

SIMPLE INTEREST AND COMPOUND INTEREST

①

9.1: Simple Interest Introduction

Bank gave you ₦ 50,000



repay after 2 years → ₦ 50,000 + extra amount = Amount
 ↓ ↓ ↓
 Time Period Principal interest (Simple Interest)
 (original amount) ↓
 charged as per rate of interest

$$\text{Simple Interest (SI)} = P \times T \times \frac{R}{100}$$

P = Principal

T = Time Period (Years, Months or days)

R = Rate of interest

$$SI = P \times (TR\%)$$

$$I\% = T \times R\%$$

$$T = 3 \text{ years} \quad R = 10\% \text{ p.a.}$$

$I\% = 30\%$. (Total Percentage on the principal)

Amount = Principal + Interest

$$= P + \frac{PTR}{100}$$

$$= P \left(1 + \frac{TR}{100} \right)$$

$$= P \left(\frac{100 + TR}{100} \right)$$

$$\underline{\underline{Ex}} \quad P = ₦ 50,000, \quad T = 2 \text{ years}, \quad R = 10\% \text{ p.a.}$$

$$SI = 50,000 \times 2 \times \frac{10}{100}$$

$$= ₦ 10,000$$

$$A = 50,000 + 10,000$$

$$= ₦ 60,000$$

(Or)

$$A = 50,000 \left(\frac{100 + (2 \times 10)}{100} \right)$$

$$= ₦ 60,000$$

②

Calculation of Interest and amount

(i) In terms of %

$$I\% = (T \times R\%) \times P$$

$$A\% = P + \frac{I}{100\%}$$

\downarrow \downarrow

$TR\%$

$$A\% = P(100\% + TR\%)$$

Ex $T = 2 \text{ years}, R = 15\%$

$$I\% = 2 \times 15\% \\ = 30\%$$

$$A\% = P + I \\ = 100\% + 30\% \\ = 130\%$$

* Principal is always original value, it can be assumed as 100 or (100%).

Ex $P = ₹ 50,000, T = 3 \text{ years}, R = 20\% \text{ p.a.}$

$$I\% = 3 \times 20\% = 60\%$$

$$I = 60\% \times 50,000 = ₹ 30,000$$

$$A\% = 100\% + 60\% = 160\%$$

$$A = 160\% \times P = 50,000 \times 160\% \\ = ₹ 80,000$$

$$\text{Ex} \quad T = \frac{3}{2} \text{ years}, \quad R = 20\% \text{ p.a.}$$

$$I\% = \frac{3}{2} \times 20\% = 30\%.$$

$$A\% = 130\%.$$

$$\text{Ex} \quad T = 3 \text{ years}, \quad R = 12\% \text{ p.a.}$$

$$I\% = 36\%.$$

$$A\% = 136\%.$$

$$I = 36\% \times P$$

$$A = 136\% \times P$$

(ii) in terms of ratio / fractions

$$\begin{aligned} R\% &= 20\% \text{ p.a.}, \quad T = 2 \text{ years} \\ &= \frac{1}{5} \rightarrow \text{result (interest for 1 year)} \\ &\quad \frac{1}{5} \rightarrow \text{original value (principal)} \end{aligned}$$

$$P_2 = 5$$

$$I = 1 \times 2 = 2$$

$$A = P + I = 5 + 2 = 7$$

$$\begin{aligned} \text{Ex} \quad R &= 8\frac{1}{3}\% \text{ p.a.} \\ &= \frac{1}{12} \rightarrow \text{Interest for 1 year} \\ &\quad \frac{1}{12} \rightarrow P \end{aligned}$$

(4)

Ex

$$R\% = 28 \frac{4}{7}\% \text{ p.a.}$$

$\frac{2}{7} \rightarrow \text{Int for 1 year}$
 $\frac{7}{7} \rightarrow P$

Ex

$$R\% = 16 \frac{2}{3}\% \text{ p.a.} \quad T = 3 \text{ years.}$$

$$= \frac{1}{6}$$

$$P = 6$$

$$2 \times 1 \times 3 = 3$$

$$A = 6 + 3 = 9.$$

9.2: Solved Problems

Q.1) A sum of £ 4000 is lent for 5 years at the rate of 15% p.a. Find the interest?

$$P = \text{£} 4000, T = 5 \text{ years}, R \% = 15\%$$

Concept 1

$$\begin{aligned} SI &= \frac{P \times T \times R}{100} \\ &= \frac{4000 \times 5 \times 15}{100} \\ &= \text{£} 3000 \end{aligned}$$

Concept 2

$$\begin{aligned} I \% &= T \times R \% \\ &= 75\% \\ SI &= P \times I \% \\ &= 4000 \times 75\% \\ &= \text{£} 3000. \end{aligned}$$

