB=4 KN (C), BC=2.4 KN (C), AC=2.72 KN (T)]



SOLUTION



Free body: Entire truss

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad C_x = 0 \quad \mathbf{C}_x = 0$$

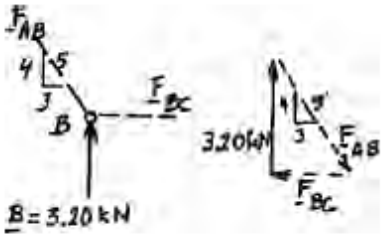
$$+\circlearrowleft \Sigma M_B = 0: \quad (1.92\ \text{kN})(3\ \text{m}) + C_y(4.5\ \text{m}) = 0$$

$$C_y = -1.28\ \text{kN} \quad \mathbf{C}_y = 1.28\ \text{kN}\downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B - 1.92\ \text{kN} - 1.28\ \text{kN} = 0$$

$$\mathbf{B} = 3.20\ \text{kN}\uparrow$$

Free body: Joint *B*:

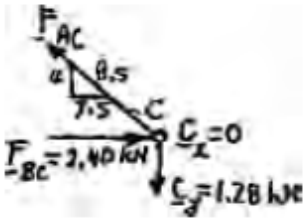


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{3.20\ \text{kN}}{4}$$

$F_{AB} = 4.00\ \text{kN} \quad C$ ◀

$F_{BC} = 2.40\ \text{kN} \quad C$ ◀

Free body: Joint *C*:



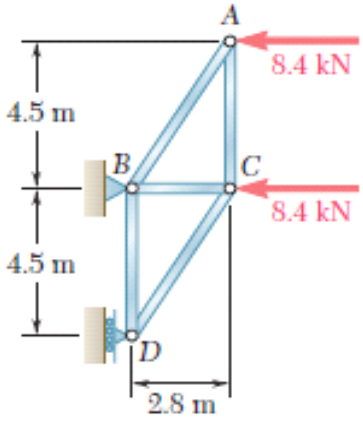
$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad -\frac{7.5}{8.5} F_{AC} + 2.40\ \text{kN} = 0$$

$$F_{AC} = +2.72\ \text{kN}$$

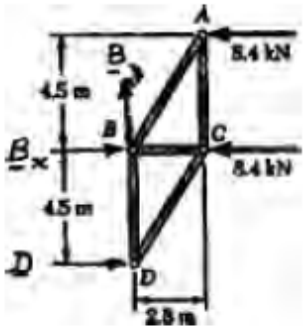
$F_{AC} = 2.72\ \text{kN} \quad T$ ◀

$$+\uparrow \Sigma F_y = \frac{4}{8.5}(2.72\ \text{kN}) - 1.28\ \text{kN} = 0 \quad \text{(Checks)}$$

3. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[R_D=8.4 KN, B_X=25.2 KN, B_Y=0]**
[AB=15.9 KN (C), AC=13.5 KN (T), CD=15.9 KN (T), BC=16.8 KN (C), BD=13.5 KN (C)]



Free body: Truss



$$+\uparrow \Sigma F_y = 0: \quad \mathbf{B}_y = 0$$

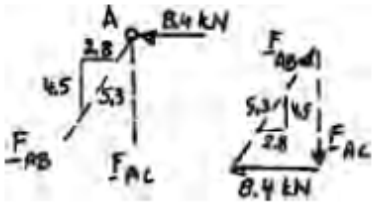
$$+\circlearrowleft \Sigma M_B = 0: \quad D(4.5\text{ m}) + (8.4\text{ kN})(4.5\text{ m}) = 0$$

$$D = -8.4\text{ kN} \qquad \mathbf{D} = 8.4\text{ kN} \leftarrow$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad B_x - 8.4\text{ kN} - 8.4\text{ kN} - 8.4\text{ kN} = 0$$

$$B_x = +25.2\text{ kN} \qquad \mathbf{B}_x = 25.2\text{ kN} \rightarrow$$

Free body: Joint *A*:

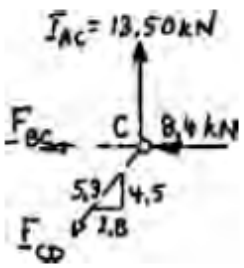


$$\frac{F_{AB}}{5.3} = \frac{F_{AC}}{4.5} = \frac{8.4\text{ kN}}{2.8}$$

$$F_{AB} = 15.90\text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 13.50\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *C*:



