AVERAGES

5.1 : Averages Introduction Part - 1

What is an average ?

Ex

Let take the data of toys sold by sales person for 5 days

Days	1st	2nd	3rd	4th	5th	Total	Avg
	45	25	30	70	80	250	$\frac{250}{5} = 50$

1) Average = $\frac{\text{Sum of all quantities}}{\text{No. of quantities}} = \frac{\Sigma x}{n}$

Ex

Dhoni batting average is 70 and played 150 matches.

Total runs = $70 \times 150 = 10,500$.

2) Total = Average * No. of quantities.

Ex

45, 25, 30, 70, 80 $\xrightarrow{\text{Avg}}$ 50.

Average always lie between lowest and highest value.

Average is also called as central value of the given data.

Properties

1) If certain data has an average 'x'

a) If each quantity is increased by 'n', then new average is 'x+n'.

Ex

20 25 10 5 $\xrightarrow{\text{Avg}}$ $\frac{60}{4} = 15$

+3 +3 +3 +3

$\xrightarrow{\text{New Avg}}$ 15+3 = 18

23 28 13 8 $\xrightarrow{\text{Avg}}$ $\frac{72}{4} = 18$

Similarly, if each quantity is decreased or multiplied or divided by 'n', then new average is 'x-n' or 'x×n' or 'x÷n' respectively.

Ex

20 25 10 5 $\xrightarrow{\text{Avg}}$ $\frac{60}{4} = 15$

-2 -2 -2 -2 $\xrightarrow{\text{New Avg}}$ 15-2 = 13

×5 ×5 ×5 ×5 $\xrightarrow{\text{New Avg}}$ = 15×5 = 75

÷5 ÷5 ÷5 ÷5 $\xrightarrow{\text{New Avg}}$ = $\frac{15}{5} = 3$

Ex In a flower shop there were 6 flowers in each boquet. If the seller has doubled the number of flowers in each boquet then the new average of flowers in each boquet is:

Old Average = 6

New Average = $6 \times 2 = 12$

Ex The average number of shirts purchased by A, B and C is 60. If all of them reached a mall and purchased 3 shirts each of them then the average number of shirts each of them now has:

Old Average = 60

New Average = $60 + 3 = 63$.

Q If the average age of a group of persons is 'x' years today then after 'n' years their average age will be $(x+n)$.

Ex

	A	B	C		
	20	35	5	Avg →	$\frac{60}{3} = 20$
After 5 years	+5	+5	+5	New Avg →	$20+5 = 25$.
	25	40	10	Avg →	$\frac{75}{3} = 25$.

Similarly, 'n' years ago their average age would have been $(x-n)$ years.

(4)

Ex The average age of 7 members of patel's family is 25 years. The average age of the same family 3 years ago is :.

Old Avg = 25 years

3 years ago ⇒ New Average = 25 − 3 = 22 years

5.2 : Averages Introduction Part 2

3) Average Speed

Distance = Speed × Time

Speed = $\frac{\text{Distance}}{\text{Time}}$

Time = $\frac{\text{Distance}}{\text{Speed}}$.

Average Speed

Case-1

If distance travelled is not equal.

Avg Speed = $\frac{\text{Total distance}}{\text{Total Time}}$

Case-2

If distance travelled is equal.

(a) If two speeds are given say s_1, s_2

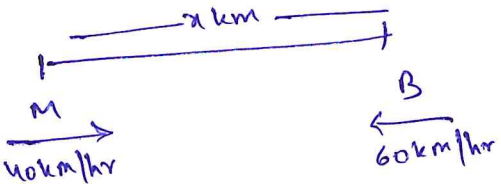
Avg. speed = $\frac{2 \cdot s_1 \cdot s_2}{s_1 + s_2}$

(b) If 3 speeds are given say s_1, s_2, s_3

Avg. speed = $\frac{3 \cdot s_1 \cdot s_2}{s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1}$

Ex

A man goes from Mysore to Banglore at a uniform speed of 40km/hr and comes back to Mysore at a uniform speed of 60km/hr. His average speed for the whole journey is



Avg. Speed = $\frac{\text{Total Distance}}{\text{Total Time}} = \frac{2x}{x/24} = 48 \text{ km/hr}.$

Total distance = $2x$

Total Time = $\frac{x}{40} + \frac{x}{60} = \frac{x}{24}$

(6)

2nd Method

Distance is equal.