Concept 3

$$\begin{aligned} R \% &= 15\% \\ &= \frac{3}{20} \rightarrow \text{Int for 1 year} \end{aligned}$$

$$\begin{array}{ccc} \frac{P}{20} & \frac{I}{5 \times 3 = 15} & \frac{A}{35} \end{array}$$

$$\begin{aligned} 4000 & \quad \frac{15}{20} \times 4000 \\ &= \text{£} 3000 \end{aligned}$$

60

Q.27 On what sum at 4% rate the simple interest will be ₹640 for 2 years?

$$R\% = 4\%, T = 2640, P = 2 \text{ years.}$$

Concept 1

$$SI = \frac{PTR}{100}$$

$$\Rightarrow 640 = \frac{PX2X4}{100}$$

$$\Rightarrow P = \text{£} 8000$$

Concept 2

$$\text{I\%} = T \times R\%$$

$$= 2 \times 4).$$

$\approx 8\%$

$$8\% \rightarrow 640$$

$$100\% \rightarrow \frac{640}{8} \times 100$$

= £8,000 (Principal)

Concept 3

$$R\% = 4\%$$

$$= \frac{1}{\sqrt{5}}$$

$$\frac{P}{Q^5} = \underline{\underline{P}}$$

640

$$\begin{aligned} \alpha &\longrightarrow 640 \\ 25 &\longrightarrow \frac{640}{25} \times 25 \\ &= \underline{\underline{\mathcal{R} 81,000 (P)}} \end{aligned}$$

Q.3) A sum of ₹ 5120 is lent for 3 years at the rate of 12.5% p.a.
Find the amount?

$$P = ₹ 5120, T = 3 \text{ years}, R = 12.5\% \text{ p.a.}$$

$$A = P + I$$

Concept 1

$$I = \frac{P \times T \times R}{100}$$

$$= \frac{5120 \times 3 \times 12.5}{100}$$

$$= ₹ 1920$$

$$A = 5120 + 1920$$

$$= ₹ 7040$$

Concept 2

$$P\% = T \times R\% = 37.5\%$$

$$A\% = 137.5\%$$

$$A = 5120 \times 137.5\%$$

$$= ₹ 7040.$$

Concept 3

$$R\% = 12.5\% = \frac{1}{8}$$

$$P = 8$$

$$T = 1 \times 3 = 3$$

$$A = 8 + 3 = 11$$

$$\begin{aligned} 8 &\longrightarrow 5120 \\ 11 &\longrightarrow \frac{5120}{8} \times 11 \\ &= ₹ 7040 \text{ (Amount)} \end{aligned}$$

8)

Q.4) A man will get ₹ 87 as simple interest on ₹ 725 @ 4% p.a. in
— years.

$$P = ₹ 725$$

$$I = ₹ 87$$

$$R = 4\%$$

Concept - 1

$$I = \frac{P \times R \times T}{100}$$

$$87 = \frac{725 \times T \times 4}{100}$$

$$T = \frac{87 \times 100}{725 \times 4}$$

$$= 3 \text{ years}$$

Concept 2

$$\text{Interest for one year} = 725 \times 4\% = 29$$

$$\begin{array}{l} 1 \text{ yr} \rightarrow 29 \\ ? \rightarrow 87 \end{array}$$

$$\frac{87}{29} \times 1 = 3 \text{ years.}$$

Q.5) If the SI on a certain sum of money at the rate of $\frac{20}{3}\%$ p.a. for 4 years is ₹ 4,400 less than the principal, find the interest.

$$R\% = \frac{20}{3}\% = 6\frac{2}{3}\%, T = 4 \text{ years.}, I = P - 4400 \Rightarrow P - I = 4400$$

$$= \frac{1}{15}$$

$$\begin{array}{l} P = 15 \\ I = 1 \times 4 = 4 \end{array}$$

$$\begin{array}{l} 11 \rightarrow 4400 \\ 4 \rightarrow \frac{4400}{11} \times 4 \\ = ₹ 1,600 (\text{Interest}) \end{array}$$

Q.6) The simple interest accrued in 9 years on a principal of ₹ 24,250 is 162% of the principal. What is the rate of interest % per annum?

$$T = 9 \text{ years}$$

$$P = ₹ 24,250$$

$$I \% = 162\%$$

$$I \% = T \times R \%$$

$$\Rightarrow 162 \% = 9 \times R \%$$

$$\Rightarrow R \% = \frac{162 \%}{9}$$

$$= 18 \% \text{ p.a.}$$

Q.7) The rate of SI for 1st 3 years is 6%, for next 4 years it is 7% and the period beyond 7 years is 7.5% p.a. If a man invest ₹ 18,800 for 11 years, find the SI earned by him?

