Chapter - 1 ⇒ NUMBER SYSTEM

1.1 → Number Introduction.

Classification

- Numbers
  - Real
    - Irrational
      - Ex:- $\sqrt{3}$, $\sqrt{5}$, π, e, ...
    - Rational
      - Integers
        - Negative Integers
        - Whole Numbers
          - Zero
          - Natural Numbers (1, 2, 3, 4, ...)
      - Fractions (p/q)
      - Decimals
        - Terminated: 0.25, 0.5
        - Repeated: 0.3333, 0.8333
  - Complex → Combination of Real & Imaginary (i)
    - Ex:- $a+ib$
    - $i^2 = -1$
    - $i = \sqrt{-1}$

Natural Numbers: All counting numbers are called natural numbers and denoted as N.

1, 2, 3, 4, 5, ...

1, 2, 3, 4, 5, ..., 30

Even Numbers:- It is a set of all integers divisible by 2. It is denoted as $2n$.

Example:- −6, −4, −2, 0, 2, 4, 6, 8, ...

Odd Numbers:- It is set of all integers not divisible by 2. We can also say that numbers which are not even are odd. It is denoted as $(2n-1)$ or $(2n+1)$

Example:- −3, −1, 1, 3, 5, 7 ......

Properties

$2 + 4 = 6$	Even + Even = Even
$2 * 4 = 8$	Even * Even = Even
$8 - 6 = 2$	Even - Even = Even
$6 \div 2 = 3$	Even ÷ Even = Odd or Even
$4 \div 2 = 2$	
$4 + 3 = 7$	Even + Odd = Odd
$4 * 3 = 12$	Even * Odd = Even
$12 \div 3 = 4$	Even ÷ odd = Even
$7 - 4 = 3$	Even - Odd = Odd
$3 + 7 = 10$	Odd + Odd = Even
$7 - 3 = 4$	Odd - Odd = Even
$7 * 3 = 21$	Odd * Odd = Odd
$21 \div 3 = 3$	odd ÷ Odd = Odd

Odd ÷ Even → Never Possible.

Prime Number (P): Any natural number which is divisible by 1 and itself. It can only have 2 factors.

Example :- 2, 3, 5, 7, ... etc.

Example

3 → Prime Number

1 3

21 → Non Prime Number

1 3 7

Number of Prime Numbers between

1 – 100 (25) 1 – 200 (46) 100 – 200 (21) ← 46 – 25

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Properties

1) 2 is the only even prime number and lowest prime number.
2) lowest odd prime number is 3.
3) The remainder of the divison of the square of a prime number $P \geq 5$ divided by 12 or 24 is 1.

$5 = \frac{25}{12}$ or $\frac{25}{24}$ = Remainder 1

$13 = \frac{169}{12}$ or $\frac{169}{24}$ = Remainder 1

$31 = \frac{961}{12}$ or $\frac{961}{24}$ = Remainder 1

Prime numbers between 100 and 200

101	113	127	131	149	151	163	173	181	191
103			137		157	167	179		193
107			139						197
109									199

Twin Primes : A pair of prime numbers having a difference two

Example → (3,5), (5,7), (11,13), (107,109), (41,43), ...

Co-Prime : Two or more numbers that have no common factor other than 1.

↳ Other name is Relatively Prime or Mutually Prime.

Example :- (13,15), (21,22)

Properties

1) Any two consecutive natural numbers are co-prime.
(1,2), (2,3), (3,4), (71,72), ...

2) If two numbers having HCF 1, that numbers are said to be co-prime.
(15,16)

3) Any two prime numbers are said to be co-prime.
(31, 37)

4) When two numbers are said to be co-prime, the LCM of two numbers will be the product of given two number

Example :- LCM for 4,5

LCM = 4×5 = 20.

Composite Number: Any natural number having atleast one factor other than 1 and itself

Example :- 4, 6, 8, 9, ...
1×4, 2×2 → 1, 2, 4

⊛ Note :- 1 is neither prime nor composite number.

Whole Number (W): When you ~~add~~ include zero to the set of natural numbers.

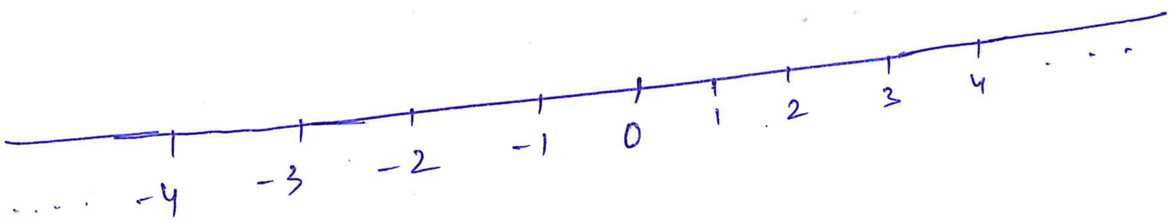
$W = [0, 1, 2, 3, 4, 5, \ldots\ldots]$

Integers (Z) ⇒ When you include negative integers to the set of whole numbers.

$Z = \{\underbrace{\infty \ldots\ldots, -4, -3, -2, -1}_{\text{Negative Integers}}, 0, \underbrace{1, 2, 3, 4, \ldots \infty}_{\text{Positive Integers}}\}$

Positive Integers → Natural Numbers

Representation of integers on Number line



⊛ $-1 > -4$

$-5 > -8$

⊛ $4 > 1$

$8 > 5$

⊛ Note ⇒ 0 is neither positive nor negative.

Non negative = $\{0, 1, 2, 3, \ldots\}$

Non Positive = $\{0, -1, -2, -3, \ldots\}$

Properties on Sign Conversions

$(a) + (b) = a + b$

$(-a) + (b) = b - a$ | Result + or -

$a + (-b) = a - b$

$(-a) + (-b) = -(a+b)$

* $(-a) * b = -ab = a * (-b)$

* $(-a) * (-b) = ab$

$-5 * -3 = 15$

$-5 * 3 = -15$

$5 * -3 = -15$

$1 + 2 = 3$

$-3 + 5 = 5 - 3 = 2$

$-5 + 3 = 3 - 5 = -2$

$+4) + (-6) = -10$

$a + (-b) \quad b + (-a)$

$4 - 6 \neq 6 - 4$

$\Rightarrow -2 \neq 2$

1.2 Number Fractions

Fractions : Any number is written in the form of p/q where $q \neq 0$.
p, q are called term.

There are 4 pieces of cake

Out of which 2 pieces was eaten $\Rightarrow$ $2/4$ of cake was eaten

Out of which 3 pieces was eaten $\Rightarrow$ $3/4$ of cake was eaten

(p/q form)

numerator (N) $\leftarrow 2/4 \rightarrow$ denominator (D)

Fraction

- Proper fraction (N < D): $2/4$, $3/5$, $11/17$
- Improper (N > D): $4/3$, $17/11$
- Mixed
  - Natural Number
  - Proper fraction

  $5\frac{3}{4}$
- Decimal: Denominator is in the form of 10ths
  $1/10$, $1/100$, $1/1000$, ...
  $7/10$, $\frac{25}{1000}$

Note :- Improper $\rightleftarrows$ Mixed

$$5\frac{3}{4} \rightarrow \frac{23}{4}$$

$$7\frac{2}{3} \rightarrow \frac{23}{3}$$

$$5\frac{3}{4} \Rightarrow 5 + 3/4$$

$$7\frac{2}{3} \Rightarrow 7 + 2/3$$

Example

$$4\frac{3}{7} + 9\frac{6}{7} = 4 + 3/7 + 9 + 6/7 = 13 + 9/7$$

$$= 13 + 1\frac{2}{7}$$

$$= 14\frac{2}{7}$$

Decimal: A number which have decimal point in it.

Example:- 2.5, 3.25, 4.173, etc

1/10 1/100 1/1000 [when decimal point increases the zeroes increases]

$2.5 \Rightarrow 2\frac{5}{10}$

(or)

$\frac{25}{10}$

Place Value Chart

Places	Ten Thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
Value	10000	1000	100	10	1	.	1/10	1/100	1/1000

Example

56327 → five digit number

Right to left

5	6	3	2	7
↓	↓	↓	↓	↓
Ten Thousands	Thousands	Hundreds	Tens	unit

Place Value

$5 \times 10000 + 6 \times 1000 + 3 \times 100 + 2 \times 10 + 7 \times 1$

$50000 + 6000 + 300 + 20 + 7$

Place Value of 3
$= 3 \times 100 = 300$

4327 → The place value of 4

$4 \times 1000 = 4000$

327.98

↓ integral part or whole number

↳ Decimal number

$3 \times 100 + 2 \times 10 + 7 \times 1 + 9 \times 1/10 + 8 \times 1/100$

24.987 . Place value of 7 = $7/1000$

10 1 10 100 1000

Place value of 2 = 2×10

⊙ Place value indicates the position and face value indicates the value.

FACE VALUE → Face value of a digit is always fixed.

8637

Face Value of 6 = 6.

Place Value of 6 = $6 \times 100 = 600$.

24397

Face Value of 9 = 9

Place Value of 9 = $9 \times 10 = 90$

Addition

23.1 1.23

$$\begin{array}{r} 23.10 \\ 1.23 \\ \hline 24.33 \end{array}$$

Substraction

$$
\begin{array}{r}
23.10 \\
-\ \ 1.23 \\
\hline
21.87
\end{array}
$$

Multiplication

$13 \times 1.1 = \boxed{13 \times 11} = 14.3$

assuming no decimal

$3.8 \times 4.6 = \boxed{38 \times 46} = 17.48$

Divison

$65 \div 1.3 = \dfrac{65}{1.3} = \dfrac{65}{13} \times 10$

$65 \div 1.32 = \dfrac{65}{1.32} = \dfrac{65}{132} \times 100$

Decimals

Terminated

$4/5 = 0.8$

$10/8 = 1.25$

Repeated or Recurring

$1/3 = 0.33333 = 0.\overline{3}$

$8/3 = 2.66666 = 2.\overline{6}$

Pure Recurring

$0.\overline{3} = 0.33333\ldots$

$2.\overline{6} = 2.66666\ldots$

$9.\overline{45} = 9.454545\ldots$

Mixed Recurring

3.67454545

$= 3.67\overline{45}$

Non Recurring

3.24678263

Convert Recurring decimal to fraction

* $0.\overline{7} = 0.7777777\ldots$

$x = 0.77777\ldots$

$10x = 7.77777\ldots$

$10x = 7 + 0.77777\ldots$

$10x = 7 + x$

$9x = 7$

$x = 7/9$

* $0.1\overline{7} = 0.1777777\ldots$

$x = 0.1777777\ldots$

$10x = 1.77777\ldots$

$100x = 17.77777\ldots$

$100x = 16 + 1.77777\ldots$

$100x = 16 + 10x$

$90x = 16$

$x = \frac{16}{90}$.

Trick

* $0.\overline{7} = 7/9$

* $0.\overline{54} = \frac{54}{99}$

* $2.357357357\ldots = 2.\overline{357} = 2 + 0.\overline{357}$

$= 2 + \frac{357}{999}$

$= 2\frac{357}{999}$ (or) $\frac{2355}{999}$

* $0.1\bar{7} = \frac{17-1}{90} = \frac{16}{90}$

* $0.11\bar{7} = \frac{117-11}{900} = \frac{106}{900}$

* $0.8737373 = 0.8\overline{73} = \frac{873-8}{990} = \frac{865}{990}$

* $0.78\overline{943} = \frac{78943-78}{99900} = \frac{78865}{99900}$

* Write $6.\bar{6}$ in the form of p/q.

