DIT-FFT

DFT Computation

- Multiplication - N^2
- Addition - $N(N-1)$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

N=4 → Multiplication = 16 | Addition = 12

Twiddle Factor

- N-point DFT :

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$ For k=0,1,2.....N-1

- Twiddle factor: $W_N = e^{-\frac{j2\pi}{N}}$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$ For k=0,1,2.....N-1

Properties of twiddle factor

1] $W_N^{\ k} = W_N^{\ k+N}$

2] $W_N^{\ k+N/2} = -W_N^{\ k}$

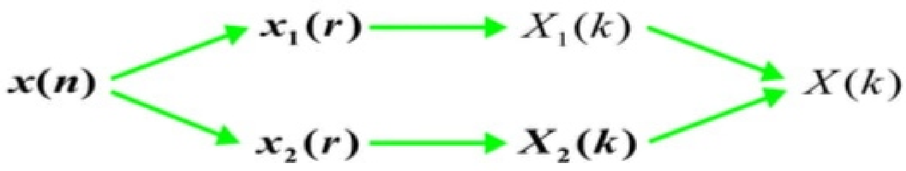
3] $W_{N/2} = W_N^{\ 2}$

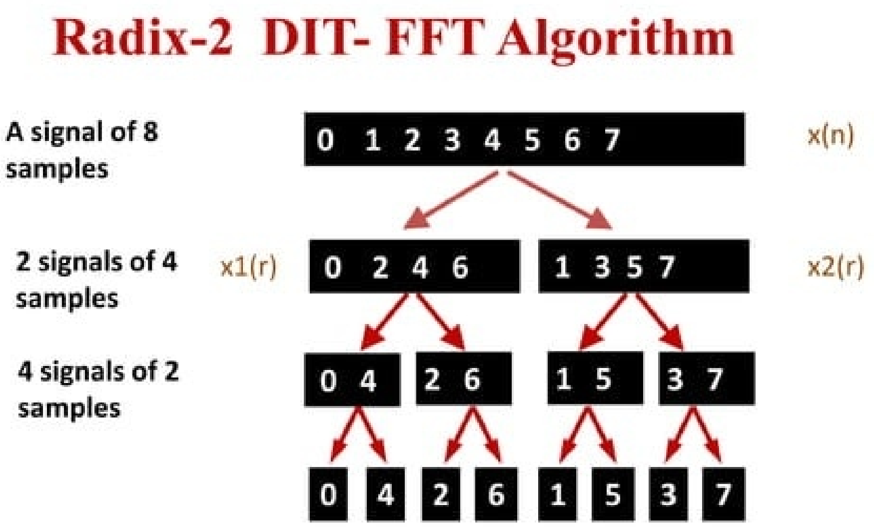
Comparison of DIT-FFT and DIF-FFT algorithms

	DIT	DIF
Input sequence	Bit reversal	Natural order
Output sequence	Natural order	Bit reversal
Computation	Same Multiplication - (N/2)log2N Addition - Nlog2N	Same Multiplication - (N/2)log2N Addition - Nlog2N
Number of sets or sections of butterflies in each stage	2^(M-m)	2^(m-1)
Exponent repeat factor (ERF)	2^(M-m)	2^(m-1)

DIT-FFT:

Decimation in time





The mathematical background:

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}}$$

$$= \sum_{n=\text{even}} x(n) W_N^{kn} + \sum_{n=\text{odd}} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_N^{k2n} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{k(2n+1)}$$

$$= \sum_{n=0}^{N/2-1} f_1(n) W_{N/2}^{kn} + \sum_{n=0}^{N/2-1} f_2(n) W_{N/2}^{kn} \cdot W_N^{k}$$

$$X(k) = F_1(k) + W_N^{k} F_2(k), \quad k = 0, 1, \ldots \frac{N}{2}-1 \quad \text{— I}$$