$$P = ₹ 18,800$$

$$T = 11 \text{ years}$$

$$1^{\text{st}} 3 \text{ years} \rightarrow 6 \% \text{ p.a.}$$

$$4 \text{ years} \rightarrow 7 \% \text{ p.a.}$$

$$\text{Beyond 7 years} \rightarrow 7.5 \% \text{ p.a.}$$

for 1st 3 years @ 6% p.a.

$$I = \frac{18800 \times 3 \times 6}{100} = 188 \times 18 = ₹ 3384$$

for next 4 years @ 7% p.a.

$$I = \frac{18800 \times 4 \times 7}{100} = 188 \times 28 = ₹ 5264$$

for next 4 years @ 7.5% p.a

$$\text{SI} = \frac{18800 \times 4 \times 7.5}{100} = 188 \times 30 = \text{₹ } 5640$$

$$\therefore \text{Total interest} = \text{₹ } (3384 + 5264 + 5640) = \text{₹ } 14,288$$

(Alternative)

$$\text{D% for 1st 3 years} = 3 \times 6\% = 18\%$$

$$\text{D% for next 4 years} = 4 \times 7\% = 28\%$$

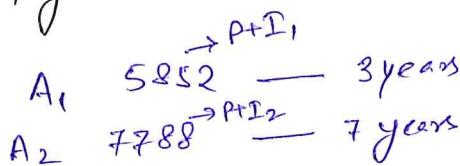
$$\text{D% for last 4 years} = 4 \times 7.5\% = 30\%$$

$$\text{Total I% for 11 years} = 18\% + 28\% + 30\% = 76\%$$

$$\text{SI for 11 years} = 18800 \times 76\%$$

$$= \text{₹ } 14,288$$

Q.8) A sum of money at ₹1 amounts to ₹5852 in 3 years and ₹7788 in 7 years. What is the rate of interest p.a.?



(Difference in amount is Interest)

$$A_2 - A_1 = \text{Interest}$$

$$7788 - 5852 = \text{₹ } 1936 \Rightarrow 4 \text{ years interest}$$

↓
(7 - 3)

$$4 \longrightarrow 1936$$

$$3 \longrightarrow \frac{1936}{4} \times 3 = \text{₹ } 1452$$

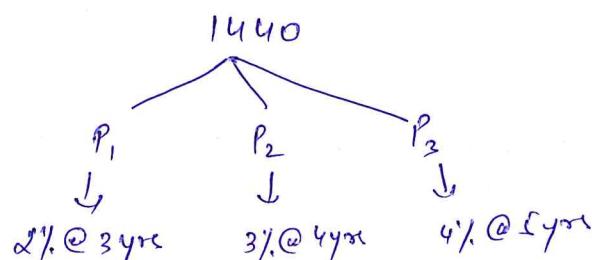
$$P = A - \text{SI}$$

$$= 5852 - 1452 = \text{₹ } 4400$$

$$81 = \frac{PTR}{100} \Rightarrow 1452 = \frac{4400 \times 3 \times R}{100} \Rightarrow R > 11\%$$

Q.3: Solved Problems

Q.9) A sum of ₹ 1440 is lent out in three parts in such a way that the interests on first part at 2% for 3 years, second part at 3% for 4 years and third part at 4% for 5 years are equal. Then find the difference between the largest and the smallest sum.



$$I_1 = I_2 = I_3$$

$$6\% \times P_1 = 12\% P_2 = 20\% P_3$$

$$\therefore 6P_1 = 12P_2 = 20P_3$$

$$P_1 : P_2 : P_3 = \frac{1}{6} : \frac{1}{12} : \frac{1}{20}$$

$$= \frac{1}{3} : \frac{1}{6} : \frac{1}{10}$$

$$\text{LCM } (3, 6, 10) = 30$$

$$P_1 : P_2 : P_3 = 10 : 5 : 1$$



$$10 + 5 + 1 = 16$$

$$16 - 1 = 9$$

$$\begin{aligned}
 16 &\longrightarrow 1440 \\
 9 &\longrightarrow \frac{1440}{16} \times 9 \\
 &= ₹ 810 \quad (\text{Ans})
 \end{aligned}$$

12

$$\mathbb{P}_1 = \mathbb{P}_2 = \mathbb{P}_3$$

$$P_1 : P_2 : P_3 : P_n = \frac{1}{\alpha_1 t_1} : \frac{1}{\alpha_2 t_2} : \frac{1}{\alpha_3 t_3} : \dots : \frac{1}{\alpha_n t_n}$$

