Q2.

Let X is an exponential r.v. with rate λ. Use Chebyshev's inequality to find an upper bound for

(1) $\mathbb{P}(|X - E(X)| \geq a)$, $a > 0$.

Sol-n.

[Preamble]. Let

(2) $X \sim \exp(\lambda) \Rightarrow f(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$ is the pdf. $\Rightarrow$

(3) $E(X) = 1/\lambda$ &

(4) $Var(X) = 1/\lambda^2 \Rightarrow \sigma^2 = \frac{1}{\lambda^2} \Rightarrow \boxed{\sigma = \frac{1}{\lambda}}$, then

$$\mathbb{P}(|X - \underbrace{E(X)}_{=\frac{1}{\lambda}}| \geq a) = \mathbb{P}\left(\left|X - \frac{1}{\lambda}\right| \geq a\right)$$

$$= \mathbb{P}\left(\left|X - \frac{1}{\lambda}\right| \geq a \cdot \frac{\lambda}{\lambda}\right)$$

$$= \mathbb{P}\left(\left|X - \frac{1}{\lambda}\right| \geq a\lambda \cdot \overbrace{\frac{1}{\lambda}}^{\sigma}\right), \text{ let } \boxed{k := a\lambda}$$

Chebyshev's Ineq-y:

(5) $\boxed{\mathbb{P}(|X - E(X)| \geq k\sigma) \leq \frac{1}{k^2}}$

$$= \mathbb{P}\left(\left|X - \frac{1}{\lambda}\right| \geq \overbrace{(a\lambda)}^{k} \cdot \overbrace{\frac{1}{\lambda}}^{\sigma}\right)$$

$$\leq \frac{1}{\underbrace{(a\lambda)}_{k}^2}.$$ Thus

(6) $\mathbb{P}(|X - E(X)| \geq a) \leq \frac{1}{(a\lambda)^2}$. $\blacksquare$