$6.\bar{6} = 6+0.\bar{6}$

$= 6+6/9$

$= 6\,6/9$

$= \frac{60}{9}$

$= \frac{20}{3}.$

(or) $6.\bar{6} = \frac{66-6}{9}$

$= \frac{60}{9}$

* $0.\overline{63} + 0.\overline{37}$

$= \frac{63}{99} + \frac{37}{99}$

$= \frac{100}{99}$

(*) Write $3.\overline{24}$ in the form of p/q

$$3.\overline{24} = 3 + 0.\overline{24} = \frac{324-3}{99} = \frac{321}{99}$$

(*) $12.\overline{34} + 28.\overline{26}$

$$= \frac{1234-12}{99} + \frac{2826-28}{99}$$

$$= \frac{1222}{99} + \frac{2798}{99}$$

$$= \frac{4020}{99}$$

$$= \frac{1340}{33}$$

1.3 : Rational and Irrational Numbers

Rational Number : A number which denotes in the form of p/q where, p, q are integers & $q \neq 0$.

Integers, fractions

Set of rational numbers encloses the set of integers and fractions.

Rational number can be written as ratio (ratio, ie, simple fraction).

$$1.5 = \frac{15}{10} = \frac{3}{2} \text{ (or) } (3:2)$$

$$7 = 7/1$$

$$2/3 = 0.6666 \text{ (repeating)}$$

Few numbers cannot be expressed in ratio or fractions ⇒ Irrational Numbers.

$\sqrt{2}$, π, e

$e \rightarrow 2.7182818\ldots\ldots$

$\pi \rightarrow 3.142859\ldots\ldots$

Note:- $\sqrt{2} * \sqrt{2} = 2$ (rational)

$\sqrt{5} \times \sqrt{10} = \sqrt{50} = 5\sqrt{2}$ (irrational)

$\sqrt{10} \times \sqrt{10} = 10$ (rational)

Real Numbers : All numbers represented on the number line are called real numbers.

$2, 3, \sqrt{3}, \sqrt{2}, -3, 3/2$, etc.

Complex Numbers :- It is combination of Real & Imaginary numbers. $a+ib$, where a and b are real number ; i is an imaginary.

$$i^2 = -1$$

$$i = \sqrt{-1}$$

$a + ib$

$\underset{\text{Real}}{\underline{3}} + \underset{\text{imaginary}}{\underline{5i}}$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = (i)(i^2)$

$= -i$

$i^4 = (i^2)^2$

$= (-1)^2$

$= 1$

$i^{240} = 1$

$i^{39} = i^{36} * i^3$

$= 1 * -i$

$= -i$

$i^{69} = i^{68} \times i^1$

$= i$

Conjugate of Complex Number

$a + bi \rightarrow$ complex number

$a - bi \rightarrow$ conjugate of complex number

$(a+bi)(a-bi) = a^2 - abi + abi - b^2 i^2 = a^2 - b^2 i^2$

$$\boxed{(a+bi)(a-bi) = a^2 + b^2}$$

Division

$$\frac{3+5i}{2-3i}$$

$$\frac{3+5i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{(3+5i)(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{6+9i+10i+15i^2}{4+9}$$

$$= \frac{19i-9}{13} \quad (\text{or}) \quad \frac{-9}{13} + \frac{19}{13}i$$

What is the value of $(\sqrt{-1})^{4n+1} + (\sqrt{-1})^{4n+3}$, where n is a natural number.

$$(\sqrt{-1})^{4n+1} + (\sqrt{-1})^{4n+3}$$

$$(\sqrt{-1})^{4n+1} + (\sqrt{-1})^{4n+1} \times (\sqrt{-1})^2$$

$$(\sqrt{-1})^{4n+1}\left(1 + (\sqrt{-1})^2\right)$$

$$(\sqrt{-1})^{4n+1}[1+-1]$$

$$= 0$$

Consecutive Number : A series of numbers in which the next number is 1 more than the previous number.

10, 11, 12, 13, 14, ...

31, 32, 33, 34, ...

Three types

1) Consecutive Natural Numbers 21, 22, 23, 24, ...

2) Consecutive Even Numbers $\underbrace{20, 22}_{2}, \underbrace{22, 24}_{2}, \underbrace{24, 26}_{2}, 28, \ldots$

3) Consecutive Odd Numbers $\underbrace{13, 15}_{2}, \underbrace{15, 17}_{2}, \underbrace{17, 19}_{2}, 21, \ldots$

5 Consecutive Natural Numbers

1st way = $x, x+1, x+2, x+3, x+4$

2nd way = $x+2, x+1, x, x-1, x-2$

5 Consecutive Even Numbers

1st way = $x, x+2, x+4, x+6, x+8$

2nd way = $x+4, x+2, x, x-2, x-4$

The sum of 5 consecutive odd numbers is 155. What is the largest number?

$x, x+2, x+4, x+6, x+8$

$x+x+2+x+4+x+6+x+8 = 155$

$5x+20 = 155$

$5x = 135$

$x = 27$

$x+8 = 35$

(OR)

$x-4, x-2, x, x+2, x+4$

$x-4+x-2+x+x+2+x+4 = 155$

$5x = 155$

$x = 31$

$x+4 = 31+4 = 35.$

Properties

1) The product of any 3 consecutive numbers is always divisible by 6.

$\frac{10 \times 11 \times 12}{6}$, $\frac{\overset{7}{\cancel{21}} \times \overset{11}{\cancel{22}} \times 23}{\cancel{6}}$

2) The product of any r consecutive integers (numbers) is divisible by $r!$.

5 consecutive Integers $\frac{11 \times 12 \times 13 \times 14 \times 15}{5!}$

5!

$= \frac{11 \times \overset{3}{\cancel{12}} \times 13 \times \overset{7}{\cancel{14}} \times \cancel{15}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

1.4 : Factors and Multiples

$$\begin{array}{c} 12 \\ 1 * 12 \\ 2 * 6 \\ 3 * 4 \end{array}$$

factor { 1, 2, 3 }   { 12, 6, 4 } factor

Factor :- Factor is the number which divides the given number.

Every number has $\underbrace{\text{1 and itself}}_{\text{Improper factors}}$ as factors

factors { 2, 3, 4, 6 → Proper factor
1, 12 → Improper factor

Multiple :- Given number divided by other number.

$\underbrace{24, 36, 48, 60}_{\text{Multiples}}$ are multiples of $\underset{\text{factor}}{12}$

factor = divides
Multiple = Divided by

Prime factorization : Product should be in the combination of Prime Number
Factors

$$\begin{array}{c} 12 \\ 1 * 12 \\ 2 * 6 \\ 3 * 4 \end{array}$$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$12 = 2^2 * 3$

(*) for the number 120, find

(i) Number of factors

(ii) Sum of factors

(iii) Number of even factors

(iv) Number of odd factors

120

1 * 120
2 * 60
3 * 40
4 * 30
5 * 24
6 * 20
8 * 15
10 * 12

Factors

(1, 120) (2, 60) (3, 40) (4, 30) (5, 24)

(6, 20) (8, 15) (10, 12)

(i) 16 factors

(ii) 1+120+2+60+3+40+4+30+5+24 + 6+20+8+15+10+120 = 360

(iii) 120, 2, 60, 40, 4, 30, 24, 6, 20, 8, 10, 12

Total = 12 factor

(iv) 16 − 12 = 4 factors / 1, 3, 5, 15

Trick

Step 1 :- Prime factorization.

$120 = 24 \times 5 = 8 \times 3 \times 5 = 2^3 \times 3 \times 5$

P_1, P_2, P_3 are different prime numbers

$N = P_1^a \times P_2^b \times P_3^c \times \ldots$

No. of factors $= (a+1)+(b+1)+(c+1)$

$= (3+1)(1+1)(1+1)$

$= 4 \times 2 \times 2 = 16$

$$\text{Sum of factors} = \frac{(P_1^{a+1}-1)\,(P_2^{b+1}-1)\,(P_3^{c+1}-1)\ldots}{(P_1-1)\,(P_2-1)\,(P_3-1)\ldots}$$

Sum of factors $= \frac{2^{3+1}-1}{2-1} * \frac{3^{1+1}-1}{3-1} * \frac{5^{1+1}-1}{5-1}$

$= 15 * 8/2 * 24/4$

$= 360$

(or)

$(2^0+2^1+2^2+2^3)(3^0+3^1)(5^0+5^1)$

$= (15)(4)(6)$

$= 360$

No. of even factor :

e = even prime number
p = odd prime number

$120 = 2^3 * 3^1 * 5^1$

$N = e^x * P_1^a * P_2^b * \dots$

No. of even factors $= x * (a+1) * (b+1) * \dots$

$= 3 * (1+1) * (1+1)$

$= 3 * 2 * 2$

$= 12$

No. of odd factors = Just eliminate even factors.

$= (a+1)(b+1)$

$= (1+1)(1+1)$

$= 2 * 2$

$= 4$

(or)

Total Factors = 16

Total Odd factors = 4

Total even factors = 16 - 4 = 12.

⊛ For the number 7200. find

(i) No. of factors

(ii) Sum of factors

(iii) No. of even factors

(iv) No. of odd factor.

$7200 = 72 \times 100$

$= 8 \times 9 \times 25 \times 4$

$= 2^3 \times 3^2 \times 5^2 \times 2^2$

$= 2^5 \times 3^2 \times 5^2$

$N = p_1^a \times p_2^b \times p_3^c$

No. of factors $= (a+1)(b+1)(c+1)$

$= (5+1)(2+1)(2+1)$

$= 6 \times 3 \times 3 = 54$

Sum of factors $= \dfrac{p_1^{a+1} - 1}{p_1 - 1} \times \dfrac{p_2^{b+1} - 1}{p_2 - 1} \times \dfrac{p_3^{c+1} - 1}{p_3 - 1}$

$= \dfrac{2^6 - 1}{2-1} \times \dfrac{3^3 - 1}{3-1} \times \dfrac{5^3 - 1}{5-1}$

$= 63 \times \dfrac{26}{2} \times \dfrac{124}{4}$

$= 25,389$

No. of odd factors $= (2+1)(2+1) = 9$

No. of even factors = Total factors − Odd factors = $54 - 9 = 45$

(or)

$(5)(2+1)(2+1)$

$= 5 \times 3 \times 3$

$= 45.$

1.5: Number Divisibility Rules

1) Divisible by 2: If the last/unit digit is even. ie, 0, 2, 4, 6, 8.

Ex:- 7392

2) Divisible by 3: If the sum of the digits of the given number is divisible by 3.

Ex:- 6393 = 6+3+9+3 = 21

3) Divisible by 4: If the last two digits is divisible by 4.

Ex:- 2300, 1996

4) Divisible by 5: If the unit digit is '0' or '5'.

Ex:- 995, 750

5) Divisible by 6: The number should simultaneously divisible by 2 and 3.

Ex:- 528 → divisible by 2

5+2+8 = 15 → divisible by 3

÷6 ✓

Ex:- 508 ✓ ÷2

X ÷ 3

X ÷ 6

7) Divisible by 7:

1071 ÷ 7

1071 = 107 − 1×2

= 105

105 = 10 − 5×2

= 0

1071 is divisible by 7

939715 ÷ 7

939
715

224

224 is divisible by 7

Hence, 939715 is divisible by 7.

7) Divisible by 8 : If the last 3 digits is divisible by 8.

Ex:- 3954048

$\downarrow$ 48 ÷ 8 ✓

÷ 8 ✓

34728

$\downarrow$ 728 ÷ 8 = 91

÷ 8 ✓

8) Divisible by 9 : If the sum of the digits is divisible by 9.

Ex:- 732987 = 7+3+2+9+8+7

$\downarrow$ = 36 ÷ 9 ✓

÷ 9 ✓

528 = 5+2+8

$\downarrow$ = 15 ÷ 9 ✗

÷ 9 ✗

9) Divisible by 11 :

1 2 3 4 5 6
6 0 3 5 5 9

Sum of odd places = 6+3+5 = 14

Sum of even places = 0+5+9 = 14

After doing the sum, we need to find the difference of the above two

Here difference is 14 - 14 = 0.

So if difference is 0 or multiple of 11 then we say that it is divisible by 11.

86845

Sum of odd place = 8+8+5 = 21

Sum of even place = 6+4 = 10

Difference = 21 − 10 = 11

Hence divisible by 11.