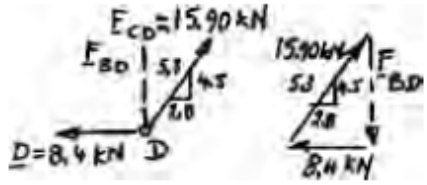
$$+\uparrow \Sigma F_y = 0: \quad 13.50\text{ kN} - \frac{4.5}{5.3}F_{CD} = 0$$

$$F_{CD} = +15.90\text{ kN} \qquad F_{CD} = 15.90\text{ kN} \quad T \blacktriangleleft$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad -F_{BC} - 8.4\text{ kN} - \frac{2.8}{5.3}(15.90\text{ kN}) = 0$$

$$F_{BC} = -16.80\text{ kN} \qquad F_{BC} = 16.80\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *D*:



$$\frac{F_{BD}}{4.5} = \frac{8.4\text{ kN}}{2.8}$$

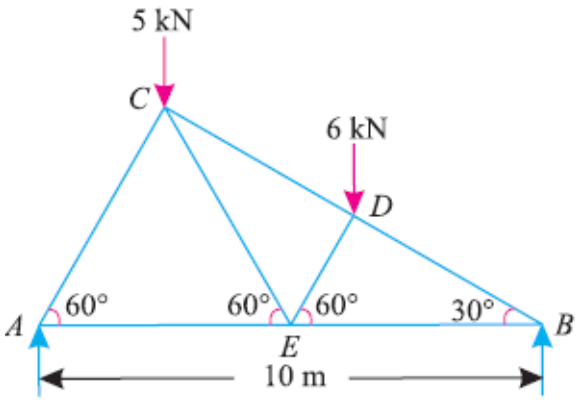
$$F_{BD} = 13.50\text{ kN} \quad C \blacktriangleleft$$

We can also write the proportion

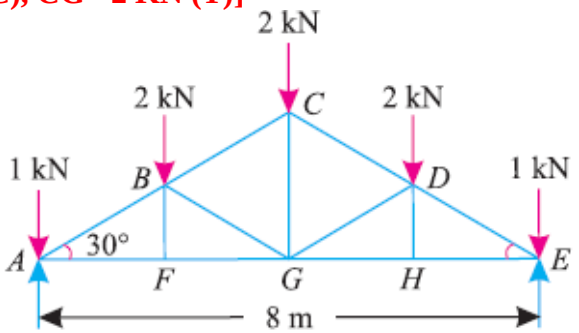
$$\frac{F_{BD}}{4.5} = \frac{15.90\text{ kN}}{5.3}$$

$$F_{BD} = 13.50\text{ kN} \quad C \quad \text{(Checks)}$$

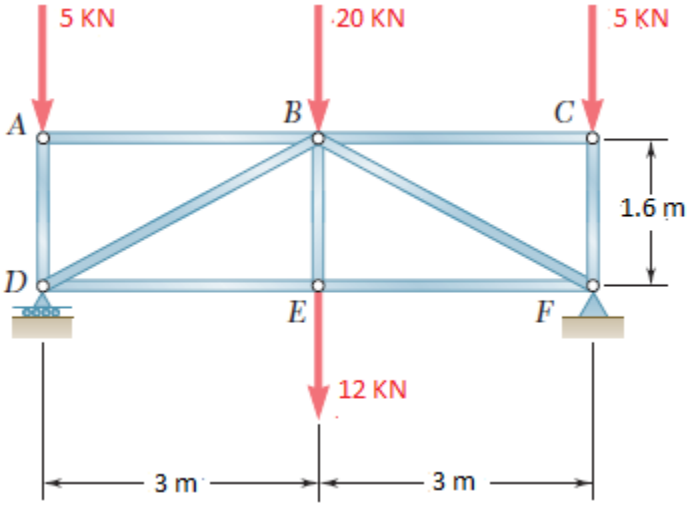
4. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[AC=6.92 KN (C), AE=3.46 KN (T), BD=10 KN (C), BE=8.66 KN (T), CD=7 KN (C), ED=5.2 KN (C), CE=5.2 KN (T)]**