$$X(k+N/2) = F_1(k+N/2) + W_N^{k+N/2} F_2(k+N/2)$$

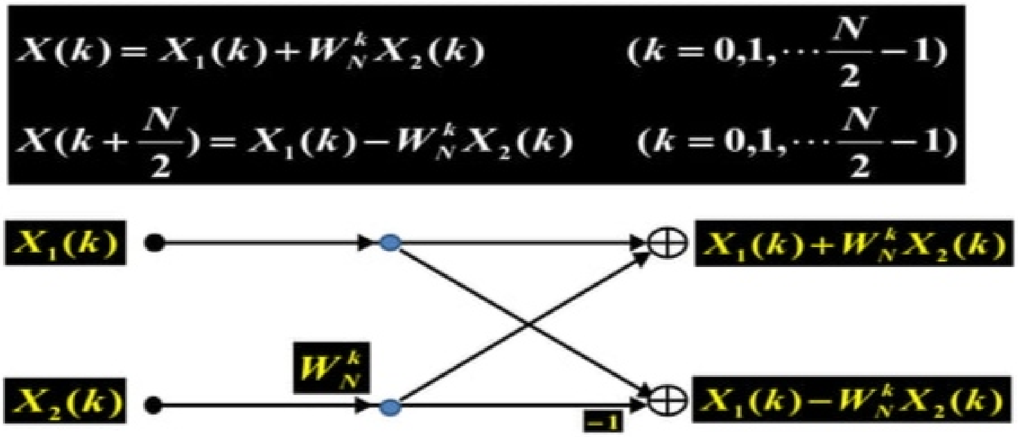
$$= F_1(k) - W_N^{k} F_2(k) \quad \text{— II}$$

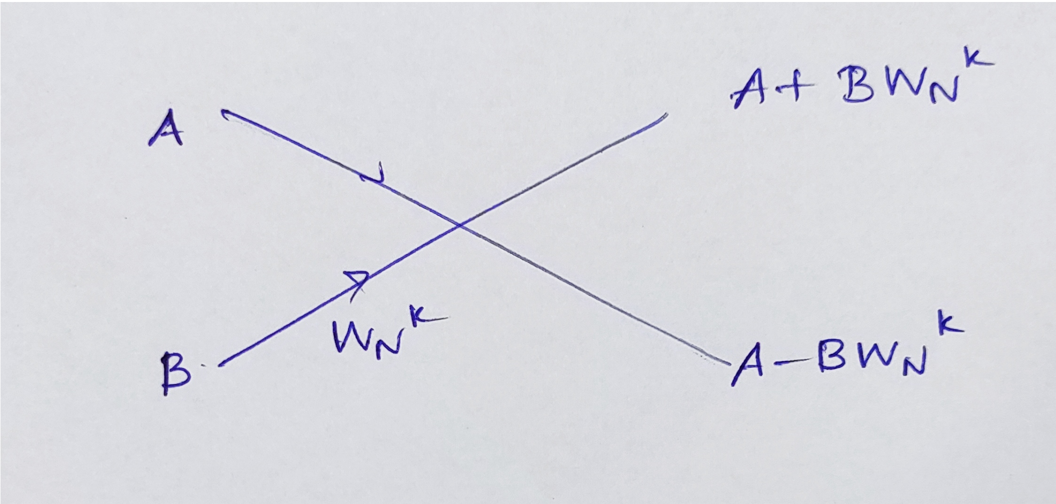
- The boxed formula is the general DFT formula
- In DIT, we split the sequence into odd and even sequences
    - Odd sequence → Has values at odd samples

- Even sequence -. Has values at even samples

- Let even function be represented as $x(2n)$ and odd function be prepresented as $x(2n+1)$
- Let them be denoted as $f1(n)$ and $f2(n)$.

The butterfly diagram is as follows:





<u>Problem:</u>

Compute the DFT for the sequence x[n] = {1,2,1,2,3,4,3,4} using DIT-FFT method

Sample Index	Binary representation	Bit reversed	Resulting index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