(*) Find the value of K, if K35624 is divisible by 11?

K35624

Sum of odd places = K+5+2 = 7+K

Sum of even places = 3+6+4 = 13

Difference should be either 0 or multiple of 11.

$K+7-13 = 0$

$K = 6$

(*) If 42573K is divisible by 72 then the value of K is

a) 4 b) 5 c) 6 d) 7

$$\frac{42573K}{72} = \frac{42573K}{8 \times 9}$$

$\frac{73K}{8} =$ (a) $\frac{734}{8}$ × (c) $\frac{736}{8} = 92$ ✓

(b) $\frac{735}{8}$ × (d) $\frac{737}{8}$ ×

(c)

∴ K = 6.

$4+2+5+7+3+K = \frac{21+K}{9} = \frac{21+6}{9} = \frac{27}{9}$ ∴ K = 6.

Find the value of K if 97215K6 is divisible by 11

97215K6

Sum of odd places = $9+2+5+6 = 22$

Sum of even places = $7+1+K = 8+K$

Difference can either be 0 or multiple of 11.

$22-(8+K) = 0$

$14-K = 0$

$K = 14$ (Not possible as single digit is required)

$22-(8+K) = 11$

$8+K = 11$

$K = 3$

1.6 : Properties on Divisibility

Divisibility Rules

$30 \div 7$

Divisor

$$7\overset{\uparrow}{)}\ \overset{\nearrow \text{Dividend}}{30}\ (\ 4 \rightarrow \text{Quotient}$$
$$\underline{28}$$
$$(2) \rightarrow \text{Remainder}$$

$$D\,)\ N\ (\ Q$$
$$\underline{\quad}$$
$$(R)$$

$$30 = 7 \times 4 + 2$$

Number (Dividend) = Divisor × Quotient + Remainder

$-30 \div 7$

$$7\,)\ -30\ (\ 4$$
$$\underline{28}$$
$$(-2)$$

Remainder is Non-negative.

In this case remainder is $7 - 2 = 5$

$42 \div 5$

$$5\,)\ 42\ (\ 8$$
$$\underline{40}$$
$$(2)$$

$$\frac{42}{5} = 8.4$$

8 is integral part (Quotient)

0.4 is Decimal part

$4/10 = 2/5$ (numerator 2 is the remainder)

Propeorties on Divisibility

1) If a is divisible by b then ac is also divisible by b.

Ex:- 54 is divisible by 3

$378 = 54 \times 7$ is also divisible by 3.

2) If a is divisible by b and b is divisible by c then a is divisible by c.

Ex:- 875 is divisible by 35

35 is divisibly by 7

Hence 875 should also be divisible by 7.

$$
\begin{array}{r}
7\,)\,875\,(125 \\
\underline{7} \\
17 \\
\underline{14} \\
35 \\
\underline{35} \\
(0)
\end{array}
$$

3) If n is divided by d and m is divided by same divisor d then (n+m) and (n-m) are both divisible by d.

Ex:- Let 28 and 742 are divisible by 7

$(28+742) = 770$

$(742-28) = 714$

$\frac{770}{7} = 110$, $\frac{714}{7} = 102$.

(4) Two different numbers when divided by the same divisor, leaves remainder x and y respectively, and when their sum is divided by the same divisor, remainder is z, then the divisor is $x+y-z$.

(or)

Divisor = Sum of Remainders − Remainder when sum is divided.

Note:- In order for property to hold, the sum of the remainders $x+y$ should be greater than divisor.

Ex:- $5\overline{)8}(1$, remainder $\boxed{3}$; $5\overline{)9}(1$, remainder $\boxed{4}$; $5\overline{)17}(3$, remainder $\boxed{2}$

$$5\overline{)\,8\,}(1 \qquad \frac{5}{\boxed{3}}$$

$$5\overline{)\,9\,}(1 \qquad \frac{5}{\boxed{4}}$$

$$5\overline{)\,17\,}(3 \qquad \frac{15}{\boxed{2}}$$

Suppose we do not know the divisor and the remainders are knowing as follows

$$d\overline{)\,8\,}(1 \qquad \frac{5}{\boxed{3}}$$

$$d\overline{)\,9\,}(1 \qquad \frac{5}{\boxed{4}}$$

$$d\overline{)\,17\,}(3 \qquad \frac{15}{\boxed{2}}$$

divisor $= 3+4-2 = 5$.

(5) What is the divisor when dividend is 345, the remainder is 5 and quotient is 20?

divisor?

$N = 345$
$R = 5$
$Q = 20$

$N = d \times Q + R$

$345 = d \times 20 + 5$

$d = 17$.

Q) In a divison sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 48, the dividend is

$d = 10Q$

$d = 5R$

$R = 48$

$N = ?$

$N = d \times Q + R$

$d = 5 \times 48 = 240$

$5 \times 48 = 10Q$

$Q = \frac{5 \times 48}{10}$

$Q = 24$

$N = 240 \times 24 + 48$

$= 5808.$

Two different numbers when divided by the same divisor, left remainder 10 and 15 respectively, and when their sum was divided by the same divisor, remainder was 3. What is the divisor?

a) 22 b) 25 c) 23 d) 24

$d \overline{) N_1 (} x$ $d \overline{) N_2 (} y$ $d \overline{) N_1 + N_2 (}$
$\quad 10 \qquad\qquad 15 \qquad\qquad 3$

$d \overline{) 25 (}$ $22 \overline{) 25 (} 1$
$(3) \qquad\qquad \frac{22}{(3)}$

$N_1 = d \times x + 10$

$N_2 = d \times y + 15$

$N_1 + N_2 = dx + dy + 10 + 15$

$= d(x+y) + 25$

$\frac{N_1 + N_2}{d} = \frac{d(x+y) + 25}{d} = \frac{d(x+y)}{d} + \frac{25}{d}$

$= R \to 0 + R \to 3$

Trick

$d = x + y - z$

$= 10 + 15 - 3$

$= 22.$

A number when divided by 221 gives a remainder 43, what remainder will be obtained by dividing the same number by 17?

$221\overline{)N}($ $17\overline{)N}($

$\quad (43)$ $(?)$

$N = 221 \times x + 43$

$$\frac{221x + 43}{17}$$

$$= \frac{221x}{17} + \frac{43}{17}$$

$$= 13x + \frac{43}{17}$$

$$17\overline{)43}(2$$
$$\underline{34}$$
$$9$$

$R = 9$

Trick :-

$d_1 = 221$

$d_2 = 17$

If $d_1 \div d_2$, the required remainder will be $R_1 \div 17$.

$$17\overline{)43}(2$$
$$\underline{34}$$
$$(9)$$

1.7: Number Remainder Theorem

$$8\overline{)40}(5$$
$$\frac{40}{(0)}$$

$R = 0$, the dividend is perfectly divided

$$8\overline{)42}(5$$
$$\frac{40}{(2)}$$

$R = 2$

Remainder should always be whole number, i.e, R cannot be negative. (non-negative).

* $\frac{17 \times 23}{12} = \frac{391}{12}$

$$12\overline{)391}(32$$
$$\frac{36}{31}$$
$$\frac{24}{(7)}$$

$R = 7$.

OR

$$\frac{17 \times 23}{12} = \frac{17}{12} \times \frac{23}{12}$$
$$= \frac{5 \times 11}{12}$$
$$= \frac{55}{12} = \underset{R}{\underset{\Downarrow}{7}}$$

* $\frac{11 \times 12 \times 13 \times 14}{5} = \frac{1 \times 2 \times 3 \times 4}{5}$

$$\Rightarrow \frac{24}{5}$$
$$= 4 = R$$

(*) find the remainder when 73+75+78 is divided by 34?

$$\frac{73+75+78}{34} = \frac{226}{34}$$

$$34\overline{)226}(6$$
$$\underline{206}$$
$$(22)$$

$$\frac{73}{34}+\frac{75}{34}+\frac{78}{34} = \frac{5+7+10}{34} = \frac{22}{34} \quad \therefore R \rightarrow 22.$$

(*) find the remainder when 14+15 is divisible by 8

$$\frac{14+15}{8} = \frac{29}{8}$$

$$8\overline{)29}(3$$
$$\underline{24}$$
$$(5)$$

Remainder theorem,

$$\frac{14}{8}+\frac{15}{8} = \frac{6+7}{8} = \frac{13}{8} \quad \therefore R \rightarrow 5$$

(*) find the last two digits of the expression

$$22 \times 31 \times 44 \times 27 \times 37 \times 43$$

last two digits can be taken out by dividing by 100.

$$12 \times 15 = 180$$
$$\div 100$$
$$100\overline{)180}(1$$
$$\underline{100}$$
$$(80)$$

$$\frac{22 \times 31 \times \cancel{44}^{11} \times 27 \times 37 \times 43}{\cancel{100}\ 25}$$

$$= \frac{22 \times 6 \times 11 \times 2 \times 12 \times 18}{25}$$

$$= \frac{132 \times 22 \times 216}{25}$$

$$= \frac{7 \times 22 \times 16}{25} \xrightarrow{R} \frac{154 \times 16}{25} \xrightarrow{R} \frac{4 \times 16}{25} \xrightarrow{R} \frac{64}{25} \xrightarrow{R} 14 \times 4 \rightarrow$$

last to digits is 56.

(✗) Find the remainder when 43^{197} is divided by 7.

$$\frac{43^{197}}{7} = \frac{43 \times 43 \times 43 \times \ldots \times 43 \text{ (197 times)}}{7}$$

$$\frac{43}{7} \xrightarrow{R} 1$$

$$= \frac{1 \times 1 \times 1 \times 1 \ldots \times 1 \text{ (197 times)}}{7}$$

$$\frac{43^2}{7} \rightarrow 1$$

$$= R \rightarrow 1$$

(✗✗) Divisor as prime number

$$\frac{43^{197}}{7} \xrightarrow{R} 1$$

$$\frac{43^{197}}{7} = \frac{43}{7} \xrightarrow{R} 1$$

$$\frac{1^{197}}{7} \xrightarrow{R} 1$$

(✗) Find the remainder when 59^{28} is divided by 7.

$$\frac{59^{28}}{7} = \frac{59}{7} \xrightarrow{R} 3$$

$$\frac{3^{28}}{7}$$

$$\frac{3^1}{7} \xrightarrow{R} 3$$

$$\frac{3^2}{7} \xrightarrow{R} 2$$

$$\frac{3^3}{7} \xrightarrow{R} 6$$

$$\frac{3^4}{7} \rightarrow \frac{3^3 \times 3}{7} \xrightarrow{R} \frac{6 \times 3}{7} = \frac{18}{7} \xrightarrow{R} 4$$

$$\frac{3^5}{7} \xrightarrow{R} 5$$

$$\frac{3^6}{7} \xrightarrow{R} 1$$

$$\frac{3^{28}}{7} = \frac{3^{24} \cdot 3^{4}}{7}$$

$$= \frac{1 \times 4}{7}$$

$$\frac{3^{28}}{7} \xrightarrow{R} 4 \rightarrow \frac{59^{28}}{7}$$

Properties

1) Remainder of $\left[\frac{(ax+1)^n}{a}\right] = 1$ for all values of n

$$\frac{85}{7} = \frac{7 \times 12 + 1}{7} = \frac{7 \times 12}{7} + \frac{1}{7}$$

$= R \rightarrow 0 + R \rightarrow 1$

$= 1$

$7 \overline{)85} (12$, 84, $\underline{1}$

$$\frac{85^8}{7} \xrightarrow{R} 1$$

$$\frac{100}{9} = \frac{9 \times 11 + 1}{9} \xrightarrow{R} 1$$

$$\frac{100^{100}}{9} \xrightarrow{R} 1$$

Q) find the remainder when 100^{99} is divisible by 11?

$$\frac{(100)^{99}}{11} = \frac{(11 \times 9 + 1)^{99}}{11} = \frac{(ax+1)^n}{a} \xrightarrow{R} 1$$

$$\frac{(11 \times 9 + 1)^{99}}{11} \xrightarrow{R} 1.$$

24) $\frac{a^n}{a+1}$ leaves a remainder of

a if n is odd

1 if n is even

$\frac{8^{24}}{9} = \frac{8}{9} \xrightarrow{R} 8$

$\frac{8^2}{9} \xrightarrow{R} 1$

n is $\div$ by 2

$\frac{24}{2} \xrightarrow{R} 0$

$\frac{8^{24}}{9} \xrightarrow{R} 1$

$\frac{a^n}{a+1}$

$\frac{8^{24}}{8+1}$

n = even

$\therefore \frac{8^{24}}{9} \xrightarrow{R} 1.$

$\frac{15^{45}}{16}$

$\frac{15}{16} \xrightarrow{R} 15$

$\frac{15^2}{16} \xrightarrow{R} 1$

$n \div 2$

$\frac{15^{44}}{16} \xrightarrow{R} 1$

$R \rightarrow \frac{15^{44}}{16} \times \frac{15}{16}$

$R \rightarrow 1 \times 15 \rightarrow 15$

$\frac{15^{45}}{15+1}$

n is odd

$a = 15$

$\frac{15^{45}}{16} \xrightarrow{R} 15$

3) Remainder of $\frac{a^n+b^n+c^n+d^n+\ldots}{a+b+c+d+\ldots} = 0$

when (i) $a, b, c, d, \ldots$ are in A.P.