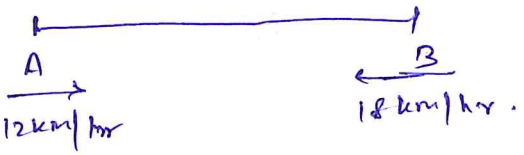
Speeds = 2 speeds.

Avg Speed = $\frac{2 \cdot S_1 \cdot S_2}{S_1 + S_2}$

$= \frac{2 \times 40 \times 60}{40 + 60}$

$= 48$ km/hr.

Ex A man goes from place A to B at a speed of 12 km/hr and returns B to A at a speed of 18 km/hr. The average speed for the whole journey is



Distance travelled is same.

2 speeds.

Avg speed = $\frac{2 \cdot S_1 \cdot S_2}{S_1 + S_2}$

$= \frac{2 \times 12 \times 18}{12 + 18}$

$= \frac{72}{5}$

$= 14\frac{2}{5}$ km/hr.

Eg: A man completes 30 km of a journey at a speed of 6 km/hr and remaining 40 km of the journey in 5 hours. His average speed for the whole journey is

$$\text{Avg Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Total Distance = 30 + 40 = 70 kms.

Total Time = $\frac{30}{6} + 5$

= 5 + 5 = 10 hours

Avg. Speed = $\frac{70}{10}$ = 7 km/hr.

4) Weighted Average

When the average of groups, instead of individuals having different number of elements is being calculated, then it is called weighted average.

Eg:

Lets take class 8

	8A	8B
Avg Marks	30	70
No. of Students	60	40

$$\text{Avg. Marks} = \frac{30\times60 + 70\times40}{60+40} = \frac{1800+2800}{100} = 46$$

$$W_A = \frac{x_1 n_1 + x_2 n_2 + x_3 n_3 + \ldots\ldots + x_n n_n}{n_1 + n_2 + n_3 + \ldots\ldots + n_n}$$

(19)

If 2 quantities are given

$$W_A = \frac{x_1 n_1 + x_2 n_2}{n_1 + n_2}$$

If 3 quantities are given

$$W_A = \frac{x_1 n_1 + x_2 n_2 + x_3 n_3}{n_1 + n_2 + n_3}$$

Ex = The average height of 30 boys, out of a class of 50 is 160 cm. If the average height of the remaining boys is 165 cm, the average height of the whole class is?

Avg. height of 30 boys = 160 cm

Avg. height of 20 boys = 165 cm.

$$W_A = \frac{x_1 n_1 + x_2 n_2}{n_1 + n_2}$$

$$= \frac{30 \times 160 + 20 \times 165}{30 + 20}$$

Avg height of 50 boys = 162 cm.

5.3 : Averages Introduction Part 3

5) Average of first 'n' numbers

a) Sum of first 'n' natural numbers = $\frac{n(n+1)}{2}$

Average of first 'n' natural numbers : $\frac{\text{Sum}}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n(n+1)}{2n}$

$= \frac{n+1}{2}$

b) Sum of first 'n' even natural numbers = $n(n+1)$

Average of first 'n' even natural numbers = $(n+1)$

c) Sum of first 'n' odd numbers = n^2

Average of first 'n' odd numbers = n.

d) Sum of squares of first 'n' natural numbers = $\frac{n(n+1)(2n+1)}{6}$

$1^2+2^2+3^3+\ldots\ldots+n^2$

Average of squares of first 'n' natural number = $\frac{(n+1)(2n+1)}{6}$

e) Sum of cubes of first 'n' natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$

$1^3+2^3+3^3+\ldots\ldots+n^3$

Average of cubes of first 'n' natural numbers = $n\left(\frac{n+1}{2}\right)^2$

f) Sum of 'n' terms of series having common difference same = $\frac{n}{2}\left[\text{first term} + \text{last term}\right]$

$2+4+8+\ldots$

Average of 'n' terms of series having common difference same = $\frac{\text{first term} + \text{last term}}{2}$

Ex The average of first 100 positive integers is.

avg of first 'n' natural numbers $= \frac{n+1}{2} = \frac{100+1}{2} = 50.5$

Ex The average of the squares of first ten natural numbers is

Avg for $1^2+2^2+3^2+\ldots\ldots+10^2$

$$\frac{(n+1)(2n+1)}{6} = \frac{11 \times 21}{6} = 38.5$$

Ex The mean of first 10 whole numbers is

$0+1+2+3+\ldots\ldots+9$

$= 1+2+3+\ldots\ldots+9$

= Sum of first 9 natural numbers.