When amounts are equal

$$\text{Principal ratio} = \frac{1}{100 + \alpha_1 t_1} : \frac{1}{100 + \alpha_2 t_2} : \frac{1}{100 + \alpha_3 t_3} : \dots : \frac{1}{100 + \alpha_n t_n}$$

Q.10) A sum of money doubles itself in 10 years at \$1. What is the rate of interest?

$$\text{Let } P = \$1$$

$$\text{Amount} = \$2$$

$$T = 10 \text{ years}$$

$$R\% = ?$$

$$\text{Interest} = A - P \\ = \$1$$

$$D = \frac{P \times T \times R}{100}$$

$$1 = \frac{1 \times 10 \times R}{100}$$

$$R = 10\%.$$

(Trick)

$$P = 100$$

$$A = 200$$

$$T = 10 \text{ years}$$

$$D = 100$$

$$D \text{ for 1 year} = \frac{100}{10} = 10$$

$$R\% = \frac{10}{100} \times 100 = 10\%$$

Q.11) A sum of money becomes four times at the simple interest rate of 5% p.a. At what rate percent will it become seven fold?

$$P = 1$$

$$A = 4$$

$$R\% = 5\%$$

$$T = ?$$

$$\begin{aligned} I &= 4 - 1 \\ &= 3 \end{aligned}$$

$$I = \frac{PTR}{100}$$

$$3 = \frac{1 \times T \times 5}{100}$$

$$T = 60 \text{ years.}$$

$$P = 1$$

$$A = 7$$

$$R\% = ?$$

$$T = 60 \text{ years.}$$

$$\begin{aligned} I &= 7 - 1 \\ &= 6 \end{aligned}$$

$$I = \frac{PTR}{100}$$

$$6 = \frac{1 \times 60 \times R}{100}$$

$$R = 10\%.$$

(Trick)

$$1 \xrightarrow[5\%]{3} 4$$

$$1 \xrightarrow[?]{6} 7$$

$$3 \longrightarrow 5\%$$

$$6 \longrightarrow \frac{5}{3} \times 6$$

$$\approx 10\%.$$

(14)

Q.12) The simple interest on a sum of money is $\frac{1}{9}$ of the principal and the number of years is equal to the rate percent per annum. Find the rate %?

$$T = R\%$$

$$I = \frac{P \times T \times R}{100}$$

$$\Rightarrow I = \frac{P \times R \times R}{100}$$

$$\Rightarrow \frac{I}{P} = \frac{R^2}{100}$$

$$\Rightarrow R^2 = \frac{I}{P} \times 100$$

$$\Rightarrow R^2 = \sqrt{\frac{I}{P} \times 100}$$

$$I = \frac{1}{9} \times P$$

$$\Rightarrow \frac{I}{P} = \frac{1}{9}$$

$$R = \sqrt{\frac{I}{P} \times 100}$$

$$= \sqrt{\frac{1}{9} \times 100}$$

$$= \frac{10}{3}\%$$

$$= 3\frac{1}{3}\%$$

Installment - Discharge a debt Problems

$$\text{Total Debt} = \left[I + \left(I + \frac{I \times r \times 1}{100} \right) + \left(I + \frac{I \times r \times 2}{100} \right) + \cdots + \left(I + \frac{I \times r \times n-1}{100} \right) \right]$$

I = Installment

n = no. of years / no. of installments

$r\%$ = rate of interest

Q.13) What annual installment will discharge a debt of £ 6450 due in 4 years at 5% SI?

$$\text{Total Debt} = £ 6450$$

$$T = 4 \text{ years}$$

$$\text{Let Installment (I)} = 100$$

$$\begin{aligned} 1^{\text{st}} \text{ year} &= 100 \\ 2^{\text{nd}} \text{ year} &= 100 + 5 = 105 \quad (\text{one year interest}) \end{aligned}$$

$$3^{\text{rd}} \text{ year} = 110$$

$$4^{\text{th}} \text{ year} = \frac{115}{430}$$

$$430 \longrightarrow 6450$$

$$100 \longrightarrow \frac{6450}{430} \times 100$$

$$= £ 1,500 \text{ -- (annual installment)}$$

(16)

9.4: Compound Interest

Compound interest is



Interest on interest

Compound interest will be used in (Population increment, Depreciation, Successive % change)

$$\text{Amount} = P \left(1 + \frac{\alpha}{100}\right)^n$$

$$CI = A - P$$

$$= P \left(1 + \frac{\alpha}{100}\right)^n - P$$

(i) Compounded annually → interest added every year

(ii) Compounded half-yearly → interest added every 6 months

(iii) Compounded quarterly → interest is added every 3 months

(iv) Compounded monthly → interest added every month

$$A = P \left(1 + \frac{\alpha/t}{100}\right)^{nt}$$

Case-1 ⇒ when $t=1$, compounded annually $A = P \left(1 + \frac{\alpha}{100}\right)^n$