(ii) n is odd.

$\underbrace{2, 3}_{1}, \underbrace{4, 5}_{1}, 6, \Rightarrow$ AP.

$32, 34, 36, 38 \Rightarrow$ AP (difference 2)

$\frac{16^3+17^3+18^3+19^3}{70}$, find the remainder?

$n = 3$ (odd)

$a = 16$, $b = 17$, $c = 18$, $d = 19$

$a+b+c+d = 16+17+18+19 = 70$

$R \to 0$.

4) The remainder when $f(x) = a+bx+cx^2+dx^3+\ldots$ is divided by $(x-a)$ is $f(a)$.

$$\frac{2+3x+4x^2+6x^3}{x-2}$$

$R \to f(2)$

Ⓐ Find the remainder when $f(x) = x^3-7x-6$ is divided by $x-4$

$R \to f(a) \to f(4)$

$\to (4)^3 - 7(4) - 6$

$\Rightarrow 64 - 28 - 6$

$\to 64 - 34$

$R \to 30$

1.8 Numbers Unit Digit

Unit Digit

Ex:- $17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63$

$7 \times 2 \times 6 \times 4 \times 7 \times 1 \times 3$

Unit digit → 6

Ex:- Find the unit digit for 5^{256}

$5^1 = ...5$ $\qquad 5^n = ...5$

$5^2 = ...5$

$5^3 = ...5$ $\qquad \therefore 5^{256} = ...5$

$5^4 = ...5$

Rules

I) 1) $(.....1)^n = ...1$ $\qquad 36^2 = 1296$

2) $(.....5)^n = ...5$ $\qquad 6^3 = 216$

3) $(.....6)^n = ...6$

For 1, 5 and 6 at the end as a unit digit then

1^n, 5^n and 6^n = unit digit → number itself.

$3^{128} = ...1$

II) 1) $(.....3)^{4n} = ...1$

2) $(.....7)^{4n} = ...1$ $\qquad 7^{27} = ...3$

$\downarrow$

3) $(.....9)^{2n} = ...1$ $\qquad 7^{24} . 7^3$

$3^1 = ...3$ $\qquad 9^1 = ...9$

$3^2 = ...9$ $\qquad 9^2 = ...1$

$3^3 = ...7$ $\qquad 9^3 = ...9$

$3^4 = ...1$ $\qquad 9^4 = ...1$

$3^5 = ...3$

iii) 1) $2^1 = \ldots 2$

$2^2 = \ldots 4$

$2^3 = \ldots 8$

$2^4 = \ldots 6$

$2^5 = \ldots 2$

$2^{4n} = \ldots 6$

2) $4^1 = \ldots 4$

$4^2 = \ldots 6$

$4^3 = \ldots 4$

$4^4 = \ldots 6$

$4^{2n} = \ldots 6$

3) $8^1 = \ldots 8$

$8^2 = \ldots 4$

$8^3 = \ldots 2$

$8^4 = \ldots 6$

$8^5 = \ldots 8$

$8^{4n} = \ldots 6$

<u>Cycle</u>

3 7 2 8	ends at 4
4 9	ends at 2

Ex: Unit Place in $(659)^{56} \times (329)^{79}$

$9^{2n} = \ldots 1$

unit digit for $(659)^{56} = \ldots 1$

unit digit for $(329)^{79} = (329)^{78} \times (329)^{1} = \ldots 1 \times \ldots 9 = \ldots 9$

$\therefore$ Unit Place for $(659)^{56} \times (329)^{79} = (\ldots 1) \times (\ldots 9) = \ldots 9$

Ex:-

Unit Place in $(321)^{321} \times (325)^{726}$

$\ldots 1 \times 5 = \ldots 5$

Ex:- $(37)^{23} \times (43)^{144} \times (57)^{226} \times (32)^{127} \times (52)^{51}$

find the unit digit for above expression

$7^{4n} = \ldots 1$

$3^{4n} = \ldots 1$

$2^{4n} = \ldots 6$

$(37)^{23} = 37^{20} \times 37^{3} = \ldots 3$

$(43)^{144} = \ldots 1$

$(57)^{226} = (57)^{224} \times (57)^{2} = \ldots 9$

$(32)^{127} = (32)^{124} \times (32)^{3} = \ldots 6 \times \ldots 8 = \ldots 8$

$(52)^{51} = (52)^{48} \times (52)^{3} = \ldots 6 \times \ldots 8 = \ldots 8$

Unit digit for given expression $= \ldots 3 \times \ldots 9 \times \ldots 1 \times \ldots 8 \times \ldots 8$

$= \ldots 8$

Ex :- find the unit digit for $38^{212} + 222^{38}$

$8^{4n} = ...6$

$2^{4n} = ...6$

$(38)^{212} = (38)^{220} \times (38)^{2} = ...6 \times ....4 = ...4$

$(222)^{38} = (222)^{36} \times (222)^{2} = ...4$

$....4 + ...4 = ...8$

1.9 Base System

Decimal System

Base = 10

Digits used : (0 to 9)

Base is called Radix

Digits cannot be greater than the base

Binary

Base = 2

Digits used = (0 and 1)

Octal

Base = 8

Digits used = (0 to 7)

Hexadecimal

Base = 16

Digits used = (0 to 9, A, B, C, D, E, F)

A → 10, B → 11, C → 12, D → 13, E → 14, F → 15

(*) Base must be greater than any integer of the number.

Ex: $(1201)_2$ → This is wrong representation.

This is because for base 2 we can use 0 and 1 only.

Ex $(1101)_2$ → This is correct representation

Conversion of decimal to any base

Ex:- Convert $(17)_{10}$ to binary system.

Base for binary = 2

$$\begin{array}{r|l}
2 & 17 \\
2 & 8 - 1 \\
2 & 4 - 0 \\
2 & 2 - 0 \\
2 & 1 - 0 \\
 & 0 - 1
\end{array}$$

$(10001)_2$

Ex:- Convert $(35)_{10}$ to binary system.

Base for binary = 2

$$\begin{array}{r|l}
2 & 35 \\
2 & 17 - 1 \\
2 & 8 - 1 \\
2 & 4 - 0 \\
2 & 2 - 0 \\
2 & 1 - 0 \\
 & 0 - 1
\end{array}$$

$(100011)_2$

Remainders from bottom to up.

Ex:- Convert $(35)_{10}$ to base 3.

$$\begin{array}{r|l}
3 & 35 \\
3 & 11 - 2 \\
3 & 3 - 2 \\
3 & 1 - 0 \\
 & 0 - 1
\end{array}$$

$(1022)_3$

Ex/ Convert $(169)_{10}$ into base 7.

$$\begin{array}{r|l} 7 & 169 \\ 7 & 24-1 \\ 7 & 3-3 \\ & 0-3 \end{array}$$

$(331)_7$

Ex Convert $(567)_{10}$ into base 8

$$\begin{array}{r|l} 8 & 567 \\ 8 & 70-7 \\ 8 & 8-6 \\ 8 & 1-0 \\ & 0-1 \end{array}$$

$(567)_{10} = (1067)_8$

Conversion from any base into decimal

Ex $(100011)_2$ to decimal.

$$\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array} \leftarrow$$

$2^0 \times 1 + 2^1 \times 1 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 0 + 2^5 \times 1 = 1 + 2 + 0 + 0 + 0 + 32$

$= (35)_{10}$.

Ex Convert $(5642)_8$ to decimal

$$\begin{array}{cccc} 5 & 6 & 4 & 2 \\ 8^3 & 8^2 & 8^1 & 8^0 \end{array} \leftarrow$$

$8^0 \times 2 + 8^1 \times 4 + 8^2 \times 6 + 8^3 \times 5 = 2 + 32 + 384 + 2560$

$= (2978)_{10}$

80 The value of x when $(25)_x = (85)_{10}$ is :

(a) 2 (b) 8 (c) 40 (d) Can't be determined.

Option (a) is not possible because the digits are equal and greater than the base.

$(25)_x$

$$\begin{array}{cc} 2 & 5 \\ x^1 & x^0 \end{array}$$

$x^0 \times 5 + x^1 \times 2 = (85)_{10}$

$5 + 2x = 85$

$2x = 80$

$x = 40.$

$\therefore (25)_{40} = (85)_{10}.$

Alternative

By using options.

(b) $(25)_8$

$$\begin{array}{cc} 2 & 5 \\ 8^1 & 8^0 \end{array}$$

$8^0 \times 5 + 8^1 \times 2 = 5 + 16 = 21$

(c) $(25)_{40}$

$$\begin{array}{cc} 2 & 5 \\ 40^1 & 40^0 \end{array}$$

$40^0 \times 5 + 40^1 \times 2 = 5 + 80 = 85.$

Ex:- Which one is invalid number?

(a) $(325)_7$ (b) $(345)_5$ (c) $(543)_6$ (d) None

Ans:- option (b). This is because the number cannot be equal or greater than the base.

Ex:- $(52)_7 + (46)_8 = (?)_{10}$

(a) $(75)_{10}$ (b) $(50)_{10}$ (c) $(39)_{(39)}$ (d) $(28)_{10}$

First convert into decimal system for doing any mathematical operation.

$(52)_7 = 7^0 \times 2 + 7^1 \times 5 = 2 + 35 = (37)_{10}$

$(46)_8 = 8^0 \times 6 + 8^1 \times 4 = 6 + 32 = (38)_{10}$

$(37)_{10} + (38)_{10} = (75)_{10}$

Ex:- $(23)_5 + (47)_9 = (?)_8$

$(23)_5 = 5^0 \times 3 + 5^1 \times 2 = 3 + 10 = (13)_{10}$

$(47)_9 = 9^0 \times 7 + 9^1 \times 4 = 7 + 36 = (43)_{10}$

$(13)_{10} + (43)_{10} = (56)_{10}$

$(56)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 56 \\ \hline 8 & 7-0 \\ \hline & 0-7 \end{array}$$

$(70)_8$

$(56)_{10} = (70)_8$.

Ex:- $(24)_5 \times (32)_5 = (?)_5$

(a) 1423 (b) 1422 (c) 1420 (d) 1323

$(24)_5 = 5^0 \times 4 + 5^1 \times 2 = 4 + 12 = (14)_{10}$

$(32)_5 = 5^0 \times 2 + 5^1 \times 3 = 2 + 15 = (17)_{10}$

$(14)_{10} \times (17)_{10} = (238)_{10}$

$$\begin{array}{r|l} 5 & 238 \\ \hline 5 & 47 - 3 \\ \hline 5 & 9 - 2 \\ \hline 5 & 1 - 4 \\ \hline & 0 - 1 \end{array}$$

$(1423)_5$

$(238)_{10} = (1423)_5$

1.10: Indices and Surds Part-1

Indices

* $a \times a \times a \times a \times \ldots\ldots a = a^n$

In a^n = n is called index (or) exponent (or) power

a is called base.