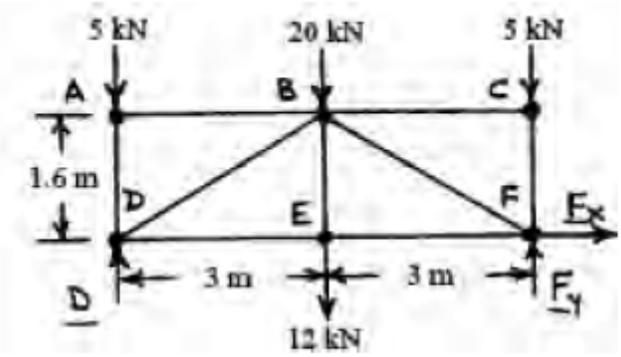


5. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[AC=DE= 6 KN (C), AF=EH=5.2 KN (T), FG=GH=5.2 KN (T), BF=DH=0, BG=DG= 2 KN (C), BC=CD= 4 KN (C), CG= 2 KN (T)]**



6. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[F_X=0, F_Y= 21 KN, R_D= 21 KN]**
[AB=0, AD=5 KN (C), BD= 34 KN (C), DE= 30 KN (T), BE= 12 KN (T)]





$$\overset{+}{\circlearrowleft}\Sigma M_D = 0: \quad F_y(6) - (20+12)(3) - (5)(6) = 0$$

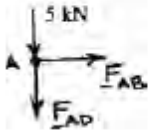
$$\mathbf{F}_y = 21\text{ kN}\uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{F}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D - (5+20+5+12) + (21) = 0$$

$$\mathbf{D} = 21\text{ kN}\uparrow$$

Joint *A*:

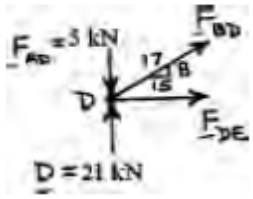


$$\Sigma F_x = 0: \quad F_{AB} = 0 \qquad F_{AB} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -5 - F_{AD} = 0$$

$$F_{AD} = -5\text{ kN} \qquad F_{AD} = 5.00\text{ kN} \quad C \blacktriangleleft$$

Joint *D*:



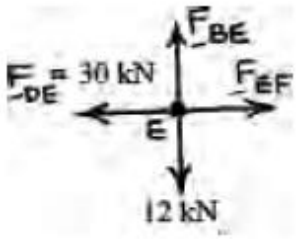
$$+\uparrow \Sigma F_y = 0: \quad -5 + 21 + \frac{8}{17}F_{BD} = 0$$

$$F_{BD} = -34\text{ kN} \qquad F_{BD} = 34.0\text{ kN} \quad C \blacktriangleleft$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{15}{17}(-34) + F_{DE} = 0$$

$$F_{DE} = +30\text{ kN} \qquad F_{DE} = 30.0\text{ kN} \quad T \blacktriangleleft$$

Joint *E*:

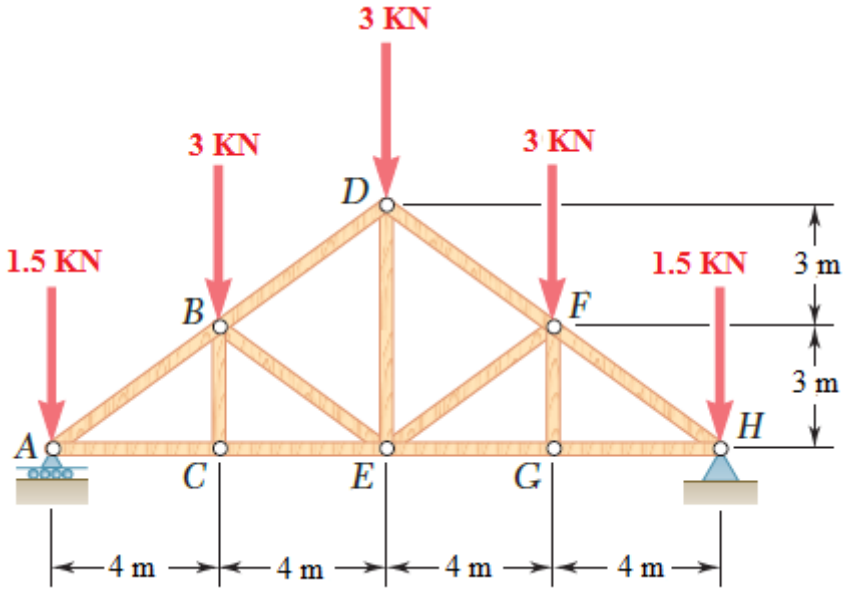


$$+\uparrow \Sigma F_y = 0: \quad F_{BE} - 12 = 0$$

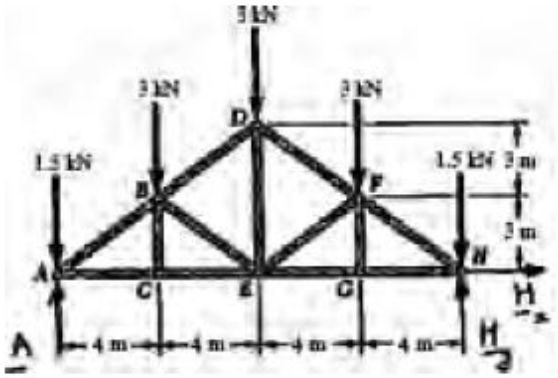
$$F_{BE} = +12\text{ kN} \qquad F_{BE} = 12.00\text{ kN} \quad T \blacktriangleleft$$

Truss and loading symmetrical about ℄

7. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[R_A=6 KN, H_Y= 6 KN, H_X=0]**
[AB=7.5 KN (C), AC=6 KN (T), CE= 6 KN (T), BD= 5 KN (C), BE=2.5 KN (C), DF= 5 KN (C), DE= 3KN (T), EF=2.5 KN (C), EG= 6 KN (T), EG=0, FH= 7.5 KN (C), GH= 6 KN (T)]



Free body: Truss



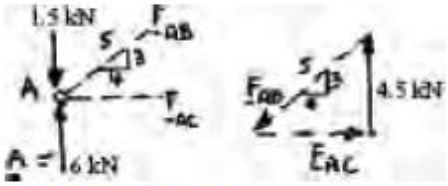
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading:

$$A = H_y = \frac{1}{2} \text{ Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 6 \text{ kN} \uparrow$$

Free body: Joint *A*:



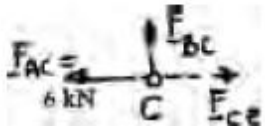
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{4.5 \text{ kN}}{3}$$

$\mathbf{F}_{AB} = 7.50 \text{ kN} \quad C$ ◄

$\mathbf{F}_{AC} = 6.00 \text{ kN} \quad T$ ◄

Free body: Joint *C*:

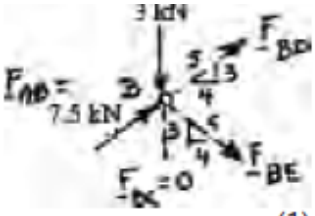
BC is a zero-force member



$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 6.00 \text{ kN} \quad T$ ◄

Free body: Joint *B*:



$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(7.5\text{ kN}) = 0$$

or $$F_{BD} + F_{BE} = -7.5\text{ kN} \qquad (1)$$

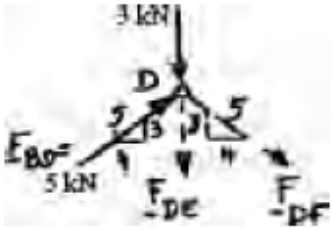
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(7.5\text{ kN}) - 3\text{ kN} = 0$$

or $$F_{BD} - F_{BE} = -2.5\text{ kN} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -10\text{ kN}$ $\quad F_{BD} = 5.00\text{ kN} \quad C \blacktriangleleft$

Subtract (2) from (1): $2F_{BE} = -5\text{ kN}$ $\quad F_{BE} = 2.50\text{ kN} \quad C \blacktriangleleft$

Free Body: Joint *D*:



$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{4}{5}(5\text{ kN}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -5\text{ kN}$ $\quad F_{DF} = 5.00\text{ kN} \quad C \blacktriangleleft$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(5\text{ kN}) - \frac{3}{5}(-5\text{ kN}) - 3\text{ kN} - F_{DE} = 0$$