Case-2 ⇒ when $t=2$, compounded half yearly $A = P \left(1 + \frac{\alpha/2}{100}\right)^{2n}$

Case-3 ⇒ when $t=4$, compounded quarterly $A = P \left(1 + \frac{\alpha/4}{100}\right)^{4n}$

Case-4 ⇒ when $t=12$, compounded monthly $A = P \left(1 + \frac{\alpha/12}{100}\right)^{12n}$

Ex
 10,000 10% 3 years Compounded annually

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{100}\right)^t \\
 &= 10,000 \left(1 + \frac{10}{100}\right)^3 \\
 &= 10,000 \times \left(1 + \frac{1}{10}\right)^3 \\
 &= 10,000 \times \left(\frac{11}{10}\right)^3 \\
 &= 10,000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \\
 &= 13,310
 \end{aligned}$$

$$\begin{aligned}
 C_1 &= 13,310 - 10,000 \\
 &= \$ 3,310
 \end{aligned}$$

(Alternative)

| | | |
|------------|------------|------------|
| <u>1st</u> | <u>2nd</u> | <u>3rd</u> |
| 10,000 | 11,000 | 12100 |
| 1000 | 1100 | 1210 |
| | | = 3310 |

(OR)

| | | |
|-------------|------------|------------|
| <u>1st</u> | <u>2nd</u> | <u>3rd</u> |
| 1000 | 1000 | 1000 |
| <u>1000</u> | <u>100</u> | <u>100</u> |
| | | 1210 |
| | | = 3310 |

Ex

$P = ₹ 2160$, $R\% = 16\frac{2}{3}\%$, $T = 3$ years. Find U .

$$\begin{array}{r}
 R = 16\% \\
 \underline{15} \\
 \alpha^{10} \\
 \hline
 360 \\
 60 \\
 \hline
 360 \\
 60 \\
 \hline
 420 \\
 70 \\
 \hline
 490 \\
 \hline
 1270 \Rightarrow C.I.
 \end{array}$$

$$\text{Q10} \quad P = ₹ 20,000, R = 20\%, T = 1 \frac{1}{2} \text{ years}, \text{ Find CI. Compounded half-yearly.}$$

$$\begin{aligned}
 A_2 &= P \left(1 + \frac{2\%}{100} \right)^{2n} \\
 &= 20000 \left(1 + \frac{20\%}{100} \right)^{2 \times 3/2} \\
 &\approx 201000 \left(1 + \frac{10}{100} \right)^3 \\
 &\approx 205000 \left(\frac{11}{10} \right)^3
 \end{aligned}$$

$$= \text{£} 26,620$$

$$CI = 24620 - 20,000 = -2,462$$

$\frac{1}{6}$ year = 18 months (3 x 6)

$$\text{COR} \quad P = 20,000, \quad \frac{\sigma}{2} = 10\%, \quad \frac{20}{2} = 10\% \quad \text{To } \frac{1}{2} \text{ year} = 18 \text{ months}$$

| | | |
|---|---|---|
| $\frac{20,000}{6 \text{ months}} = 2,000$ | $\frac{22,000}{6 \text{ months}} = 2,200$ | $\frac{24,200}{6 \text{ months}} = 2,420$ |
| | | $\Sigma 6,620/-$ |
| | | C.I. |

80

$P = \text{₹}16000$, $R\% = 20\%$, $T = 9 \text{ months}$. find CI, compounded quarterly.

$$A = P \left(1 + \frac{R}{100}\right)^t$$

$T = 9 \text{ months}$

$$= \frac{9}{12} \text{ years} = \frac{3}{4} \text{ years.}$$

$$\begin{aligned} A &= 16000 \left(1 + \frac{20/4}{100}\right)^{4 \times 3/4} \\ &= 16000 \left(1 + \frac{5}{100}\right)^3 \\ &= \text{₹}18,522 \end{aligned}$$

$$\begin{aligned} CI &= 18522 - 16000 \\ &= \text{₹}2,522. \end{aligned}$$

(Alternative)

$P = 16000$, $R/4 = \frac{20}{4} = 5\%$, $T = 9 \text{ months}$ (3×3)

| <u>3months</u> | <u>3months</u> | <u>3months</u> |
|----------------|----------------|----------------|
| 800 | 800 | 800 |
| | | 40 |
| | 840 | 882 |

$$CI = \text{₹}2,522/-$$

(20)

Rate of interest different for different years.