$2^3 \Rightarrow$ 3 index
2 base

Law of indices

1) If m and n are positive integers

(a) $a^m \times a^n = a^{m+n}$

(b) $\dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0,\ m \geq n)$

(c) $(ab)^n = a^n \times b^n$

(d) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

(e) $(a^m)^n = a^{mn}$ $\left[a^{m^n} \neq (a^m)^n\right]$

(f) $a^0 = 1$

Some more results

1) If $a^x = k$ then $a = k^{1/x}$

2) If $a^{1/x} = k$ then $a = k^x$

3) If $a^x = b^y$ then $a = b^{y/x}$ (or) $b = a^{x/y}$

4) If $a^x = a^y$ then $x = y$

5) $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$ $\quad [2^{-1} = 1/2, \; \frac{1}{2^3} = 2^{-3}]$

6) $a^{b^c} \neq (a^b)^c$

Ex $(-2)^3 \times 5^2$

$\cancel{-2 \times -2 \times -2}$

$-8 \times 25 = -200$

Ex $\left(\frac{-1}{343}\right)^{-2/3}$

$\left(\frac{-1}{343}\right)^{-2/3} = \left(\frac{-1}{7^3}\right)^{-2/3}$

$= (-7^{-3})^{-2/3}$

$= (-7)^2$

$= 49$

Ex $4^{2x+1} = 8^{x+3}$ then $x = ?$

$(2^2)^{2x+1} = (2^3)^{x+3}$

$(a^b)^c = a^{bc}$

$2^{4x+2} = 2^{3x+9}$

$4x + 2 = 3x + 9$

$x = 7.$

Ex $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$ is equal to :

(a) 0 (b) 1 (c) abc (d) None

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c$$

$$(a^m)^n = a^{mn}$$

$$x^{ab-ac} \times x^{bc-ab} \times x^{ac-bc}$$

$$a^m \times a^n = a^{m+n}$$

$$x^{ab-ac+bc-ab+ac-bc}$$

$$x^0 = 1$$

Ex $\left[\left(x+\frac{1}{y}\right)^a \left(x-\frac{1}{y}\right)^b\right] \div \left[\left(y+\frac{1}{x}\right)^a \left(y-\frac{1}{x}\right)^b\right]$ is equal to :

(a) $\left(\frac{x}{y}\right)^{a+b}$ (b) $\left(\frac{y}{x}\right)^{a+b}$ (c) $\frac{x^a}{y^b}$ (d) $(xy)^{a+b}$

$$\frac{\left(x+\frac{1}{y}\right)^a \left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a \left(y-\frac{1}{x}\right)^b} = \frac{\left(\frac{xy+1}{y}\right)^a \left(\frac{xy-1}{y}\right)^b}{\left(\frac{xy+1}{x}\right)^a \left(\frac{xy-1}{x}\right)^b}$$

$$= \frac{\frac{(xy+1)^a}{y^a} \times \frac{(xy-1)^b}{y^b}}{\frac{(xy+1)^a}{x^a} \times \frac{(xy-1)^b}{x^b}}$$

$$= \frac{(xy+1)^a (xy-1)^b}{y^{a+b}} \times \frac{x^{a+b}}{(xy+1)^a (xy-1)^b}$$

$$= \frac{x^{a+b}}{y^{a+b}}$$

$$= \left(\frac{x}{y}\right)^{a+b}$$

Ex If $a^x = b$, $b^y = c$ and $c^z = a$ then the value of xyz is:

(a) 0 (b) 1 (c) $x+y+z$ (d) abc

$a^x = b$

$\Rightarrow (c^z)^x = b$

$\Rightarrow c^{zx} = b$

$\Rightarrow (b^y)^{zx} = b$

$\Rightarrow (b)^{xyz} = (b)^1$

$\therefore xyz = 1.$

Ex $2^{2^x} = 16^{2^{3x}}$ then x is equal to:

(a) -1 (b) 0 (c) 1 (d) None

$2^{2^x} = 16^{2^{3x}}$

$2^{2^x} = (2^4)^{2^{3x}}$

$2^{2^x} = (2^{2^2})^{2^{3x}}$

$2^{2^x} = 2^{2^{3x+2}}$

$2^x = 2^{3x+2}$

$x = 3x + 2$

$2x = -2$

$x = -1$

Ex $3^{x+y} = 81$ and $81^{x-y} = 3$, value of x is

$3^{x+y} = 81$

$3^{x+y} = 3^4$

$x+y = 4$

$81^{x-y} = 3$

$(3^4)^{x-y} = 3^1$

$4x - 4y = 1$

$(x+y = 4) \times 4$

$4x - 4y = 1$

$4x + 4y = 16$

$4x - 4y = 1$

$8x = 17$ $\therefore x = 17/8$.

Ex Value of $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$ is :

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}+\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$\frac{1}{\frac{x^a+x^b+x^c}{x^a}}+\frac{1}{\frac{x^b+x^a+x^c}{x^b}}+\frac{1}{\frac{x^c+x^b+x^a}{x^c}}$$

$$\frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^b+x^a+x^c}+\frac{x^c}{x^c+x^b+x^a}$$

$$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$$

$$= 1$$

1.11 Indices and surds Part 2

Surds

$\sqrt{4} = 2 \rightarrow$ rational number (not a surd)

$3 \rightarrow$ Rational Number

$\sqrt{3} \rightarrow$ Surd

$\sqrt{9} \rightarrow$ Not a surd

$\sqrt{10} \rightarrow$ Surd

$2+4\sqrt{3} \rightarrow$ Surd $\rightarrow$ Mixed Surd

$2+4\sqrt{3}$ $\quad$ $2-4\sqrt{3}$
$\hookrightarrow$ Conjugate surd of $2+4\sqrt{3}$

Conjugate for $3+\sqrt{2} = 3-\sqrt{2}$

Conjugate for $2\sqrt{3}+5\sqrt{3} = 2\sqrt{3}-5\sqrt{3}$

Properties of Surds

1) If $a+\sqrt{b} = c+\sqrt{d}$ (or) $a-\sqrt{b} = c-\sqrt{d}$ then, $a=c$ and $b=d$

Ex $\quad a+\sqrt{b} = 2+\sqrt{3}$

$\therefore a=2$

$b=3$

2) If $a \pm \sqrt{b} = 0$, then $a=0, b=0$

3) If $\sqrt{a+\sqrt{b}} = \sqrt{c}+\sqrt{d}$, then $\sqrt{a-\sqrt{b}} = \sqrt{c}-\sqrt{d}$

Ex $\quad \sqrt{10+2\sqrt{24}} = \sqrt{6}+\sqrt{4}$

$\Downarrow$ $\quad$ (or)

$\sqrt{10+\sqrt{96}}$ $\quad$ $\sqrt{4}+\sqrt{6}$

4) Only similar surds can be simplified.

$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

$2\sqrt{3} + \sqrt{2}$ cannot be simplified

$2\sqrt{3} \times 5\sqrt{3} = 10 \times 3 = 30$

$2\sqrt{3} \times 5\sqrt{2} = 10\sqrt{3 \times 2} = 10\sqrt{6}$

$7\sqrt{3} - 5\sqrt{3} = 2\sqrt{3}$

5) Rationalization

Making two or more surds a rational number by multiplication is called rationalization.

$a^2 - b^2 = (a+b)(a-b)$

Ex1. $(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$

$(9\sqrt{3} - 5\sqrt{3})(9\sqrt{3} + 5\sqrt{3}) = 81(3) - 25(3)$

$= 168$

$$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$$

Basic formulas

1) $(a+b)^2 = a^2 + b^2 + 2ab$
2) $(a-b)^2 = a^2 + b^2 - 2ab$
3) $(a^2 - b^2) = (a+b)(a-b)$

Ex:- find the value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{(2)^2+(\sqrt{3})^2+2.2.\sqrt{3}}{4-3}$$

$$= 4+3+4\sqrt{3}$$

$$= 7+4\sqrt{3}$$

Ex: Which one is the smallest out of $\sqrt[3]{2}$ and $\sqrt[4]{3}$?

$(2)^{1/3}$

$(3)^{1/4}$

to compare we need to make the powers equal.

LCM of 3, 4 = 12

$$(2)^{1/3} = (2^4)^{1/12} = (16)^{1/12}$$

$$(3)^{1/4} = (3^3)^{1/12} = (27)^{1/12}$$

$$(27)^{1/12} > (16)^{1/12}$$

$$\therefore (3)^{1/4} > (2)^{1/3}$$

Ex Which one is greatest out of $\sqrt[3]{5}$, $\sqrt[4]{3}$, $\sqrt[3]{4}$?

$\sqrt[3]{5} = (5)^{1/3}$

$\sqrt[4]{3} = (3)^{1/4}$

$\sqrt[3]{4} = (4)^{1/3}$

LCM of 3, 4 = 12

$5^{1/3} = (5^4)^{1/12} = (625)^{1/12}$

$3^{1/4} = (3^3)^{1/12} = (27)^{1/12}$

$4^{1/3} = (4^4)^{1/12} = (256)^{1/12}$

$\therefore$ $\sqrt[3]{5}$ is greatest.

Ex Arrange the following in descending order

$\sqrt[3]{4}$, $\sqrt[4]{6}$, $\sqrt[6]{15}$, $\sqrt[12]{245}$

LCM of 3, 4, 6, 12 = 12

$(4)^{1/3} = (4^4)^{1/12} = (256)^{1/12}$

$(6)^{1/4} = (6^3)^{1/12} = (216)^{1/12}$

$(15)^{1/6} = (15^2)^{1/12} = (225)^{1/12}$

$(245)^{1/12} \quad (245)^{1/12} = (245)^{1/12}$

$\sqrt[3]{4} > \sqrt[12]{245} > \sqrt[6]{15} > \sqrt[4]{6}$

Ex: If $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{a+2}$, then the value of a is:

$$\frac{5 \cdot 5^{1/2} \times 5^3}{5^{-3/2}} = 5^{a+2}$$

$$\frac{1}{a^{-n}} = a^n$$

$$5 \times 5^{1/2} \times 5^3 \times 5^{3/2} = 5^{a+2}$$

$$5^{(1+1/2+3+3/2)} = 5^{a+2}$$

$$5^6 = 5^{a+2}$$

$$6 = a+2$$

$$a = 4$$

Ex: Find the square root of $7+2\sqrt{10}$

$$\sqrt{7+2\sqrt{10}} = \sqrt{a}+\sqrt{b}$$

factors: $5+2$; $5 \times 2 = 5+2 = 7$

$10 \times 1 = 10+1 = 11$

$\sqrt{5}+\sqrt{2}$ (or) $\sqrt{2}+\sqrt{5}$

If $\sqrt{7+4\sqrt{10}}$

then,

$\sqrt{7+2\sqrt{4\times10}}$

$\sqrt{7+2\sqrt{40}}$

this 2 is compulsory to apply the trick

If $\sqrt{7-2\sqrt{10}} = \sqrt{a}-\sqrt{b}$

$5+2$; 5×2

$\sqrt{5}-\sqrt{2}$.

Ex: Find the square root of $27-10\sqrt{2}$

$$\sqrt{27-10\sqrt{2}} = \sqrt{27-2\times5\sqrt{2}} = \sqrt{27-2\sqrt{25\times2}}$$

$25+2$

$$= \sqrt{25}-\sqrt{2}$$

$$= 5-\sqrt{2}$$

Ex Find the square root of $30+12\sqrt{6}$

$\sqrt{30+12\sqrt{6}}$

$=\sqrt{30+2\times 6\sqrt{6}}$

$=\sqrt{30+2\times\sqrt{36\times 6}}$

$=\sqrt{\underbrace{30}_{18+12}+2\sqrt{\underbrace{216}_{18\times 12}}}$

$=\sqrt{18}+\sqrt{12}$

$=3\sqrt{2}+2\sqrt{3}.$

Ex Value of $\sqrt{182+\sqrt{182+\sqrt{182+\cdots}}}$ is

$\sqrt{\underbrace{182}_{14\times 13}+\sqrt{182+\sqrt{182+\cdots}}}$

Factors
↳ Consecutive co-prime.