$F_{DE} = +3\text{ kN}$ $\quad F_{DE} = 3.00\text{ kN} \quad T \blacktriangleleft$

Because of the symmetry of the truss and loading, we deduce that

$F_{EF} = F_{BE}$ $\quad F_{EF} = 2.50\text{ kN} \quad C \blacktriangleleft$

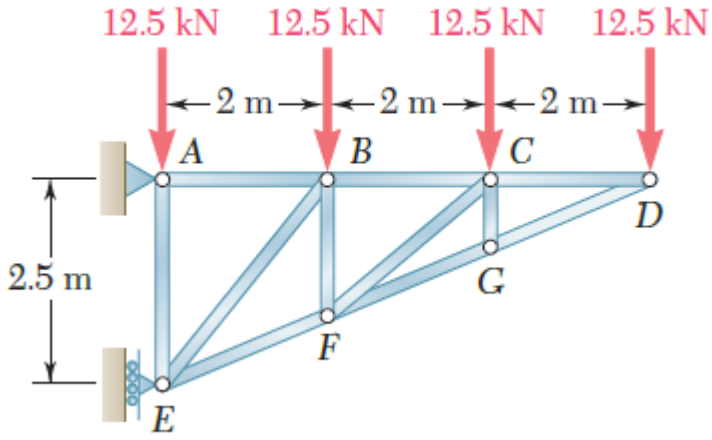
$F_{EG} = F_{CE}$ $\quad F_{EG} = 6.00\text{ kN} \quad T \blacktriangleleft$

$F_{FG} = F_{BC}$ $\quad F_{FG} = 0 \blacktriangleleft$

$F_{FH} = F_{AB}$ $\quad F_{FH} = 7.50\text{ kN} \quad C \blacktriangleleft$

$F_{GH} = F_{AC}$ $\quad F_{GH} = 6.00\text{ kN} \quad T \blacktriangleleft$

8. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[CD=30 KN (T), DG=32.5 KN (C),CG=0, FG=32.5 KN (C), CF=19.53 KN (C), BC=45 KN (T), EF=48.8 KN (C), BF=6.25 KN (T), BE= 24 KN (C), AB= 60 KN (T), AE=37.5 KN (T)]**



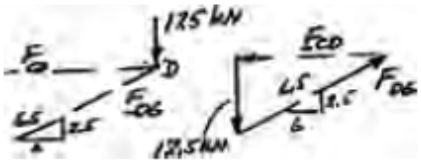
SOLUTION

Joint *D*:

$$\frac{12.5\text{ kN}}{2.5} = \frac{F_{CD}}{6} = \frac{F_{DG}}{6.5}$$

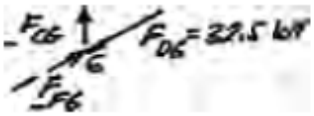
$F_{CD} = 30\text{ kN}$ T ◀

$F_{DG} = 32.5\text{ kN}$ C ◀



Joint *G*:

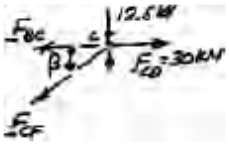
$\nwarrow \Sigma F = 0: \quad F_{CG} = 0$ ◀



$\nearrow \Sigma F = 0: \quad F_{FG} = 32.5\text{ kN} \quad C$ ◀

Joint *C*:

$$BF = \frac{2}{3}(2.5\text{ m}) = 1.6667\text{ m} \qquad \beta = \angle BCF = \tan^{-1}\frac{BF}{2} = 39.81°$$



$+\uparrow \Sigma F_y = 0: \quad -12.5\text{ kN} - F_{CF}\sin\beta = 0$

$-12.5\text{ kN} - F_{CF}\sin 39.81° = 0$

$F_{CF} = -19.526\text{ kN}$

$F_{CF} = 19.53\text{ kN}$ C ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad 30\text{ kN} - F_{BC} - F_{CF}\cos\beta = 0$

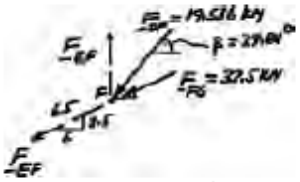
$30\text{ kN} - F_{BC} - (-19.526\text{ kN})\cos 39.81° = 0$

