HW2

Answer 1(a)

There are 6 people in a room : 3 doctors, 2 engineers and 1 mathematician.

⓪ Pick two persons one-by-one sequentially and randomly. So, the choices are exhaustive.

X is a random variable, $X : \Omega \to \mathbb{R}$ s.t.

each $D = +1$
each $E = -1$
each $M = 0$

So, there are following choices:

$$
\begin{aligned}
DD &\longrightarrow X(\omega_1) = 2 \\
DE &\longrightarrow X(\omega_2) \\
ED &\longrightarrow X(\omega_3) \Big\} = 0 \\
DM &\longrightarrow X(\omega_4) \\
MD &\longrightarrow X(\omega_5) \Big\} = 1 \\
EE &\longrightarrow X(\omega_6) = -2 \\
EM &\longrightarrow X(\omega_7) \\
ME &\longrightarrow X(\omega_8) \Big\} = -1
\end{aligned}
$$

① Sample space, $\Omega = \{\omega_i\}_{i=1}^{8}$

② σ-algebra, $\mathcal{F} = \mathcal{P}(\Omega)$

(3) $\mathbb{P}(D) = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}(E) = \frac{2}{6} = \frac{1}{3}$

$\mathbb{P}(M) = \frac{1}{6}$

$\therefore \mathbb{P}(DD) = \mathbb{P}(X(\omega_1)) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$

$\mathbb{P}(DE) = \mathbb{P}(X(\omega_2)) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$

$\mathbb{P}(ED) = \mathbb{P}(X(\omega_3)) = \left(\frac{2}{6}\right)\left(\frac{3}{5}\right) = \frac{1}{5}$

$\mathbb{P}(DM) = \mathbb{P}(X(\omega_4)) = \left(\frac{3}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{10}$

$\mathbb{P}(MD) = \mathbb{P}(X(\omega_5)) = \left(\frac{1}{6}\right)\left(\frac{3}{5}\right) = \frac{1}{10}$

$\mathbb{P}(EE) = \mathbb{P}(X(\omega_6)) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{15}$

$\mathbb{P}(EM) = \mathbb{P}(X(\omega_7)) = \left(\frac{1}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{15}$

$\mathbb{P}(ME) = \mathbb{P}(X(\omega_8)) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{15}$

$\therefore$ Probability Space : $\{\underline{\Omega}, \mathcal{F}, \mathbb{P}_x\}$

and Random variable : $X(\omega) = x \in \{-2, -1, 0, 1, 2\}$

Answer 1(b)

Probability mass function (PMF):

$$\cancel{\mathbb{P}(X(\omega)=x)}$$

$$\mathbb{P}(X(\omega)=x) = \begin{cases} \frac{1}{5}, & \text{if } X(\omega)=2 \\ \frac{1}{5}+\frac{1}{5}=\frac{2}{5} & \text{if } X(\omega)=0 \\ \frac{1}{10}+\frac{1}{10}=\frac{1}{5} & \text{if } X(\omega)=1 \\ \frac{1}{15} & \text{if } X(\omega)=-2 \\ \frac{1}{15}+\frac{1}{15}=\frac{2}{15} & \text{if } X(\omega)=-1 \\ 0 & \text{, otherwise} \end{cases}$$

Cumulative distribution function (CDF):

$$F(X) = \begin{cases} 0, & x<-2 \\ \frac{1}{15}, & -2 \le x < -1 \\ \frac{1}{15}+\frac{2}{15}=\frac{1}{5} & -1 \le x < 0 \\ \frac{1}{5}+\frac{2}{5}=\frac{3}{5} & 0 \le x < 1 \\ \frac{3}{5}+\frac{1}{5}=\frac{4}{5} & 1 \le x < 2 \\ \frac{4}{5}+\frac{1}{5}=1 & 2 \le x \end{cases}$$

xpectation, $E(X) = \sum_{n \in I} x_n \mathbb{P}(X = x_n)$

$= (-2)\left(\frac{1}{15}\right) + (-1)\left(\frac{2}{15}\right) + 0\left(\frac{2}{5}\right) + (1)\left(\frac{1}{\cancel{1}5}\right) + 2\left(\frac{1}{5}\right)$

$= 0.33$

To calculate Variance, first we have to calculate $E(X^2$

$E(X^2) = \sum x_n^2 \, \mathbb{P}(X = x_n)$

$= (-2)^2\left(\frac{1}{15}\right) + (-1)^2\left(\frac{2}{15}\right) + 0^2\left(\frac{2}{5}\right) + (1)^2\left(\frac{1}{5}\right) + (2)^2\left(\frac{1}{\cancel{1}5}\right)$

$= 1.4$

$\therefore \operatorname{var}(X) = \cancel{E(X)} \; E(X^2) - (E(X))^2$

$= 1.4 - (0.33)^2$

$= 1.29$