Ans = 14

$\sqrt{182+\sqrt{182+\sqrt{182+\cdots}}}=x$

Squaring both sides

$182+\sqrt{182+\sqrt{182+\sqrt{182+\cdots}}}=x^2$

$182+x=x^2$

$x^2-x-182=0$

$x^2-14x+13x-182=0$

$x(x-14)+13(x-14)=0$

$x=14$ or $x=-13$

$\sqrt{\underbrace{182}_{14\times 13}-\sqrt{182-\sqrt{182-\cdots}}}$

Ans = 13

Ex $\sqrt{30+\sqrt{30+\sqrt{30}\ldots}}$

$\underset{5\times 6}{\wedge}$

Ans = 6.

Ex

$\sqrt{3\sqrt{3.\sqrt{3\ldots}}}$ is equal to

Ans = 3.

$\sqrt{3\sqrt{3\sqrt{3}}} = x$

Squaring both sides

$3\sqrt{3\sqrt{3\ldots}} = x^2$

$3x = x^2$

$x = 3$

Ex find the value of $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$7+4\sqrt{3} = \sqrt{\underset{4+3}{7} + 2\sqrt{4\times 3}} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

$\sqrt{19+8\sqrt{3}}$

$= \sqrt{\underset{16+3}{19}+2\sqrt{16\times 3}}$

$= \sqrt{16}+\sqrt{3}$

$\sqrt{-\sqrt{3}+\sqrt{3+8(2+\sqrt{3})}}$

$\sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}} = \sqrt{-\sqrt{3}+\sqrt{19+8\sqrt{3}}}$

$= \sqrt{-3+\sqrt{16}+\sqrt{3}}$

$= \sqrt{\sqrt{16}} = \sqrt{4} = 2$

Ex $\dfrac{2}{\sqrt{7}+\sqrt{5}} + \dfrac{7}{\sqrt{12}-\sqrt{5}} - \dfrac{5}{\sqrt{12}-\sqrt{7}} = ?$

$\dfrac{2}{\sqrt{7}+\sqrt{5}} \times \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{2(\sqrt{7}-\sqrt{5})}{7-5} = \sqrt{7}-\sqrt{5}$

$\dfrac{7}{\sqrt{12}-\sqrt{5}} \times \dfrac{\sqrt{12}+\sqrt{5}}{\sqrt{12}+\sqrt{5}} = \dfrac{7(\sqrt{12}+\sqrt{5})}{12-5} = \sqrt{12}+\sqrt{5}$

$\dfrac{5}{\sqrt{12}-\sqrt{7}} \times \dfrac{\sqrt{12}+\sqrt{7}}{\sqrt{12}-\sqrt{7}} = \dfrac{5(\sqrt{12}+\sqrt{7})}{12-7} = \sqrt{12}+\sqrt{7}$

$\sqrt{7}-\sqrt{5}+\sqrt{12}+\sqrt{5}-(\sqrt{12}+\sqrt{7})$

$\sqrt{7}-\sqrt{5}+\sqrt{12}+\sqrt{5}-\sqrt{12}-\sqrt{7} = 0$

Ex If a, b are rationals and $a\sqrt{2}+b\sqrt{3} = \sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$ then the values of a, b are.

$a\sqrt{2}+b\sqrt{3} = \sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$

$= \sqrt{49\times2}+\sqrt{36\times3}-\sqrt{16\times3}-\sqrt{36\times2}$

$= 7\sqrt{2}+6\sqrt{3}-4\sqrt{3}-6\sqrt{2}$

$a\sqrt{2}+b\sqrt{3} = \sqrt{2}+2\sqrt{3}$

$a=1,\ b=2.$

Ex

Value of $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ is

$(\sqrt{9})$ $(\sqrt{4})$

$$\frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

$$3+\sqrt{8} - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2)$$

$$3+\sqrt{8}-\sqrt{8}-\sqrt{7}-\sqrt{7}-\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$3+2 = 5$$

Ex If $x = 15+2\sqrt{56}$ then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is

$$\sqrt{x} = \sqrt{\underset{8+7}{15} + 2\sqrt{\underset{8\times 7}{56}}}$$

$$\sqrt{x} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}-\sqrt{7}}{8-7} = \sqrt{8}-\sqrt{7}$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{8}+\sqrt{7} - (\sqrt{8}-\sqrt{7})$$

$$= \sqrt{8}+\sqrt{7}-\sqrt{8}+\sqrt{7}$$

$$= 2\sqrt{7} \text{ (or) } \sqrt{28}$$

1.12 : Progressions

Numbers are said to be in progression, when numbers are arranged in a particular order.

$$\underbrace{3 \quad 6}_{3} \quad \underbrace{\quad 9}_{3} \quad \underbrace{\quad 12}_{3} \quad \underbrace{\quad 15}_{3}$$

Three types of Progression
1) Arithmetic Progression (AP)
2) Geometric Progression (GP)
3) Harmonic Progression (HP)

AP → When every two consecutive numbers having same common difference then the sequence is said to be Arithmatic Progression.

Ex

3	6	9	12	15
↓	↓	↓	↓	↓
a	$a+d$	$a+2d$	$a+3d$	$a+4d$

$n = 5$

a = first term

d = common difference

n = number of terms

n^{th} term $= t_n = a + (n-1)d$

$t_5 = a + 4d$

Sum of n terms $= \frac{n}{2}\left[2a + (n-1)d\right]$

(or)

$\frac{n}{2}\left[\text{first term} + \text{last term}\right]$

Ex

8, 2, −4, 10, −16

$\underbrace{8,2}_{-6}\ \underbrace{2,-4}_{-6}\ \underbrace{-4,10}_{-6}\ \underbrace{10,-16}_{-6}$

GP

2 4 8 16 32

$\underbrace{4 \quad 8}_{8/4=2} \quad \underbrace{8 \quad 16}_{16/8=2} \quad \underbrace{16 \quad 32}_{\frac{32}{16}=2}$

2 ↓ first term

$\frac{4}{2} = 2$

Common Ratio = 2.

Every number/term is multiplied by a same number.

a = first term

r = common ratio

n = number of terms

$a \quad a.r \quad ar^2 \quad ar^3 \quad ar^4 \quad \ldots \quad a.r^n$

n^{th} term = $a.r^{n-1}$

Sum of n terms $\Rightarrow \dfrac{a.(1-r^n)}{1-r}$, where $r<1$

$\Rightarrow \dfrac{a(r^n-1)}{r-1}$ where $r>1$

Sum of infinite terms (S_∞) = $\dfrac{a}{1-r}$

Ex Find the 10th term of the following series

$7, 10, 13, 16, \ldots$

Common difference $= 10 - 7 = 3$

$a =$ 1st term $= 7$
$d = 3$

$T_{10} = a + (n-1)d$
$= a + 9d$
$= 7 + 9(3)$
$= 7 + 27$
$= 34$

Ex Find the sum of $11 + 12 + 13 + 14 + \ldots\ldots + 30$

$n = 30 - 11 + 1$
$= 20$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
$= \frac{n}{2}\left[\text{first term} + \text{last term}\right]$
$= \frac{20}{2}\left[11 + 30\right]$
$= 10\left[41\right]$
$= 410.$

Ex Find the 6th term of $2, 4, 8, \ldots\ldots$

Common Ratio $= 4/2 = 2$; $8/4 = 2$.

$a = 2$, $r = 2$

$t_6 = a \cdot r^{n-1}$
$= 2 \cdot 2^{6-1}$
$= 2^6 = 64$

<u>8)</u> Find the sum of 7 terms of the following series

1, 3, 9, 27, ...

Common Ratio $(r) = 3$

$$S_7 = \frac{a \cdot (r^n - 1)}{r - 1}$$

$$= 1 \cdot \frac{3^7 - 1}{3 - 1}$$

$$= \frac{2187 - 1}{2}$$

$$= \frac{2186}{2} = 1093.$$

<u>Some more rules</u>

1) Sum of first n natural numbers $= \frac{n(n+1)}{2}$
$(1+2+3+4+ \dots + n)$

2) Sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
$(1^2+2^2+3^2+4^2+ \dots + n^2)$

3) Sum of cubes of first n natural numbers $= \left(\frac{n(n+1)}{2}\right)^2$
$(1^3+2^3+3^3+4^3+ \dots + n^3)$

4) Sum of first n even numbers $= n(n+1)$
$(2+4+6+8+ \dots + 2n)$

5) Sum of first n odd numbers $= n^2$
$(1+3+5+7+ \dots + 2n-1)$

6) Sum of first n terms of the following series

$\underbrace{1, 3}_{2}, \underbrace{3, 6}_{3}, \underbrace{6, 10}_{4}, \underbrace{10, 15}_{5}, \underbrace{15, 21}_{6}, \dots$ is $= \frac{n(n+1)(n+2)}{6}$

Ex Find the sum of first 48 natural numbers

Sum of first n natural numbers $= \frac{n(n+1)}{2}$

$n = 48$ $= \frac{48(48+1)}{2}$

$= 24 \times 49$

$= 1176.$

Ex Find the value of the expression

$1 - 6 + 2 - 7 + 3 - 8 + \ldots$ to 100 terms

(a) -250 (b) -500 (c) -450 (d) -300

$(1+2+3+4+\ldots 50 \text{ terms}) - (6+7+8+9+\ldots 50 \text{ terms})$

$a = 6,\ d = 1,\ T_{50} = 6 + 49 \times 1 = 55$

$\frac{n(n+1)}{2} - \frac{n}{2}[1^{st} + \text{last}]$

$\frac{50 \times 51}{2} - \frac{50}{2}[6 + 55]$

$25 \times 51 - 25 \times 61$

$25(51 - 61)$

-250

(or)

$1 - 6 + 2 - 7 + 3 - 8 + \ldots$ to 100 terms

$\Rightarrow -5 + -5 + -5 + \ldots\ldots$ upto to 50 terms as -5

$50 \times -5 = -250.$

Ex find the sum of all numbers divisible by 6 in between 100 to 400.

Nearest number to 100 (above) divisible by 6 = 102

$$6 \overline{)400}(66$$
$$\frac{36}{40}$$
$$\frac{36}{4}$$

$\therefore$ 102, 108, 114, 120, ...

Nearest number to 400 (below) divisible by 6 = 396

$102 + 108 + 114 + 120 + \dots + 396$

Sum of n terms = $\frac{n}{2}[1^{st} + last]$

$$n = \frac{396 - 102}{6} + 1$$

$$= \frac{294}{6} + 1$$

$$= 49 + 1$$

$$= 50$$

$\therefore$ Sum of n terms = $\frac{50}{2}[102 + 396]$

$$= 25 \times 498$$

$$= 12450.$$

1.3 Previous Year GATE Questions 2010

Q.1) If $137 + 276 = 435$ how much is $731 + 672$? [GATE 2010]

(a) 534 (b) 1408 (c) 1623 (d) 1513

$137 + 276 = 435$

$$\begin{array}{r} 137 \\ +\ 276 \\ \hline 3 \end{array}$$

Here the base is not 10.

We assume that, the base can be 8.

