Stat 544. HW #2.

Q1. "Design an Experiment" and define its probability space.

(a) Use this probability space to define your own random variable X.

(b) Compute the pmf (or the pdf), cdf, $E(X)$, and $Var(X)$.

Sol-n.

Let me bring the experiment from HW #1, Q6:

(1) "Sending a packaged item to Arizona by USPS."

The experiment has the following outcomes:

(2)
$$\begin{cases} A_1 = \text{"The item is delivered on time"} \\ A_2 = \text{"The item is delivered Later than scheduled"} \\ A_3 = \text{"The item is stolen".} \end{cases}$$

(3) $\Omega_1 = \{A_1, A_2, A_3\}$.

(4) $\mathcal{F}_1 = \{\emptyset, \Omega, A_3, \{A_1, A_2\}\} =$ σ-algebra

(5)
$$\begin{cases} \mathbb{P} : \mathcal{F} \to [0, 1] : \\ P(\emptyset) = 0 \\ P(\Omega_1) = 1 \\ P(A_3) = 0.1 \\ P(\{A_1, A_2\}) = 0.9 \\ P(A_1) = 0.7, \quad P(A_2) = 0.2 \end{cases}$$

Prob. measure of $\mathcal{F}_1$.

Then

(6) $\{\Omega_1, \mathcal{F}_1, \mathbb{P}\}$ is the probability space.

Now let random variable (r.v.) X to be possible outcomes of sending 2 packages separately to Arizona by post in two different times of the day. (" Outcome of one package is completely indepen-t of that of another).

X is a random variable with $X: \Omega \to \mathbb{R}$ where

(7) $\begin{cases} A_1 = \text{"delivery on time"} = +1 \\ A_2 = \text{"late delivery"} = 0 \\ A_3 = \text{"item stolen"} = -1 \end{cases}$,

then we have the following sample space :

	1st package	2nd package	$X(w_i)$
w_1	A_1	A_1	$1+1=2$
w_2	A_1	A_2	$1+0=1$
w_3	A_1	A_3	$1+(-1)=0$
w_4	A_2	A_1	$0+1=1$
w_5	A_2	A_2	$0+0=0$
w_6	A_2	A_3	$0+(-1)=-1$
w_7	A_3	A_1	$(-1)+(1)=0$
w_8	A_3	A_2	$(-1)+(0)=-1$
w_9	A_3	A_3	$(-1)+(-1)=-2$

(8) $\Omega = \{w_i\}_{i=1}^{9}$

(9) $\mathcal{F} = \mathcal{P}(\Omega)$

Then

(10) $\{\Omega, \mathcal{F}, \mathbb{P}\}$ is the probability space s.t.

Then we have, by (5),

$\mathbb{P}(A_1) = 0.7$

$\mathbb{P}(A_2) = 0.2$

$\mathbb{P}(A_3) = 0.1$, then probabilities for 2 packages:

$$
(11)\quad \left\{ \begin{aligned}
\mathbb{P}(A_1 A_1) &= \mathbb{P}(X(\omega_1)) = 0.7 \cdot 0.7 = 0.49 \\
\mathbb{P}(A_1 A_2) &= \mathbb{P}(X(\omega_2)) = 0.7 \cdot 0.2 = 0.14 \\
\mathbb{P}(A_1 A_3) &= \mathbb{P}(X(\omega_3)) = 0.7 \cdot 0.1 = 0.07 \\
\mathbb{P}(A_2 A_1) &= \mathbb{P}(X(\omega_4)) = 0.2 \cdot 0.7 = 0.14 \\
\mathbb{P}(A_2 A_2) &= \mathbb{P}(X(\omega_5)) = 0.2 \cdot 0.2 = 0.04 \\
\mathbb{P}(A_2 A_3) &= \mathbb{P}(X(\omega_6)) = 0.2 \cdot 0.1 = 0.02 \\
\mathbb{P}(A_3 A_1) &= \mathbb{P}(X(\omega_7)) = 0.1 \cdot 0.7 = 0.07 \\
\mathbb{P}(A_3 A_2) &= \mathbb{P}(X(\omega_8)) = 0.1 \cdot 0.2 = 0.02 \\
\mathbb{P}(A_3 A_3) &= \mathbb{P}(X(\omega_9)) = 0.1 \cdot 0.1 = 0.01
\end{aligned} \right.
$$

and

random variable:

(12) $X(\omega) = x \in \{-2, -1, 0, 1, 2\}$

$\square$

(18) Probability mass function (pmf, pdf):

$$(13)\quad \mathbb{P}(X(\omega)=x)=\begin{cases} 0.49 & \text{if } X(\omega)=2 \\ 0.14+0.14=0.28 & \text{if } X(\omega)=1 \\ 0.07+0.04+0.07=0.18 & \text{if } X(\omega)=0 \\ 0.02+0.02=0.04 & \text{if } X(\omega)=-1 \\ 0.01 & \text{if } X(\omega)=-2 \\ 0 & \text{otherwise} \end{cases}$$

Cumulitive distribution function (CDF):

$$(14)\quad F(X)=\begin{cases} 0, & x<-2 \\ 0.01, & -2\le x<-1 \\ 0.01+0.04=0.05, & -1\le x<0 \\ 0.05+0.18=0.23, & 0\le x<1 \\ 0.23+0.28=0.51, & 1\le x<2 \\ 0.51+0.49=1, & 2\le x \end{cases}$$

$$(15)\quad E(X)=\sum_{n=1} x_n \mathbb{P}(X=x_n)=0.49\cdot 2+0.28\cdot 1+0.18\cdot 0+0.04\cdot(-1)+0.01\cdot(-2)=$$
$$=1.2\ ;$$

$$(16)\quad E(X^2)=\sum_{n=1} x_n^2\cdot \mathbb{P}(X=x_n)=0.49\cdot(2)^2+0.28\cdot(1)^2+0.18\cdot(0)^2+0.04\cdot(-1)^2+0.01\cdot(-2)^2=2.32\ ;$$

$$(17)\quad Var(X)=E(X^2)-(E(X))^2=2.32-(1.2)^2=0.88\ ;$$