$= \frac{9(9+1)}{2} = \frac{9 \times 10}{2} = 45.$

Here we cannot use Avg of first 'n' natural numbers because $n \neq 10$.

$$\text{Avg} = \frac{\text{Sum}}{n} = \frac{45}{10} = 4.5.$$

Trick

$0+1+2+\ldots\ldots+9$

common difference = 1

$$\text{Avg} = \frac{1\text{st term} + \text{last term}}{2}$$

$$= \frac{0+9}{2} = 4.5.$$

5.4! Averages Introduction Part 4

6) Averages on Consecutive Numbers

(a) Consecutive Natural Numbers

21, 22, (23), 24, 25 → odd places
↳ Avg is middle number

34, 35, 36, 37, 38, 39 → even places
↓
$\frac{36+37}{2} = 36.5$ (Avg.)

How to assume numbers!

3 consecutive numbers

one way :- $x, x+1, x+2$

(b) Consecutive Even numbers

42, 44, (46), 48, 50 → odd places
↳ Middle number (avg.)

14, 16, 18, 20, 22, 24 → even places

$\frac{18+20}{2} = 19$ (avg.)

How to assume the numbers

3 consecutive even numbers

one way :- $x, x+2, x+4$

2nd way :- $x+2, x, x-2$

} Same rule will be applicable for consecutive odd numbers

5 consecutive even numbers

$x+4, x+2, x, x-2, x-4$

(12)

Ex The average of 7 consecutive numbers is 20. The largest of these numbers is:

7 consecutive natural numbers

$x, x+1, x+2, x+3, x+4, x+5, x+6$

Avg of 7 number = 20

Total = Avg × No. of observation

$= 7 \times 20$

$= 140$

$x + x+1 + x+2 + x+3 + x+4 + x+5 + x+6 = 140$

$7x = 119 \Rightarrow x = 17.$

$\therefore x+6 = 17+6 = 23.$

2nd way

Average will be the middle number

Let the number be

$x+3, x+2, x+1, x, x-1, x-2, x-3$

Avg = 20

$\therefore x = 20$

Largest number $= x + 3$

$= 20 + 3$

$= 23.$

8s The sum of the middle number of 3 consecutive odd numbers and the middle number of 3 consecutive even numbers is 8. What will be the sum of the total of these odd numbers and total of those three even numbers?

		Total
3 consecutive odd number	$x+2, x, x-2$	$3x$
3 consecutive even numbers	$y+2, y, y-2$	$3y$

$\therefore x+y = 8$

$3x+3y = 3(x+y)$

$= 3 \times 8$

$= 24.$

5.5 Solved Problems

Q.1) Find the average of first 9 prime numbers.

$$\text{Avg} = \frac{\text{Sum}}{n}$$

Prime numbers = 2, 3, 5, 7, 11, 13, 17, 19

Sum = 2+3+5+7+11+13+17+19

= 77

$$\text{Avg} = \frac{77}{9} = 8.5$$

Q.2) The avg. of 6 numbers is 8. What is the 7th number so that average becomes 10.

Avg. of 6 numbers = 8

Avg = 8, n = 6

Total = Avg × n

= 8 × 6

= 48.

Avg of 7 number = 10

Total = 10 × 7 = 70

Sum of 6 number = 48

Sum of 7 number = 70.

Difference = 70 − 48 = 22 → 7th number.

2nd way

Sum of 6 numbers = 48

7th number = n

$$\text{Avg of 7 numbers} = \frac{48+n}{7} = 10$$

$$\therefore n = 22.$$

Q3) A man bought 13 pens for ₹50 each, 15 markers of ₹60 each and 12 books at ₹65 each. Find the average expenditure?

13 pens @ ₹50 each = 13×50 = 650

15 markers @ ₹60 each = 15×60 = 900

12 books @ ₹65 each = 12×65 = 780

40 — 2330

$$\bar{x}_A = \frac{x_1 n_1 + x_2 n_2 + x_3 n_3}{n_1 + n_2 + n_3}$$

$$= \frac{2330}{40}$$

$$= ₹58.25$$

Q4) The average of 11 numbers is 21. If 3 is added to each given number, what will be the new average?