$(137)_8 + (276)_8 = (435)_8$

$(1\times8^2 + 3\times8^1 + 7\times8^0) + (2\times8^2 + 7\times8^1 + 6\times8^0)$

$(64 + 24 + 7) + (128 + 56 + 6)$

$(95)_{10} + (190)_{10} = (285)_{10}$

$$\begin{array}{r|l} 8 & 285 \\ 8 & 35 - 5 \\ 8 & 4 - 3 \\ & 0 - 4 \end{array}$$

$(285)_{10} = (435)_8$

$\therefore (731)_8 + (672)_8 = (?)_8$

$(7\times8^2 + 3\times8^1 + 1\times8^0) + (6\times8^2 + 7\times8^1 + 2\times8^0)$

$(448 + 24 + 1)_{10} + (384 + 56 + 2)_{10}$

$(473)_{10} + (442)_{10} = (915)_{10} = (1623)_8$

$$\begin{array}{r|l} 8 & 915 \\ 8 & 114 - 3 \\ 8 & 14 - 2 \\ 8 & 1 - 6 \\ & 0 - 1 \end{array}$$

Octal Addition

(0 – 7)

```
  1  1
  1  3  7
  2  7  6
---------
  4  3  5
```

$7 + 6 = 13 > 8$

$8 \,\underline{|\,13}$
1 – 5

$1 + 3 + 7 = 11 > 8$

$8 \,\underline{|\,11}$
1 – 3

$1 + 1 + 2 = 4 < 8$

```
 1  1
    7  3  1
    6  7  2
-----------
 1  6  2  3
```

$1 + 2 = 3 < 8$

$3 + 7 = 10 > 8$

$8 \,\underline{|\,10}$
1 – 2

$1 + 7 + 6 = 14 > 8$

$8 \,\underline{|\,14}$
1 – 6

$1 < 8$

$(731)_8 + (672)_8 = (1623)_8$

Ex **Octal Addition**

$$\begin{array}{ccc} {}^{1} & {}^{1} & \\ 1 & 4 & 7 \\ 2 & 6 & 1 \\ \hline 4 & 3 & 0 \end{array}$$

$(430)_8$

$1+7=8 \geq 8$

$8 \,\underline{|\, 8}$
$\quad 1 - 0$

$1+4+6 = 11 > 8$

$8 \,\underline{|\, 11}$
$\quad 1 - 3$

$1+1+2 = 4 < 8$

Octal Subtraction

$(0-7)$

Ex

$$\begin{array}{cccc} & \cancel{3}^{2} & \cancel{5}^{4\,8} & 6^{8} \\ - & 1 & 5 & 7 \\ \hline & (1 & 7 & 7)_8 \end{array}$$

$8+6-7 = 7 < 8$

$8+4-5 = 7 < 8$

$2-1 = 1 < 8$

Ex $(3254)_8 - (2763)_8 = ?$

$$\begin{array}{cccc} \cancel{3}^{2} & \cancel{2}^{1\,8} & 5^{8} & 4 \\ 2 & 7 & 6 & 3 \\ \hline & 2 & 7 & 1 \end{array}$$

$4-3 = 1 < 8$

$8+5-6 = 7 < 8$

$8+1-7 = 2 < 8$

$(271)_8$

Q.2) If $(1.001)^{1259} = 3.52$, $(1.001)^{2062} = 7.85$, then $(1.001)^{3321}$?

(a) 2.23 (b) 4.33 (c) 11.37 (d) 27.64

[GATE 2012]

Solution

$(1.001)^{1259} = 3.52$

$(1.001)^{2062} = 7.85$

$(1.001)^{3321} = ?$

$$(1.001)^{1259} \times (1.001)^{2062} = (1.001)^{1259+2062}$$

$$3.52 \times 7.85 = (1.001)^{3321}$$

$$\therefore (1.001)^{3321} = 27.632 \simeq 27.64$$

Q.3) A number is as much greater than 75 as it is smaller than 117. The number is.

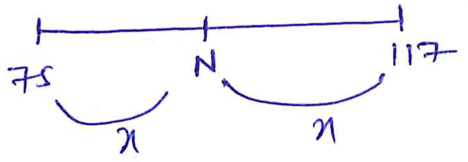
(GATE 2013)

(a) 91 (b) 93 (c) 89 (d) 96

Solution

$N \to > 75 \to 75 + x$

$N \to < 117 \to 117 - x$



$75 + x = 117 - x$

$2x = 117 - 75$

$x = \frac{42}{2} = 21$

$75 + 21 = 96$

Q.4) Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots\cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

(GATE 2013)

(a) 7 (b) 8 (c) 9 (d) 10

Solution

$$\frac{1}{\sqrt{79}+\sqrt{80}} + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$\frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} = \sqrt{2}-\sqrt{1}$$

$$\cancel{\sqrt{2}} - \sqrt{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cdots\cdots + \cancel{\sqrt{80}} - \cancel{\sqrt{79}} + \sqrt{81} - \cancel{\sqrt{80}}$$

$$-\sqrt{1} + \sqrt{81}$$

$$= 9 - 1 = 8$$

Q.5) Consider the equation

$(7526)_8 - (Y)_8 = (4364)_8$, when $(X)_N$ stands for X to the base N. Find Y.

(GATE 2014)

(a) 1634 (b) 1737 (c) 3142 (d) 3162

Solution

$$(7526)_8 - (Y)_8 = (4364)_8$$

$$(Y)_8 = (7526)_8 - (4364)_8$$

$$\begin{array}{cccc} & {}^{4} & {}^{8} & \\ 7 & \cancel{5} & 2 & 6 \\ 4 & 3 & 6 & 4 \\ \hline \end{array}$$

$$(3\ 1\ 4\ 2)_8$$

$6 - 4 = 2 < 8$

$8 + 2 - 6 = 4 < 8$

$4 - 3 = 1 < 8$

1.14 : Previous Year GATE Questions 2014

Q.6) If $\left(z+\frac{1}{z}\right)^2 = 98$, Compute $\left(z^2+\frac{1}{z^2}\right)$ (GATE 2014)

Solution

$$(a+b)^2 = a^2+b^2+2ab$$

$$\left(z+\frac{1}{z}\right)^2 = z^2+\frac{1}{z^2}+2\cdot z\cdot\frac{1}{z} = 98$$

$$\Rightarrow z^2+\frac{1}{z^2}+2 = 98$$

$$\Rightarrow z^2+\frac{1}{z^2} = 98-2$$

$$= 96.$$

Q.7) Operators $\square$, $\diamond$ and $\rightarrow$ are defined by :

$$a \,\square\, b = \frac{a-b}{a+b} \; ; \quad a \diamond b = \frac{a+b}{a-b} \; ; \quad a \rightarrow b = ab$$

(GATE 2015)

Find the value of $(66 \,\square\, 6) \rightarrow (66 \diamond 6)$

(a) -2 (b) -1 (c) 1 (d) 2

Solution

$$\underset{a}{66} \,\square\, \underset{b}{6} = \frac{66-6}{66+6}$$

$$= \frac{60}{72} = \frac{5}{6}$$

$$\underset{a}{66} \diamond \underset{b}{6} = \frac{66+6}{66-6} = \frac{72}{60} = \frac{6}{5}$$

$$\underset{a}{(66 \,\square\, 6)} \rightarrow \underset{b}{(66 \diamond 6)} = \frac{5}{6} \times \frac{6}{5}$$

$$= 1$$

OR

$$(a \,\square\, b) \rightarrow (a \diamond b)$$

$$\frac{a-b}{a+b} \times \frac{a+b}{a-b} = 1$$

Q.8) The value of $\sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$ is (GATE 2014)

(a) 3.464 (b) 3.932 (c) 4.000 (d) 4.444

Solution

$$\sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$$

$$\underbrace{12}_{3\times 4}$$

Ans = 4

Suppose $\sqrt{12-\sqrt{12-\sqrt{12-\cdots}}}$

Ans = 3

Q.9) The numerical in the units position of $211^{870}+146^{127}\times 3^{424}$ is ______. (GATE 2016)

Solution

$$(\ldots 1)^n = (\ldots 1)$$

$$(\ldots 6)^n = (\ldots 6)$$

$$(\ldots 3)^{4n} = (\ldots 1)$$

$$(\ldots 1)^{870} + (\ldots 6)^{127} \times (\ldots 3)^{424}$$

$$1 + 6 \times 1 = 7$$

Q.10) If $q^{-a} = \frac{1}{r}$ and $r^{-b} = \frac{1}{s}$ and $s^{-c} = \frac{1}{q}$, the value of abc is ?

(GATE 2016)

(a) $(rqs)^{-1}$ (b) 0 (c) 1 (d) $r+q+s$

Solution

$q^{-a} = \frac{1}{r}$	$r^{-b} = \frac{1}{s}$	$s^{-c} = \frac{1}{q}$
$\frac{1}{q^a} = \frac{1}{r}$	$\frac{1}{r^b} = \frac{1}{s}$	$\frac{1}{s^c} = \frac{1}{q}$
$r = q^a$	$s = r^b$	$q = s^c$

$q^a = r$	$q^{ab} = s$	$q^{abc} = q^1$
$(q^a)^b = r^b$	$(q^{ab})^c = s^c$	$\therefore abc = 1$. $[\because a^m = a^n \Rightarrow m = n]$
$q^{ab} = r^b$	$q^{abc} = s^c$	

Q.11) Find the smallest number y such that $y \times 162$ is a perfect cube?

(GATE 2017)

(a) 24 (b) 27 (c) 32 (d) 36

Solution

$$162 = 2 \times 81$$
$$= 2 \times 3^3 \times 3$$
$$= (2^2 \times 3^2) \times (2 \times 3^3 \times 3)$$
$$= 2^3 \times 3^3 \times 3^3$$
$$y = 2^2 \times 3^2$$
$$= 4 \times 9 = 36$$

Q. 124 If a and b are integers and $a-b$ is even, which of the following must always be even? (GATE 2017)

(a) ab (b) a^2+b^2+1 (c) a^2+b+1 (d) $ab-b$

Solution

$a-b$ = even

odd - odd = even ... case 1

(or)

even - even = even .... case 2

For case 1

$a = 3$, $b = 1$

(a) $ab = 3 \times 1 = 3$

(b) $a^2+b^2+1 = 3^2+1^2+1 = 9+1+1 = 11$

(c) $a^2+b+1 = 3^2+1+1 = 9+1+1 = 11$

(d) $ab-b = (3\times1)-1 = 3-1 = 2$ ✓

For case 2

$a = 4$, $b = 2$

(a) $ab = 4\times2 = 8$ ✓

(b) $a^2+b^2+1 = 4^2+2^2+1 = 16+4+1 = 21$

(c) $a^2+b+1 = 4^2+2+1 = 16+2+1 = 19$

(d) $ab-b = (4\times2)-2 = 8-2 = 6.$ ✓

Ans => $ab-b$.

Q.13) The last digit of $(2171)^7 + (2172)^9 + (2173)^{11} + (2174)^{13}$ is

(GATE 2017)

(a) 2 (b) 4 (c) 6 (d) 8

Solution

$(\ldots 1)^n = \ldots 1$

$(\ldots 3)^{4n} = \ldots 1$

$(\ldots 2)^{4n} = \ldots 6$

$(\ldots 4)^{2n} = \ldots 6$

$(2171)^7 + (2172)^9 + (2193)^{11} + (2174)^{13}$

$(\ldots 1)^7 + (\ldots 2)^9 + (\ldots 3)^{11} + (\ldots 4)^{13}$

$\ldots 1 + \ldots 2^8 \times \ldots 2^1 + \ldots 3^8 \times \ldots 3^3 + \ldots 4$

$\ldots 1 + \ldots 2 + \ldots 7 + \ldots 4$

$= \ldots 4$

Q.14) What is the value of x when $81 \times \left(\frac{16}{25}\right)^{x+2} \div \left(\frac{3}{5}\right)^{2x+4} = 144$?