$F_{BC} = +45.0\text{ kN}$

$F_{BC} = 45.0\text{ kN}$ T ◀

Joint F:

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0: \quad -\frac{6}{6.5}F_{EF} - \frac{6}{6.5}(32.5\text{ kN}) - F_{CF}\cos\beta = 0$$



$$F_{EF} = -32.5\text{ kN} - \left(\frac{6.5}{6}\right)(19.526\text{ kN})\cos 39.81°$$

$$F_{EF} = -48.75\text{ kN} \qquad F_{EF} = 48.8\text{ kN} \quad C \blacktriangleleft$$

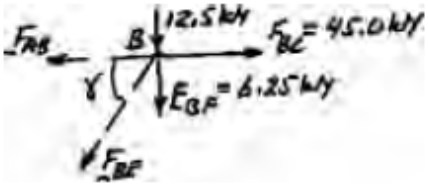
$$+\uparrow\Sigma F_y = 0: \quad F_{BF} - \frac{2.5}{6.5}F_{EF} - \frac{2.5}{6.5}(32.5\text{ kN}) - (19.526\text{ kN})\sin 39.81° = 0$$

$$F_{BF} - \frac{2.5}{6.5}(-48.75\text{ kN}) - 12.5\text{ kN} - 12.5\text{ kN} = 0$$

$$F_{BF} = +6.25\text{ kN} \qquad F_{BF} = 6.25\text{ kN} \quad T \blacktriangleleft$$

Joint B:

$$\tan\alpha = \frac{2.5\text{ m}}{2\text{ m}}; \quad \gamma = 51.34°$$



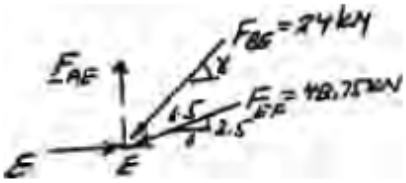
$$+\uparrow\Sigma F_y = 0: \quad -12.5\text{ kN} - 6.25\text{ kN} - F_{BE}\sin 51.34° = 0$$

$$F_{BE} = -24.0\text{ kN} \qquad F_{BE} = 24.0\text{ kN} \quad C \blacktriangleleft$$

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0: \quad 45.0\text{ kN} - F_{AB} + (24.0\text{ kN})\cos 51.34° = 0$$

$$F_{AB} = +60\text{ kN} \qquad F_{AB} = 60.0\text{ kN} \quad T \blacktriangleleft$$

Joint E:

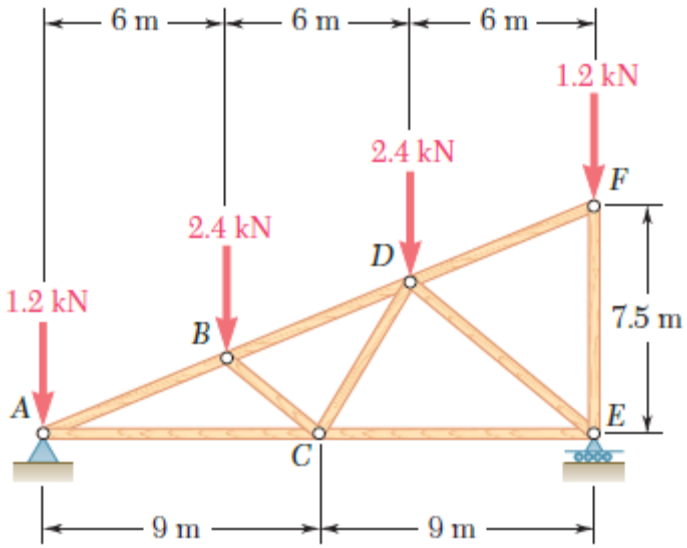


$$\gamma = 51.34°$$

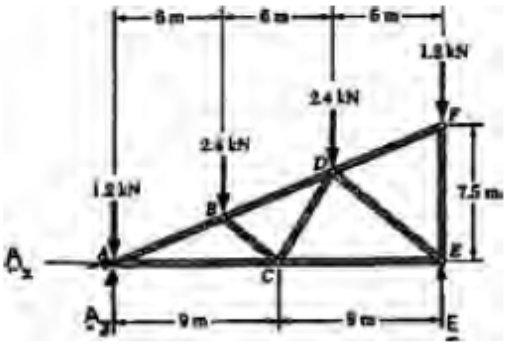
$$\stackrel{+}{\rightarrow}\Sigma F_y = 0: \quad F_{AE} - (24\text{ kN})\sin 51.34° - (48.75\text{ kN})\frac{2.5}{6.5} = 0$$

$$F_{AE} = +37.5\text{ kN} \qquad F_{AE} = 37.5\text{ kN} \quad T \blacktriangleleft$$

9. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[A_X=0, A_Y=R_E= 3.6 KN]**
[DF=0, EF= 1.2 KN (C), AB= 6.24 KN (C), AC= 2.76 KN (T), BD= 4.16 KN (C), BD= 2.5 KN (C), CD= 1.867 KN (T), CE= 2.88 KN (T), DE= 3.75 KN (C)]


6 m
6 m
6 m
1.2 kN
2.4 kN
2.4 kN
F
D
1.2 kN
B
7.5 m
A
E
C
9 m
9 m

SOLUTION



Free body: Truss

$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of loading:

$$A_y = E = \frac{1}{2}\text{Total load}$$

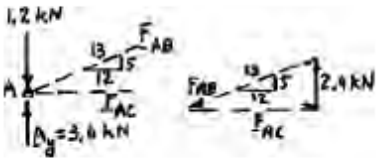
$$A_y = E = 3.6\ \text{kN} \uparrow$$

We note that DF is a zero-force member and that EF is aligned with the load. Thus

$$F_{DF} = 0 \blacktriangleleft$$

$$F_{EF} = 1.2\ \text{kN} \quad C \blacktriangleleft$$

Free body: Joint A:

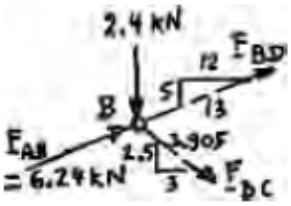


$$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4\ \text{kN}}{5}$$

$$F_{AB} = 6.24\ \text{kN} \quad C \blacktriangleleft$$

$$F_{AC} = 2.76\ \text{kN} \quad T \blacktriangleleft$$

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{3}{3.905}F_{BC} + \frac{12}{13}F_{BD} + \frac{12}{13}(6.24\ \text{kN}) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2.5}{3.905}F_{BC} + \frac{5}{13}F_{BD} + \frac{5}{13}(6.24\ \text{kN}) - 2.4\ \text{kN} = 0 \qquad (2)$$

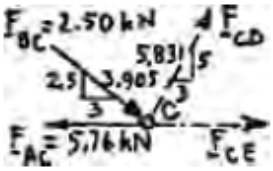
Multiply (1) by 2.5, (2) by 3, and add:

$$\frac{45}{13}F_{BD}+\frac{45}{13}(6.24\text{ kN})-7.2\text{ kN}=0,\quad F_{BD}=-4.16\text{ kN},\qquad F_{BD}=4.16\text{ kN}\quad C\blacktriangleleft$$

Multiply (1) by 5, (2) by −12, and add:

$$\frac{45}{3.905}F_{BC}+28.8\text{ kN}=0,\quad F_{BC}=-2.50\text{ kN},\qquad F_{BC}=2.50\text{ kN}\quad C\blacktriangleleft$$

Free body: Joint *C*:



$$+\uparrow\Sigma F_y=0:\quad \frac{5}{5.831}F_{CD}-\frac{2.5}{3.905}(2.50\text{ kN})=0$$

$$F_{CD}=1.867\text{ kN}\quad T\blacktriangleleft$$

$$\xrightarrow{+}\Sigma F_x=0:\quad F_{CE}-5.76\text{ kN}+\frac{3}{3.905}(2.50\text{ kN})+\frac{3}{5.831}(1.867\text{ kN})=0$$

$$F_{CE}=2.88\text{ kN}\quad T$$

Free body: Joint *E*:

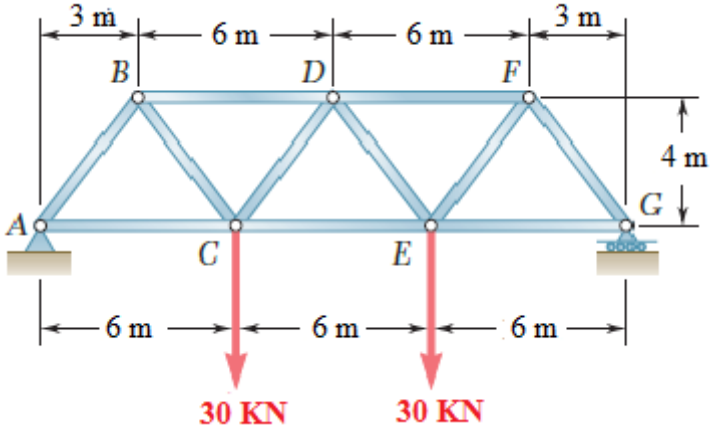
$$+\uparrow\Sigma F_y=0:\quad \frac{5}{7.81}F_{DE}+3.6\text{ kN}-1.2\text{ kN}=0$$

$$F_{DE}=-3.75\text{ kN}\qquad F_{DE}=3.75\text{ kN}\quad C\blacktriangleleft$$

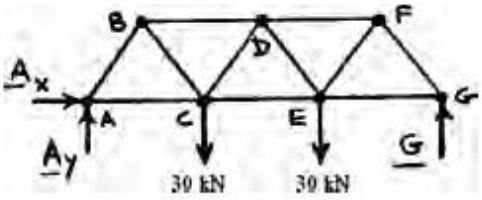
$$\xrightarrow{+}\Sigma F_x=0:\quad -F_{CE}-\frac{6}{7.81}(-3.75\text{ kN})=0$$

$$F_{CE}=+2.88\text{ kN}\qquad F_{CE}=2.88\text{ kN}\quad T\quad\text{(Checks)}$$

10. Determine the force in each member of the truss shown. State whether each member is in tension or compression **[A_X= 0, A_Y= R_G= 30 KN]**
[AB= 37.5 KN (C), AC= 22.5 KN (T), BC= 37.5 KN (T), BD= 45 KN (C), CD=0, CE= 45 KN (T)]



SOLUTION



Free body: Truss $\Sigma F_x = 0: \quad A_x = 0$

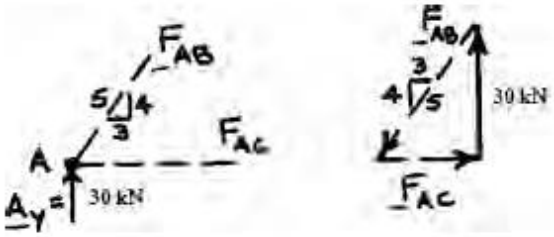
Due to symmetry of truss and loading

$$A_y = G = \frac{1}{2}\text{Total load} = 30\text{ kN}\uparrow$$

Free body: Joint *A*: $\dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{3} = \dfrac{30}{4}\text{kN}$

$F_{AB} = 37.5\text{ kN} \quad C$ ◀

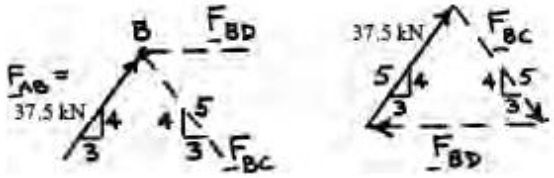
$F_{AC} = 22.5\text{ kN} \quad T$ ◀



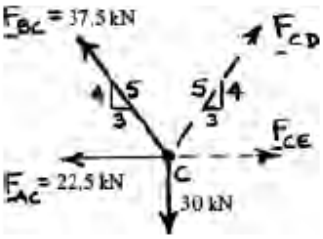
Free body: Joint *B*: $\dfrac{F_{BC}}{5} = \dfrac{F_{BD}}{6} = \dfrac{37.5}{5}\text{kN}$

$F_{BC} = 37.5\text{ kN} \quad T$ ◀

$F_{BD} = 45\text{ kN} \quad C$ ◀



Free body: Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(37.5) + \frac{4}{5}F_{CD} - 30 = 0$$

$$F_{CD} = 0 \blacktriangleleft$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad F_{CE} - 22.5 - \frac{3}{5}(37.5) = 0$$

$$+\uparrow F_{CE} = +45 \text{ kN}$$

$$F_{CE} = 45 \text{ kN} \quad T \blacktriangleleft$$

Truss and loading symmetrical about ℄