Avg of 11 numbers = 21

New Avg : 21+3 = 24.

Q5) If the mean of 5 observations is 20 and when a constant c is added to each observation, the mean becomes 22, the value of c is?

Avg. of 5 observations = 20

New Avg = 22.

$\therefore 20 + c = 22$

$c = 2.$

(16)

Q.6 The average age of 34 boys in a class is 14 years. If the teacher's age is included, the average age of the boys and the teacher becomes 15 years. What is the teacher's age?

Avg. age of 34 boys = 14 years

Sum of 34 boys age = Avg × n

= 14 × 34 = 476

Let teacher's age be x.

476 + x = total age of 35 people.

Avg age of 35 people $\Rightarrow \frac{476+x}{35} = 15$

$\Rightarrow 476 + x = 15 \times 35$

$\Rightarrow x = 525 - 476$

$= 49$ years.

2nd way

difference in teacher

34 boys $\xrightarrow{\text{avg}}$ 14 years → Total 476 (34×15)

35 people $\xrightarrow{\text{avg}}$ 15 years → Total 525 (35×15)

$= 49$

$\Rightarrow$ teacher age.

Trick

Old avg + New People Strength × difference in average. = Teacher's age

14 + 35 × 1 = 49 years.

Q.7) The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average reduces by 1 year. What is the age of the class teacher?

Avg. age of 25 people = 16 years

Sum of 25 people ages = 16 × 25
= 400

Avg age of 24 people = 16 − 1 = 15 years.

Sum of 24 people ages = 15 × 24
= 360.

∴ Teacher's age = 400 − 360 = 40 years.

Trick

16 + 24 × 1 = 40 years.

Q.8) There are 20 students with an average height of 105 cms in a class. Then 10 students with an average height of 120 cms join the class. What will be the average height of the class now?

(a) 100 cms (b) 110 cms (c) 120 cms (d) 130 cms

New average ↑ between 105 ↔ 120

option (a) and (d) can be eliminated.

option (c) can also be eliminated.

Average height of 20 students = 105 cms

Total height = 105 × 20 = 2100 cms

$$W_A = \frac{x_1 n_1 + x_2 n_2}{n_1 + n_2}$$

$$= \frac{20 \times 105 + 10 \times 120}{20 + 10}$$

$$= \frac{2100 + 1200}{30} = \frac{3300}{30}$$

= 110 cms

(OR)

20 — 105
10 — 120
30

15 more per person

10 × 15 = 150 → shared by 30 people

each person = $\frac{150}{30}$ = 5

∴ 105 + 5 = 110 cms.

5.6: Solved Problems Part - 2

Q.8) There are 20 students with an average height of 105 cms in a class. Then 10 students with an average height of 120 cms join the class. What will be the average height of the class now?

(a) 100 cms (b) 110 cms (c) 120 cms (d) 130 cms

Avg. height of class is 105 cms

+10 joined avg 120 cms

∴ New Avg ↑ will be between 105 - 120

5.6: Solved Problems Part-2

Q.9) A student on her first 3 tests received an average score of N points if she exceeds her previous average score by 20 points on her fourth test, then what is the average score for the first 4 tests?

(a) N+20 (b) N+10 (c) N+4 (d) N+5

Average of 3 tests = N

Total of 3 tests = Avg. × N

= N × 3 = 3N

Fourth test marks = N+20

Avg. of 4 tests = $\frac{3N+N+20}{4}$

= $\frac{4N+20}{4}$ = N+5

Q.10) The average weight of 8 persons increases by 1.5 kg, if a person weighing 65 kg is replaced by a new person. What could be the weight of the new person?

(a) 75kg (b) 65kg (c) 77kg (d) 82kg

Avg. weight of 8 persons = x. Let new person weight = y.

Total weight of 8 person = 8x.

Total New weight = 8x − 65 + y

New Avg = $\frac{8x-65+y}{8}$

$x+1.5 = \frac{8x-65+y}{8}$ ⇒ $8x+12 = 8x-65+y$

⇒ y = 77kg.

(19)

2nd Method

65kg. Avg ↑ 1.5kg → shared by 8 people

New Person weight = 65 + 1.5×8

= 65 + 12

= 77kg.

Replaced Weight ± Increased/Decreased Average × No. of persons

Q.11) The average of 17 numbers is 45. The average of first 9 of these numbers is 51 and the last 9 of these numbers is 36. What is the ninth number?