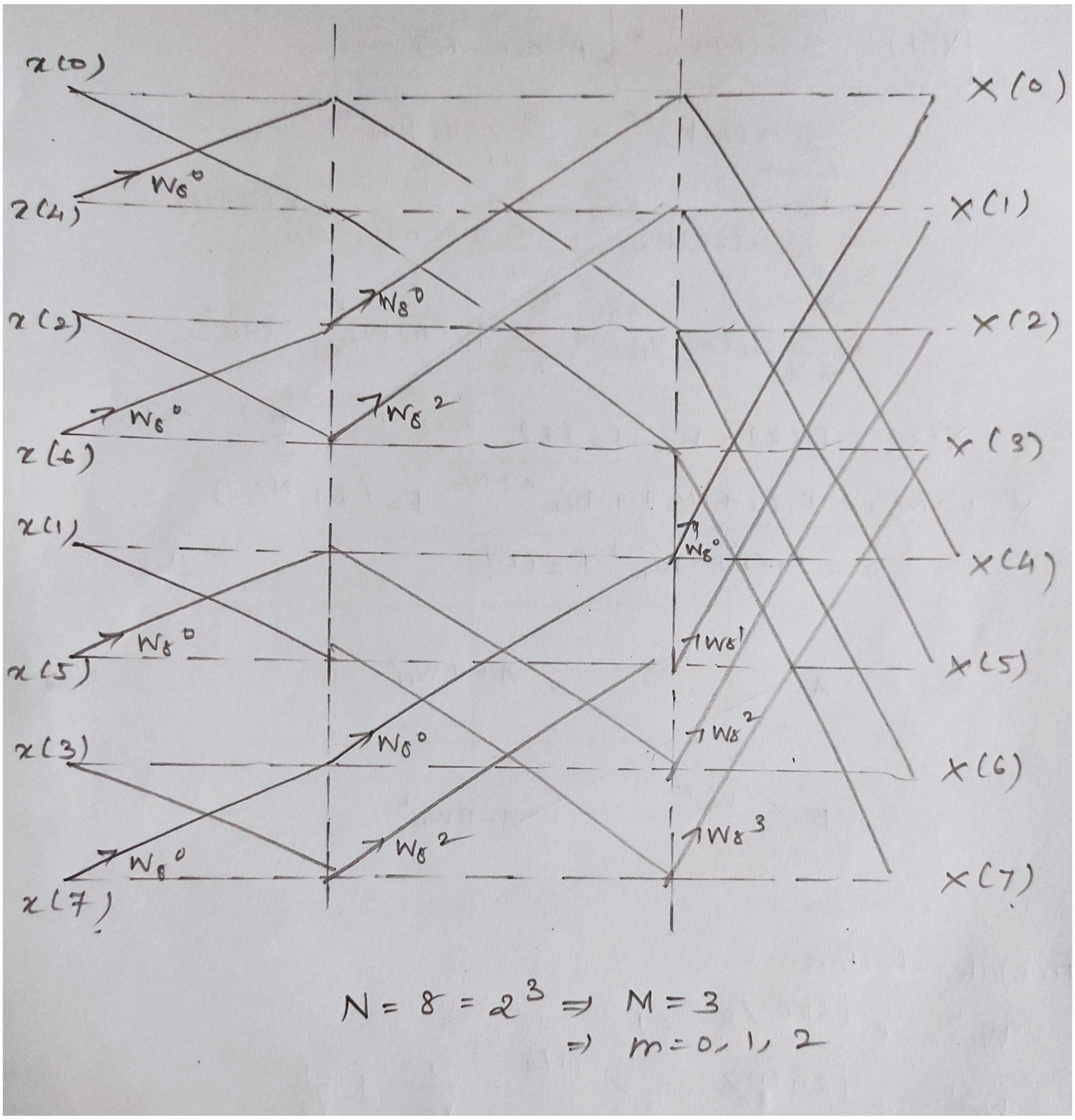
Twiddle factors :

$$W_8^0 = e^{-j2\pi 0/8} = 1$$

$$W_8^1 = e^{-j2\pi(1)/8} = e^{-j\pi/4} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^2 = e^{-j2\pi(2)/8} = e^{-j\pi/2} = -j$$

$$W_8^3 = e^{-j2\pi(3)/8} = e^{-j\pi 3/4} = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$


x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
W8 0
W8 0
W8 2
W8 0
W8 0
W8 0
W8 2
W8 0
W8 1
W8 2
W8 3
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
N = 8 = 2 3 ⇒ M = 3
⇒ m = 0, 1, 2

	Stage 1	Stage 2	Stage 3	
$x(0)=1$	$1+1(3)=4$	8	20	$X(0)$
$x(4)=3$	$1-1(3)=-2$	$-2+2j$	$-2+4.828j$	$X(1)$
$x(2)=1$	$1+1(3)=4$	0	0	$X(2)$
$x(6)=3$	$1-1(3)=-2$	$-2-2j$	$-2-0.828j$	$X(3)$
$x(1)=2$	$2+1(4)=6$	12	-4	$X(4)$
$x(5)=4$	$2-1(4)=-2$	$-2+2j$	$-2-4.828j$	$X(5)$
$x(3)=2$	$2+1(4)=6$	0	0	$X(6)$
$x(7)=4$	$2-1(4)=-2$	$-2-2j$	$-2+0.828j$	$X(7)$

$$X(k) = \{20, -2+4.828j, 0, -2-0.828j, -4, -2-4.828j, 0, -2+0.828j\}$$

Source for pictures:

Slideshare and Vinoth sir notes 🫡