1st year $r_1 \%$

2nd year $r_2 \%$

3rd year $r_3 \%$

\vdots

$A = P \left(1 + \frac{r}{100}\right)^n$ when r is equal for all year

$A = P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right) \cdots$

Ex

$P = ₹ 4,000, R_1 = 8\% \text{ for } 1^{\text{st}} \text{ year}$, find amount after 2 years.

$R_2 = 10\% \text{ for } 2^{\text{nd}} \text{ year.}$

$$A = 4000 \left(1 + \frac{8}{100}\right) \left(1 + \frac{10}{100}\right)$$

$$= 4000 \times \frac{108}{100} \times \frac{110}{100}$$

$$= ₹ 4752$$

$$\text{CI} = 4752 - 4000 = ₹ 752.$$

(Alternative)

$P = ₹ 4000, r_1 = 8\%, A = ?$
 $r_2 = 10\%.$

1st
 $\underline{8\%}$

$$4000 \times 8\%$$

$$= 320$$

2nd 10\%

$$4000 \times 10\%$$

$$= 400$$

$$\begin{array}{r} 320 \\ + 400 \\ \hline 720 \end{array}$$

$$= ₹ 752$$

If time is given in fraction say $3\frac{2}{5}$ years.

$$A = P \left(1 + \frac{r}{100}\right)^3 \left(1 + \frac{\frac{2}{5}r}{100}\right)$$

Ex

$P = \text{₹ } 8,000$, $r = 15\%$, $T = 2$ years 4 months, find CI.

$$4 \text{ months} = \frac{4}{12} = \frac{1}{3} \text{ year}$$

$$A = 8000 \left(1 + \frac{15}{100}\right)^2 \left(1 + \frac{\frac{1}{3} \times 15}{100}\right)$$

$$= \text{₹ } 11,109.$$

$$CI = 11,109 - 8,000 = \text{₹ } 3,109.$$

(Alternative)

| <u>1st</u> | <u>2nd</u> | <u>3rd</u> |
|-----------------------|-----------------------|-----------------------|
| $8000 \times 15\%$ | 1200 | 1200 |
| $= 1200$ | 1200 | 1200 |
| — | $1200 \times 15\%$ | 180 |
| 1200 | $= 180$ | $1380 \times 15\%$ |
| — | 1380 | 207 |
| | | $\frac{1587}{1587}$ |

$$12 \text{ months} \longrightarrow 1587$$

$$4 \text{ months} \longrightarrow \frac{1587}{12} \times 4 = \text{₹ } 529$$

$$CI = 1200 + 1380 + 529$$

$$= \text{₹ } 3,109.$$

(22)

Ex A sum of money at 1% amounts to thrice itself in three years. In how many years will it be 9 times itself?

$$P = 1 \quad P = 1$$

$$A = 3 \quad A = 9$$

$$T = 3 \text{ years} \quad R\%$$

$$R\%$$

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$3 = 1 \left(1 + \frac{r}{100} \right)^3$$

$$\left(1 + \frac{r}{100} \right)^3 = 3$$

$$9 = 1 \left(1 + \frac{r}{100} \right)^n$$

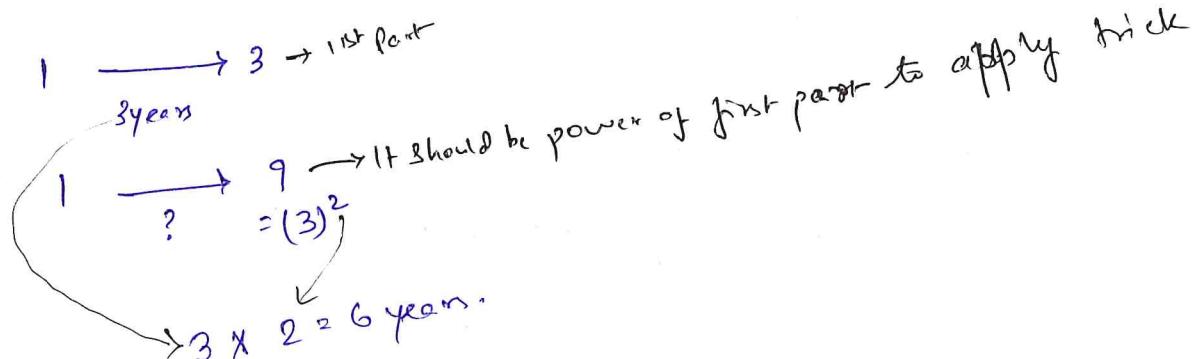
$$(3)^2 = \left(1 + \frac{r}{100} \right)^n$$

$$\left(\left(1 + \frac{r}{100} \right)^3 \right)^2 = \left(1 + \frac{r}{100} \right)^n$$

$$\left(1 + \frac{r}{100} \right)^6 = \left(1 + \frac{r}{100} \right)^n$$

$$n = 6 \text{ years.}$$

(Alternative)