9th number = Total of first nine + Total of last nine − Total of 17 numbers

= (9×51) + (9×36) − (17×45)

= 9(51+36) − (17×45)

= (9×87) − (17×45)

= 783 − 765

= 18

Q.12) The average of 11 results is 30, that of first five is 25 and that of last five is 28. Find the value of the 6th number?

first 5
last 5
10

Total = 11 numbers

1 number is missing

∴ 6th number = Total of 11 numbers − [Total of first 5 + Total of last 5]

= (30×11) − [(5×25) + (5×28)]

= 330 − (125+140)

= 330 − 265

= 65

Q.13) The average of the first and the second of three numbers is 15 more than the average of the second and the third of these numbers. What is the difference between the first and the third of these three numbers?

Let the numbers be N_1, N_2 and N_3

$$\frac{N_1+N_2}{2} = 15 + \frac{N_2+N_3}{2}$$

$$\Rightarrow N_1+N_2 = 30 + N_2 + N_3$$

$$\Rightarrow N_1 - N_3 = 30$$

Q.14) The average weight of a group of 20 boys was calculated to be 89.4 kg and it was discovered that one weight was misread as 78 kg instead of the correct one of 87 kg. The correct average weight is?

(a) 88.95 kg (b) 89.25 kg (c) 89.55 kg (d) 89.85 kg

Avg. weight of 20 boys = 89.4 kg

Wrong one = 78 kg. ↓, decreased by 9 kg.

Correct one = 87 kg

∴ Average was also decreased.

Decrease in Avg = $\frac{9}{20} = 0.45$.

Actual Avg = $89.4 + 0.45$

$= 89.85$ kg.

2nd way

Total weight = 89.4×20

New Total Weight = $89.4 \times 20 - 78 + 87 = 1797$

New Avg = $\frac{1797}{20} = 89.85$ kg.

(21)

Q.15) The average of any 5 consecutive odd natural numbers is K. If two more such numbers, just next to the previous 5 numbers are added, the new average becomes;

(a) $\frac{2}{7}(k+1)$ (b) $2k-3$ (c) $2k+1$ (d) $k+2$

Let 5 consecutive odd numbers be 3, 5, 7, 9, 11

$$\text{Avg} = \frac{3+5+7+9+11}{5} = \frac{35}{5} = 7.$$

3, 5, 7, 9, 11, 13, 15

$$\text{Avg} = \frac{35+28}{7} = \frac{63}{7} = 9$$

$\therefore 7+2 \Rightarrow k+2.$

5.7: Previous Year Gate Questions on Averages

Q1) In the summer of 2012 in Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was? (GATE 2013)

Avg. temperature Mon to Wed = 41°C

Total Temperature = 41 × 3 = 123°C

Avg. temperature Tue to Thur = 43°C

Total Temperature = 43 × 3 = 129°C

Mon : Thur = 100 : 115

= 20 : 23

Mon + Tue + Wed = 123 ... ①

Tue + Wed + Thur = 129 ... ②

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① - ② ⇒ Mon - Thur = -6

⇒ Thur - Mon = 6

Mon : Thur

20 : 23

(3)

∴ 3 → 6

23 → $\frac{6}{3} \times 23 = 46°C$.

Q.2) What is the average of all multiples of 10 from 2 to 198?
(GATE 2014)

Multiples of 10 = $10, 20, 30, \ldots\ldots, 190$

Common difference = 10

$\therefore$ Avg = $\frac{\text{first term} + \text{last term}}{2}$

$= \frac{10+190}{2} = 100.$

Q.3) The sum of eight consecutive odd numbers is 656. The average of four consecutive even numbers is 87. The sum of the smallest odd number and second largest even number is:
(GATE 2014)

The numbers are $x-6 \quad x-4 \quad x-2 \quad x \quad x+2 \quad x+4 \quad x+6 \quad x+8$

$\therefore 8x + 8 = 656$

$\Rightarrow 8x = 648$

$\Rightarrow x = 81.$

Smallest odd number = $x-6$
$= 81-6$
$= 75$

4 consecutive even number : $y-4 \quad y-2 \quad y \quad y+2$

$4y - 4 = 87 \times 4$

$y = 87+1$

$= 88$

Second largest even number = $y = 88$

$\therefore$ Sum = $75+88 = 163.$