(GATE 2017)

(a) 1 (b) -1 (c) -2 (d) Can't be determined

Solution

$81 \times \left(\frac{16}{25}\right)^{x+2} \div \left(\frac{3}{5}\right)^{2x+4} = 144$

$81 \times \left(\left(\frac{4}{5}\right)^2\right)^{x+2} \times \left(\frac{5}{3}\right)^{2x+4} = 144$

$\left(\frac{4}{5}\right)^{2x+4} \times \left(\frac{5}{3}\right)^{2x+4} = \frac{144}{81}$

$\left(\frac{4}{3}\right)^{2x+4} = \left(\frac{12}{9}\right)^2 = \left(\frac{4}{3}\right)^2$

$\therefore 2x+4 = 2$

$x = -1$

Q.15) $\underbrace{a+a+a+\dots\dots a}_{n\text{ times}} = a^2b$ and $\underbrace{b+b+b+\dots\dots+b}_{m\text{ times}} = ab^2$, where a, b, n and m are natural numbers. What is the value of

$$\underbrace{(m+m+m+\dots\dots m)}_{n\text{ times}}\underbrace{(n+n+n+\dots\dots n)}_{m\text{ times}} \;?$$

(a) $2a^2b^2$ (b) a^4b^4 (c) $ab(a+b)$ (d) a^2+b^2

Solution

$\left[\begin{array}{l} 2+2+2 = 6 \\ 2n \quad \therefore n = 3 \end{array}\right]$

$an = a^2b$ | $bm = ab^2$

$n = ab$ | $m = ab$

$(mn) \times (nm) = m^2n^2$

$= (ab)^2(ab)^2 = (ab)^4$

$= a^4b^4$

1.15: Previous Year GATE Questions 2018

Q.16) A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged. What is the number? (GATE 2018)

(a) 63 (b) 72 (c) 81 (d) 90

Solution

$$12$$
$$1\times10+2\times1$$
$10x+y \rightarrow$ Two digit number
$100x+10y+z \rightarrow$ Three digit number

$10x+y \rightarrow$ original number
x and y are the numbers

$$x+y=9$$
$$(10x+y)-45 = 10y+x$$
$$9x-9y = 45$$
$$(x-y)=5$$

$$x+y=9$$
$$x-y=5$$
$$2x=14$$
$$x=7$$

$$y=9-7$$
$$=2.$$

$$10(7)+2= 72$$

Easy to solve through options.

1.16 ! Previous Year GATE Questions 2018.

Q.17) Given that a and b are integers and $a+a^2b^3$ is odd, which one of the following statement is correct?

(a) a and b are both odd

(b) a and b are both even

(c) a is even and b is odd

(d) a is odd and b is even.

Solution

(a) $a=1$, $b=-1$

$a+a^2b^3$

$1+1^2(-1)^3$

$1-1=0$ (even) ×

(b) $a=2$, $b=2$

$2+2^2 2^3$

$2+2^5 = 34$ (even) ×

(c) $a=-2$ $b=1$

$-2+(2)^2(1)^3$

$-2+4=2$ (even) ×

(d) is correct.

$a=1$, $b=2$

$1+1^2 2^3$

$1+8=9$ (odd) ✓

Q.18) If the number 715 □ 423 is divisible by 3 (□ denotes the missing digit in the thousands place), then the smallest whole number in the place of □ is: (GATE 2018)

(a) 0 (b) 2 (c) 5 (d) 6

Solution

$$\frac{715\square423}{3}$$

Divisibility rule for 3

Sum of the digit should be multiple of 3.

$7+1+5+x+4+2+3 = 22+x$

(a) $0 = 22+0 = 22 \div 3$ ✗

(b) $2 = 22+2 = 24 \div 3$ ✓

Q.19) For what value of K given below is $\frac{(K+2)^2}{K-3}$ an integer? (GATE 2018)

(a) 4, 8, 18 (b) 4, 10, 16 (c) 4, 8, 28 (d) 8, 26, 28

Solution

Integer

- Negative integer: ..., −1, −2, −3, ...
- Zero: 0
- Positive integer: 1, 2, 3, 4, 5, ...

(a) 4, 8, 18

$\frac{(4+2)^2}{4-3} = 36$ ✓

$\frac{(8+2)^2}{8-3} = \frac{(10)^2}{5} = 20$ ✓

$\frac{(18+2)^2}{18-3} = \frac{(20)^2}{15} = \frac{400}{15} = \frac{80}{3}$ ✗

(c) $\frac{(28+2)^2}{28-3} = \frac{(30)^2}{25} = \frac{900}{25} = 36$

Ans ⇒ (C)

Q. 20) A house has a number which needs to be identified. The following 3 statements are given that can help in identifying the house number.

(i) If the house number is a multiple of 3, then it is a number from 50 to 59.

(ii) If the house number NOT a multiple of 4, then it is a number from 60 to 69.

(iii) If the house number NOT a multiple of 6, then it is a number from 70 to 79.

(GATE 2018)

What is the house number?

(a) 54 (b) 65 (c) 66 (d) 76

Solution

(a) 54 (i) ✓
(ii) ✗

(b) 65 (i) ✓
(ii) ✓
(iii) ✗

(c) 66 (i) ✗

(d) 76 (i) ✓
(ii) ✓
(iii) ✓

Ans ⇒ option (d)

Q.21) x is a 30 digit number starting with the digit 4 followed by the digit 7. Then the number x^3 will have (GATE 2017)

(a) 90 digits (b) 91 digits (c) 92 digits (d) 93 digits

Solution

$4777 = 4.777 \times 1000$
$= 4.777 \times 10^3$

$x = 47777\ldots\ldots7$

$x = 4.7777\ldots\ldots \times 10^{29}$

$x^3 = (4.777\ldots \times 10^{29})^3$

$= (4.777\ldots)^3 \times (10^{29})^3$

$= (4.777\ldots)^3 \times 10^{87}$ $(4.7 \sim 5)$

$= 87 + 3 = 90$ digits

1.17 : Previous Year GATE Questions 2013

Q.224 The sum of n terms of the series 4 + 44 + 444 + ... is

(a) $\frac{4}{81}(10^{n+1}-9n-1)$ (b) $\frac{4}{81}(10^{n-1}-9n-1)$ (GATE 2011)

(c) $\frac{4}{81}(10^{n+1}-9n-10)$ (d) $\frac{4}{81}(10^{n}-9n-10)$

Solution

$$4 + 44 + 444 + \ldots$$

$$4(1 + 11 + 111 + \ldots)$$

$$\frac{4}{9}[9 + 99 + 999 + \ldots]$$

$$\frac{4}{9}[10-1 + 100-1 + 1000-1 + \ldots]$$

$$\frac{4}{9}[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$$

$$10 + 100 + 1000 + \ldots$$

$$r = \frac{100}{10} = 10, \quad \frac{1000}{100} = 10$$

$$a = 10$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{10(10^n - 1)}{10-1} = \frac{10(10^n-1)}{9}$$

$$1 + 1 + 1 + \ldots + n = n$$

$$\frac{4}{9}\left[\frac{10(10^n-1)}{9} - n\right] = \frac{4}{9}\left[\frac{10^{n+1} - 10 - 9n}{9}\right] = \frac{4}{81}\left[10^{n+1} - 9n - 10\right]$$

Option (c)

Q.23) What will be the maximum sum of 44, 42, 40, ... ?

(GATE 2013)

(a) 502 (b) 504 (c) 506 (d) 500

Solution :-

$$\underbrace{44, 42}_{-2}, \underbrace{40}_{-2}, \underbrace{\ldots}_{-2}, 0$$

0, 2, 4, ~~6~~, 8, ..., 40, 42, 44

first term (a) = 0

Common difference (d) = 2

$$S_n = \frac{n}{2}[a + l]$$

$$n = \left(\frac{44-0}{2}\right) + 1 = 23$$

$$S_{45} = \frac{23}{2}[0 + 44]$$

$$= \frac{23 \times 44}{2}$$

$$= 23 \times 22$$

$$= 506.$$

(OR)

0, 2, 4, 6, ... 44

⇒ 2, 4, 6, ... 44

Sum of first n even natural numbers

$n(n+1)$

$22(23)$

$= 506.$

Q.24) Find the sum to n terms of the series $10+84+734+\ldots$

(a) $\frac{9(9^n+1)}{10}+1$ (b) $\frac{9(9^n-1)}{8}+1$

(GATE 2013)

(c) $\frac{9(9^n-1)}{8}+n$ (d) $\frac{9(9^n-1)}{8}+n^2$

Solution

$10+84+734+\ldots$

$(9+1)+(9^2+3)+(9^3+5)+\ldots$

$(9+9^2+9^3+\ldots n)+(1+3+5+\ldots n)$

$S_n = \frac{a(r^n-1)}{r-1}$ Sum of first n odd numbers

$= \frac{9\cdot(9^n-1)}{9-1} + n^2$

$= \frac{9(9^n-1)}{8} + n^2$

Q.25) In a sequence of 12 consecutive odd numbers, the sum of the first 5 number is 425. What is the sum of the last 5 numbers in the sequence? (GATE 2014)

Solution Let 5 numbers be $x-4,\ x-2,\ x,\ x+2,\ x+4$

$5x = 425 \Rightarrow x = 85$

$T_{12} = a+11d$

$= 81+11(2) = 81+22 = 103$

103, 101, 99, 97, 95 $\therefore \frac{5}{2}[95+103] = \frac{5}{2}\times 198 = 5\times 99 = 495$

Q. 26) Consider a sequence of numbers $a_1, a_2, a_3, \dots, a_n$ where $a_n = \frac{1}{n} - \frac{1}{n+2}$, for each integer $n > 0$. What is the sum of the first 50 terms. (GATE 2018)

(a) $\left(1+\frac{1}{2}\right) - \frac{1}{50}$

(b) $\left(1+\frac{1}{2}\right) + \frac{1}{50}$

(c) $\left(1+\frac{1}{2}\right) - \left(\frac{1}{51}+\frac{1}{52}\right)$

(d) $1 - \left(\frac{1}{51}+\frac{1}{52}\right)$

Solution

$$a_1 = \frac{1}{1} - \frac{1}{1+2}$$

$$= 1 - \frac{1}{3}$$

$$a_2 = \frac{1}{2} - \frac{1}{4}$$

$$a_3 = \frac{1}{3} - \frac{1}{5}$$

$$a_4 = \frac{1}{4} - \frac{1}{6}$$

$\vdots$

$$a_{48} = \frac{1}{48} - \frac{1}{50}$$

$$a_{49} = \frac{1}{49} - \frac{1}{51}$$

$$a_{50} = \frac{1}{50} - \frac{1}{52}$$

$$a_1 + a_2 + a_3 + \dots + a_{48} + a_{49} + a_{50}$$

$$1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{1}{3}} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \dots + \frac{1}{48} - \cancel{\frac{1}{50}} + \frac{1}{49} - \frac{1}{51} + \cancel{\frac{1}{50}} - \frac{1}{52}$$

$$1 + \frac{1}{2} - \frac{1}{51} - \frac{1}{52}$$

$$\left(1+\frac{1}{2}\right) - \left(\frac{1}{51}+\frac{1}{52}\right)$$ option (c)

Q.27) What is the value of $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$?

(GATE 2018)

(a) 2 (b) $7/4$ (c) $3/2$ (d) $4/3$

Solution

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots \infty$$

$a = 1$

$r = \frac{1}{4} \div 1 = 1$; $\frac{1}{16} \div \frac{1}{4} = \frac{1}{16} \times 4 = \frac{1}{4}$

$$S_\infty = \frac{a}{1-r} \quad (r<1)$$

$$= \frac{1}{1-1/4}$$

$$= \frac{1}{3/4} = 4/3$$

option (d)

Q.28) The sum of the squares of successive integers 8 to 16, both inclusive will be

(GATE 2018)

(a) 1126 (b) 1174 (c) 1292 (d) 1356

Solution

$8^2 + 9^2 + 10^2 + 11^2 + \dots + 16^2$ (sum)

Sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$[1^2 + 2^2 + 3^2 + \dots + 16^2] - [1^2 + 2^2 + 3^2 + \dots + 7^2]$$

$$\frac{16 \times 17 \times 33}{6} - \frac{7 \times 8 \times 15}{6} = 1496 - 140 = 1356$$

option (d)