Ex A sum of money placed at CI doubles itself in 5 years. In how many years will it amount to 8 times itself?

$$\begin{array}{ccc} 1 & \xrightarrow{\hspace{1cm}} & 2 \\ & \text{5 years} & \\ 1 & \xrightarrow{\hspace{1cm}} & 8 \\ & & = (2)^3 \end{array}$$

$$5 \times 3 = 15 \text{ years.}$$

Difference between SI & CI

P = 1000, R = 10%, T = 3 years

| <u>SI</u> | P | I | A | <u>CI</u> | P | I | A |
|-----------------|------|-----|------|-----------------|------|-----|------|
| 1 st | 1000 | 100 | 1100 | 1 st | 1000 | 100 | 1100 |
| 2 nd | 1000 | 100 | 1200 | 2 nd | 1100 | 110 | 1210 |
| 3 rd | 1000 | 100 | 1300 | 3 rd | 1210 | 121 | 1331 |

1) Interest is same for 1st year in SI & CI.

2) Interest is same for every year in SI.

3) Interest is increasing for every year in CI.

4) When consecutive interest or amounts are given in question, we can find the rate of interest without principal in CI but same is not possible in SI.

(24)

Difference between SI and CI

1) for 2 years

$$d = P \left(\frac{r}{100} \right)^2$$

2) for 3 years

$$d = P \left(\frac{r}{100} \right)^2 \left(3 + \frac{r}{100} \right)$$

Ex The difference between SI and CI compounded annually on a certain sum of money for 2 years at 4% p.a. is ₹ 1. The sum (in ₹) is -

$$d = 1$$

$$r = ?$$

$$R = 4\%$$

$$T = 2 \text{ years}$$

$$d = P \left(\frac{r}{100} \right)^2$$

$$1 = P \left(\frac{4}{100} \right)^2$$

$$\Rightarrow 1 = P \left(\frac{1}{25} \right)^2$$

$$\Rightarrow P = ₹ 625/-$$

Ex The difference between CI and SI on a certain principal in 3 yrs is ₹ 1525. If the rate is 5%. Find the principal.

$$d = ₹ 1525, R = 5\%, T = 3 \text{ years.}$$

$$d = P \left(\frac{r}{100} \right)^2 (3 + \frac{r}{100})$$

$$\Rightarrow 1525 = P \left(\frac{5}{100} \right)^2 (3 + \frac{5}{100})$$

$$\Rightarrow 1525 = P \left(\frac{1}{20} \right)^2 (3 + \frac{1}{20})$$

$$\Rightarrow P = ₹ 200,000/-$$

Ex The CI on a sum of money for 2 years is £ 410 and the SI on the same sum is £ 400. Find the rate of interest.

$$SI = £ 400 \text{ for 2 years}$$

$$\begin{array}{c} \text{1st year} \\ \hline £ 200 \\ \text{2nd year} \\ \hline £ 200 \end{array}$$

$$\begin{array}{c} CI \\ = \\ \begin{array}{c} £ 200 \\ \curvearrowright \\ £ 210 \end{array} \\ \curvearrowright \\ £ 10. \end{array}$$

$$\frac{10}{200} \times 100 = 5\% \text{ (Ans)}$$

Ex The SI on a certain sum of money for 2 years at 10% p.a. is £ 80. The CI on the same sum for the same period is

$$SI \text{ for 2 years} = £ 80$$

$$\begin{array}{c} \text{1st} \\ \hline 40 \\ \text{2nd} \\ \hline 40 \\ \text{CI} \\ \hline 40 \\ \quad \quad \quad 40 \times 10\% + 40 \\ \quad \quad \quad = 44 \end{array}$$

$$\begin{aligned} CI &= 40 + 44 \\ &= 84/- \end{aligned}$$

(26)

9.5: Installments in Compound Interest

Installments in CI - Discharge a debt

$$A = P \left(1 + \frac{r}{100} \right)^n \rightarrow \text{no. of installments/no. of years}$$

\downarrow
debt
equal amount
installments

n = 2 years

$$D = P_1 \left(1 + \frac{r}{100} \right)^1$$

$$D = P_2 \left(1 + \frac{r}{100} \right)^2$$

$$P_1 + P_2 + P_3 + \dots + P_n = \text{Total debt}$$

Installment will be same every year

$$\text{Total Debt} = D \left[\frac{100}{100+r} + \left(\frac{100}{100+r} \right)^2 + \left(\frac{100}{100+r} \right)^3 + \dots + \left(\frac{100}{100+r} \right)^n \right]$$

Ex A sum of ₹ 2,100 to be paid back in 2 equal installments. How much is each installment if the interest is compounded annually at 10% p.a.?

