MATH 588 - Finite Element Method

Final project
Date assigned: March 30, 2025
Due date: **April 28 , 2025**

PROBLEM:
Consider the following BVP:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f(x), \;\; 0 < x < 1 \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = u_0 \end{cases} \tag{1}$$

where

$$f(x) = 0.$$

and

$$u(x,0) = \begin{cases} 2x \text{ for } x \in [0, 1/2], \\ 2 - 2x \text{ for } x \in [1/2, 1]. \end{cases}$$

- Derive semidiscrete variational formulation of this BVP.
- Use forward Euler method to discretize () in time
- Write down structure of stiffness matrix A, mass matrix B, and load vector F, assuming uniform mesh step size h.
- Numerically solve the problem
  - Either write your own code using programming language of your choice **OR**
  - Modify provided Python code. The provided code solves stationary problem

$$\begin{cases} -\frac{d^2 u}{dx^2} = 1, \;\; 0 < x < 1 \\ u(0) = u(1) = 0 \end{cases}$$

and uses hard coded number of elements. So, some modifications will be required.