Total debt = ₹ 2,100, No. of installments = 2, R% = 10%.

$$\text{Total debt} = D \left[\frac{100}{100+r} + \left(\frac{100}{100+r} \right)^2 \right]$$

$$2100 = D \left[\frac{100}{100+10} + \left(\frac{100}{110} \right)^2 \right]$$

$$2100 = D \left[\frac{10}{11} + \left(\frac{10}{11} \right)^2 \right]$$

$$D = ₹ 1,210$$

(Trick)

| <u>P</u> | <u>A</u> | $R = 10\% = \frac{1}{10} \rightarrow \text{Principal} \xrightarrow{\text{Interest}}$ |
|-------------|---------------------|--|
| <u>Debt</u> | <u>Installment</u> | |
| 10 | $10+1=11 \times 11$ | |
| 1st $T=1$ | 121 | |
| 2nd $T=2$ | 121 | |
| 110 | 121 | |
| <u>210</u> | <u>121</u> | equal annual installment |

$$210 \rightarrow 2100$$

$$121 \rightarrow \frac{2100}{210} \times 121 \\ = \text{₹}1,210/-$$

Ex A man borrowed some money and paid back in 3 equal annual installments of ₹ 2,160 each. What sum did he borrow, if the rate of interest charged by the money lender was 20%, p.a. compounded annually?

$$R\% = 20\% = \frac{1}{5}, I = \text{₹}2,160, \text{No. of installments} = 3$$

| | <u>Debt</u> | <u>Installment</u> |
|----------|-------------|--------------------|
| 1st year | 5 | $5+1=6 \times 36$ |
| 2nd year | 25 | 36 |
| 3rd year | 125 | 216 |
| | 180 | 216 |
| | 155 | 216 |
| | <u>455</u> | <u>216</u> |

$$216 \rightarrow 2160 \\ 455 \rightarrow \frac{2160}{216} \times 455 = \text{₹}4,500/-$$

(28)

Q.6: GATE Previous Year Questions on Simple Interest and Compound Interest

Q.1) Leila aspires to buy a car worth ₹ 10,00,000 after 5 years. What is the minimum amount in ₹ that should deposit now in bank which offers 10% annual rate of interest, if the interest was compounded annually?

(GATE 2018)

- (a) 5,00,000 (b) 6,21,000 (c) 6,66,667 (d) 7,50,000

$$A = ₹ 10,00,000$$

$$T = 5 \text{ years}$$

$$R\% = 10\%$$

$$A = P \left(1 + \frac{R}{100}\right)^T$$

$$10^5 = P \left(1 + \frac{10}{100}\right)^5$$

$$10^5 = P \left(1 + \frac{1}{10}\right)^5$$

$$10^5 = P \left(\frac{11}{10}\right)^5$$

$$P = \frac{10^5 \times 10}{11^5}$$

$$= \frac{10^1}{11^5}$$

$$= 6,20921 \approx ₹ 6,21,000$$

Trick)

$$A = ₹ 10,00,000$$

SI for 5 yrs 10%.

50%.

CI 10%.

50%.

- (a) 5,00,000

$$\frac{2,50,000}{7,50,000} \times$$

- (b) 6,21,000

$$\frac{3,101,500}{9,31,500} \times$$

(Ans)

- (c) 6,66,667
3,33,333
1000001 ×

(Q3)

Q.2) The population of new city is 5 million and is growing at 20% annually. How many years would it take to double at this growth rate? (GATE 2014)

- (a) 3-4 yrs (b) 4-5 yrs (c) 5-6 yrs (d) 6-7 yrs

$$P = 1$$

$$A = 2$$

$$R\% = 20\%$$

$$2 = 1 \times \left(1 + \frac{20}{100}\right)^n$$

$$2 = \left(\frac{6}{5}\right)^n$$

$$2 = (1.2)^n$$

Now check with option

$$(1.2)^3 = 1.728$$

$$(1.2)^4 = 2.07$$

Ans \Rightarrow 3-4 yrs.

(Alternative)

$$\begin{array}{r} 50,00,000 \\ \times 20\% \\ \hline 10,00,000 \\ \hline 60,00,000 \end{array}$$

$$\begin{array}{r} 60,00,000 \\ \times 20\% \\ \hline 12,00,000 \\ \hline 72,00,000 \end{array}$$

$$\begin{array}{r} 72,00,000 \\ \times 20\% \\ \hline 14,40,000 \\ \hline 86,40,000 \end{array}$$

$$\begin{array}{r} 86,40,000 \\ \times 20\% \\ \hline 17,28,000 \\ \hline 103,68,000 \end{array}$$

Between 3-4 years it becomes double.

